See also arXiv:1607.01495

Reaching higher gradients in bulk Nb with nano-layer coating

KEK (High Energy Accelerator Research Organization), and
The Graduate University for Advanced Studies, SOKENDAI

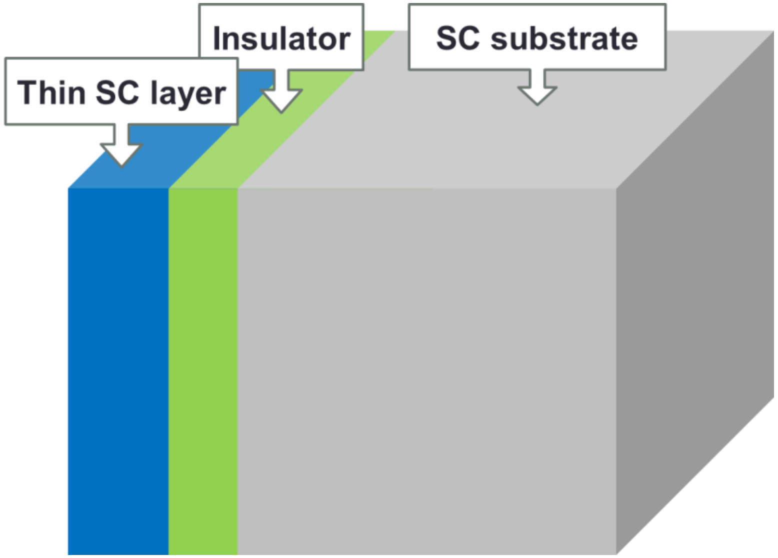
Takayuki Kubo

http://researchmap.jp/kubotaka/

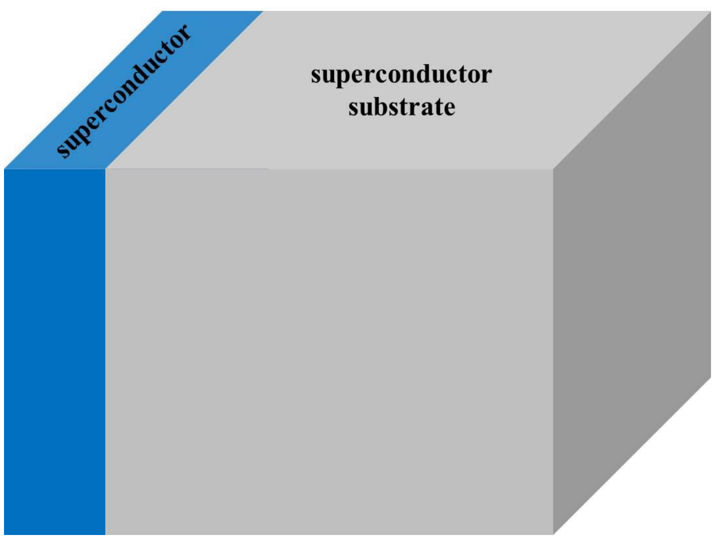
This work was supported by Japan Society for the Promotion of Science KAKENHI:
Grant-in-Aid for Young Scientists (B), Number 26800157,
Grant-in-Aid for Challenging Exploratory Research, Number 26600142,
and Photon and Quantum Basic Research Coordinated Development Program from the Ministry of Education, Culture, Sports, Science and Technology, Japan.

See today's arXiv

arXiv:1607.01495



- In the multilayer approach for high gradients, the insulator layer is the **essential constituent**, which stop penetration of vortices and suppress vortex dissipation.

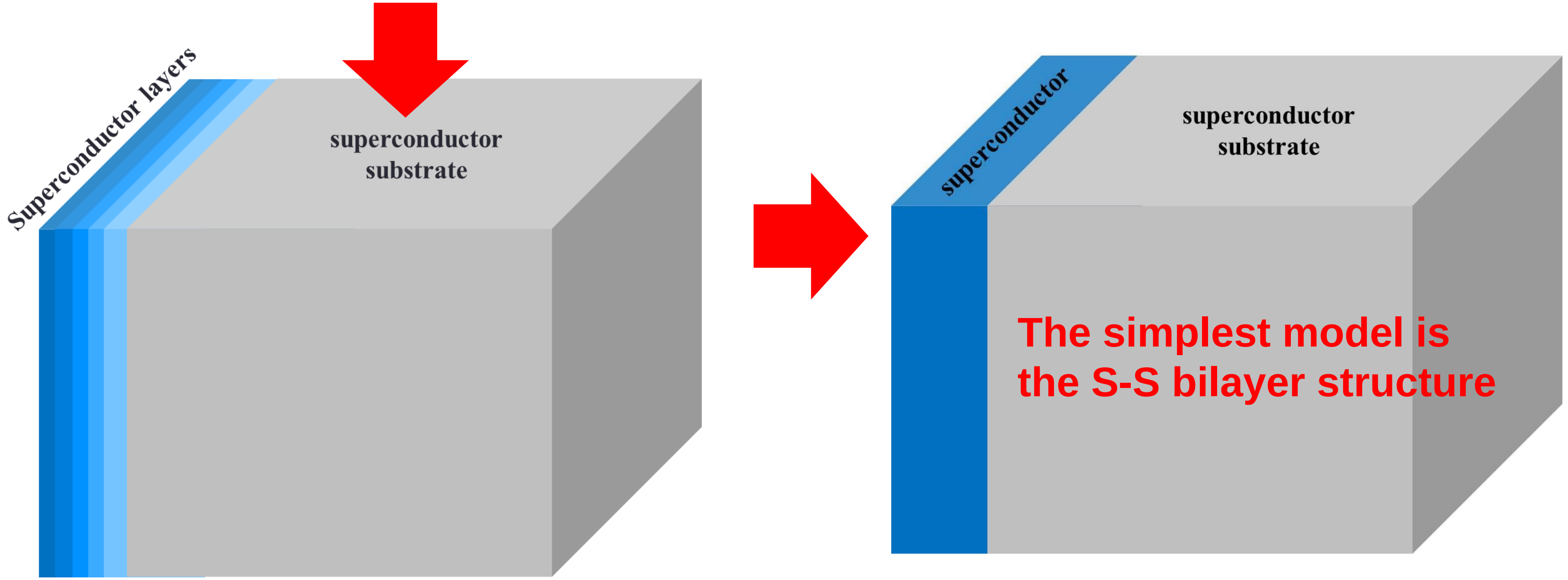


- In this talk, however, I will talk about the multilayer structure without insulator layers. This system is **also interesting and worth studying** because of the following reasons.

First, **it can be regarded as a model of the surface of a Nb**

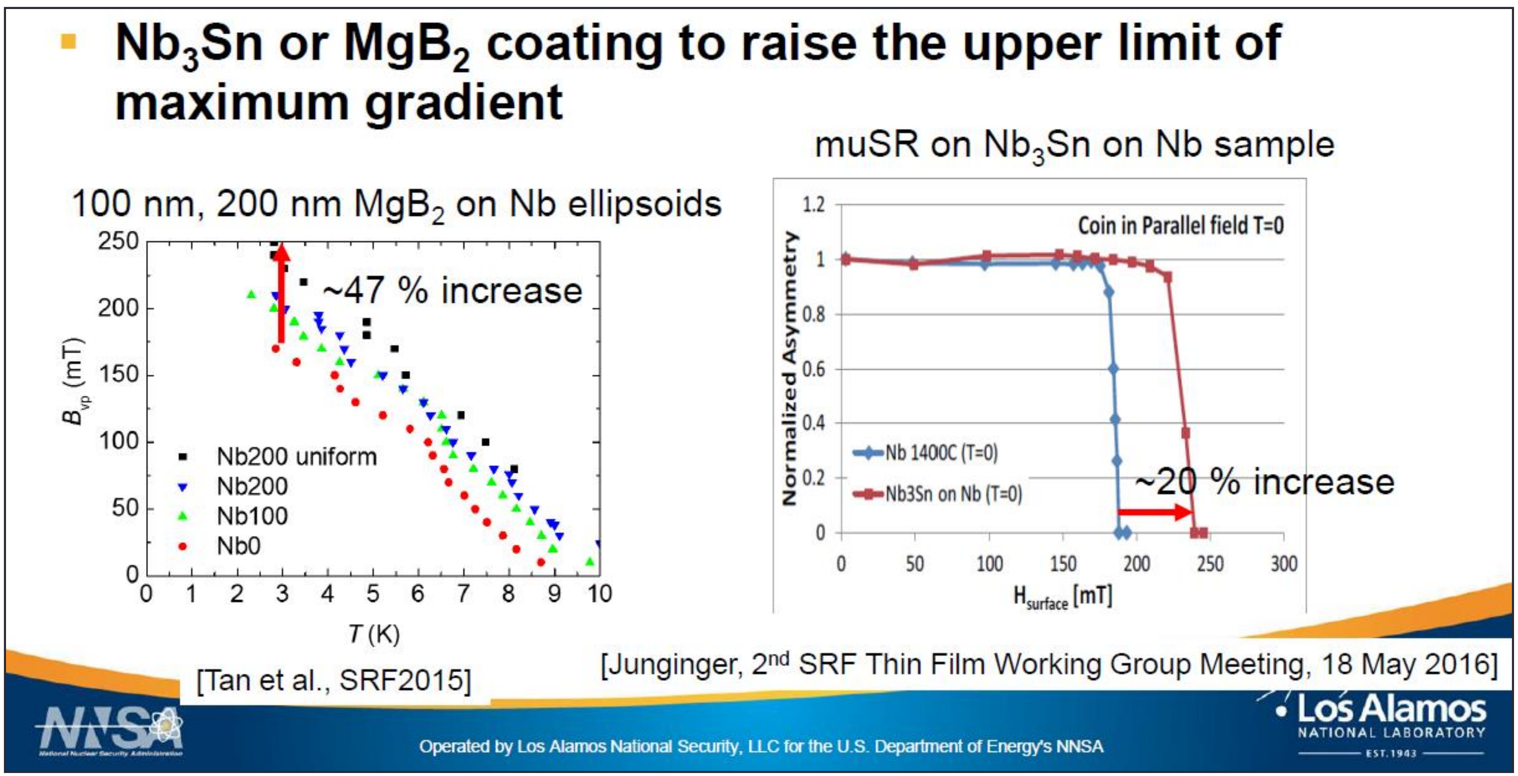
The Nb surface after the low temperature baking has a depth dependent mean-free path, which can be described by an infinite number of thin superconductors continuously piled up on a substrate.

- T. Kubo**,** in proceedings of LINAC14, Geneva, Switzerland (2014), p. 1026, THPP074,
- The discussion section of T. Kubo, Progress of Theoretical and Experimental Physics **2015**, 063G01 (2015)
- M. Checcin, in proceedings of IPAC16, Busan, Korea (2016)



Second, some researchers have made S-S bilayer structures such as MgB_2-Nb and Nb_3Sn-Nb and have carried out sample testing, which should be understood from theoretical view point.

Theoretically, this had already been studied in [T.Kubo, in proceedings of LINAC14, Geneva, Switzerland (2014), p. 1026, THPP074], which had already been applied to a discussion of MgB_2-Nb system. [see a figure in p.16 of the slide of T. Tan in *SRF2015,* TUBA06].



T. Tajima

Contents

Some features of the S-S bilayer structure had already been investigated through the studies of the S-I-S structure.

Contents

Some features of the S-S bilayer structure had already been investigated through the studies of the S-I-S structure.

① **The theoretical field limit** is given by the same formula as the S-I-S. Note that this fact does not necessarily mean that we can achieve it. We need a gimmick to stop vortex penetration.

- T. Kubo**,** in proceedings of LINAC14, Geneva, Switzerland (2014), p. 1026, THPP074
- A. Gurevich, AIP Advance **5**, 017112 (2015) [see 4th paragraph in p.4]

Contents

Some features of the S-S bilayer structure had already been investigated through the studies of the S-I-S structure.

① **The theoretical field limit** is given by the same formula as the S-I-S. Note that this fact does not necessarily mean that we can achieve it. We need a gimmick to stop vortex penetration.

- T. Kubo, in proceedings of LINAC14, Geneva, Switzerland (2014), p. 1026, THPP074
- A. Gurevich, AIP Advance **5**, 017112 (2015) [see 4th paragraph in p.4]

② **Such a gimmick** exists in the S-S bilayer? For the case of S-I-S, the insulator layer is the robust instrument to stop vortex penetration and suppress the vortex dissipation. In the S-S bilayer, the S-S boundary may play a role of a trap to prevent vortex penetration.

- T. Kubo, in proceedings of LINAC14, Geneva, Switzerland (2014), p. 1026, THPP074

Contents

Some features of the S-S bilayer structure had already been investigated through the studies of the S-I-S structure.

① **The theoretical field limit** is given by the same formula as the S-I-S. Note that this fact does not necessarily mean that we can achieve it. We need a gimmick to stop vortex penetration.

- T. Kubo, in proceedings of LINAC14, Geneva, Switzerland (2014), p. 1026, THPP074
- A. Gurevich, AIP Advance **5**, 017112 (2015) [see 4th paragraph in p.4]

② **Such a gimmick** exists in the S-S bilayer? For the case of S-I-S, the insulator layer is the robust instrument to stop vortex penetration and suppress the vortex dissipation. In the S-S bilayer, the S-S boundary may play a role of a trap to prevent vortex penetration.

- T. Kubo, in proceedings of LINAC14, Geneva, Switzerland (2014), p. 1026, THPP074

③ **The surface resistance** is given by the similar formula as the S-I-S structure.

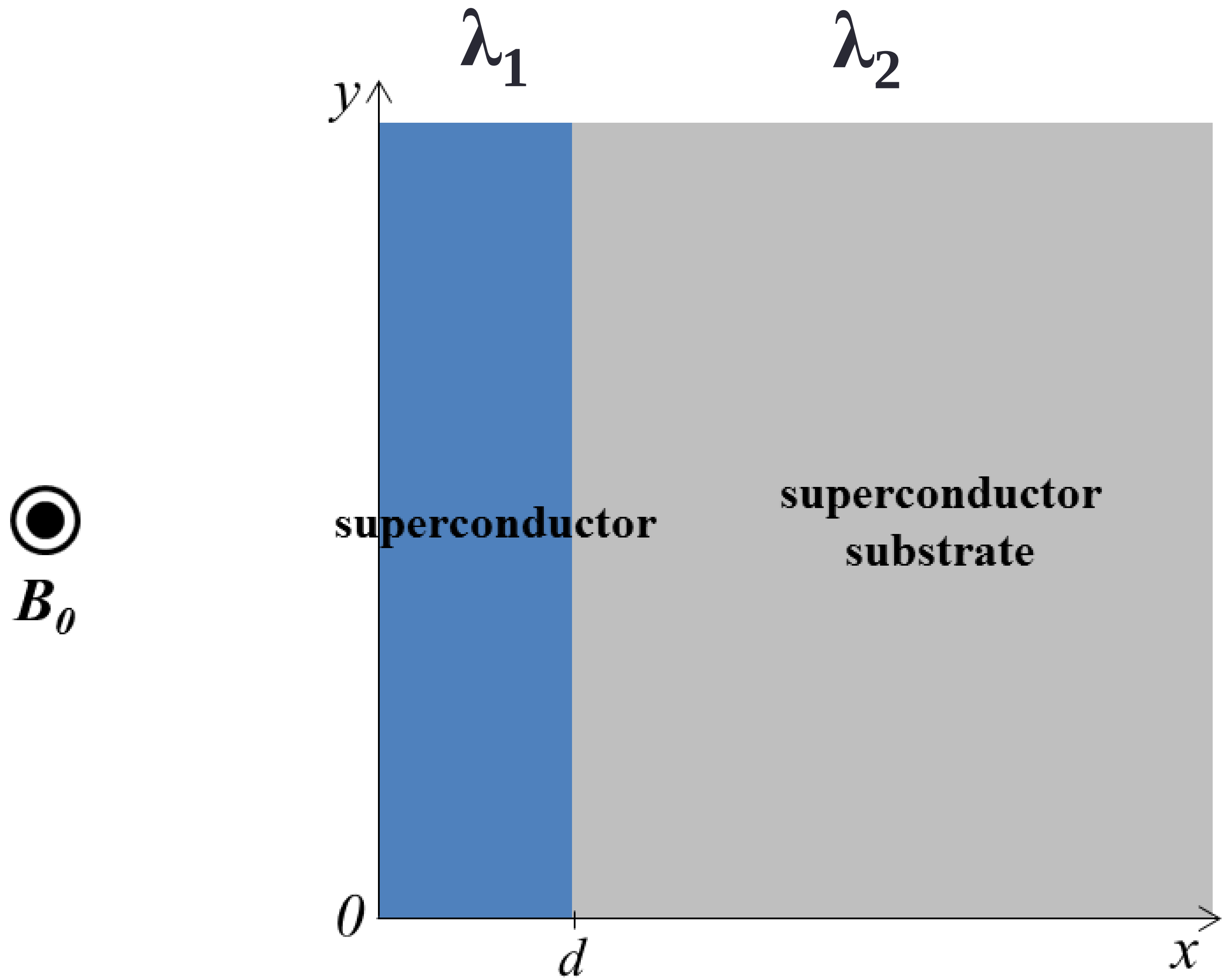
- A. Gurevich, AIP Advance **5**, 017112 (2015)
- T. Kubo, arXiv:1607.01495 [physics.acc-ph]

①Theoretical field limit of the S-S bilayer structure

This topic is studied in

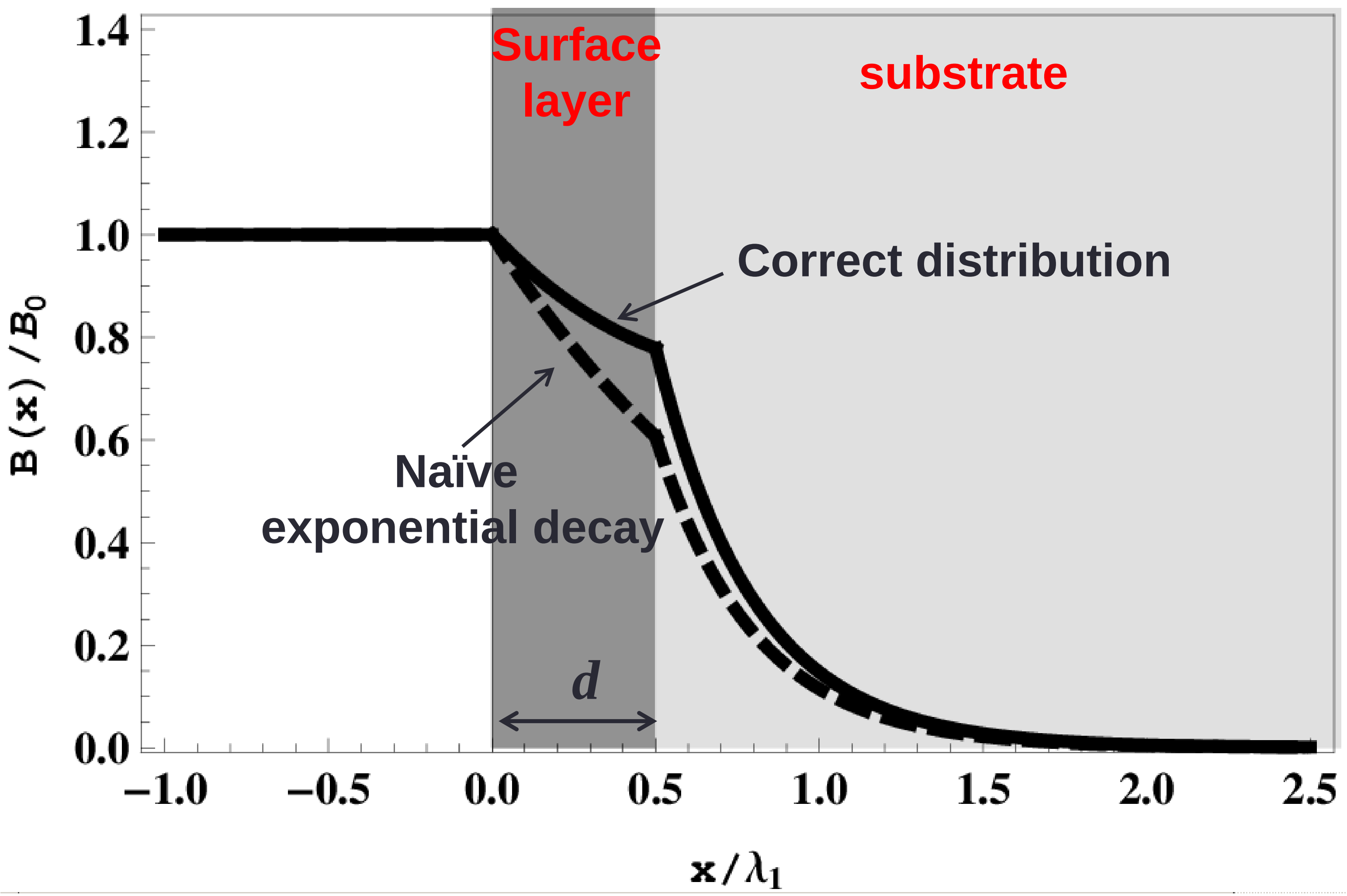
- T. Kubo, in proceedings of LINAC14, Geneva, Switzerland (2014), p. 1026, THPP074
- A. Gurevich, AIP Advance **5**, 017112 (2015) [see 4th paragraph in p.4]

Review of the theoretical field limit of "S-I-S" is given in the presentation file of [T. Kubo, SRF2015, Whistler, Canada (2015), TUBA07], which is also useful to understand that of the S-S bilayer.

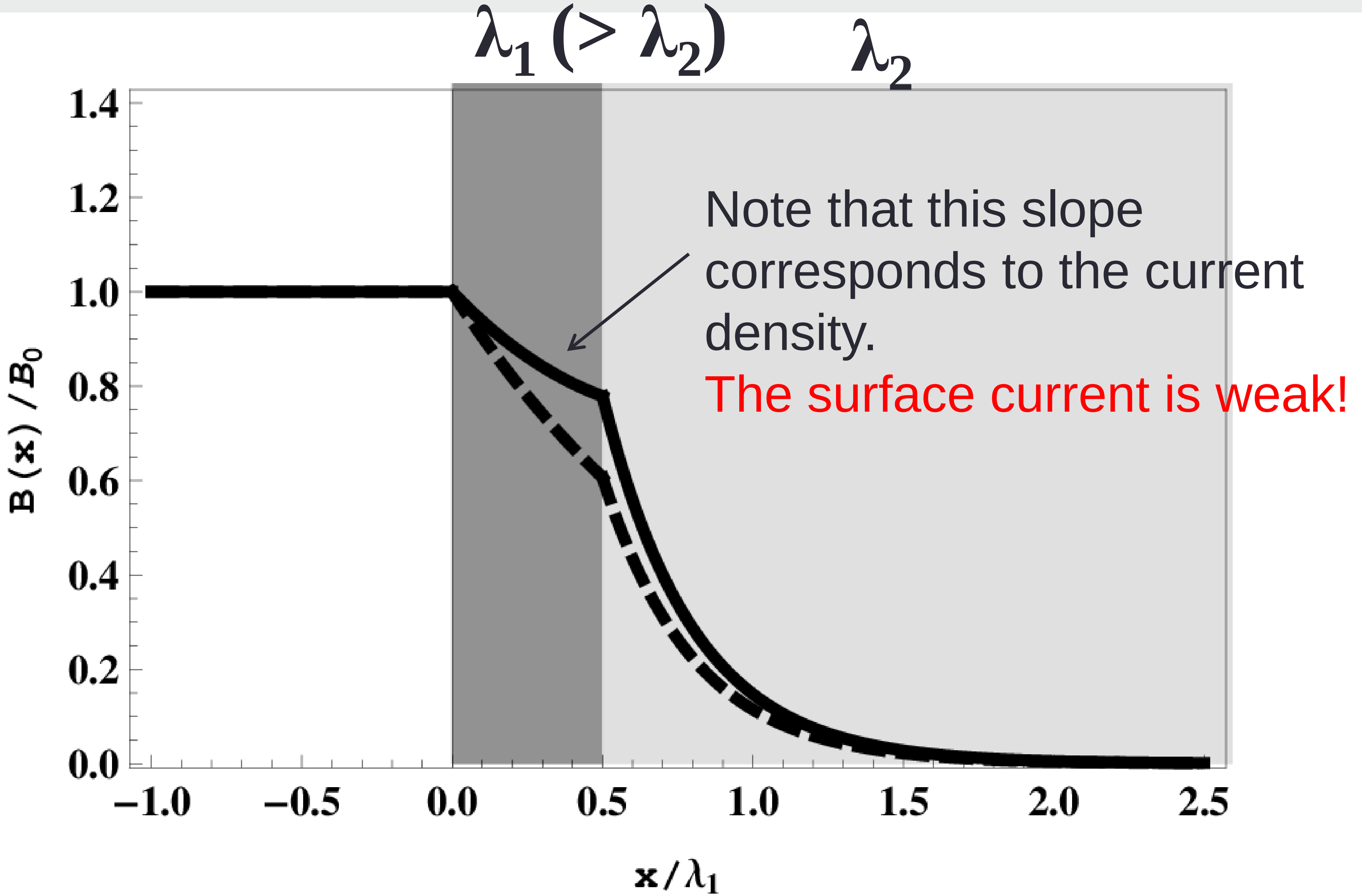


λ1
λ2
y
superconductor
superconductor substrate
B0
0
d
x

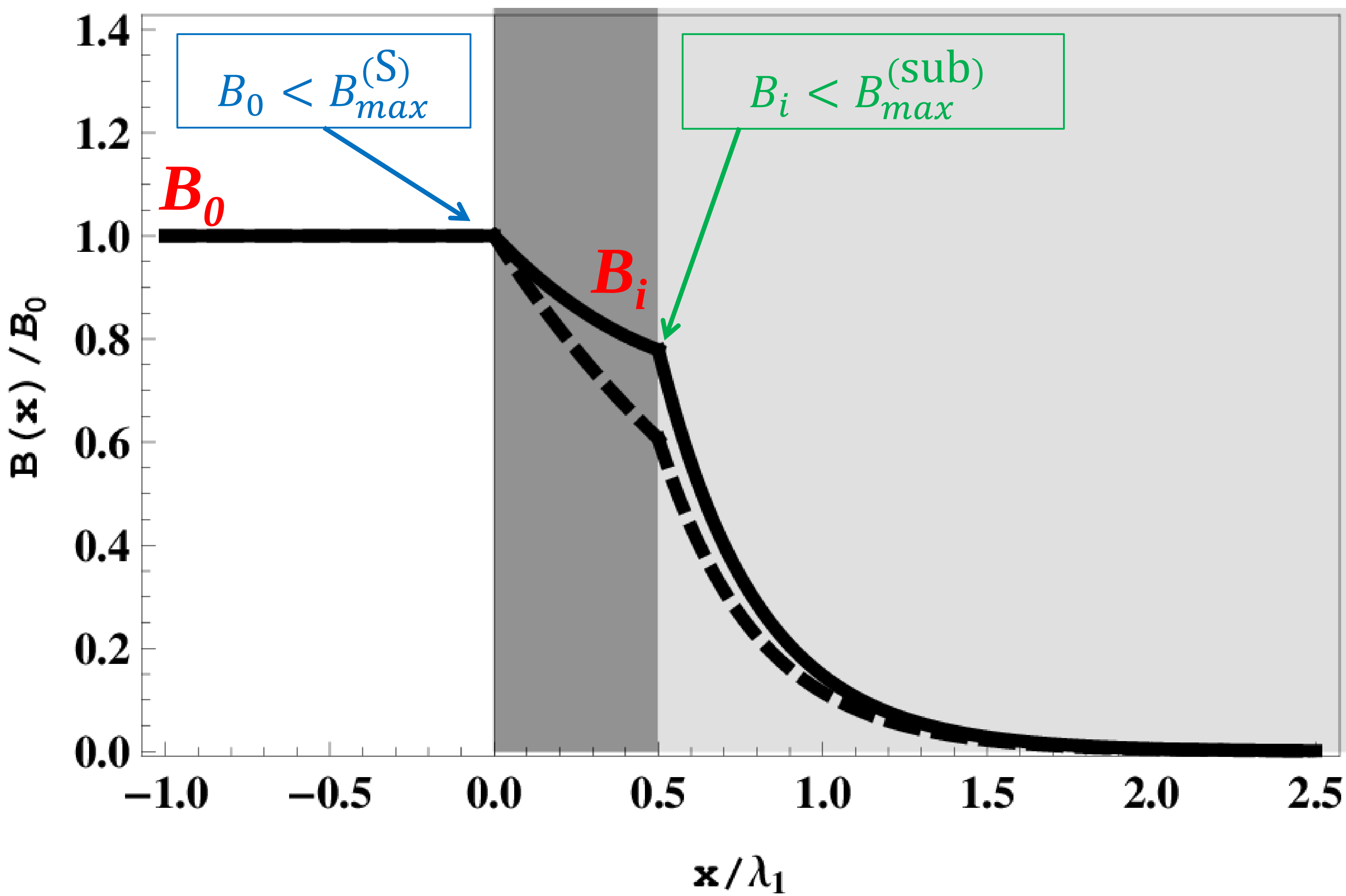
1. The magnetic field distribution (and thus the screening current distribution $J \propto dB/dx$) in the surface layer is different from the naïve exponential decay due to the counter flow induced by the substrate: **the same situation as the S-I-S structure**.



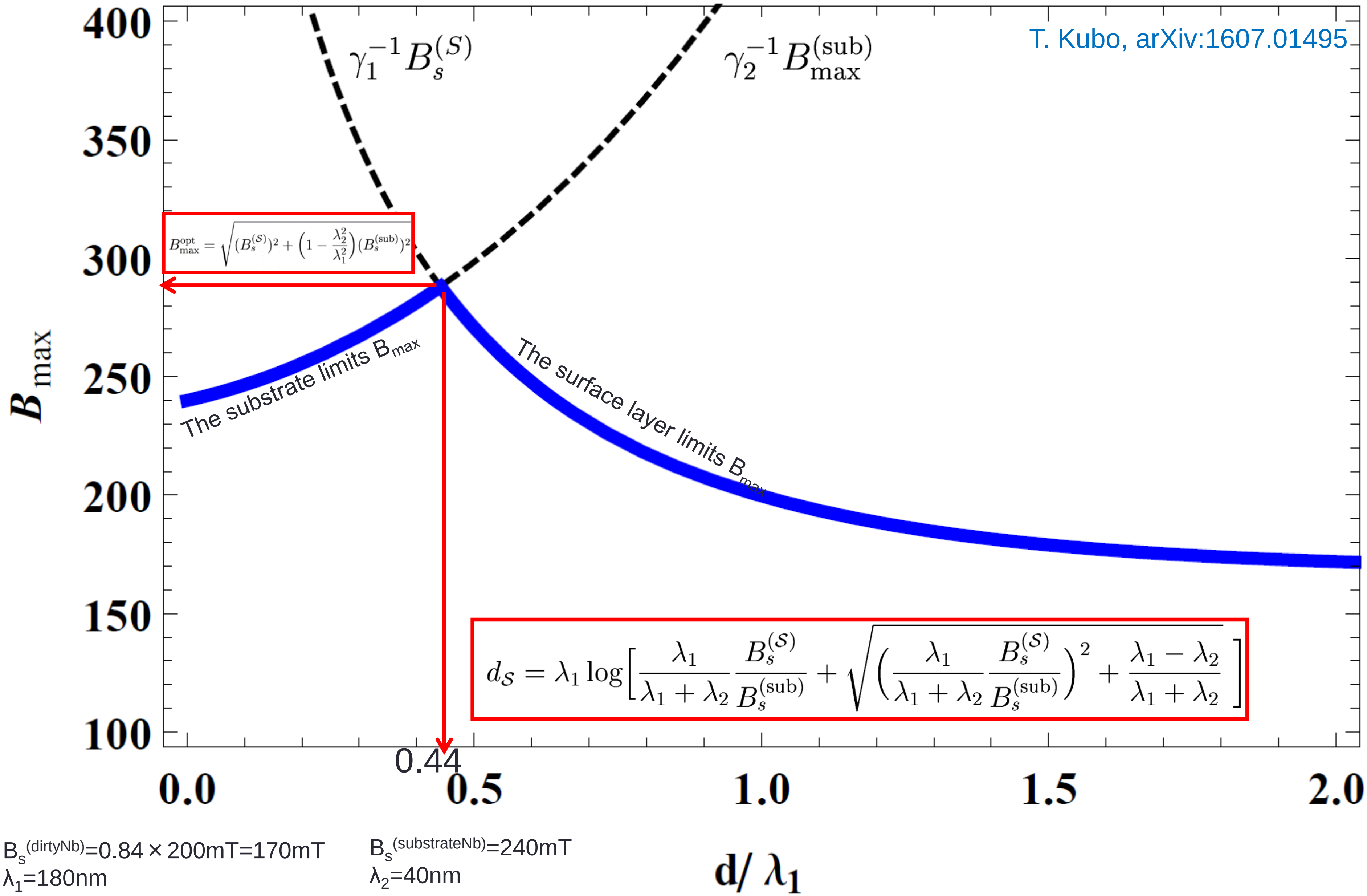
2. When d is thin enough and $\lambda_1 > \lambda_2$, the screening current in the surface layer is suppressed, and the surface field can exceed superheating field of the surface material.



3. However, an extremely thin d can not protect the substrate. Thus the *surface* layer must have some thickness to decay the magnetic field and protect the substrate.



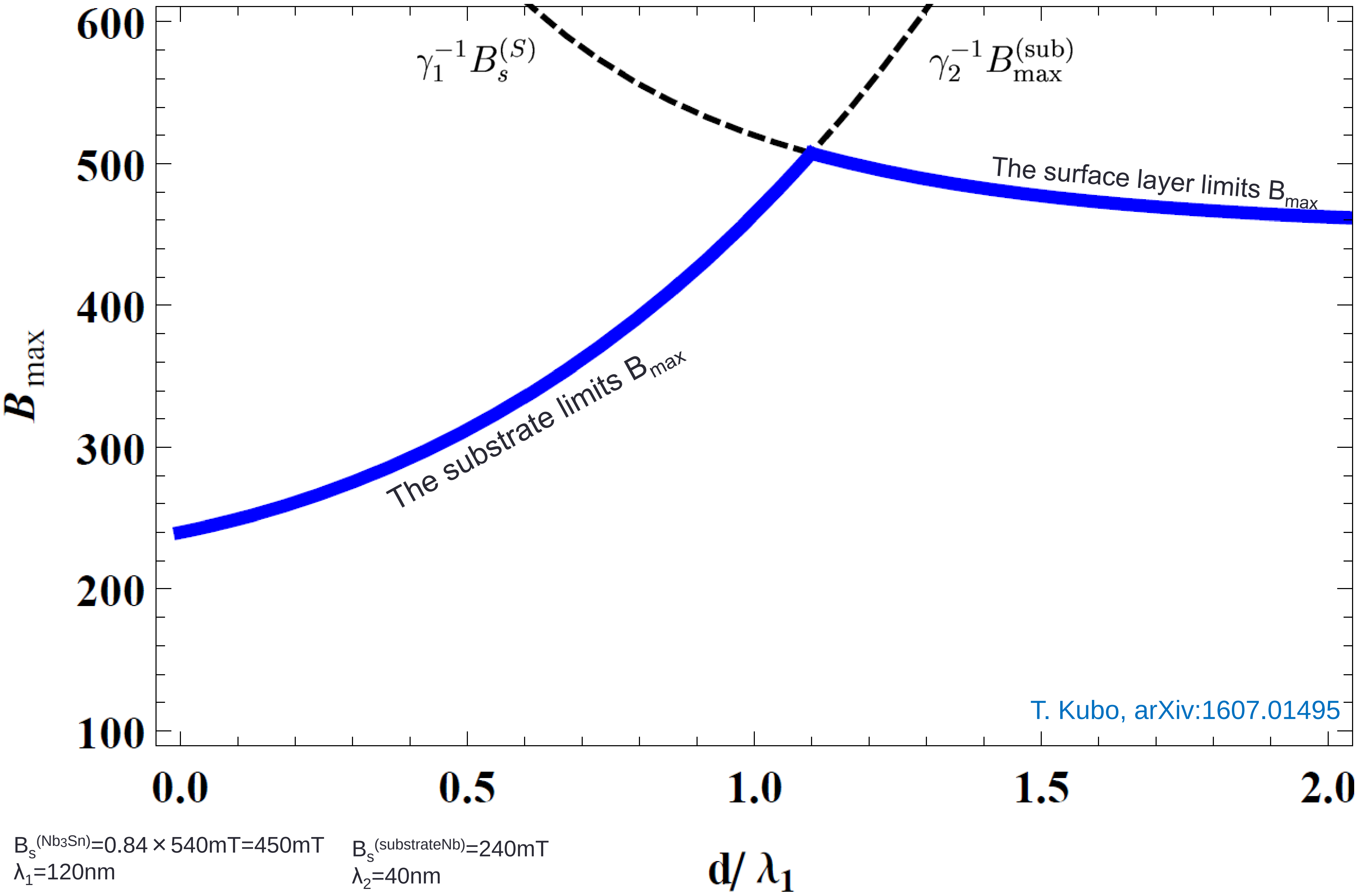
Example1 Dirty Nb layer on Nb substrate



$B_s^{(\mathrm{dirtyNb})}$=0.84 × 200mT=170mT
λ_1=180nm

$B_s^{(\mathrm{substrateNb})}$=240mT
λ_2=40nm

Example2 Nb_3Sn layer on Nb substrate

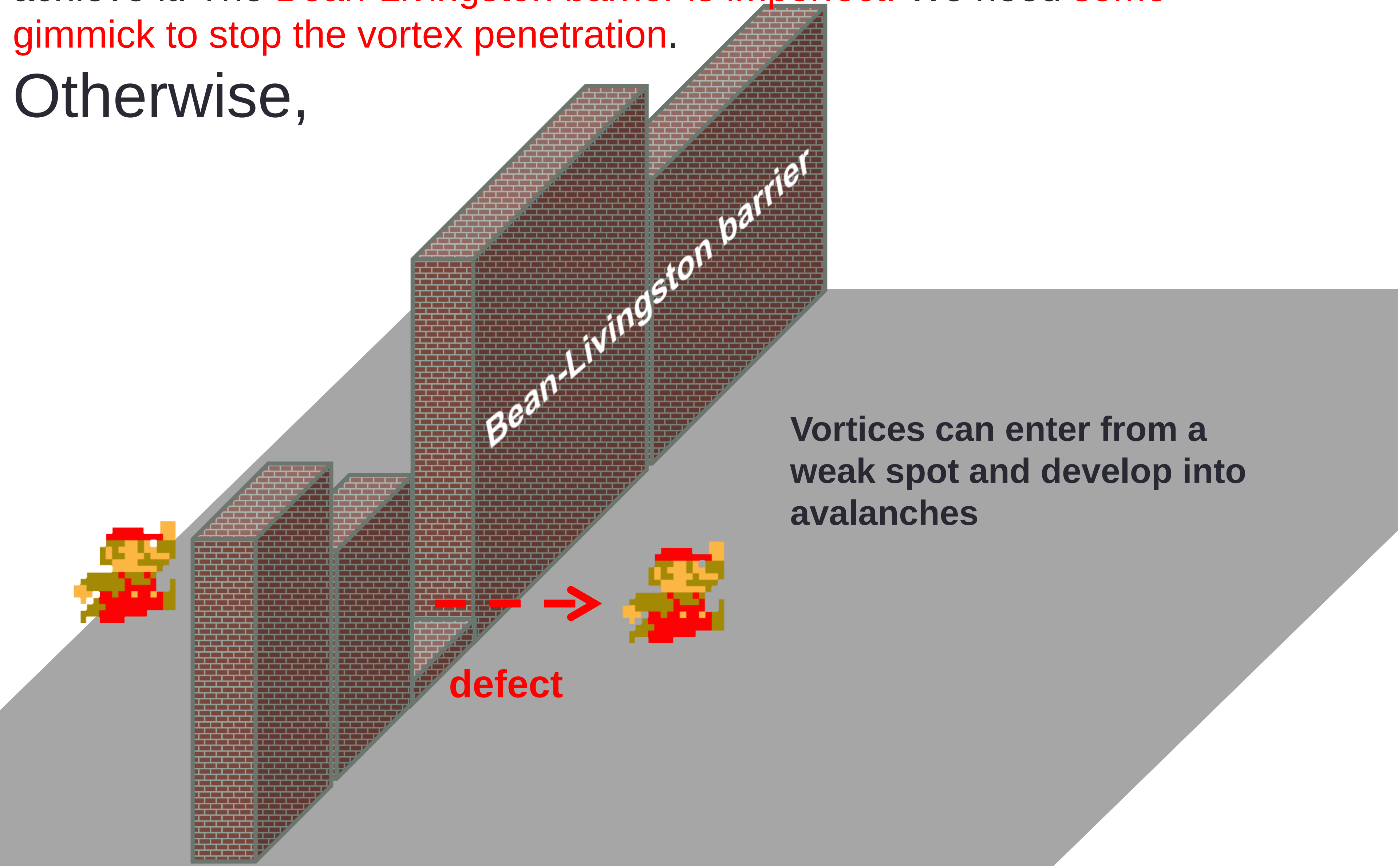


$B_s^{(Nb3Sn)}$=0.84 × 540mT=450mT
λ_1=120nm

$B_s^{(substrateNb)}$=240mT
λ_2=40nm

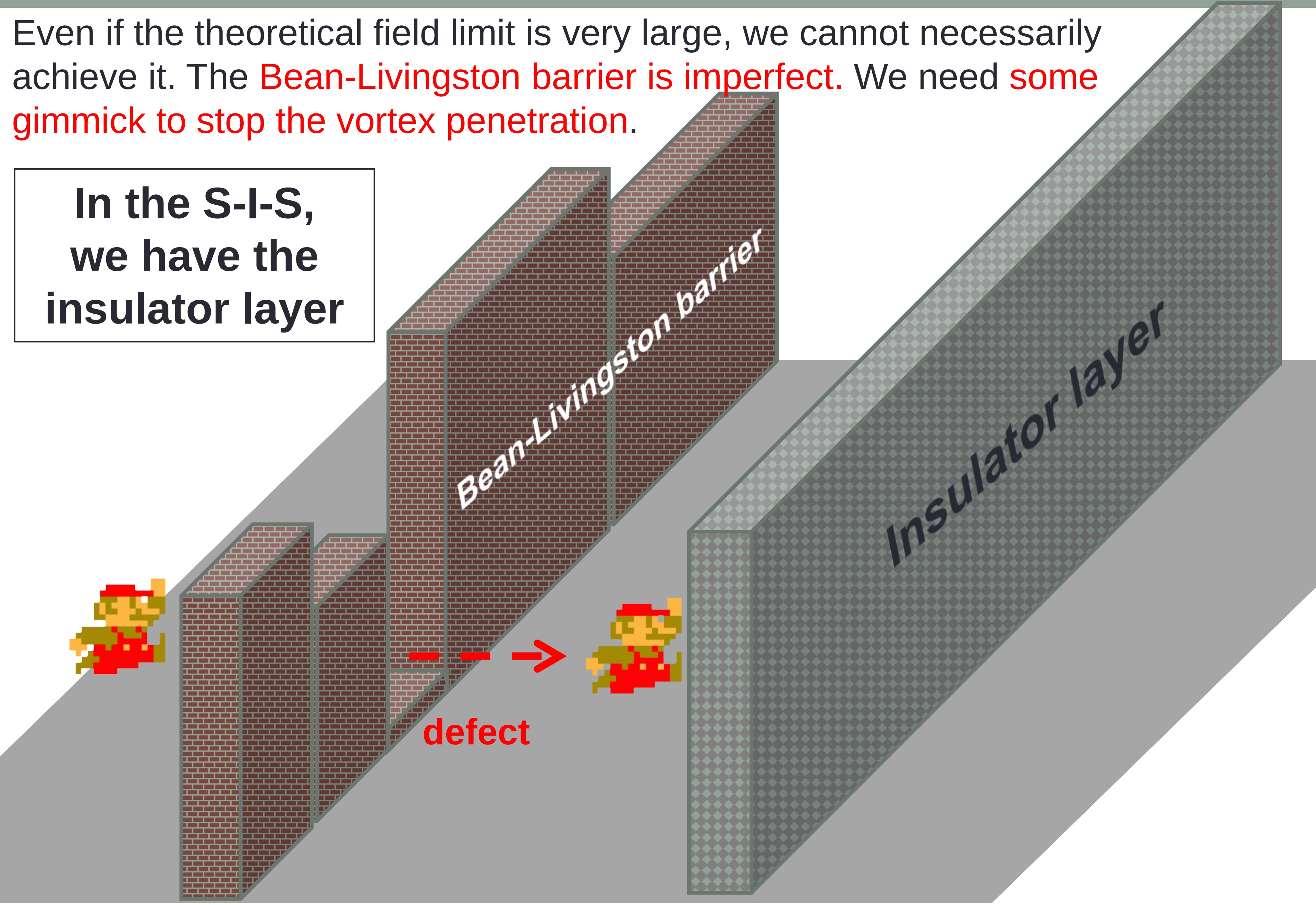
Even if the theoretical field limit is very large, we cannot necessarily achieve it. The Bean-Livingston barrier is imperfect. We need some gimmick to stop the vortex penetration.

Otherwise,



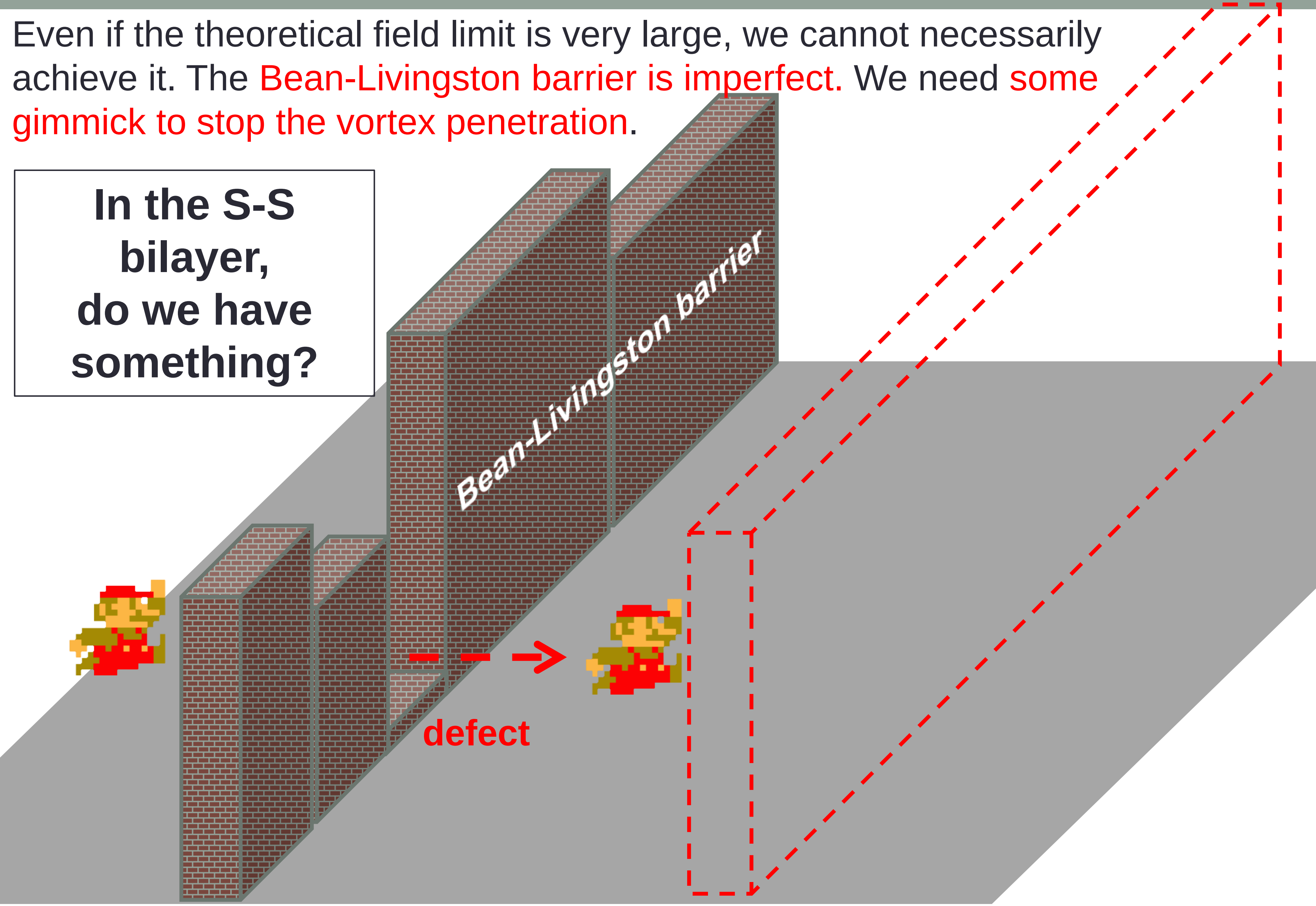
Even if the theoretical field limit is very large, we cannot necessarily achieve it. The Bean-Livingston barrier is imperfect. We need some gimmick to stop the vortex penetration.

In the S-I-S, we have the insulator layer

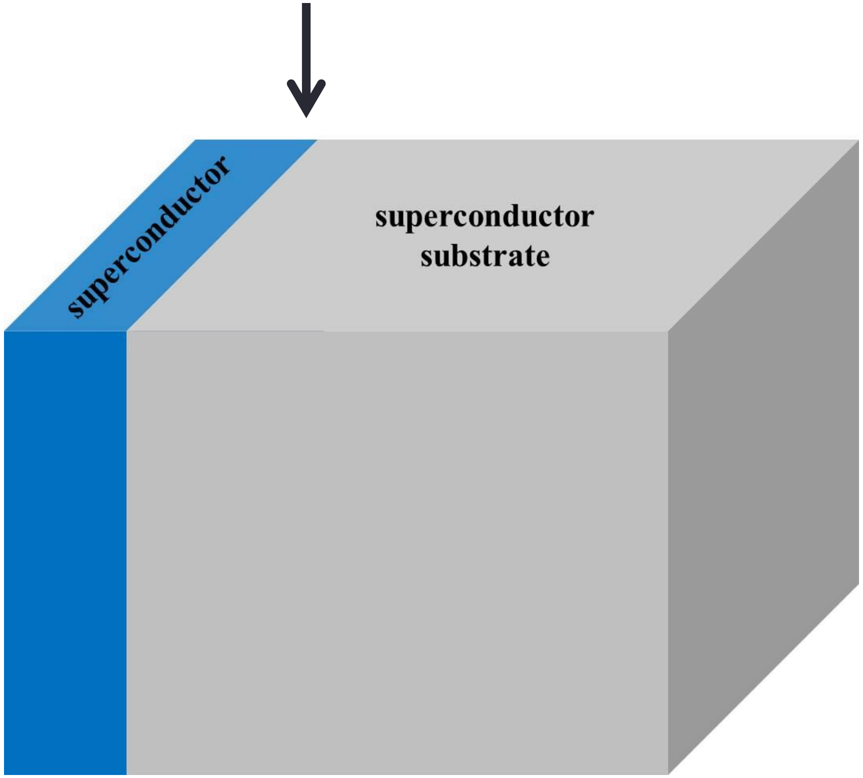


Even if the theoretical field limit is very large, we cannot necessarily achieve it. The Bean-Livingston barrier is imperfect. We need some gimmick to stop the vortex penetration.

In the S-S bilayer, do we have something?



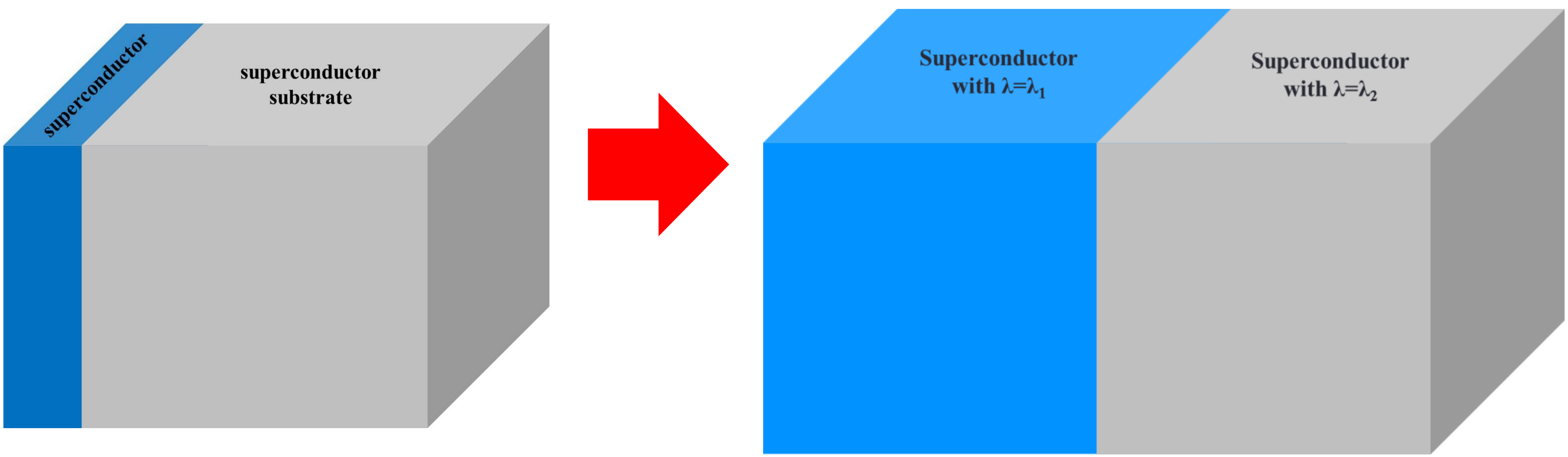
②the role of the S-S boundary



T. Kubo, in proceedings of LINAC14, Geneva, Switzerland (2014), p. 1026, THPP074

②-1
Infinite superconductor with two regions as an instructive exercise

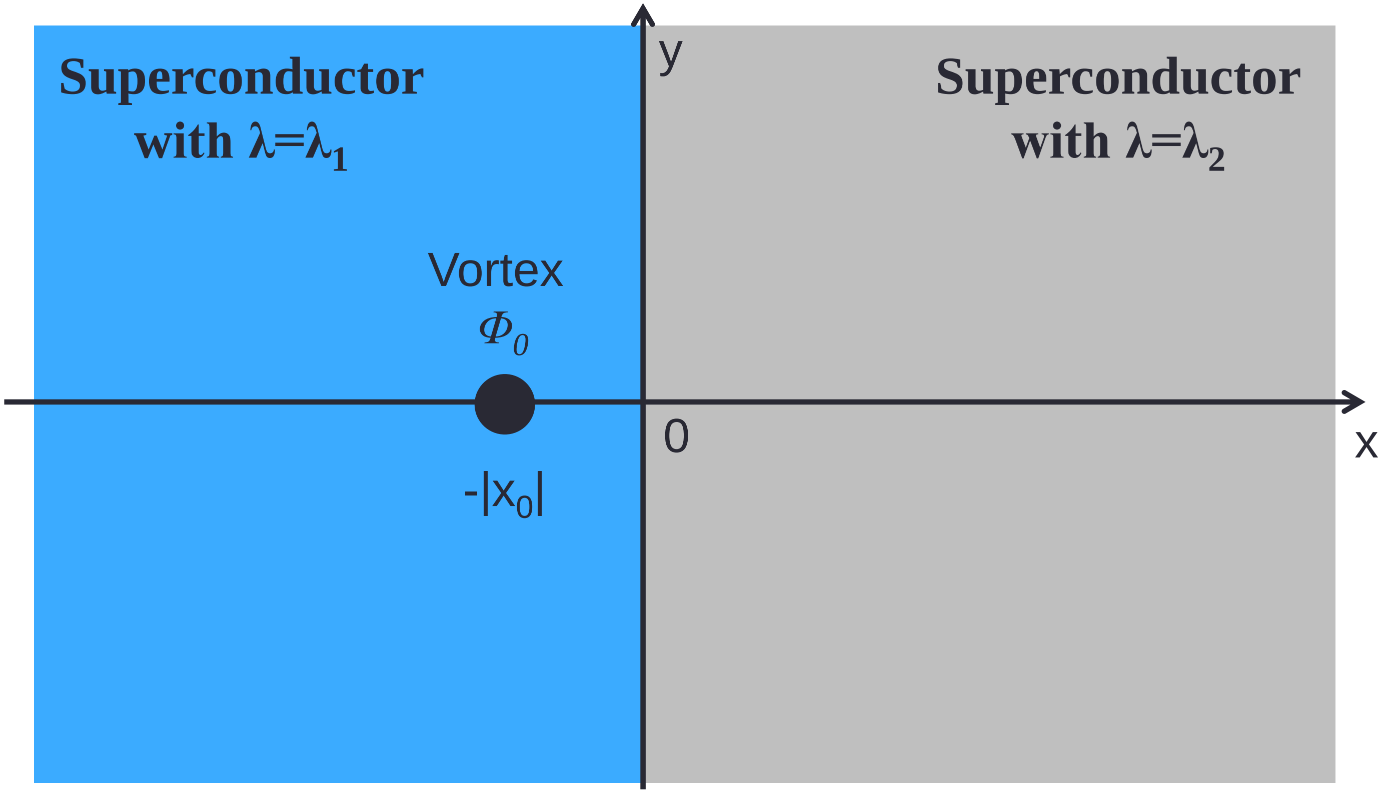
Let us begin with this system



- T. Kubo, in proceedings of LINAC14, Geneva, Switzerland (2014), p. 1026, THPP074
- G. S. Mkrtchyan, F. R. Shakirzyanova, E. A. Shapoval, and V. V. Shmidt, Zh. Eksp. Theor. Fiz. **63**, 667 (1972).

Suppose a vortex is placed at $x = x_0 = -|x_0|$.

We can examine the interaction between the vortex and the S-S boundary by directly solving the modified London equation and can obtain an analytical expression of the force acting on the vortex.

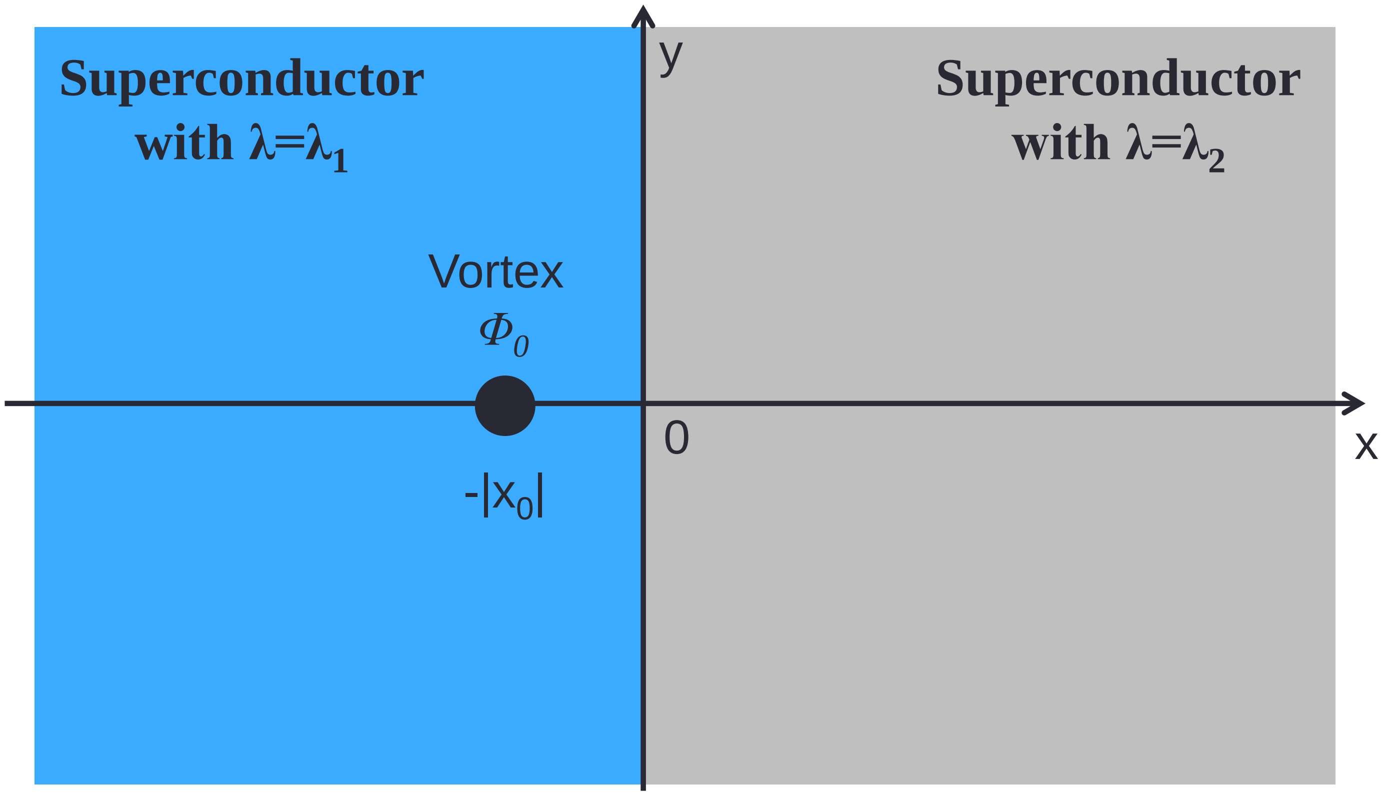


The force acting on the vortex is given by

$$f = -\frac{\phi_0^2}{2\pi\mu_0\lambda_1^2}\int_0^\infty dk\,\frac{p_1\lambda_1^2 - p_2\lambda_2^2}{p_1\lambda_1^2 + p_2\lambda_2^2}e^{2p_1x_0} \qquad p_1 = \sqrt{k^2+\lambda_1^{-2}} \qquad p_2 = \sqrt{k^2+\lambda_2^{-2}}$$

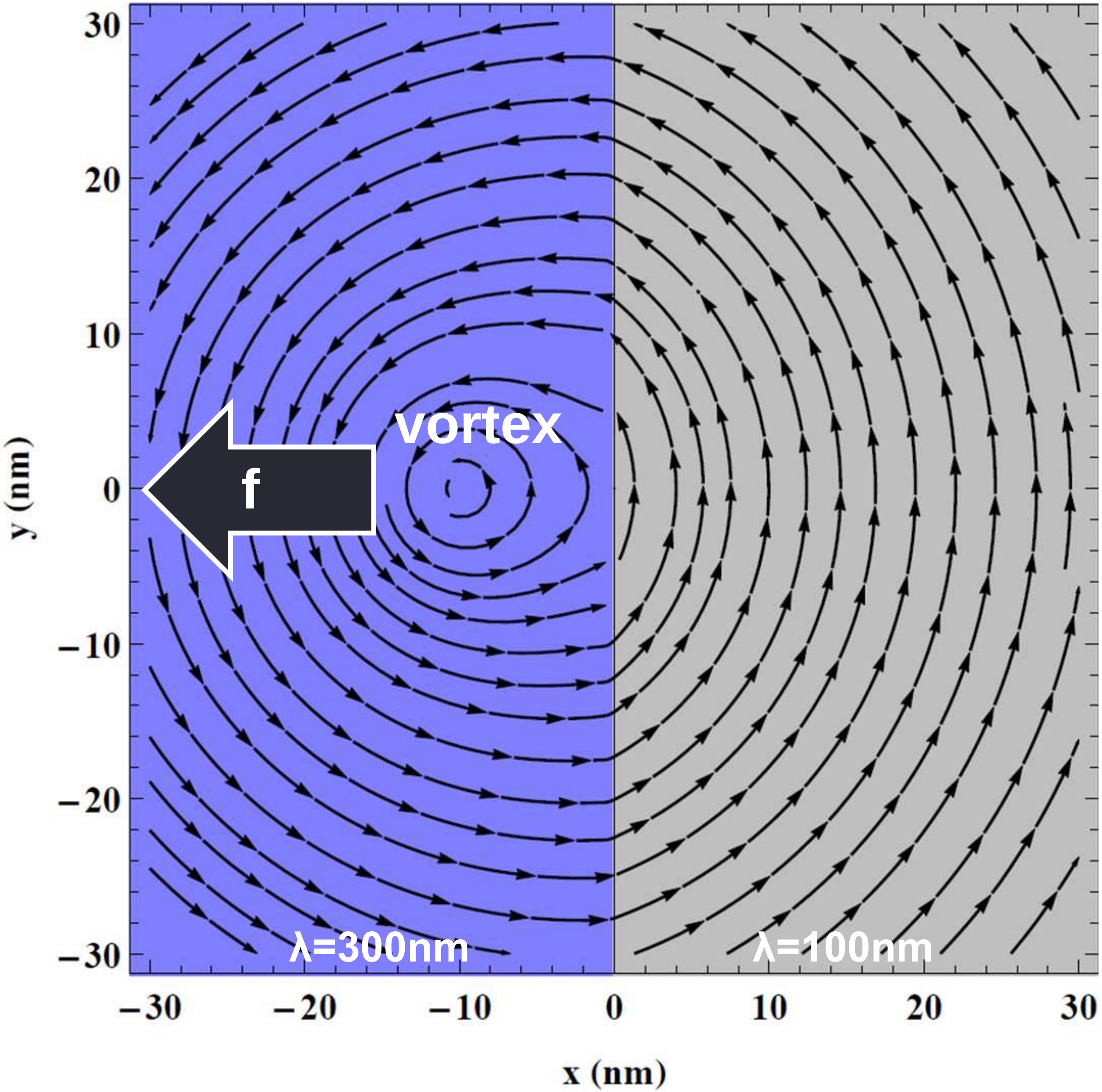
When $|x_0| << \lambda$

$$f = -\frac{\phi_0\phi_1}{4\pi\mu_0\lambda_1^2|x_0|}. \qquad \phi_1 = \eta\phi_0 \qquad \eta = \frac{\lambda_1^2-\lambda_2^2}{\lambda_1^2+\lambda_2^2}$$



The derivation process is explained in detail in Appendix of T. Kubo, arXiv:1607.01495

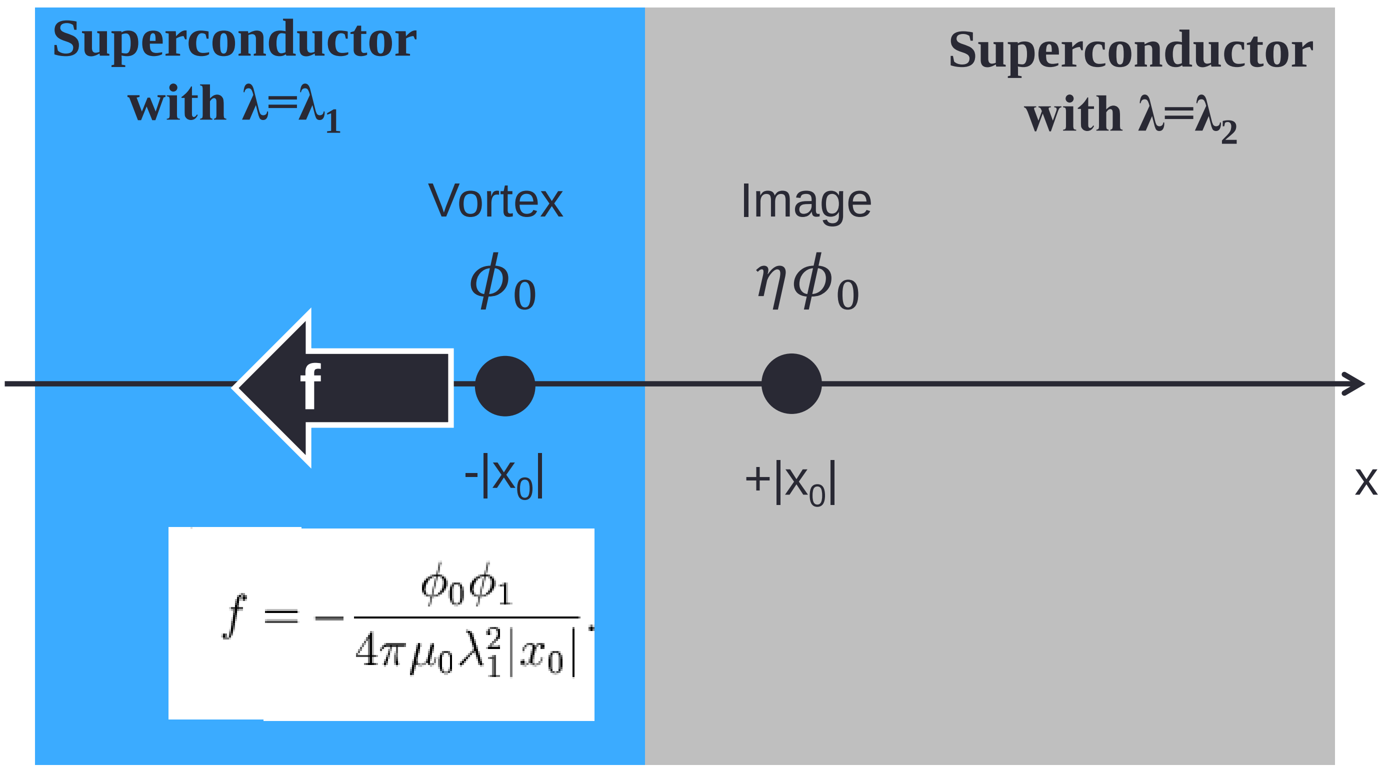
The vortex is pushed by the S-S boundary to the direction of the material with a larger λ.



T.Kubo, in proceedings of LINAC14, Geneva, Switzerland (2014), p. 1026, THPP074

G. S. Mkrtchyan, F. R. Shakirzyanova, E. A. Shapoval, and V. V. Shmidt, Zh. Eksp. Theor. Fiz. **63**, 667 (1972).

Instead of the brute-force approach (solving the differential equation), the same result can be obtained by using the method of images.
The appropriate image is a vortex with a flux $\phi_1 = \eta\phi_0$ $\quad \eta = \frac{\lambda_1^2 - \lambda_2^2}{\lambda_1^2 + \lambda_2^2}$

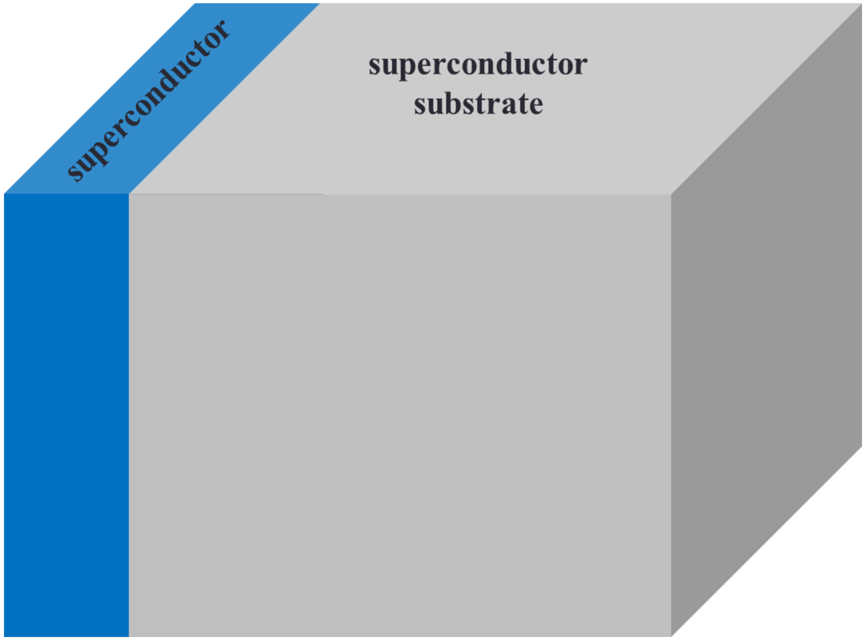


$$f = -\frac{\phi_0\phi_1}{4\pi\mu_0\lambda_1^2|x_0|}.$$

T. Kubo, in proceedings of LINAC14, Geneva, Switzerland (2014), p. 1026, THPP074

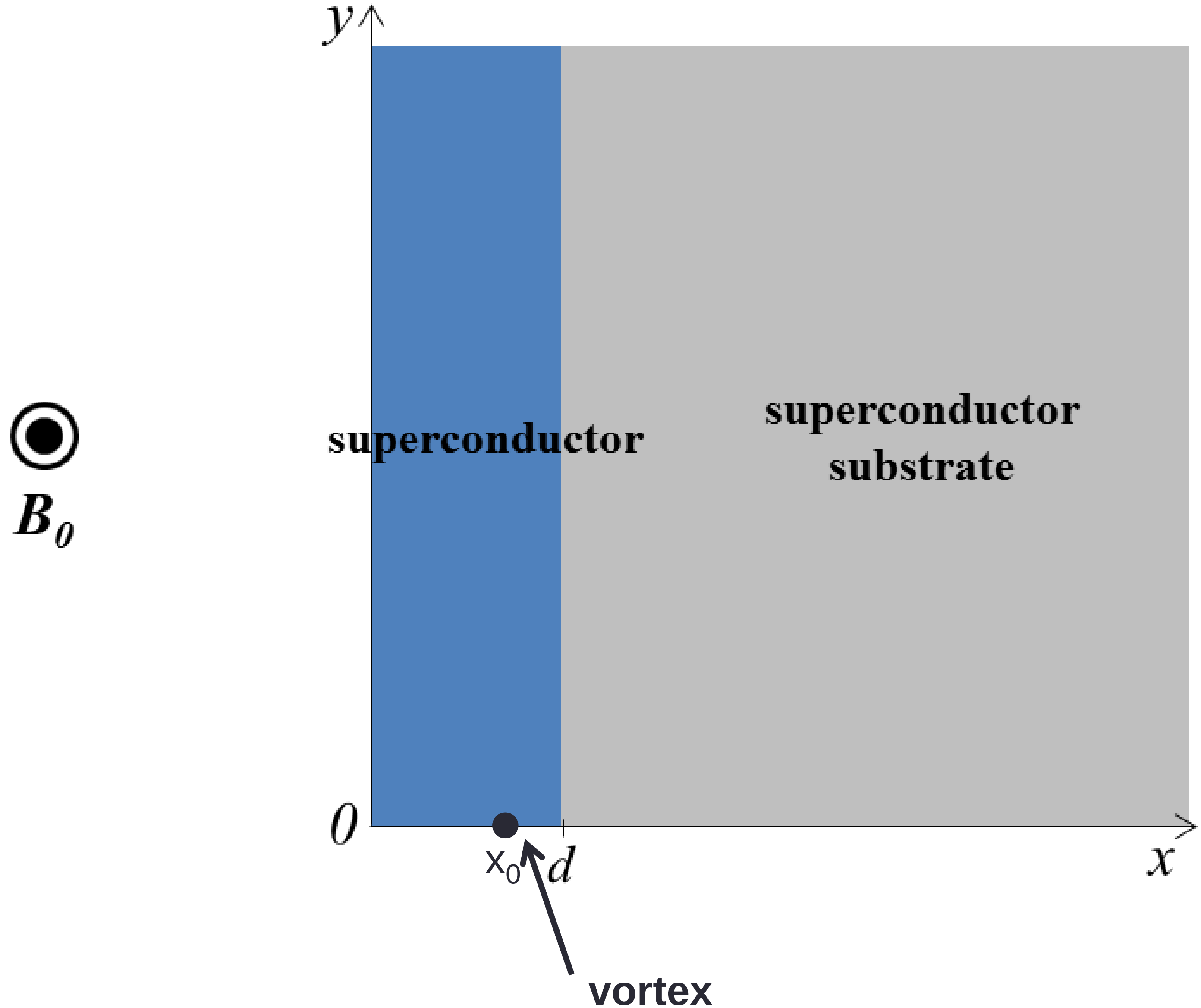
②-2
Thin superconductor on a superconductor substrate

Let us apply the result obtained in the above to this system.

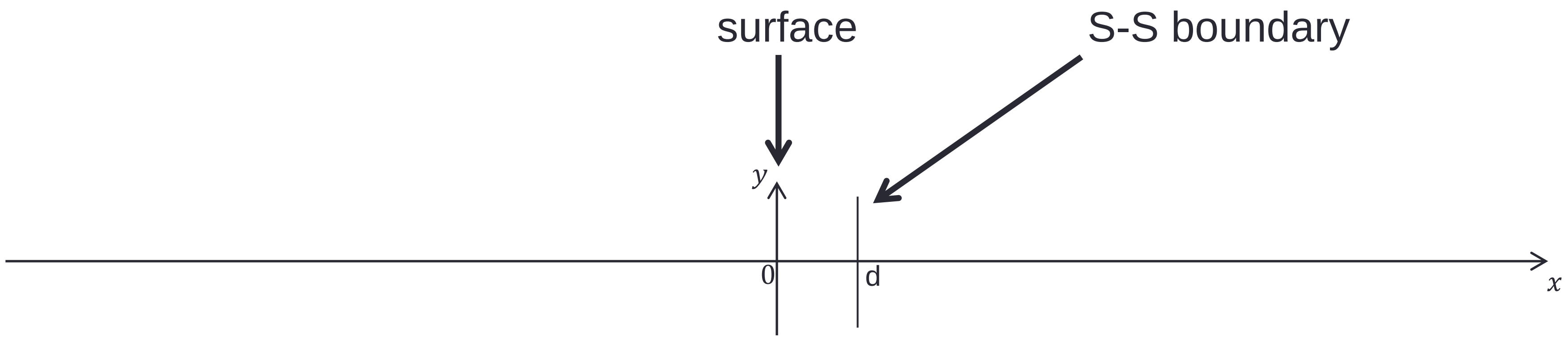


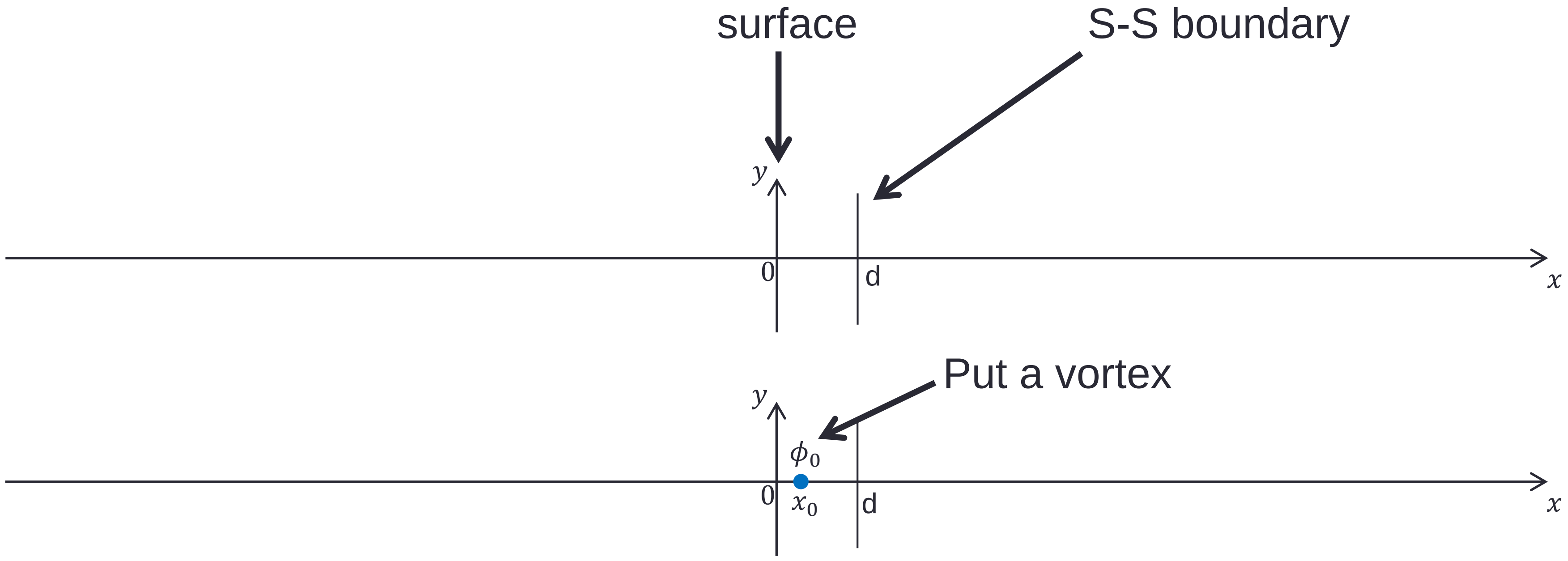
- T. Kubo, in proceedings of LINAC14, Geneva, Switzerland (2014), p. 1026, THPP074

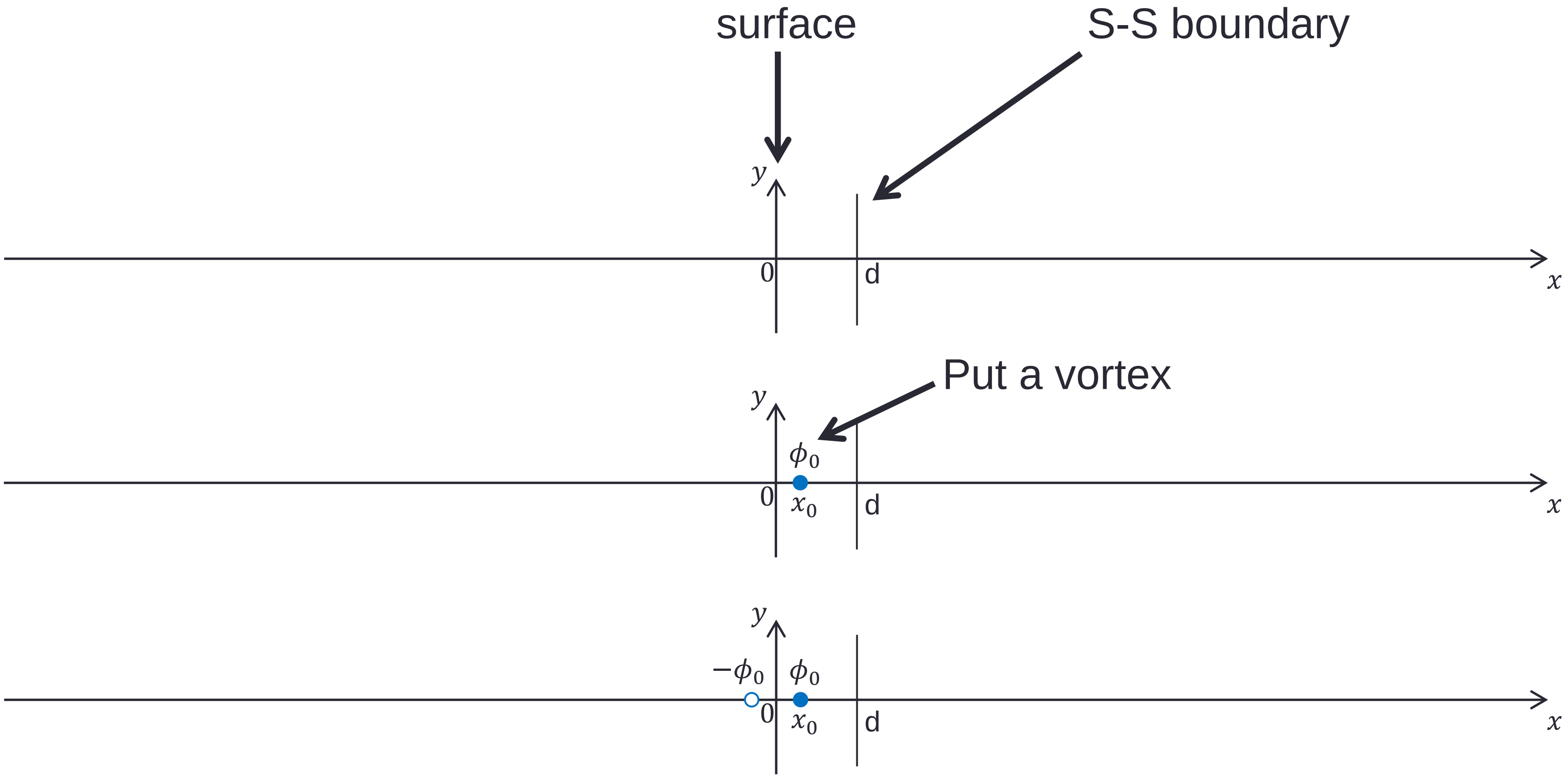
Suppose a vortex is placed at $x = x_0$ in the surface layer (blue region).
Let us examine **the interaction among the vortex, surface, and S-S boundary**.

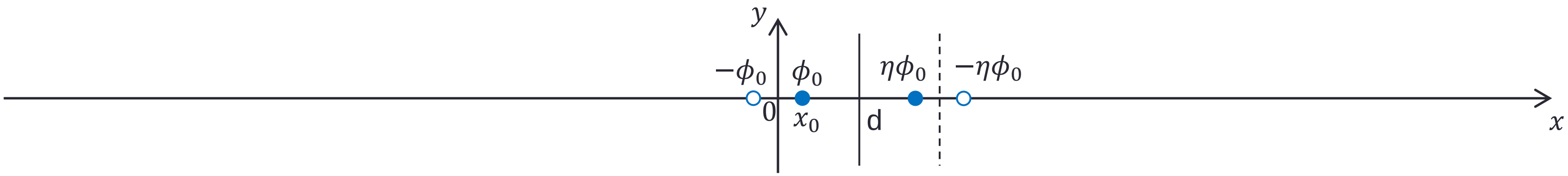


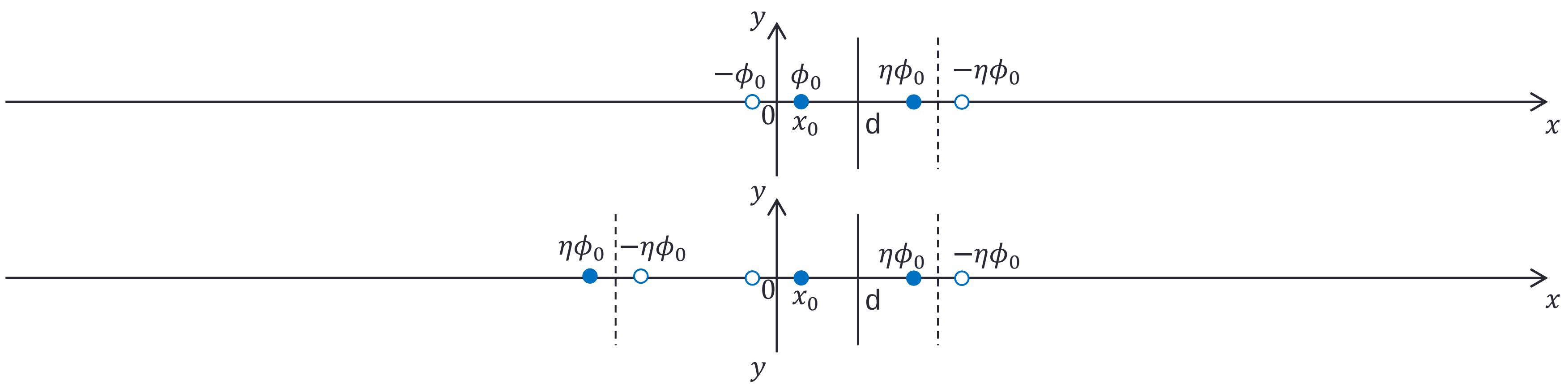
The easiest way is to use the method of images as usual.

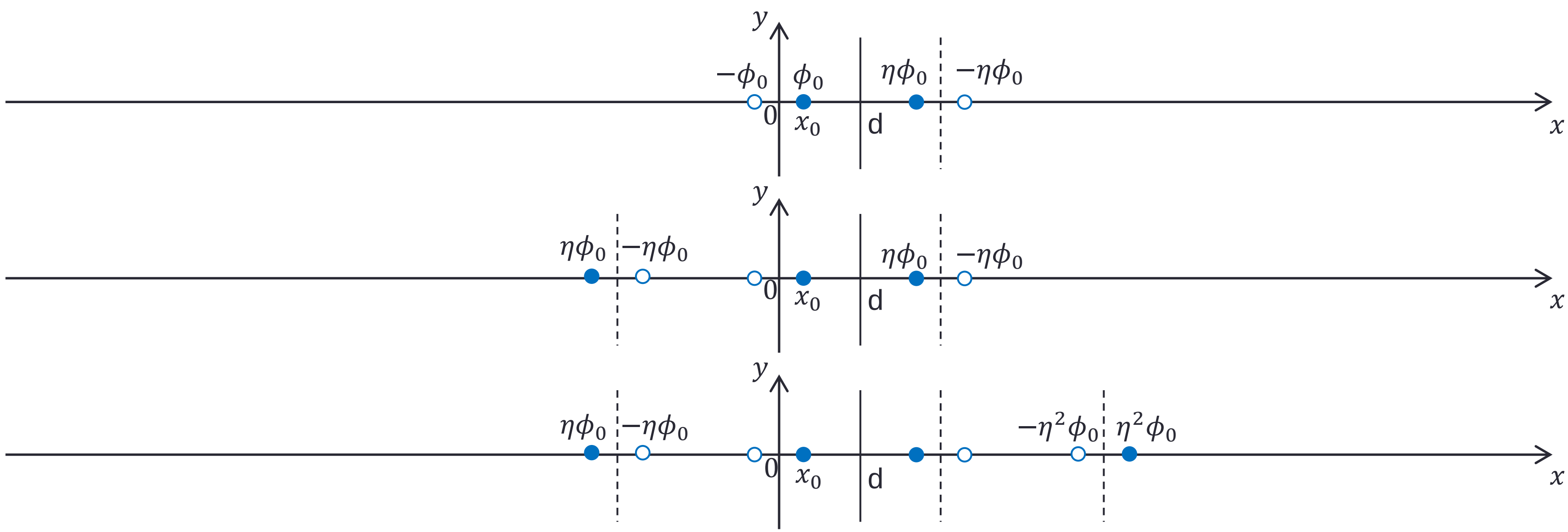


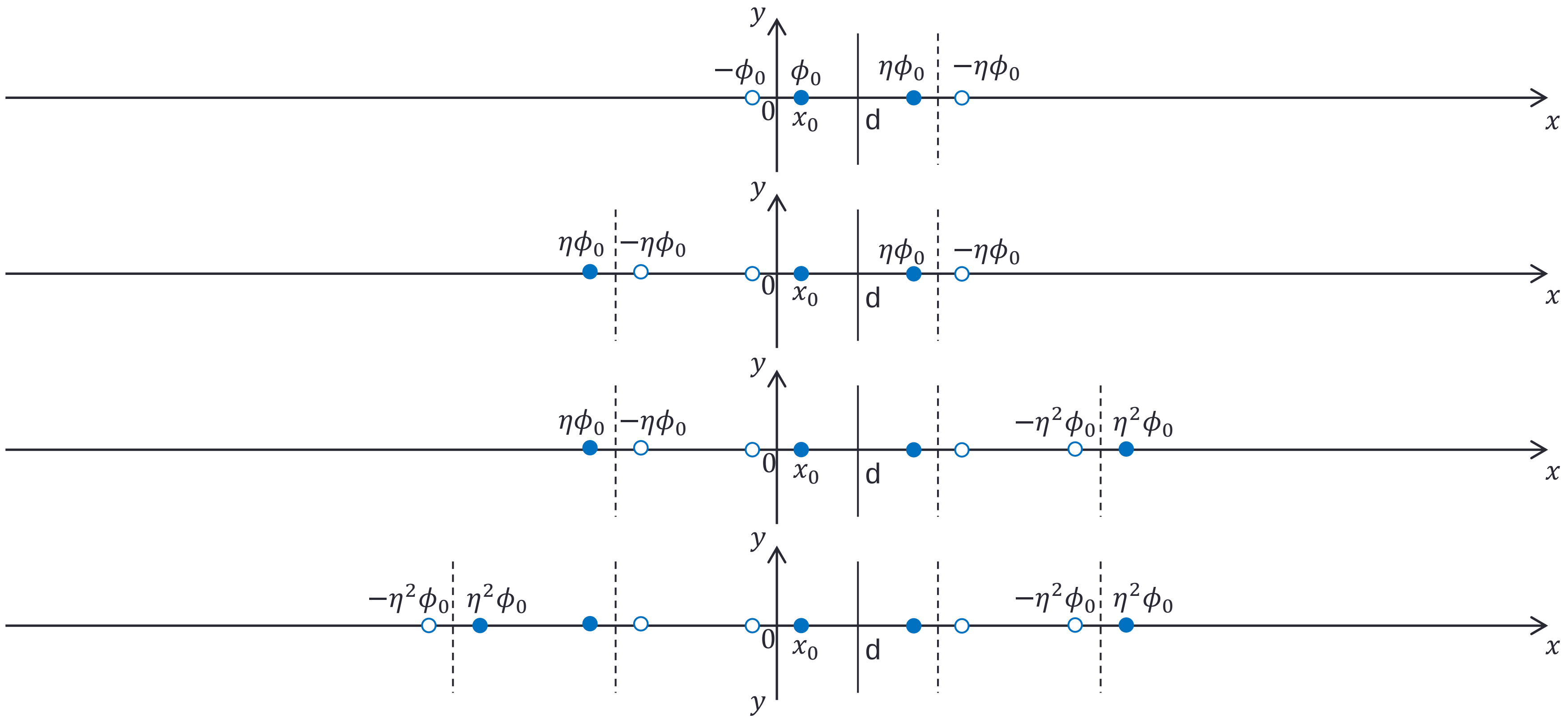

surface
S-S boundary
y
0
d
x
Put a vortex
y
ϕ_0
0
x_0
d
x


surface
S-S boundary
y
0
d
x
Put a vortex
y
ϕ_0
0
x_0
d
x
y
$-\phi_0$
ϕ_0
0
x_0
d
x

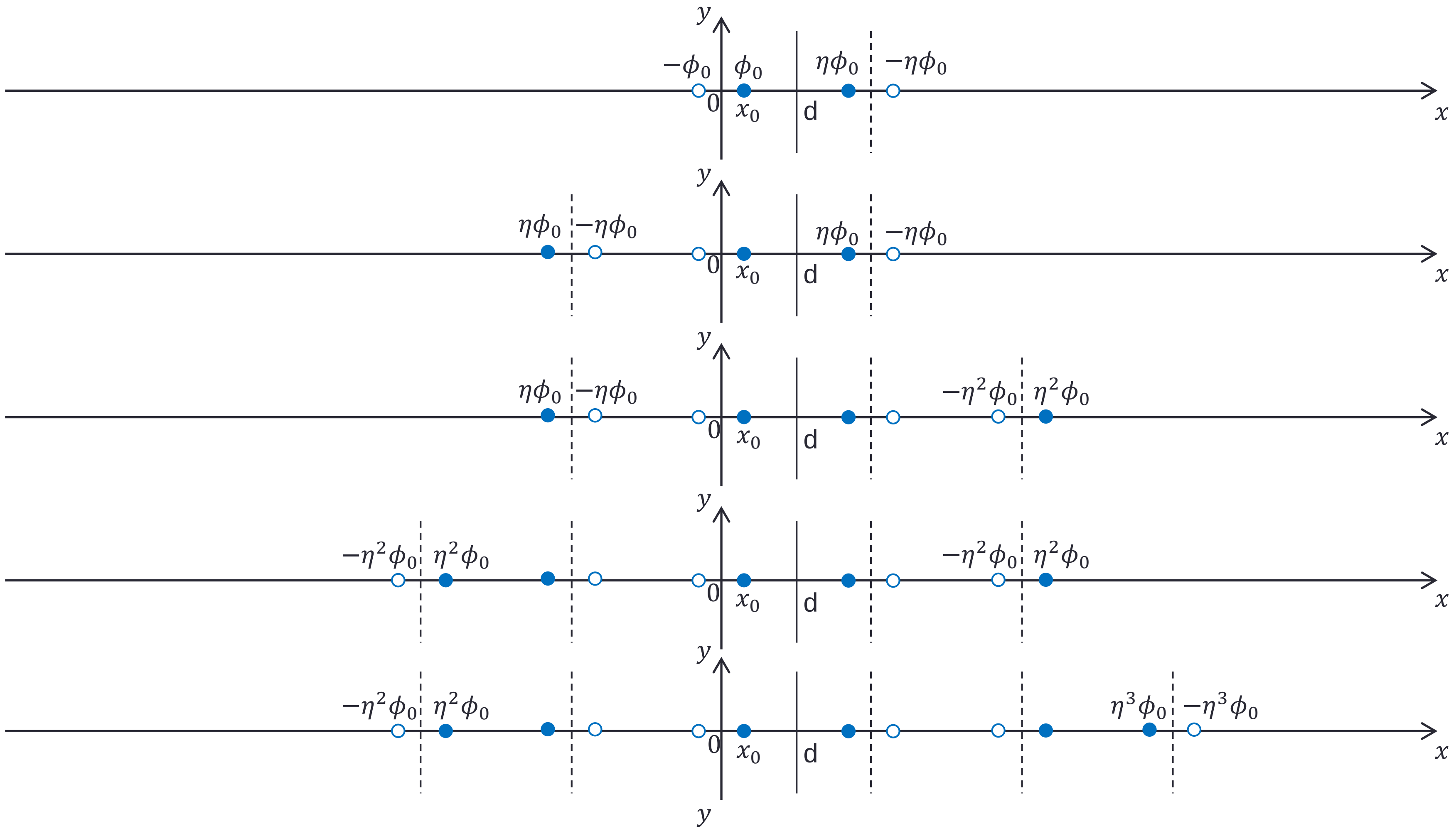

y
$-\phi_0$
ϕ_0
$\eta\phi_0$
$-\eta\phi_0$
0
x_0
d
x

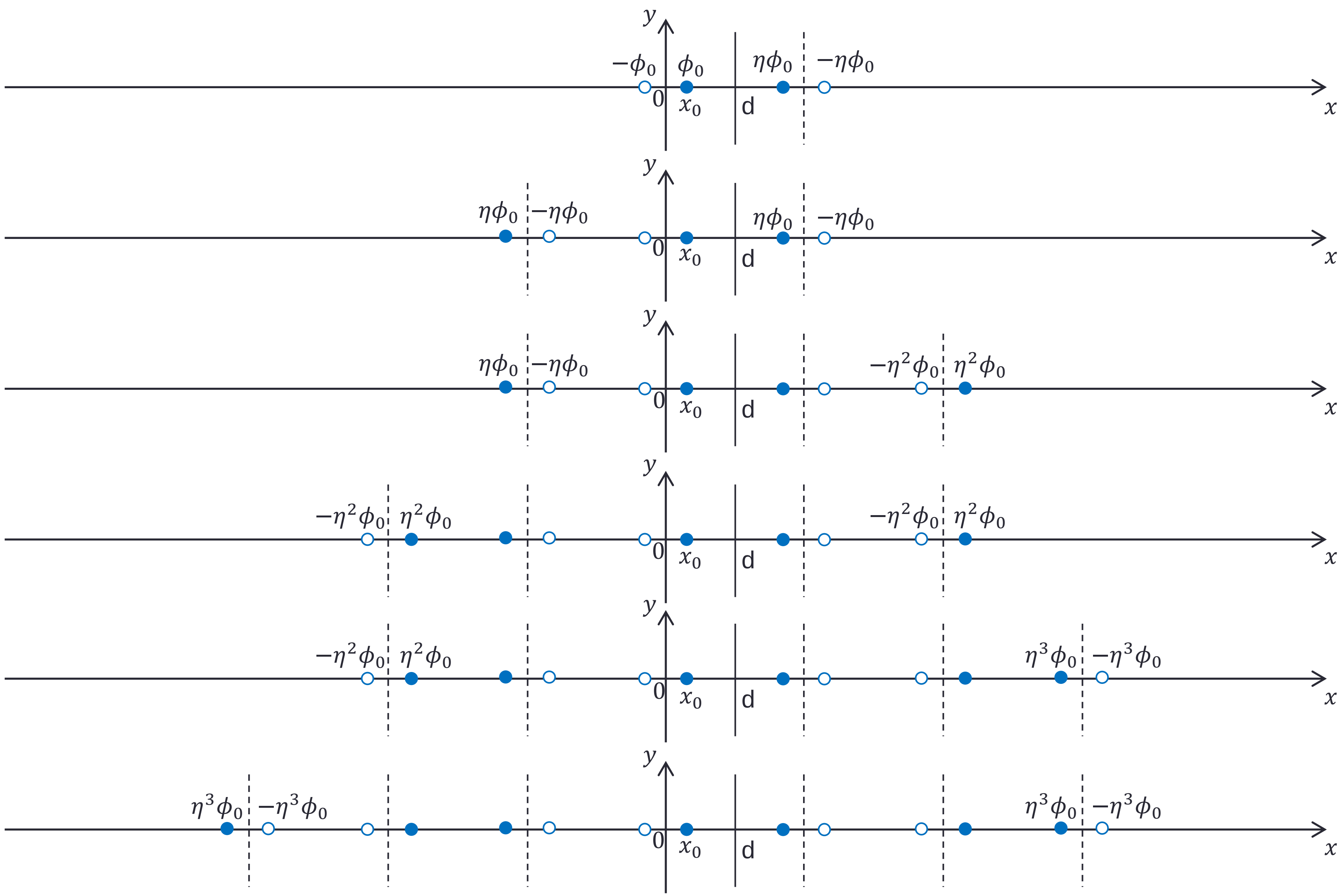

y
$-\phi_0$
ϕ_0
$\eta\phi_0$
$-\eta\phi_0$
0
x_0
d
x
y
$\eta\phi_0$
$-\eta\phi_0$
$\eta\phi_0$
$-\eta\phi_0$
0
x_0
d
x
y


y
$-\phi_0$
ϕ_0
$\eta\phi_0$
$-\eta\phi_0$
0
x_0
d
x
y
$\eta\phi_0$
$-\eta\phi_0$
$\eta\phi_0$
$-\eta\phi_0$
0
x_0
d
x
y
$\eta\phi_0$
$-\eta\phi_0$
$-\eta^2\phi_0$
$\eta^2\phi_0$
0
x_0
d
x

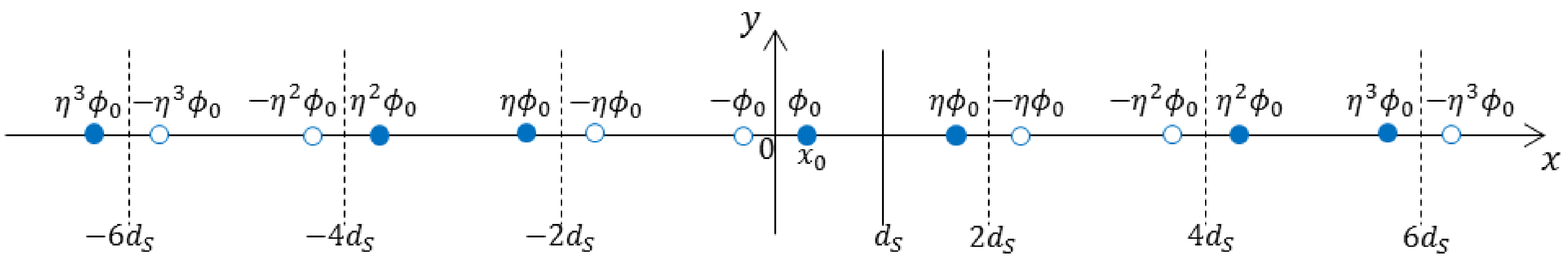


y
$-\phi_0$
ϕ_0
$\eta\phi_0$
$-\eta\phi_0$
0
x_0
d
x
$\eta\phi_0$
$-\eta\phi_0$
$-\eta^2\phi_0$
$\eta^2\phi_0$


y
$-\phi_0$
ϕ_0
$\eta\phi_0$
$-\eta\phi_0$
0
x_0
d
x
$-\eta^2\phi_0$
$\eta^2\phi_0$
$\eta^3\phi_0$
$-\eta^3\phi_0$

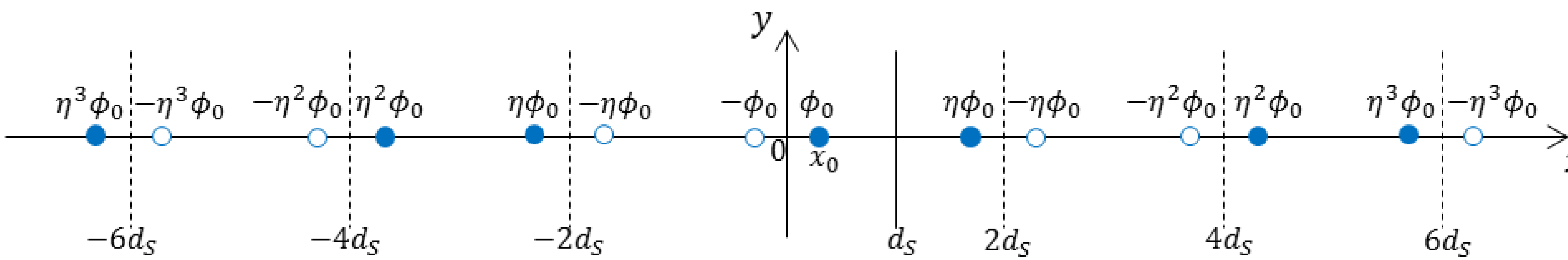

y
$-\phi_0$ ϕ_0 $\eta\phi_0$ $-\eta\phi_0$
0 x_0 d
x
$\eta\phi_0$ $-\eta\phi_0$
$-\eta^2\phi_0$ $\eta^2\phi_0$
$\eta^3\phi_0$ $-\eta^3\phi_0$

We need an infinite number of images!



The total force acting on the vortex is given by the summation of all contribution from the images.

We need an infinite number of images!



The total force acting on the vortex is given by the summation of all contribution from the images. When d<λ, we obtain

$$f_{\rm B} = \hat{\mathbf{x}}\frac{\phi_0^2}{4\pi\mu_0\lambda_1^2}\Bigl[-\frac{1}{x_0}+\sum_{n=1}^{\infty}(-1)^n\eta^n\Bigl(\frac{1}{nd_S-x_0}-\frac{1}{nd_S+x_0}\Bigr)\Bigr]$$

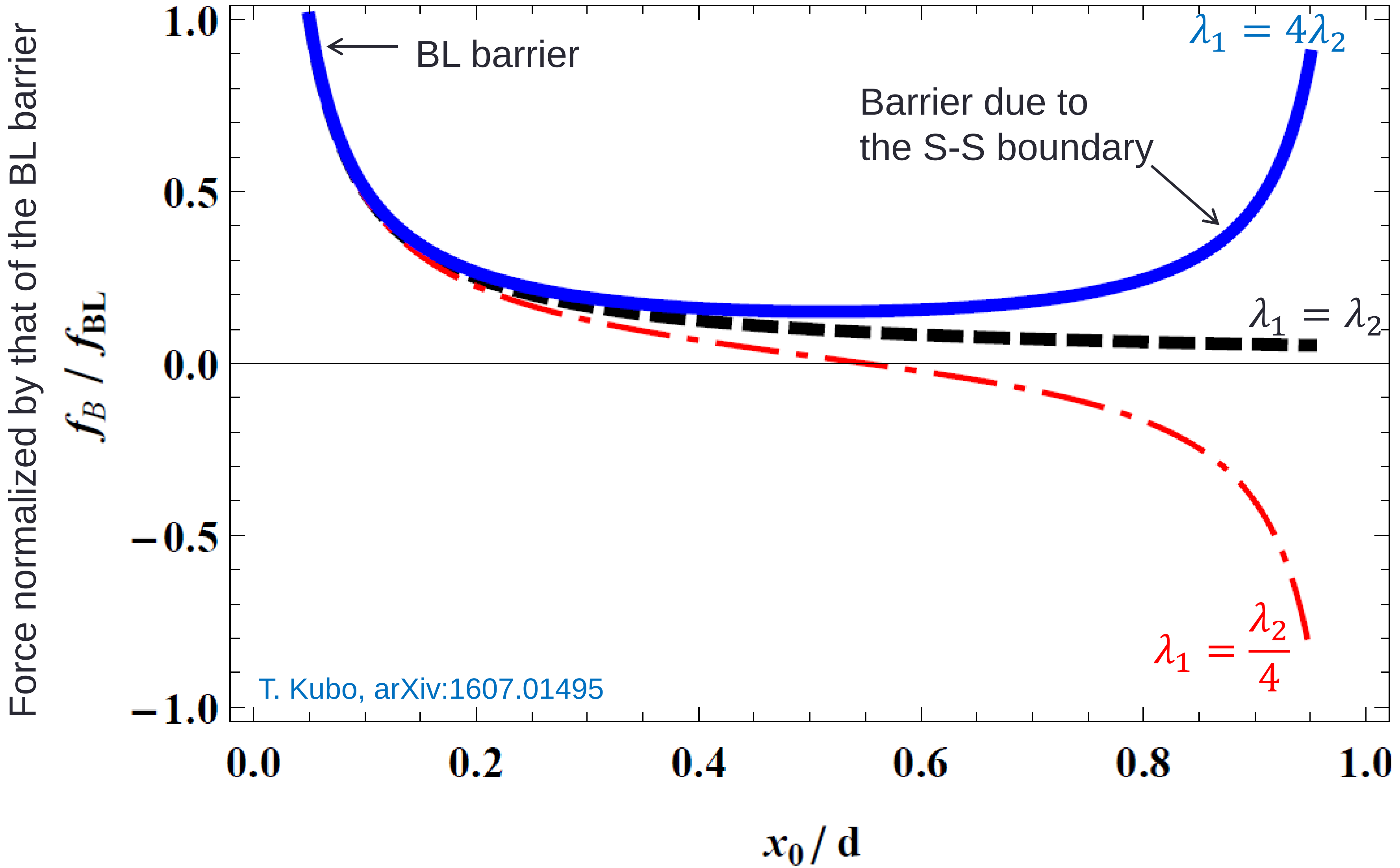
$$= -\frac{\phi_0^2}{4\pi\mu_0\lambda_1^2 d}\Bigl[\frac{1}{a}F(1,a;1+a;-\eta)+\frac{\eta}{1-a}F(1,1-a;2-a;-\eta)\Bigr]$$

F is the Gaussian hypergeometric function $F(a,b;c;z) = [\Gamma(c)/\Gamma(b)\Gamma(c-b)]\int_0^1 dt(1-tz)^{-a}t^{b-1}(1-t)^{c-b-1}$

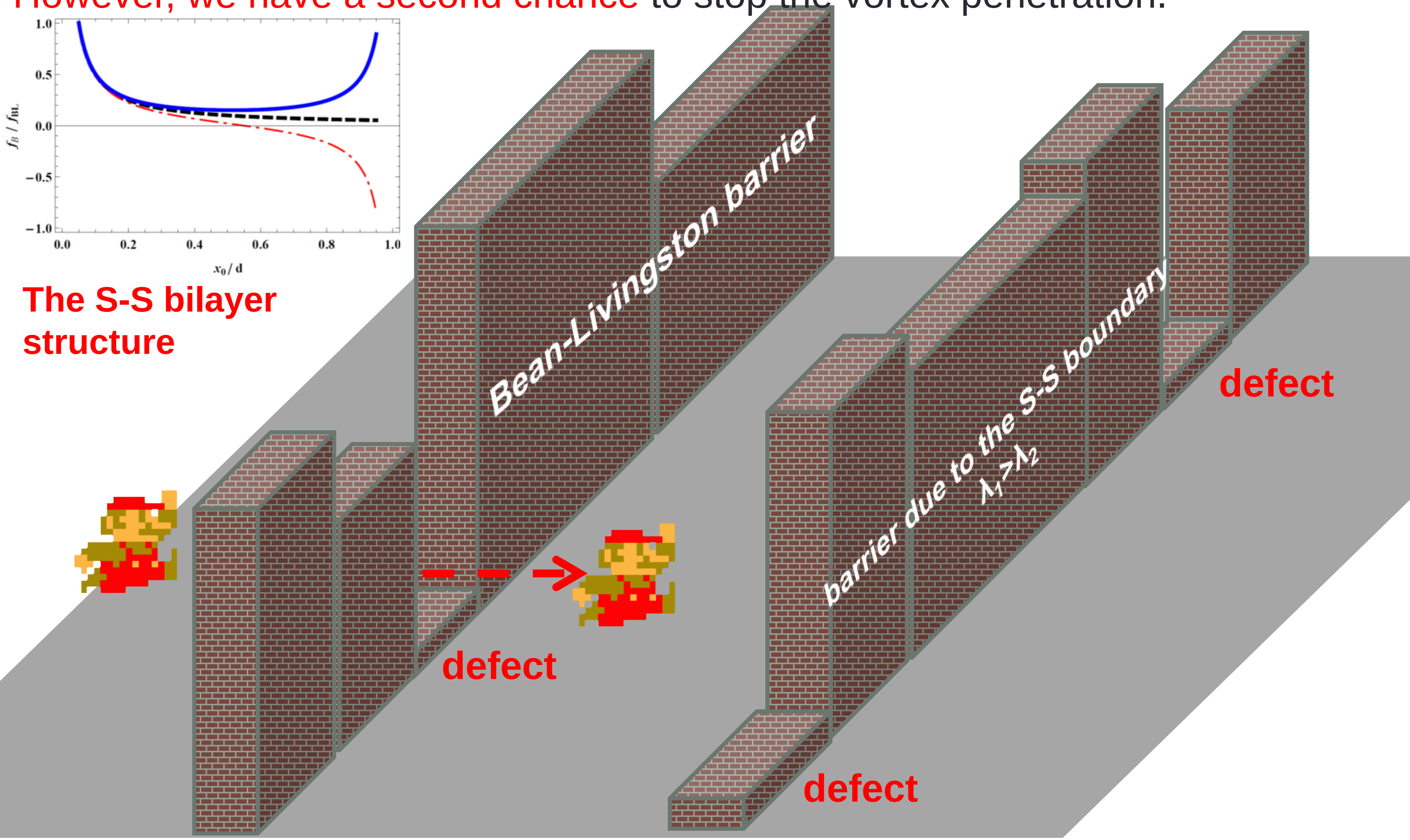
Of course, we can obtain the same result by directly solving the London equation under the appropriate boundary conditions.

The derivation process is explained in detail in “T. Kubo, arXiv:1607.01495”

$$f_{\rm B} = -\frac{\phi_0^2}{4\pi\mu_0\lambda_1^2 d}\left[\frac{1}{a}F(1,a;1+a;-\eta) + \frac{\eta}{1-a}F(1,1-a;2-a;-\eta)\right]$$



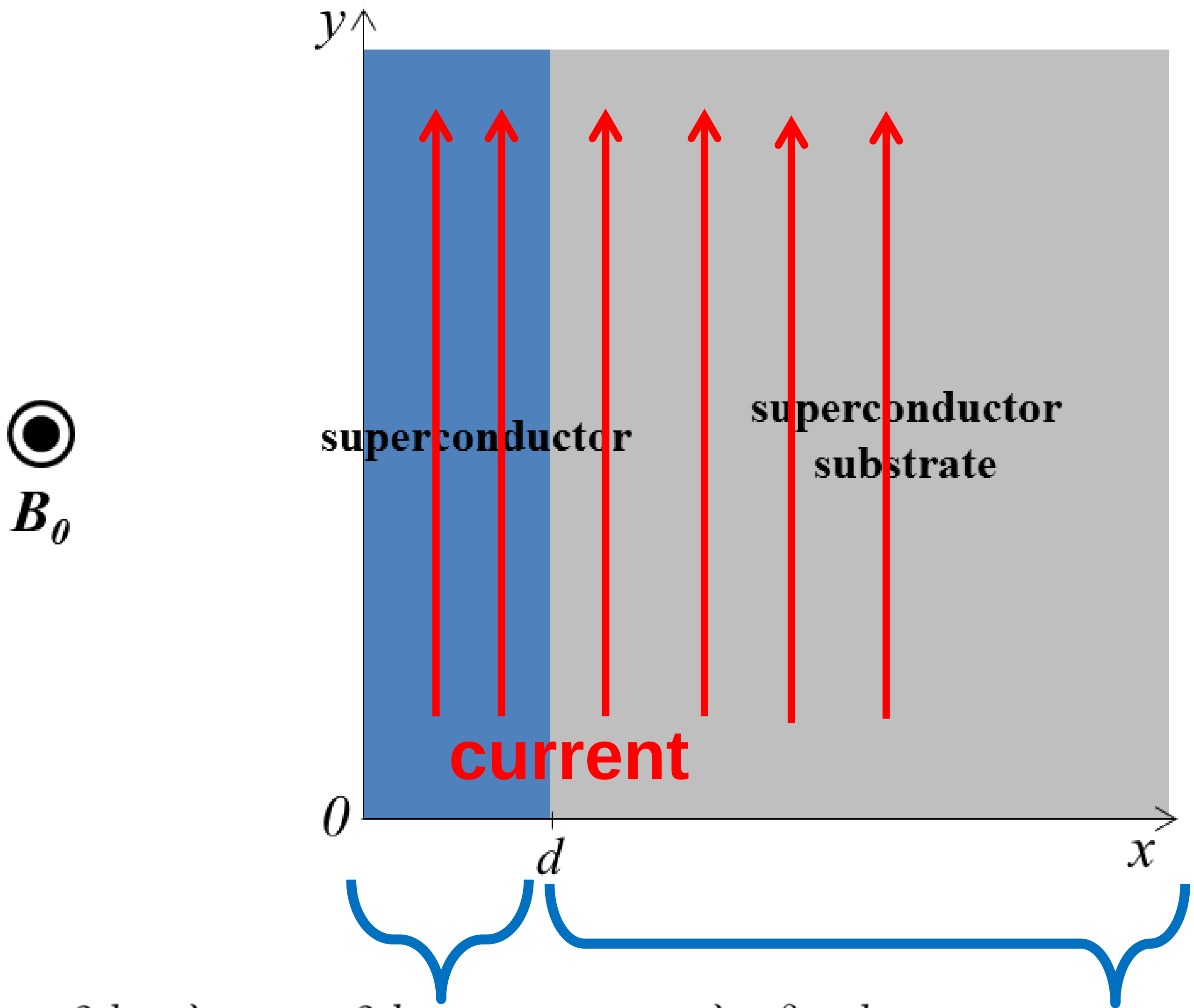
In addition to the BL barrier, we have the second barrier due to the S-S boundary. The second barrier is also imperfect: easily weakened by defects. However, we have a second chance to stop the vortex penetration.



③the surface resistance of the S-S bilayer structure

- A. Gurevich, AIP Advance **5**, 017112 (2015) [for the S-I-S]
- T. Kubo, arXiv:1607.01495

A part of the screening current flows in the surface layer.



$$R_s = \left[\frac{1+\left(\frac{\lambda_2}{\lambda_1}\right)^2}{2}\sinh\frac{2d}{\lambda_1} + \frac{\lambda_2}{\lambda_1}\left(\cosh\frac{2d}{\lambda_1} - 1\right) - \left\{1 - \left(\frac{\lambda_2}{\lambda_1}\right)^2\right\}\frac{d}{\lambda_1}\right]\gamma_2^2 R_s^{(S)} + \gamma_2^2 R_s^{(\mathrm{sub})}$$

The surface resistance formula for the S-S bilayer

$$R_s = \left[\frac{1+(\frac{\lambda_2}{\lambda_1})^2}{2}\sinh\frac{2d}{\lambda_1} + \frac{\lambda_2}{\lambda_1}\left(\cosh\frac{2d}{\lambda_1} - 1\right) - \left\{1 - \left(\frac{\lambda_2}{\lambda_1}\right)^2\right\}\frac{d}{\lambda_1}\right]\gamma_2^2 R_s^{(S)} + \gamma_2^2 R_s^{(\mathrm{sub})}$$

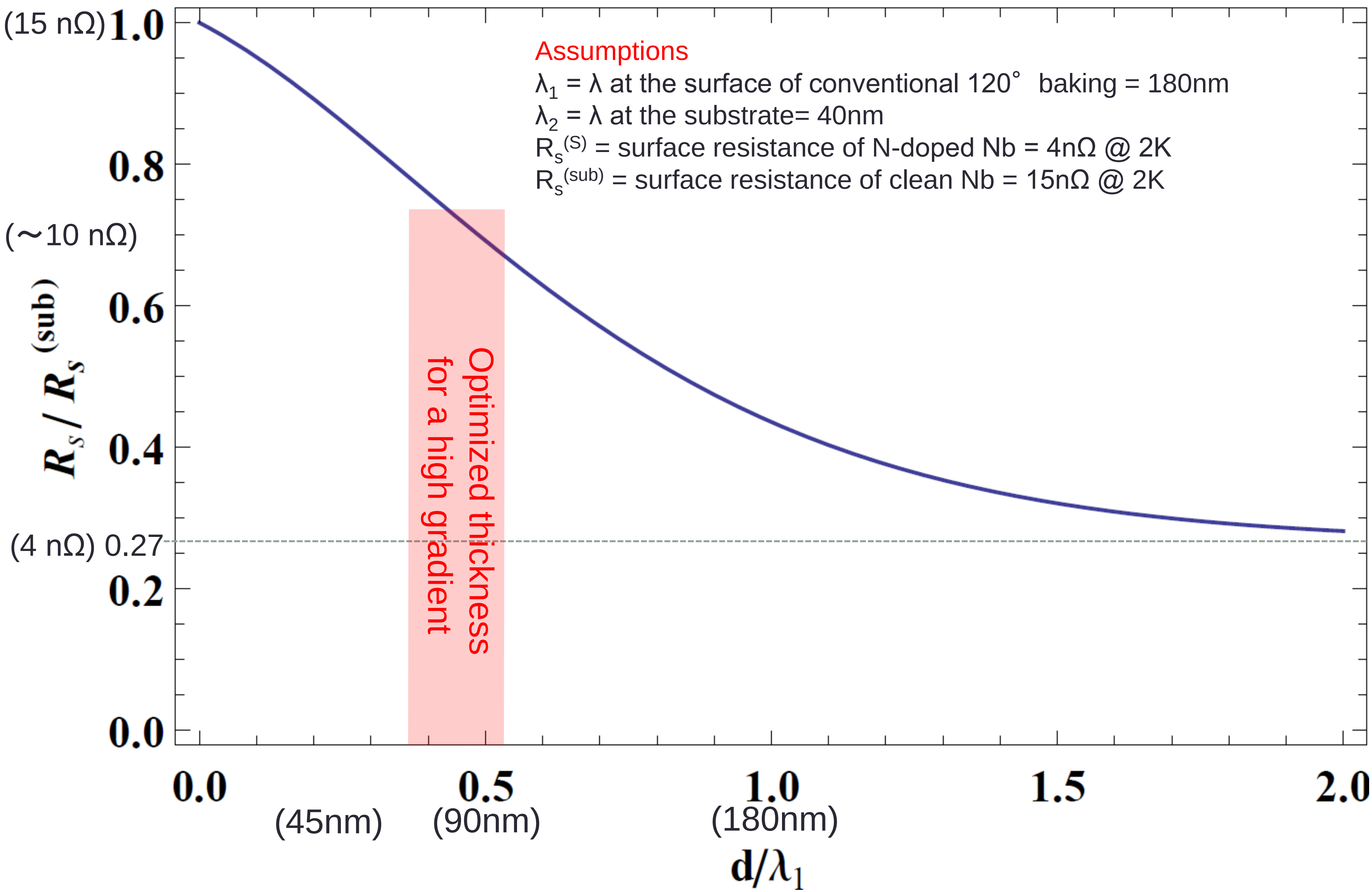
$$\gamma_2 \equiv \frac{1}{\cosh\frac{d}{\lambda_1} + \frac{\lambda_2}{\lambda_1}\sinh\frac{d}{\lambda_1}}.$$

The input parameters are

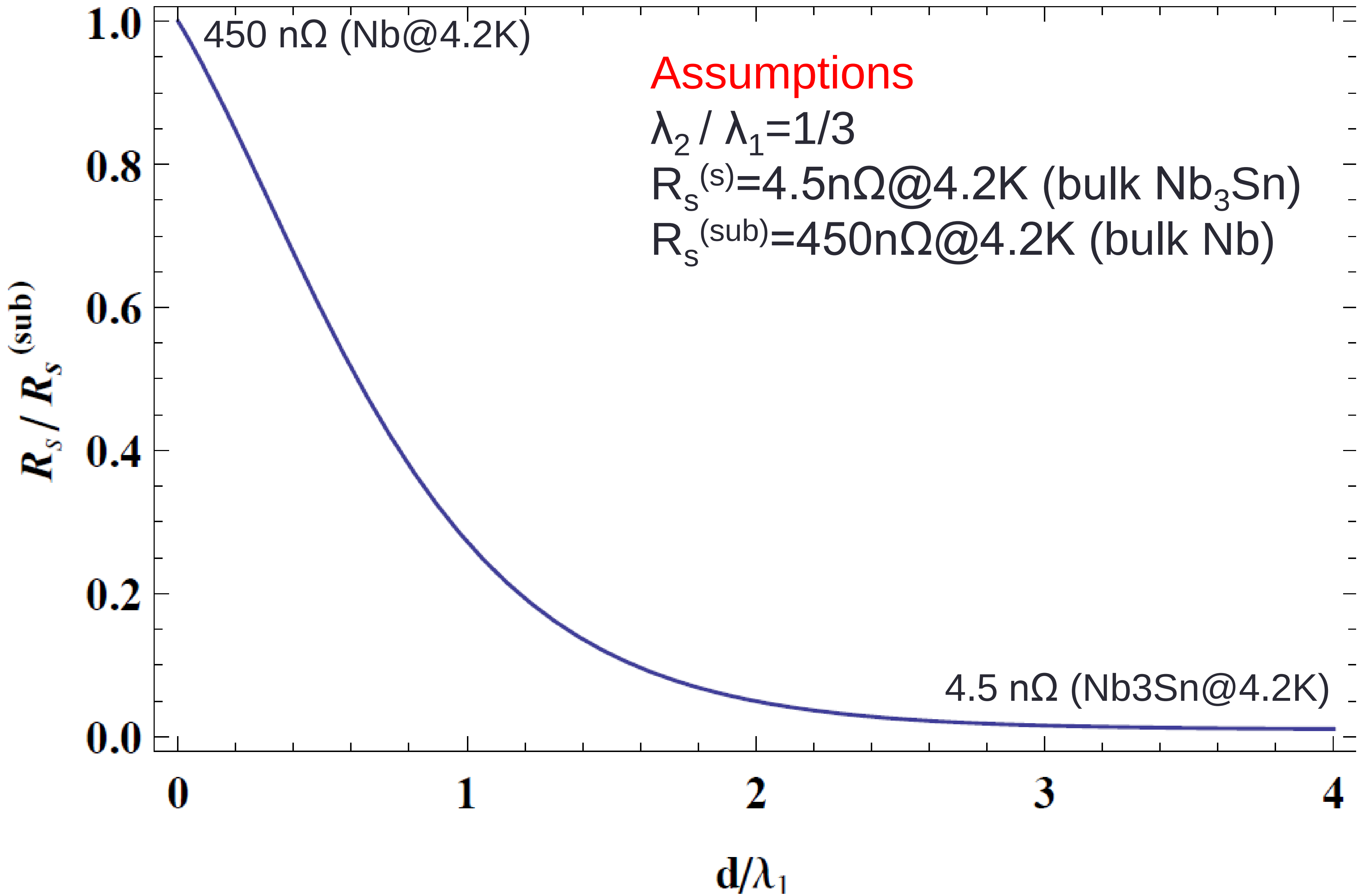
λ_1 , λ_2, d, $R_s^{(S)}$, $R_s^{(sub)}$

- λ_1: λ of the surface layer
- λ_2: λ of the substrate
- d: surface layer thickness
- $R_s^{(S)}$: The surface resistance of the surface layer material with an infinite thickness
- $R_s^{(sub)}$: The surface resistance of the substrate

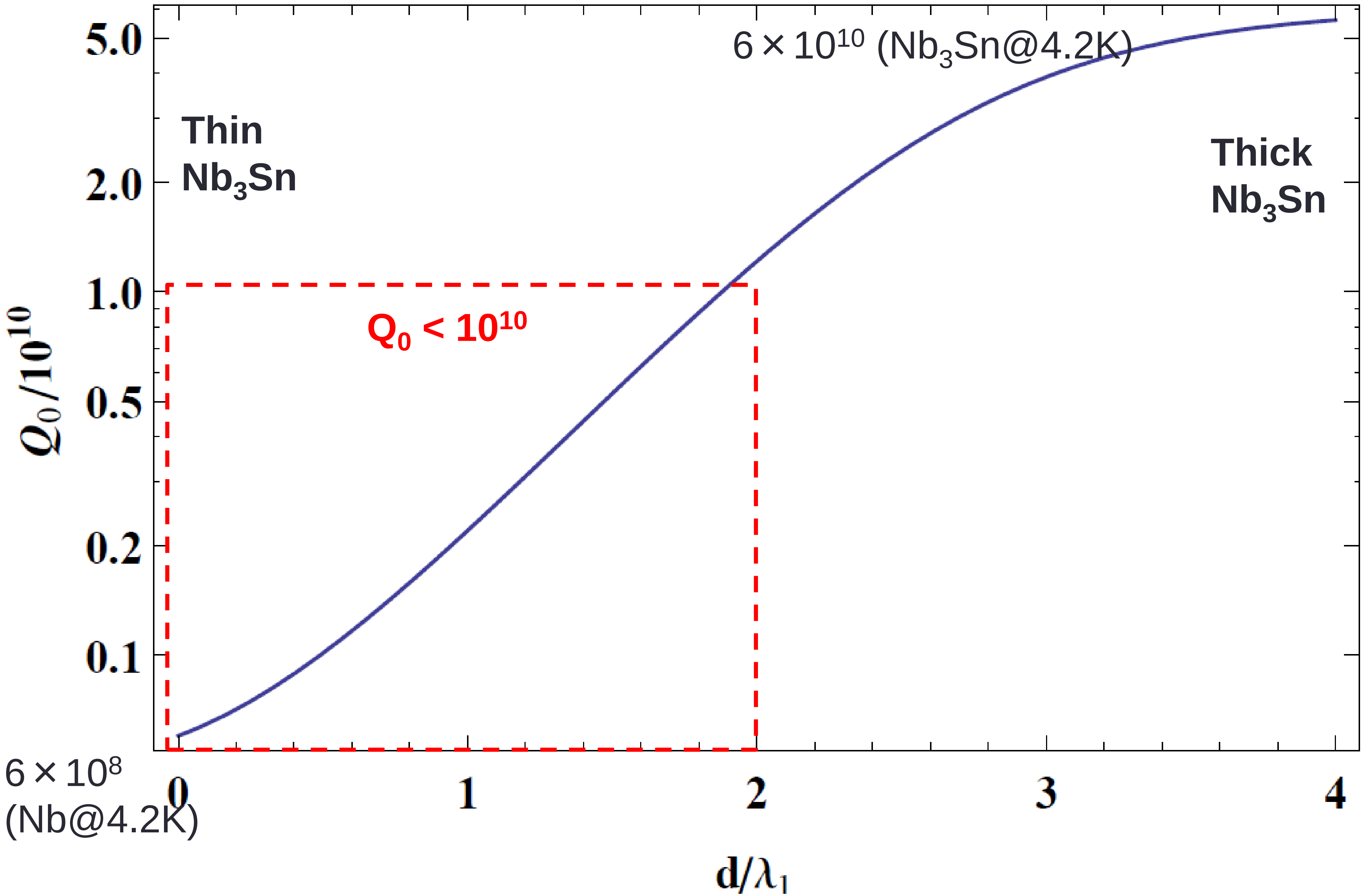
Example1: thin N rich Nb layer on clean Nb substrate



Example2: Nb_3Sn layer on clean Nb substrate@4.2K



Example2: Nb_3Sn layer on clean Nb substrate@4.2K



Summary

Some features of the S-S bilayer structure had already been investigated through the studies of the S-I-S structure.
In this talk, I have reviewed

① **The theoretical field limit.** Note that just a theoretical limit for ideal case. A gimmick to stabilize the Meissner state is necessary to achieve such a high field. That is the following second topic.

- T. Kubo**,** in proceedings of LINAC14, Geneva, Switzerland (2014), p. 1026, THPP074
- A. Gurevich, AIP Advance **5**, 017112 (2015) [see 4th paragraph in p.4]

② **The role of S-S boundary.** While it is not as robust as the insulator of S-I-S, it would contribute to preventing penetration of vortices.

- T.Kubo**,** in proceedings of LINAC14, Geneva, Switzerland (2014), p. 1026, THPP074

③ **The surface resistance of the S-S bilayer structure**

- A. Gurevich, AIP Advance **5**, 017112 (2015)
- T. Kubo, arXiv:1607.01495

All the contents in this talk are contained in

arXiv:1607.01495