

SUSY (and other BSM?) Messages for ILC and CLIC

Philip Bechtle



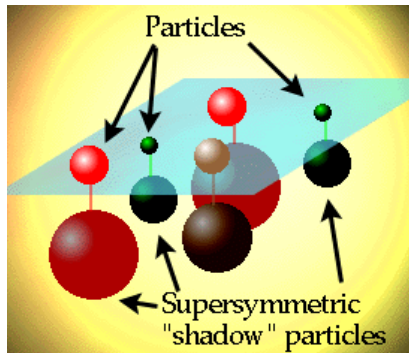
September 9th 2016

- 1 What do we know about Supersymmetry, and from where?
- 2 Excluding Supersymmetric Models
- 3 Regardless of the GUT-model, what scenario could still be realized?
- 4 Other than SUSY

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Supersymmetry

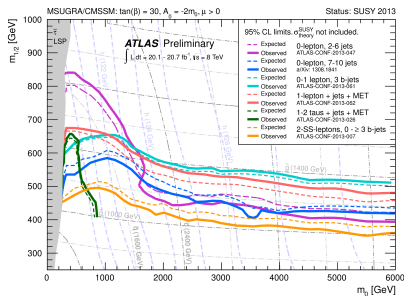
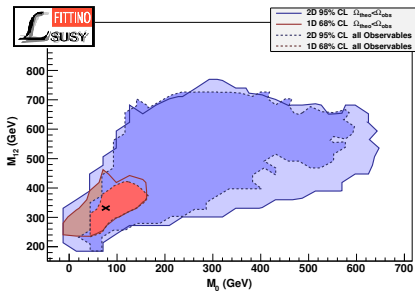
- We need Dark Matter, a stable Higgs mass and an explanation for EWSB...



In any case: $m_{Hlike} < 1 \text{ TeV}$
 $m_{SUSY} \leq \mathcal{O}(\text{TeV})$
 \Rightarrow Terascale

- Introduce shadow world:
One SUSY partner for each SM d.o.f.
- Nice addition for free: If R -parity conserved, automatically the Lightest SUSY Particle (LSP) is a stable DM candidate
- But: Where are all those states?
- SUSY breaking introduces a lot of additional parameters
Understand model: Measure parameters!

An Incomplete Overview of the Current Situation



e.g. [arXiv:0907.2589](https://arxiv.org/abs/0907.2589) [hep-ph]

- There obviously is a problem.
- Can we predict SUSY for the ILC at all?

The status of the CMSSM



healthy?

The status of the CMSSM



healthy?



pretty dull?

The status of the CMSSM



healthy?



pretty dull?



almost dead?

The status of the CMSSM



healthy?



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almost dead?



buried?

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Measurements

Measurements

$\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$	$(3.20 \pm 1.50 \pm 0.76) \times 10^{-9}$
$\mathcal{B}(B^\pm \rightarrow \tau^\pm \nu)$	$(0.72 \pm 0.27 \pm 0.11 \pm 0.07) \times 10^{-4}$
$\mathcal{B}(b \rightarrow s \gamma)$	$(3.43 \pm 0.21 \pm 0.07 \pm 0.23) \times 10^{-4}$
Δm_s	$(17.719 \pm 0.043 \pm 4.200) \text{ ps}^{-1}$
$a_\mu - a_\mu^{\text{SM}}$	$(28.7 \pm 8.0 \pm 2.0) \times 10^{-10}$
Ωh^2	0.1187 ± 0.0017
m_W	$80.385 \pm 0.015 \pm 0.010$
m_t	$(173.18 \pm 0.94) \text{ GeV}$
$\sin^2 \theta_{\text{eff}}$	0.23113 ± 0.00021

+ all kinds of limits

+ Higgs mass and rate information

Higgs, Searches and Astrophysics

Direct searches for sparticles and Higgs Bosons

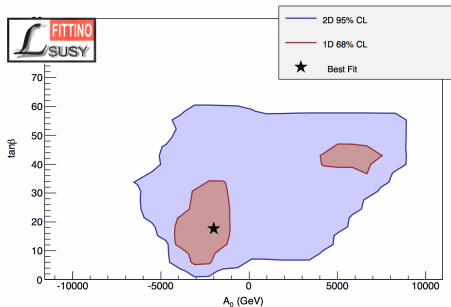
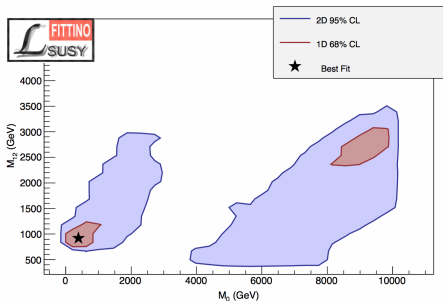
- Higgs limits via **HiggsBounds**
- Higgs signals via **HiggsSignals**
- LEP chargino mass limit
- ATLAS MET + jets + 0 lepton search (20fb^{-1})

Astrophysical observables

- We require χ_1^0 to be the LSP
- Dark matter relic density:
 $\Omega_{\text{CDM}} h^2 = 0.1187 \pm 0.0017 \pm 0.0119$ (Planck '13)
- Direct detection limit LUX ('13)

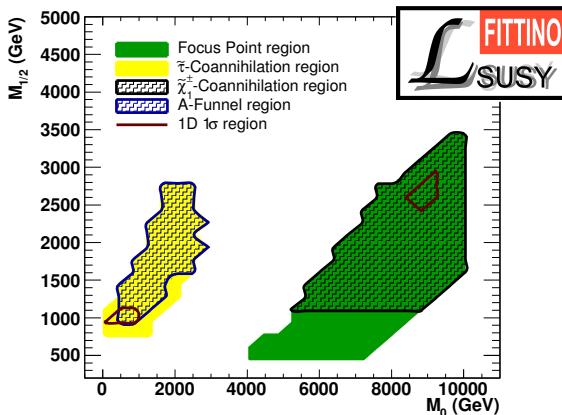
“Allowed” Parameter Range in the Fit

The following results from <http://arxiv.org/abs/1508.05951>

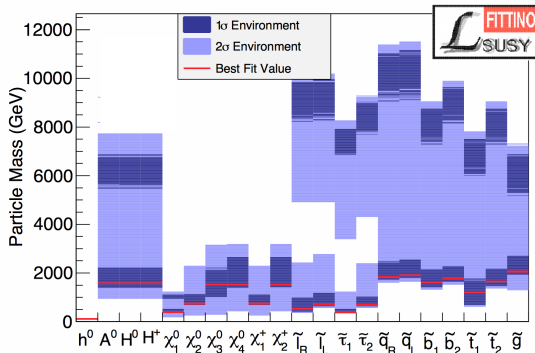
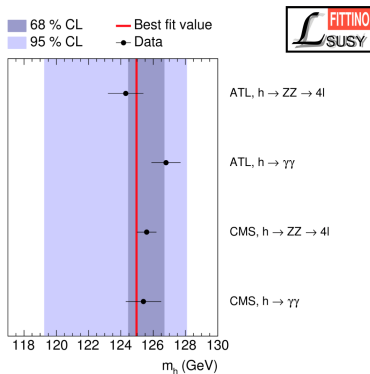


Classifying the “Allowed” Regions

- $\tilde{\tau}_1$ coannihilation: $m_{\tilde{\tau}_1}/m_{\tilde{\chi}_1^0} - 1 < 0.15$
- \tilde{t}_1 coannihilation: $m_{\tilde{t}_1}/m_{\tilde{\chi}_1^0} - 1 < 0.2$
- $\tilde{\chi}_1^\pm$ coannihilation: $m_{\tilde{\chi}_1^\pm}/m_{\tilde{\chi}_1^0} - 1 < 0.1$
- A/H funnel: $|m_A/2m_{\tilde{\chi}_1^0} - 1| < 0.2$
- focus point region: $|\mu/m_{\tilde{\chi}_1^0} - 1| < 0.4$

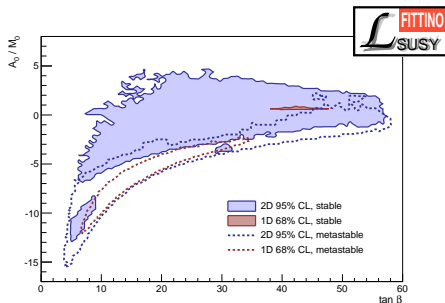
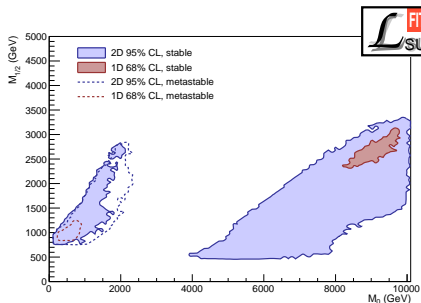


For ILC/CLIC: Predicted Ranges of SUSY Particle Masses

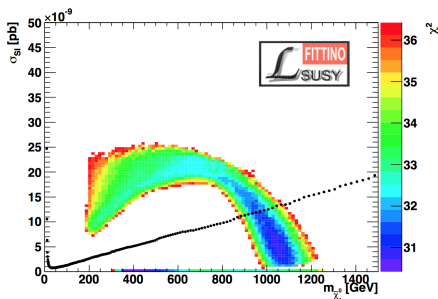
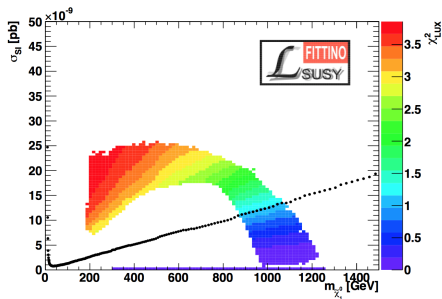


But is that stable on cosmological timescales?

- Using **VeVacious** [arXiv:1307.1477](https://arxiv.org/abs/1307.1477) [hep-ph]
- All minima in stable or metastable (lifetime \gg age of the universe) regions



Sensitivity of Direct Detection Experiments

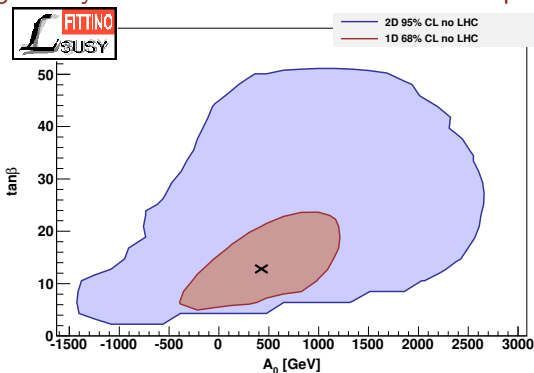


Contributions from Direct Detection
No contributions from Indirect Detection

Why are global fits of SUSY so CPU-consuming?

- ... and impossible with **naively** employing Minuit?
- This is an old result – just for education!

Looking at any correlations for all other allowed parameters:

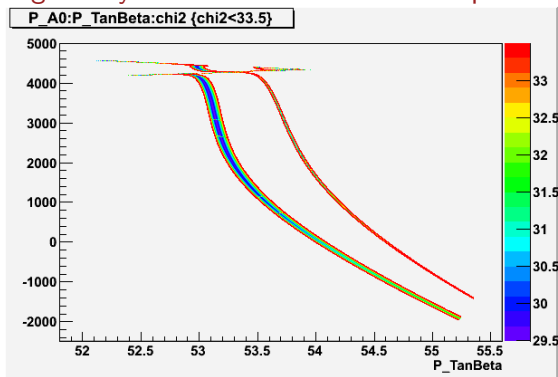


Looks OK

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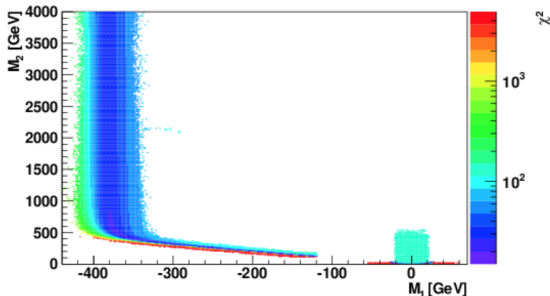


Looks Terrible

Is that a problem of GUT-scale models only?

- ... and impossible with **naively** employing Minuit?
- This is a work-in-progress result

Testing the pMSSM11 for M_1 and M_2 only:

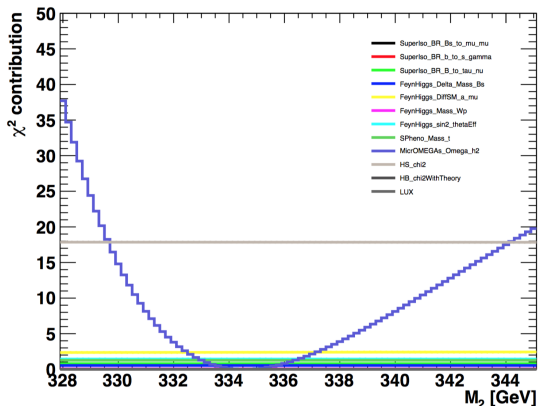


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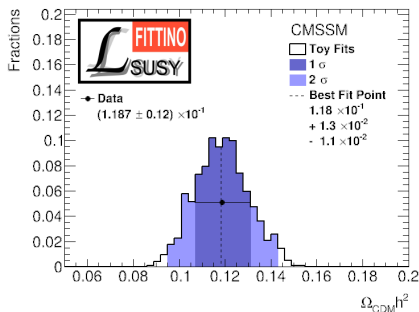
- ... and impossible with **naively** employing Minuit?
- This is a work-in-progress result

From which observable does that come?

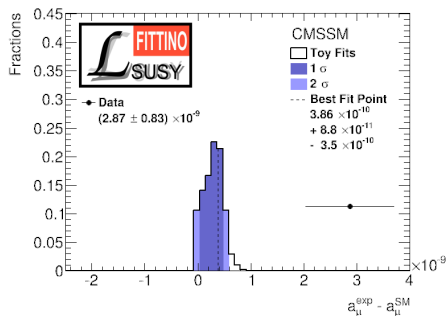
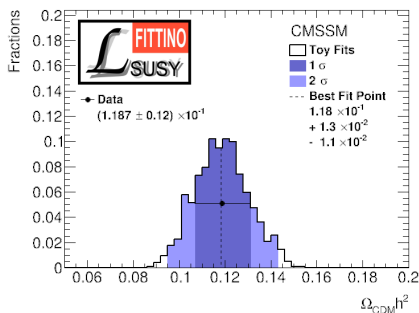


Measuring Ωh^2 precisely makes fits harder

Can we predict anything for the ILC in the cMSSM?

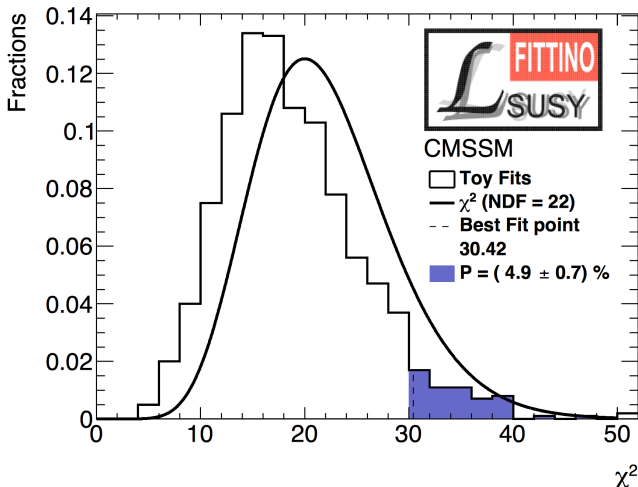


Can we predict anything for the ILC in the cMSSM?



- Most observables are fitted fine in the CMSSM, but not $(g - 2)_{\mu}$

What is the \mathcal{P} -value of the CMSSM?

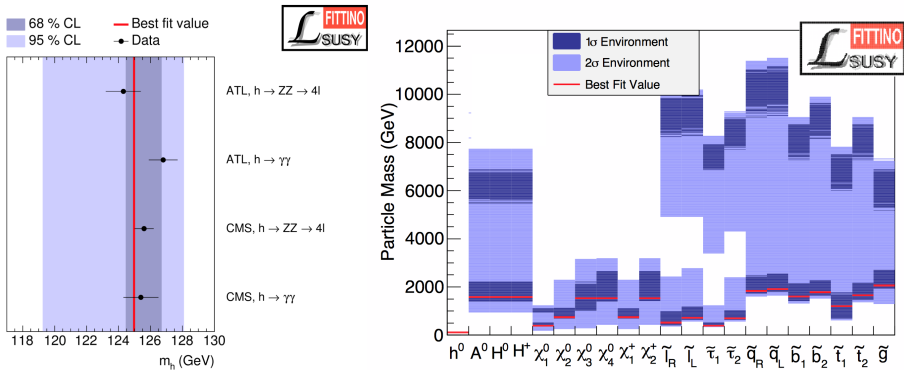


- For the first time, it has conclusively been shown that the most constrained popular SUSY model can be excluded
- Without $(g-2)_\mu$, the \mathcal{P} -value with the given observable set is $51 \pm 3\%$

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For ILC/CLIC: Predicted Ranges of SUSY Particle Masses

We are still working on the pMSSM11, so let's use another result on the pMSSM10 here...



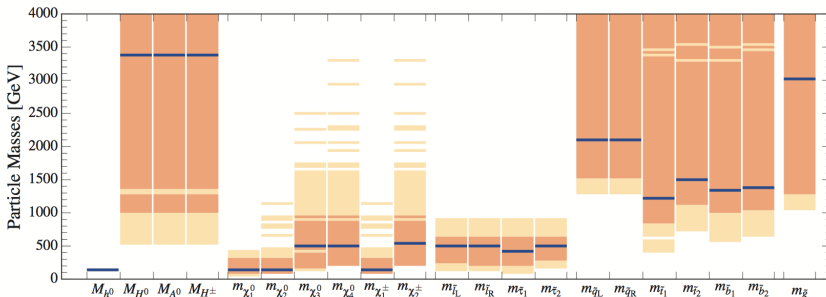
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pMSSM10 prediction: best-fit masses

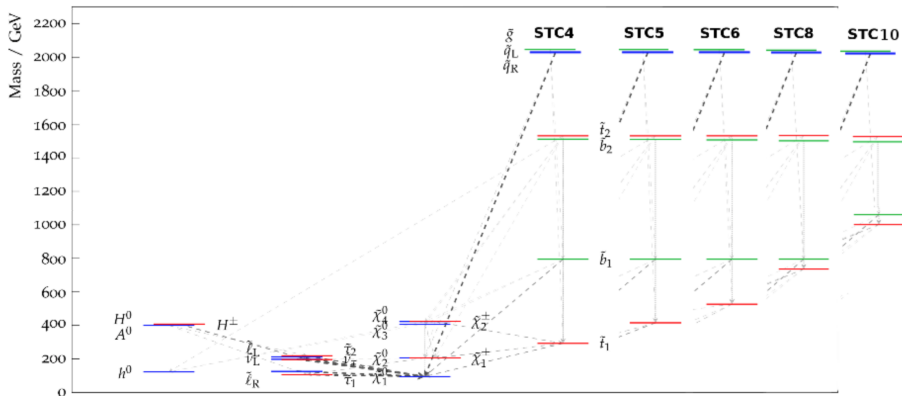


[2015]



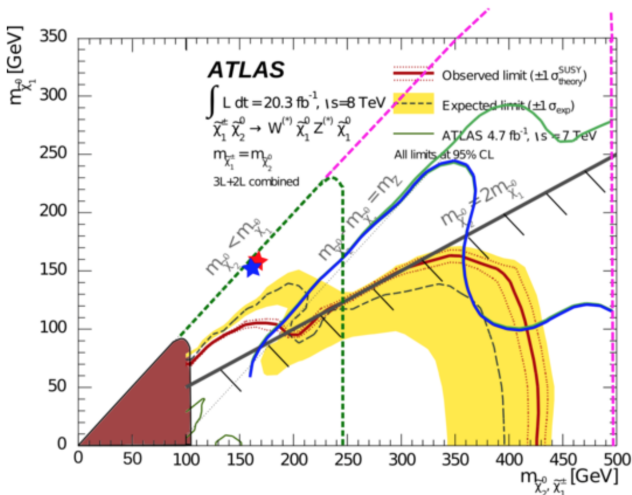
Proposals for ILC/CLIC Benchmark Models

By H. Baer, J. List [arXiv:1307.0782](https://arxiv.org/abs/1307.0782) [hep-ph]



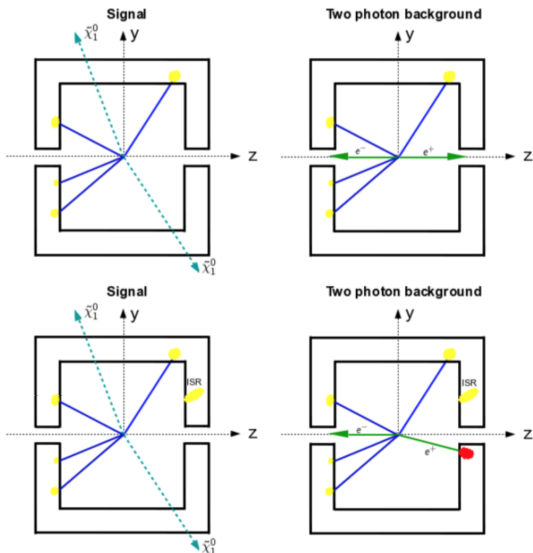
Measuring such a Scenario

E.g. by M. Berggren *Nucl. and Particle Physics Proc.* 273-275 (2016) 577-583



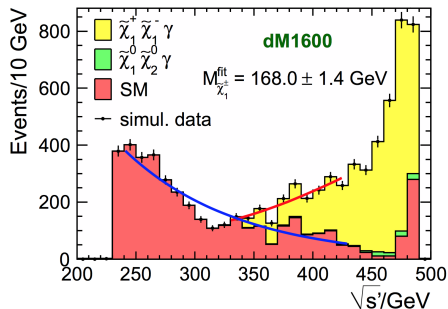
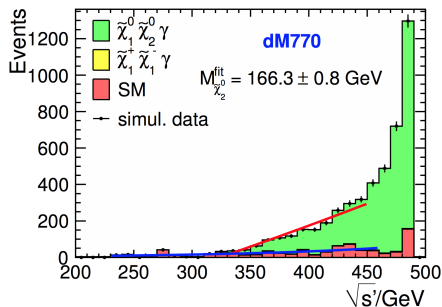
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(2016) 577-583



Cross-check Ωh^2 in such a scenario at the ILC?

By S. Lehtinen, M. Berggren and J. List

<http://arxiv.org/pdf/1602.08439.pdf>

observable	uncertainty	observable	uncertainty
$m_{\tilde{e}_R}$	0.17%	$m_{\tilde{\mu}_R}$	0.40%
$m_{\tilde{e}_L}$	1%	$m_{\tilde{\mu}_L}$	1%
$m_{\tilde{\tau}_1}$	0.16%	$m_{\tilde{\tau}_2}$	2.5%
θ_τ	1%	A_τ	20%
$m_{\tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau}$	1%	$m_{\tilde{\chi}_1^\pm}$	1%
$m_{\tilde{\chi}_1^0}$	0.15%	$m_{\tilde{\chi}_2^0}$	0.5%
$N_{12,13,14}$	1% each	U_{mix}, V_{mix}	20% each
$m_{\tilde{\chi}_3^0, \tilde{\chi}_4^0}$	1%	$m_{\tilde{\chi}_2^\pm}$	1%
m_{H_0, A_0, H^\pm}	1%		

Table 4: STC8 particles observed at the 1 TeV ILC

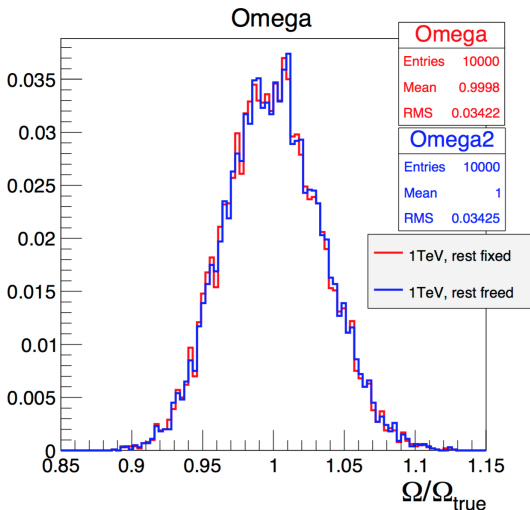
observable	range	observable	range
$m_{\tilde{d}_L, \tilde{u}_L, \tilde{s}_L, \tilde{c}_L}$ all equal	$1 \rightarrow 50$ TeV	$m_{\tilde{d}_R, \tilde{u}_R, \tilde{s}_R, \tilde{c}_R}$	$= m_{\tilde{d}_L} - 100$ GeV
$m_{\tilde{t}_1, \tilde{t}_2, \tilde{b}_1, \tilde{b}_2}$ independent	$0.6 \rightarrow 50$ TeV	$m_{\tilde{g}}$	$1 \rightarrow 50$ TeV
$\theta_{t,b}$	$0 \rightarrow \pi/2$	$A_{t,b}$	$0 \rightarrow -5000$

Table 5: STC8 particles not observed at the 1 TeV ILC

Cross-check Ωh^2 in such a scenario at the ILC?

By S. Lehtinen, M. Berggren and J. List

<http://arxiv.org/pdf/1602.08439.pdf>



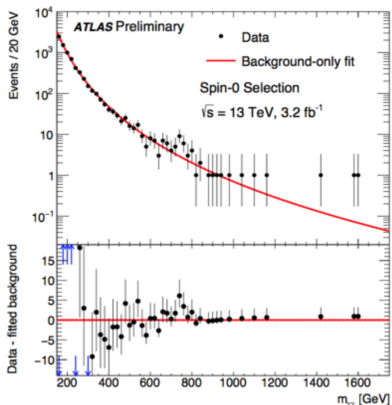
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A little dream (unfortunately not come true)

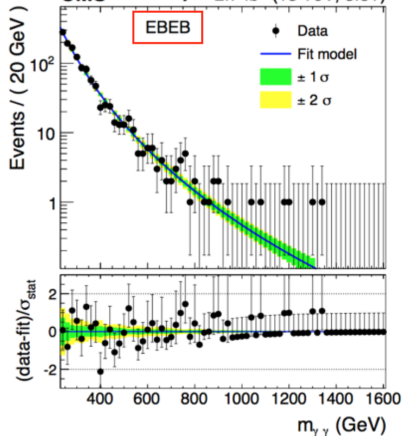
Many nice other BSM constraints at ILC/CLIC, from top precision couplings to TGCs, but hard to predict from current measurements...

By F. Richard *et al.* [arXiv:1607.03829v2](https://arxiv.org/abs/1607.03829v2) [hep-ph]

ATLAS-CONF-2016-018

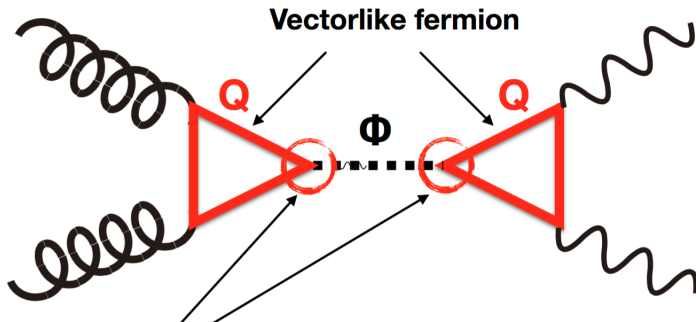


CMS Preliminary 2.7 fb⁻¹ (13 TeV, 3.8T)



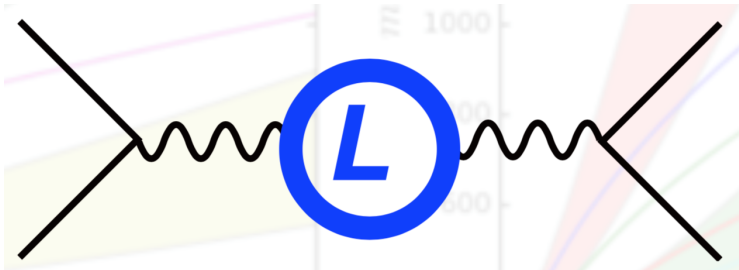
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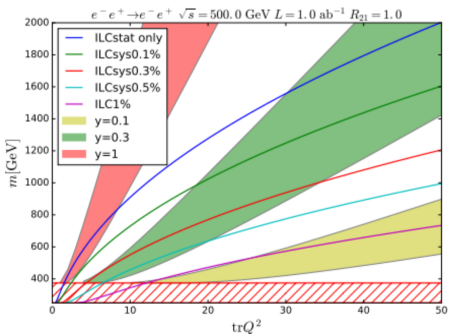
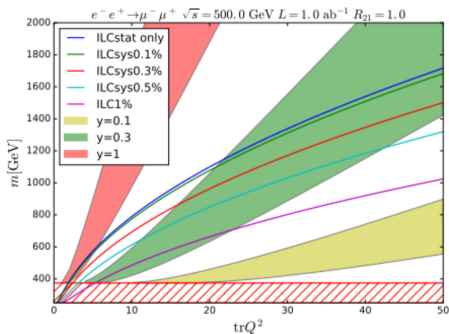
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Conclusions

- We have the Higgs, it needs to be explained somewhat naturally
- We have the LHC, we desperately would like it to find something else
- We want the ILC, we know it could do great physics on the Higgs, but would it find something else?
- We want to find something like SUSY, but . . .

Conclusions

- We have the Higgs, it needs to be explained somewhat naturally
- We have the LHC, we desperately would like it to find something else
- We want the ILC, we know it could do great physics on the Higgs, but would it find something else?
- We want to find something like SUSY, but . . .
- The cMSSM is somewhere between extremely dull and completely dead
- More general SUSY is still possible, and we could re-connect to cosmology by precisely predicting the relic density from ILC measurements!

Backup Slides

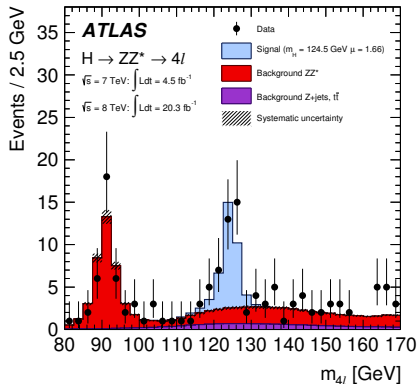
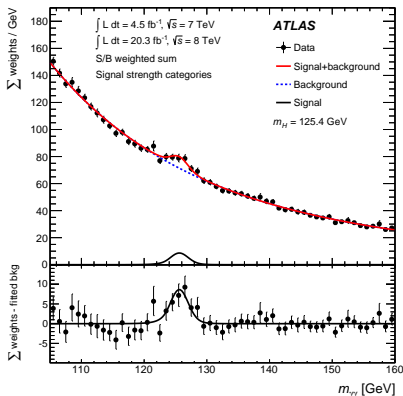
Particle Physics 2015 – Exhilarating or Dull?

The LHC Experiments are an incredible success



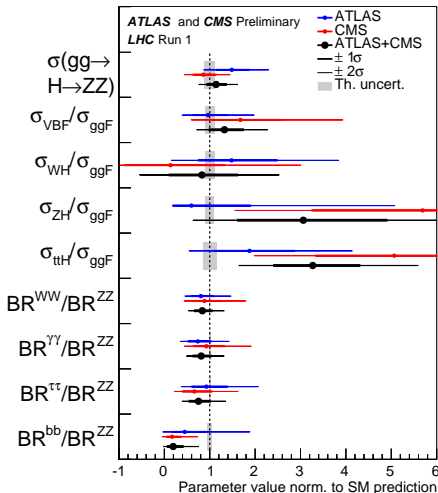
Particle Physics 2015 – Exhilarating or Dull?

We found a Higgs Boson



Particle Physics 2015 – Exhilarating or Dull?

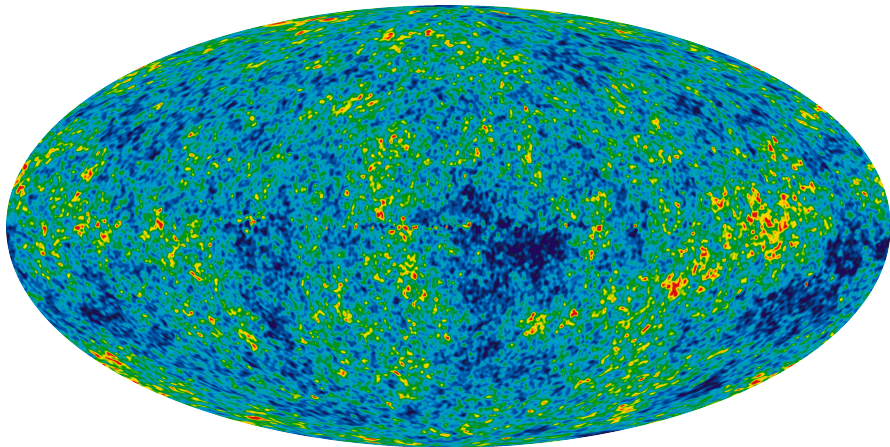
Its properties are consistent with the SM Higgs



ATLAS-CONF-2015-044, <http://cds.cern.ch/record/2052552>

Particle Physics 2015 – Exhilarating or Dull?

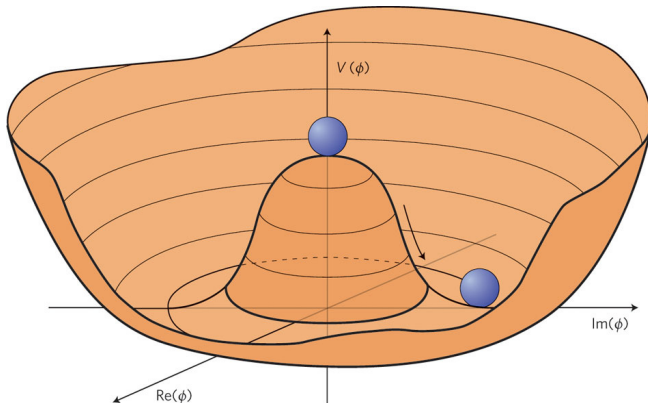
The Elephant outside the room



Somehow that can't be dull ...

The Greatest Mystery of the Standard Model

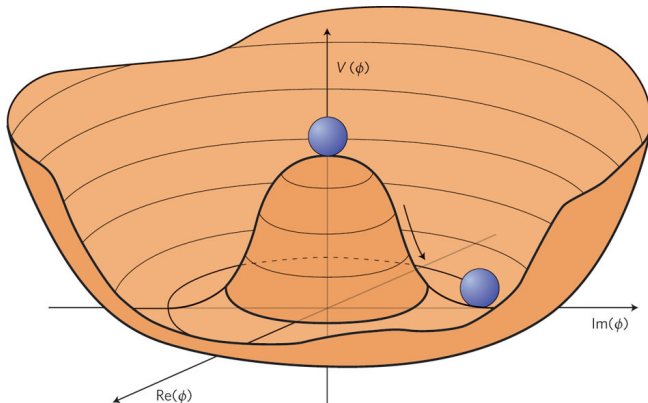
- Sure, the Higgs explains the mass



- $V(\Phi) = -\mu^2|\Phi|^2 + \lambda|\Phi|^4$

The Greatest Mystery of the Standard Model

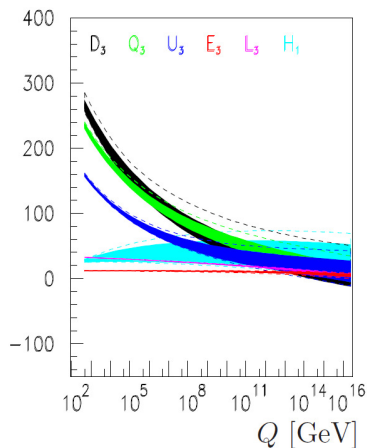
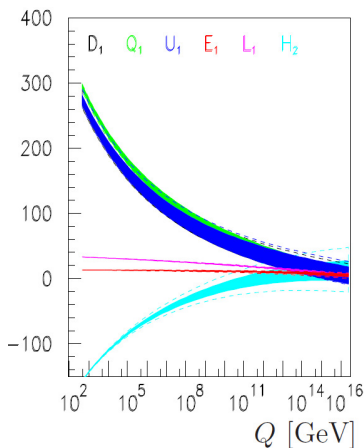
- Sure, the Higgs explains the mass



- $V(\Phi) = -\mu^2|\Phi|^2 + \lambda|\Phi|^4$
- But come on, is that an explanation? Also, the mass is not even stable under radiative corrections...

Explaining the Higgs Potential

- Naturally include $V(\Phi) = -\mu^2|\Phi|^2 + \lambda|\Phi|^4$ through RGE running for large m_t



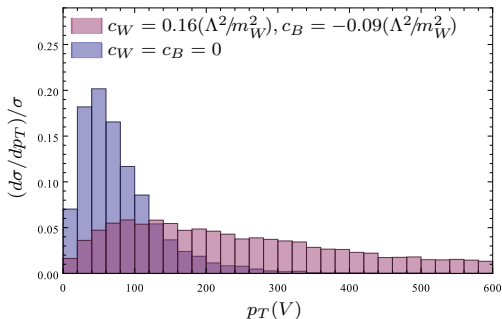
- Example from [arXiv:hep-ph/0511006v2](https://arxiv.org/abs/hep-ph/0511006v2)

Why do it differently: Kinematic (p_T) distributions

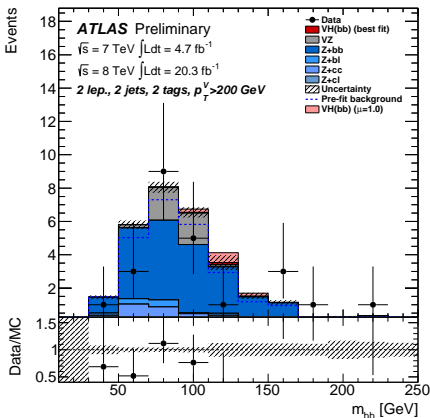
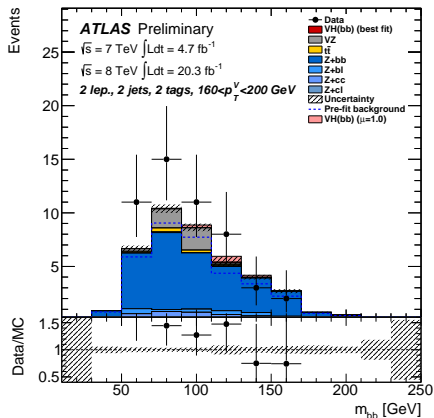
- In EFT approach: can have operators with different tensor structure
 \Rightarrow potential changes in kinematic distributions
 (while inclusive rate might be unaffected)

Look at the ATLAS search $pp \rightarrow VH \rightarrow V(b\bar{b})$

[Biekötter, Knochel, Krämer, Liu, Riva, 1406.7320]



What can be used?



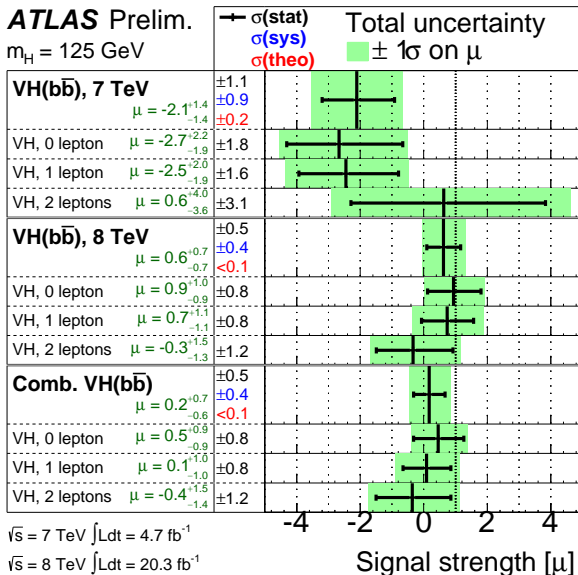
ATL-CONF-2013-079

What can be used?

2-jet, 2-tag sample													
Process	0-lepton			1-lepton					2-lepton				
	E_T^{miss} [GeV]			p_T^W [GeV]					p_T^Z [GeV]				
	120-160	160-200	>200	0-90	90-120	120-160	160-200	> 200	0-90	90-120	120-160	160-200	>200
$Z \rightarrow \nu\nu$	1.6	0.9	1.0	<0.1	<0.1	<0.1	<0.1	<0.1	<0.1	<0.1	<0.1	<0.1	<0.1
$Z \rightarrow \ell\ell$	<0.1	<0.1	<0.1	0.2	<0.1	<0.1	<0.1	<0.1	2.1	0.5	0.4	0.2	0.2
$W \rightarrow \ell\nu$	0.4	0.2	0.2	7.6	1.7	1.2	1.0	1.1	<0.1	<0.1	<0.1	<0.1	<0.1
VH total	2.0	1.1	1.1	7.8	1.8	1.2	1.1	1.1	2.1	0.5	0.4	0.2	0.2
VH expected	11	5.8	6.1	42	9.5	6.6	5.6	6.1	11	2.7	2.2	1.1	1.2
Top	159	33	8	2763	729	359	113	40	166	32	8.0	0.5	<0.1
W+c, light	21	5.3	2.7	616	65	27	12	7.8	<0.1	<0.1	<0.1	<0.1	<0.1
W+b	30	10	6.1	909	106	49	25	19	<0.1	<0.1	<0.1	<0.1	<0.1
Z+c, light	23	8.1	5.2	22	2.1	0.5	0.3	0.1	91	12	5.6	1.6	1.0
Z+b	226	71	39	97	13	3.9	1.8	0.5	938	146	64	14	8.3
WW	0.5	0.1	0.1	11	1.0	0.7	0.3	0.2	<0.1	<0.1	<0.1	<0.1	<0.1
VZ	26	11	10	145	20	12	7.6	6.5	60	8.6	4.5	2.2	2.1
Multijet	4.8	1.1	0.7	1306	45.6	8.7	4.8	0.4	<0.1	<0.1	<0.1	<0.1	<0.1
Total Bkg.	491 ± 10	141 ± 3	72 ± 2	5869 ± 64	981 ± 16	460 ± 9	165 ± 4	74 ± 3	1255 ± 24	199 ± 4	82 ± 2	18 ± 1	11.4 ± 0.5
Data	502	143	90	5916	990	458	162	79	1282	204	70	22	6
S/B	0.004	0.008	0.02	0.001	0.002	0.003	0.006	0.02	0.002	0.003	0.005	0.01	0.02

ATL-CONF-2013-079

What could we compare to, just as a simple test?



How to make use of kinematic distributions?

Example: ATLAS search for $pp \rightarrow VH \rightarrow V(b\bar{b})$ ATLAS-CONF-2013-079

- different event selections / kinematic regions:
(2 or 3 jets) \otimes (0, 1 or 2 leptons) \otimes (3 E_T^{miss} or 5 p_T^V bins)

\Rightarrow 26 categories: N_{obs} , N_{BG} , ΔN_{BG} , N_S^{SM} publicly available (Table 5)

- But: no coherent information on correlations
- Just for testing: Layman calculation:

$$\mu_i = \frac{N_{\text{obs}}^i - N_{\text{BG}}^i}{N_S^{\text{SM},i}},$$

$$\delta\mu_i = \frac{\sqrt{N_{\text{obs}}^i + \Delta N_{\text{BG}}^i{}^2}}{N_S^{\text{SM},i}} \oplus \frac{\Delta N_S^{\text{SM}}}{N_S^{\text{SM}}} \cdot \mu_i$$

- combination of μ^i (neglecting correlations):

$$\mu_{0\ell} = 1.15 \pm 1.06 \quad (\text{ATLAS: } 0.5 \pm 0.9)$$

$$\mu_{1\ell} = 0.20 \pm 0.93 \quad (\text{ATLAS: } 0.1 \pm 1.0)$$

$$\mu_{2\ell} = -1.70 \pm 1.79 \quad (\text{ATLAS: } -0.4 \pm 1.5)$$

\Rightarrow unfortunately unable to reproduce

What would be necessary?

- This is only a very rough first test, maybe others have made more thorough studies
- Still, it has been independently tested by 4 people, with the same result

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- This is only a very rough first test, maybe others have made more thorough studies
- Still, it has been independently tested by 4 people, with the same result
- Of course you can say that it is not necessary that phenomenologists can use our kinematic distributions in fits.
- Unfolded distributions might improve the situation, but correlations would still be lacking, so still (other?) challenges for independent fits
- I can only speculate about the concrete **minimal** additional information which would improve this situation, but a full set of μ 's in all 26 subchannels with a full experimental covariance matrix for bg and signal uncertainties (seperately) might be a starting point?

The obvious Likelihood Based Solution

- Let's provide a rather complex function:

$$\mathcal{L}(d, P) = p_{\mu}(d|m_h, \mu_c, c, N_{jet}, p_T; \vec{\eta}_b, \vec{\eta}_s) p(\vec{\eta}_b|\hat{\vec{\eta}}_b) p(\vec{\eta}_s|\hat{\vec{\eta}}_s)$$

where p_{μ} contains all **correlations** between all subchannels and all kinematic subdivisions, and where

- c : subchannel
- $\vec{\eta}$: scale factor for the theory uncertainties on b, s
- $\hat{\vec{\eta}}$: input scale factor for the theory uncertainties on b, s chosen by the user
- These must be vectors, separately for α_s , pdf's, \dots , and for different production modes, decay modes, etc.
- Could maybe be handled. All **internal** nuisance parameters of the experiments would be profiled out.
- Correlations between experimental nuisance parameters and theory nuisance parameters are ignored (probably rightfully so)**
- Should be fast. FULL parametrization of the outcome of the PL fit after profiling out all experimental systematics.
- Provide all acceptances, efficiencies, compositions of all subchannels**
- After we formulated that: Turned out to be practically what Kyle, Tilman et al. already proposed**

The not so obvious gaussian approximation

- Just a short overview here:
- Provide all acceptances, efficiencies, compositions of all subchannels
- In principle it's easy: N measurements of $\hat{\mu}_i$ ($i = 1, \dots, N$) in subchannels, kinematic bins, etc.
- Has a covariance matrix $\mathbf{C} = C_{ii'} = \rho_{ii'} \sigma_i \sigma_{i'}$
- **But:** $\mathbf{C}_{ii'}$ needs to be decomposed into different error sources
- Idea (only roughly written here): Decompose $\mathbf{C}_{ii'}$ into individual matrices

$$\mathbf{C}_{ii'} = \sum_j \mathbf{C}_{ii'}^j$$

where the $\mathbf{C}_{ii'}^j$ represent the uncertainty for each individual error source for each component (e.g. ggF might have a different scaling of its theory error in a new physics model than VBF, same for final states, etc)

- Then, the uncertainties in the individual matrices can be scaled
- Looks simple, but fully formulated it can become a bit ugly, too.

For the Future: The Likelihood Based Solution

- Let's provide a rather complex function:

$$\mathcal{L}(d, P) = p_{\mu}(d|m_h, \vec{\mu}_c, N_{jet}, p_T, \dots; \vec{\eta}_b, \vec{\eta}_s) p(\vec{\eta}_b|\hat{\vec{\eta}}_b) p(\vec{\eta}_s|\hat{\vec{\eta}}_s)$$

where p_{μ} contains all **correlations** between all subchannels and all kinematic subdivisions, and where

- c : subchannel, including kinematic bins, etc
- $\vec{\eta}$: scale factor for the theory uncertainties on b, s
- $\hat{\vec{\eta}}$: input scale factor for the theory uncertainties on b, s chosen by the user
- These must be vectors, separately for α_s , pdf's, \dots , and for different production modes, decay modes, etc.
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HiggsSignals

The program HiggsSignals

(PB, S. Heinemeyer, O. Stal, T. Stefaniak, G. Weiglein,
arXiv:1305.1933, arxiv:1403.1582)

- evaluates the total χ^2 for both the signal strengths and/or the mass measurements,
- featuring two distinct χ^2 methods (peak- and mass-centered),
- includes correlations among the major **externally accessible** systematic uncertainties (cross sections, branching ratios, luminosity, theory mass uncertainty),
- includes many more features:
 - It finds best assignment of Higgs bosons to the signal and automatically combines signal rates of Higgses overlapping within mass resolution,
 - Framework to include signal efficiencies,
 - New (even hypothetical) signals can be implemented by the user,
 - Toy measurements can be given to existing observables for statistical studies,
 - Signal rate uncertainties can be scaled for future projections,
 - ...

HiggsSignals is a stand-alone program using the HiggsBounds libraries. Coding language is Fortran90/2003.

HiggsSignals: The basic idea

- 1 Take model-predictions of a given (arbitrary) Higgs sector for

$$m_k, \quad \Gamma_k^{\text{tot}}, \quad \sigma_i(pp \rightarrow H_k), \quad \text{BR}(H_k \rightarrow XX),$$

with $k = 1, \dots, N$, $i \in \{\text{ggH}, \text{VBF}, \text{WH}, \text{ZH}, t\bar{t}H\}$

for N neutral Higgs bosons as the **program's user input**.

Optional input: **Theo. uncertainties for mass, cross sections and BR's.**

- 2 Calculate the predicted signal strength μ for every observable.
- 3 Perform a χ^2 **test** of model predictions against all available data from Tevatron and LHC, using **signal rate** and **mass measurements**.

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The aim is to be as

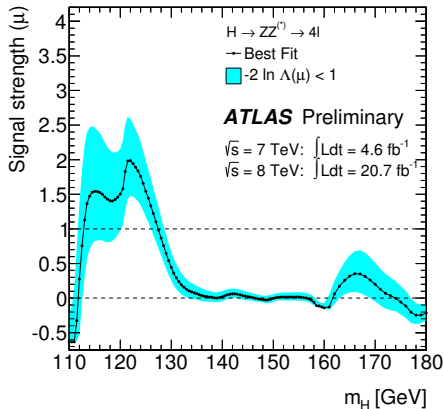
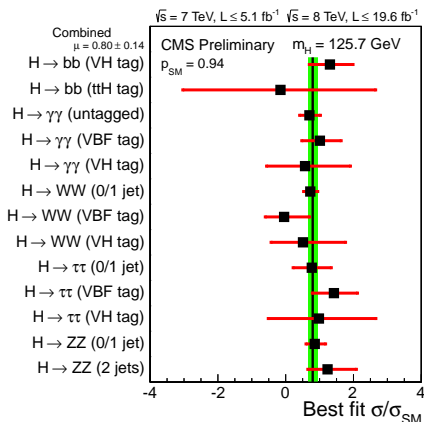
- **model-independent as possible**,
- **precise as possible** (given the limited public information available)

Experimental input

- Signal strength measurements:

$$\mu_{H \rightarrow XX} = \frac{\sum_i \epsilon_i^{\text{model}} [\sigma_i(pp \rightarrow H) \times \text{BR}(H \rightarrow XX)]_{\text{model}}}{\sum_i \epsilon_i^{\text{SM}} [\sigma_i(pp \rightarrow H) \times \text{BR}(H \rightarrow XX)]_{\text{SM}}},$$

with $i \in \{\text{ggH}, \text{VBF}, \text{WH}, \text{ZH}, t\bar{t}H\}$ and efficiencies ϵ_i .



Experimental input

The user can directly add/remove/edit observables via text files:

```
# Published at Moriond 2013.
# Data read in from Fig. 25a.
# No efficiencies are given (for this inclusive result)
# Mass uncertainty contains 0.6 GeV (stat) and 0.5 GeV (syst) error.
#(Gauss: 0.8, linear: 1.1)
2013013101 201301301 1
ATL-CONF-2013-013
LHC, ATL, ATL
(pp)->h->ZZ->4l
8 25.3 0.036
1 1
1.1
124.3 124.3 0.1
4 -1
13 23 33 43

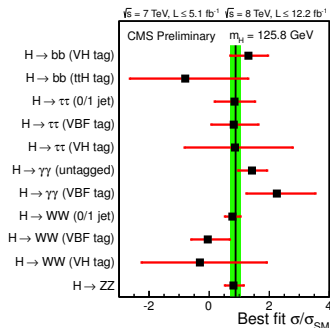
124.3          1.293          1.697          2.194
```

Peak-centered χ^2 method

- Tests agreement between model and data at the observed mass.
- Define observables by the best-fit signal strength, $\hat{\mu}_i$, at a hypothetical Higgs mass \hat{m}_i .
- The total χ^2 consists of a signal strength and a Higgs mass part,

$$\chi_{total}^2 = \chi_{\mu}^2 + \sum_{\text{assigned Higgses } i} \chi_{m_i}^2$$

- Only analyses with a good mass measurement enter $\chi_{m_i}^2$ ($H \rightarrow \gamma\gamma, ZZ$)
- Can be evaluated at different \hat{m}_i for each measurement
- Assign carefully chosen penalties if predicted Higgs m_i is too far off from \hat{m}_i



Good method to get a global picture on Higgs coupling properties.

Efficiencies

Essential information! Is included in **HiggsSignals** if available.

Expected signal and estimated background									
Event classes		SM Higgs boson expected signal ($m_H=125\text{ GeV}$)							Background $m_{\gamma\gamma} = 125\text{ GeV}$ (ev./GeV)
		Total	ggH	VBF	VH	ttH	σ_{eff} (GeV)	FWHM/2.35 (GeV)	
7 TeV 5.1 fb ⁻¹	Untagged 0	3.2	61.4%	16.8%	18.7%	3.1%	1.21	1.14	3.3 ± 0.4
	Untagged 1	16.3	87.6%	6.2%	5.6%	0.5%	1.26	1.08	37.5 ± 1.3
	Untagged 2	21.5	91.3%	4.4%	3.9%	0.3%	1.59	1.32	74.8 ± 1.9
	Untagged 3	32.8	91.3%	4.4%	4.1%	0.2%	2.47	2.07	193.6 ± 3.0
	Dijet tag	2.9	26.8%	72.5%	0.6%	–	1.73	1.37	1.7 ± 0.2
8 TeV 19.6 fb ⁻¹	Untagged 0	17.0	72.9%	11.6%	12.9%	2.6%	1.36	1.27	22.1 ± 0.5
	Untagged 1	37.8	83.5%	8.4%	7.1%	1.0%	1.50	1.39	94.3 ± 1.0
	Untagged 2	150.2	91.6%	4.5%	3.6%	0.4%	1.77	1.54	570.5 ± 2.6
	Untagged 3	159.9	92.5%	3.9%	3.3%	0.3%	2.61	2.14	1060.9 ± 3.5
	Dijet tight	9.2	20.7%	78.9%	0.3%	0.1%	1.79	1.50	3.4 ± 0.2
	Dijet loose	11.5	47.0%	50.9%	1.7%	0.5%	1.87	1.60	12.4 ± 0.4
	Muon tag	1.4	0.0%	0.2%	79.0%	20.8%	1.85	1.52	0.7 ± 0.1
	Electron tag	0.9	1.1%	0.4%	78.7%	19.8%	1.88	1.54	0.7 ± 0.1
	E_T^{miss} tag	1.7	22.0%	2.6%	63.7%	11.7%	1.79	1.64	1.8 ± 0.1

An interface to insert *relative efficiency scale factors* $\zeta^i \equiv \epsilon_{\text{model}}^i / \epsilon_{\text{SM}}^i$ per tested parameter point and analysis is provided since **HiggsSignals-1.1**. This in principle *really* allows arbitrary Higgs sectors

The χ^2 evaluation

In the χ^2 evaluation, we try to take into account the **correlations of the major systematic uncertainties**, that are publicly known. These are

- correlated **luminosity uncertainty**: $\Delta\mathcal{L}$,
- correlated **theoretical rate uncertainties**: $\Delta\sigma_i$, ΔBR_i .

Other correlations of systematics could be easily incorporated if they were public.

The global χ^2 for the signal strength measurements is then given by

$$\chi_\mu^2 = (\hat{\mu} - \mu)^T \mathbf{C}_\mu^{-1} (\hat{\mu} - \mu).$$

A similar calculation is done for the mass observables $\Rightarrow \chi_m^2$.

Complications with multiple neutral Higgs bosons

Any neutral Higgs boson could be responsible for the observed signal.

- Higgs boson i is *assigned* to the observable α , if its mass is close enough to observed signal position:

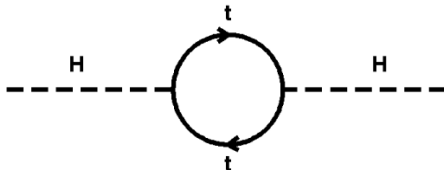
$$|m_i - \hat{m}_\alpha| \leq \Lambda \sqrt{(\Delta m_i)^2 + (\Delta \hat{m}_\alpha)^2} \quad \Rightarrow \quad \text{Higgs } i \text{ assigned}$$

with tuning parameter $\Lambda \simeq 1$ (assignment range).

- If multiple Higgs bosons are assigned, their signal strengths are added incoherently: $\mu_\alpha = \sum_i \mu_{\alpha,i}$
- If **no** Higgs boson is assigned to an observable α , its χ^2 contribution is evaluated for **zero predicted signal strength**, $\mu_\alpha = 0$.

Supersymmetry

- Even if we have found the Higgs, we still have a problem . . .



$$m_h^2 \sim \Lambda^2$$

in the presence of gravity:
natural

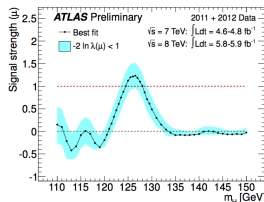
$$m_h = \Lambda = M_{Planck} \approx 10^{19} \text{ GeV}$$

Finetuning at M_{Planck} :

$$m_{h,obs}^2 = m_{h,bare}^2 + (\text{fine-tuned difference of couplings} \approx M_{Planck}^{-2}) \times M_{Planck}^2$$

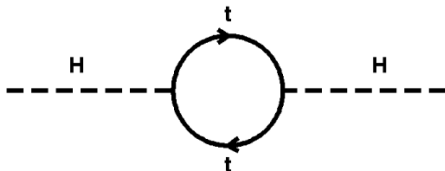
- If the new particle is the Higgs:

$$m_h \approx 126 \text{ GeV}$$



Supersymmetry

- Even if we have found the Higgs, we still have a problem . . .



only SM: $m_h^2 \sim \Lambda^2$

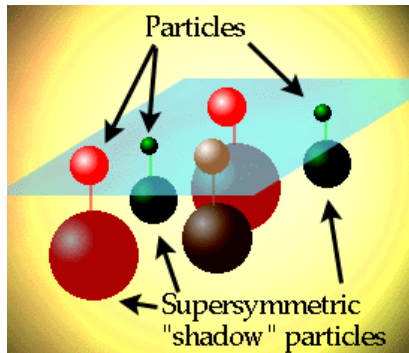


SUSY: $m_h \sim m \ln M_{SUSY}^2 / \mu^2$

- If the new particle is the Higgs:
 $m_h \approx 126 \text{ GeV}$
- To prevent quadratic divergencies:
Introduce shadow world:
One SUSY partner for each SM d.o.f.
- Nice addition for free: If R -parity conserved, automatically the Lightest SUSY Particle (LSP) is a stable DM candidate
- **But: Where are all those states?**

Supersymmetry

- Even if we have found the Higgs, we still have a problem . . .



In any case: $m_{Hlike} < 1 \text{ TeV}$
 $m_{SUSY} \leq \mathcal{O}(\text{TeV})$
 \Rightarrow Terascale

- If the new particle is the Higgs:
 $m_h \approx 126 \text{ GeV}$
- To prevent quadratic divergencies:
 Introduce shadow world:
 One SUSY partner for each SM d.o.f.
- Nice addition for free: If R -parity conserved, automatically the Lightest SUSY Particle (LSP) is a stable DM candidate
- But: Where are all those states?
- SUSY breaking introduces a lot of additional parameters
 Understand model: Measure parameters!

HiggsSignals main Ideas

HiggsSignals (PB, S. Heinemeyer, O. Stal, T. Stefaniak, G. Weiglein, arXiv:1305.1933, arxiv:1403:1582)

- Evaluates a χ^2 using a gaussian approximation of the μ measurements in all subchannels (can be asymmetric gaussians, often already quite good approximation)
- Model-independent input
- (Originates from before the collaborations published 'almost' likelihoods)
- One of the main distinctive features: Can handle **any** number of Higgs bosons, and as long as user is prepared to re-evaluate channel efficiencies: Can handle **arbitrary** Higgs sectors
- Works well as long as statistics in each subchannel is low, such that experimental correlations between subchannels are not yet too dominant

HiggsSignals Inputs

User Input (From Theory):

- Take model-predictions of a given (arbitrary) Higgs sector for

$$m_k, \quad \Gamma_k^{\text{tot}}, \quad \sigma_i(pp \rightarrow H_k), \quad \text{BR}(H_k \rightarrow XX),$$

with $k = 1, \dots, N$, $i \in \{\text{ggH}, \text{VBF}, \text{WH}, \text{ZH}, t\bar{t}H\}$

for N neutral Higgs bosons as the **program's user input**.

- Optional input:* Theo. uncertainties for mass, cross sections and BR's
This is important for all **New Physics** models

Experimental Input:

- m_h measurements
- Signal strength measurements:

$$\mu_{H \rightarrow XX_j} = \frac{\sum_i \epsilon_{\text{model}}^{ij} [\sigma_i(pp \rightarrow H) \times \text{BR}(H \rightarrow XX)]_{\text{model}}}{\sum_i \epsilon_{\text{SM}}^{ij} [\sigma_i(pp \rightarrow H) \times \text{BR}(H \rightarrow XX)]_{\text{SM}}},$$

with $i \in \{\text{ggH}, \text{VBF}, \text{WH}, \text{ZH}, t\bar{t}H\}$ and efficiencies ϵ_i .

- Efficiencies** of each production mode i in each subchannel j
- 1D μ measurements allow for easier deconvolution of theory uncertainties than 2D
- But it is much more difficult to account for experimental systematics **in between subchannels**

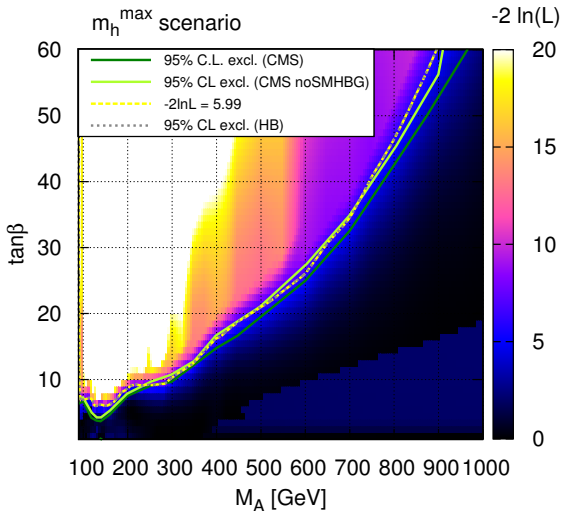
HiggsBounds Let's not forget the Limits

HiggsBounds

(PB, S. Heinemeyer, O. Brein, O. Stal, T. Stefaniak, G. Weiglein, K. Williams
arXiv:0811.4169, arXiv:1102.1898, arXiv:1311.0055)

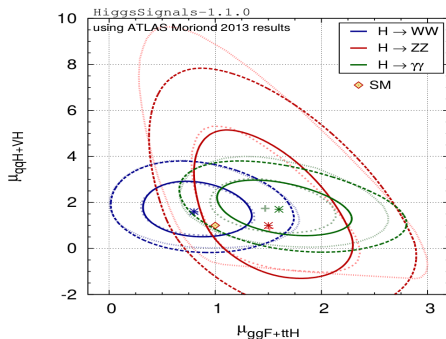
- Limits continue to be of great relevance! Let's not forget that we do not know for sure that there is only one Higgs!
- We are talking about likelihoods for measurements! Why not finally publish likelihoods for exclusions?
- Also: SM Higgs search combinations in the full mass range remain important. As far as we know, the last of such combinations was published at HCP 2012 by CMS, using the 4.8fb⁻¹ / 12.2fb⁻¹ of 7/8 TeV data.
- Equally important as for the signal rate measurements is the publication of signal efficiencies for the limits (if necessary, mass-dependent).
- CMS made a nice approach to publish likelihood information for a single resonance toy model in the non-standard $H \rightarrow \tau\tau$ search. Extremely useful e.g. in global BSM fits.

HiggsBounds **Let's not forget the Limits**

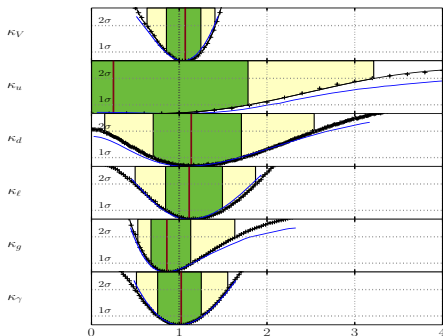


CMS $H \rightarrow \tau^+ \tau^-$ works extremely well! Yellow should reproduce green

Validation against ATLAS and CMS (Moriond 2013)



ATL-CONF-2013-034



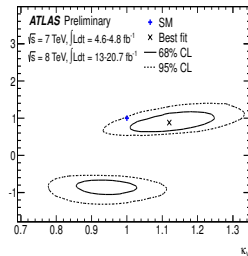
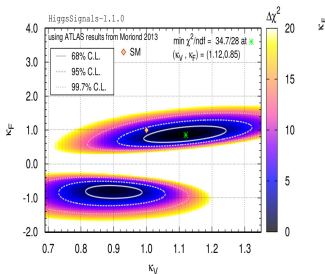
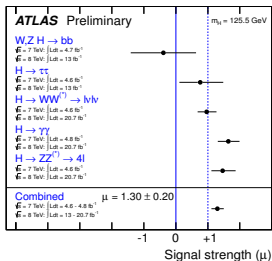
CMS-PAS-13-005

Generally good agreement Main limiting factors / challenges:

- Missing public information on signal efficiencies,
- Missing public information on correlations of exp. systematics,
- some measurements are performed at different m_H values than validation.

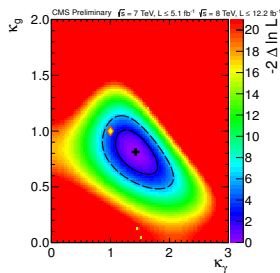
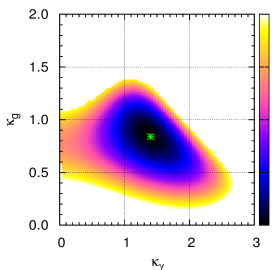
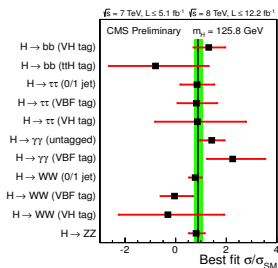
Test using ATLAS and κ_F, κ_V

- Test simple 2D effective coupling benchmark models, proposed in **LHC Higgs Cross Section Working Group, Sep.12, [1209.0040]**
- Scale fermion couplings by κ_F and vector boson couplings by κ_V
- non-trivial scaling of loop-induced $H\gamma\gamma$ coupling.
- loop-induced Hgg coupling scales with κ_F (effectively a fermion loop).
- No special treatment of negative μ_i

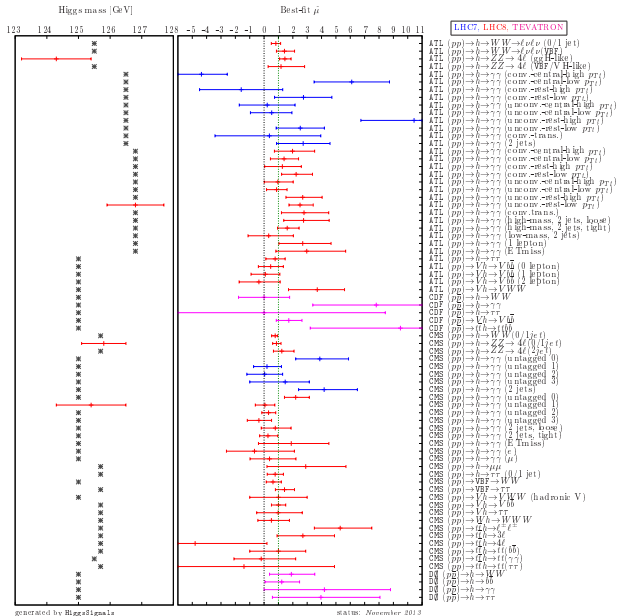


Test using CMS and κ_g, κ_γ

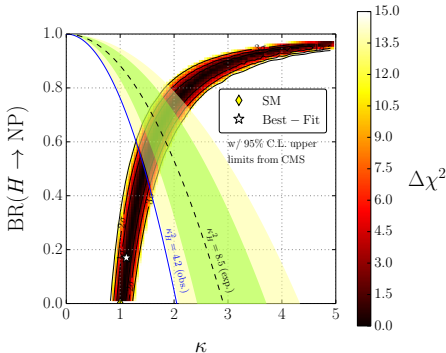
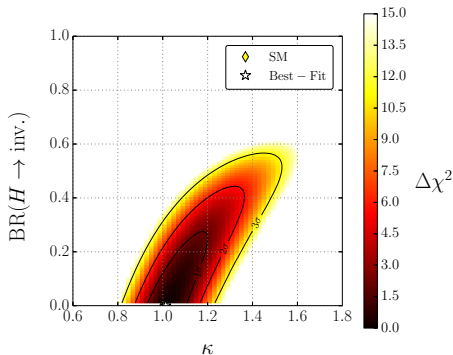
- Test simple 2D effective coupling benchmark models, proposed in **LHC Higgs Cross Section Working Group, Sep.12, [1209.0040]**
- scale loop-induced gluon couplings by κ_g and photon couplings by κ_γ . (keep tree-level couplings at their SM value)
- probing new physics contributions to loop-induced couplings.
- No special treatment of negative μ_i



Default set of observables (in HiggsSignals-1.1.0)



The Minimal Visible Rate

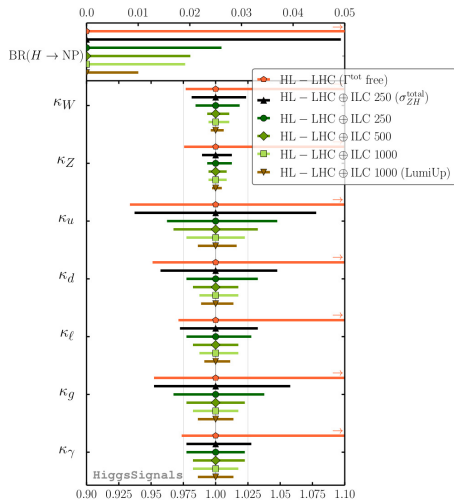
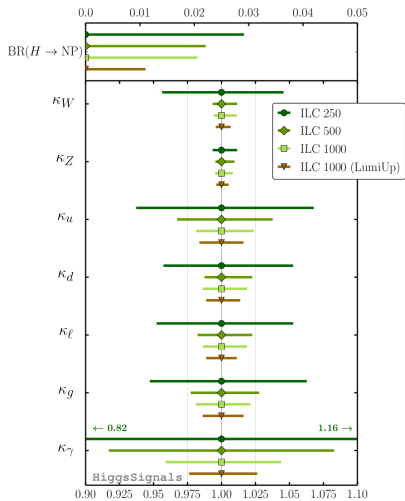


using CMS-PAS-HIG-14-002

$$\kappa_{H,\text{limit}}^2 = 40 \text{ (10)} \quad \rightarrow \quad \kappa \leq 2.51 \text{ (1.78)} \quad \text{and} \quad \mathcal{B}(h \rightarrow \text{NP}) \leq 84\% \text{ (68\%)}$$

Example: Ultimate Precision at the ILC

Just as an example to show why this sort of input is very flexible for all kind of studies



Indirect Constraints

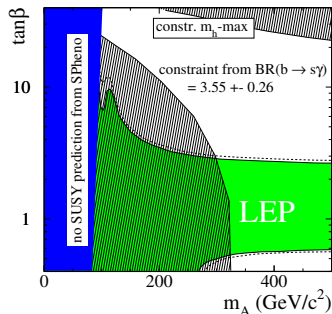
SM “radiative penguin”

SUSY penguin

SUSY or 2HDM penguin

- BaBar (SLAC) and Belle (KEK)
- Example for sensitivity for large M at low $\sqrt{s} = 10.58 \text{ GeV}$
- New Physics at the same loop level as in the SM
- Radiative Penguin decay

$$\mathcal{B}^{SM}(b \rightarrow s\gamma) = (3.15 \pm 0.23) \cdot 10^{-4}$$

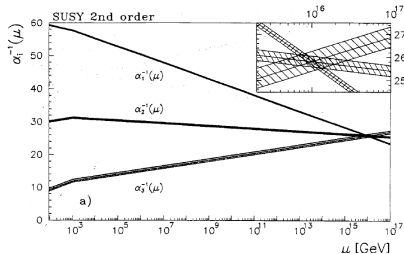
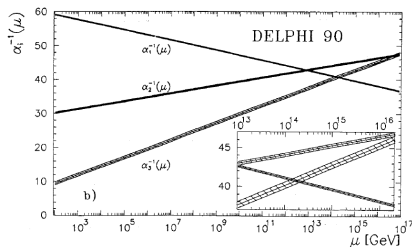


Why try (trust?) SUSY?

Wim de Boer *et al.* (1991):

It was shown that the evolution of the coupling constants within the minimal Standard Model with one Higgs doublet does not lead to Grand Unification, but if one adds five additional Higgs doublets, unification can be obtained at a scale below $2 \cdot 10^{14}$ GeV. However, such a low scale is excluded by the limits on the proton lifetime.

On the contrary, the minimal supersymmetric extension of the Standard Model leads to unification at a scale of $10^{16.0 \pm 0.3}$ GeV. Such a large unification scale is compatible with the present limits on the proton lifetime of about 10^{32} years. Note that the Planck mass (10^{19} GeV) is well above the unification scale of 10^{16} GeV, so presumably quantum gravity does not influence our results.

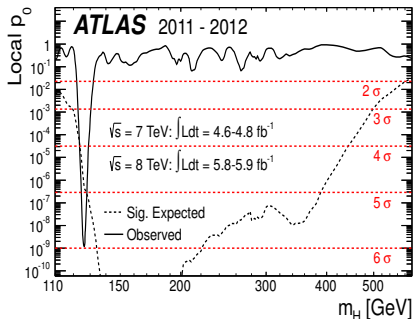


„Prediction“ of $\sin^2 \theta_W$:

$$\sin^2 \theta_W^{SUSY} = 0.2335(17),$$

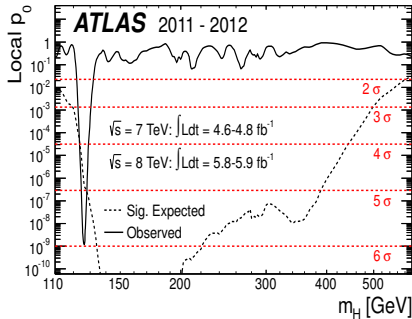
$$\sin^2 \theta_W^{exp} = 0.2315(02)$$

So near ... and yet so far ...

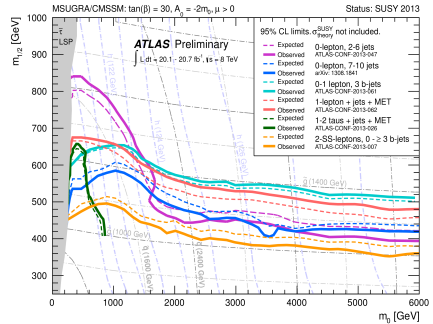


We found a SM-like Higgs ...

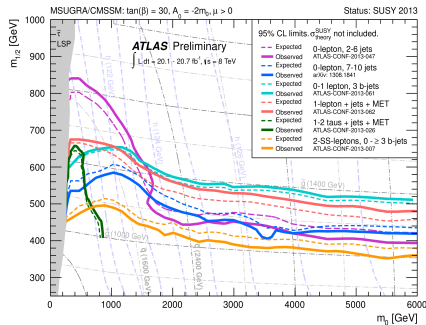
So near ... and yet so far ...



We found a SM-like Higgs ...



But we did not find anything else.



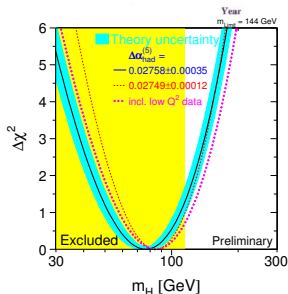
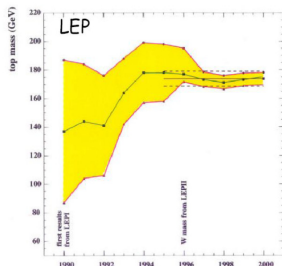
But we did not find anything else.

- **How** can we learn from the Higgs discovery for **any** model of physics beyond the SM?
- **What** can we learn from everything we know about SUSY?

Experiments at highest precision

- Babar
- Belle
- BES, CLEO
- Aleph
- Delphi
- L3
- OPAL
- SLC
- $(g - 2)_\mu$
- m_W, m_t, B bei CDF
- m_W, m_t, B bei D0
- Indirect Detection, Satellites
- SuperBelle
- ILC?

Multi-Messenger



Experimente at highest energies

- SUSY-search at Tevatron
- Higgs-search at Tevatron
- ATLAS
- CMS
- Cosmology of the early universe
- ...

Fitting the CMSSM

Using $HS(,HB)$ + other input

see e.g. [arXiv:1204.4199](#), [arXiv:1310.3045](#), and [arXiv:1410.6035](#), [arxiv:1508.05951](#)

CMSSM is experimentally constrained by

- indirect constraints from low energy precision measurements
- direct searches for sparticles and Higgs bosons
- astrophysical observations

To evaluate the corresponding model predictions we use:

- **SPheno** for spectrum calculation
- **FeynHiggs** for Higgs properties, $(g-2)_\mu$ & Δm_s
- **SuperIso** for other B-Physics observables
- **Prospino**, **Herwig++**, **Delphes** for direct sparticle searches
- **MicrOMEGAs** for dark matter relic density
- **DarkSUSY** via **Astrofit** for direct detection cross section

Higgs, Searches and Astrophysics

Direct searches for sparticles and Higgs Bosons

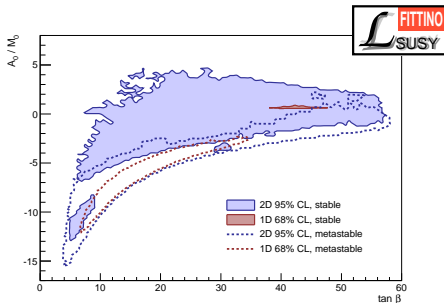
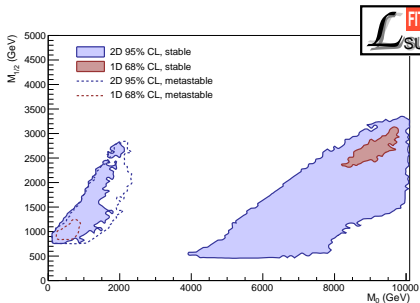
- Higgs limits via **HiggsBounds**
- Higgs signals via **HiggsSignals**
- LEP chargino mass limit
- ATLAS MET + jets + 0 lepton search (20fb^{-1})

Astrophysical observables

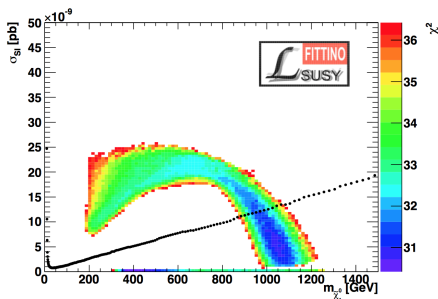
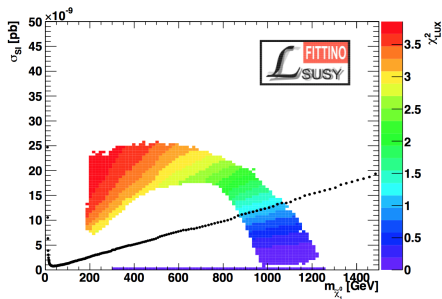
- We require χ_1^0 to be the LSP
- Dark matter relic density:
 $\Omega_{\text{CDM}} h^2 = 0.1187 \pm 0.0017 \pm 0.0119$ (Planck '13)
- Direct detection limit from 225 live days of Xenon100 ('12)

But is that stable on cosmological timescales?

- Using VeVacious
- All minima in stable or metastable (lifetime \gg age of the universe) regions

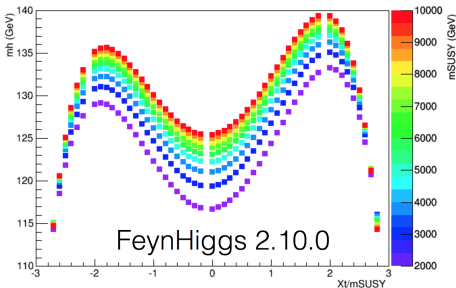
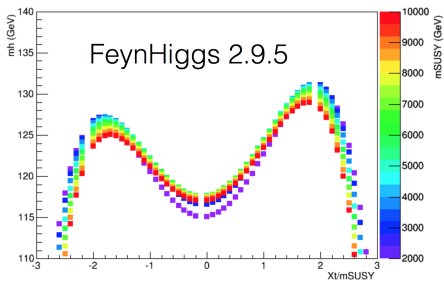


Sensitivity of Direct Detection Experiments



Contributions from **Direct Detection**
 No contributions from **Indirect Detection**

The Higgs Mass



In latest codes, there is enough headroom to accommodate the 125 GeV Higgs boson in the cMSSM

The E821 Experiment at Brookhaven

Storage ring @ BNL

250" diameter ring
Superconducting coils
 $B = 1.45\text{T}$
 $p_{\mu} = 3.1\text{GeV}/c$



The E821 Experiment at Brookhaven

$$a_\mu = a_\mu^{QED} + a_\mu^{QCD} + a_\mu^{QFD}$$

	VALUE ($\times 10^{-11}$) UNITS
QED ($\gamma + \ell$)	$116\,584\,718.853 \pm 0.022 \pm 0.029_\alpha$
HVP(lo)*	$6\,923 \pm 42$
HVP(ho)**	-98.4 ± 0.7
H-LBL [†]	105 ± 26
EW	153.6 ± 1.0
Total SM	$116\,591\,802 \pm 42_{\text{H-LO}} \pm 26_{\text{H-HO}} \pm 2_{\text{other}} (\pm 49_{\text{tot}})$

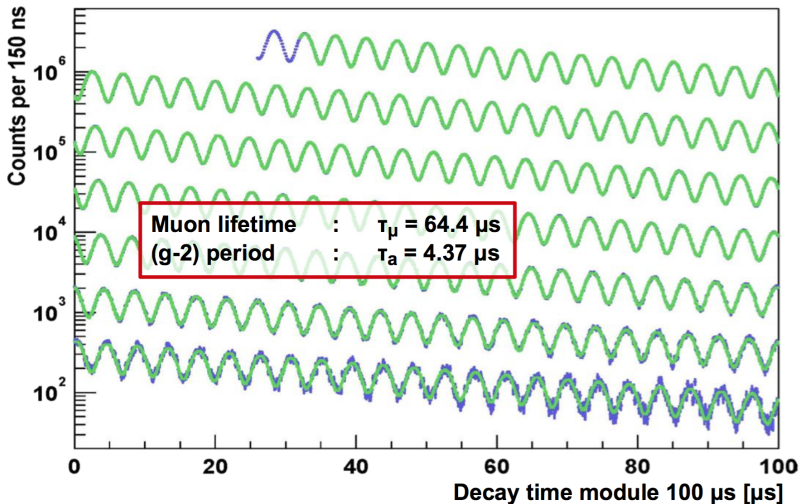
Significant
work ongoing

$$\sigma_{\text{exp}}^{\text{today}} = \pm 63$$

- * Davier *et al.*, Eur. Phys. J. C (2011) 71:1515
- ** Hagiwara *et al.*, J. Phys. G38, 085003 (2011).
Prades *et al.* Lepton Moments

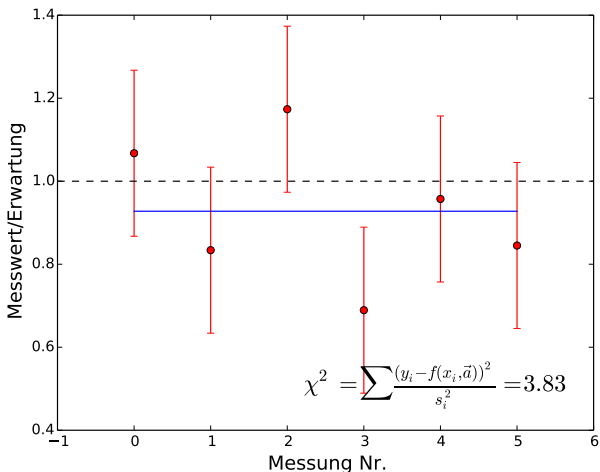
The E821 Experiment at Brookhaven

$$f(t) = N_0 e^{-t/\tau_\mu} [1 + A \cos \omega_a t + \varphi]$$



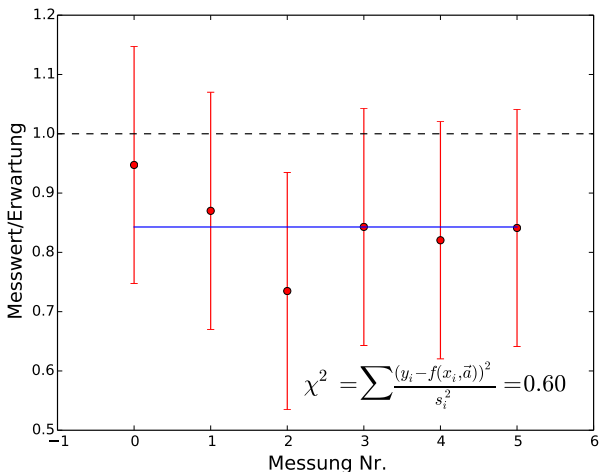
$\sim 10^{10}$ events, ~ 10 lifetimes

Now we know where the cMSSM would be



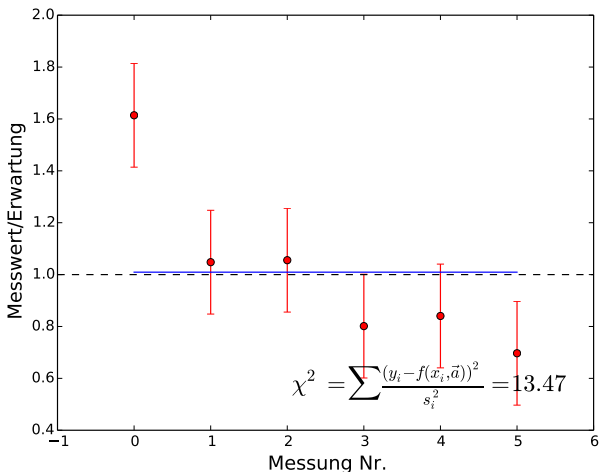
A small abstract example:
6 independent measurements

Now we know where the cMSSM would be



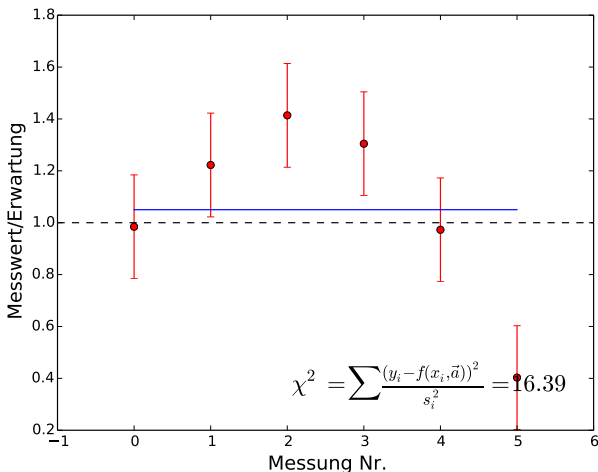
A small abstract example:
6 independent measurements

Now we know where the cMSSM would be



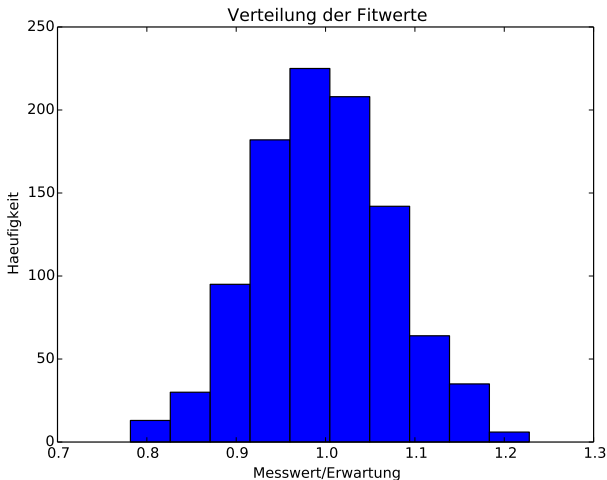
A small abstract example:
6 independent measurements

Now we know where the cMSSM would be



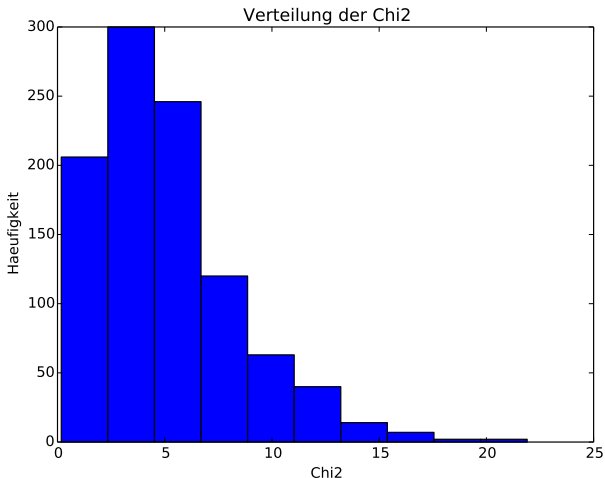
A small abstract example:
6 independent measurements

But how do we find whether it can be there?



Repeat these measurements each in a simulation: “Toys”

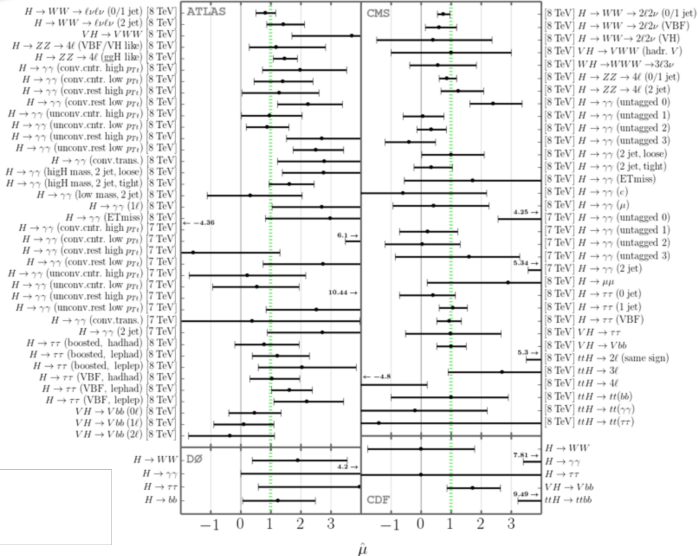
But how do we find whether it can be there?



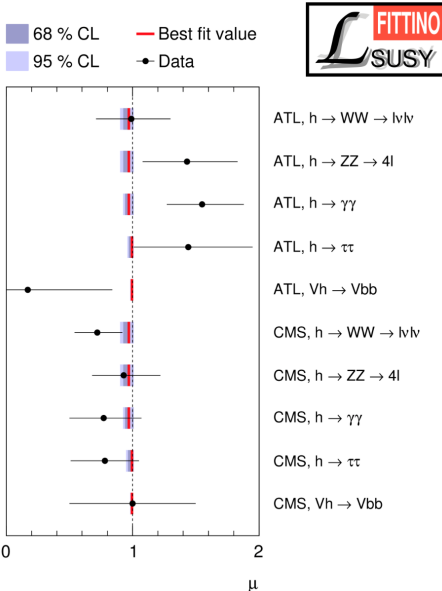
Repeat these measurements each in a simulation: “Toys”

To which Higgs Maesurent Set do we Fit best?

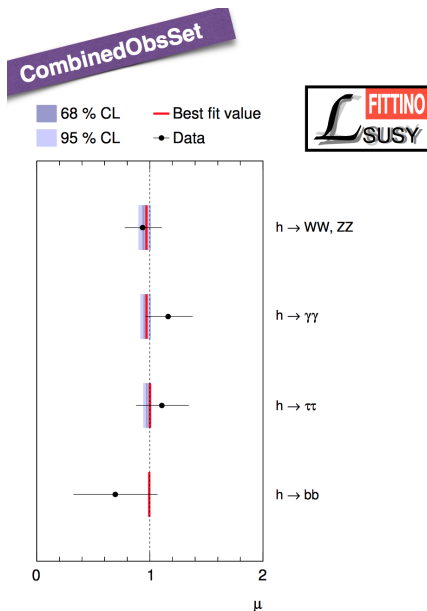
LargeObsSet



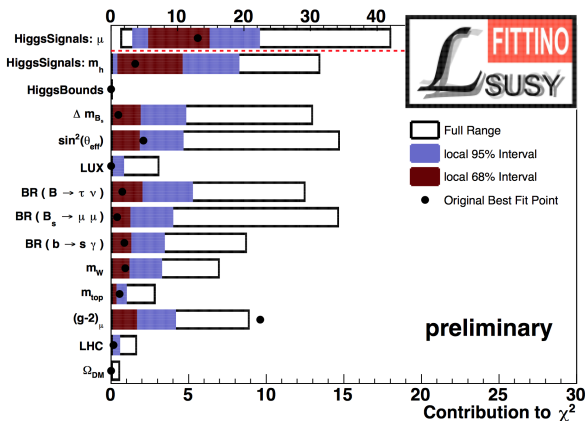
To which Higgs Measurement Set do we Fit best?



To which Higgs Measurement Set do we Fit best?

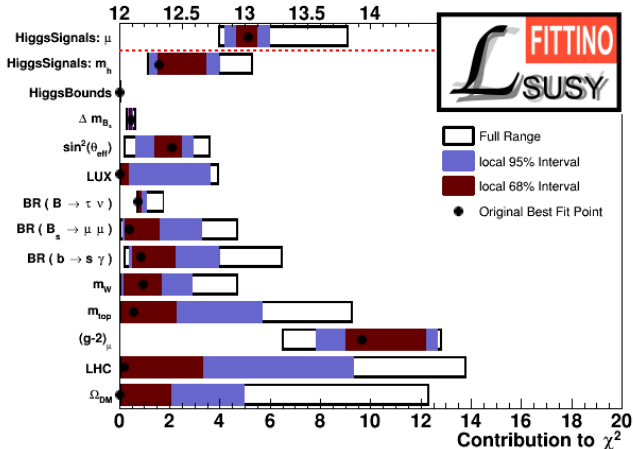


So does the Higgs do anything?



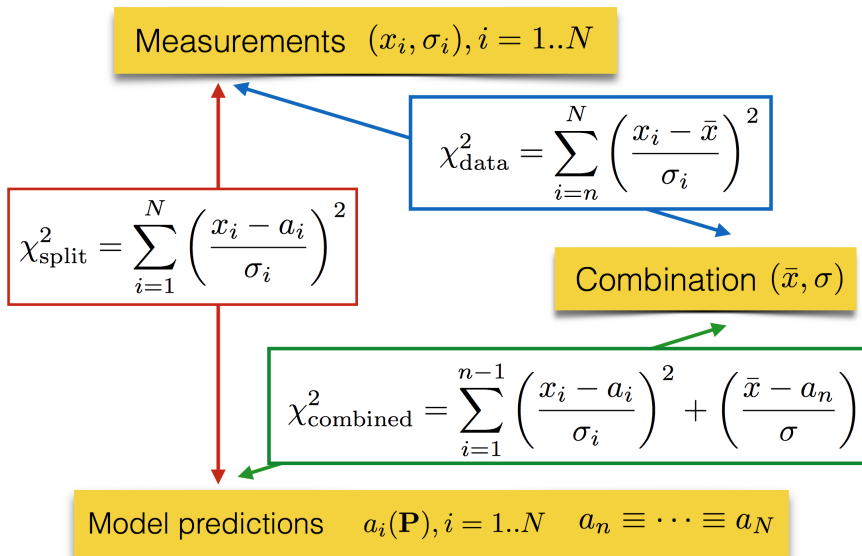
- This plot shows the variation of the χ^2 contributions for all toy fits, calculated with respect to the **smeared values**
- If the colored band is small: Observable has no effect on the fit
- m_h obviously has an effect, μ 's a bit.

So does the Higgs do anything?

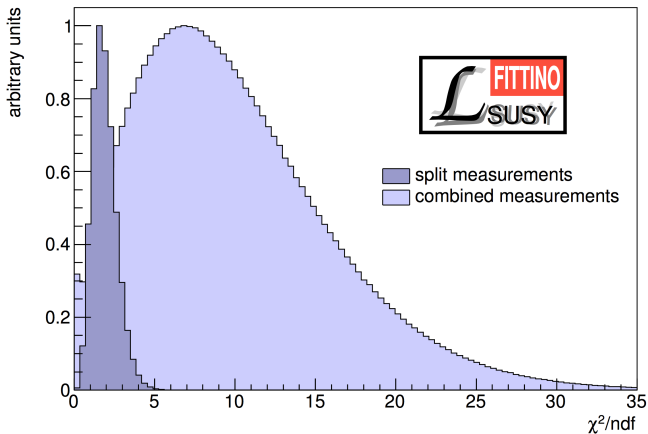


- This plot shows the variation of the χ^2 contributions for all toy fits, calculated with respect to the **measured values**
- If the colored band is small: Observable has no effect on the fit
- m_h obviously has an effect, μ 's a bit.

Effect of the Combination on the \mathcal{P} -value



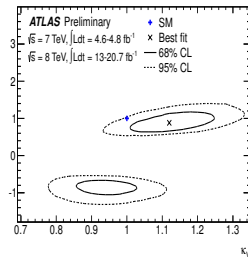
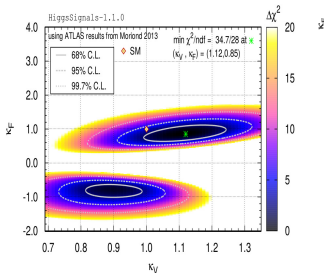
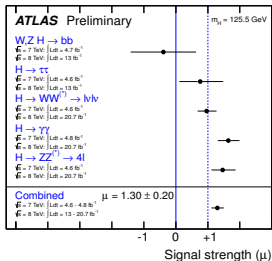
Effect of the Combination on the \mathcal{P} -value



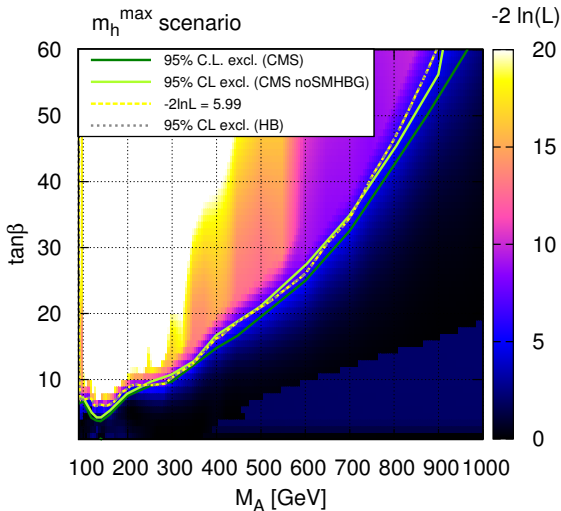
$n = 1, \quad N = 10, \quad 3\sigma \text{ deviation}$

HiggsSignals Test using ATLAS and κ_F, κ_V

- Test simple 2D effective coupling benchmark models, proposed in **LHC Higgs Cross Section Working Group, Sep.12, [1209.0040]**
- Scale fermion couplings by κ_F and vector boson couplings by κ_V
- non-trivial scaling of loop-induced $H\gamma\gamma$ coupling.
- loop-induced Hgg coupling scales with κ_F (effectively a fermion loop).
- No special treatment of negative μ_i



HiggsBounds **Let's not forget the Limits**



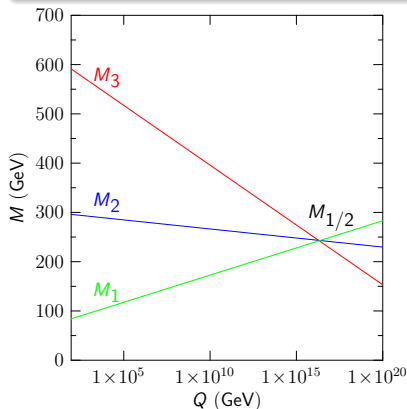
CMS $H \rightarrow \tau^+\tau^-$ works extremely well! Yellow should reproduce green

Constraint SUSY Models

Reduction of the number of parameters: Assume unification at M_{GUT}

Example

Universal gaugino mass $M_{1/2}$



Unified model: CMSSM

4 additional continuous parameters

- universal gaugino mass $M_{1/2}$
- universal scalar mass M_0
- universal trilinear coupling A_0
- $\tan \beta = v_2/v_1$
- $\text{sign} \mu$

Typical model point for older studies:
SPS 1a

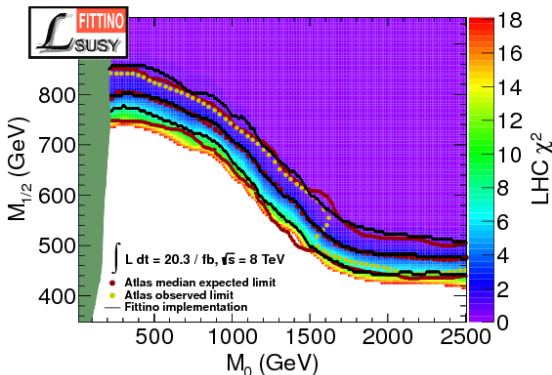
$M_{1/2} = 250$ GeV, $M_0 = 100$ GeV,
 $A_0 = -100$ GeV, $\tan \beta = 10$, $\text{sign} \mu = 1$

χ^2 contributions

At each parameter point \vec{P} calculate:

$$\chi^2 = \left(\vec{O}_{\text{meas}} - \vec{O}_{\text{pred}}(\vec{P}) \right)^T \text{cov}^{-1} \left(\vec{O}_{\text{meas}} - \vec{O}_{\text{pred}}(\vec{P}) \right) + \chi_{\text{limits}}^2$$

- An example for a limit: The ATLAS 0-lepton generic SUSY search



\mathcal{P} -values for different Observable Sets

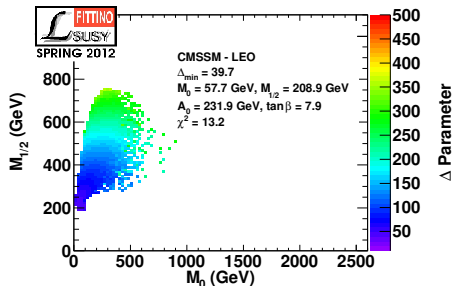
Observable Set	χ^2/ndf	naive p -value (%)	toy p -value (%)
Small	27.1/16	4.0	1.9 ± 0.4
Medium	30.4/22	10.8	4.9 ± 0.7
Combined	17.5/13	17.7	8.3 ± 0.8
Medium (Focus Point)	30.8/22	10.0	7.8 ± 0.8
Medium without (g-2)	18.1/21	64.1	51 ± 3
No Higgs rates	15.5/9	7.8	1.3 ± 0.4

from PB et al., <http://arxiv.org/abs/1508.05951>

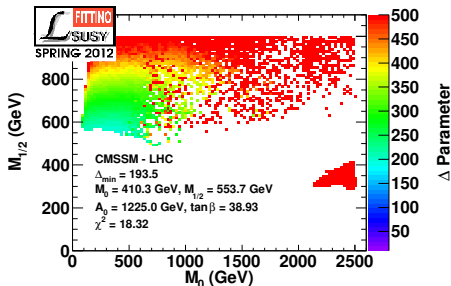
Traditional Finetuning

Older results from [arXiv:1204.4199](https://arxiv.org/abs/1204.4199)

Might be interesting to revisit them for the pMSSM?



Without LHC



Incl. ATLAS 01 SUSY results

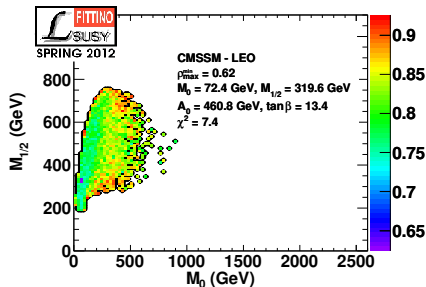
$$c_a = \left| \frac{\partial \ln M_Z^2}{\partial \ln a} \right|,$$

$$\Delta = \max(c_a).$$

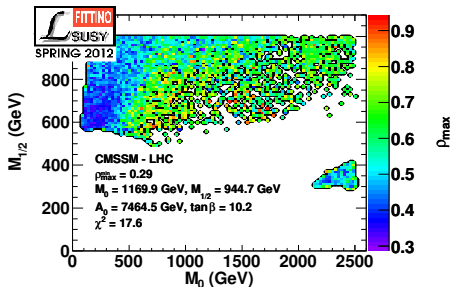
Alternative Finetuning

Older results from [arXiv:1204.4199](https://arxiv.org/abs/1204.4199)

Might be interesting to revisit them for the pMSSM?



Without LHC



Incl. ATLAS 01 SUSY results

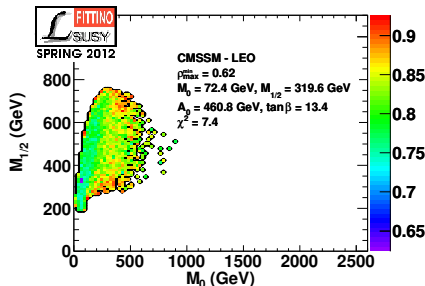
$$\varrho_{ij} = \left\langle \frac{(P_i - \langle P_i \rangle) \cdot (P_j - \langle P_j \rangle)}{\sigma_{P_i} \sigma_{P_j}} \right\rangle,$$

$$\Delta' = \max(|\varrho_{ij}|).$$

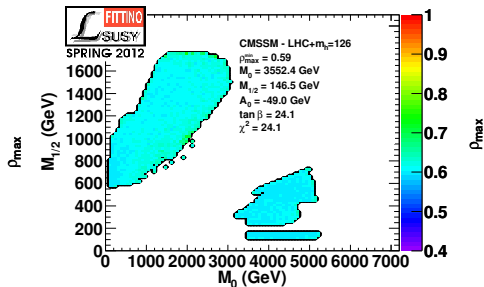
Alternative Finetuning

Older results from [arXiv:1204.4199](https://arxiv.org/abs/1204.4199)

Might be interesting to revisit them for the pMSSM?



Without LHC



Incl. ATLAS 01 SUSY results
 $+m_h = 126 \pm 2 \pm 3 \text{ GeV}$

$$\varrho_{ij} = \left\langle \frac{(P_i - \langle P_i \rangle) \cdot (P_j - \langle P_j \rangle)}{\sigma_{P_i} \sigma_{P_j}} \right\rangle,$$

$$\Delta' = \max(|\varrho_{ij}|).$$

A Warning: Apparent Finetuning

