Cosmology with clusters : going beyond ΛCDM

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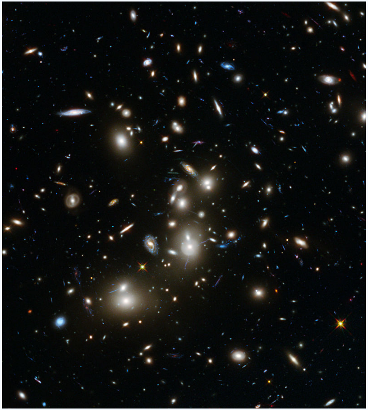
A sensitive indicator of cosmology

Clusters of galaxies



Probes of background
& **perturbed sector** $\Rightarrow$ **growth rate of structures**

A sensitive indicator of cosmology

Many questions about clusters :

- Definition ?
- Detection ?
- Measure characteristics ?
- Predictions ?
- Extract information ?
- ...

A sensitive indicator of cosmology

Many questions about clusters :

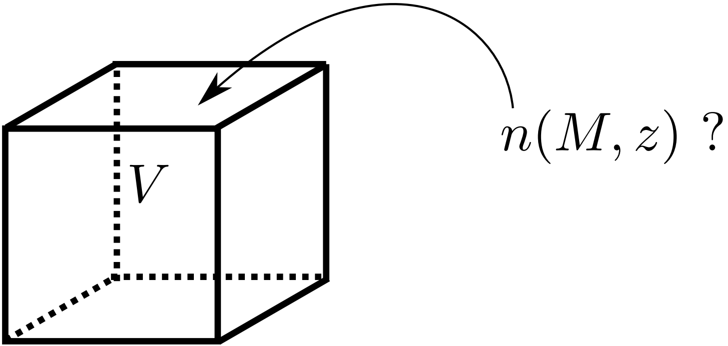
- Definition ?
- Detection ?
- Measure characteristics ?
- **Predictions ?**
- **Extract cosmological information ?**
- ...

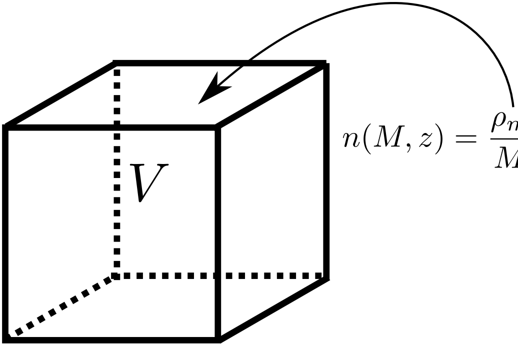
Clusters as cosmological probes

Main method :
use 1-pt statistics i.e.
compare observed and predicted n_{clusters}

For a given cosmology,
how do we predict the number of clusters ?

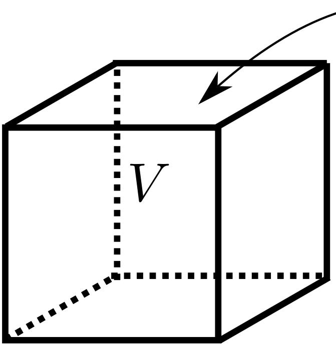
Clusters as cosmological probes





$$n(M, z) = \frac{\rho_m}{M}$$

Clusters as cosmological probes

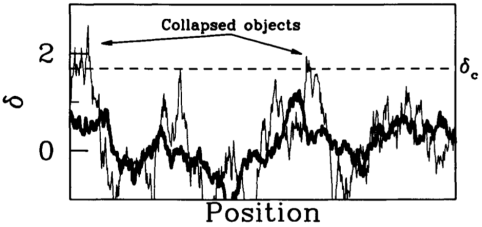


$$n(M, z) = \frac{\rho_m}{M} \times f(M, z, cosmology?)$$

$$0 < f < 1$$

Press-Schechter (1974) formalism

Main ingredient : linear P(k)



In the **linear** density field, all peaks above a threshold δ_c are **collapsed objects** in the **real** density field

$$\Rightarrow f \propto \int_{\delta_c}^{\infty} \exp\left(-\delta^2/2\sigma^2\right) \text{ with } \sigma(M,z)^2 = \int P(k,z)W^2(M)dk$$

In CDM and ΛCDM, $\delta_c \sim 1.686$
but : link with clusters ? i.e. collapsed objects with $\delta \sim 100s$? ?

Spherical collapse : classical picture

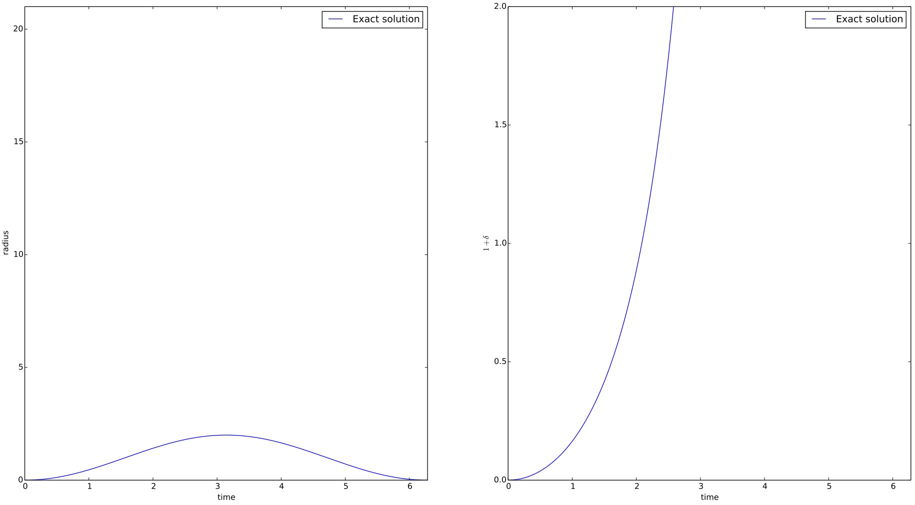
In the CDM paradigm :

- Start with a small overdensity $\delta_{initial} << 1$ at early times
- Consider it as separate, closed universe
- Evolution given by Friedman equations

$$\frac{1}{a}\frac{da}{dt} = H_0(\Omega_m a^{-3} + (1-\Omega_m)a^{-2})^{1/2}$$

- Get $r(t)$ and $\delta(t)$

Spherical collapse : classical picture



Spherical collapse : classical picture

We can compute $t_{collapse}$ (when $R = 0$ and $\delta = \infty$)
... and then ?

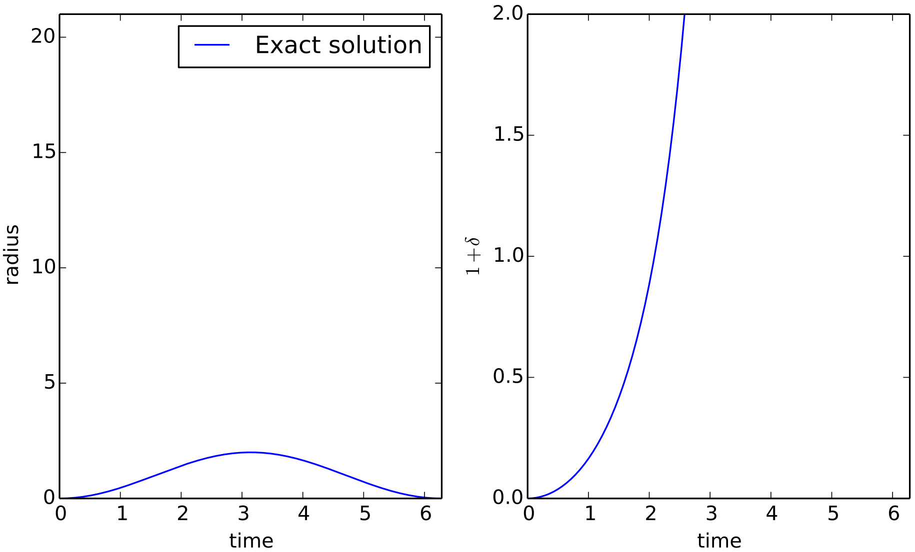
Link back to what we know well : linear sector

- Initally at $t = t_{ini}$: $\delta = \delta_{ini}$
- At collapse at $t = t_{coll}$: $\delta = \infty$
- **But** in linear theory : $\delta(t_{coll}) = \delta_{ini} D(t_{coll}) \sim 1.686$

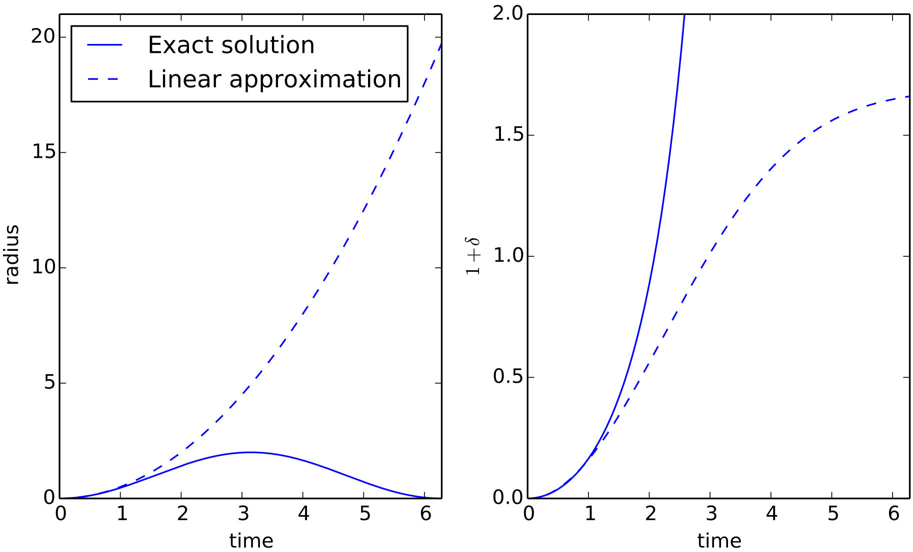
$\Rightarrow$ At any t, $\delta \geq 1.686$ in the linear field correspond to collapsed objects in the real field

We can use a theoretical prediction from linear theory (i.e. $P_{lin}(k)$) to make prediction on non-linear objects

Spherical collapse : classical picture



Spherical collapse : classical picture



Spherical collapse : classical picture

N.B.

- In reality, δ does not reach ∞
- Virialization occurs : $\delta \rightarrow \delta_{vir} \equiv \Delta_v$
- Using virial theorem, we can compute Δ_v (~ 178 for CDM)
- **Used when defining what is a cluster in data**

Spherical collapse : not-so-classical picture

What happens when cosmology$\neq$CDM ?

In a universe with smooth components like dark energy driving the expansion but not participating in collapse we cannot consider spherical collapse to be a separate universe

In ΛCDM

Already different : δ_c depends on redshift & cosmological parameters
Fitting formula :

$$\delta_c(z,\Omega) = 3\frac{(12\pi)^{2/3}}{20}(1 + 0.0123\log(\Omega_m(z)))$$

In more general cosmologies, what do we do ?

Go back to the continuity and Euler equation to derive the general equation

$$\begin{aligned} \frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla \cdot (1+\delta)\mathbf{v} &= 0\,, \\ \frac{\partial \mathbf{v}}{\partial t} + \frac{1}{a}(\mathbf{v}\cdot\nabla)\mathbf{v} + H\mathbf{v} &= -\frac{1}{a}\nabla\Psi\,, \end{aligned} \tag{1}$$

which is true for any type of dark energy or even metric modified gravity

Spherical collapse : not-so-classical picture

Then :

- Consider again a top-hat perturbation
- Equation on δ :

$$\frac{d^2\delta}{dt^2} - \frac{4}{3}\frac{1}{1+\delta}\frac{d\delta}{dt}^2 + 2H\frac{d\delta}{dt} = \frac{1+\delta}{a^2}\nabla^2\Psi$$

- Poisson : $\nabla^2\Psi = 4\pi G a^2 \bar{\rho}_m \delta$ (top-hat remain a top-hat)
- G can be changed to $G_{eff} = G_{Newton} + \Delta G$
- Use conservation of mass to convert δ to r

Spherical collapse : not-so-classical picture

Equation on radius

$$\frac{1}{r}\frac{d^2r}{dt^2} = -\frac{4\pi G_{Newton}}{3}(\bar{\rho}_m + (1+3w_{eff})\rho_{eff}) - \frac{4\pi G_{eff}}{3}\bar{\rho}_m\delta$$

Computing the collapse

- G_{eff} can be computed from any ($\sim$) given DE/MG theory
- As before, we compute $t_{collapse}$
- $D(t)$ can be computed from a given DE/MG theory
- We can get $\delta_c \Rightarrow$ **code developed**
- We can also get Δ_{vir} (with a modified virial theorem)

What do we do with this δ_c ? Inject back into P-S

Other ingredients : fitting function

Press-Schechter formalism works surprising well...
for 1980's standards !

Since then :

- Many new refinements
- % level precision

but Added parameters

and Fitted on numerical simulations (cosmology-dependent)

$\Rightarrow$ Universality ? Open question...

Other ingredients : fitting function

Table 1: Compilation of Fitting Functions

Ref.	Fitting Function $f(\sigma)$	Mass Range	Redshift Range	Cosmology Fitted
Press and Schechter (1974)	$f_{\rm PS}(\sigma) = \sqrt{\frac{2}{\pi}}\frac{\delta_c}{\sigma}\exp\left[-\frac{\delta_c^2}{2\sigma^2}\right]$	–	–	–
Sheth et al. (2001)	$f_{\rm ST}(\sigma) = A\sqrt{\frac{2a}{\pi}}\left[1+\left(\frac{\sigma^2}{a\delta_c^2}\right)^p\right]\frac{\delta_c}{\sigma}\exp\left[-\frac{a\delta_c^2}{2\sigma^2}\right]$, $A = 0.3222,\ a = 0.707,\ p = 0.3.$	–	–	Einstein-de Sitter

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Jenkins et al. (2001)	$f_{\rm J}(\sigma) = 0.315\exp\left[\lvert\ln\sigma^{-1}+0.61\rvert^{3.8}\right]$	$-1.2 < \ln\sigma^{-1} < 1.05$	0 – 5	τCDM, ΛCDM
Reed et al. (2003)	$f_{\rm R03}(\sigma) = f_{ST}(\sigma)\exp\left[\frac{-0.7}{\sigma\cosh(2\sigma)^5}\right]$	$-1.7 < \ln\sigma^{-1} < 0.9$	0 – 15	$\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$
Warren et al. (2006)	$f_{\rm W}(\sigma) = 0.7234\left(\sigma^{-1.625}+0.2538\right)\exp\left[\frac{-1.1982}{\sigma^2}\right]$	$10^{10}{\rm M}_\odot < M < 10^{15}{\rm M}_\odot$	0	ΛCDM: WMAP1

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Jenkins et al. (2001)	$f_{\rm J}(\sigma) = 0.315 \exp\left[\lvert \ln\sigma^{-1} + 0.61\rvert^{3.8}\right]$	$-1.2 < \ln\sigma^{-1} < 1.05$	$0 - 5$	τCDM, ΛCDM
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Reed et al. (2007)	$f_{\rm R07}(\sigma) = \nu \exp\left[-\frac{ca\frac{\delta_c^2}{\sigma^2}}{2} - \frac{0.03\left(\frac{\delta_c}{\sigma}\right)^{0.6}}{(n_{\rm eff}+3)^2}\right]$ $\times A\sqrt{\frac{2a}{\pi}}\left[1 + \left(\frac{1}{\frac{\delta_c^2}{\sigma^2}a}\right)^p + 0.6G_1(\sigma) + 0.4G_2(\sigma)\right]$ $n_{\rm eff} = 6\frac{d\log\sigma^{-1}}{d\log M} - 3,\ G_1(\sigma) = \exp\left[-\frac{\ln(\sigma^{-1}-0.4)^2}{0.72}\right]$, $G_2(\sigma) = \exp\left[-\frac{\ln(\sigma^{-1}-0.75)^2}{0.08}\right]$	$-1.7 < \ln\sigma^{-1} < 0.9$	$0 - 30$	ΛCDM: WMAP1
Tinker and Kravtsov (2008)	$f_{\rm T}(\sigma, z) = A\left(\left(\frac{b}{\sigma}\right)^a + 1\right)\exp\left[-\frac{c}{\sigma^2}\right]$, $A = 0.186\,(1+z)^{-0.14},\ a = 1.47\,(1+z)^{-0.06}$, $b = 2.57\,(1+z)^{-\alpha}, c = 1.19$, $\alpha = \exp\left[-\left(\frac{0.75}{\ln(\Delta_{\rm vir}/75)}\right)^{1.2}\right]$	$-0.6 < \ln\sigma^{-1} < 0.4$	$0 - 2.5$	ΛCDM: WMAP1, WMAP3+
Crocce et al. (2010)	$f_{\rm Cr}(\sigma) = A\,(\sigma^{-a} + b)\exp\left[-\frac{c}{\sigma^2}\right]$, $A = 0.58\,(1+z)^{-0.13},\ a = 1.37\,(1+z)^{-0.15}$, $b = 0.3\,(1+z)^{-0.084},\ c = 1.036\,(1+z)^{-0.024}$	$10^{10.5}{\rm M}_\odot < M < 10^{15.5}{\rm M}_\odot$	$0 - 2$	$(\Omega_M, \Omega_\Lambda, n, h, \sigma_8) = (0.25, 0.75, 0.95, 0.7, 0.8)$
Courtin et al. (2010)	$f_{\rm Co}(\sigma) = f_{\rm ST}(\sigma)$, $A = 0.348,\ a = 0.695,\ p = 0.1$	$-0.8 < \ln\sigma^{-1} < 0.7$	0	ΛCDM: WMAP5
Bhattacharya et al. (2011)	$f_{\rm B}(\sigma, z) =$ $A\sqrt{\frac{2}{\pi}}\exp\left[-\frac{a\delta_c^2}{2\sigma^2}\right]\left[1 + \left(a\frac{\delta_c^2}{\sigma^2}\right)^{-p}\right]\left(\frac{\delta_c^2}{\sigma^2}\sqrt{a}\right)^q$, $A = 0.333\,(1+z)^{-0.11},\ a = 0.788\,(1+z)^{-0.01}$, $p = 0.807,\ q = 1.795$	$10^{11.8}{\rm M}_\odot < M < 10^{15.5}{\rm M}_\odot$	$0 - 2$	wCDM+
Angulo et al. (2012)	$f_{\rm A}(\sigma) = A\left[\left(\frac{b}{\sigma}\right)^a + 1\right]\exp\left[-\frac{c}{\sigma^2}\right]$, $(A, a, b, c) = (0.201, 1.7, 2.08, 1.172)$ or $(A, a, b, c)_{\rm SUB} = (0.265, 1.9, 1.675, 1.4)$	$10^{8}{\rm M}_\odot < M < 10^{16}{\rm M}_\odot$	0	ΛCDM: WMAP1
Watson et al. (2013)	$f_{\rm W_{FOF}}(\sigma, z) = f_{\rm T}(\sigma, z)$, $A = 0.282,\ a = 1.406,\ b = 2.163,\ c = 1.21$	$-0.55 < \ln\sigma^{-1} < 1.31$	$0 - 30$	ΛCDM: WMAP5
Watson et al. (2013)	$f_{\rm W_{SO}}(\sigma, z) = \Gamma(\Delta, \sigma, z) f_{\rm T}(\sigma, z)$, $(A, a, b, c)_{z=0} = (0.194, 2.267, 1.805, 1.287)$, $(A, a, b, c)_{z>6} = (0.563, 874, 3.810, 1.453)$, $(A, a, b, c)_{0<z<6} =$ $\Omega_M(z) \times (1.907(1+z)^{-3.216} + 0.074$, $3.136(1+z)^{-3.058} + 2.349$, $5.907 \times (1+z)^{-3.599} + 2.344,\ 1.318)$, $\Gamma(\Delta, \sigma, z) = C(\Delta)\left(\frac{\Delta}{178}\right)^{d(z)}\exp\left[\frac{p\left(1-\frac{\Delta}{178}\right)}{\sigma^q}\right]$, $C(\Delta) = 0.947\exp\left[0.023\left(\frac{\Delta}{178} - 1\right)\right]$, $d(z) = -0.456\Omega_M(z) - 0.139,\ p = 0.072$, $q = 2.130.$	$-0.55 < \ln\sigma^{-1} < 1.05\ (z = 0)$, $-0.06 < \ln\sigma^{-1} < 1.024\ (z > 0)$	$0 - 30$	ΛCDM: WMAP5

Other ingredients : fitting function

More work needed to go beyond ΛCDM
....or not ?

To answer, we need numerical simulations

Other ingredients : matter power spectrum

Computing the linear $P(k)$ for DE/MG models

EFTCAMB (or MGCAMB)

Extract/predict cosmological constraints

Likelihood code

- Fitting function TBD (for now, Despali et al. 2015)
- Scaling laws (M-observable relation) : a whole other topic/problem ! (approach of Sartoris et al. 2015)
- Need some mocks to test the pipeline

Fisher matrix forecast code

- Choice (and testing) of code TBD : EFTCAMB, MGCAMB (for now, EFTCAMB)
- Retrieve outputs and feed to likelihood code

Bonus !

Outputs from EFTCAMB allow iSW calculations
$\Rightarrow$ likelihood & Fisher forecasts

Thank you for your attention !