Cosmology and the Higgs boson

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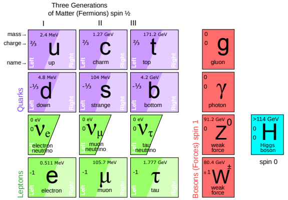
Outline

1. Standard Model and the reality of the Universe
   - Standard Model is in great shape!
   - All new physics at low scale–νMSM
   - Top-quark and Higgs-boson masses and vacuum stability
2. Stable Electroweak vacuum
3. Metastable vacuum and Cosmology
   - Safety today
   - Safety at inflation

Lesson from LHC so far – Standard Model is good



- SM works in all laboratory/collider experiments (electroweak, strong)
- LHC 2012 – final piece of the model discovered – Higgs boson
  - Mass measured $\sim 125\,\mathrm{GeV}$ – weak coupling! Perturbative and predictive for high energies
- Add gravity
  - get cosmology
  - get Planck scale $M_P \sim 1.22 \times 10^{19}$ GeV as the highest energy to worry about

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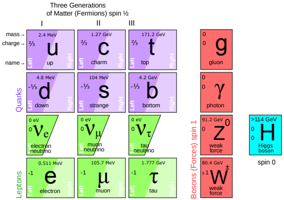

\+ Einstein gravity

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Many things in cosmology are not explained by SM

Experimental observations

- Dark Matter
- Baryon asymmetry of the Universe
- Inflation (nearly scale invariant spectrum of initial density perturbations)

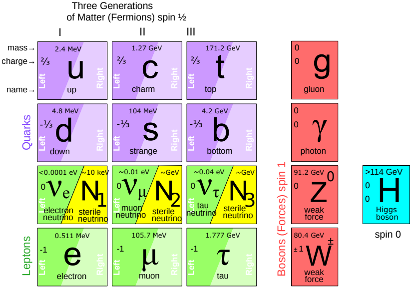
Laboratory also asks for SM extensions

- Neutrino oscillations

Nothing really points to a definite scale above EW

- Neutrino masses and oscillations (absent in SM)
  - Right handed neutrino between 1 eV and 10^{15} GeV
- Dark Matter (absent in SM)
  - Models exist from 10^{-5} eV (axions) up to 10^{20} GeV (Wimpzillas, Q-balls)
- Baryogenesys (absent in SM)
  - Leptogenesys scenarios exist from $M \sim 10$ MeV up to 10^{15} GeV

Possible: New physics only at low scales – νMSM



Role of sterile neutrinos

N_1 $M_1 \sim 1 - 50\text{keV}$: (Warm) Dark Matter,
Note: $M_1 = 7\text{keV}$ has been seen in X-rays?!

$N_{2,3}$ $M_{2,3} \sim$ several GeV:
Gives masses for active neutrinos, Baryogenesys

Asaka, Shaposhnikov'05; Asaka, Blanchet, Shaposhnikov'05

What we are left with?

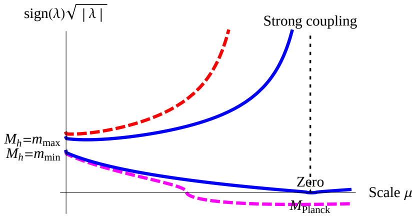
- Inflationary mechanism required
- Higgs is weakly coupled

but not completely trouble free

Standard Model self-consistency and Radiative Corrections

- Higgs self coupling constant λ changes with energy due to radiative corrections.

$$(4\pi)^2\beta_\lambda = 24\lambda^2 - 6y_t^4 + \frac{3}{8}(2g_2^4 + (g_2^2 + g_1^2)^2) + (-9g_2^2 - 3g_1^2 + 12y_t^2)\lambda$$

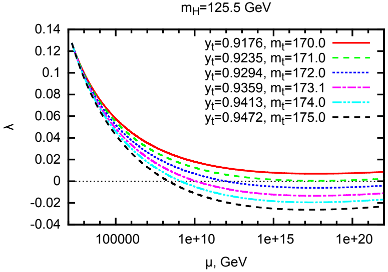


- Behaviour is determined by the masses of the Higgs boson $m_H = \sqrt{2\lambda}v$ and other heavy particles (top quark $m_t = y_t v/\sqrt{2}$)
- If Higgs is heavy $M_H > 170$ GeV – the model enters *strong coupling* at some low energy scale – new physics emerges.

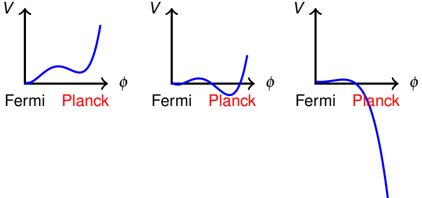
Lower Higgs masses: RG corrections push Higgs coupling to negative values

- For Higgs masses $M_H < M_{\text{critical}}$ coupling constant is negative above some scale μ_0.
- The Higgs potential may become negative!
  - Our world is not in the lowest energy state!
  - Problems at some scale $\mu_0 > 10^{10}$ GeV?

Coupling λ evolution:

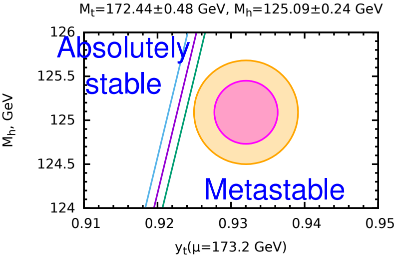


Higgs potential $V(\phi) \simeq \lambda(\phi)\frac{\phi^4}{4}$

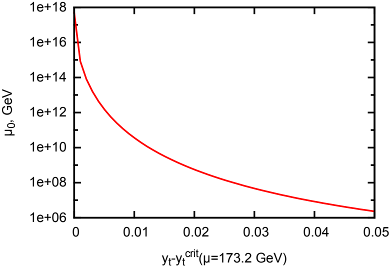


LHC result: SM is definitely perturbative up to Planck scale, and probably has metastable SM vacuum

Experimental values for y_t



Scale μ_0 for $\lambda(\mu_0) = 0$

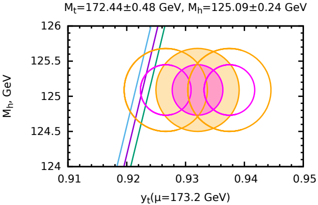


We live close to the metastability boundary – but on which side?!

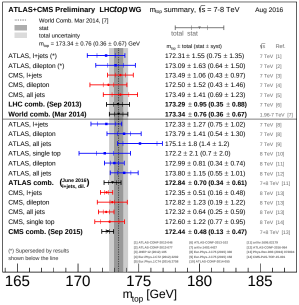
Future measurements of top Yukawa and Higgs mass are essential!

Determination of top quark Yukawa

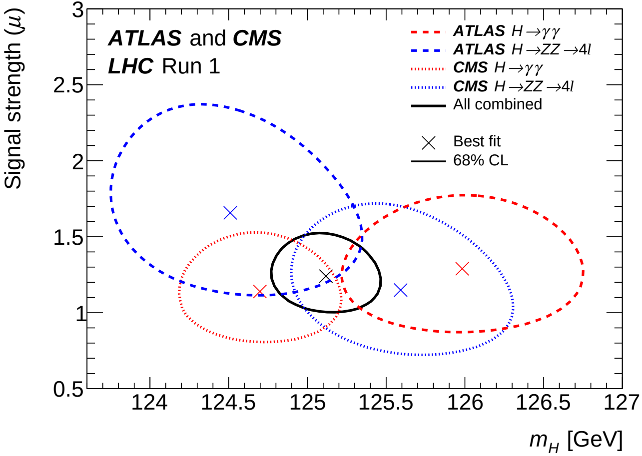
- Hard to determine mass in the events
- Hard to relate the “pole” (the same for “Mont-Carlo”) mass to the $\overline{\text{MS}}$ top quark Yukawa
  - NLO event generators
  - Electroweak corrections – important at the current precision goals!



- Build a lepton collider?
- Improve analysis on a hadron collider?



Higgs boson mass measurements



ATLAS CMS 2015

Vacuum stability – what it means?

- Stable Electroweak vacuum – looks safe
- Metastable – is it ok?

Inflation versus vacuum stability

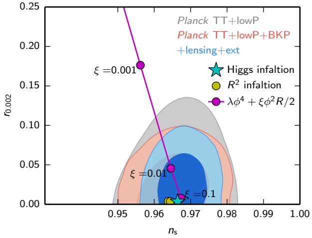
Stable SM vacuum	inflaton & Higgs independent	inflaton & Higgs interacting	inflaton = Higgs
Large r	Yes	Yes	Yes (threshold corr.)
Small r	Yes	Yes	Yes
Planck scale corections	Any	Any	Scale inv.

Metastable SM vacuum	inflaton & Higgs independent	inflaton & Higgs interacting	inflaton = Higgs
Large r	No	Yes Model dep.	No
Small r	Yes $r < 10^{-9}$	Yes Model dep.	Yes (threshold corr.)
Planck scale corections	Restricted	Model dep.	Scale inv.

Stable EW vacuum – mostly anything works

Would be a rather dull situation

- No problems throughout the whole thermal evolution of the Universe.
- Adding inflation – many examples
  - R^2 inflation
  - non-minimally coupled Higgs inflation
    - specific CMB predictions
  - Separate scalar inflaton interacting with the Higgs boson
    - Together with requirements of weak coupling and some scale symmetries often predicts hidden light or EW scale scalars



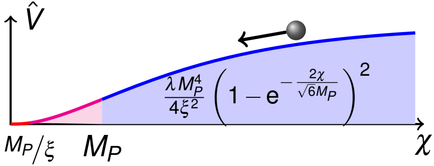
Higgs inflation at tree level

Scalar part of the (Jordan frame) action

$$S_J = \int d^4x\sqrt{-g}\left\{-\frac{M_P^2}{2}R - \xi\frac{h^2}{2}R + g_{\mu\nu}\frac{\partial^\mu h\partial^\nu h}{2} - \frac{\lambda}{4}(h^2-v^2)^2\right\}$$

To get observed $\delta T/T \sim 10^{-5}$

$$\frac{\sqrt{\lambda}}{\xi} = \frac{1}{49000}$$

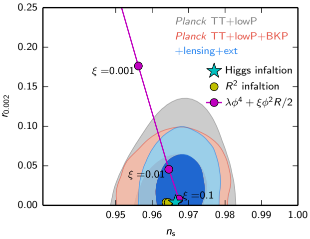


Conformal transformation: $\hat{g}_{\mu\nu} = \sqrt{1+\frac{\xi\phi^2}{M_P^2}}\,g_{\mu\nu}$,

Requirement from UV physics – No corrections $\frac{h^n}{M_P^{4-n}}$ allowed

CMB parameters are predicted

Exactly like preferred by CMB



For large ξ Higgs inflation

spectral index $\quad n \simeq 1 - \frac{8(4N+9)}{(4N+3)^2} \simeq 0.97$

tensor/scalar ratio $\quad r \simeq \frac{192}{(4N+3)^2} \simeq 0.0033$

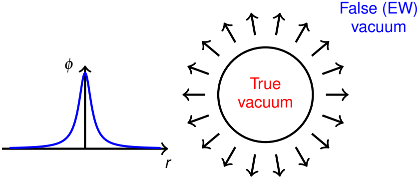
$$\delta T/T \sim 10^{-5} \quad \Longrightarrow \quad \frac{\xi}{\sqrt{\lambda}} \simeq 47000$$

Note: for very near critical top quark/Higgs masses results change and allow for larger r

What if we live in metastable vacuum?

Do not worry! At least not too much

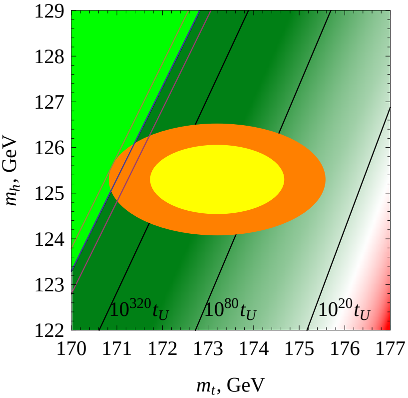
Vacuum decays by creating bubbles of true vacuum, which then expand very fast ($v \to c$)



Tunneling suppression:

$$p_{\text{decay}} \propto e^{-S_{\text{bounce}}} \sim e^{-\frac{8\pi^8}{3\lambda(h)}}$$

Lifetime $\gg$ age of the Universe!



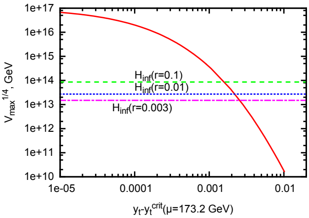
Note on Planck corrections

- Critical bubble size $\sim$ Planck scale
- Potential corrections $V_{\text{Planck}} = \pm \frac{\phi^n}{M_P^{n-4}}$ change lifetime!
  - Only '$+$' sign is allowed for Planck scale corrections!

As far as we are "safe" now (i.e. at low energies), what about Early Universe?
What happens with the Higgs boson at inflation?

- if Higgs boson is completely separate from inflation
- if Higgs boson interacts with inflaton/gravitation background
- if Higgs boson drives inflation

Metastable vacuum during inflation *is* dangerous



- Let us suppose Higgs is not at all connected to inflationary physics (e.g. R^2 inflation)
- All fileds have vacuum fluctuation
- Typical momentum $k \sim H_{\text{inf}}$ is of the order of Hubble scale
  - If typical momentum is greater than the potential barrier – SM vacuum would decay if

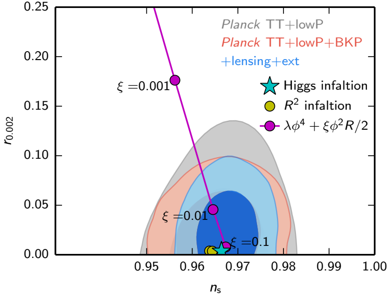
$$H_{\text{inf}} > V_{\text{max}}^{1/4}$$

Most probably, fluctuations at inflation lead to SM vacuum decay…

- Observation of tensor-to-scalar ratio *r* by CMB polarization missions would mean great danger for metastable SM vacuum!

Measurement of primordial tensor modes determines scale of inflation

$$H_{\rm inf} = \sqrt{\frac{V_{\rm infl}}{3M_P^2}} \sim 8.6 \times 10^{13}\,{\rm GeV} \left(\frac{r}{0.1}\right)^{1/2}$$



Does inflation contradict metastable EW vacuum?

- Higgs interacting with inflation can cure the problem. Examples
  - Higgs (ϕ)–inflaton (χ) interaction may stabilize the Higgs

    $$L_{\text{int}} = -\alpha\phi^2\chi^2$$

  - Higgs-gravity *negative* non-minimal coupling stabilizes Higgs in de-Sitter (inflating) space

    $$L_{\text{nm}} = \xi\phi^2 R$$

    (However, destabilises EW vacuum after inflation)
- New physics below μ_0 may remove Planck scale vacuum and make EW vacuum stable – many examples
  - Threshold effects
  - Modified λ running

New physics *above* μ_0 may solve the problem

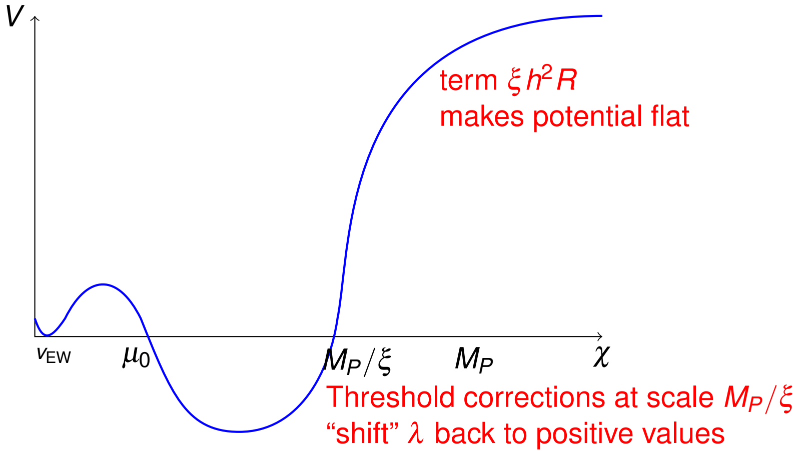
Requirements

- Minimum at Planck scale should be removed (but can remain near $\mu_0 \sim 10^{10}\,\mathrm{GeV}$)
- Reheating after inflation should be fast.

No need for new physics at "low" ($< \mu_0$) scales!
Example: Higgs inflation with threshold corrections at M_p/ξ

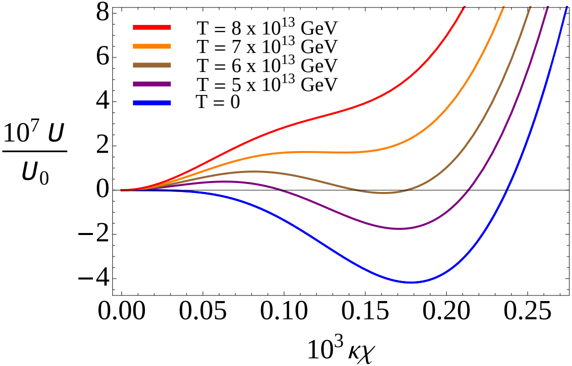
Higgs inflation and radiative corrections

$$S_J = \int d^4x\sqrt{-g}\left\{-\frac{M_P^2}{2}R - \xi\frac{h^2}{2}R + g_{\mu\nu}\frac{\partial^\mu h\partial^\nu h}{2} - \frac{\lambda}{4}(h^2-v^2)^2\right\}$$



(Not really to scale)

After inflation symmetry is restored in preheating



- Thermal potential removes the high scale vacuum
- Universe cools down to EW vacuum

Inflation versus vacuum stability

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Conclusions: Higgs potential stability

what is good and what is bad?

Bad

Predictions depend on high scale physics

Conclusions: Higgs potential stability

what is good and what is bad?

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Predictions depend on high scale physics

Good

Predictions depend on high scale physics

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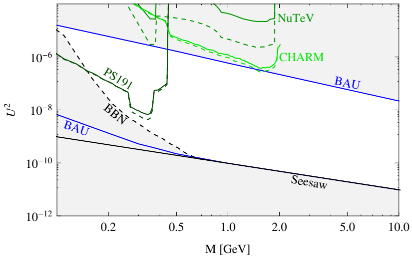
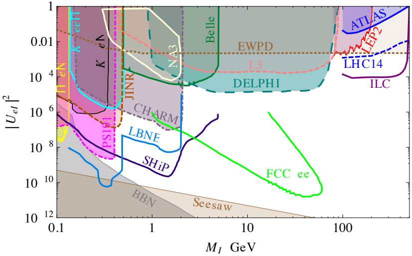
Backup

Search for $N_{2,3}$ is possible

- Leptogenesys by $N_{2,3}$
  $$\Delta M / M \sim 10^{-3}$$
- Experimental searches
  - $N_{2,3}$ production in hadron decays (LHCb):
    - Missing energy in K decays
    - Peaks in Dalitz plot
  - $N_{2,3}$ decays into SM
    - Beam target: SHiP
    - High luminosity lepton collider at Z peak





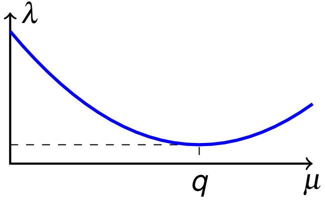
Note: Other related models (e.g. scalars for DM generation, light inflaton) also show up in such experiments

RG running indicates small λ at Planck scale

Renormalization evolution of the Higgs self coupling λ

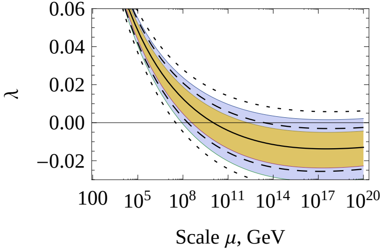
$$\lambda \simeq \lambda_0 + b \ln^2 \frac{\mu}{q}$$

$$\left.\begin{array}{l} b \simeq 0.000023 \\ \lambda_0 - \text{small} \\ q \text{ of the order } M_p \end{array}\right\} \text{depend on } M_h^*,\ m_t^*$$



$$(4\pi)^2 \frac{\partial \lambda}{\partial \ln \mu} = 24\lambda^2 - 6y_t^4 + \frac{3}{8}(2g_2^4 + (g_2^2 + g_1^2)^2) + (-9g_2^2 - 3g_1^2 + 12y_t^2)\lambda$$

Higgs mass M_h=125.3±0.6 GeV



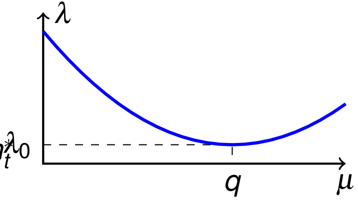
RG running indicates small λ at Planck scale

Potentials in different regimes

$$\lambda \simeq \lambda_0 + b\ln^2\frac{\mu}{q}$$

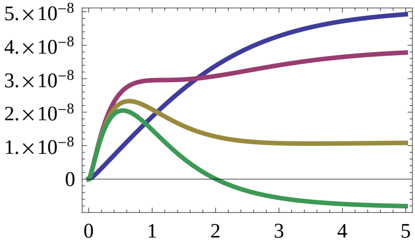
$b \simeq 0.000023$

$$\left.\begin{array}{l}\lambda_0 - \text{small}\\ q \text{ of the order } M_p\end{array}\right\} \text{depend on } M_h^*,\ m_t^*$$

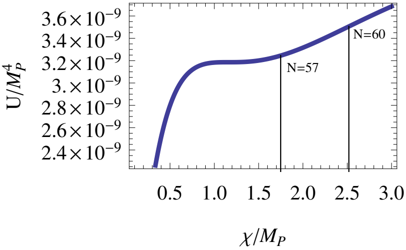


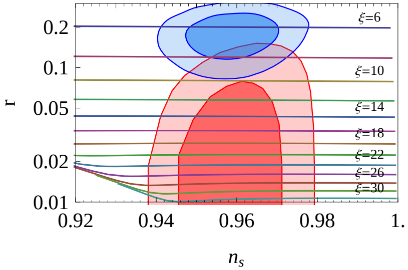
$$U(\chi) \simeq \frac{\lambda(\mu)M_P^4}{4\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2$$

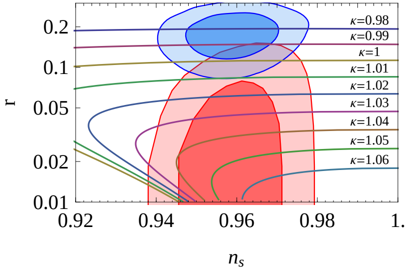
$$\mu^2 = \alpha^2\frac{y_t(\mu)^2}{2}\frac{M_P^2}{\xi}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)$$



Interesting inflation near to the critical point







Parameters in
particle physics: λ_0, q, ξ
cosmology: $\mathscr{P}_R$, r, n_s

$\kappa \sim q\frac{\sqrt{\xi}}{M_P}\frac{\sqrt{2}}{y_t}$

For given r (or ξ) very small change of κ (or M_h^*) gives any n_s

Modifying the gravity action gives inflation

Another way to get inflation in the SM

The first working inflationary model

Starobinsky'80

The gravity action gets higher derivative terms

$$S_J = \int d^4x\sqrt{-g}\left\{-\frac{M_P^2}{2}R + \frac{\zeta^2}{4}R^2\right\} + S_{SM}$$

Conformal transformation

conformal transformation (change of variables)

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \,, \qquad \Omega^2 \equiv \exp\left(\frac{\chi(x)}{\sqrt{6}M_P}\right)$$

$\chi(x)$ – new field (d.o.f.) "scalaron"

Resulting action (Einstein frame action)

$$S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} - \frac{M_P^4}{4\zeta^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2 \right\}$$

Cut off scale today

Let us work in the Einstein frame for simplicty

Change of variables: $\frac{d\chi}{dh} = \frac{M_P\sqrt{M_P^2+(\xi+6\xi^2)h^2}}{M_P^2+\xi h^2}$ leads to the higher order terms in the potential (expanded in a power law series)

$$V(\chi) = \lambda \frac{h^4}{4\Omega^4} \simeq \lambda \frac{h^4}{4} \simeq \lambda \frac{\chi^4}{4} + \# \frac{\chi^6}{(M_P/\xi)^2} + \cdots$$

Unitarity is violated at tree level

in scattering processes (eg. $2 \to 4$) with energy above the "cut-off"

$$E > \Lambda_0 \sim \frac{M_P}{\xi}$$

Hubble scale at inflation is $H \sim \lambda^{1/2}\frac{M_P}{\xi}$ – not much smaller than the today cut-off Λ_0 :(

Burgess:2009ea,Barbon:2009ya,Hertzberg:2010dc

"Cut off" is background dependent!

Classical background Quantum perturbations

$$\chi(x,t) = \bar{\chi}(t) + \delta\chi(x,t)$$

leads to background dependent suppression of operators of dim $n > 4$

$$\frac{\mathcal{O}_{(n)}(\delta\chi)}{[\Lambda_{(n)}(\bar{\chi})]^{n-4}}$$

Example

Potential in the inflationary region $\chi > M_P$:

$$U(\chi) = \frac{\lambda M_P^4}{4\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2$$

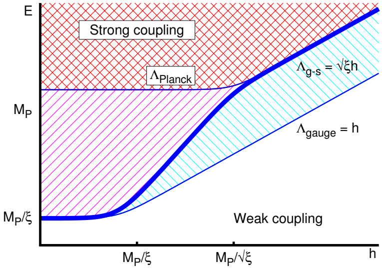
leads to operators of the form: $\frac{\mathcal{O}_{(n)}(\delta\chi)}{[\Lambda_{(n)}(\bar{\chi})]^{n-4}} = \frac{\lambda M_P^4}{\xi^2} e^{-\frac{2\bar{\chi}}{\sqrt{6}M_P}} \frac{(\delta\chi)^n}{M_P^n}$

Leading at high n to the "cut-off"

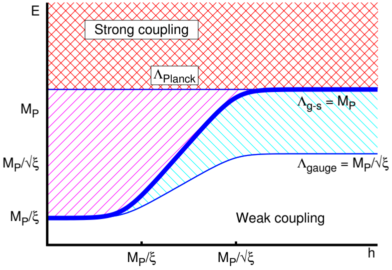
$$\Lambda \sim M_P$$

Cut-off grows with the field background

Jordan frame



Einstein frame



Relation between cut-offs in different frames:

$$\Lambda_{\text{Jordan}} = \Lambda_{\text{Einstein}} \Omega$$

Relevant scales

Hubble scale $H \sim \lambda^{1/2} \frac{M_P}{\xi}$

Energy density at inflation

$$V^{1/4} \sim \lambda^{1/4} \frac{M_P}{\sqrt{\xi}}$$

Reheating temperature $M_P/\xi < T_{\text{reheating}} < M_P/\sqrt{\xi}$

Bezrukov:2011jz

Shift symmetric UV completion allows to have effective theory during inflation

$$\mathscr{L} = \frac{(\partial_\mu \chi)^2}{2} - U_0 \left(1 + \sum u_n \mathrm{e}^{-n\cdot\chi/M}\right)$$
$$= \frac{(\partial_\mu \chi)^2}{2} - U_0 \left(1 + \sum \frac{1}{k!} \left[\frac{\delta\chi}{M}\right]^k \sum n^k u_n \mathrm{e}^{-n\cdot\bar{\chi}/M}\right)$$

Effective action (from quantum corrections of loops of $\delta\chi$)

$$\mathscr{L}_{\mathrm{eff}} = f^{(1)}(\chi)\frac{(\partial_\mu \chi)^2}{2} - U(\chi) + f^{(2)}(\chi)\frac{(\partial^2\chi)^2}{M^2} + f^{(3)}(\chi)\frac{(\partial\chi)^4}{M^4} + \cdots$$

All the divergences are absorbed in u_n and in $f^{(n)} \sim \sum f_l e^{-n\chi/M}$

UV completion requirement

Shift symmetry (or scale symmetry in the Jordan frame) is respected

$$\chi \mapsto \chi + \mathrm{const}$$

Connection of inflationary and low energy physics requires more assumptions on the UV theory

$$\lambda U(\bar{\chi}+\delta\chi) = \lambda \left(U(\bar{\chi}) + \frac{1}{2}U''(\bar{\chi})(\delta\chi)^2 + \frac{1}{3!}U'''(\bar{\chi})(\delta\chi)^3 + \cdots \right)$$

in one loop: $\lambda U''(\bar{\chi})\bar{\Lambda}^2,\ \lambda^2(U''(\bar{\chi}))^2 \log\bar{\Lambda}$,

in two loops: $\lambda U^{(IV)}(\bar{\chi})\bar{\Lambda}^4,\ \lambda^2(U''')^2\bar{\Lambda}^2,\ \lambda^3 U^{(IV)}(U'')^2(\log\bar{\Lambda})^2$,

If no power law divergences are generated

then the loop corrections are arranged in a series in λ

$$U(\chi) = \lambda U_1(\chi) + \lambda^2 U_2(\chi) + \lambda^3 U_3(\chi) + \cdots$$

A rule to fix the finite parts of the counterterm functions $U_i(\chi)$

Example – dimensional regularisation + $\overline{\text{MS}}$

RG improved potential for Higgs inflation

$$U_{\text{RG improved}}(\chi) = \frac{\lambda(\mu)}{4}\frac{M_P^4}{\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2$$

with

$$\mu^2 = \alpha^2 m_t^2(\chi) = \alpha^2 \frac{y_t^2(\mu)}{2}\frac{M_P^2}{\xi}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)$$

- Large λ – slow (logarithmic) running, no noticeable change compared to tree level potential
- Small λ – $\delta\lambda$ significant, may give interesting “features” in the potential (“critical inflation”, large r)
- Most complicated – how really λ behave in HI?

Note on the choice of μ

- μ is the scale appearing in (dimensional) regularization
- No questions asked in the "usual" case of renormalizable theories – only space/field independent choice gives regularization that is not-breaking renormalizability.
- HI is not renormalizable – multiple choices possible

The choice for this talk:

In Jordan frame: $\mu^2 \propto M_P^2 + \xi h^2$

In Einstein frame: $\mu^2 \propto \text{const}$

Adding required counterterms to the action

- In principle – HI is not renormalizable, all counterterms appear at some loop order
- Let us try to add only the *required* counterterms at each order in loop expansion

$$\mathscr{L} = \frac{(\partial\chi)^2}{2} - \frac{\lambda}{4}F^4(\chi) + i\bar{\psi}_t\partial\!\!\!/\psi_t + \frac{y_t}{\sqrt{2}}F(\chi)\bar{\psi}_t\psi_t$$

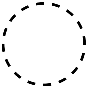
$$F(\chi) \equiv \frac{h(\chi)}{\Omega(\chi)} \approx \left\{ \begin{array}{ll} \chi & , \chi < \frac{M_P}{\xi} \\ \frac{M_P}{\sqrt{\xi}}\left(1 - \mathrm{e}^{-\sqrt{2/3}\chi/M_P}\right)^{1/2} & , \chi > \frac{M_P}{\xi} \end{array} \right\}$$

Doing quantum calculations we should add

$$\mathscr{L} + \mathscr{L}_{\text{1-loop}} + \delta\mathscr{L}_{\text{1-loop c.t.}} + \cdots$$

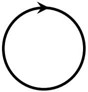
Counterterms: λ modification

Calculating vacuum energy



$$= \frac{1}{2}\,\mathrm{Tr}\ln\left[\Box - \left(\frac{\lambda}{4}(F^4)''\right)^2\right]$$

$$= \frac{9\lambda^2}{64\pi^2}\left(\frac{2}{\bar{\varepsilon}} - \ln\frac{\lambda(F^4)''}{4\mu^2} + \frac{3}{2}\right)\left(F'^2 + \frac{1}{3}F''F\right)^2 F^4,$$

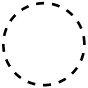


$$= -\,\mathrm{Tr}\ln\left[i\partial\!\!\!/ + y_t F\right]$$

$$= -\frac{y_t^4}{64\pi^2}\left(\frac{2}{\bar{\varepsilon}} - \ln\frac{y_t^2 F^2}{2\mu^2} + \frac{3}{2}\right)F^4$$

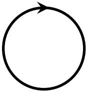
Counterterms: λ modification

Calculating vacuum energy



$$= \frac{1}{2}\,\mathrm{Tr}\ln\left[\Box - \left(\frac{\lambda}{4}(F^4)''\right)^2\right]$$

$$\delta\mathscr{L}_{\rm ct} = \frac{9\lambda^2}{64\pi^2}\left(\frac{2}{\bar{\varepsilon}} + \delta\lambda_{1a}\right)\left(F'^2 + \frac{1}{3}F''F\right)^2 F^4,$$



$$= -\,\mathrm{Tr}\ln\left[i\partial\!\!\!/ + y_t F\right]$$

$$\delta\mathscr{L}_{\rm ct} = -\frac{y_t^4}{64\pi^2}\left(\frac{2}{\bar{\varepsilon}} + \delta\lambda_{1b}\right)F^4$$

Small χ : $F'^4 F^4 \sim \chi^4 \sim F^4$

Large χ : $F'^4 F^4 \sim \mathrm{e}^{-4\chi/\sqrt{6}M_P}$, and $F^4 \sim M_P^4/\xi^2$

$\delta\lambda_{1b}$ – just λ redefinition, while $\delta\lambda_{1a}$ is not!

Modified "evolution" of $\lambda(\mu)$

For RG we should in principle write infinite series

$$\frac{d\lambda}{d\ln\mu} = \beta_\lambda(\lambda, \lambda_1, a\ldots)$$

$$\frac{d\lambda_1}{d\ln\mu} = \beta_{\lambda_1}(\lambda, \lambda_1, \ldots)$$

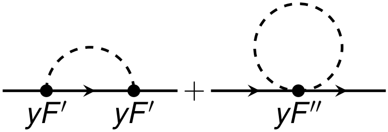
$$\ldots$$

- Assuming δ_i are small and have the same hierarchy, as the loop expansion, we truncate this to just first equation.
- Neglect change of $\delta\lambda_1$ between $\mu \sim M_P/\xi$ and $M_P/\sqrt{\xi}$

$$\lambda(\mu) \to \lambda(\mu) + \delta\lambda\left[\left(F'^2 + \frac{1}{3}F''F\right)^2 - 1\right],$$

Counterterms: Top Yukawa coupling

Calculating propagation of the top quark in the background χ



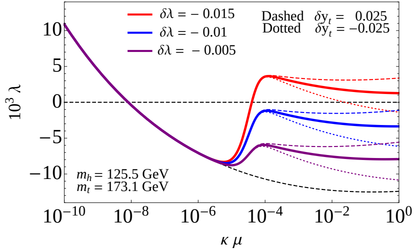
$$\begin{aligned} \delta\mathscr{L}_{\rm ct} \quad &\sim \quad \left(\# \frac{y_t^3}{\bar{\varepsilon}} + \delta y_{t1} \right) F'^2 F \bar{\psi}\psi \\ &+ \quad \left(\# \frac{y_t \lambda}{\bar{\varepsilon}} + \delta y_{t2} \right) F''(F^4)'' \bar{\psi}\psi \end{aligned}$$

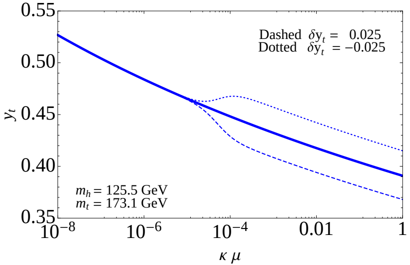
$$y_t(\mu) \to y_t(\mu) + \delta y_t \left[F'^2 - 1 \right]$$

Threshold effects at M_P/ξ summarized by two new arbitrary constants $\delta\lambda$, δy_t

$$\lambda(\mu) \to \lambda(\mu) + \delta\lambda\left[\left(F'^2 + \tfrac{1}{3}F''F\right)^2 - 1\right]$$

$$y_t(\mu) \to y_t(\mu) + \delta y_t\left[F'^2 - 1\right]$$



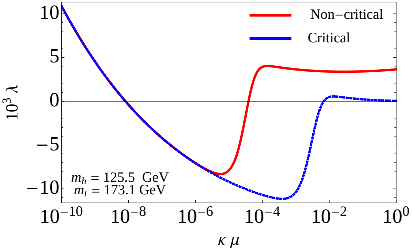


Modified λ evolution can make the potential positive again

$$\lambda(\mu) \to \lambda(\mu) + \delta\lambda \left[\left(F'^2 + \frac{1}{3} F'' F \right)^2 - 1 \right]$$

$$y_t(\mu) \to y_t(\mu) + \delta y_t \left[F'^2 - 1 \right]$$

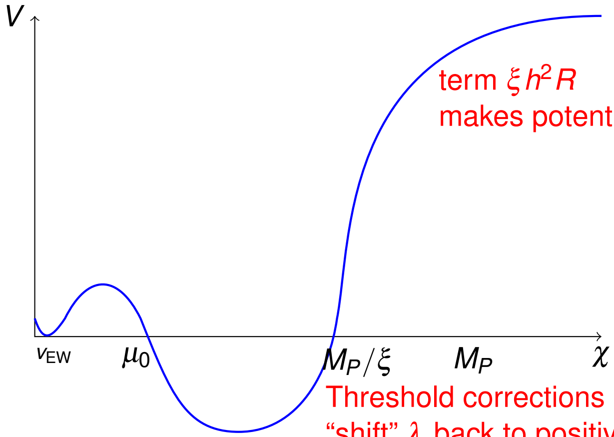
Effect on the potential



(Red curve: $\xi = 1500$, $\delta y_t = 0.025$, $\delta\lambda = -0.015$)

Higgs inflation and radiative corrections

$$S_J = \int d^4x \sqrt{-g} \left\{ -\frac{M_P^2}{2} R - \xi \frac{h^2}{2} R + g_{\mu\nu} \frac{\partial^\mu h \partial^\nu h}{2} - \frac{\lambda}{4} (h^2 - v^2)^2 \right\}$$



(Not really to scale)