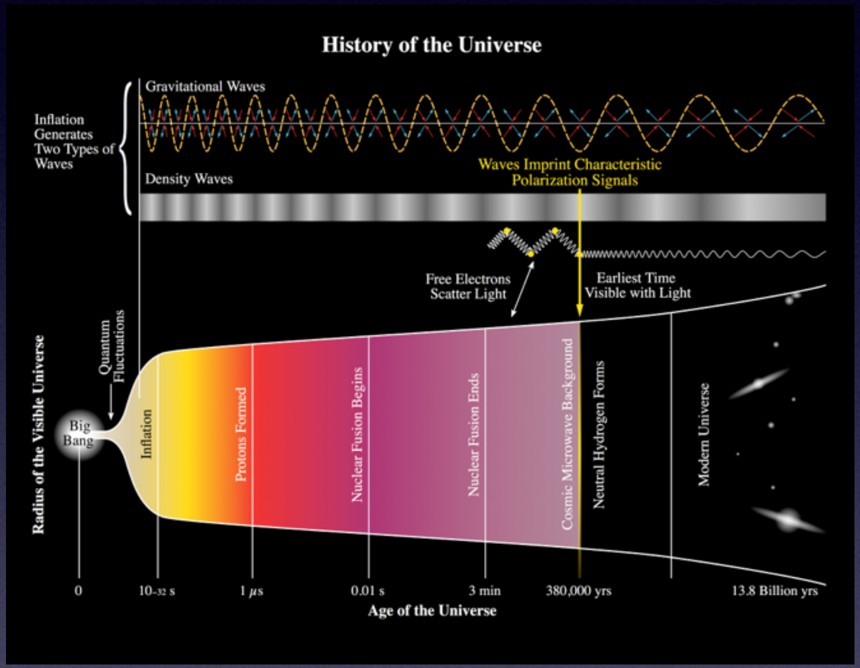
Histories of dark matter

Part II : The primordial Universe

Book, chapter 2



Yann Mambrini

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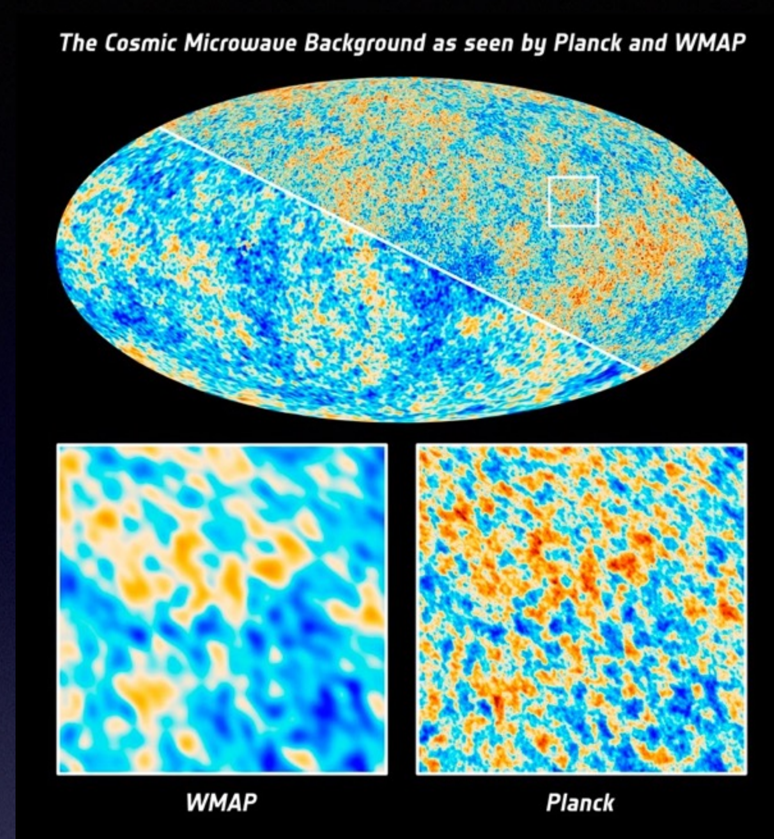
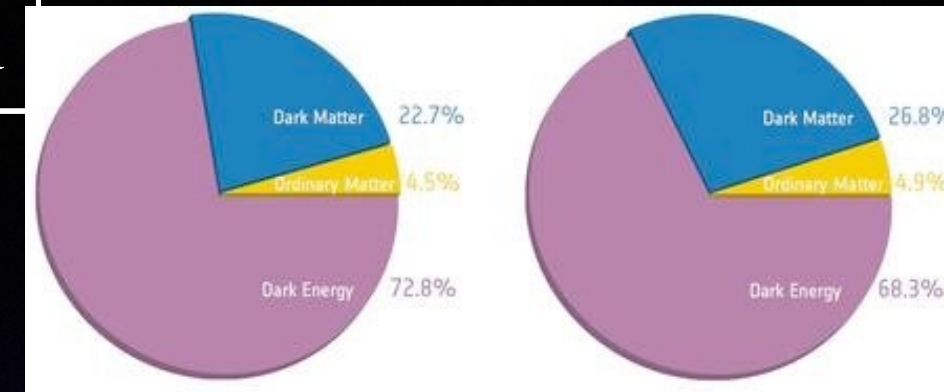
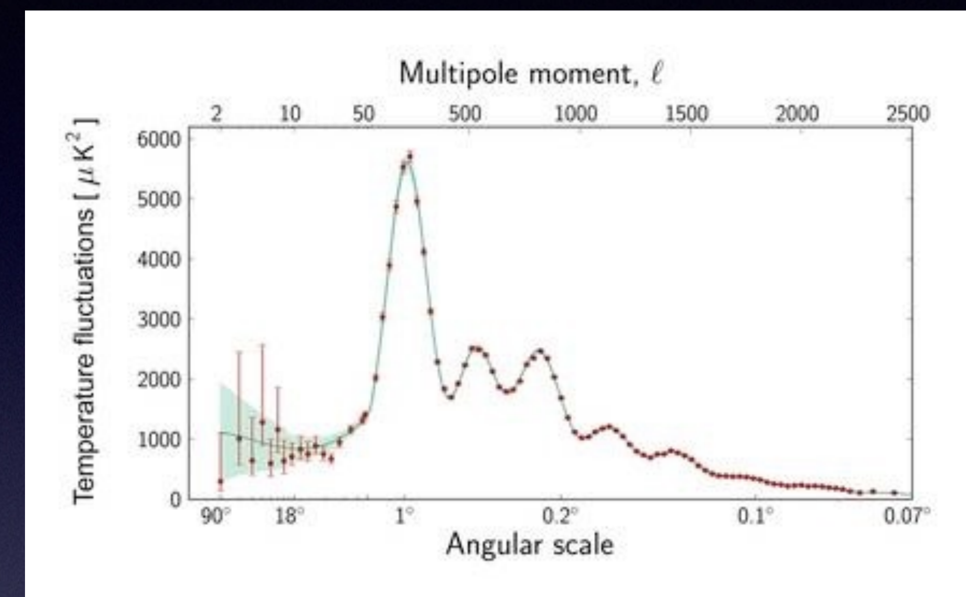
Indirect proofs

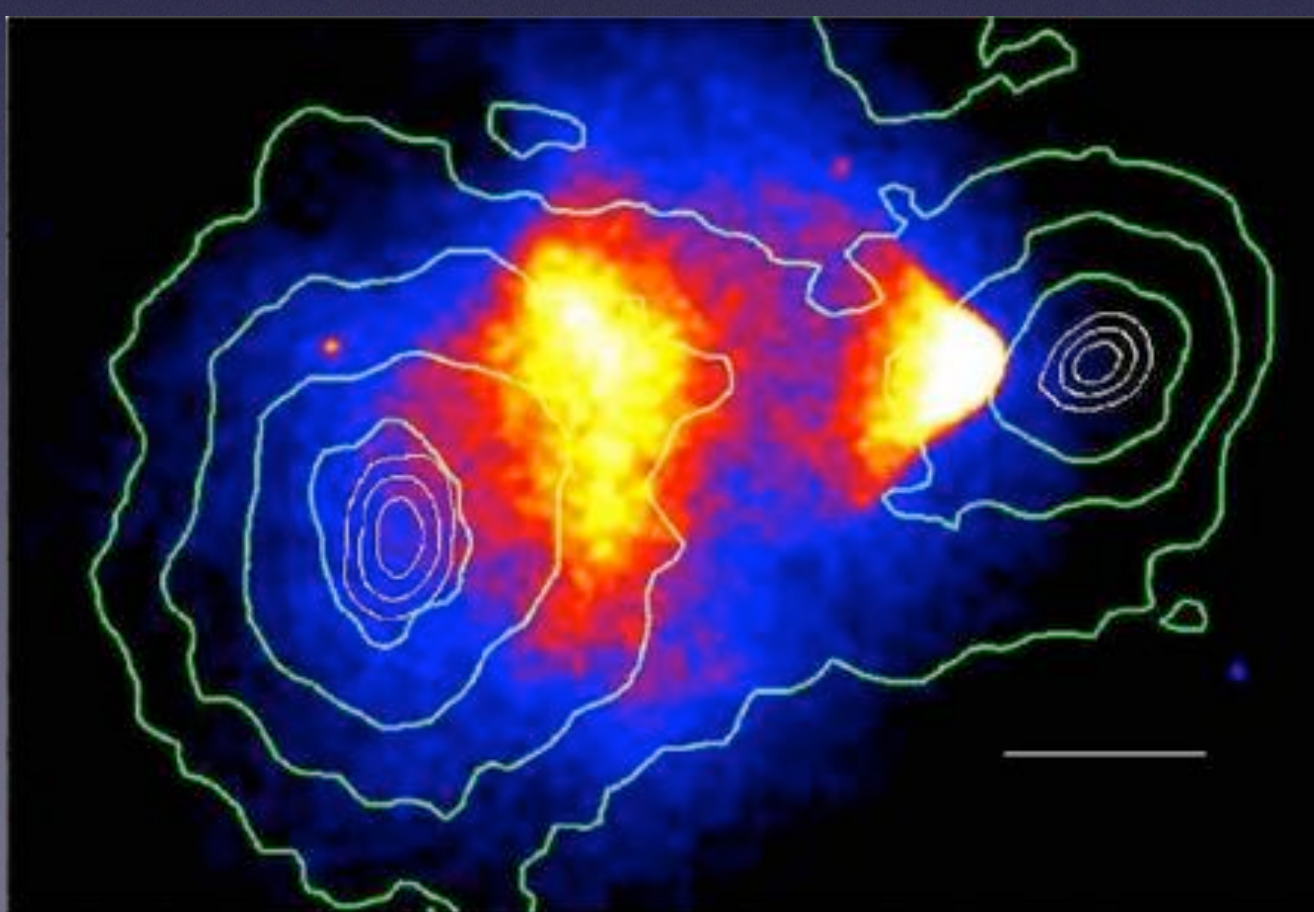
Cosmic Microwave Background

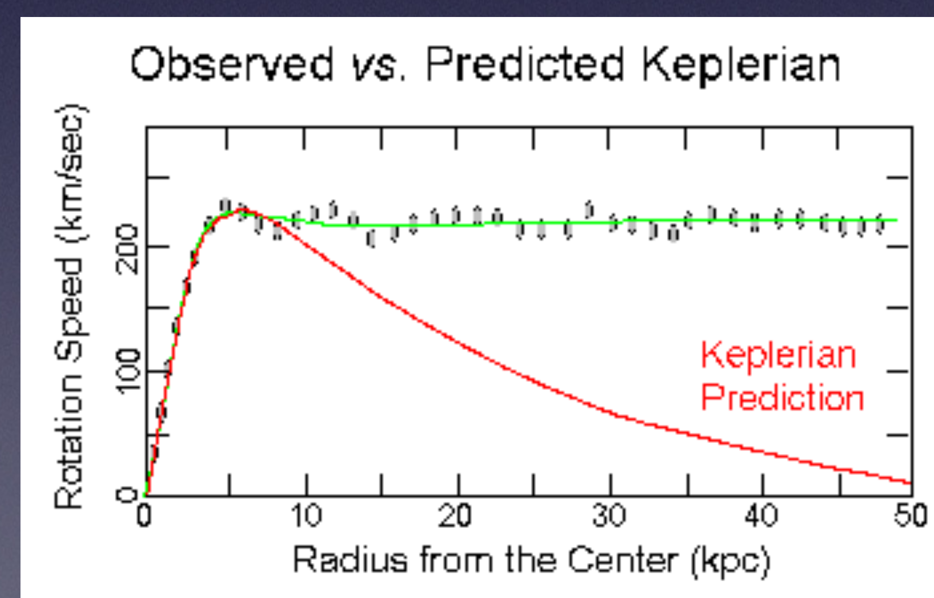






Bullet cluster





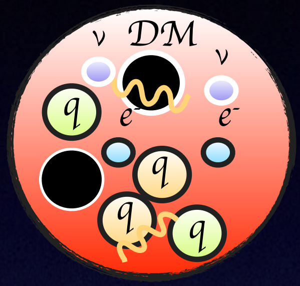
Rotation curve

A little thermal history of the Universe

Reheating temperature
TRH

time

A little thermal history of the Universe



$$f \sim \frac{e^{-\frac{E}{T}}}{1 \pm e^{-\frac{E}{T}}}$$

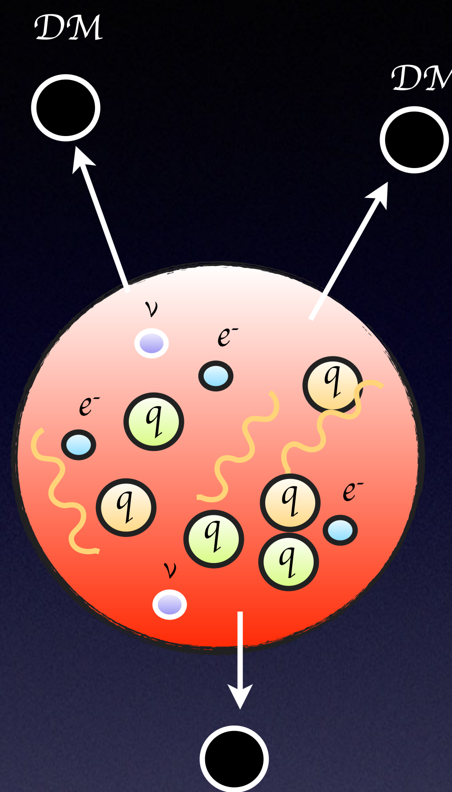
Equilibrium:
Thermal bath

10^8 GeV
10^{-22} second

Reheating temperature
TRH

time

A little thermal history of the Universe



$$f \sim \frac{e^{-\frac{E}{T}}}{1 \pm e^{-\frac{E}{T}}}$$

$$\rho_{DM} \sim e^{-\frac{m}{T}}$$

Equilibrium:
Thermal bath

Dark Matter
decoupling

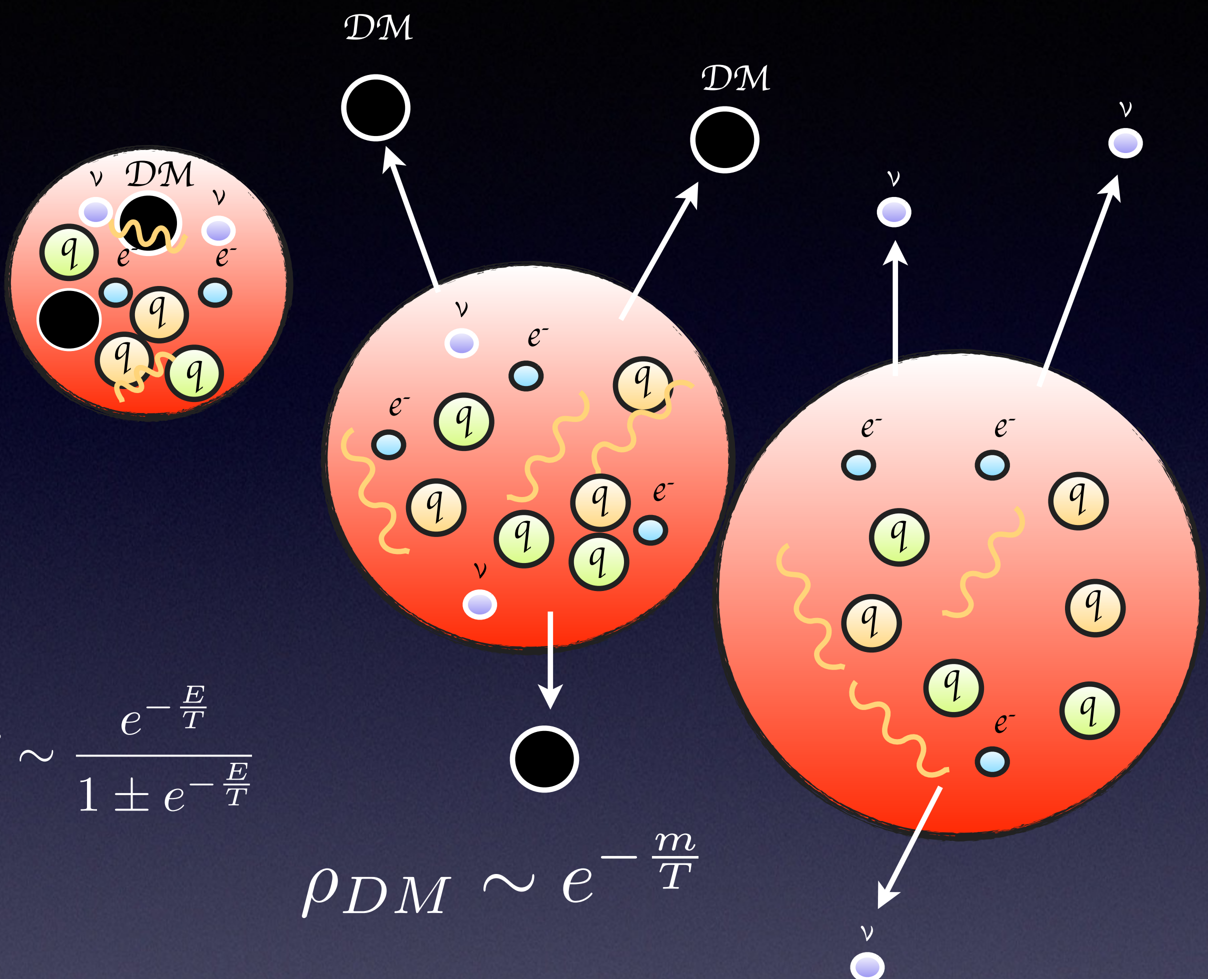
10^8 GeV
10^{-22} second

10 GeV
10^{-8} second

Reheating temperature
TRH

time

A little thermal history of the Universe



$$f \sim \frac{e^{-\frac{E}{T}}}{1 \pm e^{-\frac{E}{T}}}$$

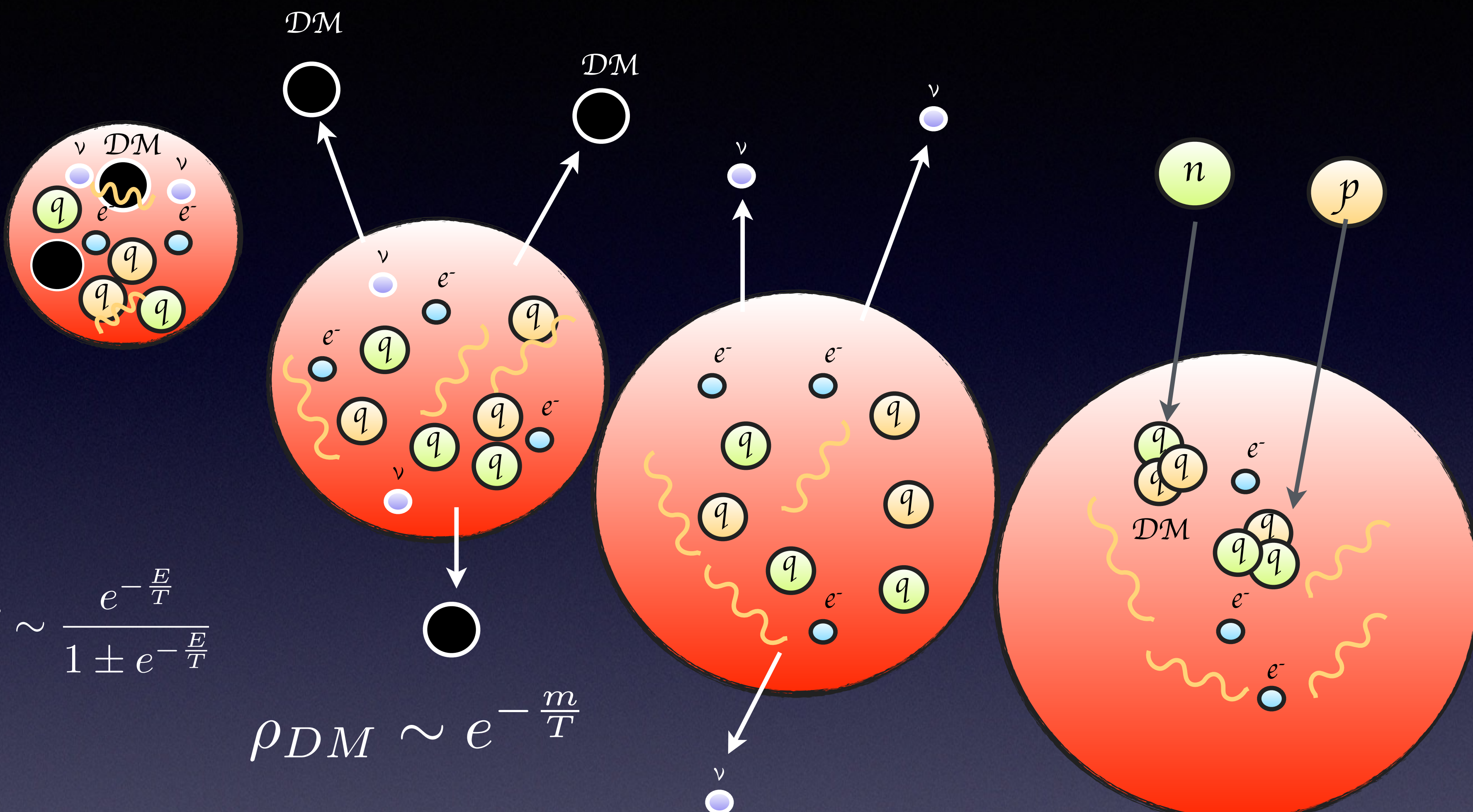
$$\rho_{DM} \sim e^{-\frac{m}{T}}$$

Equilibrium: Thermal bath	Dark Matter decoupling	Neutrino decoupling
10^8 GeV	10 GeV	2 MeV
10^{-22} second	10^{-8} second	2 seconds

Reheating temperature
TRH

time

A little thermal history of the Universe



$$f \sim \frac{e^{-\frac{E}{T}}}{1 \pm e^{-\frac{E}{T}}}$$

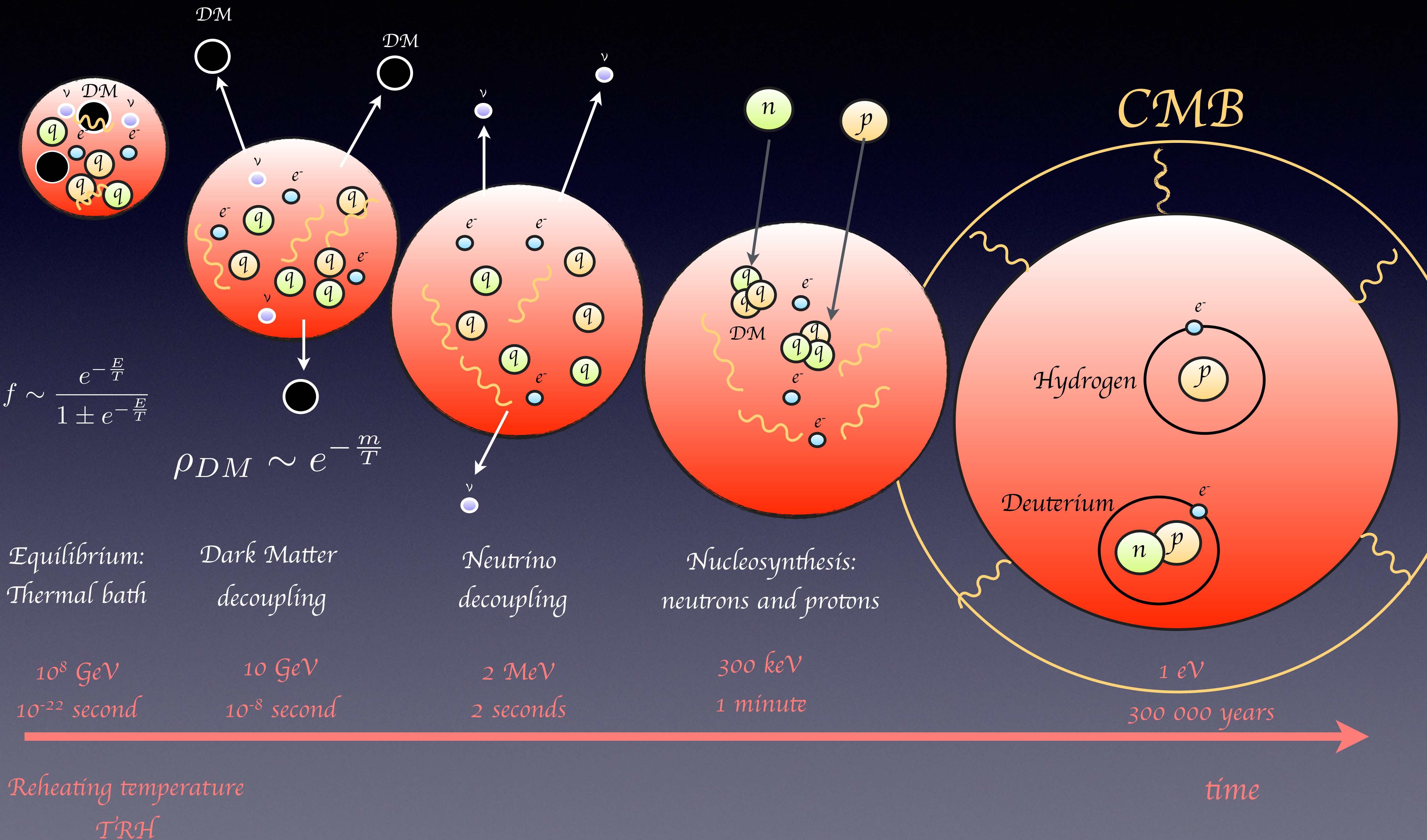
$$\rho_{DM} \sim e^{-\frac{m}{T}}$$

Equilibrium: Thermal bath	Dark Matter decoupling	Neutrino decoupling	Nucleosynthesis: neutrons and protons
10^8 GeV 10^{-22} second	10 GeV 10^{-8} second	2 MeV 2 seconds	300 keV 1 minute

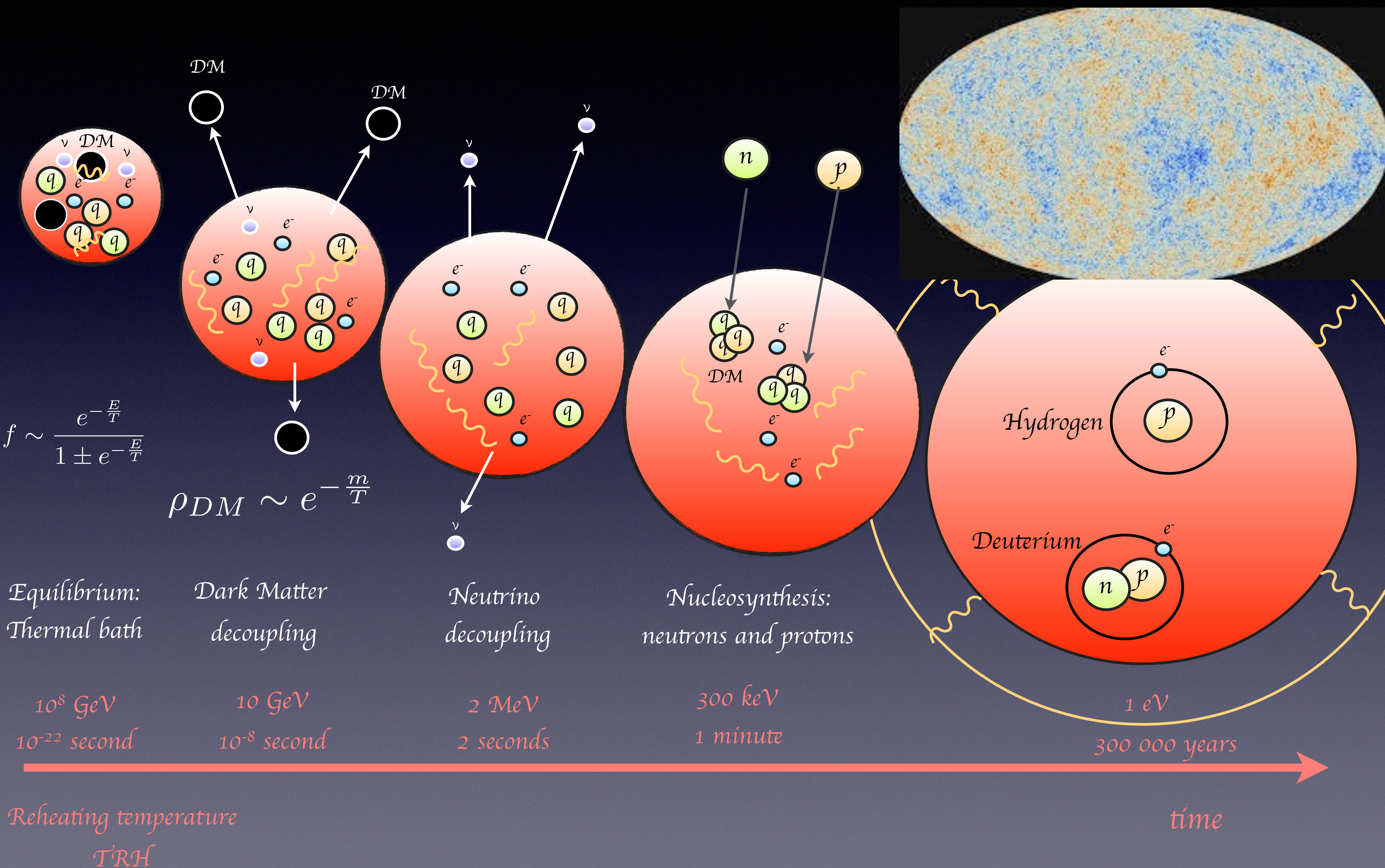
Reheating temperature
TRH

time

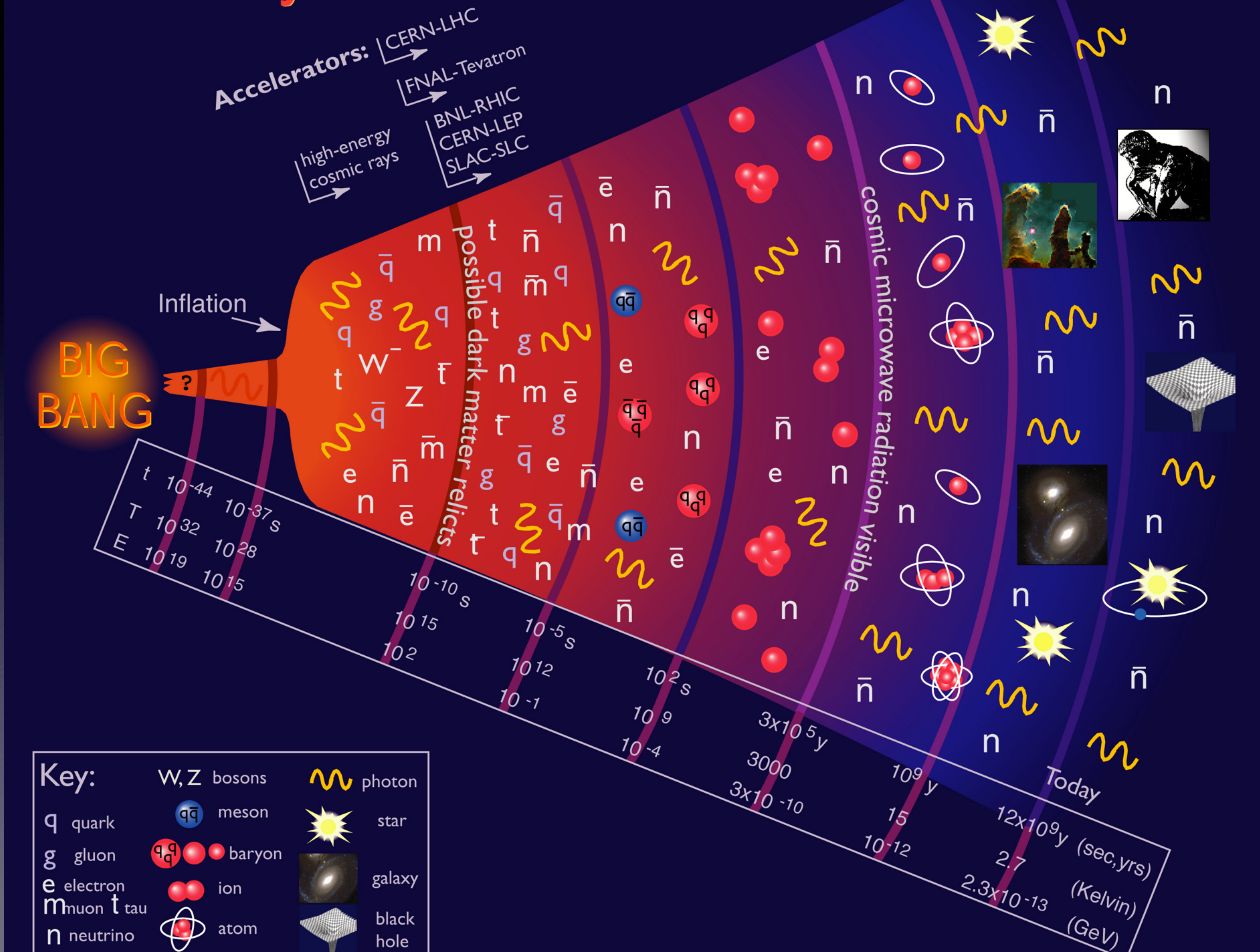
A little thermal history of the Universe



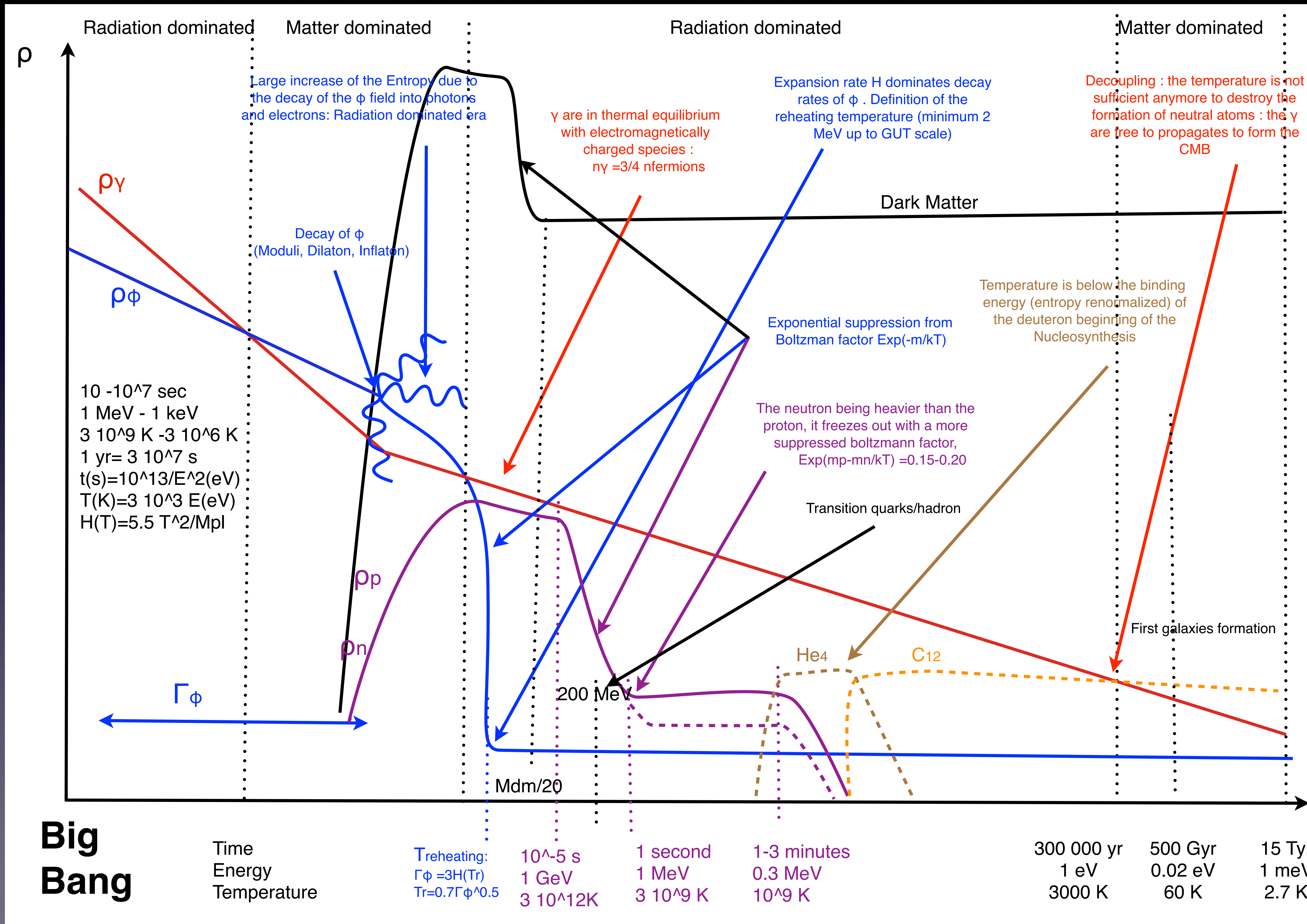
A little thermal history of the Universe



History of the Universe

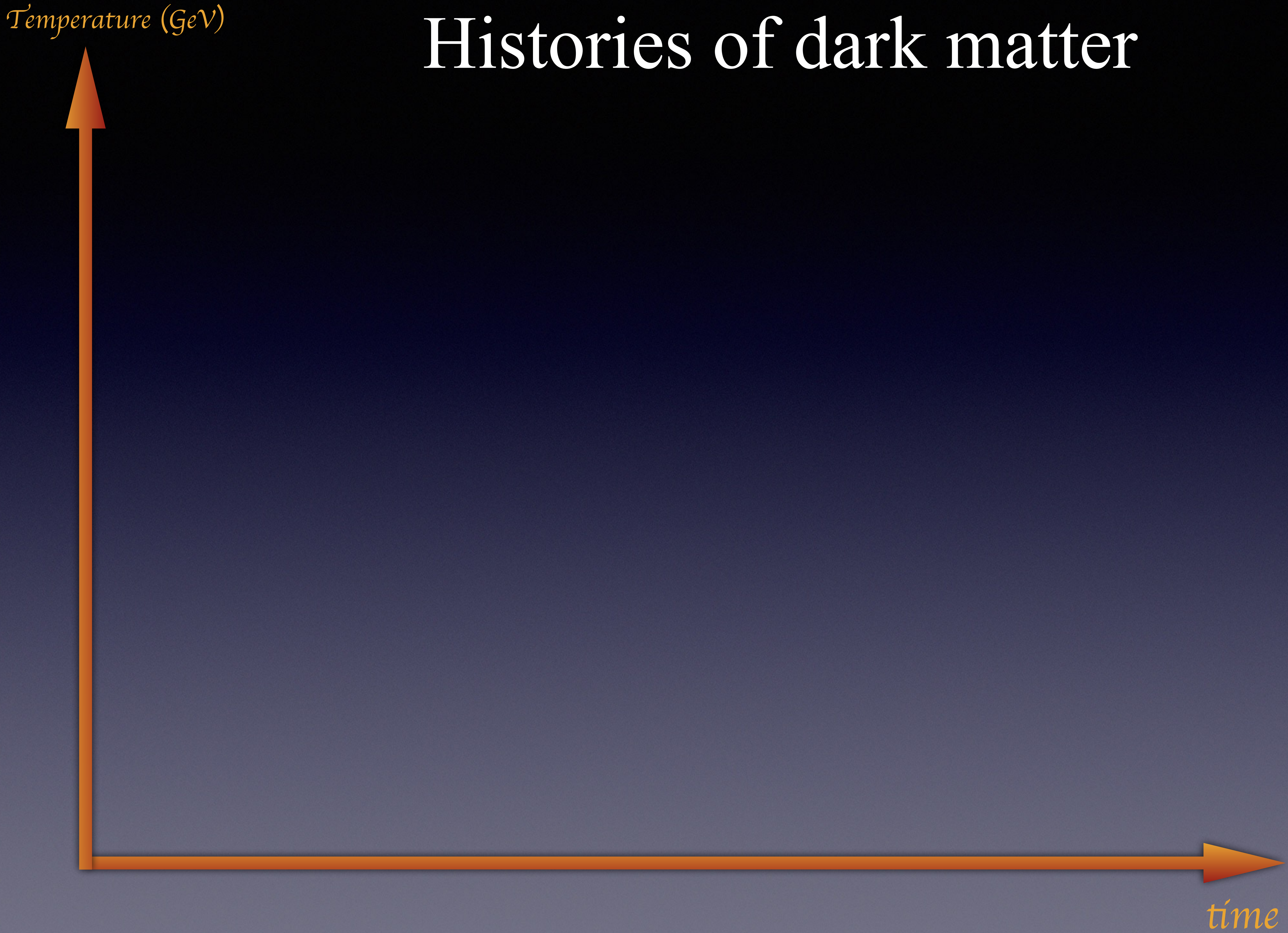


Particle Data Group, LBNL, © 2000. Supported by DOE and NSF

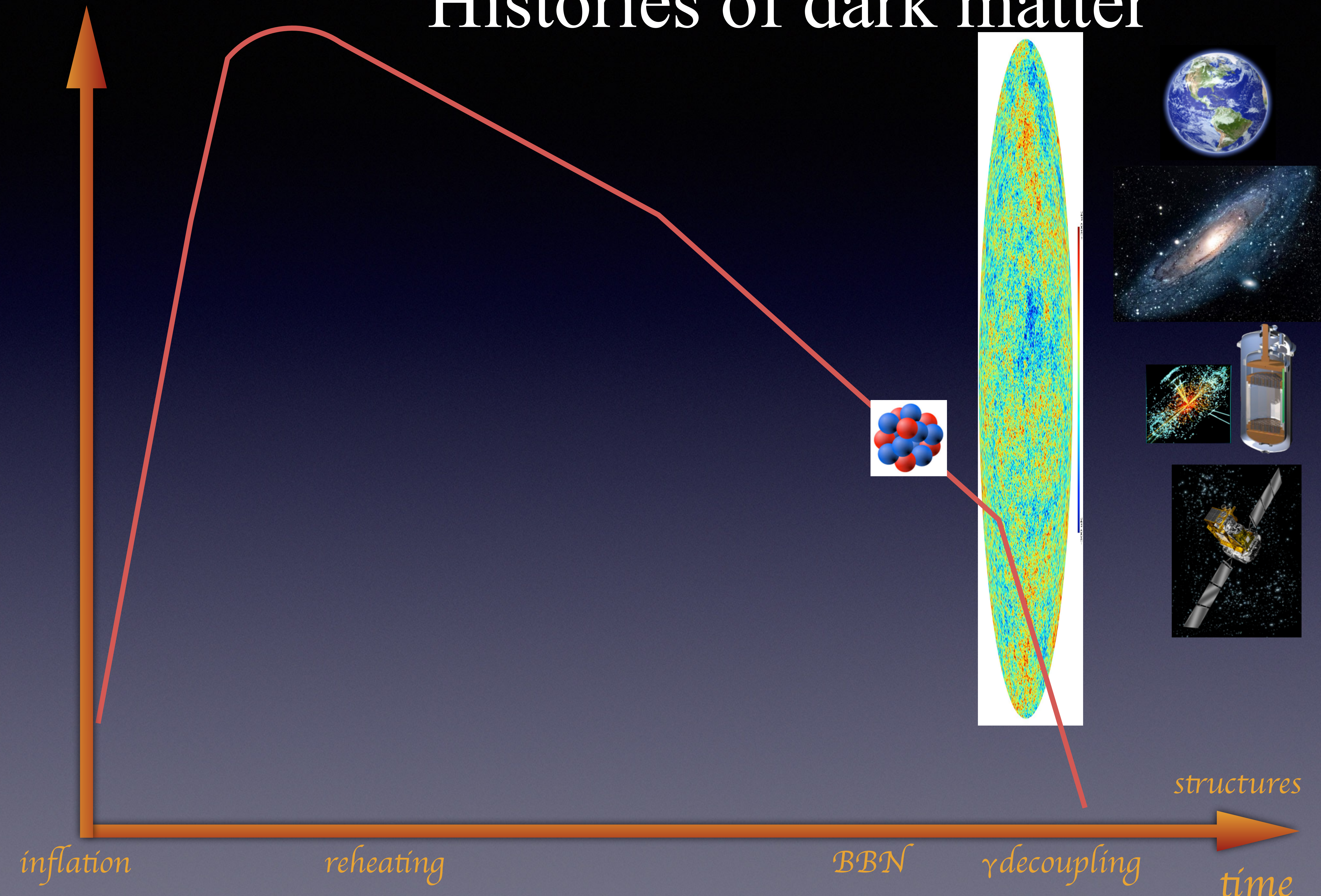


Radiation dominated
Matter dominated
Radiation dominated
Matter dominated
ρ
Large increase of the Entropy due to the decay of the ϕ field into photons and electrons: Radiation dominated era
γ are in thermal equilibrium with electromagnetically charged species : nγ =3/4 nfermions
Expansion rate H dominates decay rates of ϕ . Definition of the reheating temperature (minimum 2 MeV up to GUT scale)
Decoupling : the temperature is not sufficient anymore to destroy the formation of neutral atoms : the γ are free to propagates to form the CMB
ργ
Dark Matter
Decay of ϕ
(Moduli, Dilaton, Inflaton)
ρϕ
Temperature is below the binding energy (entropy renormalized) of the deuteron beginning of the Nucleosynthesis
Exponential suppression from Boltzman factor Exp(-m/kT)
10 -10^7 sec
1 MeV - 1 keV
3 10^9 K -3 10^6 K
1 yr= 3 10^7 s
t(s)=10^13/E^2(eV)
T(K)=3 10^3 E(eV)
H(T)=5.5 T^2/Mpl
The neutron being heavier than the proton, it freezes out with a more suppressed boltzmann factor, Exp(mp-mn/kT) =0.15-0.20
Transition quarks/hadron
ρp
ρn
First galaxies formation
He4
C12
Γϕ
200 MeV
Mdm/20
Big Bang
Time
Energy
Temperature
Treheating:
Γϕ =3H(Tr)
Tr=0.7Γϕ^0.5
10^-5 s
1 GeV
3 10^12K
1 second
1 MeV
3 10^9 K
1-3 minutes
0.3 MeV
10^9 K
300 000 yr
1 eV
3000 K
500 Gyr
0.02 eV
60 K
15 Tyr
1 meV
2.7 K

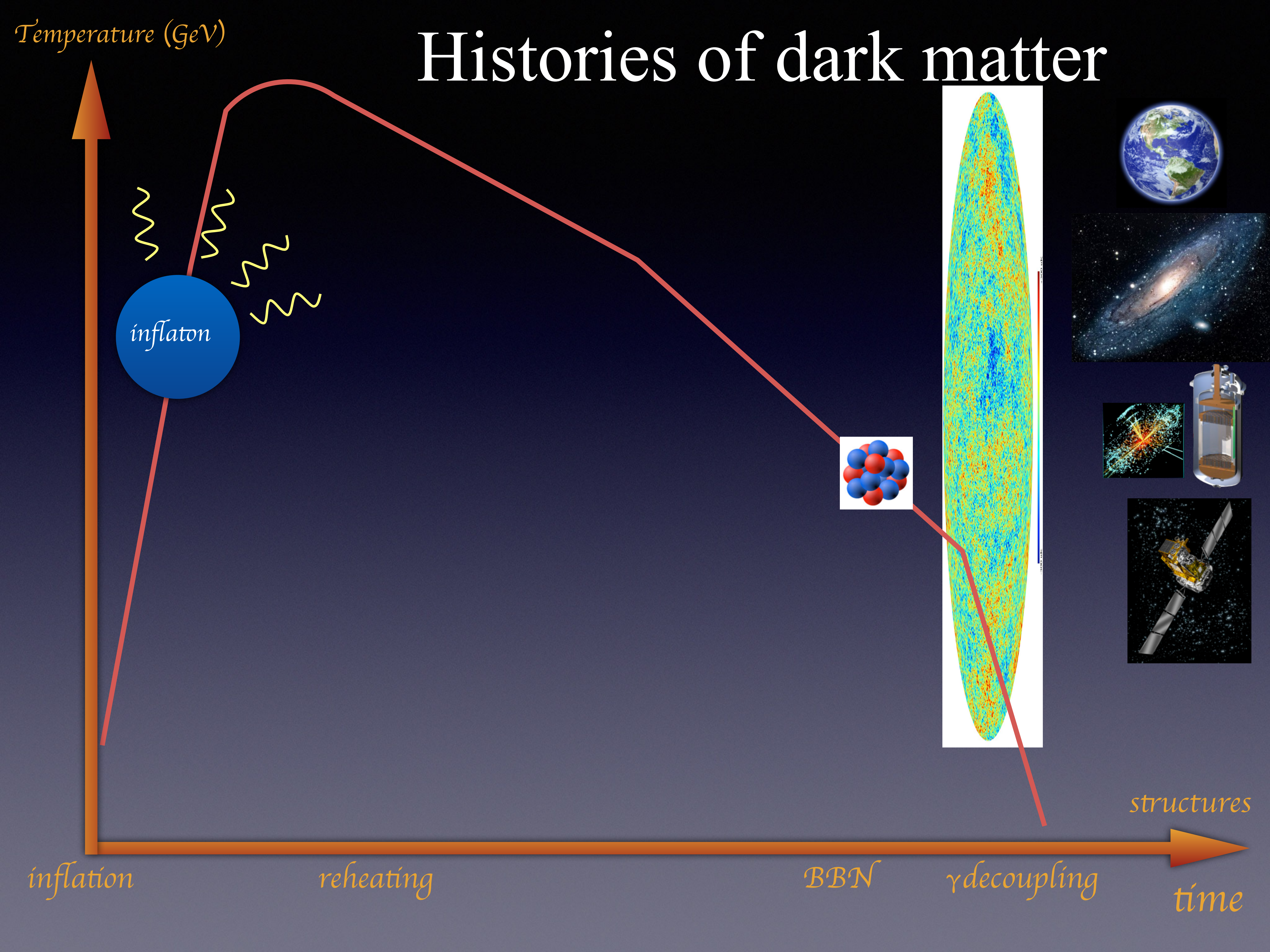
Histories of dark matter



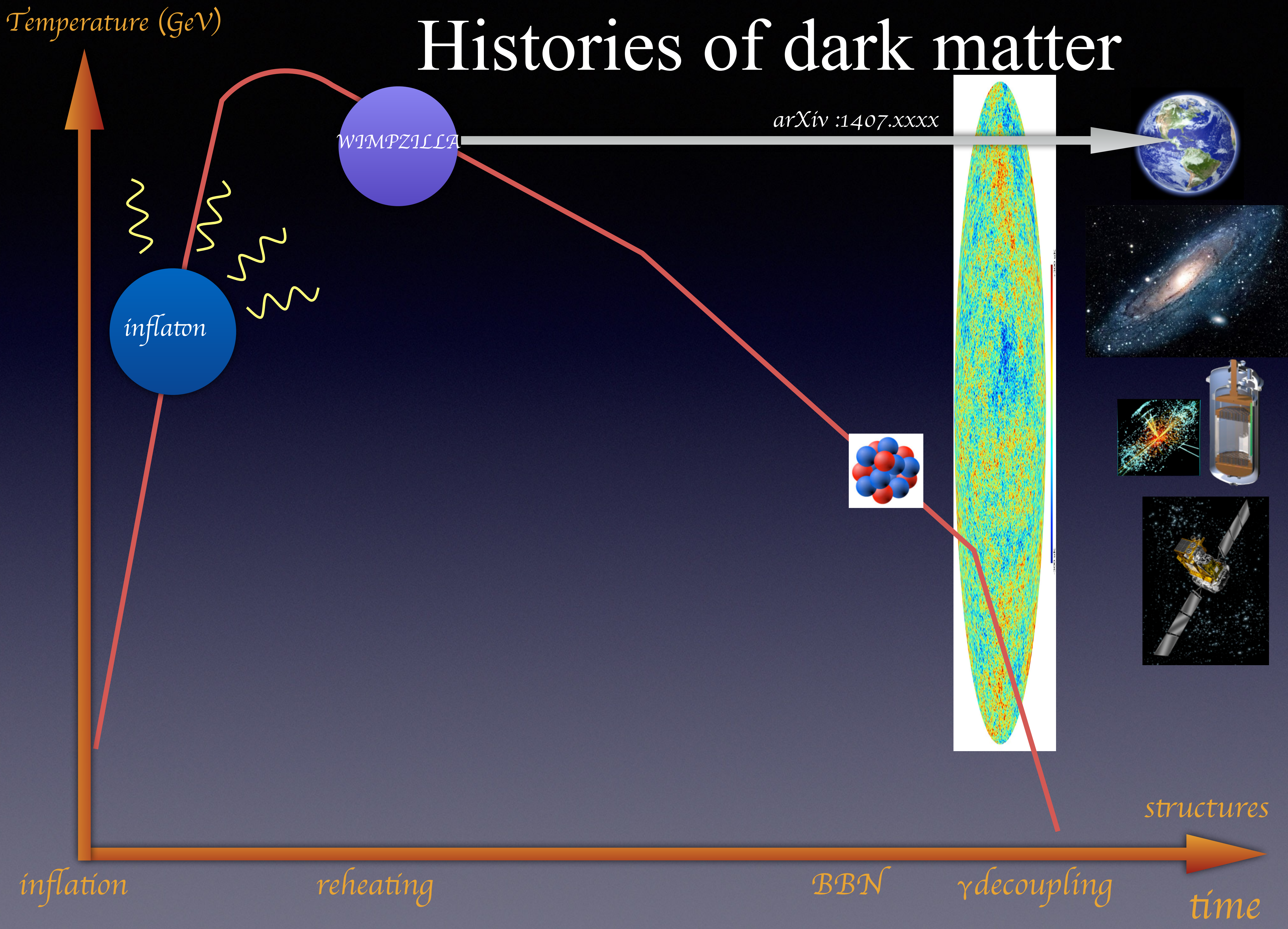
Histories of dark matter



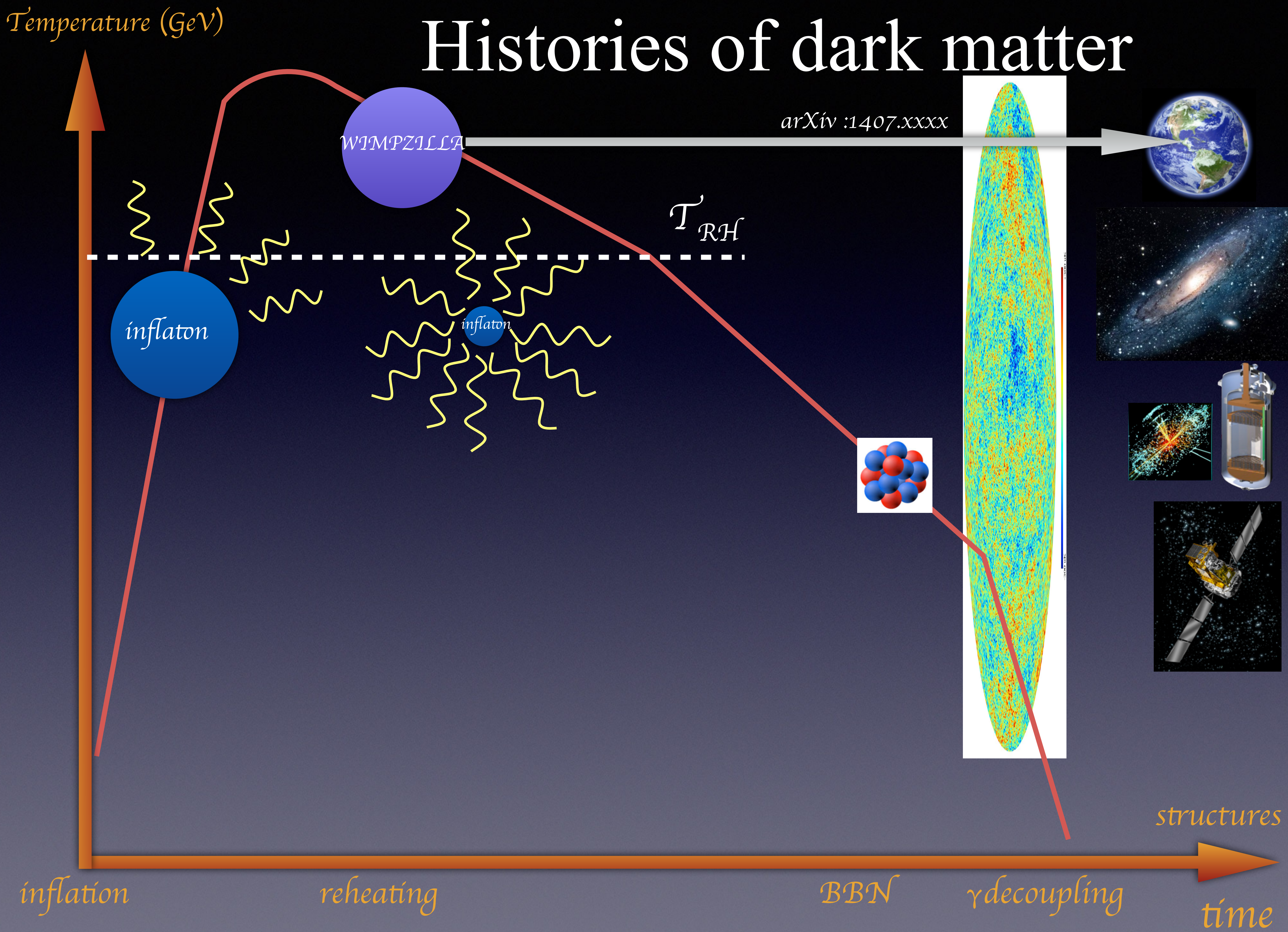
Histories of dark matter



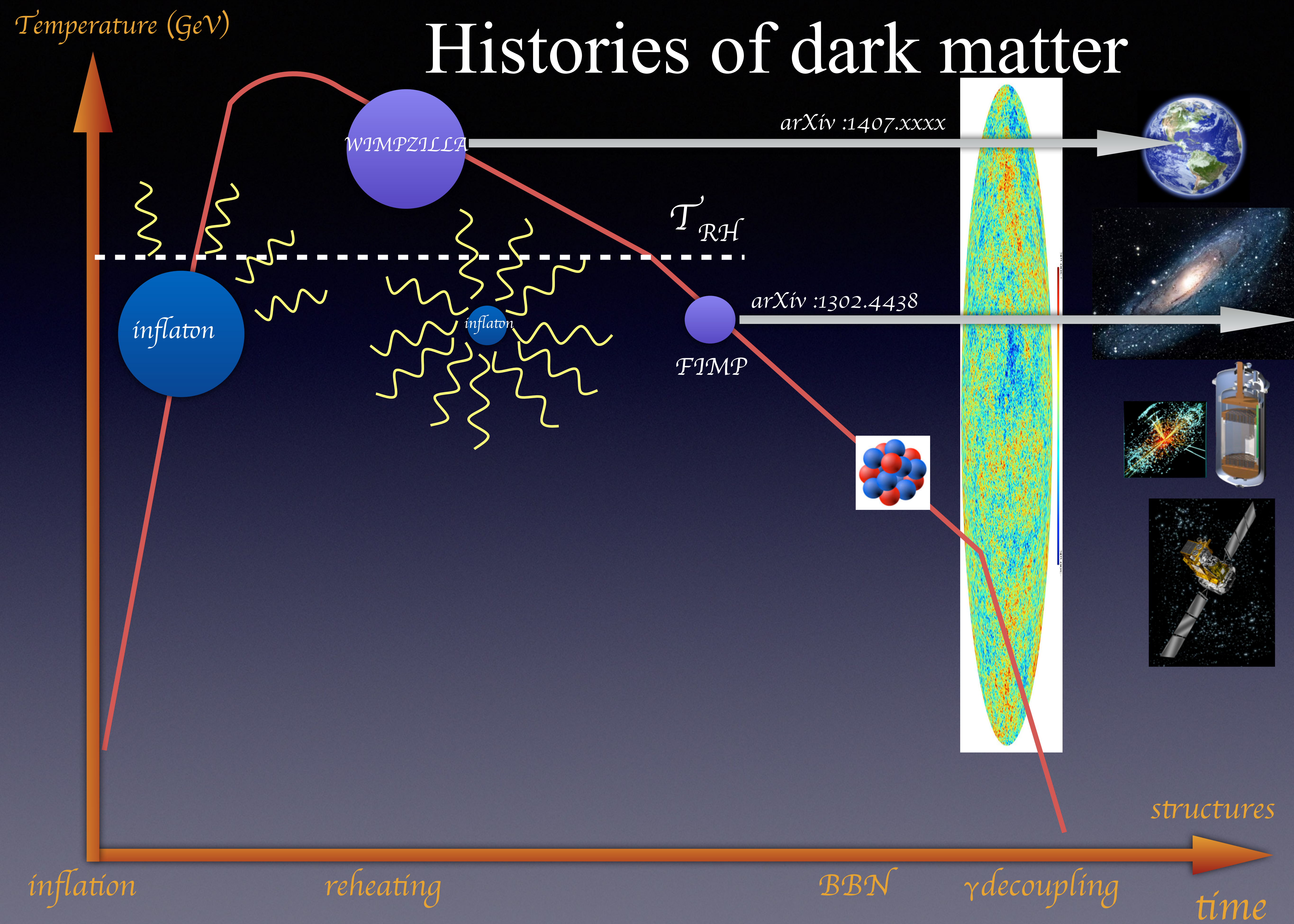
Histories of dark matter



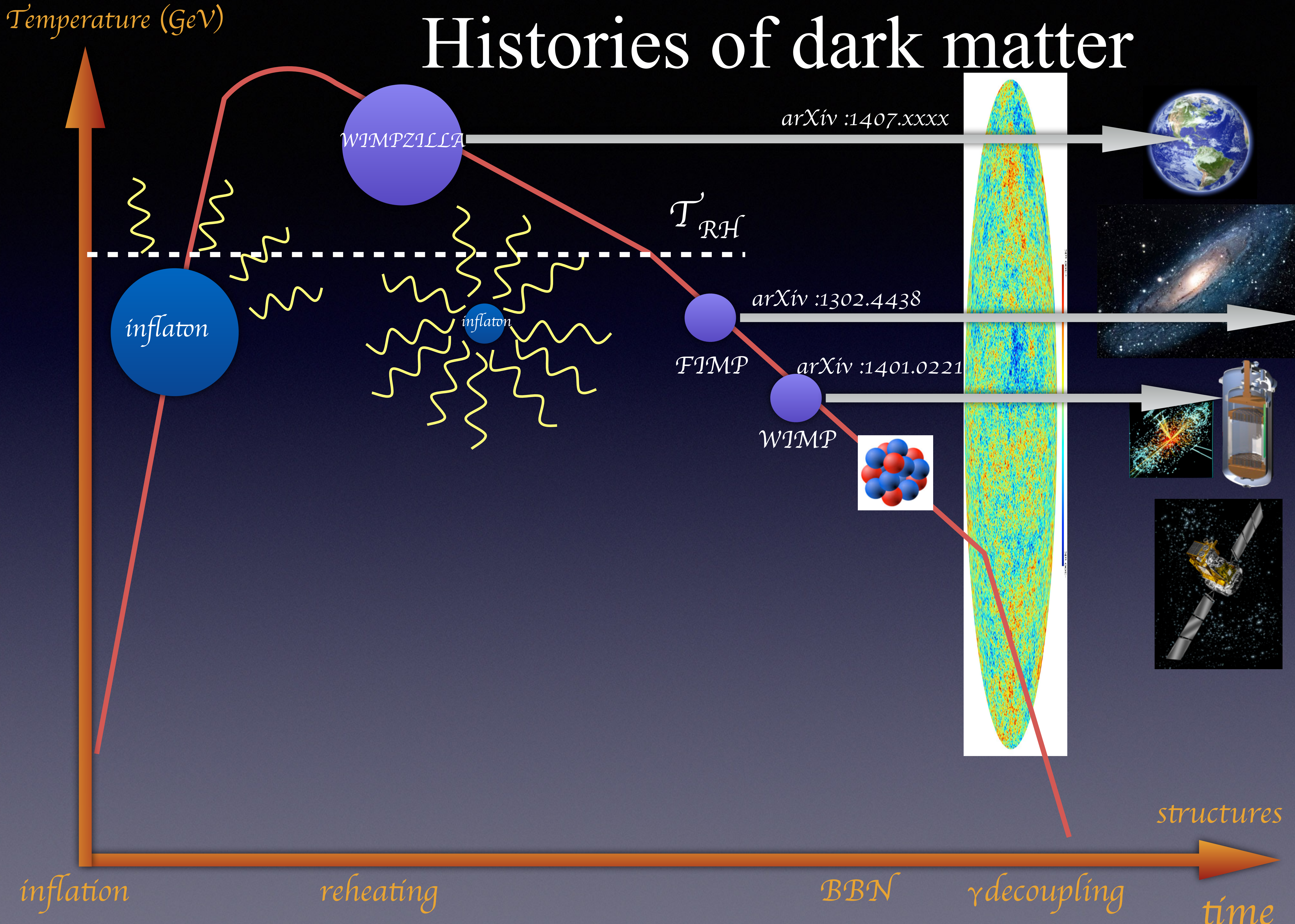
Histories of dark matter



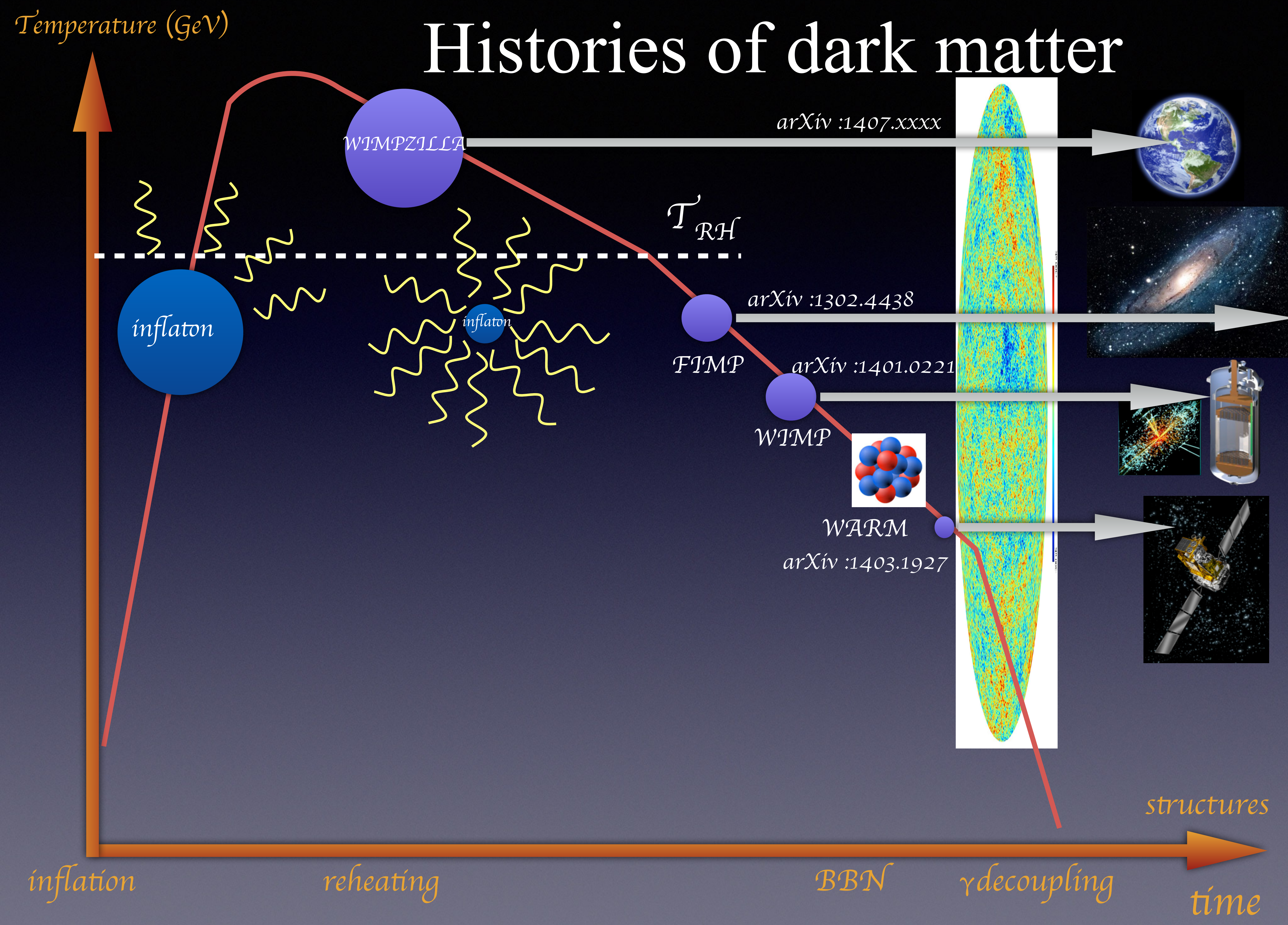
Histories of dark matter



Histories of dark matter



Histories of dark matter



Fields of interest

The meaning of equilibrium

$$\Gamma_{i\to f} = n_i(T)\langle\sigma v\rangle_{i\to f} = H(T)$$

Particle Physics

Universe in expansion

Relativistic Thermodynamics

1) Warm-up: thermodynamics

Book, chapter 2

$$\Gamma_{i\to f} = n_i(T)\langle\sigma v\rangle_{i\to f} = H(T)$$

Particle Physics

Energy distribution of a gas of a particle A, with chemical potential μ_A is:

$$f_A(\vec{p}) = \frac{g_A}{e^{(E-\mu_A)/kT} \pm 1}$$

$$\begin{aligned} n_A(T) &= \frac{g_A}{(2\pi)^3}\int f_A(\vec{p})d^3p = \frac{g_A}{2\pi^2}\int_{m_A}^{\infty}\frac{(E^2-m_A^2)^{1/2}}{e^{(E-\mu_A)/kT}\pm 1}EdE \\ &= \frac{g_A}{2\pi^2}\int_{m_A}^{\infty}\frac{p^2dp}{e^{(\sqrt{p^2+m_A^2}-\mu_A)/kT}\pm 1} \end{aligned}$$

number density

$$\begin{aligned} \rho_A(T) &= \frac{g_A}{(2\pi)^3}\int E(\vec{p})f(\vec{p})d^3p = \frac{g_A}{2\pi^2}\int_{m_A}^{\infty}\frac{(E^2-m_A^2)^{1/2}}{e^{(E-\mu_A)/kT}\pm 1}E^2dE \\ &= \frac{g_A}{2\pi^2}\int_{m_A}^{\infty}\frac{\boxed{(\sqrt{p^2+m_A^2})}\,p^2dp}{e^{(\sqrt{p^2+m_A^2}-\mu_A)/kT}\pm 1} \end{aligned}$$

energy density

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number density

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energy density

$$\rho_A(T)_{T\gg m,\mu} = \frac{\pi^2}{30}g_AT^4 \quad \text{(bosons)}$$
$$\rho_A(T)_{T\gg m,\mu} = \boxed{\frac{7}{8}}\frac{\pi^2}{30}g_AT^4 \quad \text{(fermions)}$$
$$n_A(T)_{T\gg m,\mu} = \frac{\zeta(3)}{\pi^2}g_AT^3 \quad \text{(bosons)}$$
$$n_A(T)_{T\gg m,\mu} = \boxed{\frac{3}{4}}\frac{\zeta(3)}{\pi^2}g_AT^3 \quad \text{(fermions)}$$

$$n_A(T)_{T\ll m_A} = g_A\left(\frac{m_AT}{2\pi}\right)^{3/2}e^{-(m_A-\mu_A)/T}$$
$$\rho_A(T)_{T\ll m_A} = g_Am_A\left(\frac{m_AT}{2\pi}\right)^{3/2}e^{-(m_A-\mu_A)/T}$$

$\rho_A = m_A\, n_A$ [non rel.]

$\rho_A = \pi^2/30\ T^4$ [rel.]

Energy distribution of a gas of a particle A, with chemical potential μ_A is:

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number density

energy density

$$\rho_A(T)_{T\gg m,\mu} = \frac{\pi^2}{30}g_AT^4 \quad \text{(bosons)}$$

$$\rho_A(T)_{T\gg m,\mu} = \boxed{\frac{7}{8}}\frac{\pi^2}{30}g_AT^4 \quad \text{(fermions)}$$

$$n_A(T)_{T\gg m,\mu} = \frac{\zeta(3)}{\pi^2}g_AT^3 \quad \text{(bosons)}$$

$$n_A(T)_{T\gg m,\mu} = \boxed{\frac{3}{4}}\frac{\zeta(3)}{\pi^2}g_AT^3 \quad \text{(fermions)}$$

$$n_A(T)_{T\ll m_A} = g_A\left(\frac{m_AT}{2\pi}\right)^{3/2}e^{-(m_A-\mu_A)/T}$$

$$\rho_A(T)_{T\ll m_A} = g_Am_A\left(\frac{m_AT}{2\pi}\right)^{3/2}e^{-(m_A-\mu_A)/T}$$

$\rho_A = m_A\, n_A$ [non rel.]

$\rho_A = \pi^2/30\ T^4$ [rel.]

2 remarks

$$\langle E_c\rangle_{T\ll m_A}^{per\ \ particle} = \frac{\langle E_c\rangle}{n_A(T)_{T\ll m_A}} = \frac{3}{2}T.$$

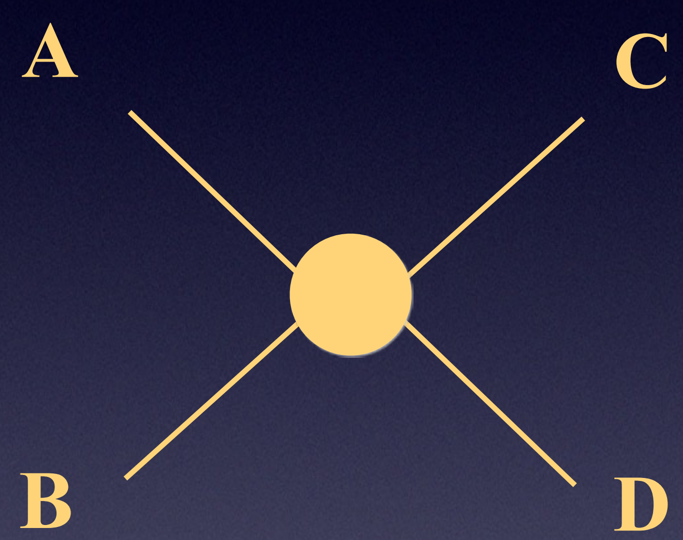
$$K_2(z) = \frac{z^2}{3}\int_1^{\infty}e^{-zt}(t^2-1)^{3/2}dt$$

For a Boltzmann distribution:

$$f_A(\vec{p}) \simeq g_Ae^{-\frac{E}{T}} \qquad n_A^{\text{Boltzmann}} = g_A\int\frac{d^3p}{(2\pi)^3}e^{-\frac{E}{T}} = g_A\frac{m_A^2T}{2\pi^2}K_2\left(\frac{m_A}{T}\right)$$

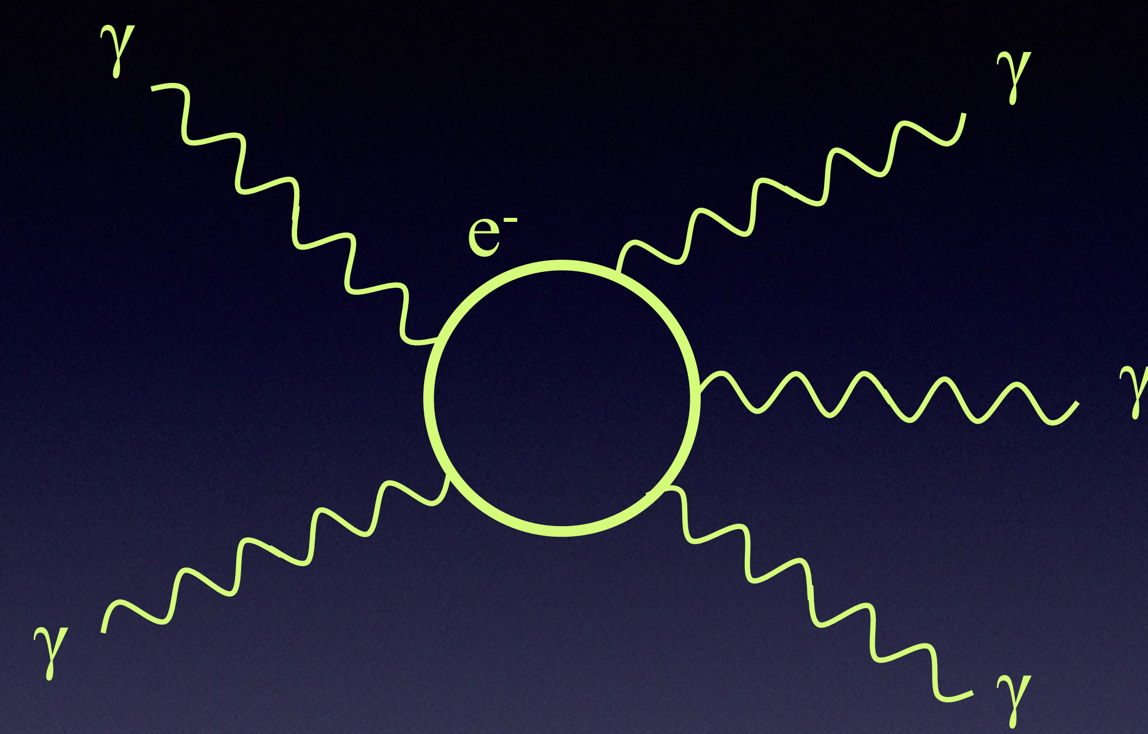
The chemical potential

$\mu_A + \mu_B > \mu_C + \mu_D$



$\mu_A + \mu_B < \mu_C + \mu_D$

$\boxed{\text{Chemical equilibrium } \mu_A + \mu_B = \mu_C + \mu_D}$



$\boxed{3\,\mu_\gamma = 2\,\mu_\gamma \Rightarrow \mu_\gamma = 0}$

Degrees of freedom

The primordial plasma is a sum of different species. However, one keeps « T » as the definition of the photon temperature

$$\rho(T) = \frac{\pi^2}{30}T^4 \sum_{i=all\ species} \frac{30}{\pi^2}\left(\frac{T_i}{T}\right)^4 \frac{g_i}{2\pi^2}\int_{x_i}^{\infty}\frac{(\epsilon^2 - x_i^2)^{1/2}\epsilon^2 d\epsilon}{e^{\epsilon-\mu_i/T}\pm 1} = g_\rho(T)\frac{\pi^2}{30}T^4$$

with $x_i = m_i/T$ and

$$g_\rho(T) = \frac{30}{\pi^2}\sum_{i=all\ species}\frac{g_i}{2\pi^2}\left(\int_{x_i}^{\infty}\frac{(\epsilon^2 - x_i^2)^{1/2}\epsilon^2 d\epsilon}{e^{\epsilon-\mu_i/T}\pm 1}\right)\left(\frac{T_i}{T}\right)^4$$

Approximating the Boltzmann function as a step function:

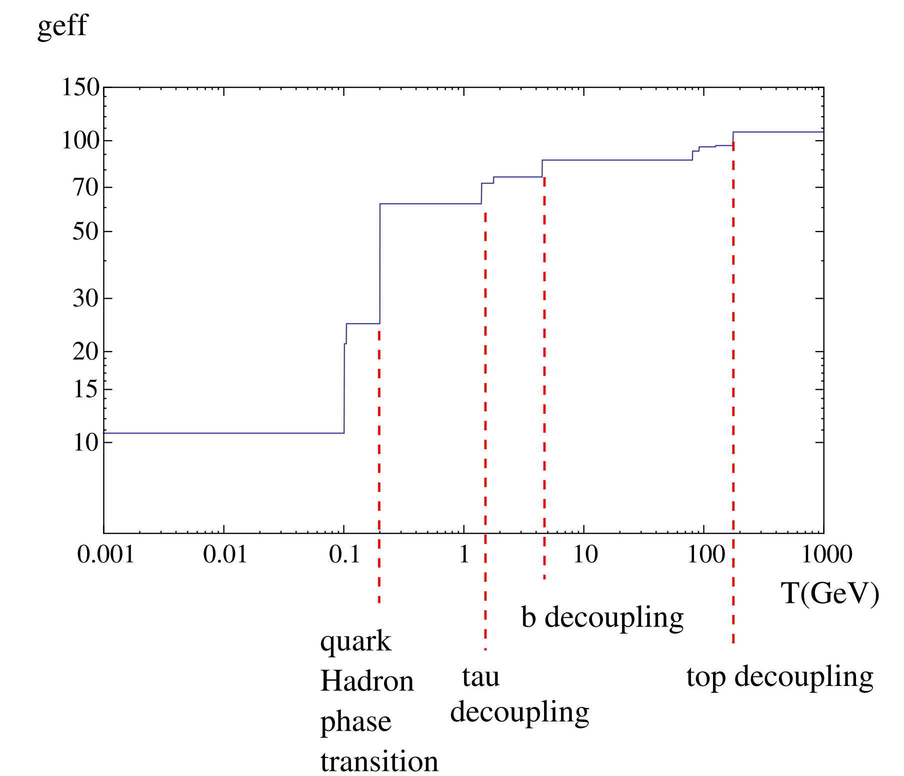
$$g_\rho(T) \simeq \sum_{b=bosons} g_b\left(\frac{T_b}{T}\right)^4 + \frac{7}{8}\sum_{f=fermions} g_f\left(\frac{T_f}{T}\right)^4$$

In thermal equilibrium, the temperatures T_b and T_f are equal to the photon temperature of the bath T_γ=T. We obtain :

$$g_\rho^{SM} = (3\times 3 + 9\times 2 + 1) + \frac{7}{8}(6\times 4\times 3 + 3\times 4 + 3\times 2) = 106.75$$

$$g_\rho^{\text{today}} = 2 + \frac{7}{8}\times 2\times 3\times\left(\frac{4}{11}\right)^{4/3} = 3.36$$

$$n_\gamma^{\text{today}} = \frac{2\zeta(3)}{\pi^2}T^3 = 394\ \text{cm}^{-3}$$



Effective degree of freedom of the primordial plasma as function of the temperature

Entropy

Book, section 2.2.7

$$\boxed{TdS = dU + PdV = \delta Q}$$

$$U = \rho V$$

$$S = sV$$

$$\boxed{d\rho - Tds = (Ts - \rho - P)\frac{dV}{V}.}$$

$$\boxed{s = \frac{\rho + P}{T}}$$

$$\boxed{P = \frac{1}{3}\int_0^\infty f(E)EdE = \frac{1}{3}\rho}$$

$$\boxed{s = \frac{2\pi^2}{45} g_s T^3}$$

$$g_s = \sum_{b=bosons} g_b \left(\frac{T_b}{T}\right)^3 + \frac{7}{8}\sum_{f=fermions}\left(\frac{T_f}{T}\right)^3$$

$$g_s^{\text{today}} = 2 + \frac{7}{8} \times 2 \times 3 \times \frac{4}{11} = 3.91$$

$$s_0 = \frac{2\pi^2}{45} g_s^{\text{today}} T_0^3 = 34.71\ \text{K}^3 = 2.1 \times 10^{-38}\ \text{GeV}^3 = 2787\ \text{cm}^{-3}$$

Example: when the neutrino decouple from the bath, $T_\nu = T_\gamma$. After the decoupling of the electrons and positrons, they « gave » their degrees of freedom to the photon, not the neutrino: T_γ increases compare to T_ν:

$$\boxed{T_\nu = (4/11)^{1/3} T_\gamma} = (4/11)^{1/3} \times 2.725 \approx 1.95\ \text{K}.$$

$$n_\nu = 109\ \text{cm}^{-3}$$

Consequences

Considering an adiabatic evolution, the total entropy S = sV should stay constant. This implies V proportional to $1/T^3$. Morever, once a particle decouple from the thermal bath, $s_{before} = s_{after}$ implies

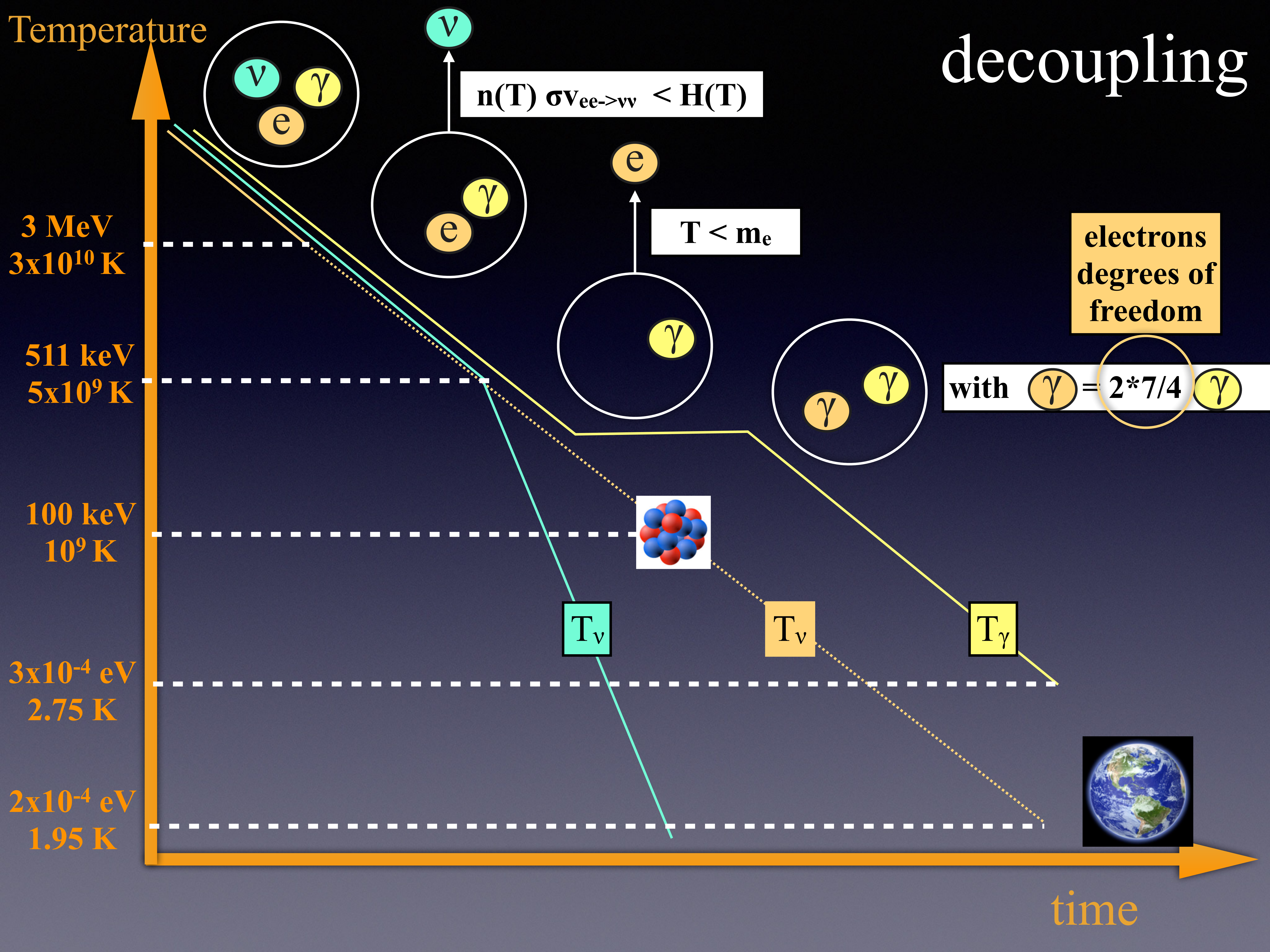
$g_s^{before} . T^3_{before} = g_s^{after} . T^3_{after}$

or

$$T_\gamma^{\text{after}} = T_\gamma^{\text{before}} \times (g_s^{\text{before}}/g_s^{\text{after}})^{1/3}.$$

before $e^+ e^-$ decoupling : $g_s = 2 + 7/8 \times 4 =$ **11/2**

after $e^+ e^-$ decoupling : $g_s =$ **2**



decoupling
Temperature
time
3 MeV
3x10^10 K
511 keV
5x10^9 K
100 keV
10^9 K
3x10^-4 eV
2.75 K
2x10^-4 eV
1.95 K
ν
γ
e
n(T) σv_ee->νν < H(T)
T < m_e
electrons degrees of freedom
with γ = 2*7/4 γ
T_ν
T_ν
T_γ

Fields of interest

The meaning of equilibrium

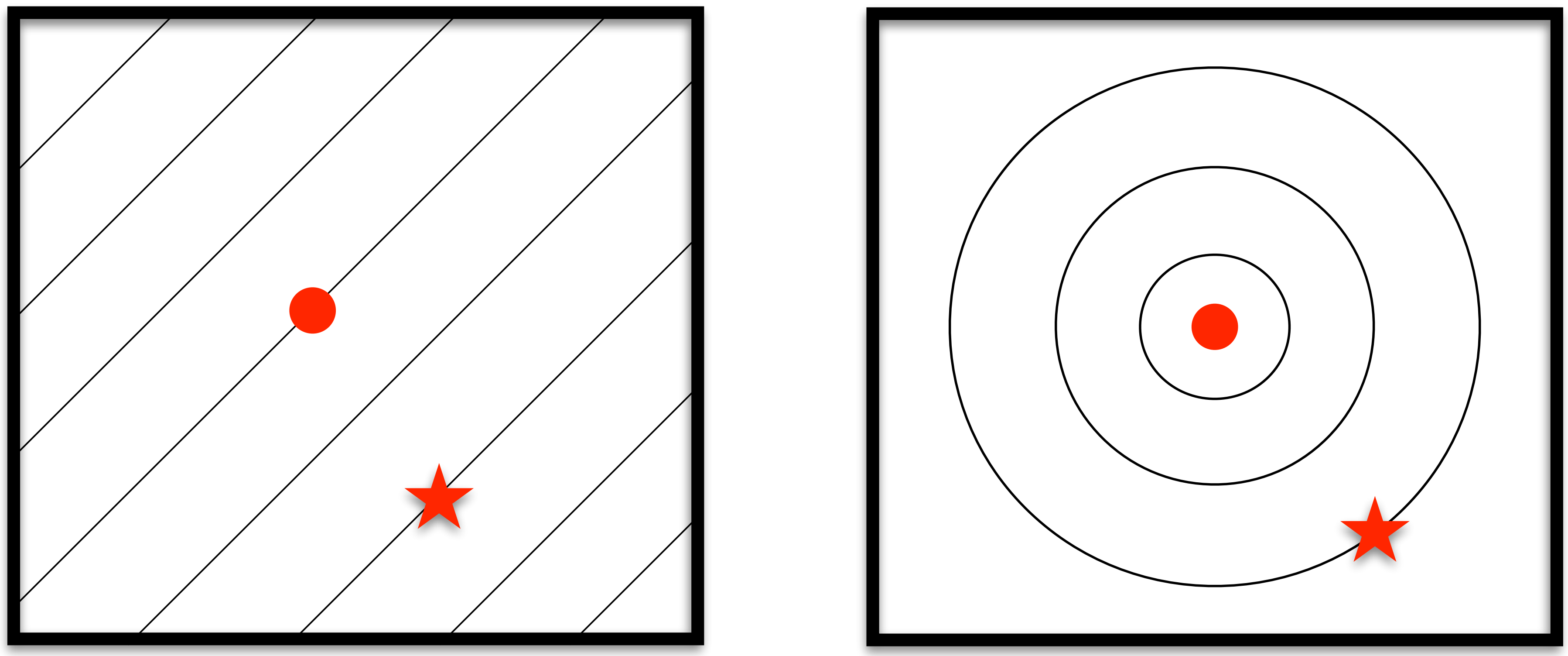
$$\Gamma_{i\to f} = n_i(T)\langle\sigma v\rangle_{i\to f} = H(T)$$

Universe in expansion

The Friedmann equations

Book, 2.2.1

A newtonian approach: we live in an isotropic and homogeneous Universe



Homogeneous non-isotropic Universe

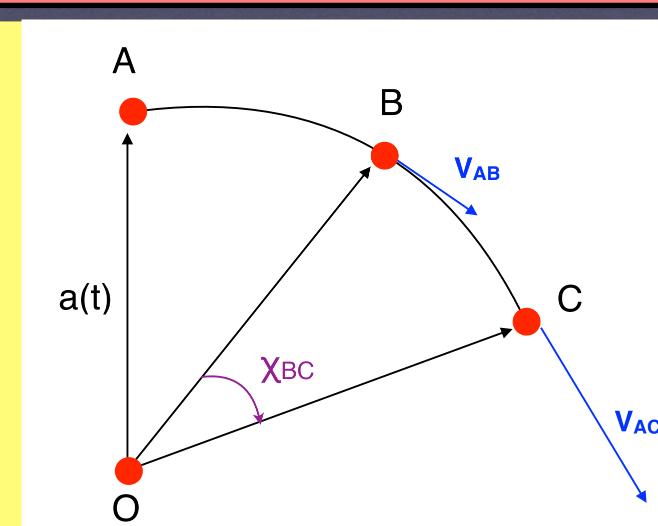
Isotropic non-homogeneous Universe

The Hubble law is the only law which describes an isotropic and homogeneous Universe (proof in the book).

$$v_{AB} = H(t) r_{AB} \quad \Rightarrow H(t) = \frac{\dot{a}(t)}{a(t)}$$

$$r_{AB}(t) = \chi_{AB} a(t)$$

$$v_{AB} = \chi_{AB} \dot{a}(t)$$



The Friedmann equations

Book, 2.2.1

If one defines ρ(t) the mass density between A and B and r = r_{AB}, on can write

$$M = \frac{4\pi}{3} r^3 \rho(t), \quad \Rightarrow \rho(t) = \rho_0 \left(\frac{a_0}{a(t)}\right)^3$$

whereas the equation of motion is given by

$$m_B \ddot{r} = m_B \chi \ddot{a}(t) = -\frac{GmM}{\chi^2 a^2(t)}, \quad \Rightarrow \ddot{a}(t) = -\frac{4\pi G}{3}\rho(t) a(t) = -\frac{4\pi G}{3}\rho_0 \frac{a_0^3}{a^2(t)}.$$

multiplying by the derivative of *a* on both side and integrating with constant *k*, on obtains

$$\frac{1}{2}\dot{a}^2 = \frac{4\pi G}{3}\rho_0 \frac{a_0^3}{a(t)} + k, \Rightarrow \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = H^2(t) = \frac{8\pi G}{3}\rho(t) + \frac{k}{a^2(t)}$$

The last equation is called the <u>Friedmann</u> equation and is also valid in a general relativity framework

The Friedman equations

A relativistic approach

The Hubble « constant »

From the Friedman equation, one can write

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3}$$

Which implies, once neglecting the curvature and cosmological constant (primordial universe, radiation dominated)

$$H^2 = \frac{8\pi G}{3}\rho(T)$$

which gives

$$H(T) = \sqrt{g_\rho}\sqrt{\frac{4\pi^3}{45}}\sqrt{G}T^2 = \sqrt{g_\rho}\sqrt{\frac{4\pi^3}{45}}\frac{T^2}{M_P} \simeq \boxed{1.66 g_\rho^{1/2}\frac{T^2}{M_P}}$$

Summary of the thermodynamical episode

γ

Relativistic particles in thermal **equilibrium**

$n_A(T) = \xi(3)/\pi^2 g_A T^3$

$\rho_A(T) = \pi^2/30\, g_A T^4$

ν1

Non Relativistic particles which decoupled **while being relativistic**

$n_A(T) = \xi(3)/\pi^2 g_A T_A^3$

$\rho_A(T) = n_A(T_A) \times m_A \;\alpha\; m_A T_A^3$

with $T_A = f(T)$

Density of **entropy**

$s(T) = 2\pi^2/45\, g_s T^3$

Non relativistic particles in thermal **equilibrium**

$n_A(T) = g_A (m_A T/2\pi)^{3/2} e^{-mA/T}$

$\rho_A(T) = n_A(T)\, m_A \;\alpha\; T^{3/2} m_A^{5/2}$

DM

Non Relativistic particles which decoupled **while being non relativistic**

$n_A(T)=s(T)\, Y_{dec} \;\alpha\; T^3/(\sigma v\, m_A M_{PL})$

$\rho_A(T) = n_A(T) \times m_A \;\alpha\; T^3/(\sigma v\, M_{PL})$

critical energy (density of energy needed to fulfill the Universe)

$\rho_c(T)=8\pi G/[3 H^2(T)]$

nowadays $\rho^0_c=8\pi G/[3 H^2(T_0)] = 10^{-5} h^2$ GeV^4

with $H(T_0)=100$ h km/s/Mpc

$H^2(T)= 8\pi G/3\, \rho + k/a^2 + \Lambda$

in **radiation dominated** universe ($T > 1$ eV)

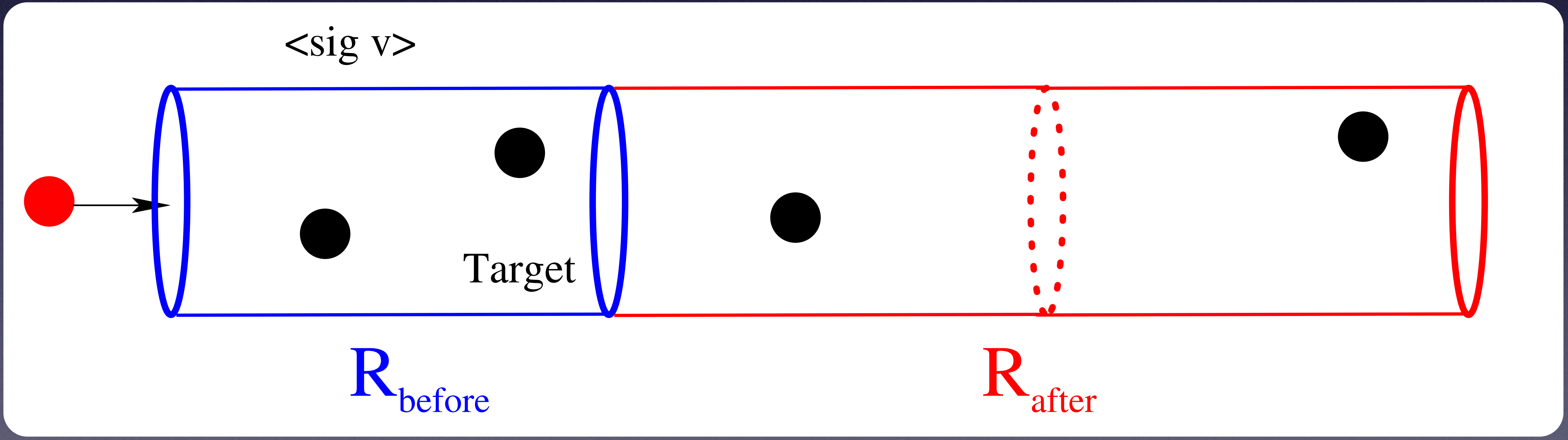
$H(T) = 1.6\, T^2/M_{PL}$
($G=1/M_{PL}^2$)

Decoupling from the bath

Suppose a particle **A** interacting in the bath with a rate per particle $\boldsymbol{\Gamma = n\langle\sigma v\rangle}$, n being the density of the target particle, and $\langle\sigma v\rangle$ the average cross section times the relative velocity. $\Delta t = 1/\Gamma$ represents the mean time between two collisions. During this time Δt the universe has expanded by a factor ΔR such as $\boldsymbol{\Delta R/R = H\Delta t = H/\Gamma}$.

In another word, when $H \simeq \Gamma$, the size of the Universe has doubled and the density n of the target has been divided by 8.

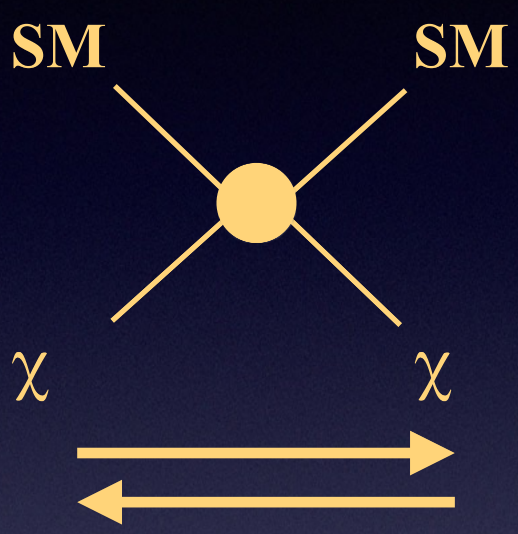
The approximation $H \simeq \Gamma$ to obtain the decoupling time of particles is usually quite accurate.



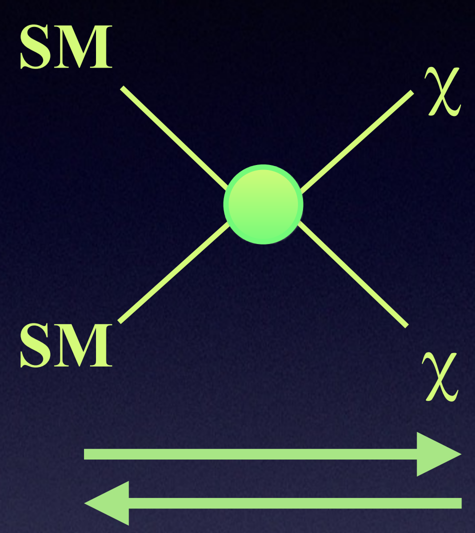
Illustrative example of the decoupling epoch when the number of interaction is divided by 2 during a time Δt due to the dilution of the target. **The volume necessary to have 2 collision (R_{before}) is now just sufficient to give one collision (R_{after})**

The meaning of thermal equilibrium

A system of particles (χ + SM) is in thermal equilibrium when both kinetic and chemical equilibrium is achieved :



kinetic equilibrium



chemical equilibrium

Usually, (except some exception/transition) both are realized simultaneously.

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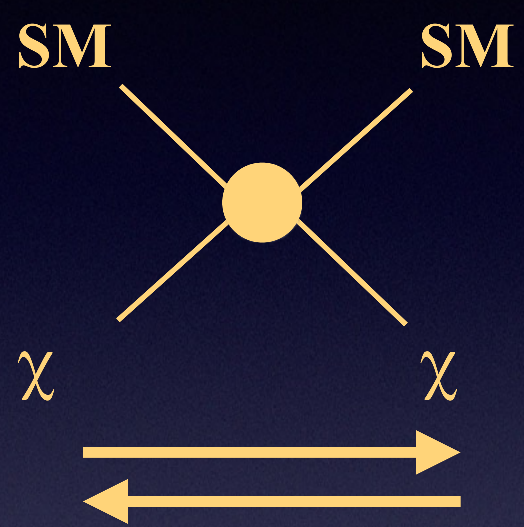
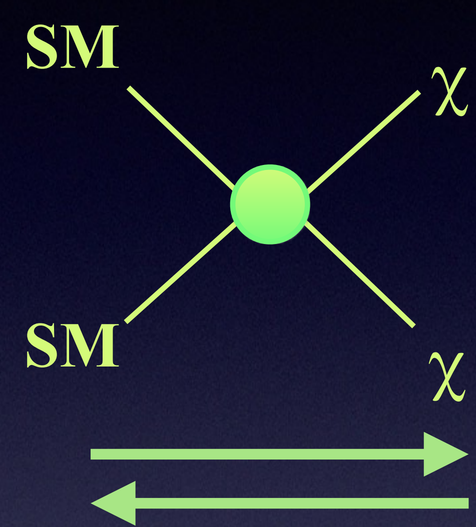
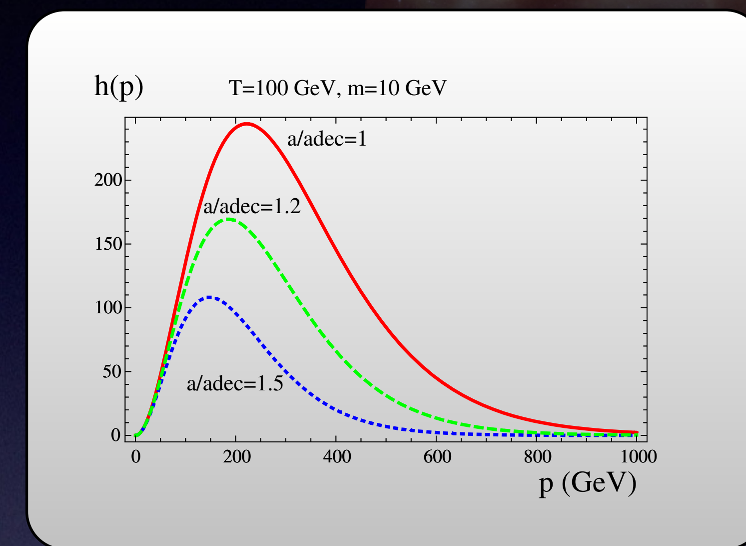


kinetic equilibrium



chemical equilibrium



This is the only way to define a temperature of the Universe, which is in fact the temperature of the photons : $n(T) \alpha T^{3}$.

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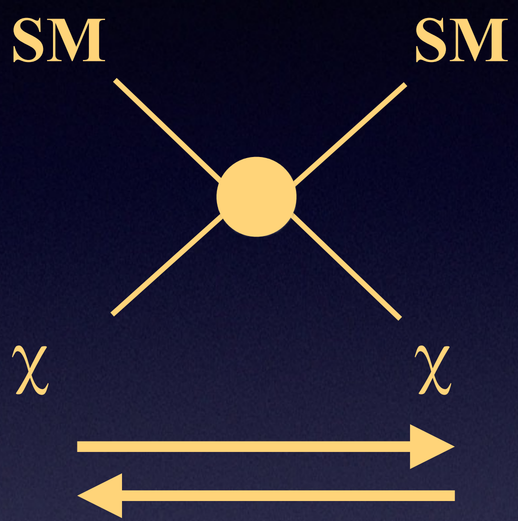
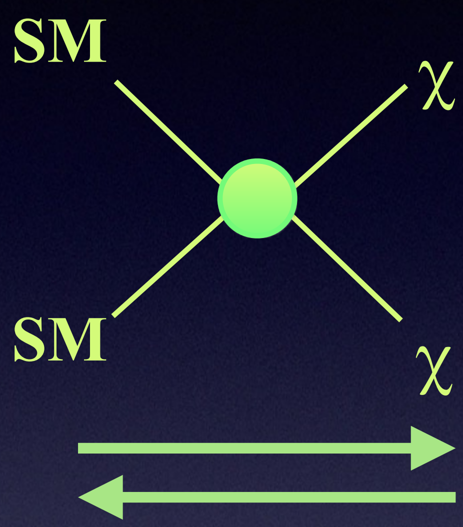
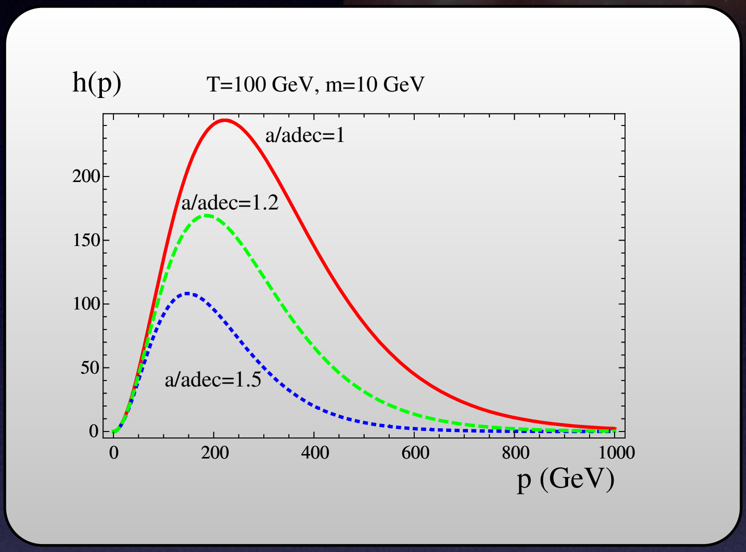


kinetic equilibrium



chemical equilibrium



This is the only way to define a temperature of the Universe, which is in fact the temperature of the photons : n(T) α T^3.

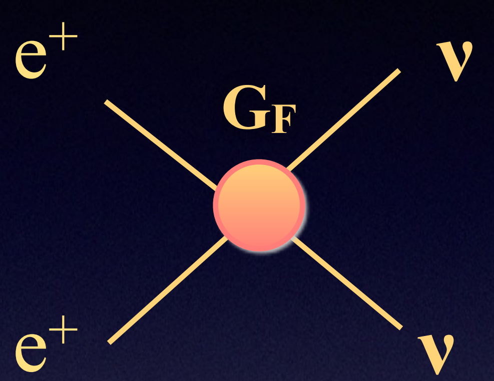
Usually, (except some exception/transition) both are realized simultaneously.

$$\Gamma_{\chi\chi\to SM\ SM}(T) < H(T) = \frac{\dot{a}(T)}{a(T)}, \quad \text{a being the scale factor}$$

1/H, (H being the Hubble parameter) is also called the « doubling time »: Universe is twice bigger after the time 1/H. The previous expression just means that the chemical equilibrium dominates over the expansion rate.

Two ways of decoupling

$n \times \langle \sigma v \rangle < H$ means n is small <u>OR</u> σv is small.



$G_F = 10^{-5}\ \mathrm{GeV}^{-2}$

<u>The massless case (neutrino)</u>

The decoupling condition

$$n \times \langle \sigma v \rangle \simeq T_\nu^3 \times G_F^2 T_\nu^2 = H(T_\nu) = \frac{T_\nu^2}{M_{\rm Pl}} \Rightarrow T_\nu \simeq \left(G_F^2 M_{\rm Pl}\right)^{-1/3} \simeq 3\ \mathrm{MeV}$$

At 3 MeV, the neutrinos decouple from the thermal bath, but being still relativistic

---

<u>Remark</u>: E = m+3/2 T => $\gamma^2 = (E/m)^2 = 1 - v^2/c^2$ => **<u>$v \simeq 0.3\ c$</u>**. The particle is « almost » non-relativistic while decoupling

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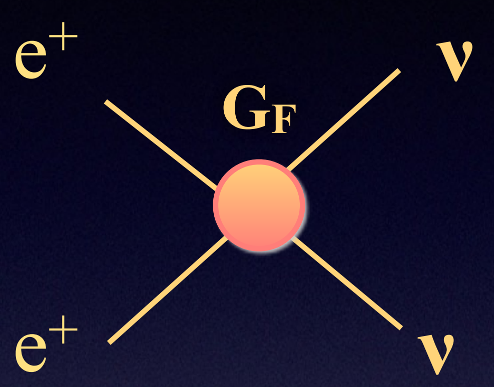

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At 3 MeV, the neutrinos decouple from the thermal bath, but being still relativistic

---

The massive case (dark matter)

The decoupling condition

$$n \times \langle \sigma v \rangle \simeq (T_\chi m_\chi)^{3/2} e^{-m_\chi/T_\chi} \times \frac{1}{T_\chi^2} = H(T_\chi) = \frac{T_\chi^2}{M_{\rm Pl}} \Rightarrow \frac{m_\chi}{T_\chi} \simeq 25$$

T_χ corresponds physically to the temperature under which the thermal bath does not have sufficient energy to produce a dark matter particle.

Remark: $E = m + 3/2\, T \Rightarrow \gamma^2 = (E/m)^2 = 1 - v^2/c^2 \Rightarrow$ **$v \simeq 0.3\, c$**. The particle is « almost » non-relativistic while decoupling

Decoupling of relativistic particles

hot / warm dark matter: no need of the Boltzmann equation

The condition of decoupling of a particle A can be written [see the neutrino case as example]

$\Gamma = n\langle\sigma v\rangle = n\langle\sigma\rangle \simeq T^3 <\sigma> \quad \simeq \quad H(T_d) = T_d^2/M_{pl} \Rightarrow T_d \simeq (<\sigma> . M_{pl})^{-1}$

At the temperature Td, the particle A will not communicate with the thermal bath of photons anymore, BUT will still preserve its Boltzmann/Maxwell distribution in density or energy

$n_A(T_A) \simeq (T_A)^3$

with $T_A = T_\gamma$ as long as the thermal bath do not loose any other particles. When other particles from the plasma decoupled from it, they will transfer their entropy, and then their temperature through their degrees of freedom (conservation of entropy, see the neutrino case) to the photons. Then T_γ differs from T_A

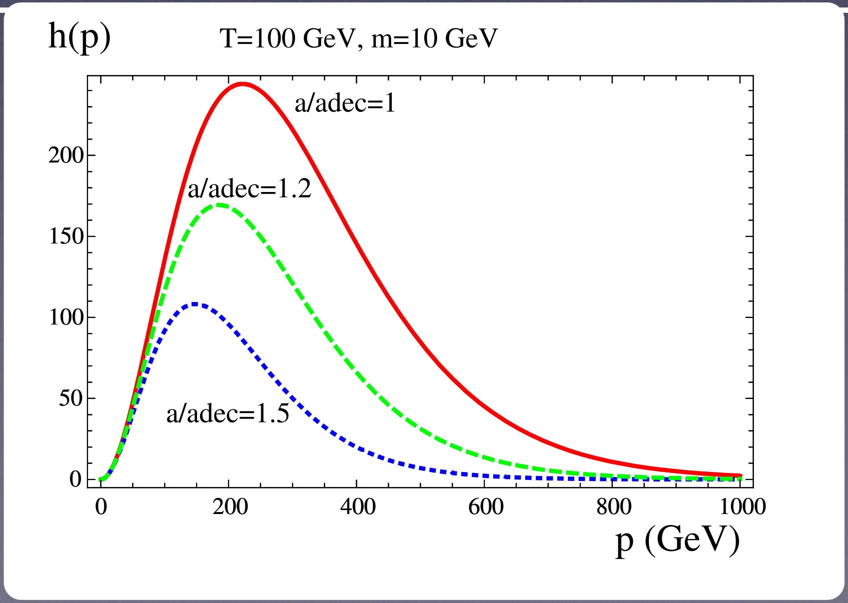
The situation is the same for the energy density which is redshifted the same way than the photon.

This situation stay EVEN when the particle becomes non relativistic anymore!

Indeed, nothing will perturb the botlzmann/Maxell distribution once decoupled:

(DEFINITION of a TEMPERATURE).

Nowadays, even if the neutrinos are massive, heavier than the CMB temperature, they still follow a relativistic distribution: $n(T) \simeq T^3$.



$$\Omega_A = \frac{\rho_A}{\rho_c^0} = \frac{n(T)m_A}{\rho_c^0} = \frac{\zeta(3) g_A (T_A^0)^3}{\pi^2 10^{-5} h^2} m_A$$

$$\Rightarrow \Omega_A h^2 = 10^5 \frac{\zeta(3) g_A (T_A^0)^3}{\pi^2} m_A$$

The Boltzmann equation in a nutshell

For the massive case, one will need to solve the Boltzmann equation as the decoupling from the thermal bath is slightly more complex.

In a close system in expansion, we have conservation of the total number of particles :

$$\frac{dN_i}{dt} = -N_i n_i \langle \sigma v \rangle_{i \to j} + N_j n_j \langle \sigma v \rangle_{j \to i}$$

and from $N_i = n_i \times a^3$ one obtains

$$\frac{dn_i}{dt} = -3\frac{\dot{a}}{a} n_i - n_i n_i \langle \sigma v \rangle_{i \to j} + n_j n_j \langle \sigma v \rangle_{j \to i}$$

noticing that when $n_i = n_j = n^{EQ}$ **the system is frozen** ($dn_i/dt=0$), one deduces:

$$\boxed{\frac{dn_i}{dt} = -3Hn_i - \langle \sigma v \rangle_{i \to j} \left[n_i^2 - (n_i^{EQ})^2 \right]}$$

This is the Boltzmann equation

Solving the equation

book 2.3.4

$$\frac{dn}{dt} = -3Hn - \langle\sigma v\rangle \left(n^2 - n_{eq}^2\right) \quad \Rightarrow \quad \frac{dn}{dT} = 3\frac{n}{T} - \frac{\langle\sigma v\rangle}{HT}\left(n_{eq}^2 - n^2\right)$$

$$\left(\frac{d}{dt} = \frac{dT}{dt}\frac{d}{dT} = -T\frac{da/dt}{a}\frac{d}{dT} = -HT\frac{d}{dT} \quad \rightarrow \quad t \propto \frac{M_{\rm Pl}}{T^2}\right)$$

defining the yield $Y = \frac{n}{S}$, with the entropy $S = \frac{2\pi^2}{45} g_s T^3$

$$\boxed{\frac{dY_\chi}{dT} = T^2 \frac{\langle\sigma v\rangle_{1,2\rightarrow\chi\chi}}{H(T)}\left(Y_\chi^2 - Y_{eq}^2\right)} \text{ with } \langle\sigma v\rangle = \int_{T_{RH}}^{T} \Pi_i \frac{d^3 p_i}{(2\pi)^{3i}} |\mathcal{M}_{1,2\rightarrow\chi\chi}|^2 e^{-E_1/T} e^{-E_2/T} \text{ and } H(T) = \frac{T^2}{M_{\rm Pl}}$$

It is easier to express everything as function of x=m/T, similar to « time » evolution

$$\frac{dY}{dx} = \frac{\langle\sigma v\rangle s}{Hx}\left(Y_{eq}^2 - Y^2\right) \simeq \frac{6.6\times 10^9}{x^2}\frac{g_s}{g_\rho^{1/2}}\left(\frac{m}{\rm GeV}\right)\left(\frac{\langle\sigma v\rangle}{3\times 10^{-26}{\rm cm^3 s^{-1}}}\right)\left(Y_{eq}^2 - Y^2\right)$$

The solution

The s-wave case: book 2.4.3

It is easier to express everything as function of x=m/T, similar to « time » evolution

$$\boxed{\frac{dY}{dx}} = \sqrt{\frac{\pi}{45}}\frac{g_s}{\sqrt{g_\rho}} m M_P \frac{\langle\sigma v\rangle}{x^2}\left(Y_{eq}^2 - Y^2\right) = \sqrt{\frac{\pi}{45}}\frac{g_s}{\sqrt{g_\rho}} m M_P \frac{\langle\sigma v\rangle}{x^2} = \boxed{\frac{\lambda}{x^2}\left(Y_{eq}^2 - Y^2\right)}$$

with $\lambda = \langle\sigma v\rangle m M_P \sqrt{\frac{\pi}{45}}\ \frac{g_s}{\sqrt{g_\rho}}$ and $g_\rho = g_s$

The solution

The s-wave case: book 2.4.3

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$$\boxed{\frac{dY}{dx}} = \sqrt{\frac{\pi}{45}}\frac{g_s}{\sqrt{g_\rho}} m M_P \frac{\langle \sigma v \rangle}{x^2}\left(Y_{eq}^2 - Y^2\right) = \sqrt{\frac{\pi}{45}}\frac{g_s}{\sqrt{g_\rho}} m M_P \frac{\langle \sigma v \rangle}{x^2} = \boxed{\frac{\lambda}{x^2}\left(Y_{eq}^2 - Y^2\right)}$$

with $\lambda = \langle \sigma v \rangle m M_P \sqrt{\frac{\pi}{45}}\, \frac{g_s}{\sqrt{g_\rho}}$ and $g_\rho = g_s$

s-wave annihilation is characterized by $\langle \sigma v \rangle = \text{cte}$

$$\frac{dY}{dx} \approx -\frac{\lambda(x)}{x^2} Y^2$$

$$\frac{1}{Y_0} - \frac{1}{Y_f} = \lambda\left(\frac{1}{x_f} - \frac{1}{x_0}\right) \quad \Rightarrow \quad \boxed{Y_0 \simeq \frac{x_f}{\lambda}}$$

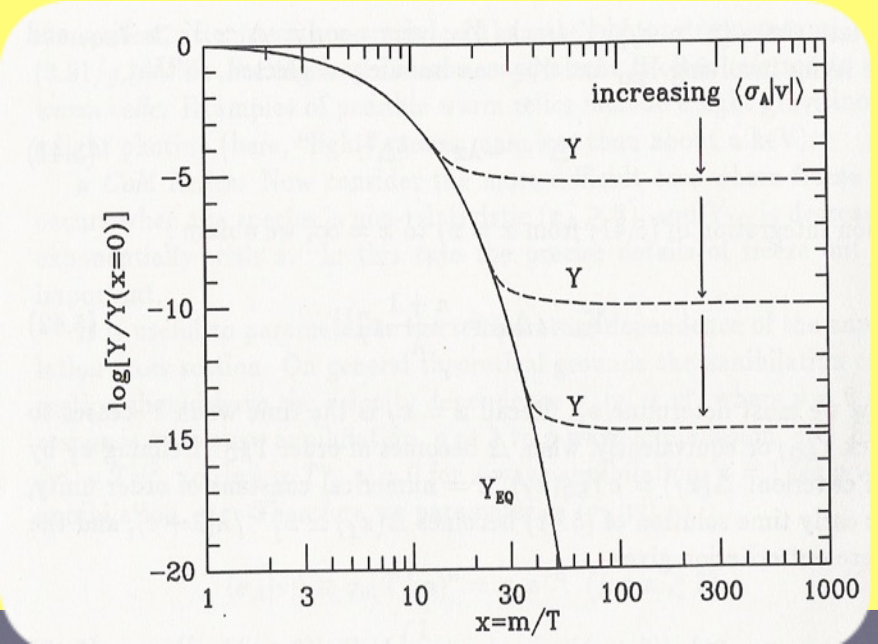
with x_f, time of freeze out obtained as a first approximation by solving

$$n_{eq}(x_f)\langle \sigma v \rangle = H(x_f)$$

$$\boxed{Y(T_0) \simeq 5 \times 10^{-10} \left(\frac{1\ \text{GeV}}{m}\right)}$$

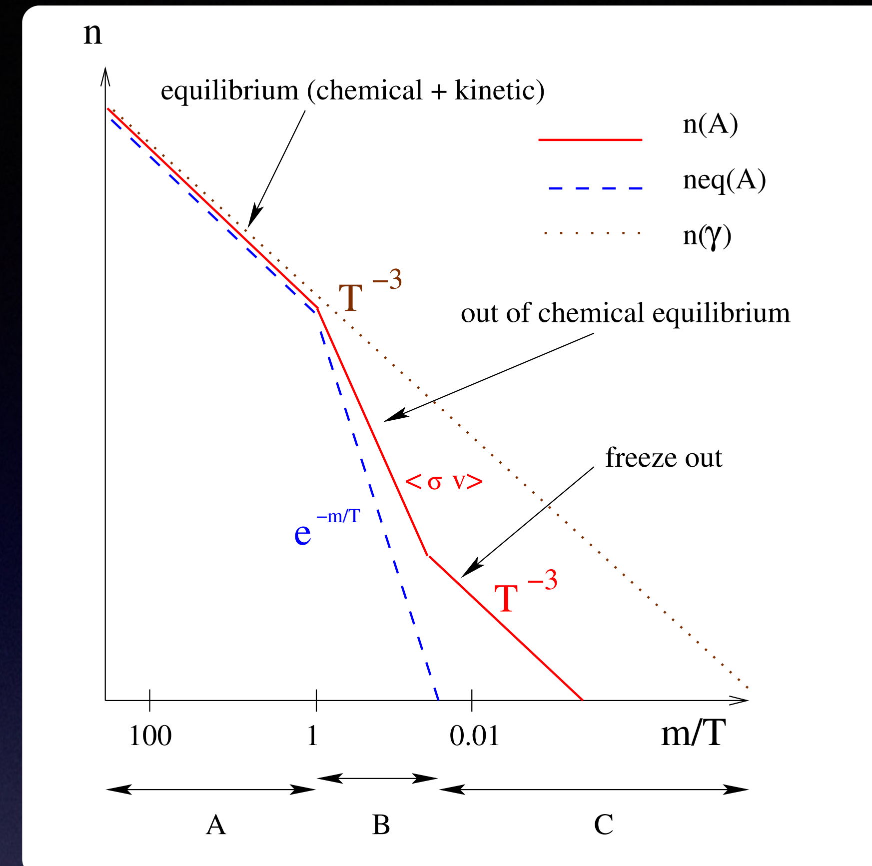
$$\Omega_A = \frac{\rho_A}{\rho_c^0} = \frac{m n_A}{\rho_c^0} = \frac{m Y_0 s_0}{\rho_c^0} = \frac{m\ x_f}{\lambda}\frac{2\pi^2 g_s^{\text{today}} T_0^3}{45 \rho_c^0}$$

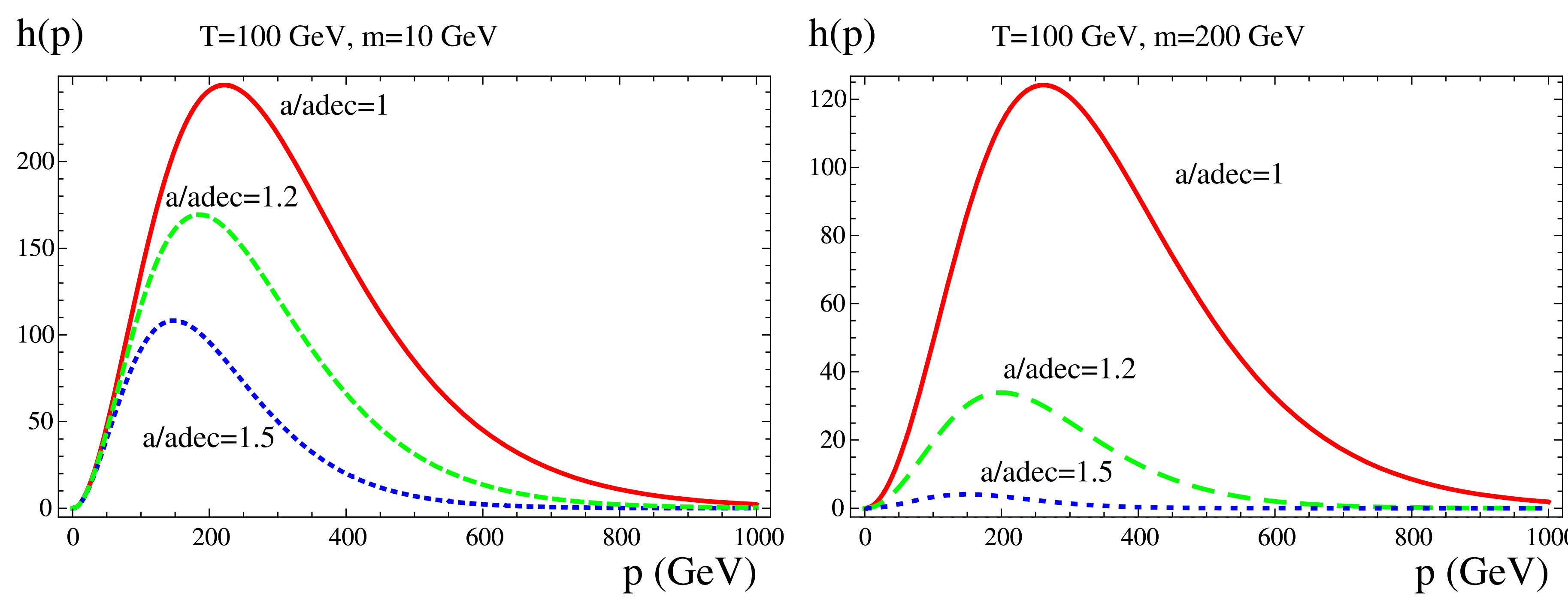
$$\boxed{\Omega_A h^2 \simeq \frac{0.17}{\frac{\langle \sigma v \rangle}{(1.2\times 10^{-26}\ \text{cm}^3\ \text{s}^{-1})}}}$$



Remarks : temperatures and energies after decoupling.

Book, 2.2.6





$$f(p, t > t_{\rm dec}) = \frac{1}{e^{E/\tilde{T}(t)} \pm 1}, \quad \text{with} \quad \tilde{T}(t) = T_{\rm dec}\frac{a(t_{\rm dec})}{a(t)} \; .$$

$$f(p, t > t_{\rm dec}) = e^{-m/T_{\rm dec}} e^{-p^2/2m\tilde{T}}, \quad \tilde{T}(t) = T_{\rm dec}\left(\frac{a(t_{\rm dec})}{a(t)}\right)^2$$

First microscopical approach

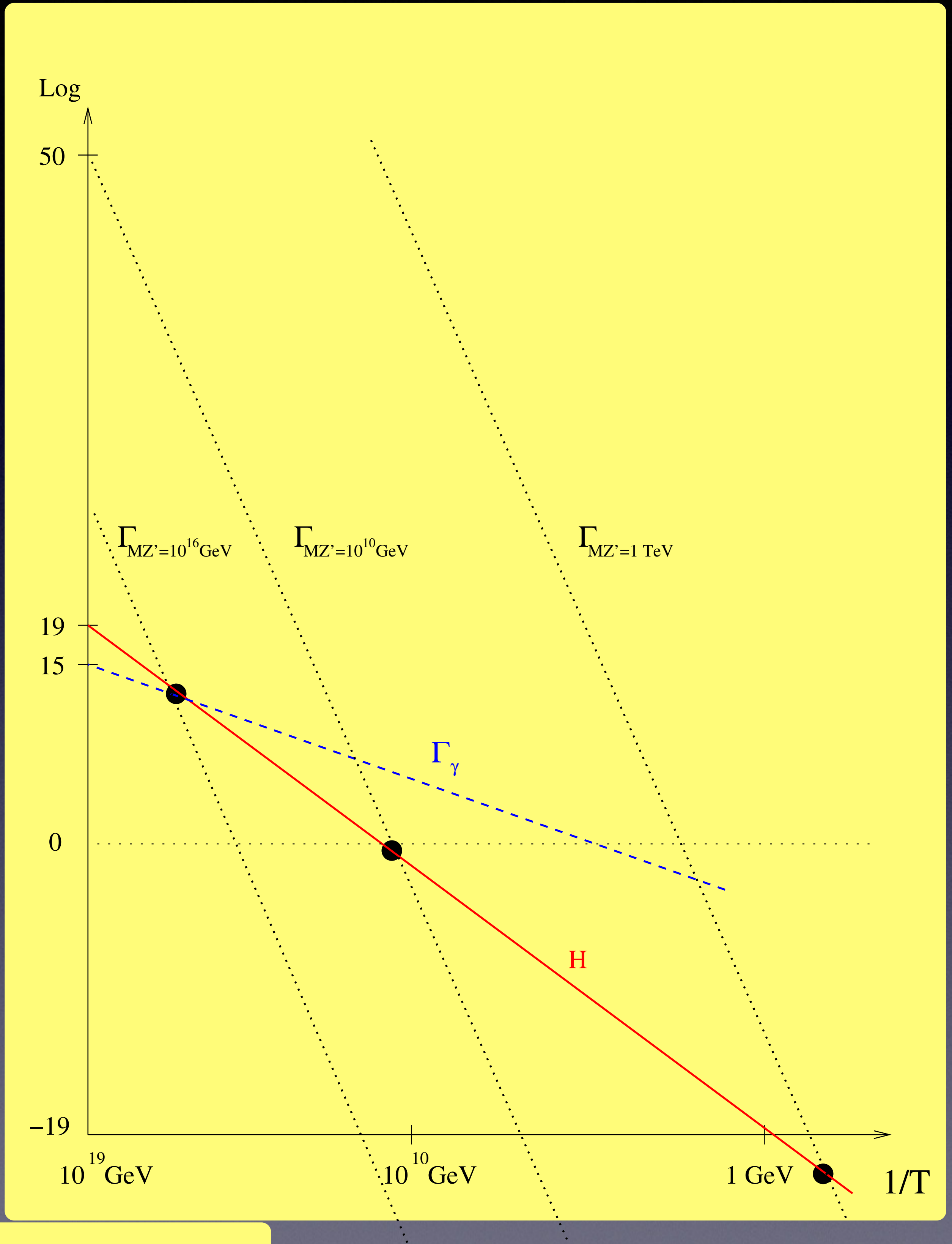
In the case of the exchange of a **massless mediator**, $\sigma \simeq g^4/T^2$ implying $\Gamma = n\langle\sigma v\rangle = n\langle\sigma\rangle \simeq g^4 T$. The particle will thus be decoupled from the primordial plasma when

$$\frac{\Gamma}{H} \lesssim 1 \Rightarrow \frac{g^4 M_P}{T} \lesssim 1 \Rightarrow T \gtrsim g^4 M_p \simeq 10^{15}\mathrm{GeV}$$

In the case of the exchange of a **massive gauge** boson Z' ($M_{Z'} > T$) the cross section of the reaction can be written $\sigma \simeq g^4T^2/M_{Z'}^4$ $\Rightarrow \Gamma \simeq g^4T^5/M_{Z'}^4$. The decoupling temperature will then be given by the usual condition:

$$\frac{\Gamma}{H} \lesssim 1 \Rightarrow \frac{g^4 M_P T^3}{M_{Z'}^4} \lesssim 1$$

$$\Rightarrow T \lesssim \left(\frac{M_{Z'}}{g}\right)^{4/3} M_p^{-1/3} \simeq 0.1 \left(\frac{M_{Z'}}{1\ \mathrm{TeV}}\right)^{4/3} \quad \mathrm{MeV}$$



$$\log H \simeq -2\log 1/T - 19; \;\; \log\Gamma_\gamma \simeq -\log 1/T - 4; \;\; \log\Gamma_{M_{Z'}} = -5\log 1/T - 4\log M_{Z'} - 4$$

Fields of interest

The meaning of equilibrium

$$\Gamma_{i\to f} = n_i(T)\langle\sigma v\rangle_{i\to f} = H(T)$$

Particle Physics

The Hut-Lee-Weinberg bound

Enrico Fermi

"Tentativo di una teoria dei raggi β",

Ricerca Scientifica, 1933

TENTATIVO DI UNA TEORIA DEI RAGGI β

Nota (1) di ENRICO FERMI

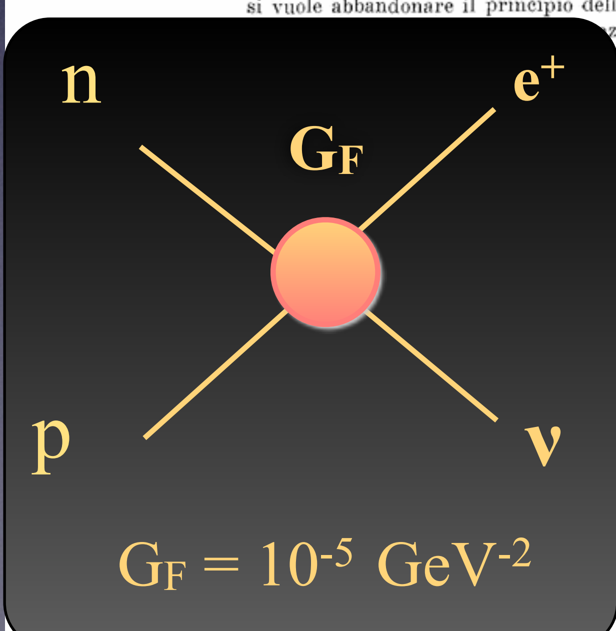
Sunto. - *Si propone una teoria quantitativa dell'emissione dei r in cui si ammette l'esistenza del « neutrino » e si tratta l'emissione elettroni e dei neutrini da un nucleo all'atto della disintegrazione un procedimento simile a quello seguito nella teoria dell'irradi per descrivere l'emissione di un quanto di luce da un atomo ec Vengono dedotte delle formule per la vita media e per la forma spettro continuo dei raggi β, e le si confrontano coi dati sperim*

Ipotesi fondamentali della teoria.

§ 1. Nel tentativo di costruire una teoria degli elettroni nucleari e dell'emissione dei raggi β, si incontrano, come è noto, due difficoltà principali. La prima dipende dal fatto che i raggi β primari vengono emessi dai nuclei con una distribuzione continua di velocità. Se non si vuole abbandonare il principio della conservazione dell'energia, si ...zione dell'energia che si libera ...rga alle nostre attuali possibi- ...sta di PAULI si può p. es. am- ...icella, il così detto « neutrino », ...a dell'ordine di grandezza di ...nette poi che in ogni processo β ...elettrone, che si osserva come all'osservazione portando seco ...teoria ci baseremo sopra l'ipo-

...ia degli elettroni nucleari, di- ...e relativistiche delle particelle ...nno una soddisfacente spiega- ...lle vengano legate in orbite di

...cerca Scientifica », 2, fasc. 12, 1933.



The Hut-Lee-Weinberg bound

TENTATIVO DI UNA TEORIA DEI RAGGI β

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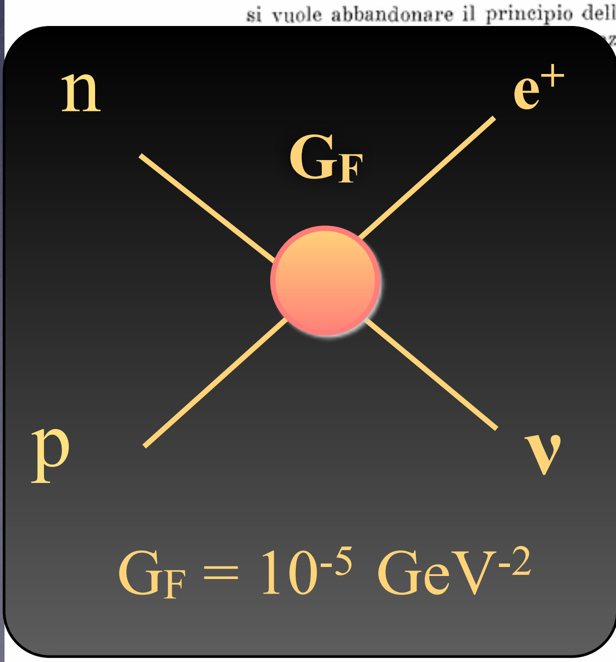
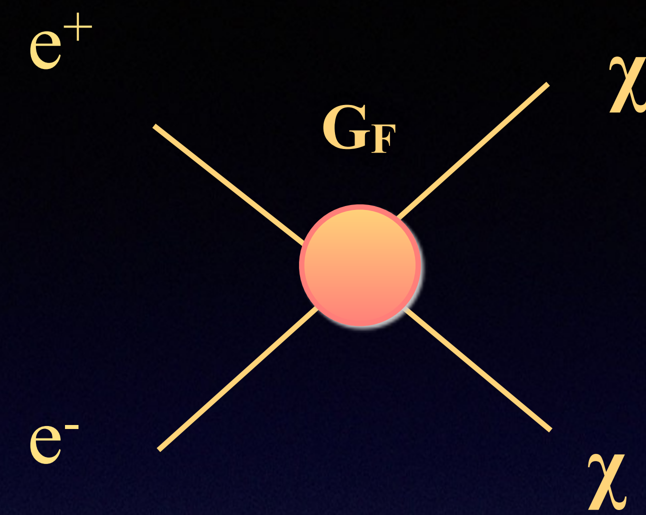
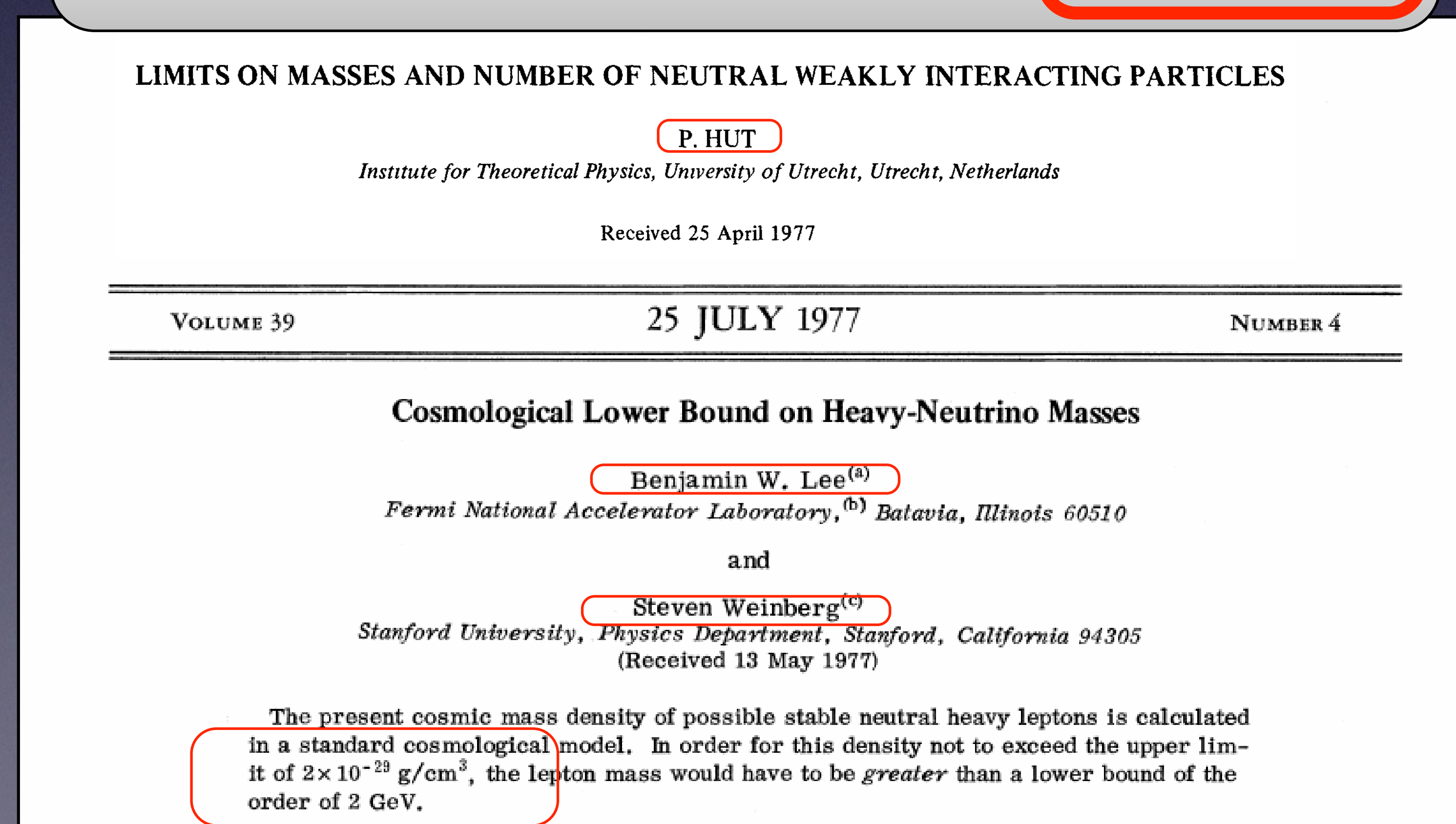


$G_F = 10^{-5}$ GeV^{-2}



The (Hut-)Lee-Weinberg bound (1977)

$$\langle \sigma v \rangle = G_F^2 m_\chi^2 > 10^{-9}\ \mathrm{GeV}^{-2} \quad \Rightarrow \quad \boxed{m_\chi > 2\ \mathrm{GeV}}$$



LIMITS ON MASSES AND NUMBER OF NEUTRAL WEAKLY INTERACTING PARTICLES

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Institute for Theoretical Physics, University of Utrecht, Utrecht, Netherlands

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Cosmological Lower Bound on Heavy-Neutrino Masses

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and

Steven Weinberg[c]
Stanford University, Physics Department, Stanford, California 94305
(Received 13 May 1977)

The present cosmic mass density of possible stable neutral heavy leptons is calculated in a standard cosmological model. In order for this density not to exceed the upper limit of 2×10^{-29} g/cm^3, the lepton mass would have to be *greater* than a lower bound of the order of 2 GeV.

The Zeldovich-Cowsik-McClelland bound

REST MASS OF MUONIC NEUTRINO AND COSMOLOGY

S. S. Gershtein and Ya. B. Zel'dovich
Submitted 4 June 1966
ZhETF Pis'ma 4, No. 5, 174-177, 1 September 1966

of the e^+e^- increases the number of quanta without changing the number of neutrinos per unit of co-moving volume [5]. At the present time we can expect

$$[e^+] + [e^-] = 0, \quad [\nu_\mu] + [\bar{\nu}_\mu] = [\nu_e] + [\bar{\nu}_e] = 0.5[\gamma].$$

At 3°K we have $[\gamma] = 550$ g/cm^3, from which we obtain for the neutrino at the present time

$$[\nu_\mu] + [\bar{\nu}_\mu] = [\nu_e] + [\bar{\nu}_e] = 300 \text{ cm}^{-3}.$$

Comparing with the density limit given above, we obtain

$$m_0(\nu_\mu) < 7 \times 10^{-31} \text{ g} = 400 \text{ eV}/c^2$$

[CMB, Penziaz 1965 + discovery ν_μ, Lederman 1962]

VOLUME 29, NUMBER 10 PHYSICAL REVIEW LETTERS 4 SEPTEMBER 1972

An Upper Limit on the Neutrino Rest Mass*

R. Cowsik† and J. McClelland
Department of Physics, University of California, Berkeley, California 94720
(Received 17 July 1972)

In order that the effect of gravitation of the thermal background neutrinos on the expansion of the universe not be too severe, their mass should be less than 8 eV/c^2.

at $T < m_e$, e^- and e^+ decouple from the thermal bath: they give their degrees of freedom (2+2=4) to the photons and not the neutrinos because the latter are already out of equilibrium since T ~ 3 MeV
$2\, n_\gamma \to (2 + 7/8\ 4)\, n_\gamma = 11/2\, n_\gamma$.

The photons are then almost 3 times more « dense » than the neutrino in the bath after the decoupling of the electrons,
resulting at T_0=2.7 K, n_γ = 400 cm^{-3} [CMB] => $n_\nu + n_{\bar\nu}$ = 200 cm^{-3}

To avoid overclosing the Universe, one needs
$\rho_\nu = m_\nu (n_\nu + n_{\bar\nu}) < \rho_{crit} = 3 H^2/8 \pi G = 2 \times 10^{-29}\, h^2$ g / cm^3
=> $m_\nu < 2\ 10^{-31}\, h^2$ g = 94 h^2 eV = 45 eV [H_0=67 km/s/Mpc]
[corrections 1/3 from Ω_m versus 1 (or 5, Zeldovich) + number of families]

[Zeldovich considered $\rho < 2 \times 10^{-28}$ g/cm^3]

A little remark

Treatment of Zeldovich is ~ok but two little mistakes has been made by Cowsik:
(the original article can be find there: http://www.ymambrini.com/My_World/History.html)

$$n_{Fi}(0) = n_{Fi}(z_{eq})\left[\frac{1}{1+z_{eq}}\right]^3 \approx 0.0913(2s_i+1)\left[\frac{T_r(0)}{\hbar c}\right]^3$$

and

$$n_{Bi}(0) \approx 0.122(2s_i+1)\left[\frac{T_r(0)}{\hbar c}\right]^3.$$

Taking $T_r(0) \approx 2.7\,^\circ\mathrm{K}$, we have

$$n_{Fi}(0) \approx 150(2s_i+1)\ \mathrm{cm}^{-3},$$

Not true. Cowsik forgot to take into account the reheating of the thermal bath (photons) due to the entropy conservation once the electrons/positrons decoupled. Factor $(4/11)^{1/3}$ (see book section 2.2.7 + Entropy slide)

$$\rho_{\mathrm{weak}} \approx \sum n_{Bi} m_i + n_{Fj} m_j \gtrsim 150(2s_i+1)m_i < \rho_{\mathrm{tot}}$$

or (6)

$$\sum(2s_i+1)m_i \lesssim 66\ \mathrm{eV}/c^2.$$

Here the summation is to be carried out over all the particle and antiparticle states of both fermions and bosons. Considering only the neutrinos and antineutrinos of the muon and electron kind each having a mass of m_ν, Eq. (6) leads to the result $m_\nu < 8\ \mathrm{eV}/c^2$.

Cowsik considered left + right handed neutrino whereas right handed neutrino does not feel weak interaction, i.e. cannot be considered as in thermal equilibrium with the left handed ones: only 2 degrees of freedom for neutrinos should be considered ($\nu_L+\underline{\nu_L}$), not 4

A little mistreatment made by Zeldovich

$$[\nu_e] + [\bar{\nu}_e] = [\nu_\mu] + [\bar{\nu}_\mu] = [e^+] + [e^-] = \boxed{2\frac{\int(e^x+1)^{-1}x^2dx}{\int(e^x-1)^{-1}x^2dx}[\gamma] = 1.5[\gamma].}$$

However, during the course of the cooling from $T > m_e c^2$ (for which these relations are written) to the present time, when $T \ll m_e c^2$, these relations change, since the annihilation

of the e^+e^- increases the number of quanta without changing the number of neutrinos per unit of co-moving volume [5]. At the present time we can expect

$$\boxed{[e^+] + [e^-] = 0, \quad [\nu_\mu] + [\bar{\nu}_\mu] = [\nu_e] + [\bar{\nu}_e] = 0.5[\gamma].}$$

At 3°K we have $[\gamma] = 550$ g/cm^3, from which we obtain for the neutrino at the present time

$$[\nu_\mu] + [\bar{\nu}_\mu] = [\nu_e] + [\bar{\nu}_e] = 300 \text{ cm}^{-3}.$$

Zeldovich considered indeed that the degrees of freedom (quanta as he wrote) of the electrons are transferred to the photons and not the neutrino (*Peebles, 1966*) he miscounted the degrees of freedoms forgetting that the fundamental quantities which is conserved is the entropy s(T)=(ρ+P/3)/T=4/3 ρ(T)/T and not n(T).

They considered that [e+] + [e-] = (3/4 x 4)=1.5 [γ] give all their 4 degrees of freedom to the photons after their decoupling, thus

$[\gamma]_{after}$ = (2 + 4)/2=3$[\gamma]_{before}$ implying ([ν] + [ν̲])$_{after}$ = 0.5 $[\gamma]_{after}$

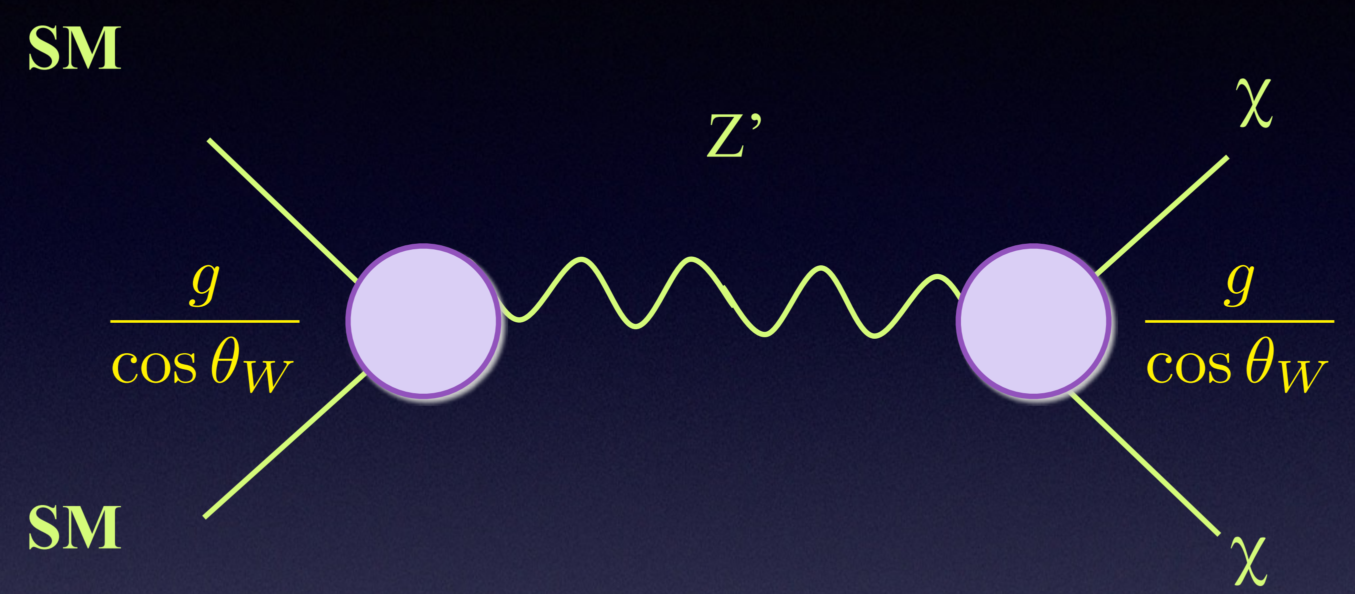
The exact treatment should be made with the entropy conservation

$[\gamma]_{after}$ = (2 + 7/8 x 4)/2= 11/4 $[\gamma]_{before}$

11/4 compared to 3, not so much differences..

Application: the Z' case

As a simple microscopic application, let suppose the case of an intermediate gauge boson Z' , mediator between the thermal Standard Model bath (SM) and the Dark Matter (DM) of mass m:



$$\sigma v = \left(\frac{g^4}{64\pi\cos^4\theta_W}\right)\frac{m^2}{(s-M_{Z'}^2)^2+\Gamma_{Z'}^2 M_{Z'}^2}$$

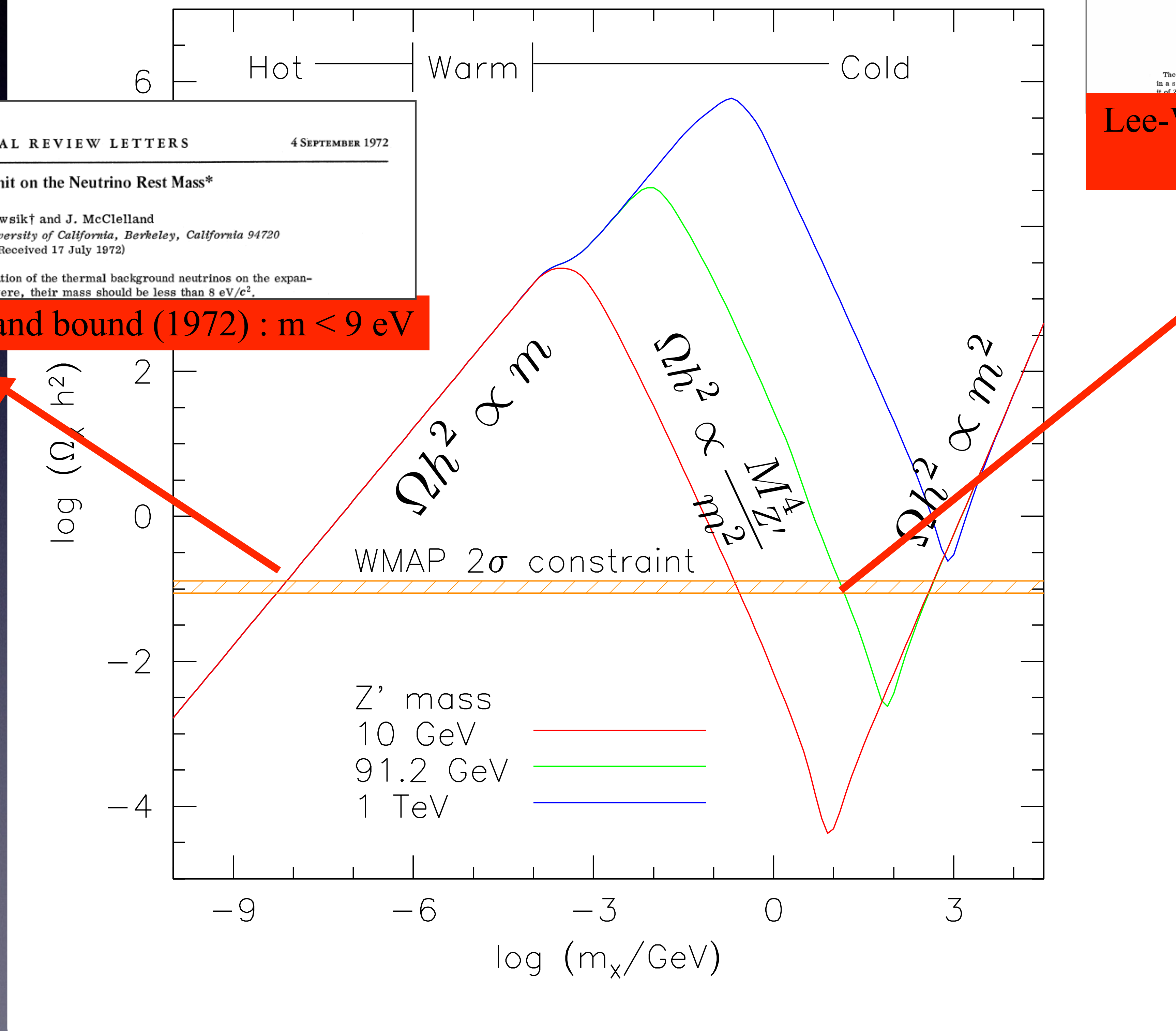
3 regimes

1) **m << $T_{decoupling}$** : relativistic case, $\boxed{\Omega \sim m}$

2) **$T_{decoupling}$ << m << $M_{Z'}$** , $\sigma \sim m^2/(M_{Z'})^4$ => $\boxed{\Omega \sim (M_{Z'})^4/m^2}$

3) **$M_{Z'}$ << m** , $\sigma \sim 1/m^2$ => $\boxed{\Omega \sim m^2}$

Application: the Z' case



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Cowsik-McClelland bound (1972) : m < 9 eV

PHYSICAL REVIEW LETTERS

VOLUME 39 25 JULY 1977 NUMBER 4

Cosmological Lower Bound on Heavy-Neutrino Masses

Benjamin W. Lee[a]
Fermi National Accelerator Laboratory,[b] *Batavia, Illinois 60510*
and
Steven Weinberg[c]
Stanford University, Physics Department, Stanford, California 94305
(Received 13 May 1977)

The present cosmic mass density of possible stable neutral heavy leptons is calculated in a standard cosmological model. In order for this density not to exceed the upper lim-

Lee-Weinberg bound (1977) : m > 2 GeV

Unitarity limit

Unitarity Limits on the Mass and Radius of Dark-Matter Particles

Kim Griest
Center for Particle Astrophysics, University of California, Berkeley, California 94720

Marc Kamionkowski
Physics Department, Enrico Fermi Institute, The University of Chicago, Chicago, Illinois 60637-1433
and NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory,
Batavia, Illinois 60510-0500
(Received 5 October 1989)

Using partial-wave unitarity and the observed density of the Universe, we show that a stable elementary particle which was once in thermal equilibrium cannot have a mass greater than 340 TeV. An extended object which was once in thermal equilibrium cannot have a radius less than 7.5×10 fm. A lower limit to the relic abundance of such particles is also found.

Boltzmann

$$\Omega_X h^2 = \frac{1.07\times10^9 (n+1) x_f^{n+1}\ \mathrm{GeV}^{-1}}{g_*^{1/2} m_{\mathrm{Pl}} \langle \sigma v_{\mathrm{rel}} \rangle_f'} \approx \frac{3\times10^{-27}\ \mathrm{cm}^3/\mathrm{sec}}{\langle \sigma v_{\mathrm{rel}} \rangle_f},$$

+

Unitarity

$$(\sigma_J)_{\max} v_{\mathrm{rel}} \approx \frac{4\pi(2J+1)}{m_X^2 v_{\mathrm{rel}}} \approx \frac{3\times10^{-22}(2J+1)\ \mathrm{cm}^3/\mathrm{sec}}{[m_X/(1\ \mathrm{TeV})]^2}.$$

+

Overclosure

$$\Omega_X h^2 \geq 1.7\times10^{-6}\sqrt{x_f}\,[m_X/(1\ \mathrm{TeV})]^2$$

=

$$m_\chi \lesssim 340\ \mathrm{TeV}$$

Summary on bounds:

Lee-Weinberg bound (1977)

$$\langle \sigma v \rangle = G_F^2 m_\chi^2 > 10^{-9}\ \mathrm{GeV}^{-2} \quad \Rightarrow \quad m_\chi > 2\ \mathrm{GeV}$$

BUT non-valid as soon as we suppose an extra mediator (Z' lighter then the Z for instance => $G'_F > G_F$

Cowsik-McClelland bound (1972)

$$\Omega_\nu h^2 = \frac{\rho_\nu}{\rho_0^c} h^2 = \frac{n_\nu\ m_\nu}{10^{-5}\ \mathrm{GeV\ cm^3}} \simeq \frac{m_\nu}{92\ \mathrm{eV}} \quad \Rightarrow \quad m_\nu \lesssim 9\ \mathrm{eV}$$

BUT be careful to the extra degrees of freedom (dark radiation)

Unitarity bound

$$m_\chi \lesssim 340\ \mathrm{TeV}$$

BUT thermal production has been imposed (FIMP or WIMPZILLA do not enter in this game)

Problem of hot dark matter: the formation of large structures

book 2.8.3

A particle when decoupled while relativistic (« hot ») collisionless will have tendency to stream out from overdense region into underdense region, smoothing out in the process inhomogenities. One should then compute its free-streaming path, and check that it is lower than the size of proto-galaxies (~Megaparsec)

$$\lambda_{FS}(t) = \int_0^t dr = \int_0^t \frac{v(t')}{a(t')}dt'$$

- Between $t = 0$ and $t = t_{nr}$, the neutrino is relativistic and thus $v = c$ and the expansion factor is proportional to $t^{1/2}$ [Eq.(B.16)], $a(t) = \alpha\, t^{1/2}$

$$\lambda_{FS}^{0-nr}(t) = \int_0^t \frac{dt'}{a(t')} = \frac{2}{\alpha}t^{1/2} = 2\frac{a(t)t_{nr}}{a_{nr}^2} \qquad [0 < \mathrm{t} < \mathrm{t_{nr}}]. \qquad (2.204)$$

- Between t_{nr} and t_{EQ}, the velocity of the particle $v(t)$ is proportional to $a^{-1}(t)$ by redshift, which means $v(t) = c\frac{a_{nr}}{a(t)} = \frac{a_{nr}}{a(t)}$ which gives

$$\lambda_{FS}^{nr-EQ}(t) - \lambda_{FS}^{nr-EQ}(t_{nr}) = \lambda_{FS}^{nr-EQ}(t) - \lambda_{FS}^{0-nr}(t_{nr}) = \int_{t_{nr}}^t \frac{a_{nr}dt'}{a^2(t')} = \frac{t_{nr}}{a_{nr}}\ln\left(\frac{t}{t_{nr}}\right)$$

$$\Rightarrow \quad \lambda_{FS}^{nr-EQ}(t) = 2\frac{t_{nr}}{a_{nr}} + 2\frac{t_{nr}}{a_{nr}}\ln\left(\frac{a(t)}{a_{nr}}\right) = 2\frac{t_{nr}}{a_{nr}}\left[1 + \ln\left(\frac{a(t)}{a_{nr}}\right)\right] \qquad [\mathrm{t_{nr}} < \mathrm{t} < \mathrm{t_{EQ}}]. \qquad (2.205)$$

- Between $t = t_{EQ}$ and $t = t_0$ the dependance on the velocity is of course still red-shifted, but the relation between time and scale factor from Eq.(B.16) is $a(t) = \alpha\, t^{2/3}$. Remarking $t_{nr} = t_{EQ}(a_{nr}/a_{EQ})^2$ we then obtain

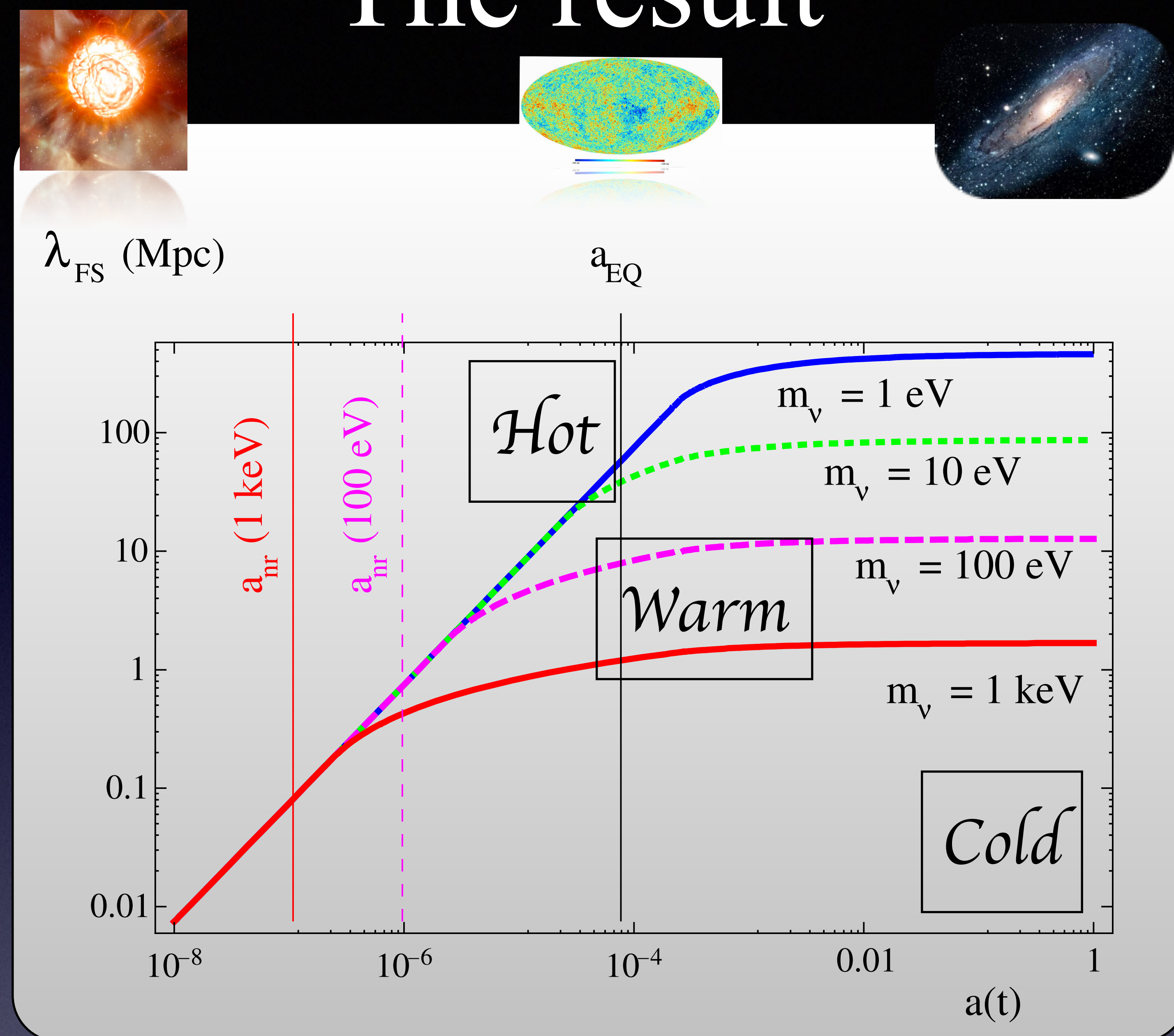
$$\lambda_{FS}^{EQ-t_0}(t) = \lambda_{FS}^{nr-EQ}(t_{EQ}) + a_{nr}\int_{t_{EQ}}^t \frac{dt'}{a^2(t')} = 2\frac{t_{nr}}{a_{nr}}\left[1 + \ln\left(\frac{a_{EQ}}{a_{nr}}\right)\right] + \frac{3t_{nr}}{a_{nr}}\left[1 - \left(\frac{a_{EQ}}{a(t)}\right)^{1/2}\right]$$

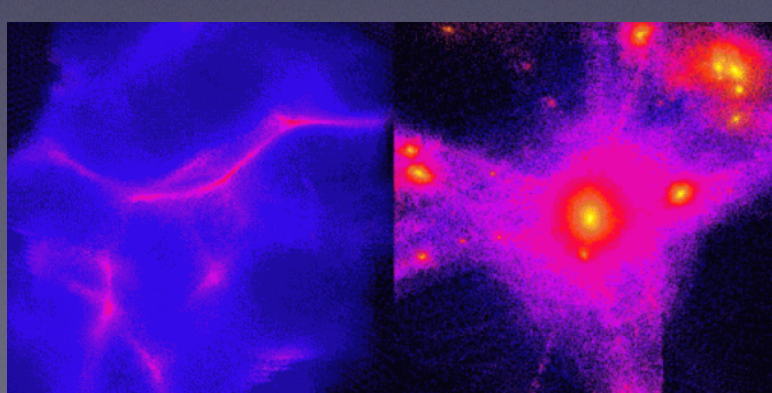
$$\Rightarrow \quad \lambda_{FS}^{EQ-t_0}(t) = \frac{2t_{nr}}{a_{nr}}\left[\frac{5}{2} + \ln\left(\frac{a_{EQ}}{a_{nr}}\right) - \frac{3}{2}\left(\frac{a_{EQ}}{a(t)}\right)^{1/2}\right] \qquad [\mathrm{t_{EQ}} < \mathrm{t} < \mathrm{t_0}]. \qquad (2.206)$$

$$\lambda_{FS}^{\infty} = \frac{M_P}{T_0 T_{nr}}\sqrt{\frac{45}{4\pi^3 g_\rho}}\left[\frac{5}{2} + \ln\left(\frac{\rho_0^R T_{nr}}{\rho_0^M T_0}\right)\right] \approx 70 \text{ Mpc}\frac{1 \text{ eV}}{T_{nr}} \approx 210 \text{ Mpc}\frac{1 \text{ eV}}{m_\nu}$$

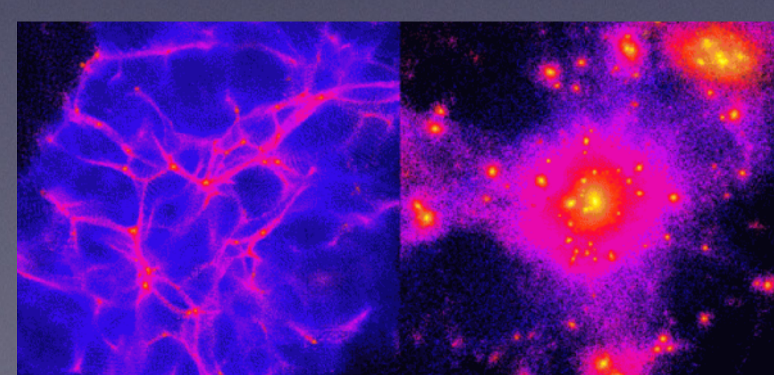
credit ITP, University of Zurich

The result

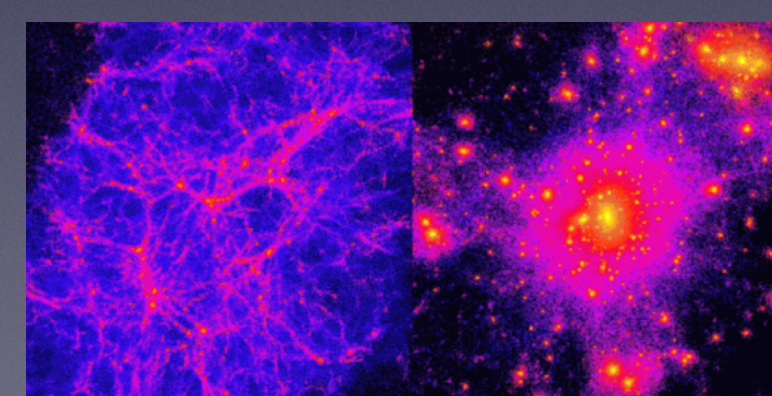




Hot dark matter (ν)



Warm dark matter (ν_R)



Cold dark matter

credit ITP, University of Zurich

Two exceptions to the Boltzmann equation

The WIMPZILLA

or how to have the eyes bigger than the stomach

The FIMP (Freeze In Massive Particle)

or how to have the stomach smaller than the eyes

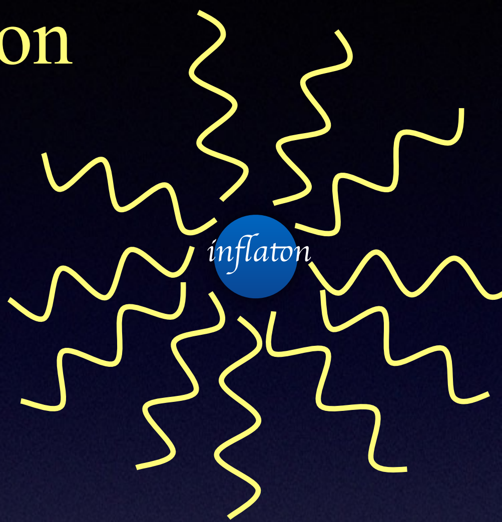
The WIMPZILLA : How to obtain a very massive (> 10^8 GeV) candidate?

$$\dot{\rho}_I + 3H\rho_I + \Gamma_I \rho_I = 0$$

Boltzmann equation for the decaying Inflaton

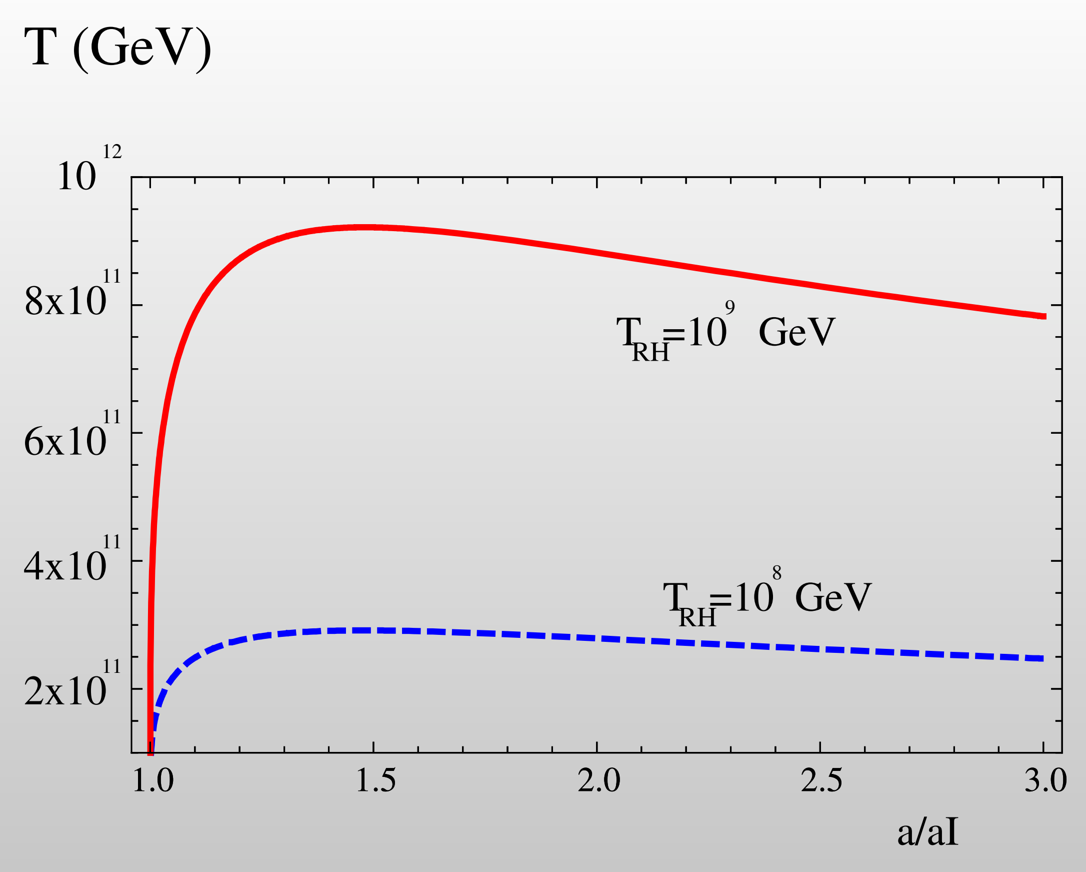
$$\dot{\rho}_R + 4H\rho_R - \Gamma_I \rho_I = 0$$

Boltzmann equation for the Radiation



$$T = \left(\frac{9}{5 g_* \pi^3}\right)^{1/8} \sqrt{T_{RH}} (H_I M_p)^{1/4} \left[\left(\frac{a}{a_I}\right)^{-3/2} - \left(\frac{a}{a_I}\right)^{-4} \right]^{1/4}$$

($\rho \propto T^4$)



The WIMPZILLA : How to obtain a very massive (> 10^8 GeV) candidate?

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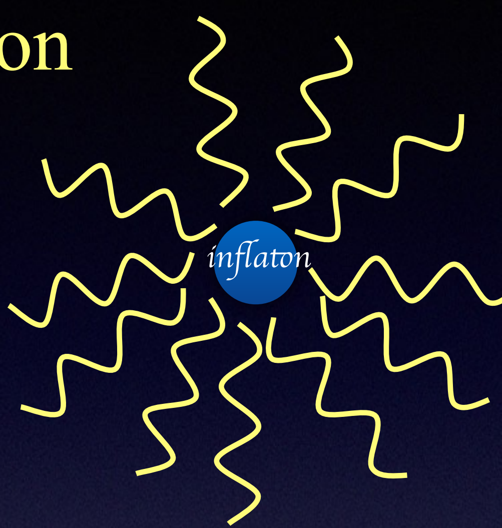
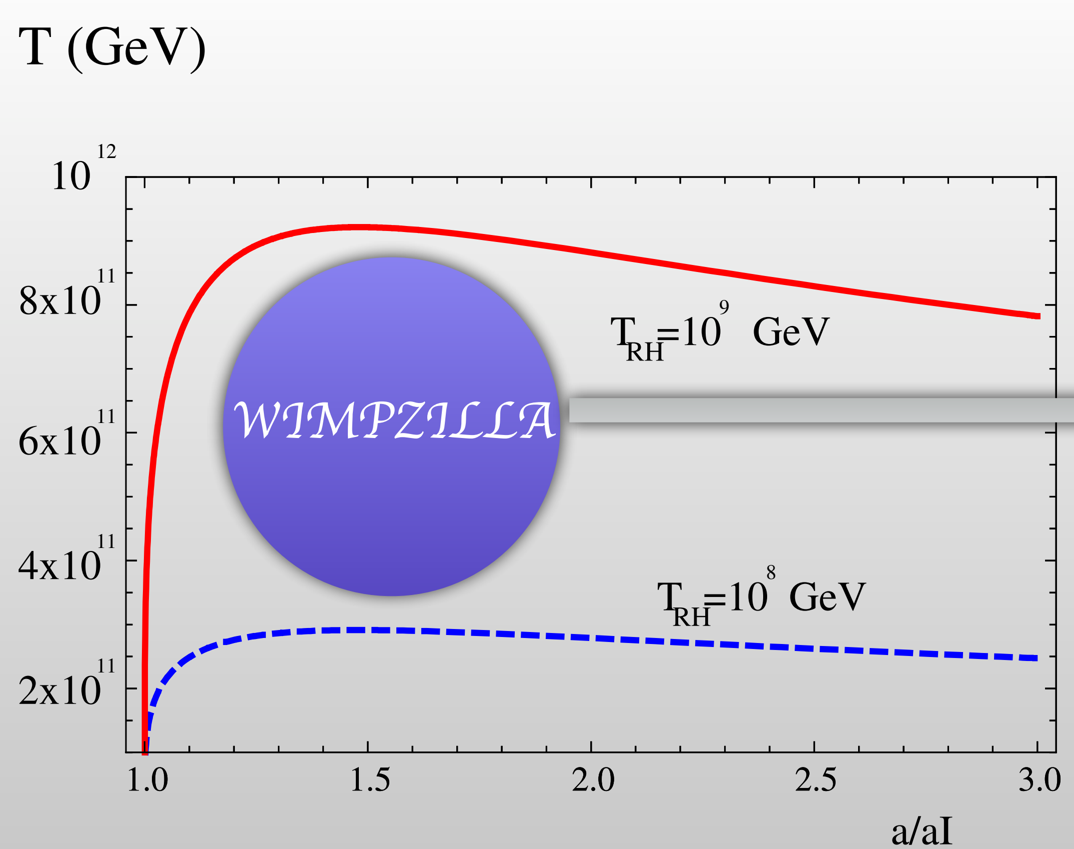
Boltzmann equation for the decaying Inflaton

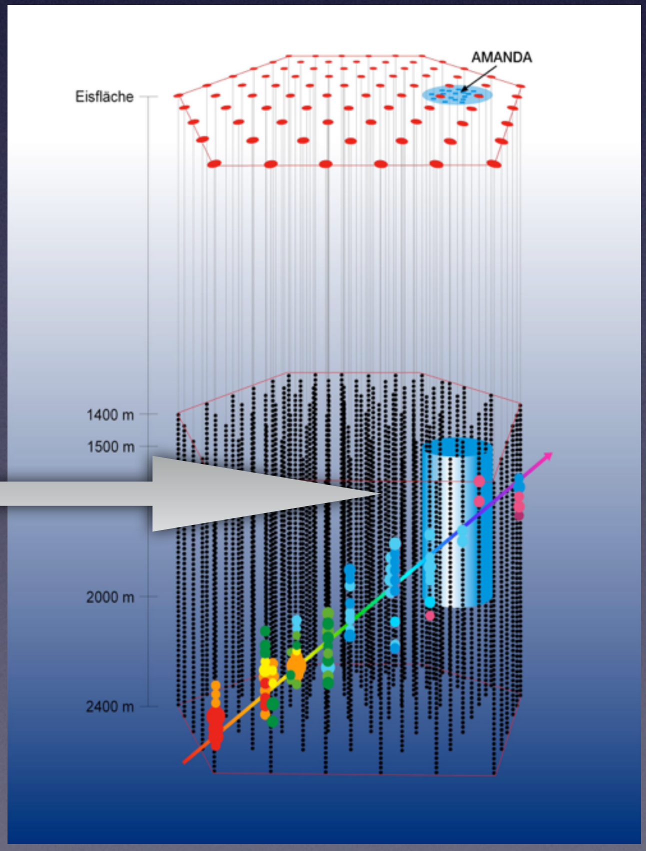
Boltzmann equation for the Radiation



$$T = \left(\frac{9}{5 g_* \pi^3}\right)^{1/8} \sqrt{T_{RH}} (H_I M_p)^{1/4} \left[\left(\frac{a}{a_I}\right)^{-3/2} - \left(\frac{a}{a_I}\right)^{-4} \right]^{1/4}$$

$(\rho \propto T^4)$

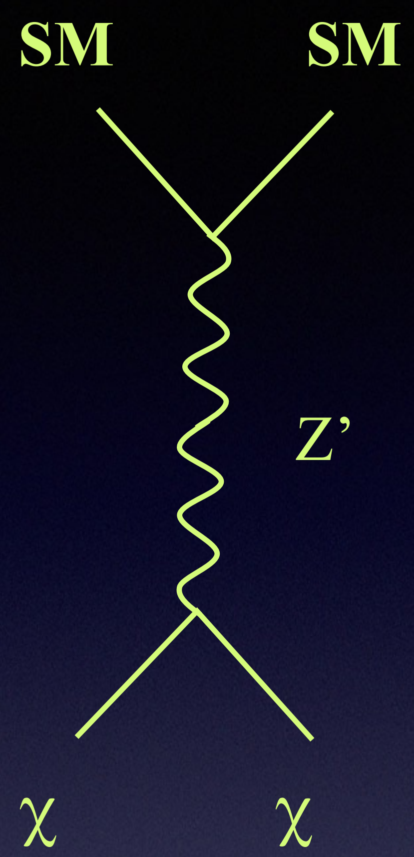




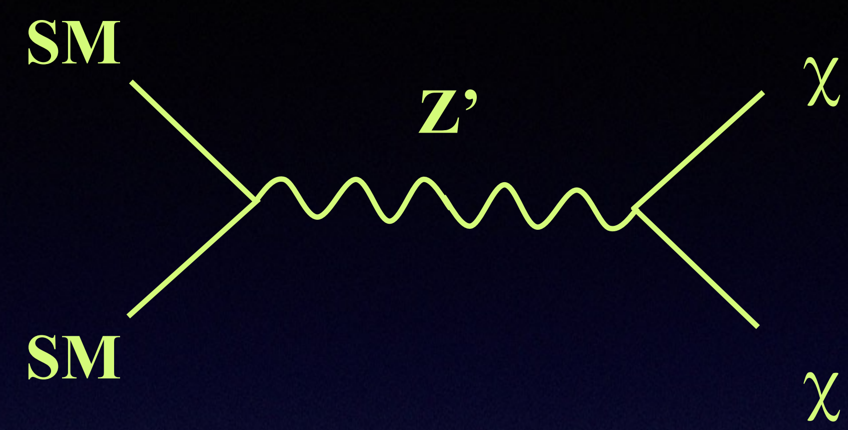
ICECUBE (South Pole)

Very feebly interacting dark matter

(freeze-in mechanism, FIMP)



Scattering process is too weak to reach kinetic equilibrium with the thermal bath

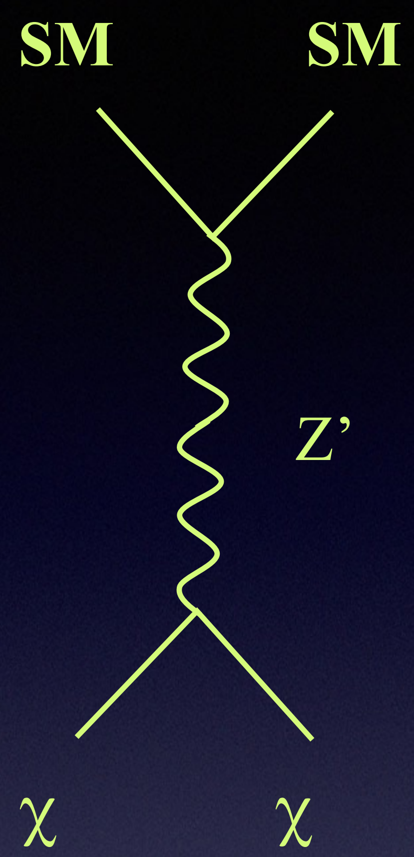


Annihilation is too weak to reach the thermal equilibrium

The dark matter is produced from the thermal bath but at a very slow rate, until the expansion rate dominates the annihilation ($H > \Gamma$)

Very feebly interacting dark matter

(freeze-in mechanism, FIMP)



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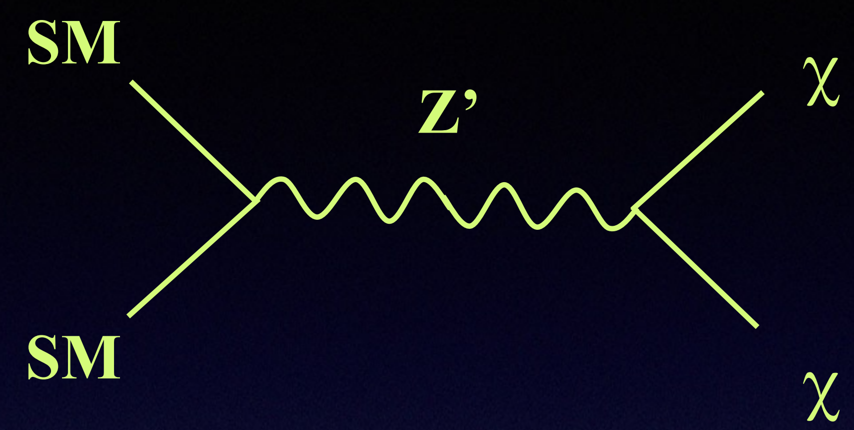

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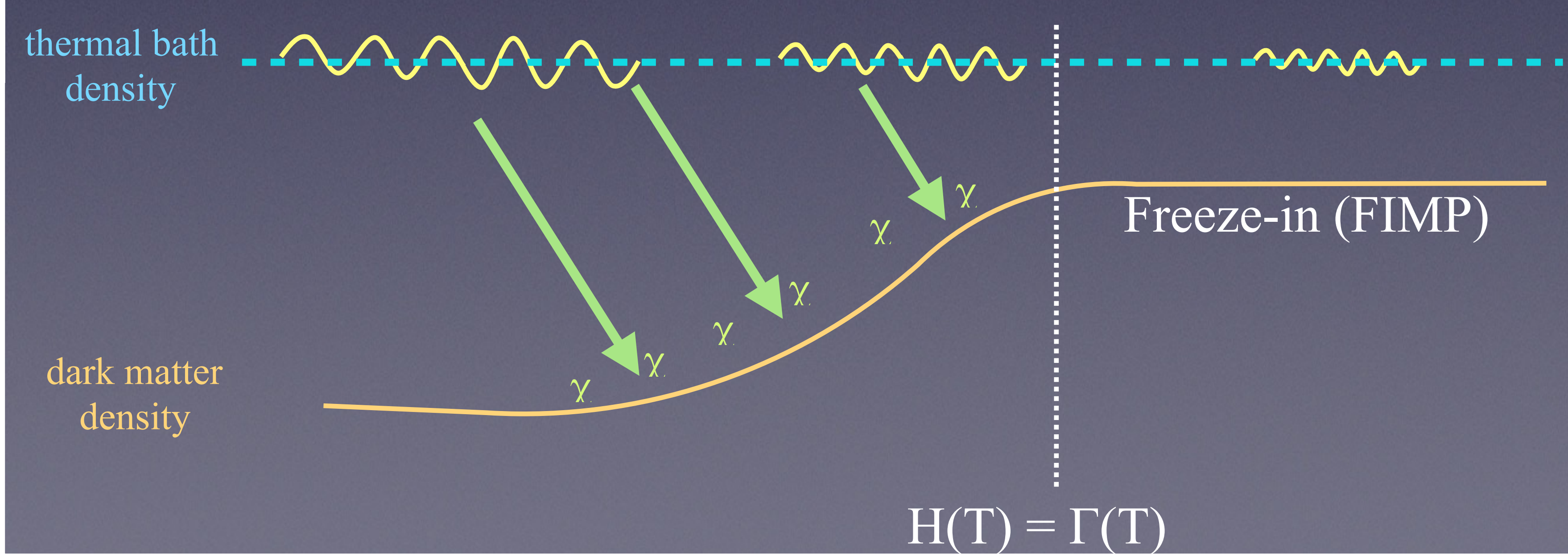
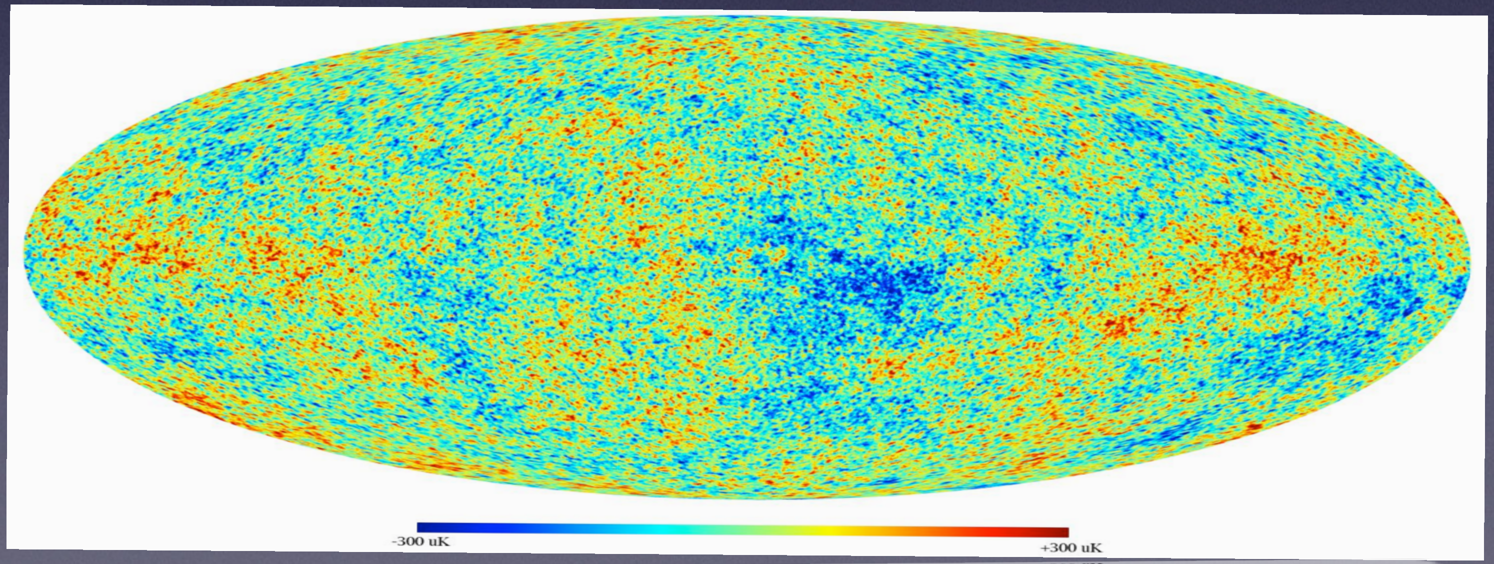
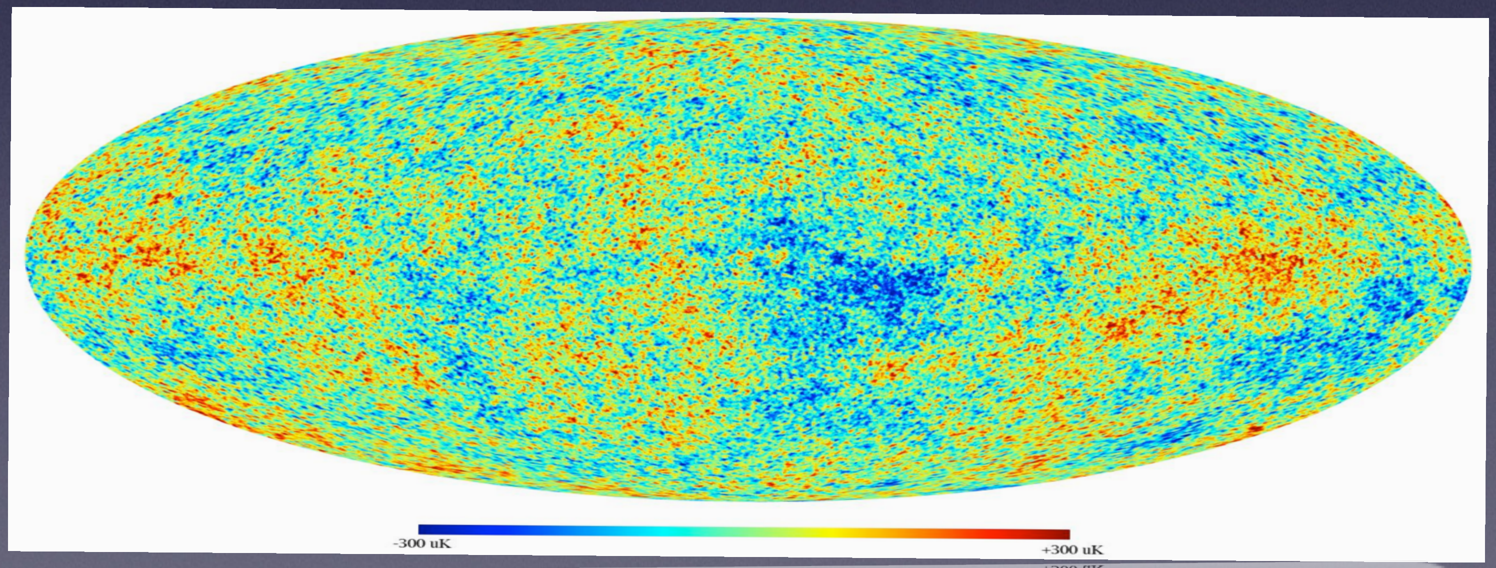


End of the primordial Universe part.



End of the primordial Universe part.

Questions?



Summary on bounds:

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$$\langle \sigma v \rangle = G_F^2 m_\chi^2 > 10^{-9}\ \mathrm{GeV}^{-2} \quad \Rightarrow \quad m_\chi > 2\ \mathrm{GeV}$$

BUT non-valid as soon as we suppose an extra mediator (Z' lighter then the Z for instance => $G'_F > G_F$

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BUT be careful to the extra degrees of freedom (dark radiation)

Unitarity bound

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BUT be careful to the extra degrees of freedom (dark radiation)

Unitarity limit

$$TdS = dU + PdV = \delta Q$$

$$\rho(T) = \frac{\pi^2}{30}T^4 \sum_{i=all\ species} \frac{30}{\pi^2}\left(\frac{T_i}{T}\right)^4 \frac{g_i}{2\pi^2}\int_{x_i}^{\infty}\frac{(\epsilon^2 - x_i^2)^{1/2}\epsilon^2 d\epsilon}{e^{\epsilon-\mu_i/T} \pm 1} = g_\rho(T)\frac{\pi^2}{30}T^4$$

with $x_i = m_i/T$ and

$$g_\rho(T) = \frac{30}{\pi^2}\sum_{i=all\ species}\frac{g_i}{2\pi^2}\left(\int_{x_i}^{\infty}\frac{(\epsilon^2 - x_i^2)^{1/2}\epsilon^2 d\epsilon}{e^{\epsilon-\mu_i/T} \pm 1}\right)\left(\frac{T_i}{T}\right)^4$$

Approximating the Boltzmann function as a step function:

$$g_\rho(T) \simeq \sum_{b=bosons} g_b \left(\frac{T_b}{T}\right)^4 + \frac{7}{8}\sum_{f=fermions} g_f \left(\frac{T_f}{T}\right)^4$$

Non-thermal production through decay

$$TdS = dU + PdV = \delta Q$$

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Non-thermal production and freeze in

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Self interacting dark matter

$$TdS = dU + PdV = \delta Q$$

$$\rho(T) = \frac{\pi^2}{30}T^4 \sum_{i=all\ \ species} \frac{30}{\pi^2}\left(\frac{T_i}{T}\right)^4 \frac{g_i}{2\pi^2}\int_{x_i}^{\infty}\frac{(\epsilon^2 - x_i^2)^{1/2}\epsilon^2 d\epsilon}{e^{\epsilon-\mu_i/T}\pm 1} = g_\rho(T)\frac{\pi^2}{30}T^4$$

with $x_i = m_i/T$ and

$$g_\rho(T) = \frac{30}{\pi^2}\sum_{i=all\ \ species}\frac{g_i}{2\pi^2}\left(\int_{x_i}^{\infty}\frac{(\epsilon^2 - x_i^2)^{1/2}\epsilon^2 d\epsilon}{e^{\epsilon-\mu_i/T}\pm 1}\right)\left(\frac{T_i}{T}\right)^4$$

Approximating the Boltzmann function as a step function:

$$g_\rho(T) \simeq \sum_{b=bosons} g_b\left(\frac{T_b}{T}\right)^4 + \frac{7}{8}\sum_{f=fermions} g_f\left(\frac{T_f}{T}\right)^4$$

The dark radiation

$$TdS = dU + PdV = \delta Q$$

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Formulae Latexit

```
\frac{dN_i}{dt} = -N_i  n_i ~\langle \sigma v
\rangle_{i \rightarrow j}
+ N_j  n_j \langle \sigma v\rangle_{j \rightarrow
i}
\\
\mathrm{and ~ from ~} N_i = n_i\times a^3 ~
\mathrm{one ~ obtains}
\\
\frac{dn_i}{dt} = -3 \frac{\dot a}{a}n_i - n_i n_i
\langle \sigma v \rangle_{i \rightarrow j} + n_j
n_j \langle \sigma v \rangle_{j \rightarrow i}
\\
\frac{dn_i}{dt} = -3 H n_i - \langle \sigma v
\rangle_{i\rightarrow j} \left[ n_i^2 -
(n_i^{EQ})^2\right]
```

Summary

The Universe reheats (waking up)

[WIMPZILLA]

Some dark matter particles leave the table early (breakfast)

[FIMP/E-WIMP]

Some are more talkative and stay until the (lunchtime)

[WIMP]

Some stay for the teatime snack

[Warm dark matter]

In any case, they should be present for the dinner

[Galactic detection]

The meaning of decoupling

The decoupling happens when the expansion rate dominates over the interaction rate:

$$\Gamma_{SM\ SM \to \chi\chi}(T_d) = n(T_d) \times \langle \sigma v \rangle_{T_d} = H(T_d)$$

In one second, the Universe has doubled is sized, but the thermal bath has produced less than 2 particles χ : the χ are diluted. T_d is called the decoupling temperature

$$T_d^3 \times \frac{|\mathcal{M}_{SM \to \chi}(T_d)|^2}{s} \simeq T_d^3 \times \frac{|\mathcal{M}_{SM \to \chi}(T_d)|^2}{T_d^2} = H(T_d) = \frac{T_d^2}{M_{\rm Pl}}$$

Decoupling of the inflaton : T_d=T_{RH} (reheating)

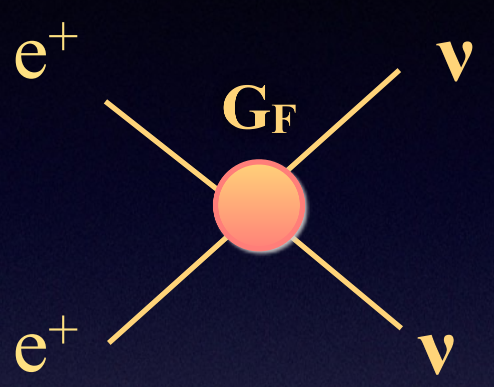
Decoupling of the dark matter: T_d=T_{FO} (freeze-out)

Decoupling of the atoms: T_d = T_{rec} (recombination/CMB)

Two ways of decoupling

$n \times \langle \sigma v \rangle < H$ means n is small <u>OR</u> σv is small.

<u>The massless case (neutrino)</u>



$G_F = 10^{-5}\ \mathrm{GeV}^{-2}$

The decoupling condition

$$n \times \langle \sigma v \rangle \simeq T_\nu^3 \times G_F^2 T_\nu^2 = H(T_\nu) = \frac{T_\nu^2}{M_{\rm Pl}} \Rightarrow T_\nu \simeq \left(G_F^2 M_{\rm Pl}\right)^{-1/3} \simeq 3\ \mathrm{MeV}$$

At 3 MeV, the neutrinos decouple from the thermal bath, but being still relativistic

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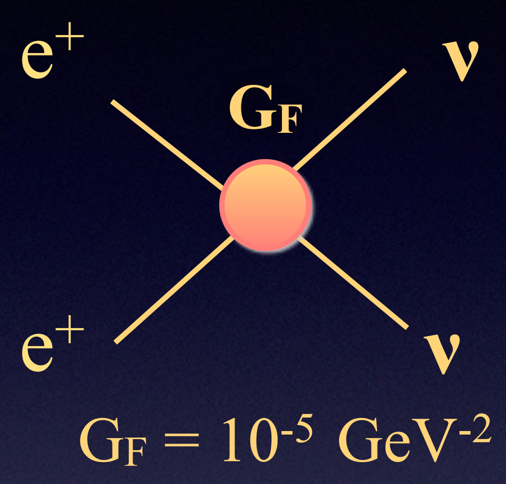

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At 3 MeV, the neutrinos decouple from the thermal bath, but being still relativistic

The massive case (dark matter)

The decoupling condition

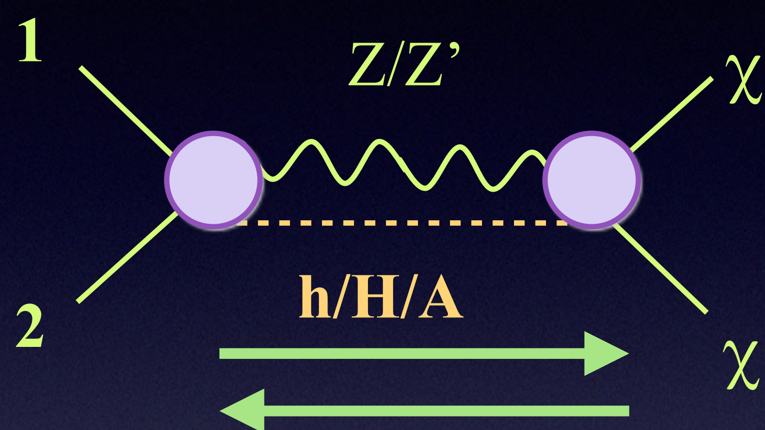
$$n \times \langle \sigma v \rangle \simeq (T_\chi m_\chi)^{3/2} e^{-m_\chi/T_\chi} \times \frac{1}{T_\chi^2} = H(T_\chi) = \frac{T_\chi^2}{M_{\rm Pl}} \Rightarrow \frac{m_\chi}{T_\chi} \simeq 25$$

T_χ corresponds physically to the temperature under which the thermal bath does not have sufficient energy to produce a dark matter particle.

The Boltzman equation

The exact way to solve precisely the problem

(even if the proceedings approximations give in 90% of the case relatively good results)



$$\frac{dn}{dt} = -3Hn - \langle \sigma v \rangle \left(n^2 - n_{eq}^2\right) \quad \Rightarrow \quad \frac{dn}{dT} = 3\frac{n}{T} - \frac{\langle \sigma v \rangle}{HT}\left(n_{eq}^2 - n^2\right)$$

$$\left(\frac{d}{dt} = \frac{dT}{dt}\frac{d}{dT} = -T\frac{da/dt}{a}\frac{d}{dT} = -HT\frac{d}{dT} \quad \rightarrow \quad t \propto \frac{M_{\rm Pl}}{T^2}\right)$$

defining the yield $Y = \dfrac{n}{S}$, with the entropy $S = \dfrac{2\pi^2}{45} g_s T^3$

$$\boxed{\frac{dY_\chi}{dT} = T^2 \frac{\langle \sigma v \rangle_{1,2\rightarrow\chi\chi}}{H(T)}\left(Y_\chi^2 - Y_{eq}^2\right)} \quad \text{with} \quad \langle \sigma v \rangle = \int_{T_{RH}}^{T} \Pi_i \frac{d^3 p_i}{(2\pi)^{3i}} |\mathcal{M}_{1,2\rightarrow\chi\chi}|^2 e^{-E_1/T} e^{-E_2/T} \quad \text{and} \quad H(T) = \frac{T^2}{M_{\rm Pl}}$$

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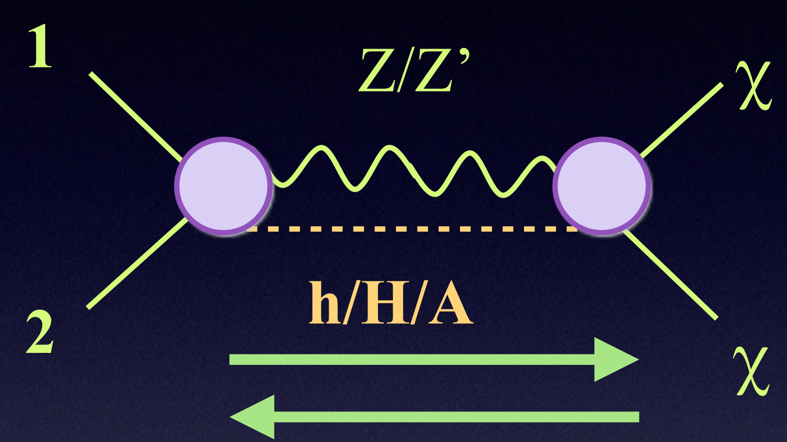

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$$\Omega h^2 \simeq \frac{3\times 10^{-27}\mathrm{cm}^3\mathrm{s}^{-1}}{\langle \sigma v \rangle}, \quad \Omega h^2_{\rm PLANCK} \simeq 0.1 \quad \Rightarrow \quad \langle \sigma v \rangle_{\rm decoupling} \simeq 3\times 10^{-26}\mathrm{cm}^3\mathrm{s}^{-1} \sim 10^{-9}\ \mathrm{GeV}^{-2}$$

$\langle \sigma v \rangle \sim G_{\mathcal{F}}^2$: « *WIMP miracle* »

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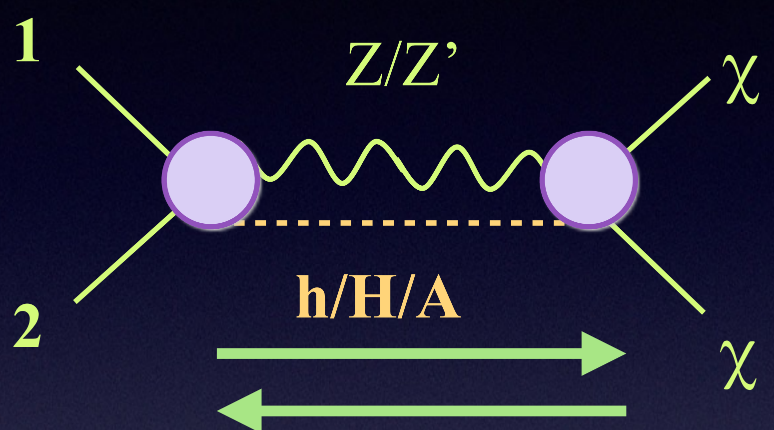

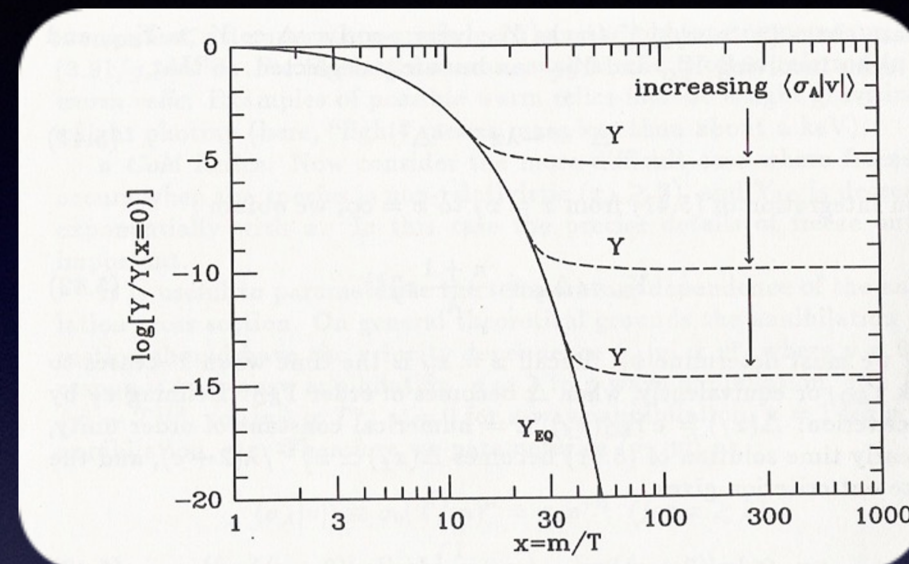

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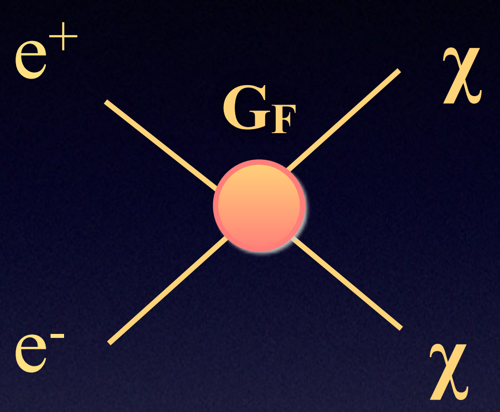
A (too?) simple application

The Lee-Weinberg bound (1977)



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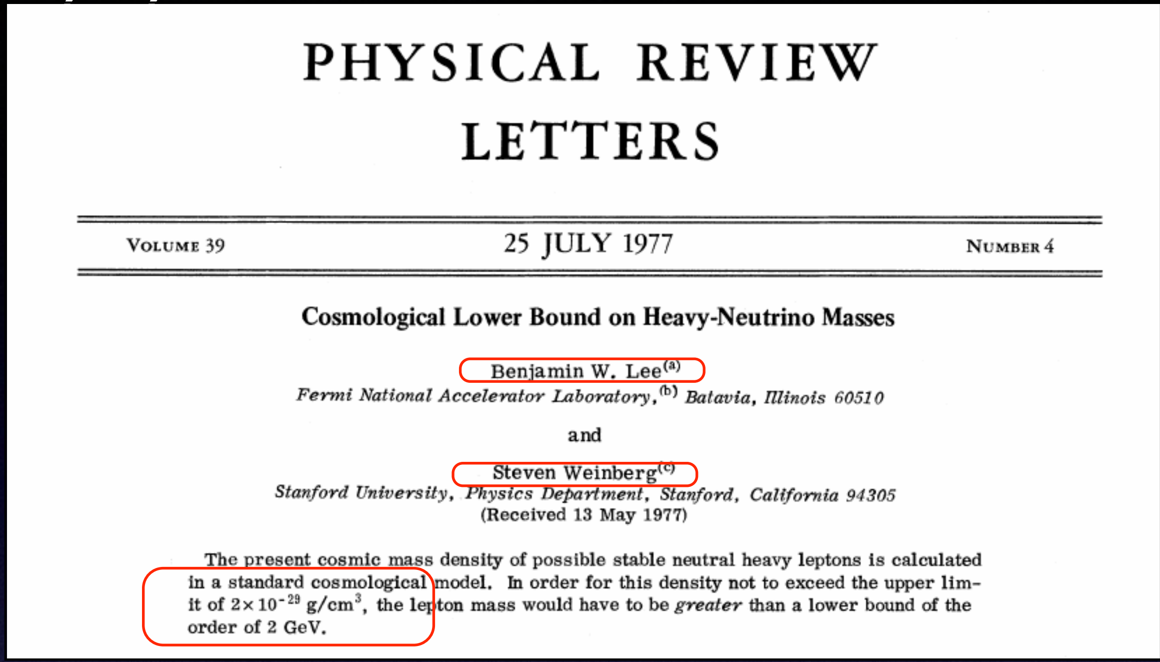


PHYSICAL REVIEW LETTERS

VOLUME 39 — 25 JULY 1977 — NUMBER 4

Cosmological Lower Bound on Heavy-Neutrino Masses

Benjamin W. Lee[(a)]
Fermi National Accelerator Laboratory,[(b)] *Batavia, Illinois 60510*

and

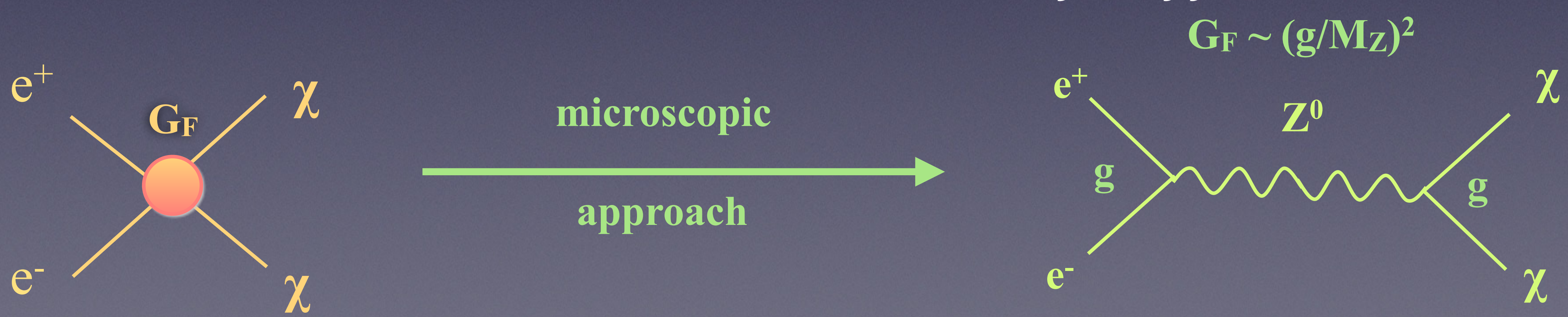
Steven Weinberg[(c)]
Stanford University, Physics Department, Stanford, California 94305

(Received 13 May 1977)

The present cosmic mass density of possible stable neutral heavy leptons is calculated in a standard cosmological model. In order for this density not to exceed the upper limit of 2×10^{-29} g/cm^3, the lepton mass would have to be *greater* than a lower bound of the order of 2 GeV.

$$\langle \sigma v \rangle = G_F^2 m_\chi^2 > 10^{-9}\ \mathrm{GeV}^{-2} \quad \Rightarrow \quad m_\chi > 2\ \mathrm{GeV}$$

Bound no valid in a microscopic approach:



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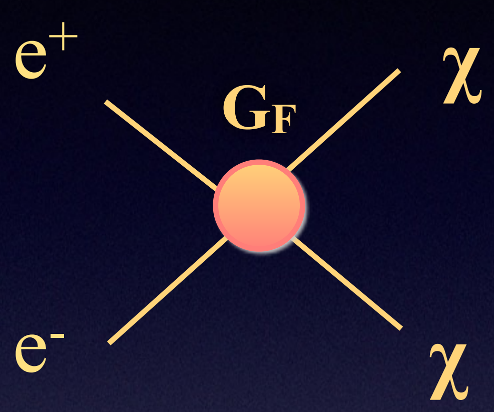

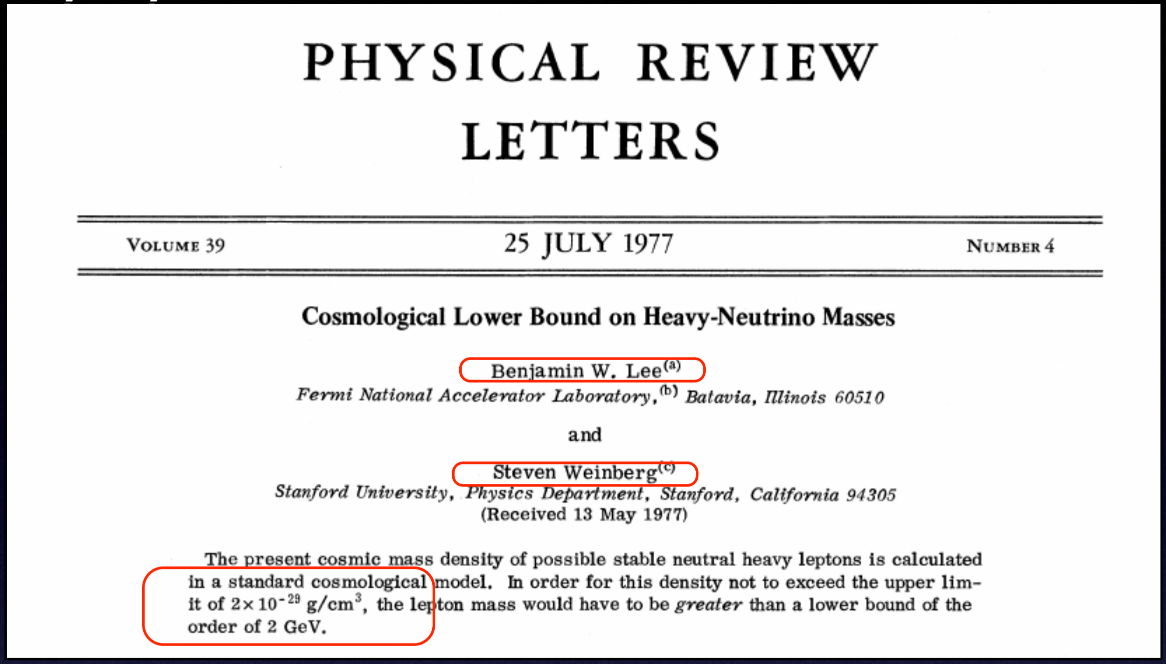

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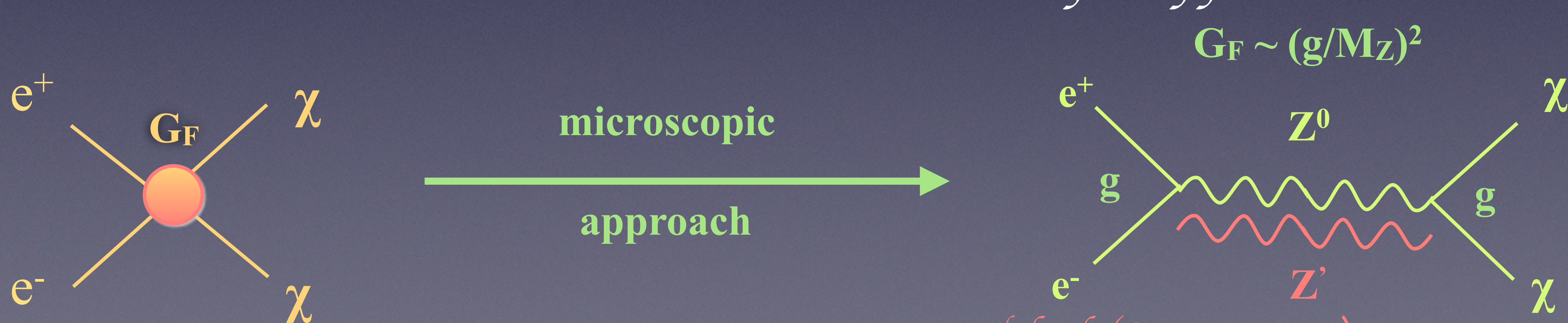

« 5th forth (P. Fayet, 1978)»

Lee-Weinberg bound not valid anymore (possibility for light dark matter)