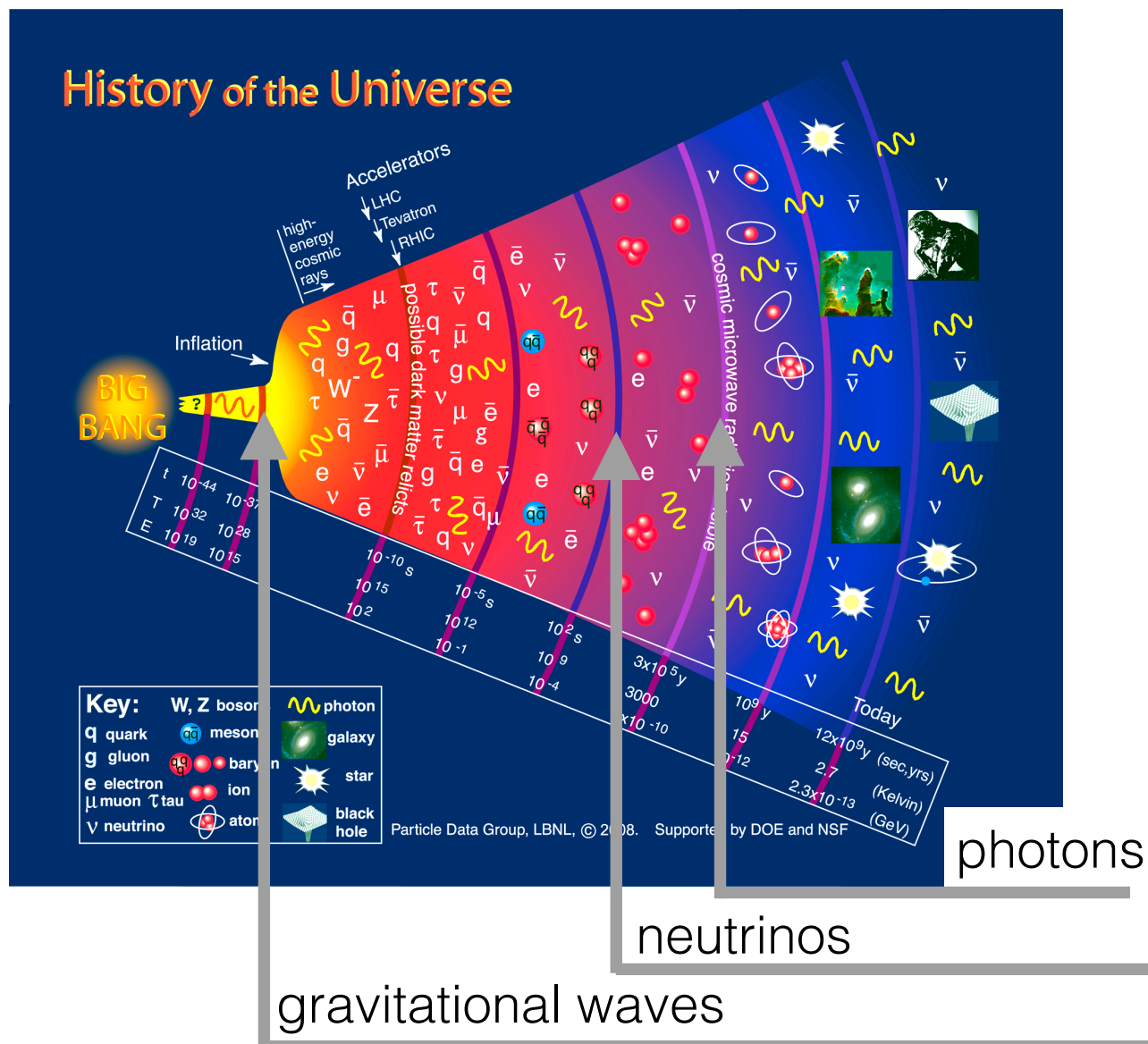


# Cosmological sources of GWs

## Probing the early universe with GWs

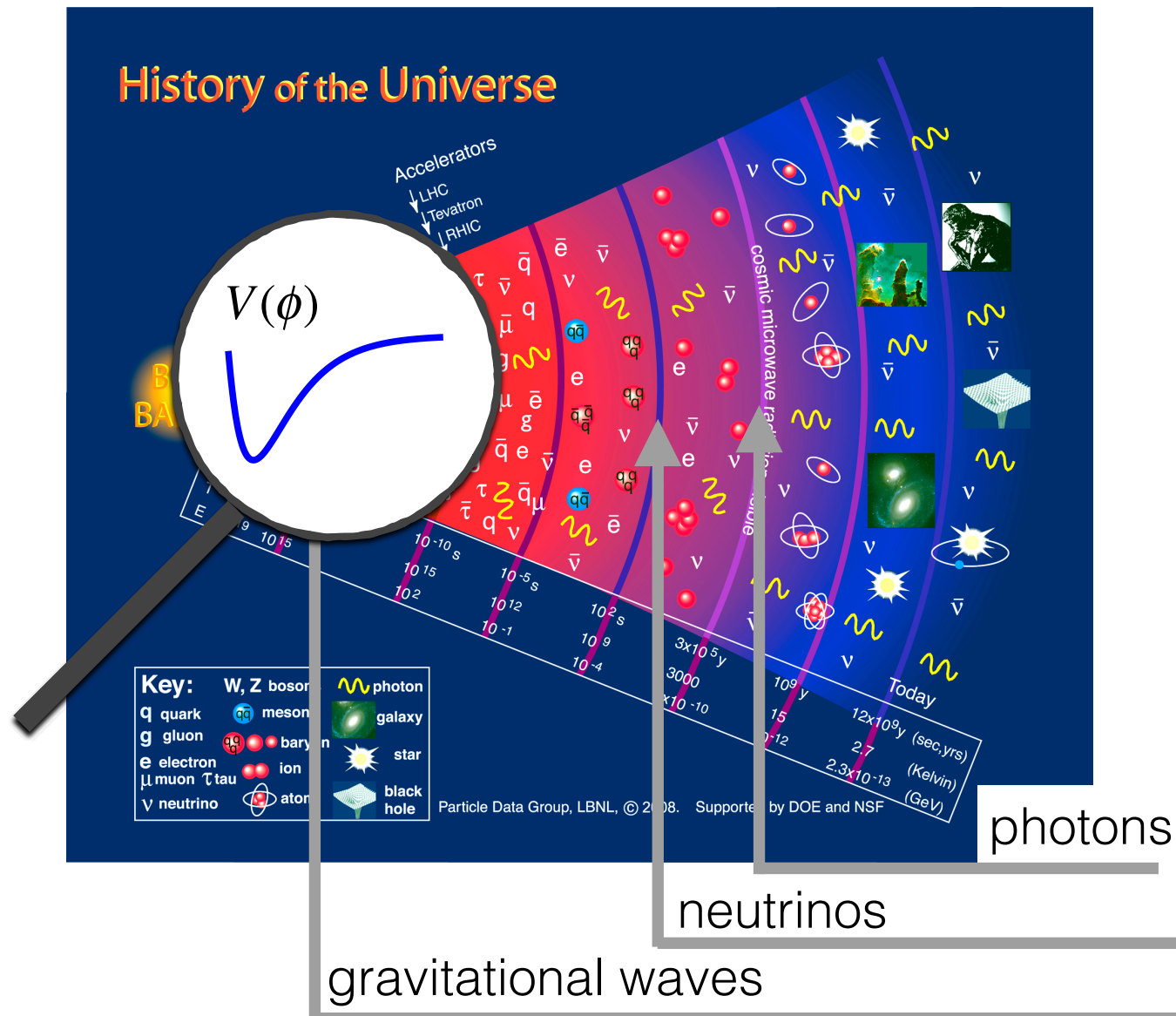


**Valerie Domcke**  
APC, Paris

IPA, 5. - 9.9.2016  
LAL, Orsay

# Cosmological sources of GWs

## Probing the early universe with GWs



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# Outline

stochastic  
GW backgrounds

- Primordial GW background

- .....GWs from cosmic inflation

- .....how to read to cosmic GW history book

- .....searching for GWs: CMB versus direct detection

- .....enhanced primordial GW background ?

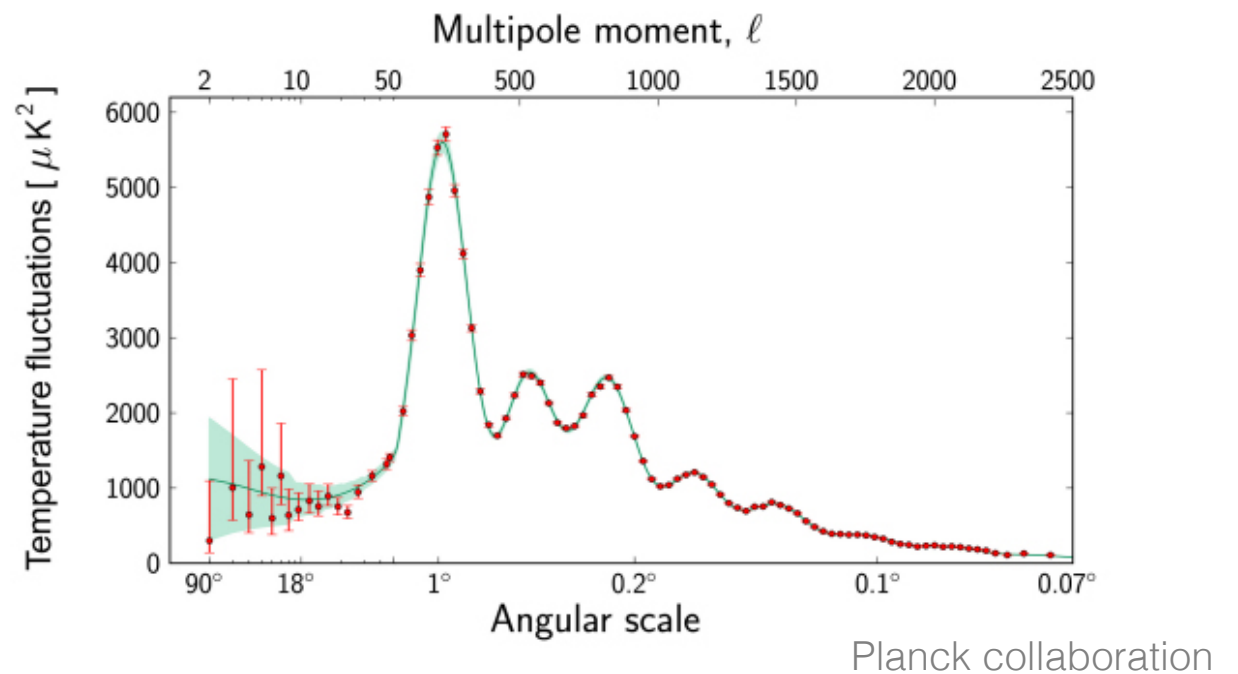
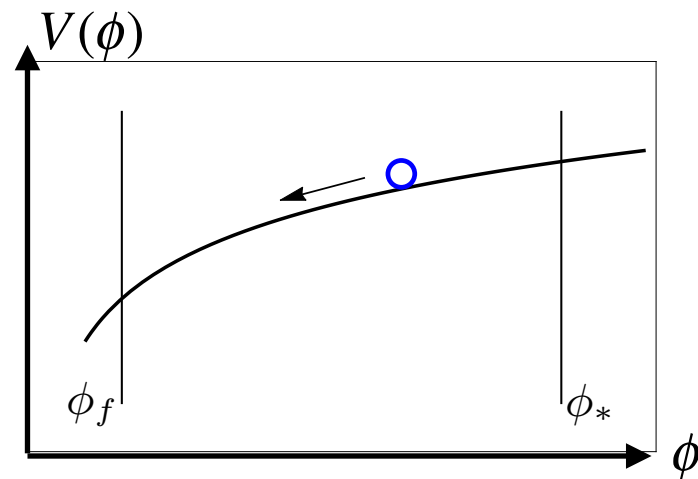
- Further GW sources in the early Universe

- .....cosmic strings

- .....first-order phase transitions

- .....primordial black holes

# The paradigm of slow-roll inflation



large vacuum energy → exponential expansion → homogeneity of CMB  
 quantum fluctuations → become classical → tiny anisotropies in the CMB

The big question:  $V(\phi) = ??$  →  $\Delta_s^2 = \frac{V(\phi)}{24\pi^2\epsilon(\phi)}$ ,  $\Delta_t^2 = \frac{2V(\phi)}{3\pi^2}$ ;  $\epsilon = \frac{\dot{\phi}^2}{2H^2} \simeq \frac{1}{2} \left( \frac{V'(\phi)}{V(\phi)} \right)^2$   
 scalar spectrum, tensor spectrum

very successful paradigm, but very many possible realizations



# Scales and horizons

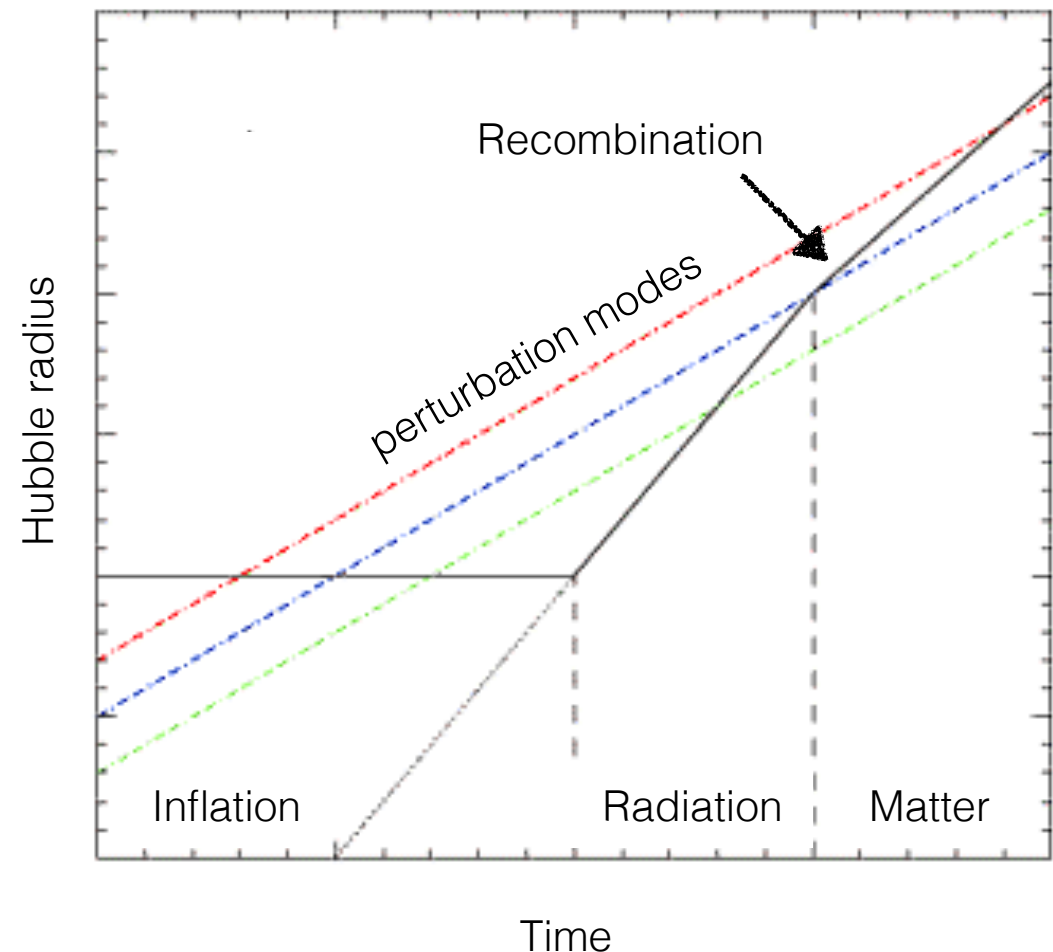
co-moving perturbation modes  
leave Hubble horizon during inflation,  
re-enter after inflation



perturbation with given frequency today  
corresponds to fixed time during inflation  
and re-entry



1:1 relation:  $f \rightarrow k \rightarrow N_k \rightarrow V(\phi_k)$



$$N = N_{\text{CMB}} + \ln \frac{k_{\text{CMB}}}{0.002 \text{ Mpc}^{-1}} - 44.9 - \ln \frac{f}{10^2 \text{ Hz}}, \quad N = \int H \, dt$$

spectrum sensitive to primordial spectrum (scalar potential) and post-inflationary expansion

# Some useful properties of GWs

perturbations of the background metric:  $ds^2 = a^2(\tau)(\eta_{\mu\nu} + h_{\mu\nu}(\mathbf{x}, \tau))dx^\mu dx^\nu$

governed by linearized Einstein equation ( $\tilde{h}_{ij} = ah_{ij}$ , TT - gauge)

$$\tilde{h}_{ij}''(\mathbf{k}, \tau) + \underbrace{\left(k^2 - \frac{a''}{a}\right)}_{\sim a^2 H^2} \tilde{h}_{ij}(\mathbf{k}, \tau) = \underbrace{16\pi G a \Pi_{ij}(\mathbf{k}, \tau)}_{\text{source term from } \delta T_{\mu\nu}}$$

source: anisotropic  
(not spherical symmetric)  
stress-energy tensor

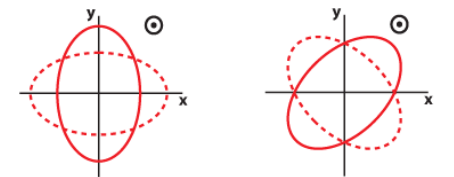
$$k \gg aH : h_{ij} \sim \cos(\omega\tau)/a, \quad k \ll aH : h_{ij} \sim \text{const.}$$

a useful plane wave expansion: 
$$h_{ij}(\mathbf{x}, \tau) = \sum_{P=+, \times} \int_{-\infty}^{+\infty} \frac{dk}{2\pi} \int d^2 \hat{\mathbf{k}} h_P(\mathbf{k}) \underbrace{T_k(\tau)}_{\sim a(\tau_i)/a(\tau)} e_{ij}^P(\hat{\mathbf{k}}) e^{-ik(\tau - \hat{\mathbf{k}}\mathbf{x})}$$

transfer function , expansion coefficients , polarization tensor  $P = +, \times$

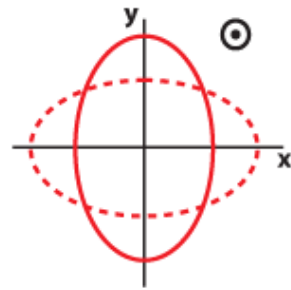
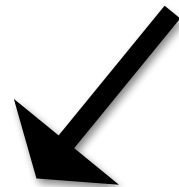
observational quantity in direct detection

$$\Omega_{\text{GW}} = \frac{1}{\rho_c} \frac{\partial \rho_{\text{GW}}(k, \tau)}{\partial \ln k}, \quad \rho_{\text{GW}}(\tau) = \frac{1}{32\pi G} \left\langle \dot{h}_{ij}(\mathbf{x}, \tau) \dot{h}^{ij}(\mathbf{x}, \tau) \right\rangle$$



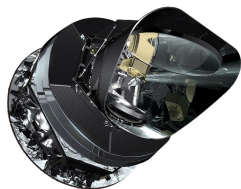
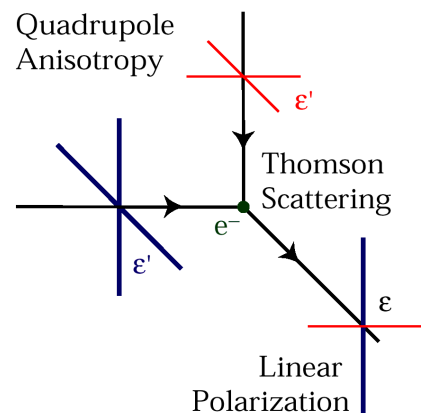
# Hunting for primordial GWs

CMB

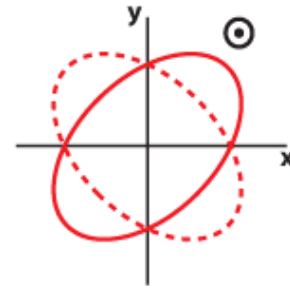


tensor anisotropies  
on last scattering surface

polarization of CMB photons  
through Thomson scattering



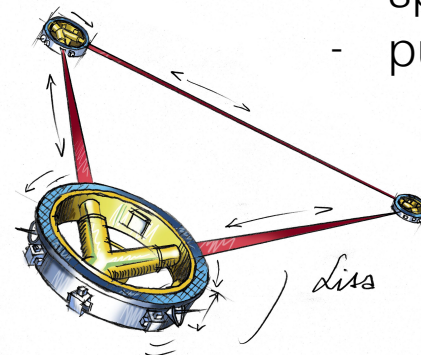
- Lensing:  $T \rightarrow E$
- dust contaminates primordial signal
- B - modes most sensitive



direct

GW travels freely until today

distortion of space as GW  
passes detector



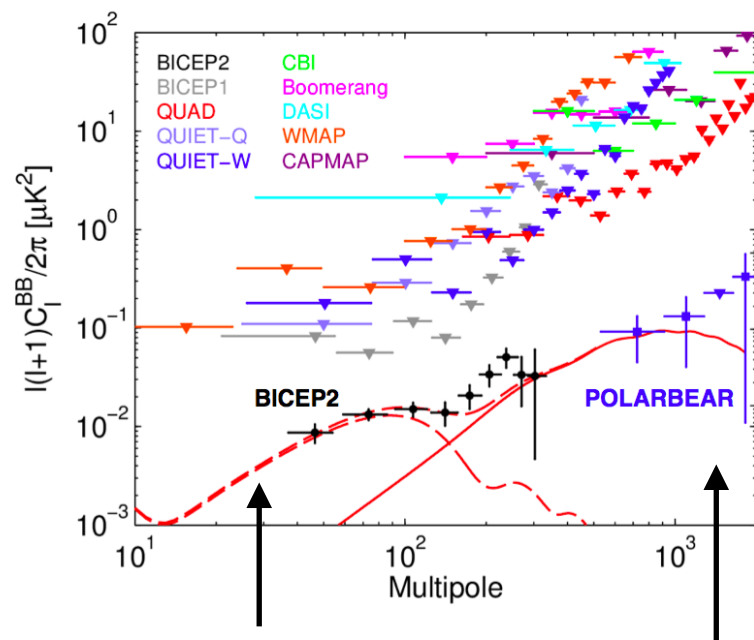
- ground-based interferometers
- space-based interferometers
- pulsar timing arrays

# Hunting for primordial GWs

CMB

$$r = \Delta t^2 / \Delta s^2$$

BICEP2 '14



hypothetical primordial  
contribution with  $r \sim 0.17$

Lensing

sensitive to CMB scales

# CMB

direct

$$\Omega_{\text{GW}}(k) = \frac{\Delta_t^2}{12} \frac{k^2}{a_0^2 H_0^2} T_k^2 \simeq \frac{\Delta_t^2}{12} \Omega_r$$

BICEP2 '14



Smith, Kamion

$k[\text{Mpc}^{-1}]$

$10^{-5}$   $10^0$   $10^5$   $10^{10}$   $10^{15}$   $10^{20}$   $10^{25}$

$10^0$

**time of horizon exit**

$10^{-5}$

msec pulsar

eLISA

LIGO

ET

BBO/DECIGO

$10^{-10}$

$10^{-15}$

inflation

$r = 0.1$

$10^{-20}$

$10^{-25}$

$10^{-30}$

**time of re-entry**

eq

RH

$f[\text{Hz}]$

$10^{-20}$   $10^{-15}$   $10^{-10}$   $10^{-5}$   $10^0$   $10^5$   $10^{10}$

$\Omega_{\text{GW}} h^2$

Valerie Domcke (APC) - IPA 2016

# But this is not the end of the story...

## Non-standard sources during inflation

scalars: spectator fields (enhanced by  $c_s < 1$ )

gauge fields: pseudoscalar inflation

phase transition(s) during inflation

Cook, Sorbo 2012  
Biagetti, Fasiello, Riotto 2014

Anber, Sorbo '06./'10/'12,  
Barnaby, Namba, Peloso '11,  
Barnaby, Pajer, Peloso '12, ...

Freese, Spolyar 2004

see also Hebecker, Jaeckel, Rompineve, Witkowski '16  
for PT just after inflation

## Non-standard evolution after inflation

stiff equation of state during reheating

Spookily '93; Joyce '96;  
Giovannini '99; Sa, Henriques '10

## Second order gravitational waves

sourced by large scalar perturbations

Assadulahi, Wands '09

Bouncing cosmologies, broken spacial diffeomorphism, ..... + your favorite model I forgot to mention

See also: eLISA inflation working group report, to appear soon;  
Guzzetti, Bartolo, Liguori, Matarrese '16



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# pseudoscalar inflation

a generic coupling for a pseudoscalar inflaton:

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi) - \frac{\alpha}{4\Lambda}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}.$$

Turner, Widrow '88,  
Garretson, Field, Carroll '92,  
Anber, Sorbo '06./'10/'12,  
Barnaby, Namba, Peloso '11,  
Barnaby, Pajer, Peloso '12 ,  
.....

resulting background equations of motion:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial\phi} = \frac{\alpha}{\Lambda}\langle\vec{E}\vec{B}\rangle.$$

$$\frac{d^2 A_\pm^a(\tau, k)}{d\tau^2} + \left[ k^2 \pm 2k \frac{\xi}{\tau} \right] A_\pm^a(\tau, k) = 0, \quad \xi = \frac{\alpha\dot{\phi}}{2\Lambda H}$$

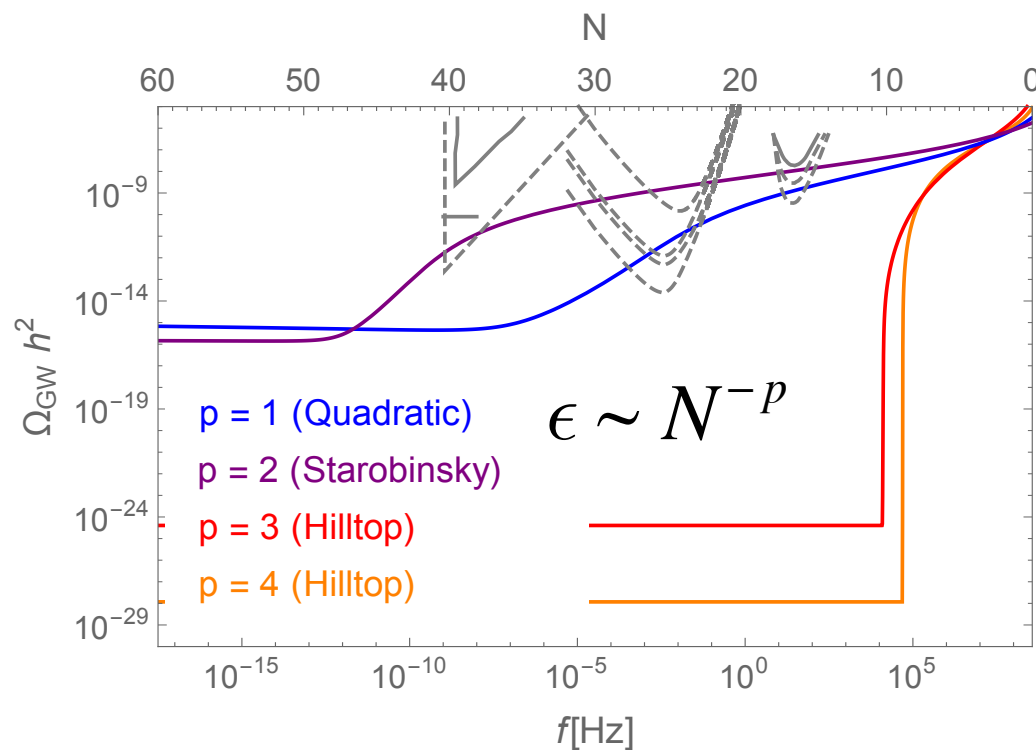
- tachyonic instability for the gauge field, controlled by  $\xi \propto \sqrt{\epsilon} = \dot{\phi}/(\sqrt{2}H)$
- exponential growth of gauge field modes towards end of inflation
- backreaction on inflaton eom, new friction term:  $\langle\vec{E}\vec{B}\rangle \simeq \mathcal{N} \cdot 2.4 \cdot 10^{-4} \frac{H^4}{\xi^4} e^{2\pi\xi}$   
+ additional source for scalar and tensor fluctuations

power spectrum of scalar and tensor perturbations affected

# GW spectrum of pseudoscalar inflation

VD, Pieroni, Binetruy 2016

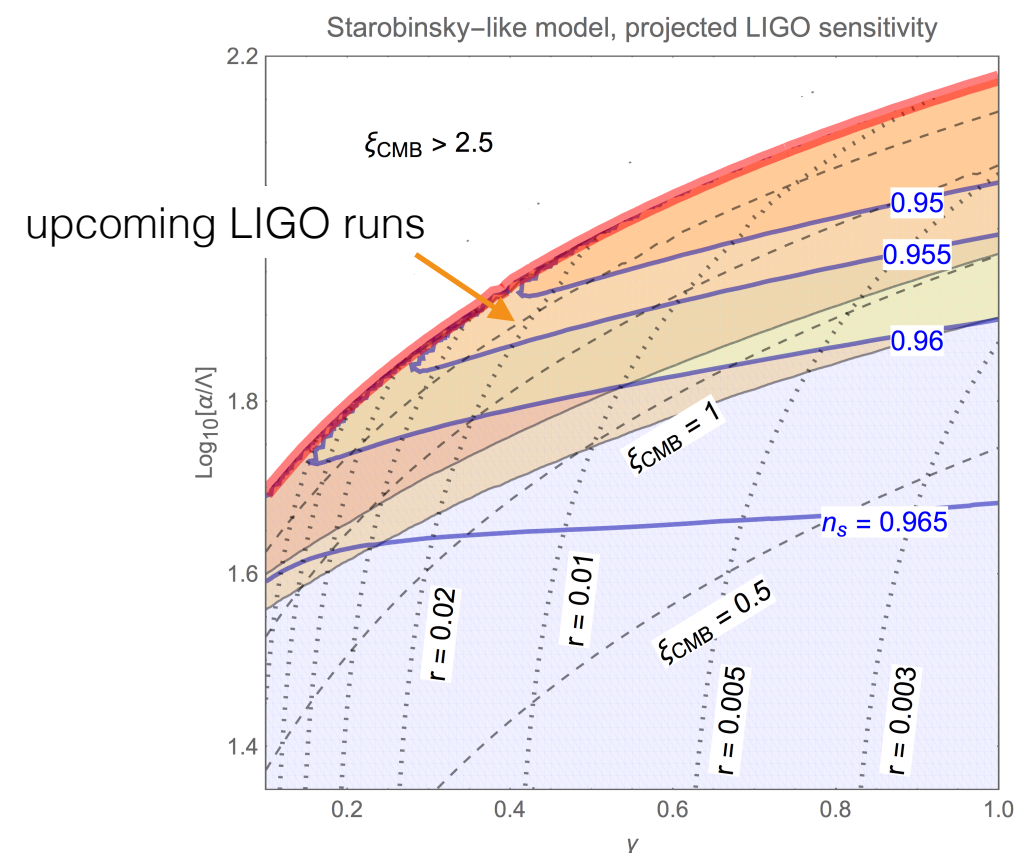
$$\Omega_{\text{GW}} = \frac{1}{12} \left( \frac{H}{\pi M_P} \right)^2 (1 + 4.3 \times 10^{(-7)} \frac{H^2}{M_P^2 \xi^6} e^{4\pi\xi})$$



- generically very blue spectrum
- low scale models feature stronger increase

Starobinsky-type model

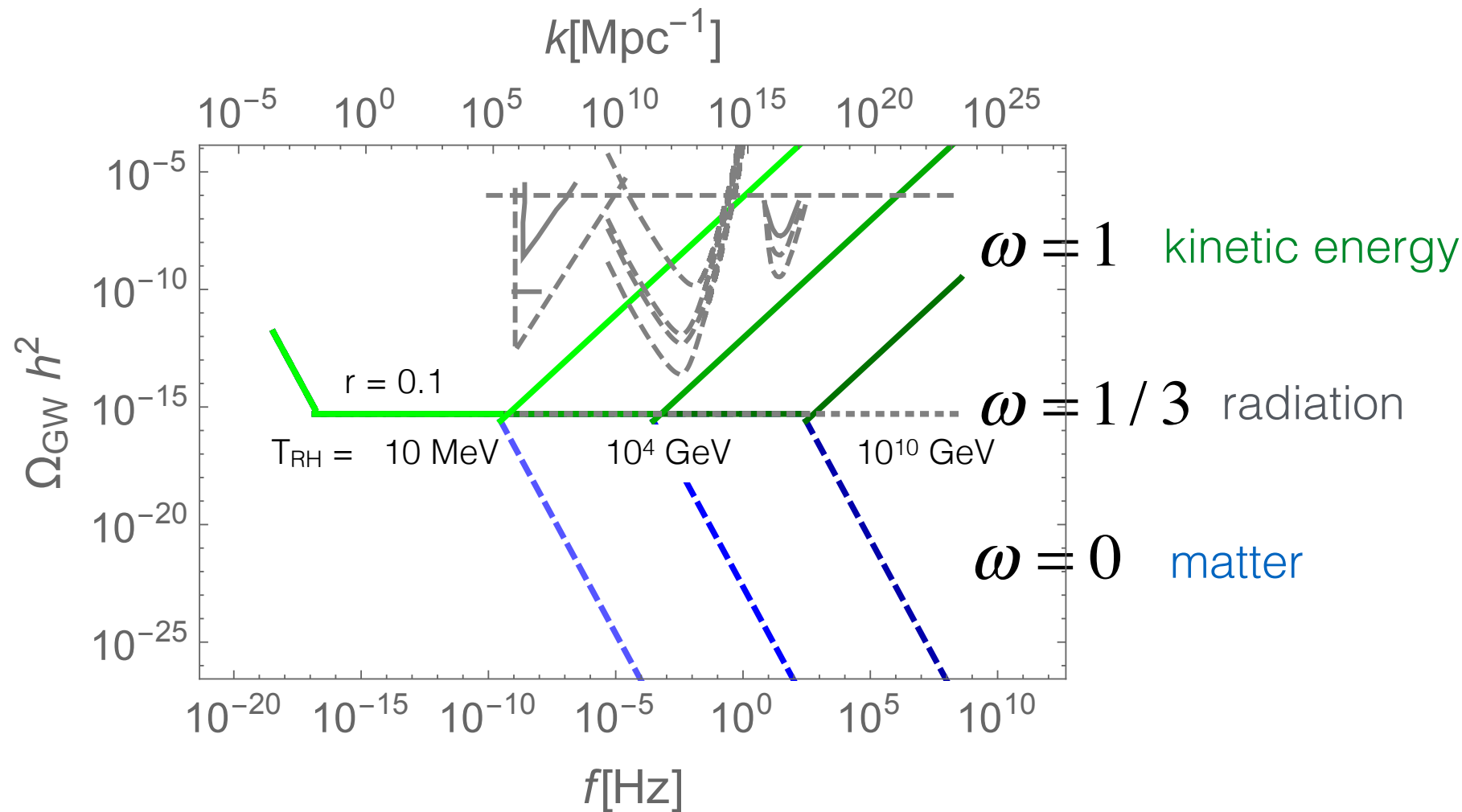
$$V(\phi) = V_0 (1 - e^{-\gamma\phi})^2$$



observable signal for direct detection, sensitive to underlying inflation model

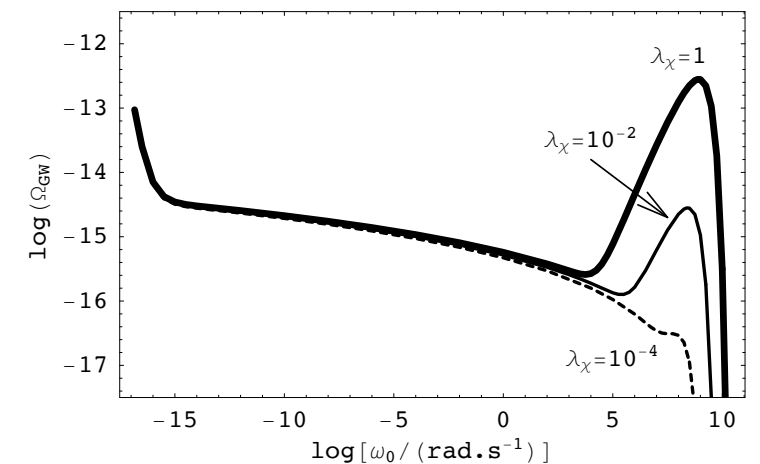
# non-standard equation of state after inflation

$$\Omega_{\text{GW}}(k) = \frac{\Delta_t^2}{12} \frac{k^2}{a_0^2 H_0^2} T_k^2, \quad T_k(t) = \frac{a(t_i)}{a(t)} = \left( \frac{t_i}{t} \right)^{\frac{2}{3(1+\omega)}} \rightarrow \Omega(f) = \Omega(f_0) \left( \frac{f}{f_0} \right)^{\frac{2(3\omega-1)}{1+3\omega}}$$



kination phase after inflation:  
Spookily '93; Joyce '96

GW production in  
(hybrid) quintessential models:  
Giovannini '99; Sa, Henriques '10

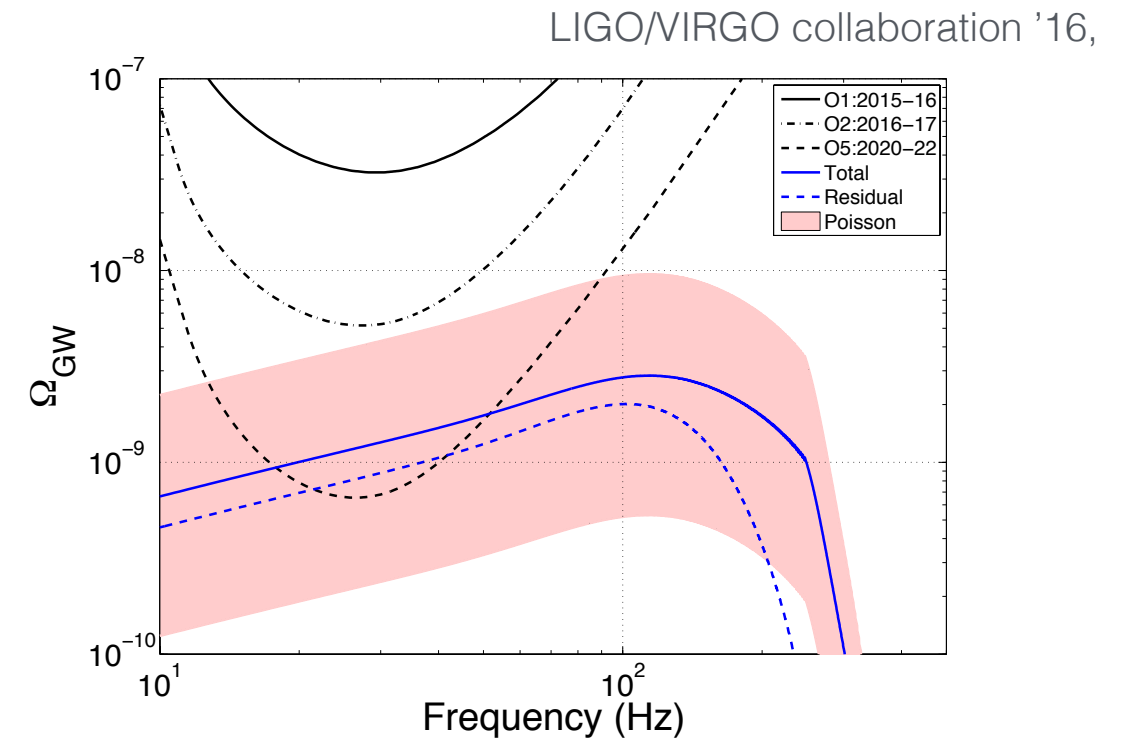
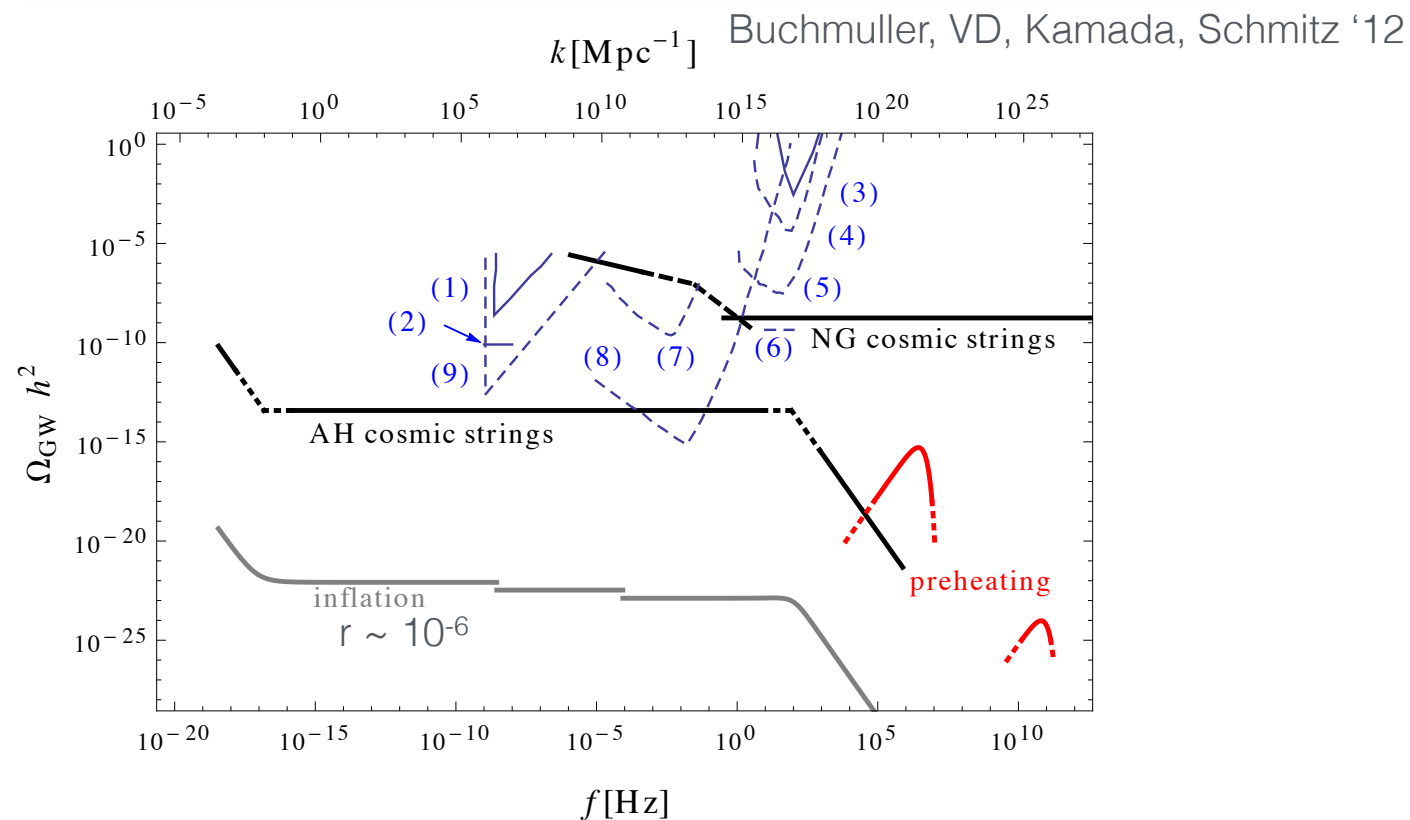


stiff equation of state during reheating can enhance primordial GW signal

# Other (stochastic) backgrounds

(incomplete list)

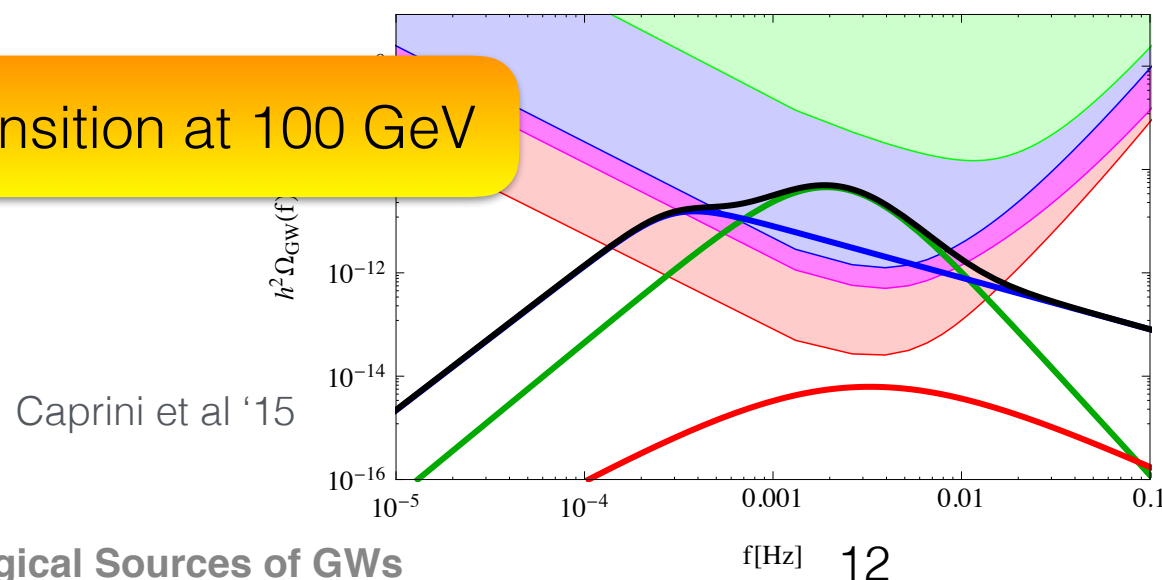
## Cosmic strings from GUT-scale phase transition



unresolved BH mergers

.... primordial black holes?

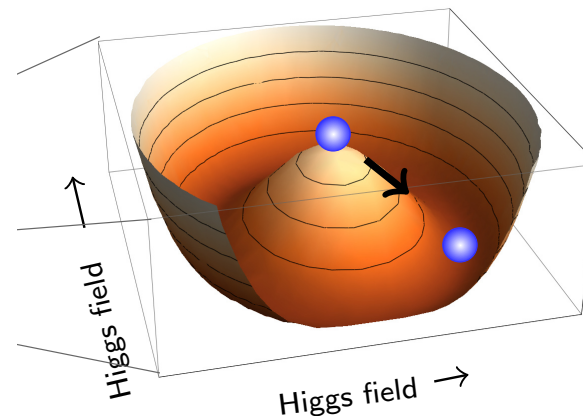
## phase transition at 100 GeV



scalar field  
sound waves  
turbulence

# Cosmic strings

U(1) phase transition in the early universe (after inflation) -> cosmic strings



Cosmic string network, topologically stable but loses energy into GWs (and particles)

Evolution of cosmic string network can be studied numerically in the “Abelian Higgs” or “Nambu Goto” model

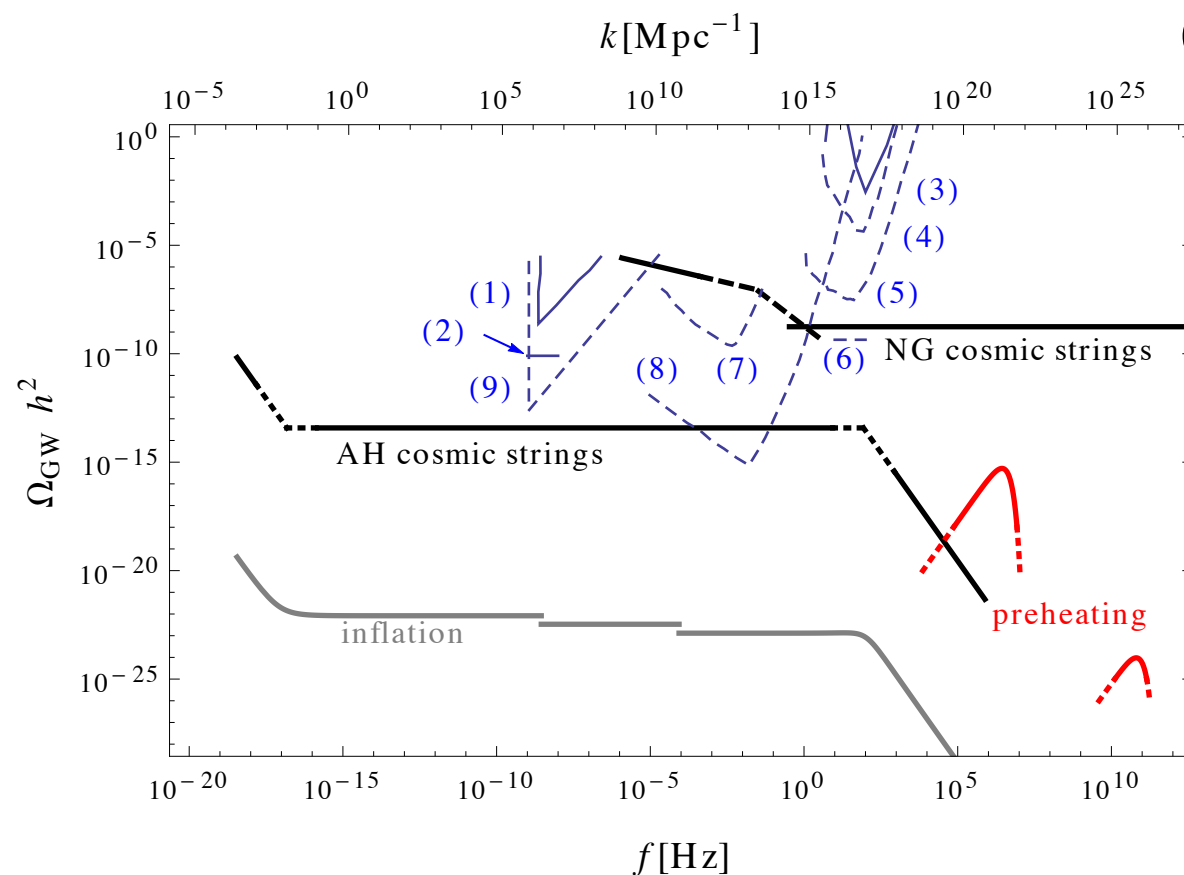
- Abelian Higgs model: Main source for GWs are horizon sized cosmic strings
- Nambu Goto model: Main source for GWs are small cosmic string loops

Vilenkin '81, Hindmarsh '12



# Cosmic strings

direct detection

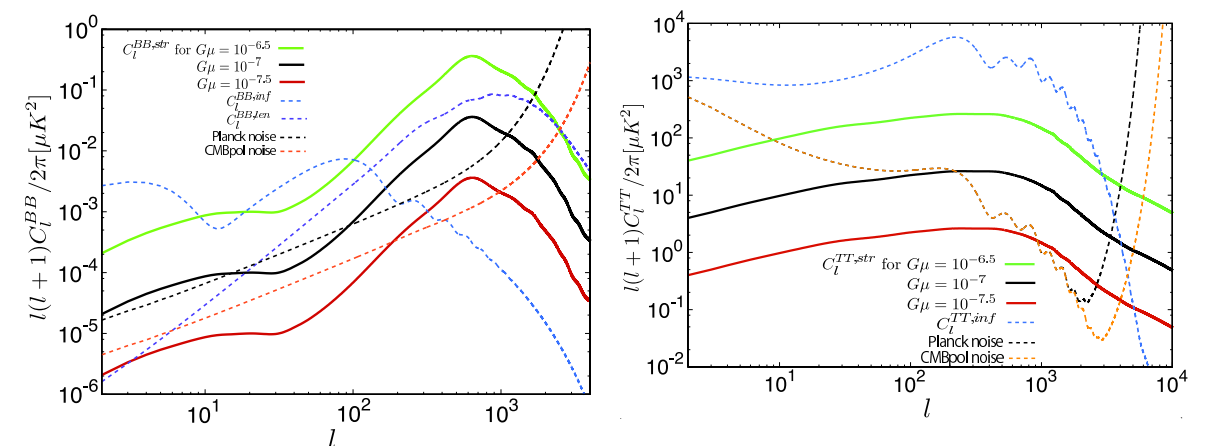


GUT-scale phase transition after hybrid inflation,  
Buchmueller, VD, Kamada, Schmitz '12

- similar to inflation signal but amplitude determined by scale of phase transition: can be strongly enhanced!
- large theoretical uncertainty
- as for inflation, sensitive to cosmological history

CMB: direkt search for cosmic strings:

Silk et al '13

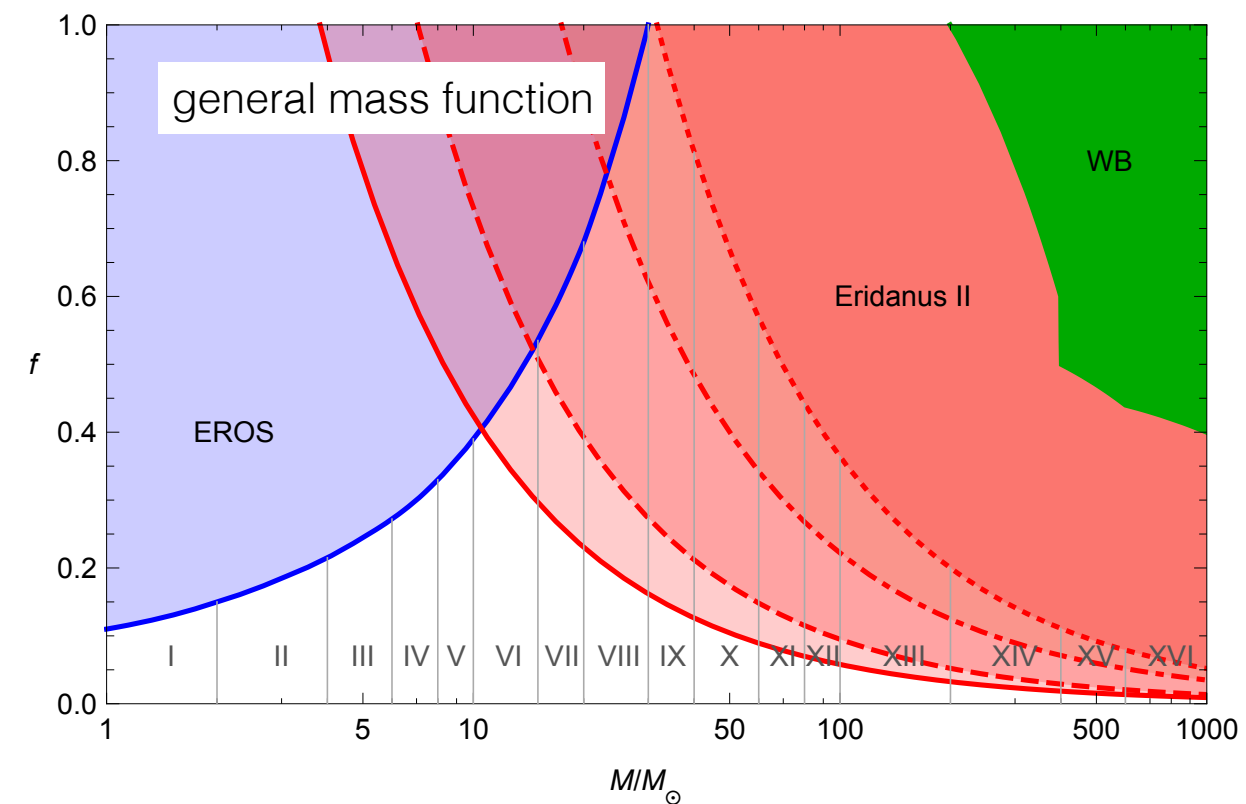
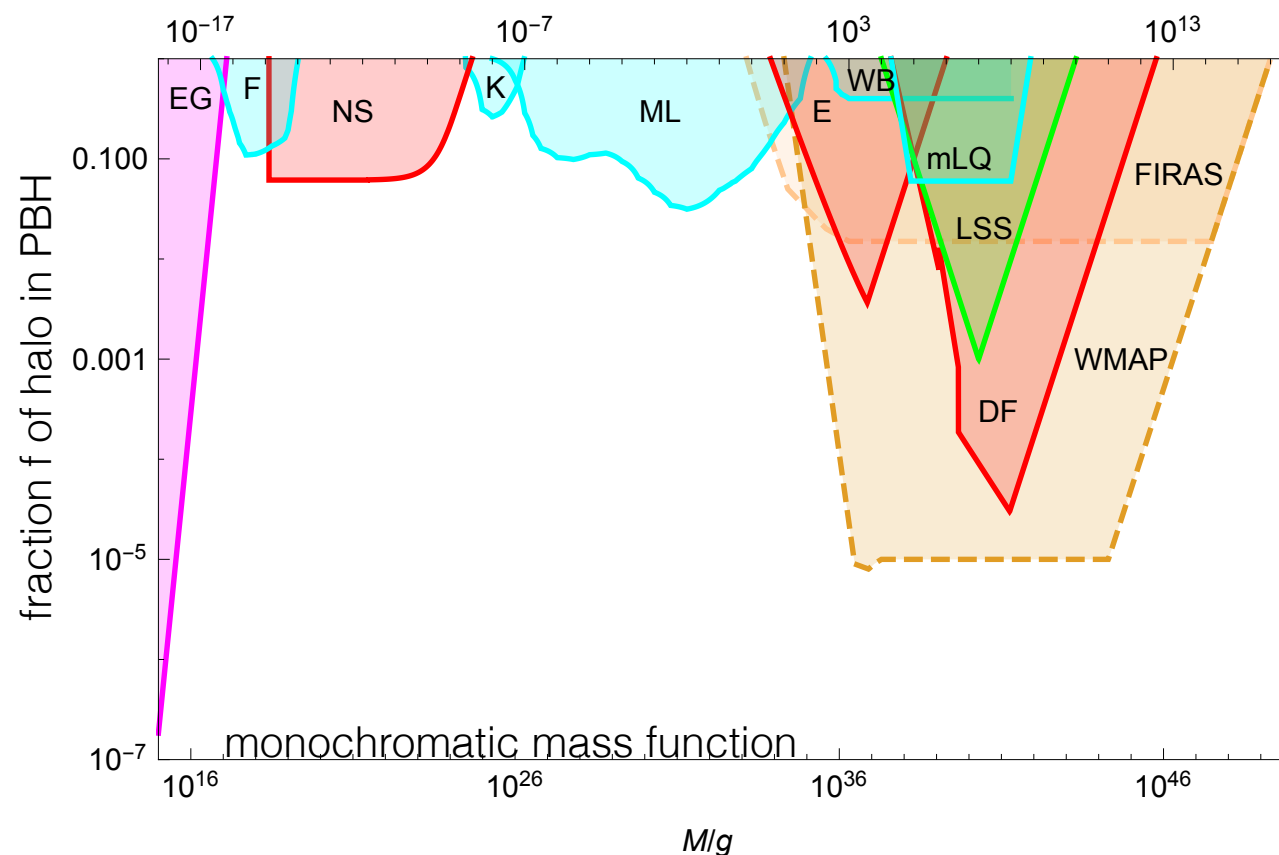


# primordial black holes

- scalar fluctuations larger than a critical value collapse into PBHs after horizon re-entry
- very large range of mass scales possible - very different to stellar BHs:

$$M \sim \frac{c^3 t}{G} \sim 10^{15} g \left( \frac{t}{10^{-23} s} \right) \sim \begin{cases} 10^{-5} g & \text{for } t = 10^{-43} s \\ 10^5 M_{\odot} & \text{for } t = 1 s \end{cases}$$

Carr, Kohri, Sendouda, Yokohama 2010,  
Carr, Kuhnel, Sandstad 2016



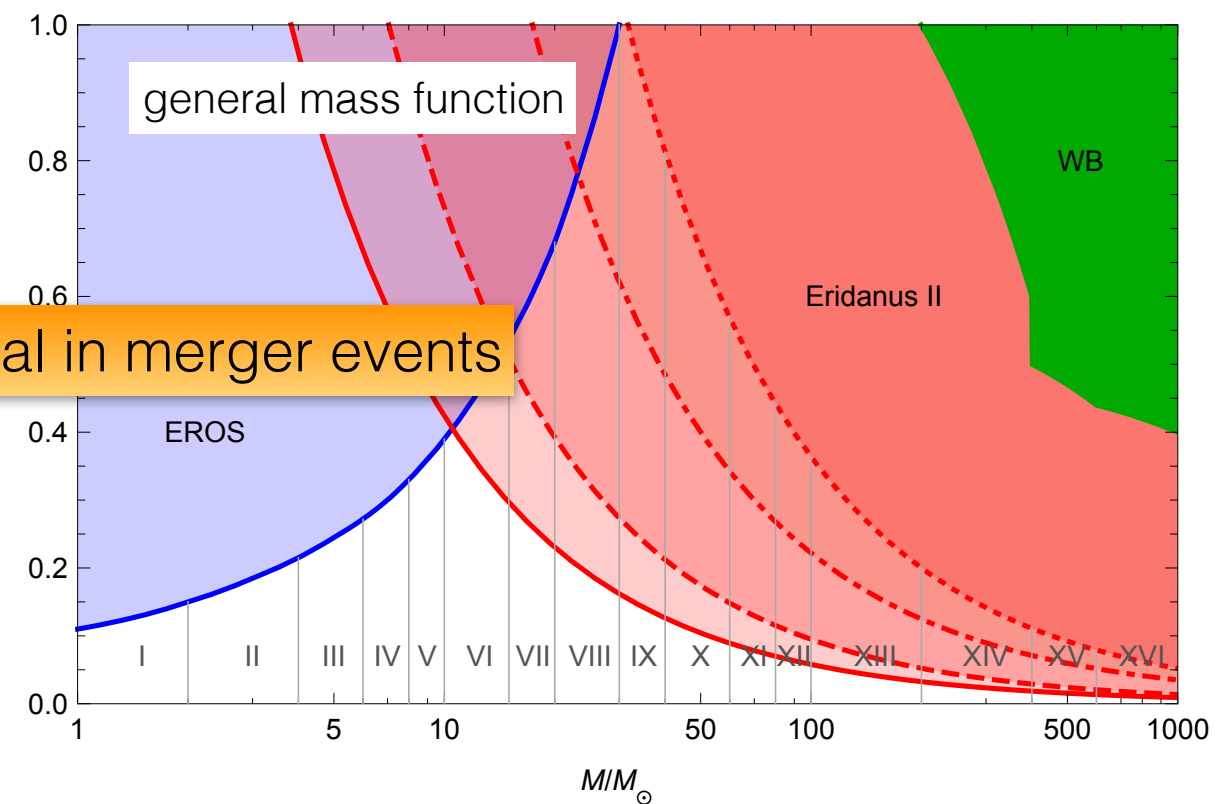
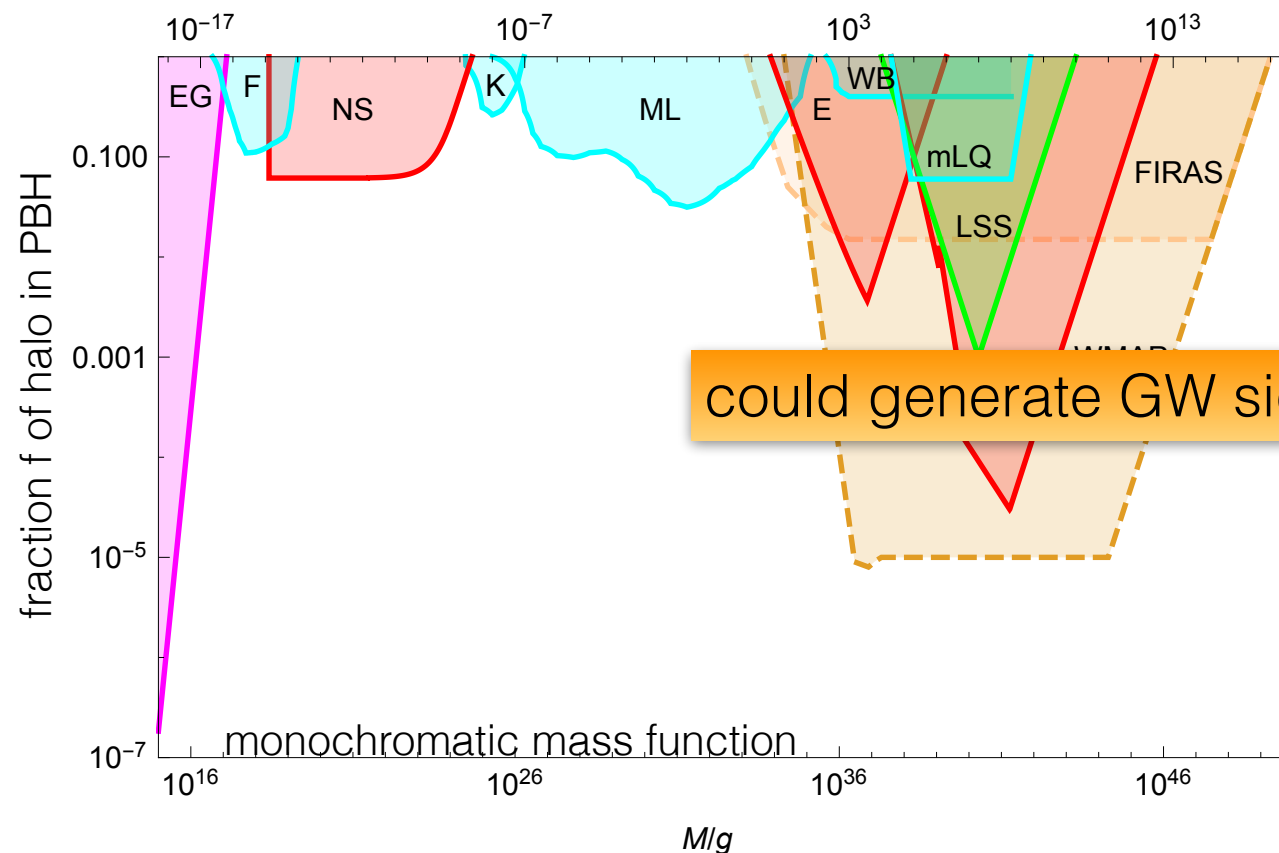
could still form 100% of DM for suitable broad mass function

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Carr, Kuhnel, Sandstad 2016



could still form 100% of DM for suitable broad mass function

# Conclusion and Outlook

- There is no guaranteed early Universe GW signal for upcoming detectors - but many interesting models will be probed
- The stochastic background of cosmic inflation is an extremely powerful tool: It would shed light on the microphysics of inflation, as well as the entire subsequent cosmological history
- The complementarity of CMB and direct GW measurements provides a powerful probe of the physics of cosmic inflation.
- For the simplest models of inflation, the primordial GW signal is unobservable by upcoming GW interferometers. But possible game changers are:
  - non-standard sources during inflation
  - stiff equation of state during reheating
  - second order tensor perturbations
- Other potential GW sources linked to the early universe are preheating, cosmic strings, merger of primordial black holes, phase transitions...

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**Thank you!**

backup slides



# second order GW production

Large scalar perturbations re-entering the horizon after inflation



grow in a matter-dominated reheating phase



source second order tensor perturbations

max. amplitude:  $\Omega_{\text{GW}}^{\text{max}} \approx \Delta_s^4 \Omega_r \left( \frac{k_{\text{inf}}}{k_{\text{RH}}} \right)^2$

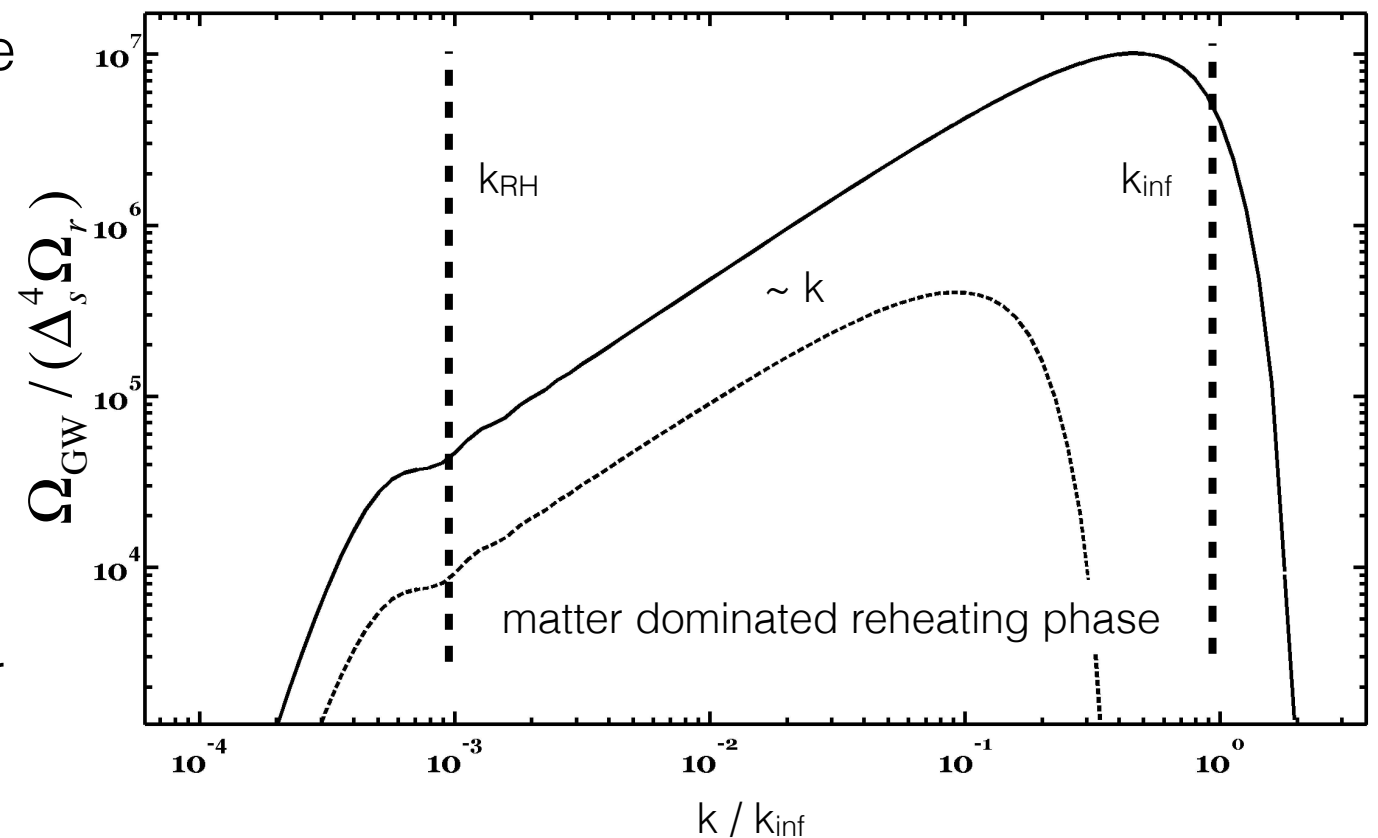
detectable signal for eLISA/LIGO/VIRGO for relatively small reheating temperatures and

$$(\Delta_s^2)_{\text{small scales}} \gg (\Delta_s^2)_{\text{CMB}}$$

note: very large  $\Delta_s^2$  on small scales leads to the formation of primordial black holes, which in turn can produce GWs in merger processes.

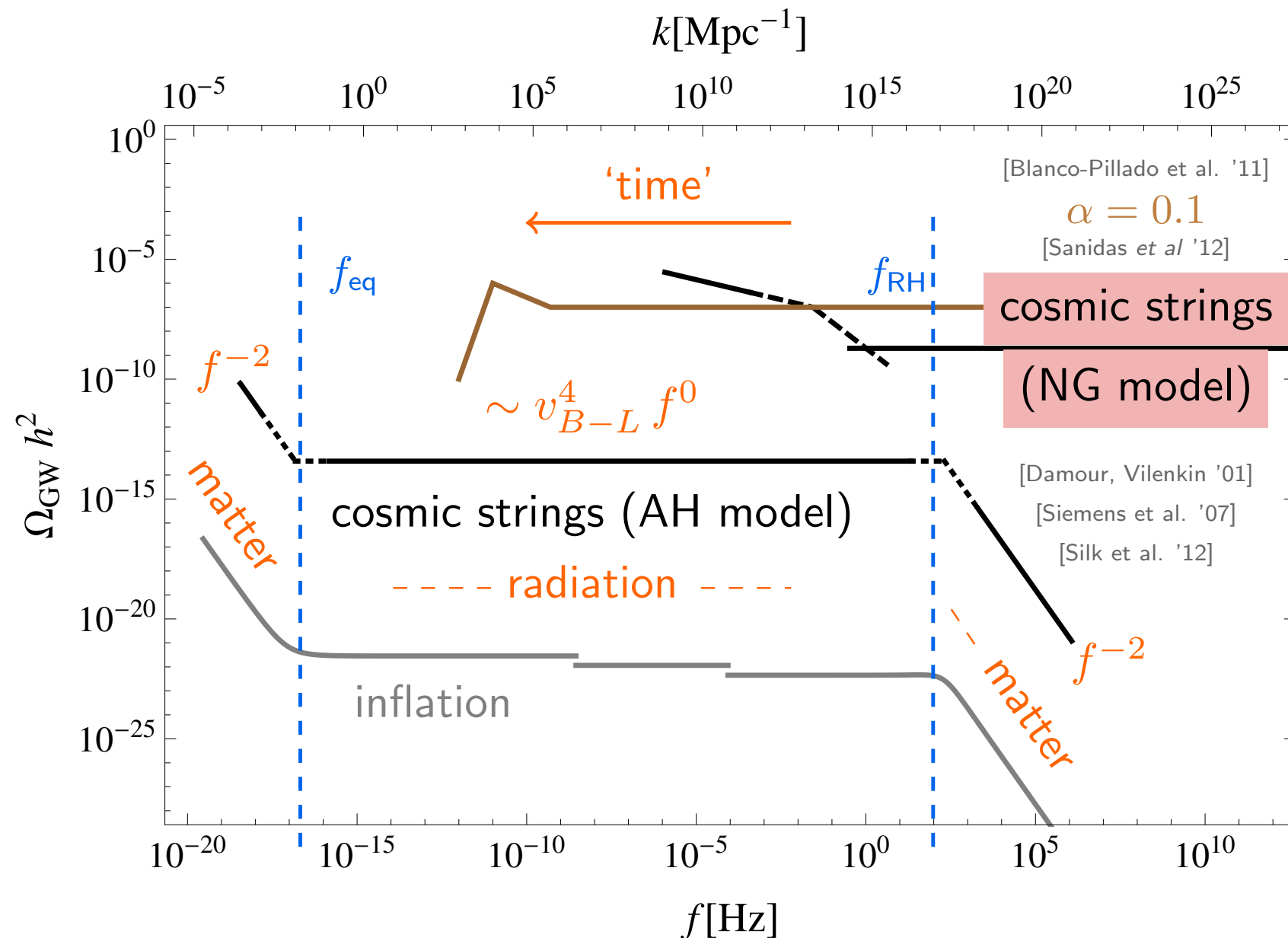
Tomita '67, .....

Assadulahi, Wands '09



primordial scalar fluctuations can source gravitational waves after inflation

# Cosmic strings (Nambu-Goto)



effect of loop-size  $\alpha$  in NG model.  $\alpha = 1$  preferred by most recent simulations.

# pseudoscalar inflation

## observational signatures

A brief overview:

- CMB: scalar and tensor fluctuations, in particular non-gaussianities, but also spectral index and  $\mu$ -distortions.
- blue GW signal (enhanced on small scales), maximally chiral. suppressed at CMB scales but interesting for LISA, LIGO/VIRGO,...
- PBH formation due to enhanced scalar power on small scales
- indirekt bound on tensor spectrum from  $N_{\text{eff}}$  in BBN and CMB

Barnaby, Namba, Peloso '11,  
Barnaby, Peloso '11,  
Barnaby, Pajer, Peloso '12,  
Meerburg, Pajer '12

Anber, Sorbo '12

Linde, Mooij, Pajer '12

Allen '96,  
Pagano, Salvati, Melchiorre '15

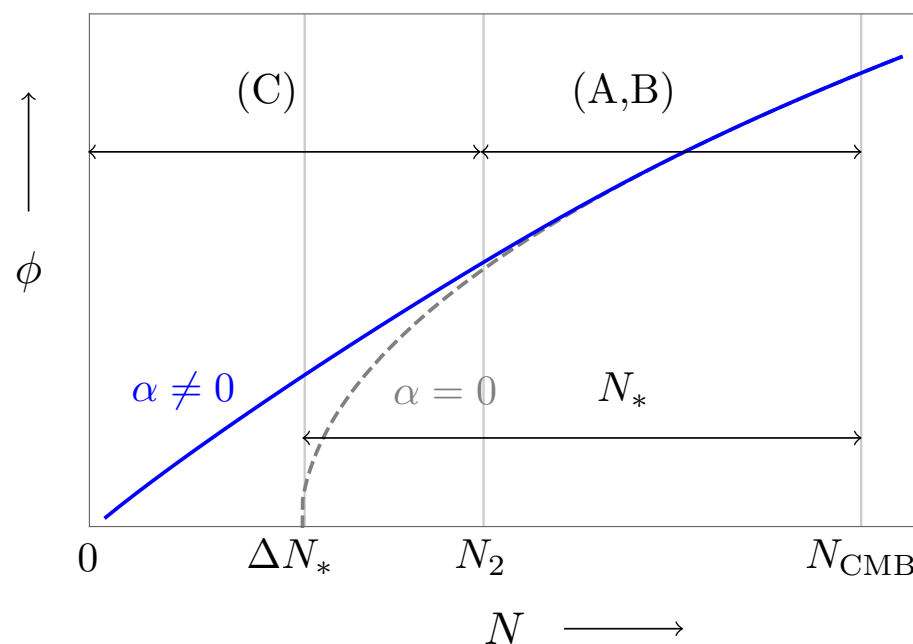
→ very interesting setup for multi-messenger analysis

main focus here on CMB and direct GW observations,  
can the inflaton gauge field coupling enable us to probe the microphysics of inflation?

# pseudoscalar inflation background dynamics

a useful classification of inflation models:  $\epsilon_V = \frac{\beta}{N^p} + \mathcal{O}(1/N^{p+1})$  Mukhanov '13  $\rightarrow$  3 parameters:  $\alpha, \beta, p$

evolution of the inflation field

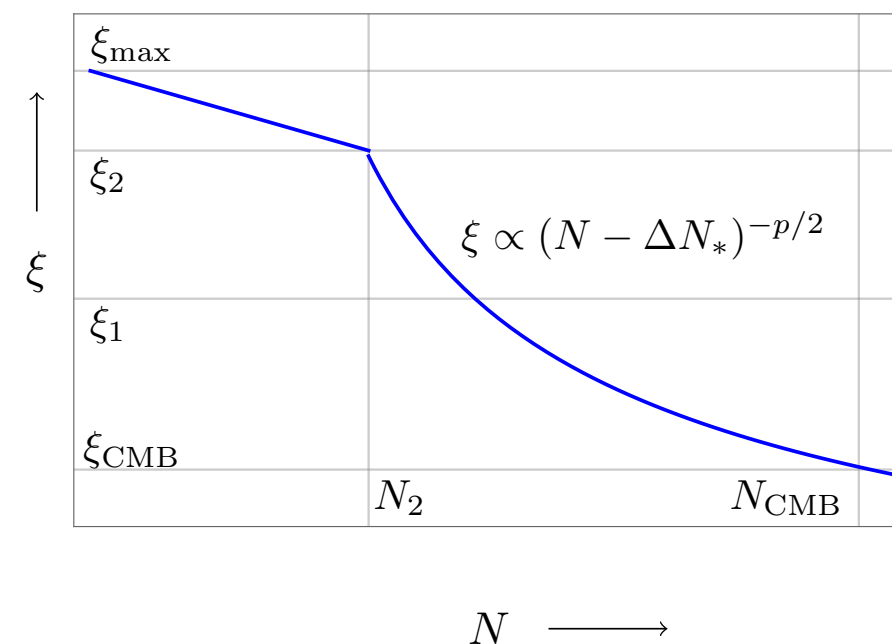


- additional friction, CMB observables evaluated at 'later' point on  $V(\phi)$

$$n_s \simeq 1 - \frac{\mathcal{O}(1)}{N_*}, \quad r = \frac{16\beta}{N_*}, \quad N_* < N_{\text{CMB}} \simeq 60$$

$n_s$  reduced,  $r$  increased

evolution of  $\xi \propto \sqrt{\epsilon} = \dot{\phi}/(\sqrt{2}H)$



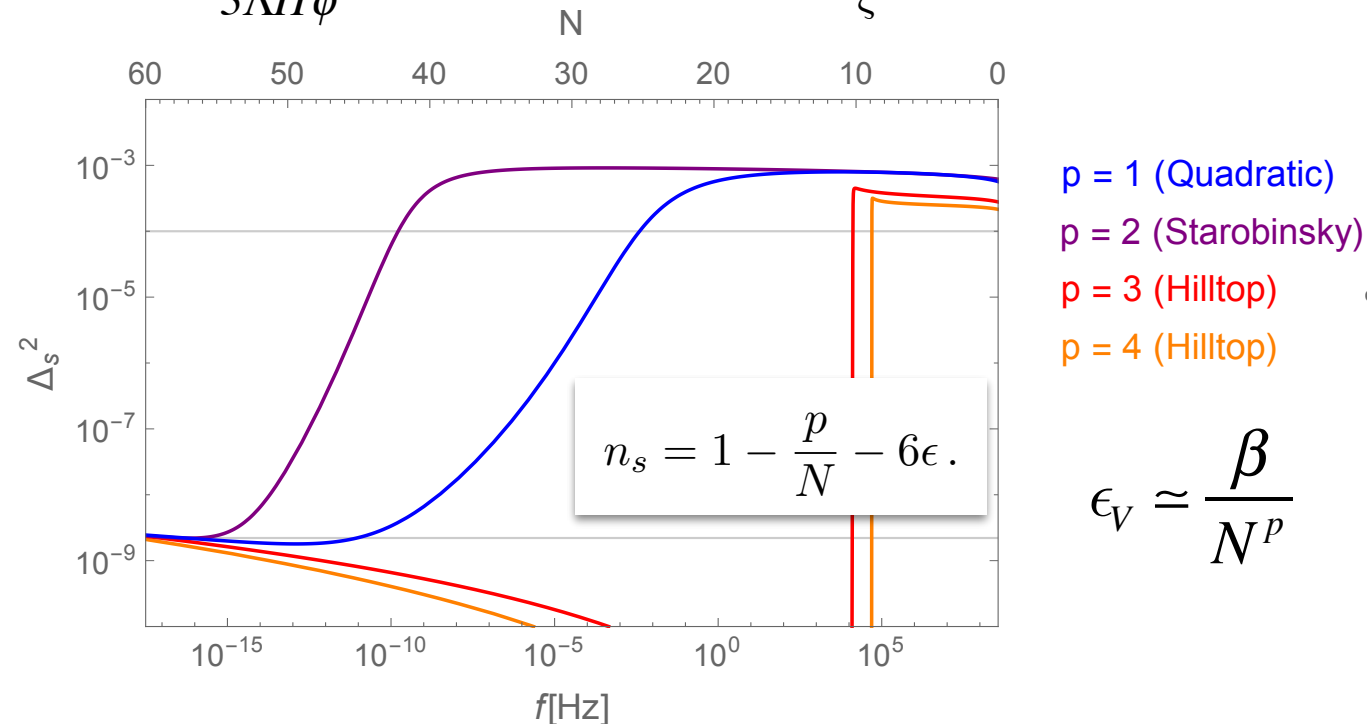
- rapid increase of  $\xi$  for large values of  $p$ , enters eom exponentially

large gauge field effects  
at the end of inflation,  
in particular for large  $p$  (small  $r$  !)

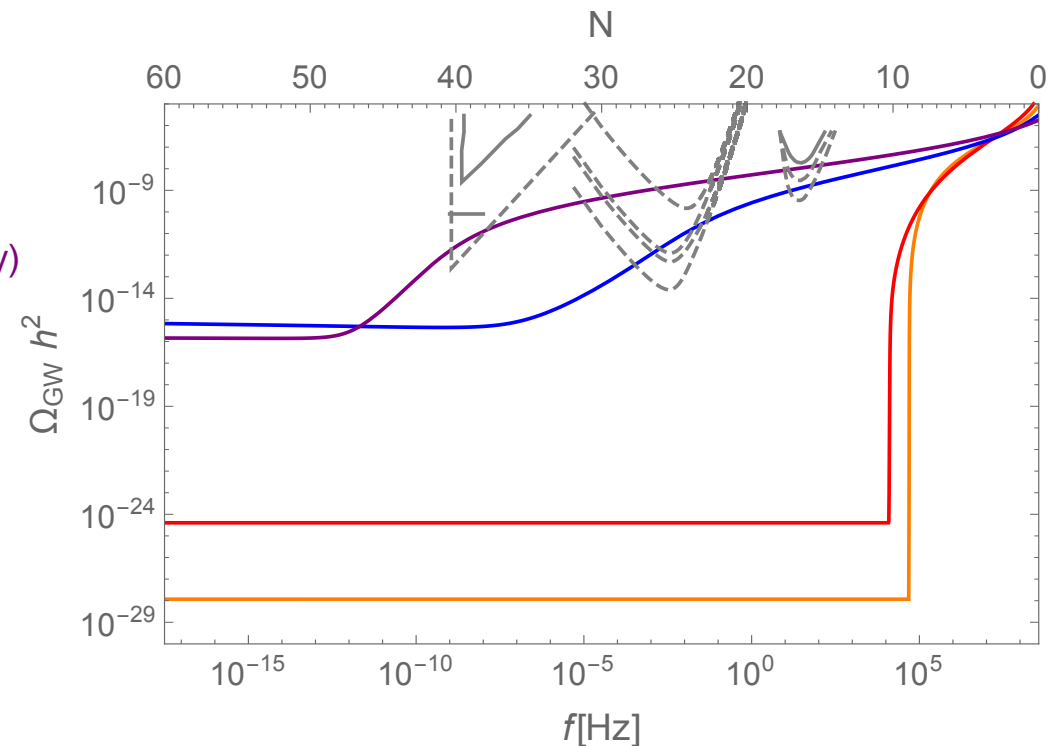
# pseudoscalar inflation perturbation power spectra

$$\Delta_s^2(k) = \Delta_s^2(k)_{\text{vac}} + \Delta_s^2(k)_{\text{gauge}} = \left( \frac{H^2}{2\pi |\dot{\phi}|} \right)^2 + \left( \frac{\alpha \langle \vec{E} \vec{B} \rangle}{3bH\dot{\phi}} \right)^2$$

$$b = 1 - 2\pi\xi \frac{\alpha \langle \vec{E} \vec{B} \rangle}{3\Lambda H \dot{\phi}}, \quad \langle \vec{E} \vec{B} \rangle \simeq \mathcal{N} \cdot 2.4 \cdot 10^{-4} \frac{H^4}{\xi^4} e^{2\pi\xi}$$



$$\Omega_{\text{GW}} = \frac{1}{12} \left( \frac{H}{\pi M_P} \right)^2 (1 + 4.3 \times 10^{(-7)} \frac{H^2}{M_P^2 \xi^6} e^{4\pi\xi})$$



- amplitude @  $N_{\text{CMB}}$  fixes one parameter
- large power on small scales  $\rightarrow$  PBHs
- nearly universal amplitude on small scales

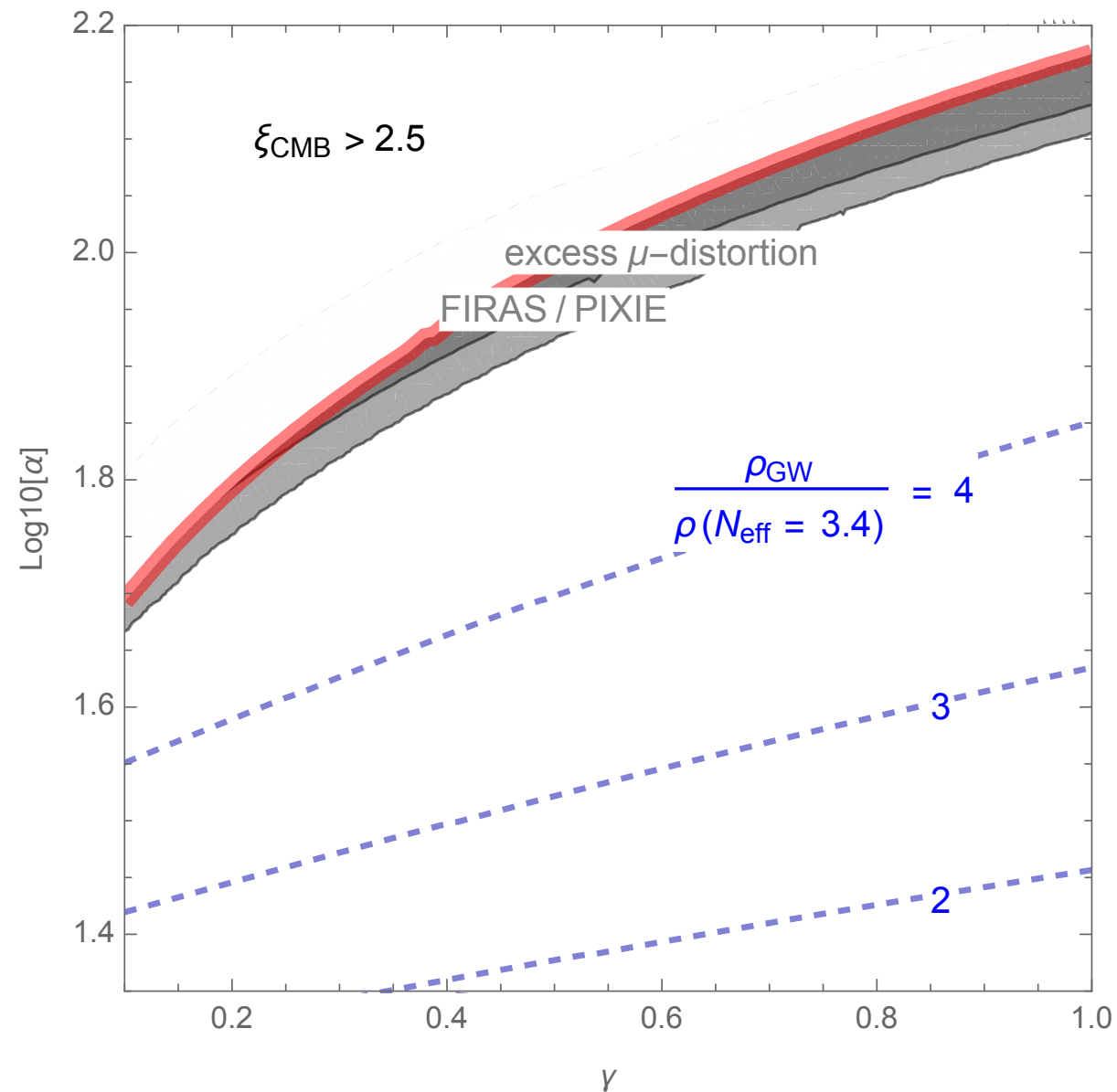
- increase at small scales to universal value
- low scale models feature stronger increase
- Onset of increase depends on coupling  $\alpha$

uncertainties in strong  
back reaction regime,  
Sloth '15, Peloso '16

we find both universal and inflation model specific features

# pseudoscalar inflation

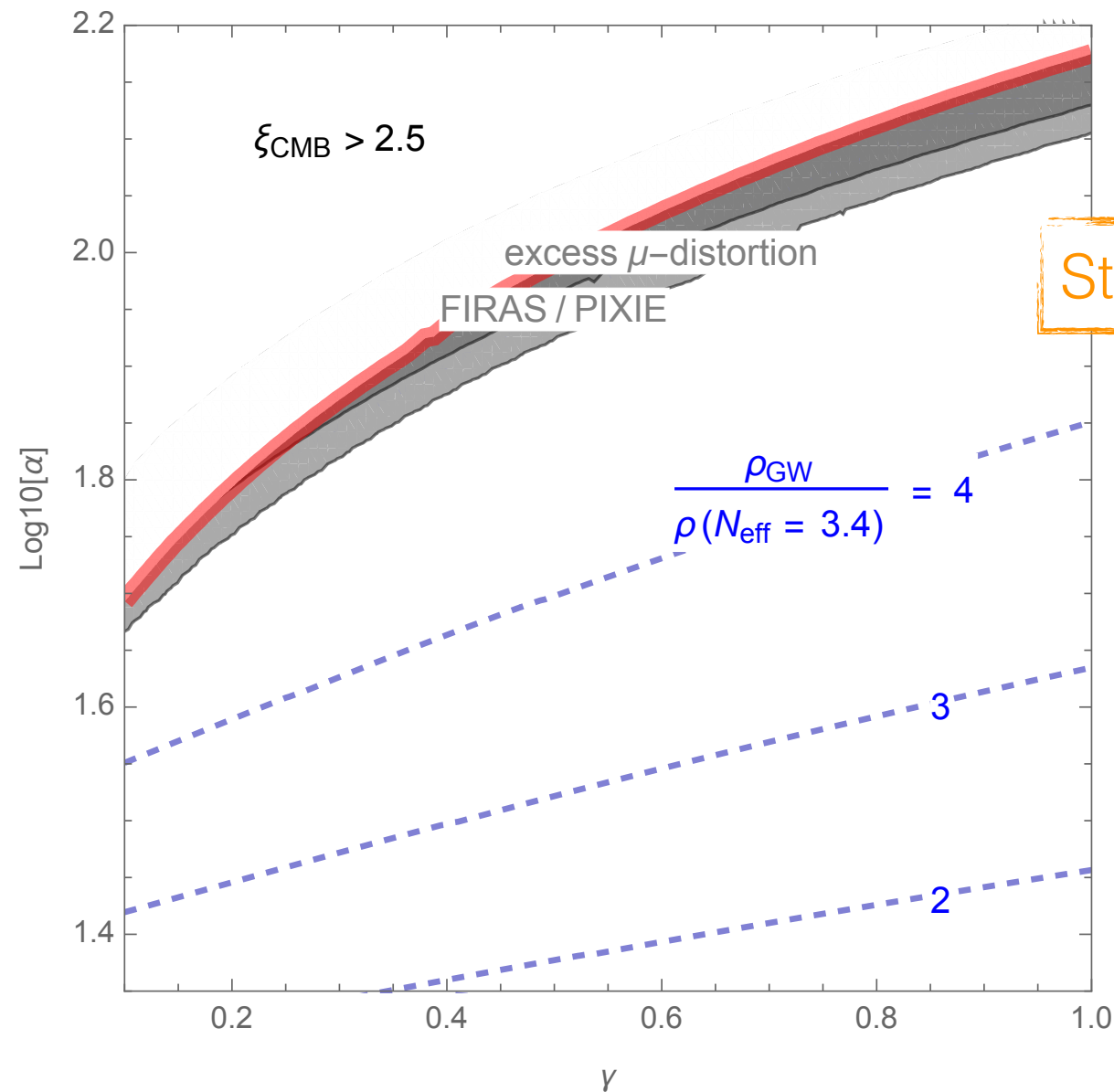
## $\mu$ - distortion and $N_{\text{eff}}$





# pseudoscalar inflation

## $\mu$ - distortion and $N_{\text{eff}}$



Starobinsky-like model