A Panorama of Modified Gravity

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The Universe accelerates: why?

Maybe a landscape of Universes?

Or not?

The acceleration of the Universe could also be due to many mundane causes:

A calculable cosmological constant and/or vacuum energy

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -8\pi G_N \Lambda^4 g_{\mu\nu}$$

A modification of General Relativity (GR)

$$F(R_{\mu\nu}, g_{\mu\nu} \ldots) = 8\pi G_N T_{\mu\nu}$$

The cosmological dynamics of fields (new matter)

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G_N T^{\rm new}_{\mu\nu}$$

The Cosmological Constant

The cosmological constant comprises two terms: a bare cosmological constant which can be seen as a boundary condition set initially at the Big Bang and the sum of all the vacuum fluctuations. Vacuum fluctuations couple to matter (the Casimir effect) but do they couple to gravity? If yes they entail a serious problem:

$$\Lambda^4 \equiv V_{\rm cc} + V_{\rm vacuum} = 3\Omega_\Lambda m_{\rm pl}^2 H_0^2 \sim 10^{-48}\ {\rm GeV}^4$$

But what is the problem exactly?

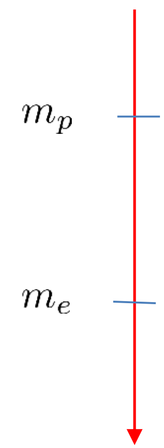
In a Lorentz invariant and causal effective field theory, the description of the late acceleration must be captured by a (very) low energy effective theory at energies well below all the mass scales of the standard model

$$S = \int d^4x \sqrt{-g}(\Lambda^4 + \frac{m^2}{2}\phi^2 + \frac{(\partial\phi)^2}{2} \dots)$$

The cosmological constant and masses correspond to renormalisable (relevant) operators at low energy. Their numerical values is sensitive to UV physics and must be determined at a given scale by comparison with experiments. This is the usual renormalisation programme where “infinities” are absorbed by “infinite” counterterms, leaving a finite value which is determined experimentally. The bare value of the cosmological plays the role of such a counterterm.

In a top-down approach, every time a particle decouples, it contributes to the vacuum energy:

$$V_{\mathrm{vacuum}} \sim \frac{1}{64\pi^4}\mathrm{Str}M^4$$



Phase transitions (EW breaking, QCD) also contribute.

The measurement of the dark energy now corresponds to the renormalisation condition at very low energy.

$$V_{\mathrm{cc}} + V_{\mathrm{vacuum}} = 3\Omega_\Lambda m_{\mathrm{pl}}^2 H_0^2 \sim 10^{-48}\ \mathrm{GeV}^4$$

So what is the problem?

From a bottom-up approach, there is no problem… the dark energy is measured at low energy and one can "integrate in" all the particles of the standard model to take into account the effective vacuum energy in different eras of the Universe (before BBN, before the EW transition etc…) . In different eras, the effective cosmological constant is subdominant.

In the early Universe, if inflation takes place and then ends, this sets the initial value of the cosmological constant to vanish and imposes that the contributions from the decoupling of the particles of the theory almost vanish.

$$\frac{1}{64\pi^4}\mathrm{Str}M^4 = 3\Omega_\Lambda H_0^2 m_{\mathrm{Pl}}^2 \qquad \text{Why?}$$

From a top-down approach, there is a hierarchy problem. Why should my "grand unified theory" at very large energy know (or care) that when all the particles of the models have decoupled the sum of the bare cosmological constant, defined by the model at high energy, and the quantum fluctuations down to the electron (neutrino) scale should be ALMOST zero?

This is a similar problem to the Higgs mass problem.

In the low energy world after BBN is there a way of guaranteeing that the vacuum energy at low energy is (nearly) zero?

Weinberg's no-go theorem:

A generic 4d Quantum Field Theory cannot admit a Poincaré invariant vacuum with vanishing vacuum energy and non-vanishing fermion masses.

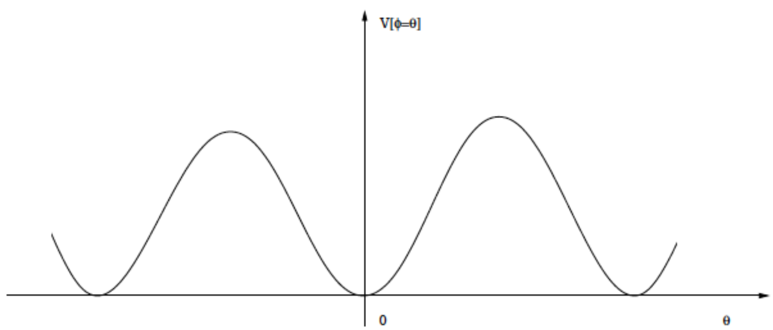
Global SUSY is a counter example.

One can violate some of the hypotheses to try to calculate the dark energy:

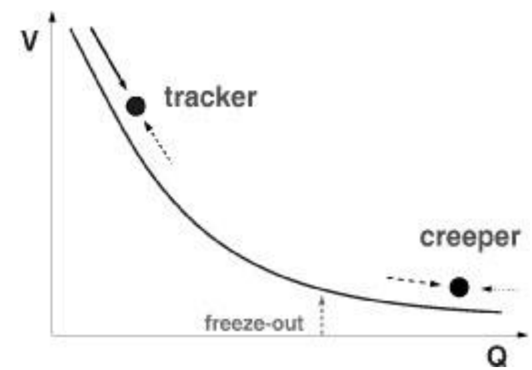
Use extra dimensions (the curvature of the extra dimensions absorbs the vacuum energy)

Use the dynamics of cosmological fields (quintessence)

DARK
ENERGY



Thawing



Freezing

A Simple Example: Pseudo-Goldstone Models

Global U(1) symmetry broken at scale f:

$$\Phi = f e^{i\varphi/\sqrt{2}f}$$

Symmetry broken by small term:

$$\mathcal{L}_{\text{breaking}} = \mu^4 \frac{\Phi}{2f} + cc$$

Low energy potential:

$$\boxed{V(\varphi) = \mu^4 \cos \frac{\varphi}{\sqrt{2}f}}$$

Cosmologically, if initially the field is small compared to f, it is frozen there until its mass becomes larger than the Hubble rate. Tuning this event to be in the recent past of the Universe and requiring that the dark energy is due to the low energy potential implies:

$$\mu \sim \Lambda, \quad f \sim m_{\text{Pl}}$$

A major drawback of these models is that the phase transition must be at the Planck scale, this requires embedding this model in theories dealing with quantum gravity… maybe embedding it in string theory although no global symmetries there…

On the other hand, this model is useful to see that low energy dark energy fields couple to matter:

$$\mathcal{L}_{\rm int} = \frac{\partial\Phi\partial\bar{\Phi}}{f^3}\bar{\psi}\psi \longrightarrow \boxed{\mathcal{L}_{\rm int} \sim \frac{(\partial\phi)^2}{f^3}\bar{\psi}\psi}$$

The derivative interaction implies no effect on static tests of gravity vs fifth forces. This an example of a disformal coupling to matter.

This is one example of coupling to matter, we will also consider conformal couplings which are tightly constrained.

$$m_\phi \sim H_0$$

Extremely small mass for the scalar field.

Modified gravity

Massive Gravity

Mass term for a graviton

The simplest modification is massive gravity (Pauli-Fierz):

$$\delta\mathcal{L} = \frac{m_G^2}{4}(h^{\mu\nu}h_{\mu\nu} - h^2)$$

Pauli-Fierz gravity is ghost free (negative kinetic energy terms) . Unfortunately, a massive graviton carries 5 polarisations when a massless one has only two polarisations. In the presence of matter, the graviton wave function takes the form:

$$h_{\mu\nu} = \frac{8\pi G_N}{p^2}(T_{\mu\nu} - \frac{1}{2}T\eta_{\mu\nu}) \to h_{\mu\nu} = \frac{8\pi G_N}{p^2 + m_G^2}(T_{\mu\nu} - \frac{1}{3}T\eta_{\mu\nu})$$

The massless limit does not give GR! (van Dam-Velman-Zakharov discontinuity). The extra polarization is lethal. Solved by Vainshtein mechanism (non-linearity).

Ghost in curved space-time!

Bimetric Gravity

One way to cure these problems is to consider a non-linear version of massive gravity with two dynamical metric:

$$S = \int d^4x \left(e_1 \frac{R_1}{16\pi G_N} + e_2 \frac{R_2}{16\pi G_N} + \Lambda^4 \sum_{ijkl} m_{ijkl} \epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma} e_\mu^{ai} e_\nu^{bj} e_\rho^{ck} e_\sigma^{dl}\right)$$

where the graviton mass is of order:

$$m_g^2 \sim \frac{\Lambda^4}{m_{\rm Pl}^2} \sim H_0^2$$

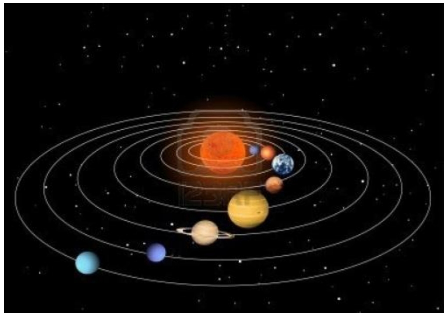
What we have learnt in the last ten years is that such extensions involve light scalar fields, why?:

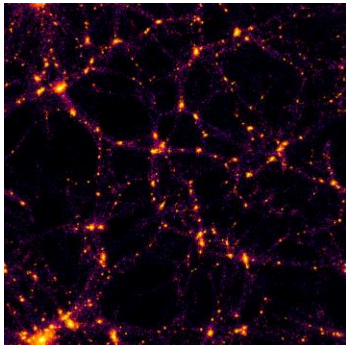
Massive gravitons have a scalar part

$$5 = 2 + 2 + 1$$

Their interaction with matter generates fifth forces which would have been seen in the laboratory and the solar system.

GRAVITY ACTS ON ALL SCALES





Looking for extensions of General Relativity valid from small to large scales.

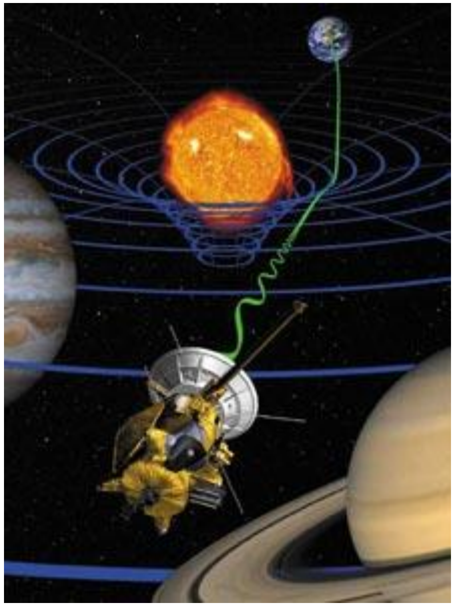
In general, these theories require a fine-tuning of the cosmological constant but have unexpected field theoretic properties which go beyond the usual framework of field theory: irrelevant operators dominate, UV completions may not be required (classicalisation)… The most general form of these theories is known in some cases (Horndeski for one scalar) but one must go into the details of the models to make them work from the solar system (or the laboratory) to large cosmological scales.

Deviations from Newton's law are parametrised by:

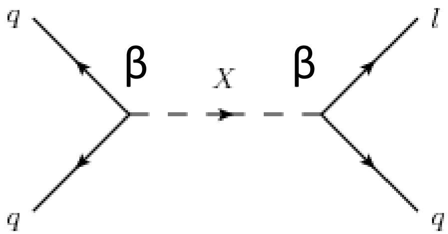
$$\phi_N = -\frac{G_N}{r}(1 + 2\beta^2 e^{-r/\lambda})$$

For large range forces with large λ, the tightest constraint on the coupling β comes from the Cassini probe measuring the Shapiro effect (time delay):

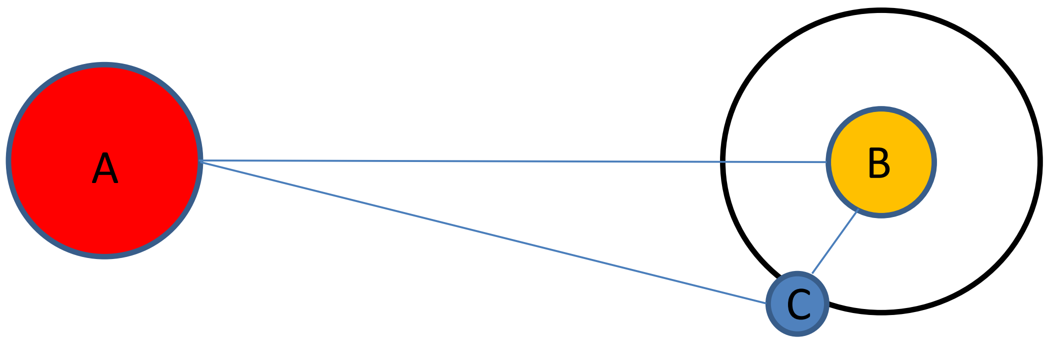
$$\beta^2 \leq 4 \cdot 10^{-5}$$



Bertotti et al. (2004)



VIOLATION OF THE STRONG EQUIVALENCE PRINCIPLE



$$\eta_{BC} \equiv \left|\frac{a_C - a_B}{a_C + a_B}\right|$$

$$\eta_{\mathrm{moon-earth}} \leq 10^{-13}$$

Lunar ranging experiment

Tight bound on the perihelion advance of the moon too.

How do we hide light scalar fields ?

SCREENING

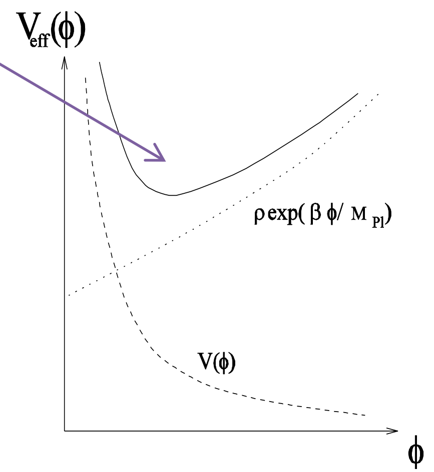
The effect of the environment

When conformally coupled to matter, scalar fields have a matter dependent effective potential

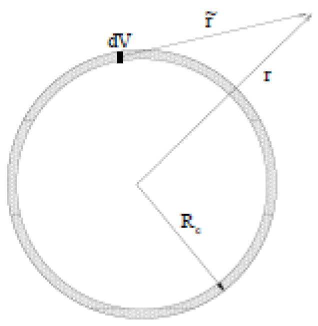
$$V_{eff}(\phi) = V(\phi) + \rho_m(A(\phi) - 1)$$

Environment dependent minimum

$$\beta(\phi) = m_{\rm Pl}\frac{d\ln A(\phi)}{d\phi}$$



Chameleon



The field generated from deep inside is Yukawa suppressed. Only a thin shell radiates outside the body. Hence suppressed scalar contribution to the fifth force.

One example: f(R) gravity

$$S_{\mathsf{MG}} = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}f(R)$$

f(R) is totally equivalent to a field theory with gravity and a scalar

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G_N}R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{L}_m(\psi_m, e^{2\phi/\sqrt{6}m_{\mathsf{Pl}}}g_{\mu\nu})\right)$$

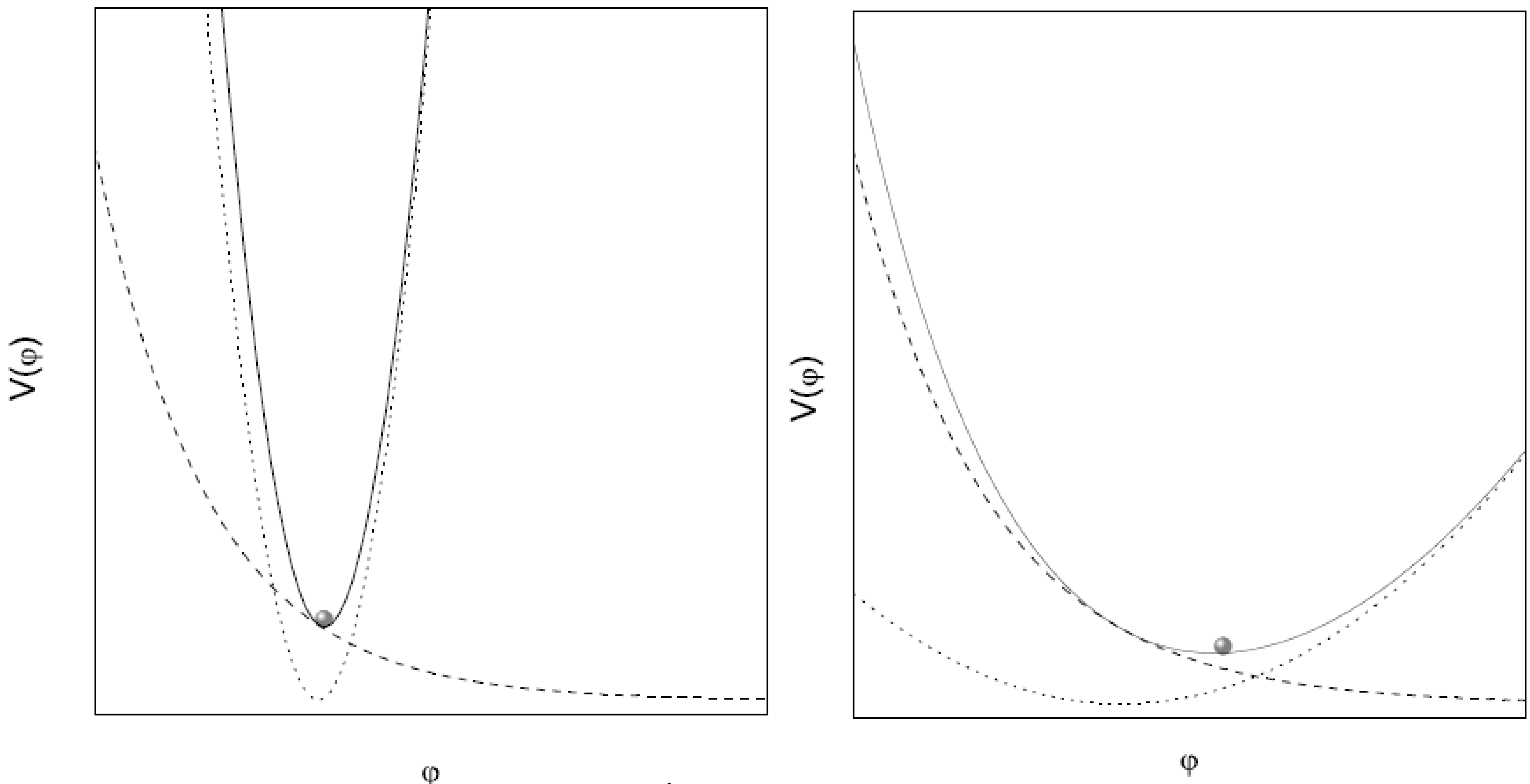
The potential V is directly related to f(R)

Crucial coupling between matter and the scalar field

$$V(\phi) = m_{\mathsf{Pl}}^2\frac{Rf' - f}{2f'^2},\ \ f' = e^{-2\phi/\sqrt{6}m_{\mathsf{Pl}}}$$

Would be ruled out if no chameleon effect

Damour-Polyakov mechanism



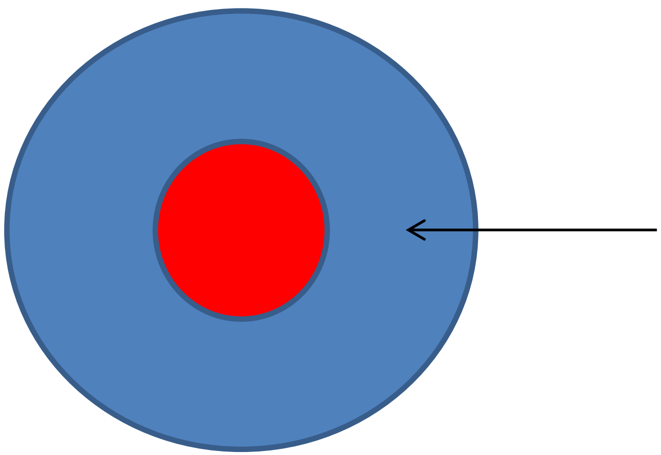
Dilaton

$$V(\phi) = V_0 e^{-\phi/m_{\rm Pl}}, \quad A(\phi) = 1 + \frac{A_2}{2m_{\rm Pl}^2}(\phi - \phi_\star)^2$$

Another simple example: the CUBIC GALILEON

$$\mathcal{L}_\phi = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2\Lambda^3}(\partial\phi)^2\partial^2\phi + \frac{\beta\phi}{M_P}T \ .$$

$\Lambda^3 = m^2 m_{\rm Pl}$ m graviton mass



$$\frac{F_\phi}{F_N} = 2\beta^2\left(\frac{r}{R_V}\right)^{3/2} \ .$$

Well inside the Vainshtein radius, Newtonian gravity is restored. Well outside gravity is modified.

The Vainshtein radius is very large for stars, typically 0.1 kpc for the sun, and a mass for the graviton of the order of the Hubble rate .

K-mouflage models

$$S = \int d^4x\sqrt{-g}(\frac{R}{16\pi G_N} + M^4 K(\chi)) + S_m(\psi, A^2(\phi)g_{\mu\nu}) \qquad \chi = -\frac{(\partial\phi)^2}{2M^4}$$

M is the dark energy scale for cosmologically interesting models. Examples that one may consider:

$$K(\chi) = -1 + \chi + K_0\chi^m$$

Particles have modified trajectories compared to Newton's law in this background:

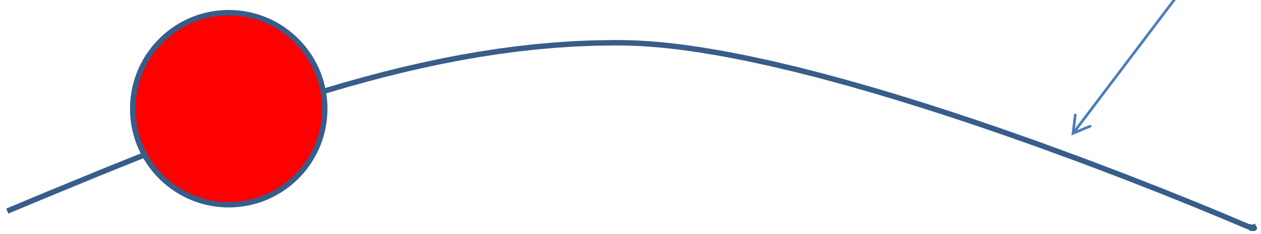
$$\frac{d^2r}{dt^2} = -\frac{G_N m}{r^2}(1 + \frac{2\beta^2}{K'})$$

Screening happens inside the K-mouflage radius where K'>>1. Still the tiny deviations depend on r and can lead to an anomalous perihelion in the earth-moon system.

These are the only four mechanisms !

Summary: the motion of massive objects in modified gravity theories:

Trajectories due to gravity + scalar

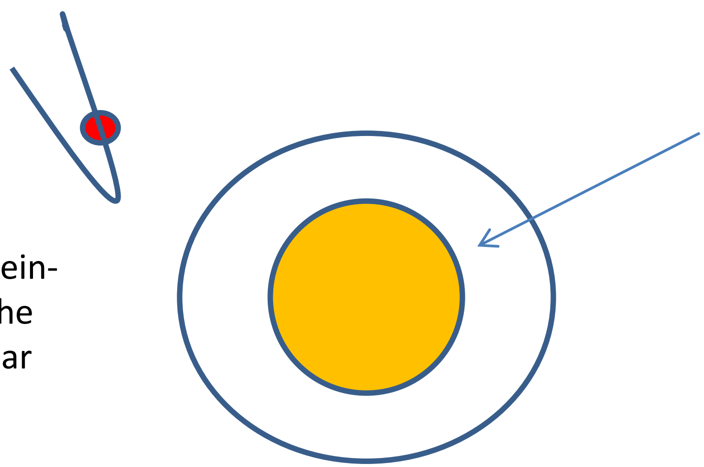


Chameleons and Damour-Polyakov: $Q \ll \beta$

$$\ddot{X} = -\nabla\Phi_N - Q\nabla\phi_{\text{ext}}$$

Unlike chameleons et al., K-mouflage and Vainshtein do not affect the charge Q: $Q = \beta$

$\nabla\phi_{\text{ext}} \ll 2\beta\nabla\Phi_N$



Outside the Vainshtein-Kmouflage radius, the object feels the scalar force

$\nabla\phi_{\text{ext}} = 2\beta\nabla\Phi_N$ outside the Vainshtein-Kmouflage radius

There is more than the conformal coupling

Matter couples to a metric which can differ from the Einstein metric involved in the Einstein-Hilbert term with a constant Newton constant:

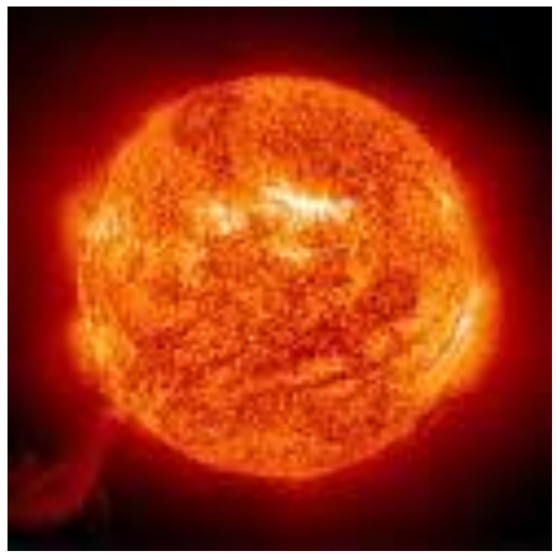
$$S = \int d^4x \sqrt{-g}\, \frac{R(g)}{16\pi G_N} + S_m(\psi^i, \tilde{g}_{\mu\nu})$$

Bekenstein (1992) showed that causality and the weak equivalence principle restricts the form of the auxiliary metric:

$$\tilde{g}_{\mu\nu} = A(\phi, X) g_{\mu\nu} + B(\phi, X) \partial_\mu \phi \partial_\nu \phi \qquad X = -\frac{1}{2}(\partial\phi)^2$$

What is the physics associated with the disformal coupling B(ϕ,X) ?

As the axions, the disformally coupled scalars can have an effect at high density and high temperature inside the inferno at the core of stars. From the gentle burning of main sequence stars, to helium burning stars and then the explosion of supernovae, the processes involve higher energies (hence shorter distances) and electromagnetic to strong interaction processes.



There can also be effects in particle physics.

The Bekenstein coupling generates all sorts of couplings of extremely light scalar field to the standard model:

$$\mathcal{L}_1 = C_1 \frac{(\partial\phi)^2}{M^4} T, \quad \mathcal{L}_2 = C_2 \frac{\partial_\mu\phi\partial_\nu\phi}{M^4} T^{\mu\nu}$$

$$\mathcal{L}_{10} = C_{10} \frac{\phi}{N} T$$

What are the constraints and which phenomena are affected?

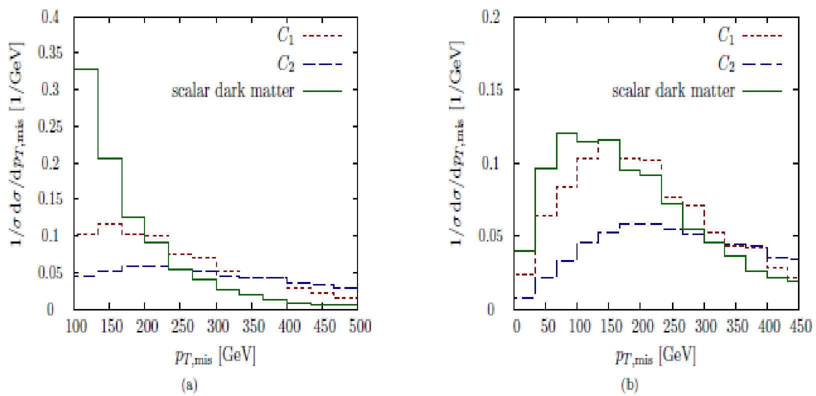
Precision tests of the standard model (all)

Missing energy signals (disformal)

Displaced vertices (conformal)

The most constrained test of dark energy at colliders comes from missing energy, for instance monojets or t-tbar.

$$\mathcal{L}_1 : 2\phi + t\bar{t}\ M \geq 237.4\mathrm{GeV}(ATLAS),\ M \geq 192.4\mathrm{GeV}(CMS)$$



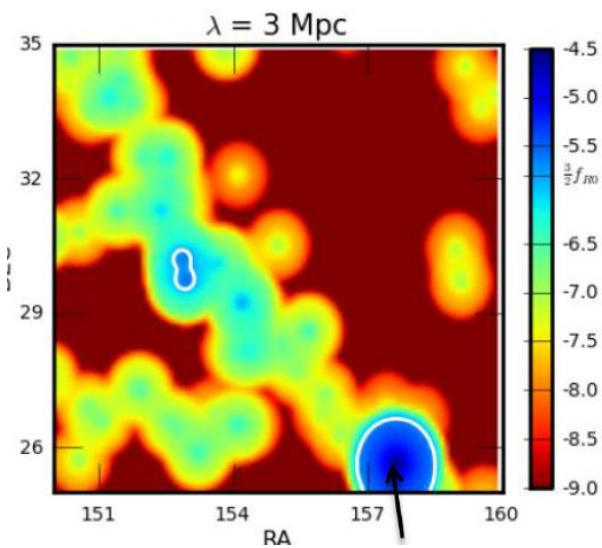
The missing pT distribution can be significantly different from a dark matter signal (in particular for jets).

$$\mathcal{L}_2 : 2\phi + jet\ M \geq 693.4\mathrm{GeV}(ATLAS),\ M \geq 822.8\mathrm{GeV}(CMS)$$

$$\mathcal{L}_2 : 2\phi + t\bar{t}\ M \geq 461.2\mathrm{GeV}(ATLAS),\ M \geq 399.8\mathrm{GeV}(CMS)$$

Where could we observe modified gravity?

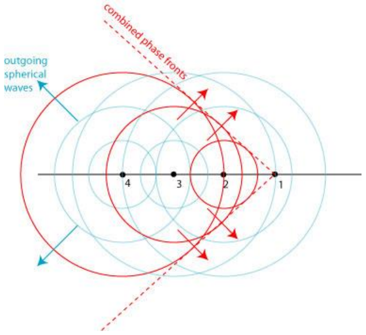
Modification of the growth of cosmological structure: future galaxy surveys (Euclid …)



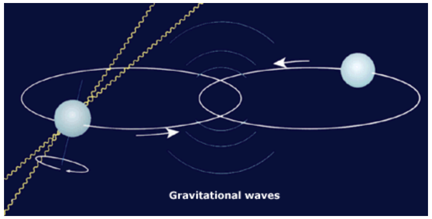
Unscreened region of the Universe

Where could we observe modified gravity?

Gravitational waves: modification of the speed of gravitational waves.



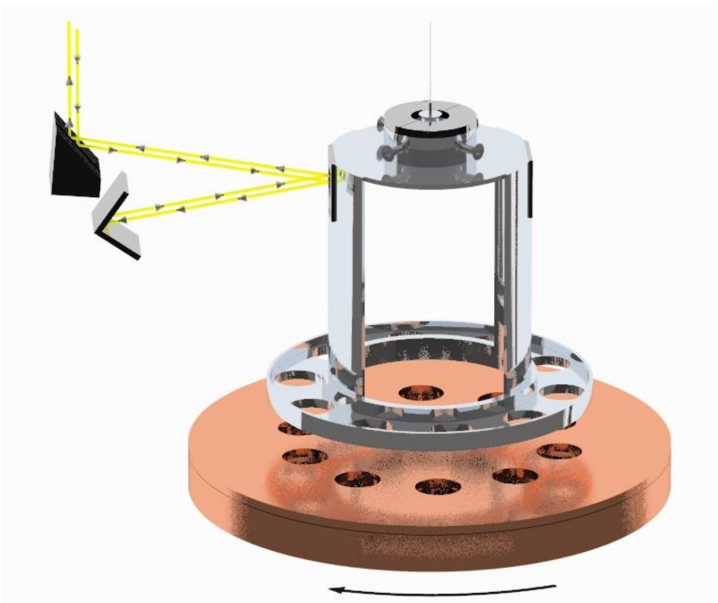
Gravitational Cerenkov effect



Binary pulsars

Where could we observe modified gravity?

Laboratory experiments: torsion pendulum, atomic interferometry, Casimir effect ….



Conclusions

Light scalar fields could be what remains from massive gravity, string theory …. They need to be screened in the local environment otherwise tight bounds would be violated.

This prompts one to study the screening mechanism from a bottom-up approach irrespective of their UV completion, if ever needed.

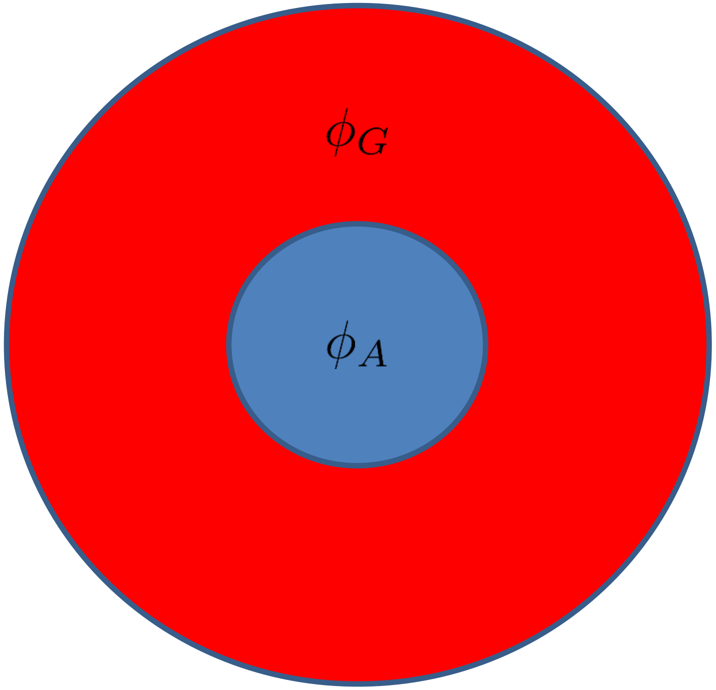
For conformally coupled scalars, there are only four mechanisms: Vainshtein, K-mouflage, chameleon and Damour-Polyakov.

Future tests from laboratory to cosmological tests.

Supplement

Chameleons:

The screening criterion for an object BLUE embedded in a larger region RED expresses the fact that the Newtonian potential of an object must be larger than the variation of the field:



Scalar charge: $$Q_A = \frac{|\phi_G - \phi_A|}{2m_{\rm Pl}\Phi_A}$$

$$Q_A \leq \beta_G$$

Self screening: large Newton potential

Blanket screening: due to the environment G

Φ_A Newton's potential at the surface

Difficult to distinguish models at the background level… how about

Large Scale Structure?

The chameleons modify the growth of structure in a scale and time dependent way.

Galileons only modify the growth in a time-dependent way. All structures up to clusters are screened so reducing possible effects.

K-mouflage modifies growth in a time-dependent way. Galaxy clusters are not screened so effect on the number of large clusters.

Chameleons:

The cosmological background evolves like in the concordance model. The main difference coming from the modification of gravity arises at the perturbation level where the Cold Dark Matter density contrast evolves like:

$$\delta'' + \mathcal{H}\delta' - \frac{3}{2}\Omega_m \mathcal{H}^2 (1 + \frac{2\beta^2}{1 + \frac{m^2 a^2}{k^2}})\delta = 0$$

Modified gravity

The new factor in the bracket is due to a modification of gravity depending on the comoving scale k. Most effects in the quasi-linear regime around Mpc scales, i.e. requires N-body or sem-analytical methods to go beyond linear perturbation theory.

This is now available for most models.

The growth of structures depends on the comoving Compton length:

$$\lambda_c = \frac{1}{ma}$$

Gravity acts in an usual way for scales larger than the Compton length (matter era)

$$\delta \sim a$$

Gravity is modified inside the Compton length with MORE growth (matter era):

$$\delta \sim a^{\frac{\nu}{2}}, \quad \nu = \frac{-1 + \sqrt{1 + 24(1 + 2\beta_c^2)}}{2}$$

When the coupling is a function of the scalar field, the growth is not power-like but still anomalous growth.

Lensing is also affected:

$$\Phi_{\mathrm{WL}} = \Phi_N$$

No modification of the lensing potential

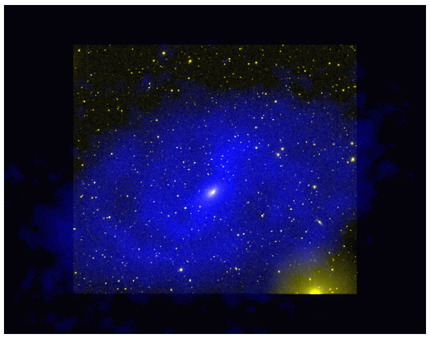
But the density contrast is modified and the effective Newton constant varies with time:

$$\Delta\Phi_N = 4\pi\bar{A}(t)G_N a^2\delta\rho$$

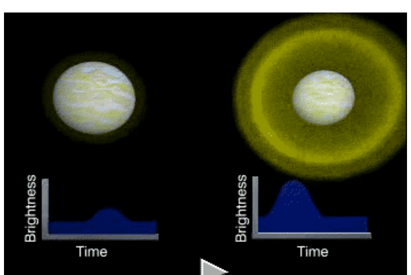
Growth and lensing are modified in all these models. Still linear scales more reliable. Comparison with available data of large scale structure and lensing has been performed giving constraints on f(R) and other models such as dilatons which are loosely restricted. For f(R), the cosmological bounds are not as good as the bounds from astrophysics but subject to fewer uncertainties.

Astrophysical tests:

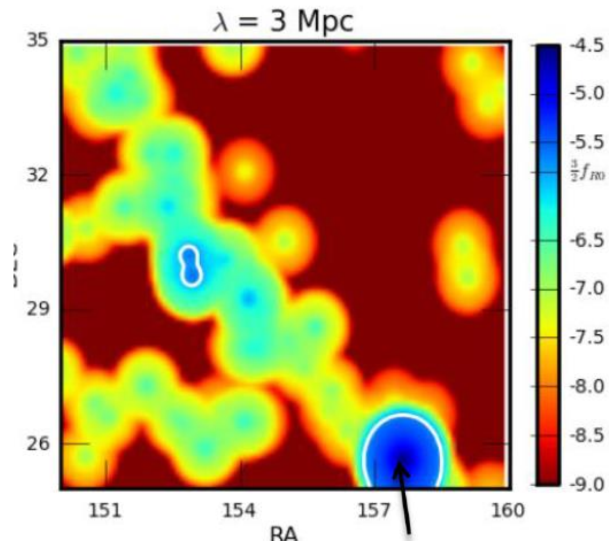
Motion of screened stars different from unscreened HI gas in unscreened dwarf galaxies.



Distance indicators for cepheids and TRGB stars in screened and unscreened dwarf galaxies are different as their luminosities vary



$\Phi_N \sim 10^{-7}$



screened

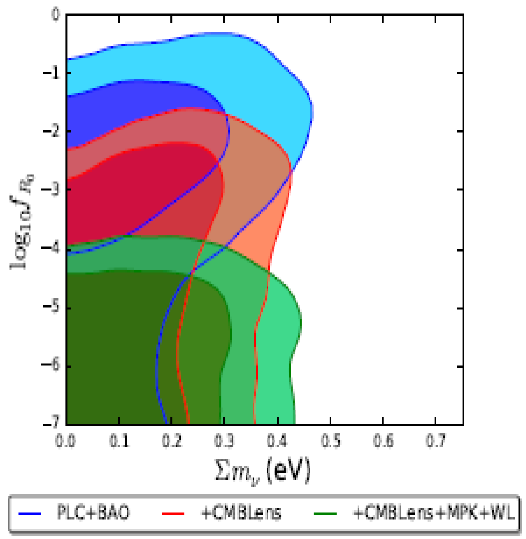
$\xi = 10^{-6}$

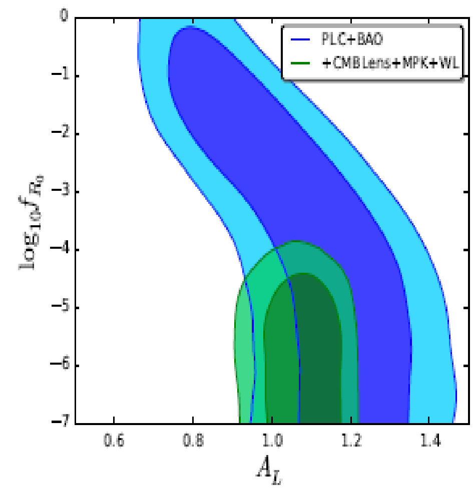
SDSS catalogue, within 200 Mpc, scalar range 1 Mpc

No effects measured so far: bound on the range of the scalar interaction.

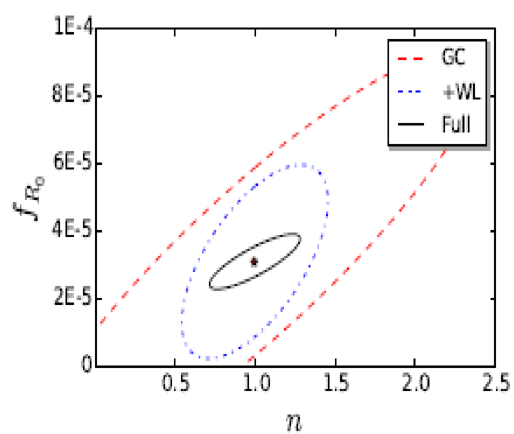
$$|f_{R_0} \leq 10^{-7}$$

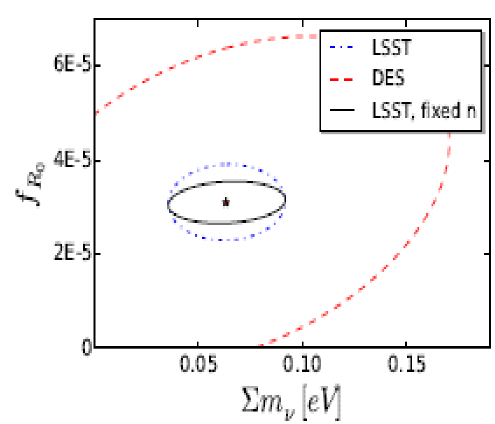
The latest contours on f(R) from cosmological observables



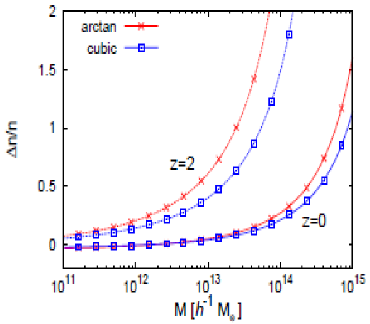


$f(R)$+A_L, fixed $\sum m_\nu$		$f(R)$+A_L, varying $\sum m_\nu$		
f_{R_0}	A_L	f_{R_0}	A_L	$\sum m_\nu$
$3\,(8)\times 10^{-5}$	$1.08^{+0.07\,(0.12)}_{-0.05\,(0.13)}$	$0.4\,(1.0)\times 10^{-4}$	$1.11^{+0.10\,(0.16)}_{-0.06\,(0.15)}$	$0.30\,(0.38)$





One of the main stumbling blocks of all these analyses is the difficulty of finding clear features which could distinguish models. One of those would be the fact that the physics of large clusters is largely affected for K-mouflage models passing the solar system tests.



More clusters for K-mouflage models on large scales

Much more analytical and numerical work will be required to chart the different modified gravity models which are compatible with local gravity tests.

In fact, around a background configuration and in the presence of matter, the Lagrangian of such extensions can be linearised and the main screening mechanisms can be schematically distinguished :

$$\mathcal{L} \supset \boxed{-\frac{Z(\phi_0)}{2}}(\partial\delta\phi)^2 - \frac{m^2(\phi_0)}{2}\delta\phi^2 + \frac{\beta(\phi_0)}{M_P}\delta\phi\delta T \ ,$$

The Vainshtein mechanism reduces the coupling by increasing Z. The K-mouflage mechanism has the same effect … while the Damour-Polyakov mechanism suppresses β and the chameleon property increases m.

The Vainshtein and K-mouflage mechanisms can be easily analysed:

Effective Newtonian potential:

$$\Psi = (1 + \frac{2\beta^2(\phi)}{Z(\phi)})\Phi_N$$

For theories with second order equations of motion:

$$Z(\phi) = 1 + a(\phi)L^2\frac{D^\mu D_\mu \phi}{m_{\rm Pl}} + b(\phi)\frac{(\partial\phi)^2}{M^4} + \dots$$

Vainshtein

K-mouflage

$$M^4 \sim 3H_0^2 m_{\rm Pl}^2, \quad L \sim H_0^{-1}$$

Cosmological choice

Vainshtein

Newtonian gravity retrieved when the curvature is large enough:

$$\nabla^2 \Phi_N \geq \frac{1}{2\beta L^2}$$

On large cosmological scales, this tells us that overdensities such as galaxy clusters are screened :

$$\delta \geq \frac{1}{3\Omega_{m0}\beta}$$

On small scales (solar system, galaxies) screening only occurs within the Vainshtein radius:

$$R_V = (\frac{3\beta m L^2}{4\pi m_{\rm Pl}^2})^{1/3}$$

K-mouflage

Newtonian gravity retrieved when the gravitational acceleration is large enough:

$$|\nabla\Phi_N| \geq \frac{M^2}{2\beta m_{\rm Pl}}$$

On large cosmological scales, this tells us that overdensities such as galaxy clusters are not screened :

$$\frac{k}{H_0} \leq \beta\delta$$

On small scales (solar system, galaxies) screening only occurs within the K-mouflage radius:

$$R_K = (\frac{\beta m}{4\pi m_{\rm Pl} M^2})^{1/2}$$

Dwarf galaxies are not screened.