Higgs -- Theory

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SM Higgs, hierarchy problem, supersymmetry, composite Higgs

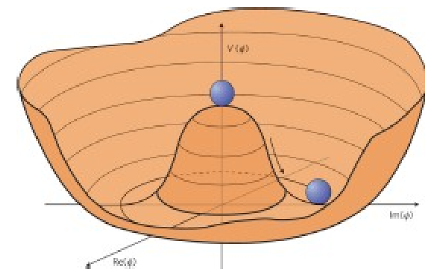
SM Higgs & Custodial Symmetry

SM-like Higgs @ 125 GeV

(and nothing else so far!!)

$$\Phi = \begin{pmatrix} \varphi_+ \\ \varphi_0 \end{pmatrix}$$

$$|D_\mu \Phi|^2 - \frac{1}{2}\lambda^2 \left(|\Phi|^2 - \frac{v^2}{2} \right)^2$$



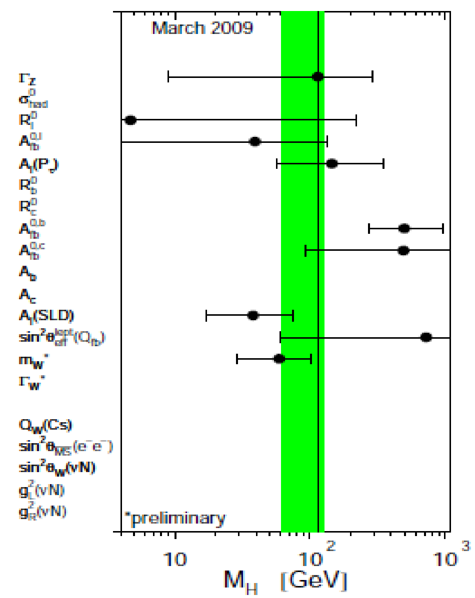
- The SM Higgs is a complex scalar doublet $\Rightarrow$ 4 real fields. Both kinetic and potential terms have $\mathrm{O}(4) = \mathrm{SU}(2) \times \mathrm{SU}(2)$ symmetry.
- One of them is $\mathrm{SU}(2)_\mathrm{L}$. Usually, the other is called $\mathrm{SU}(2)_\mathrm{R}$. Higgs carries a (2,2) rep.
- When Higgs gets a vev, $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \rightarrow \mathrm{SU}(2)_\mathrm{V}$. This is called 'custodial SU(2)'. So what?
- Consider the Lagrangian for gauge bosons after EWSB. $L = \Pi_\pm W^+ W^- + \Pi_{33} W^3 W^3 + \Pi_{3B} W^3 B + \Pi_{BB} BB$ where $\Pi_{ab} \sim < J_a J_b >$. Also $\Pi(p^2) = \Pi(0) + p^2 \Pi'(0)$.
- Each J transforms as (3,1) under $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}$ or as 3 under $\mathrm{SU}(2)_\mathrm{V}$. Recall $3 \times 3 = 1 + 3 + 5$.
  $A_i B_j : A_i B_i + (A_i B_j - A_j B_i) + \frac{1}{2}(A_i B_j + A_j B_i) - \frac{1}{3}(A.B)\delta_{ij}$
- $\Delta\rho = \alpha T \propto (\Pi_\pm(0) - \Pi_{33}(0))$, symmetric under indices and traceless, so transforms as 5 of $\mathrm{SU}(2)_\mathrm{V}$.
- Since $\mathrm{SU}(2)_\mathrm{V}$ is a symmetry of the vacuum, only singlets of $\mathrm{SU}(2)_\mathrm{V}$ can have non-vanishing expectation value. Hence $T = 0$ at leading order.

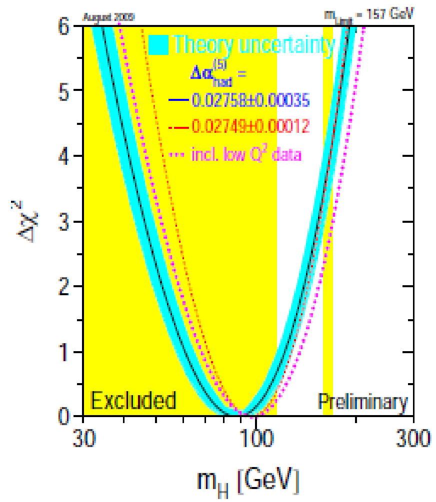
EWPT, Unitarity, Triviality, Stability

Higgs observed in the expected range

EWPT: Higgs mass < 186 GeV @ 95% CL

Non-observation at LEP2 : > 114 GeV.



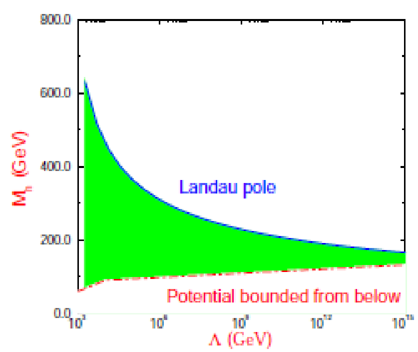


$$\Delta\rho \simeq \frac{3G_F}{8\pi^2\sqrt{2}}\left[m_t^2 - (M_Z^2 - M_W^2)\ln\left(\frac{M_h^2}{M_Z^2}\right)\right]\;,\; S = \frac{1}{6\pi}\ln\left(\frac{M_h}{M_Z}\right)$$

Unitarity: $W_L^+ W_L^- \to W_L^+ W_L^-$ scattering $\rightsquigarrow$

$$a_{(J=0)} = \frac{s}{32\pi v^2} - \frac{s}{32\pi v^2}\left(\frac{s}{s-m_h^2}\right) \to \frac{m_h^2}{32\pi v^2} < \frac{1}{2} \Rightarrow M_h < 780 \text{ GeV.}$$

$$\boxed{d\lambda/dt = (16\pi^2)^{-1}\left[24\lambda^2 + 12\lambda y_t^2 - 6y_t^4 - \ldots\right]}$$



Triviality: Avoid Landau pole ($\lambda^{-1} > 0$) $\rightsquigarrow M_h < 170$ GeV for $\Lambda = 10^{16}$ GeV.

Vacuum stability: V bounded from below ($\lambda > 0$) $\rightsquigarrow M_h > 130$ GeV for $\Lambda = 10^{16}$ GeV.

Hierarchy problem

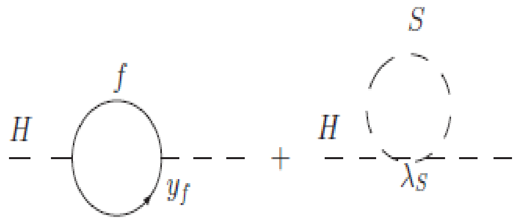
- SM is an `effective theory'. Coefficients of its operators, which we today consider as fundamental input parameters, should actually be derived phenomenological quantities, to be computed one day in terms of more fundamental BSM theory.
- It should be just like Fermi theory of weak interaction.

$$G_F = \frac{g_W^2}{4\sqrt{2}\, m_W^2}$$

Measure weak gauge coupling and W mass, compare with previously measured Fermi coupling. Perfect agreement tells us that we understand the microscopic origin of WI in terms of EW theory.

Now consider the radiative corrections to the Higgs mass.

$\Delta M_h^2(f) = -\frac{y_f^2}{16\pi^2} 2\Lambda^2 \quad ; \quad \Delta M_h^2(S) = \frac{\lambda_S}{16\pi^2}\Lambda^2$. Here Λ is the highest scale of the theory.



Remember, quadratic divergences can be removed by renormalization but `finite' corrections arising from states at cut-off remain.

Hierarchy problem – Salient features

- Physics at several orders of magnitude shorter distances is affecting weak scale dynamics. This is against the concept of effective theory description.
- Even if parameters are adjusted for cancellation in a given order (see different signs for fermion and boson loops), it is offset in the next order, so tuning needed order by order in perturbation theory.
- Since Higgs mass is not protected, the order parameter (vev) is also not protected, which destabilizes the entire theory.
- Basically, d < 4 term is not protected $c\,\Lambda_{\rm SM}^2 H^\dagger H$
- $$m_H^2 = \int_0^\infty dE\,\frac{dm_H^2}{dE} = \int_0^{\leq\Lambda_{SM}} \ldots + \int_{\leq\Lambda_{SM}}^\infty \ldots = (\delta m_H^2)_{SM} + (\delta m_H^2)_{BSM}$$

  The <u>2nd term</u> is completely unknown!
- We will never be able to calculate the Higgs mass in terms of the microscopic theory parameters if the cutoff is much above the TeV scale.

Role of symmetry

- We know of mechanisms by which particles can stay light in the presence of heavier particles.
- Chiral symmetry keeps the electron mass under control => $m\ q\ q^c$

  Suppose `m' is a field (spurion). Assign a charge such that the chiral symmetry is restored.

$$(q, q^c) \to e^{i\alpha}(q, q^c) \Rightarrow m \to e^{-2i\alpha} m$$

- Now consider the quantum correction to the electron mass, and convince yourself that the correction to the mass transforms under chiral symmetry exactly the same way as the mass itself.

  This tells us that the correction is small when mass is small. $\delta m \ = \ m\ f\left(|m^2|\right)$

- In the limit of m --> 0 we have an enhanced symmetry, the `chiral symmetry'. Remember, gauge symmetry ensures that photon mass remains zero.

- Absence of symmetry protecting Higgs mass is due to its spin-zero nature. Massless particles of spin-half or higher have 2 d.o.f, while massive ones of spin-half or higher have more than 2 d.o.f. We have to associate the Higgs with some `symmetry'.

Supersymmetry -- tension!

Tie the scalar mass to the mass of a fermion. ***Quadratic divergence cancels between loops containing different spin states.***

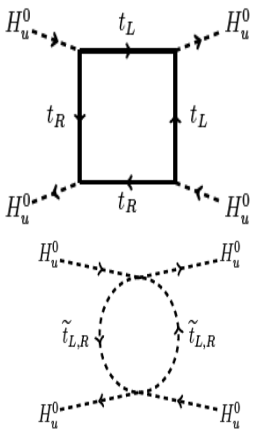
$$0.5\, M_Z^2 \simeq -|\mu|^2 - M_{H_u}^2 \simeq -|\mu^2| + \mathcal{O}(1)\, m_{\tilde{t}}^2$$

$m_h \simeq 125$ GeV $\Rightarrow m_{\tilde{t}} \sim$ few TeV $\Rightarrow$ large cancellation $\Rightarrow$ little hierarchy problem.

Quartic coupl related to gauge coupls $\Rightarrow M_h$ predictive

$$m_h^2 \simeq M_Z^2 \cos^2 2\beta + \frac{3 m_t^4}{\sqrt{2}\pi^2 v^2} \ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right)$$

Relation valid irrespective of SUSY breaking mechanism.
$m_h \simeq 125$ GeV requires the stop mass in TeV range.



For small mixing, stop > 10 TeV (1 loop), (5-10) TeV (2 loop), 3 TeV (leading 3 loop).

For stop : (1 – 10) TeV: change in Higgs mass: (18 – 31) GeV at 1 loop, (4 – 7) GeV at 2 loop, (0.5 – 3) GeV at 3 loop

Composite Higgs

- *What if the Higgs is a pseudo-Goldstone boson?*
- There is a large hierarchy in physics that is well understood, which is between the mass of proton and higher scale. QCD coupling starts off small at UV and blows up at IR creating a low physical scale by `dimensional transmutation'. $\Lambda^2 = M_{\rm UV}^2 \, e^{-4\pi/g_0^2}$
- Breaking of `chiral symmetry' would have led to a perfect description of EW breaking without any Higgs, <u>only that W-mass would be 29 MeV!</u> $m_W = \frac{g f_\pi}{2}$

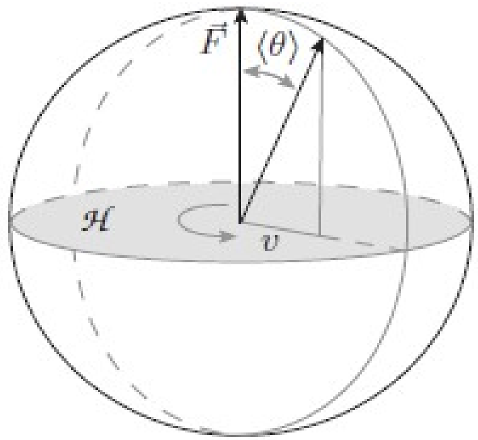
QCD coupling becomes strong, $SU(2)_L \times SU(2)_R \times U(1)_B$ gets spontaneously broken to $SU(2)_V \times U(1)_B$, resulting in 3 massless Goldstone bosons (the 3 pions of QCD). But these pions get eaten in the usual way by the EW gauge fields, since the EW gauge symmetry is broken from $SU(2)_L \times U(1)_Y$ to $U(1)_Q$, where $Q = T_L^3 + T_R^3 + B/2 = T_L^3 + Y$.

- **<u>Technicolor comes to the rescue!</u>** Another strongly coupled gauge group through slow running of coupling constant creates TeV scale.

Technicolor to Composite Higgs

- Technicolor was dead long before Higgs was discovered! It gives unacceptably large contribution to S-parameter.
- The dim-6 operator is $H^{\dagger}W_{\mu\nu}B^{\mu\nu}H \Rightarrow S \sim v^2/f_{\pi}^2 \sim 1$
- Construct a theory in which v << f ==> **Composite Higgs**
- Unlike Technicolor in which strong sector directly participates in EWSB, the role of strong sector here is to provide a set of pseudo-NGBs.

<u>TOY MODEL</u>



- G = SO(3) to H = SO(2) spontaneous breaking from **`vacuum misalignment'**. $\vec{\Phi}(x) = e^{i\theta^{\hat{a}}(x)\hat{T}^{\hat{a}}}\vec{F}$
- Due to `explicit' breaking of G, the pions develop a potential (`**shift symmetry**' violated).
- The pion vev (v) triggers H breaking (like EWSB).

$$\xi \equiv \frac{v^2}{f^2} = \sin^2\langle\theta\rangle \ll 1$$

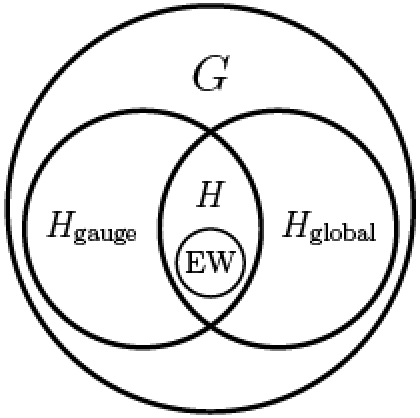
Minimal Composite Higgs

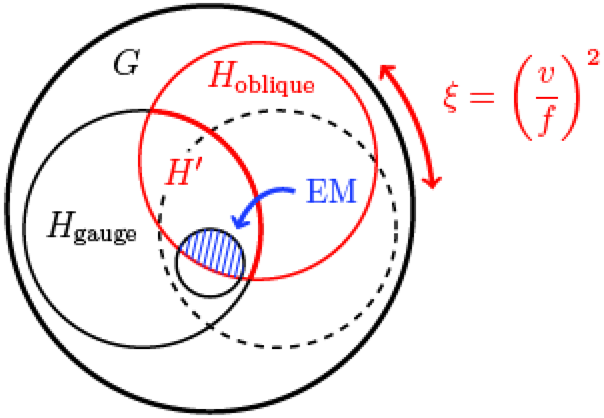
- G = SO(5), H = SO(4) (`custodial' symmetry protected).
- SSB triggered by strong dynamics => 4 NGBs.
- Gauge SU(2) X U(1) part of SO(4). Hypercharge is identified with third comp of SU(2)-Right. This explicity breaks G. $D_\mu \vec{\Phi} = \left(\partial_\mu - i\, g W_\mu^\alpha T_L^\alpha - i\, g' B_\mu T_R^3\right) \vec{\Phi}$

$$\vec{\Phi} = e^{i\frac{\sqrt{2}}{f}\Pi_i(x)\hat{T}^i} \begin{bmatrix} \vec{0} \\ f+\sigma(x) \end{bmatrix}$$

$$H = (h_u, h_d)^T$$

$$\vec{\Pi} = \begin{bmatrix} \Pi_1 \\ \Pi_2 \\ \Pi_3 \\ \Pi_4 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} -i\,(h_u - h_u^\dagger) \\ h_u + h_u^\dagger \\ i\,(h_d - h_d^\dagger) \\ h_d + h_d^\dagger \end{bmatrix}$$



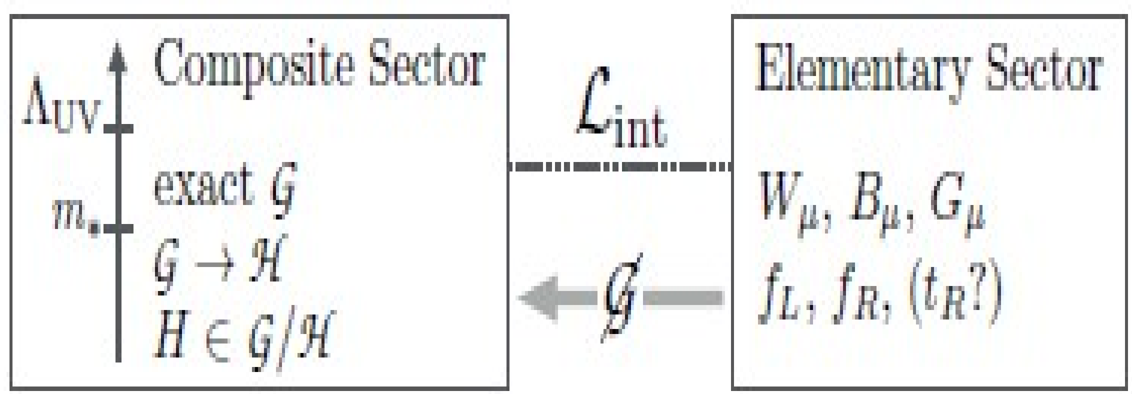


$$\frac{g^2 v^2}{4}\left(|W|^2 + \frac{1}{2c_w^2} Z^2\right)\left[2\sqrt{1-\xi}\,\frac{h}{v} + (1-2\xi)\,\frac{h^2}{v^2} - \frac{4}{3}\xi\sqrt{1-\xi}\,\frac{h^3}{v^3} + \ldots\right]$$

LHC Higgs BR measurements

Broad perspective of composite Higgs

- Holographic description: 4d strongly coupled <=> 5d (AdS) weakly coupled. Higgs mass (and potential) `finite' due non-locality of extra dimension.
- Gauge-Higgs Unification: 5th comp. of gauge field in warped scenario can be the Higgs. Massless at tree level, acquires finite mass at one loop.



$\boxed{\text{SM}} \xleftarrow{0\leftarrow\xi} \boxed{\text{Composite}} \xrightarrow{\xi\to 1} \boxed{\text{Higgsless/TC RIP!}}$

- Issues: (1) Explicit realization of G to H breaking, (2) S parameter (in IR), (3) Flavor

G --> H for Lie groups

$\mathcal{G}$	$\mathcal{H}$	C	N_G	$\mathbf{r}_{\mathcal{H}} = \mathbf{r}_{\mathrm{SU(2)\times SU(2)}}\ (\mathbf{r}_{\mathrm{SU(2)\times U(1)}})$
SO(5)	SO(4)	✓	4	$\mathbf{4} = (\mathbf{2},\mathbf{2})$
SU(3) × U(1)	SU(2) × U(1)		5	$\mathbf{2}_{\pm\mathbf{1/2}} + \mathbf{1}_{\mathbf{0}}$
SU(4)	Sp(4)	✓	5	$\mathbf{5} = (\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$
SU(4)	$[\mathrm{SU}(2)]^2 \times \mathrm{U}(1)$	✓*	8	$(\mathbf{2},\mathbf{2})_{\pm\mathbf{2}} = 2 \cdot (\mathbf{2},\mathbf{2})$
SO(7)	SO(6)	✓	6	$\mathbf{6} = 2 \cdot (\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$
SO(7)	G_2	✓*	7	$\mathbf{7} = (\mathbf{1},\mathbf{3}) + (\mathbf{2},\mathbf{2})$
SO(7)	SO(5) × U(1)	✓*	10	$\mathbf{10}_{\mathbf{0}} = (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3}) + (\mathbf{2},\mathbf{2})$
SO(7)	$[\mathrm{SU}(2)]^3$	✓*	12	$(\mathbf{2},\mathbf{2},\mathbf{3}) = 3 \cdot (\mathbf{2},\mathbf{2})$
Sp(6)	Sp(4) × SU(2)	✓	8	$(\mathbf{4},\mathbf{2}) = 2 \cdot (\mathbf{2},\mathbf{2})$
SU(5)	SU(4) × U(1)	✓*	8	$\mathbf{4}_{-\mathbf{5}} + \bar{\mathbf{4}}_{+\mathbf{5}} = 2 \cdot (\mathbf{2},\mathbf{2})$
SU(5)	SO(5)	✓*	14	$\mathbf{14} = (\mathbf{3},\mathbf{3}) + (\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{1})$
SO(8)	SO(7)	✓	7	$\mathbf{7} = 3 \cdot (\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$
SO(9)	SO(8)	✓	8	$\mathbf{8} = 2 \cdot (\mathbf{2},\mathbf{2})$
SO(9)	SO(5) × SO(4)	✓*	20	$(\mathbf{5},\mathbf{4}) = (\mathbf{2},\mathbf{2}) + (\mathbf{1}+\mathbf{3},\mathbf{1}+\mathbf{3})$
$[\mathrm{SU}(3)]^2$	SU(3)		8	$\mathbf{8} = \mathbf{1}_{\mathbf{0}} + \mathbf{2}_{\pm\mathbf{1/2}} + \mathbf{3}_{\mathbf{0}}$
$[\mathrm{SO}(5)]^2$	SO(5)	✓*	10	$\mathbf{10} = (\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{1}) + (\mathbf{2},\mathbf{2})$
SU(4) × U(1)	SU(3) × U(1)		7	$\mathbf{3}_{-\mathbf{1/3}} + \bar{\mathbf{3}}_{+\mathbf{1/3}} + \mathbf{1}_{\mathbf{0}} = 3 \cdot \mathbf{1}_{\mathbf{0}} + \mathbf{2}_{\pm\mathbf{1/2}}$
SU(6)	Sp(6)	✓*	14	$\mathbf{14} = 2 \cdot (\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{3}) + 3 \cdot (\mathbf{1},\mathbf{1})$
$[\mathrm{SO}(6)]^2$	SO(6)	✓*	15	$\mathbf{15} = (\mathbf{1},\mathbf{1}) + 2 \cdot (\mathbf{2},\mathbf{2}) + (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3})$

$$V(h) = \frac{g_{\mathrm{SM}}^2 M^2}{16\pi^2}\left(-ah^2 + \frac{b}{2f^2}h^4\right)$$

Conclusions and outlook

- Before the discovery of the `Higgs boson', `Higgs mechanism' was working too well. EWPT at LEP established the theory of p-NGB's eaten by W and Z boson on a firm footing – the theory of non-linear sigma model. Those measurements were largely insensitive to the presence of the Higgs boson which UV completes the theory.
- Higgs `elementary' or `composite'? Measurements of Higgs BR essential.
- `Supersymmetry' & `Extra Dimension / Composite Higgs' leading candidates! SUSY is also an extra-d theory in fermionic coordinates!
- `Little Higgs' was an early incarnation of `composite Higgs'. `Collective symmetry breaking' keeps the Higgs mass light. It is also the reason behind cancellation of quadratic divergence between loops containing same spin particles.

<u>TWO OTHER TYPES</u>

- `Dilatonic Higgs' arises from spontaneous breaking of scale invariance. This is a holographic dual to `radion' in RS scenario, which is a field whose vev sets the distance between UV and IR branes.
- In `Twin Higgs' scenario, the Higgs potential is protected by a discrete symmetry.

<u>MESSAGE</u>

Wait for more data from LHC, super-LHC, ILC and cross-check with flavor data from Belle, BaBar...