

# *Hindrance factors for short half-life isomers in the rare-earth axially deformed nuclei using fast timing.*

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# Overview

Transition rates in nuclei.

Fermi's Golden Rule.

Weisskopf estimates.

K-Selection rule and Isomeric transitions.

Circumnavigation of the K-Selection rule.

Short half-life isomers in the mass 170-80 region.

Potential Experiments with Nuball and LaBr<sub>3</sub> Fast timing.

## $\gamma$ -ray transition rates given by Fermi Golden Rule:

$$\lambda = \frac{2\pi}{\hbar} \left| \int \psi_f^* M(\sigma L) \psi_i d\tau \right|^2 \frac{dN}{dE}$$

Density of final states  
for photon – gives a  
transition energy  
dependence of  $E^3$

e.g. Electric transition operator:  $M(EL) \sim \frac{1}{k} (kr)^L P_L(\cos \theta)$

### The single-particle Weisskopf estimates:

$$\lambda(E1) = 1.0 \times 10^{14} A^{2/3} E^3$$

$$\lambda(E2) = 7.3 \times 10^7 A^{4/3} E^5$$

$$\lambda(E3) = 34 A^2 E^7$$

$$\lambda(E4) = 1.1 \times 10^{-5} A^{8/3} E^9$$

$$\lambda(M1) = 5.6 \times 10^{13} E^3$$

$$\lambda(M2) = 3.5 \times 10^7 A^{2/3} E^5$$

$$\lambda(M3) = 16 A^{4/3} E^7$$

$$\lambda(M4) = 4.5 \times 10^{-6} A^2 E^9$$

Large L  $\rightarrow$  small  $(kr)^L$  (as  $kr < 1$ )

## Modification of single-particle Weisskopf estimates

$$\lambda = \frac{2\pi}{\hbar} \left| \int \psi_f^* M(\sigma L) \psi_i d\tau \right|^2 \frac{dN}{dE}$$

**Transition rates can be strongly affected by:**

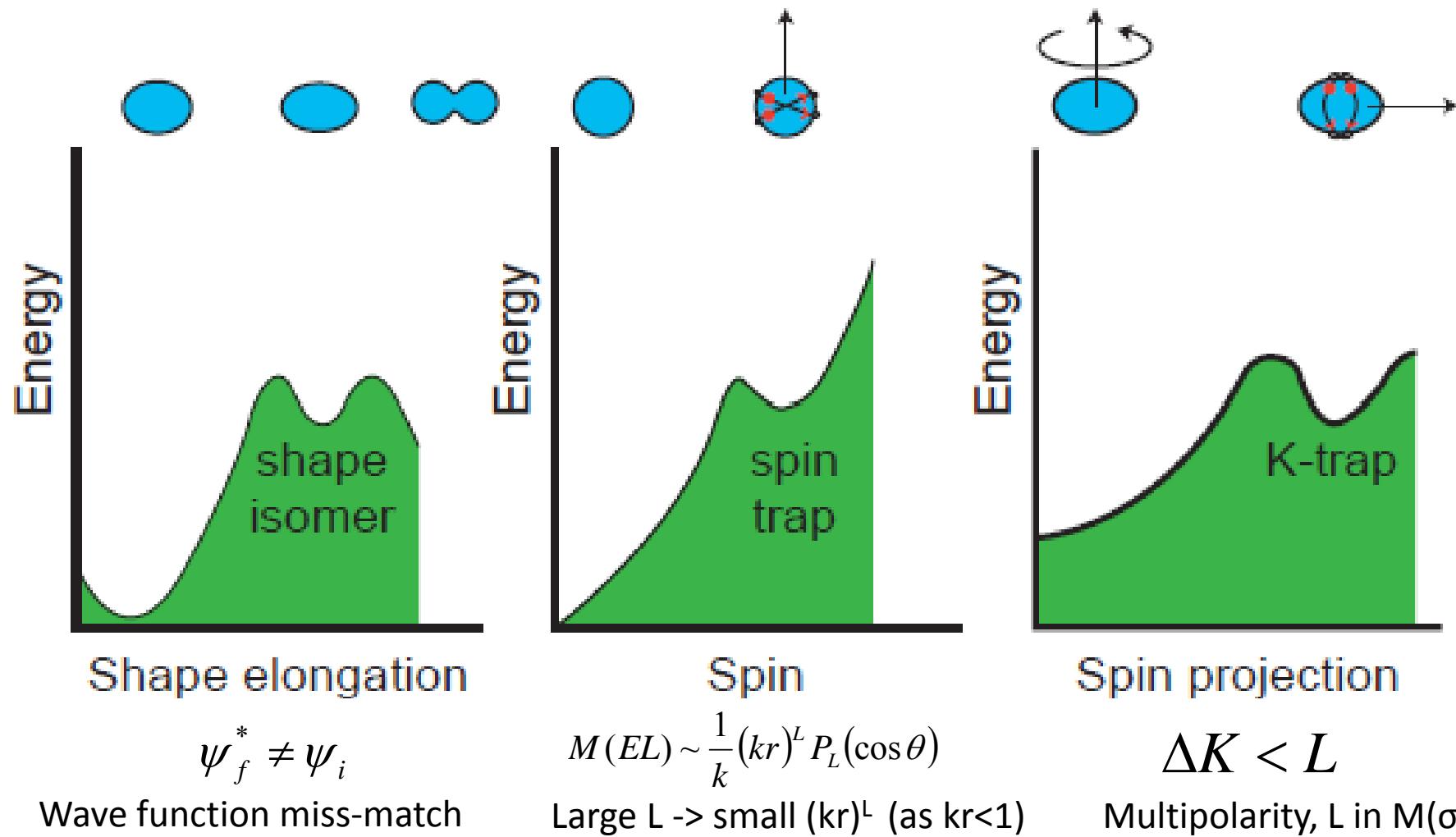
1. **Collectivity**, e.g. Rotational states have very similar underlying initial and final wave functions which **enhances** the single-particle rates (invoke effective charges to increase proton and neutron charges).
2. **Wave function miss-match** where initial and final state matrix element overlap interval is small, **hindering** the transition rates relative to the single-particle rate.
3. **The density of states**,  $dN/dE$  can **enhance** or **hinder** the transition rates.

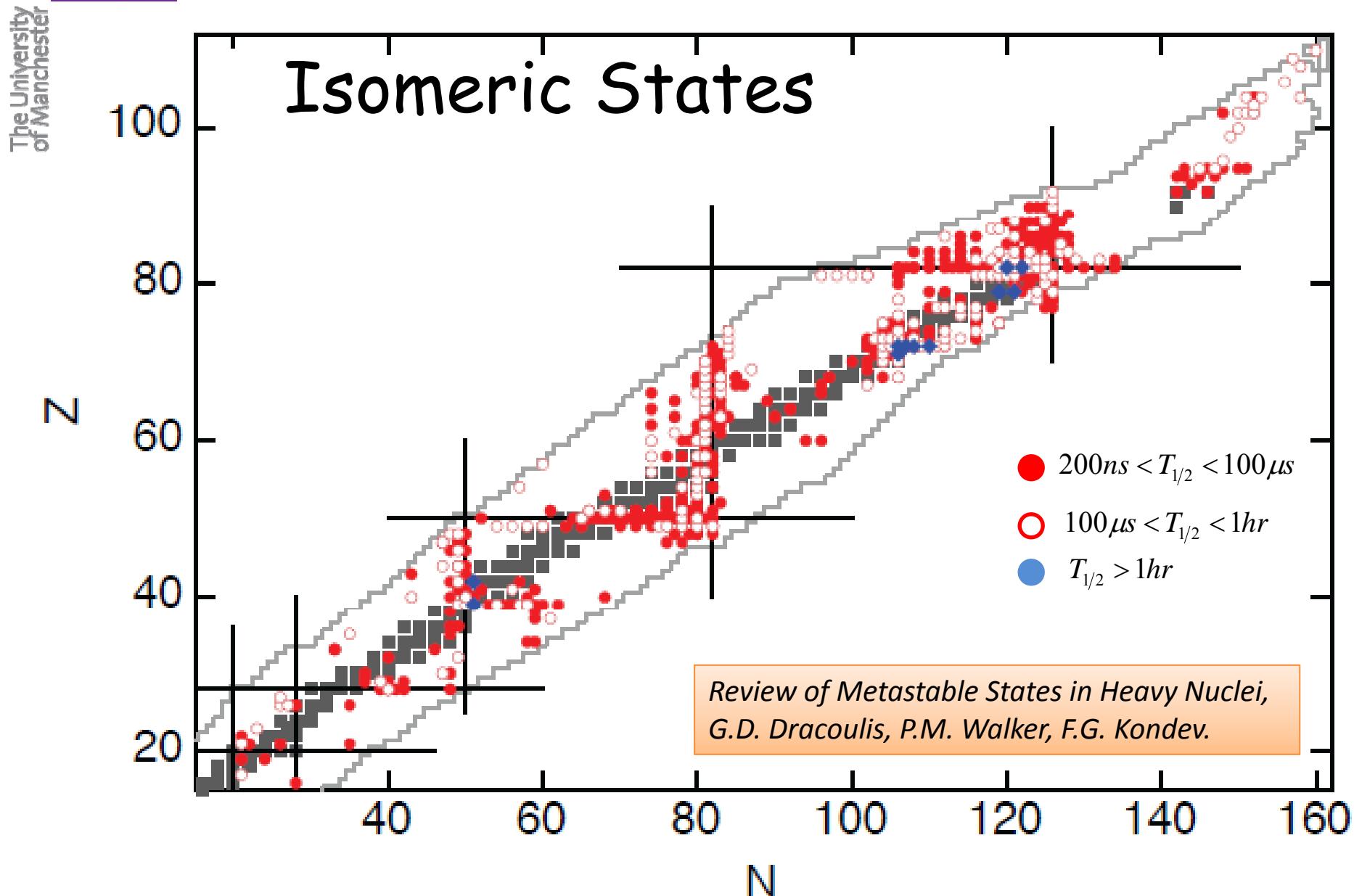
# Isomeric States

Cullen - NuBall workshop, Orsay 2016

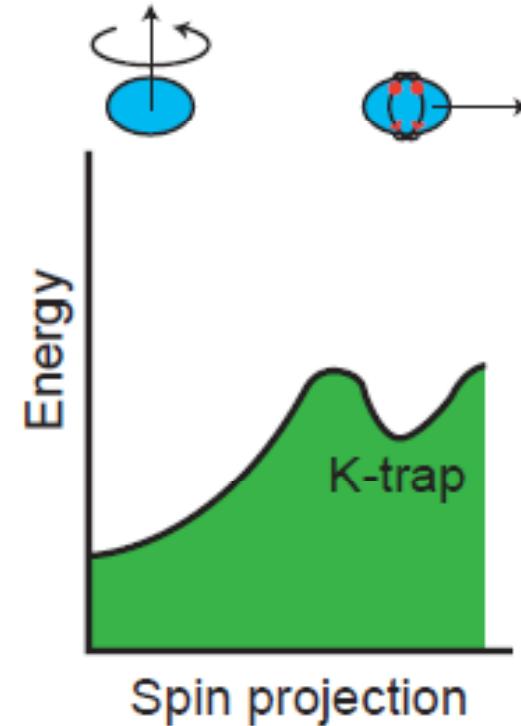
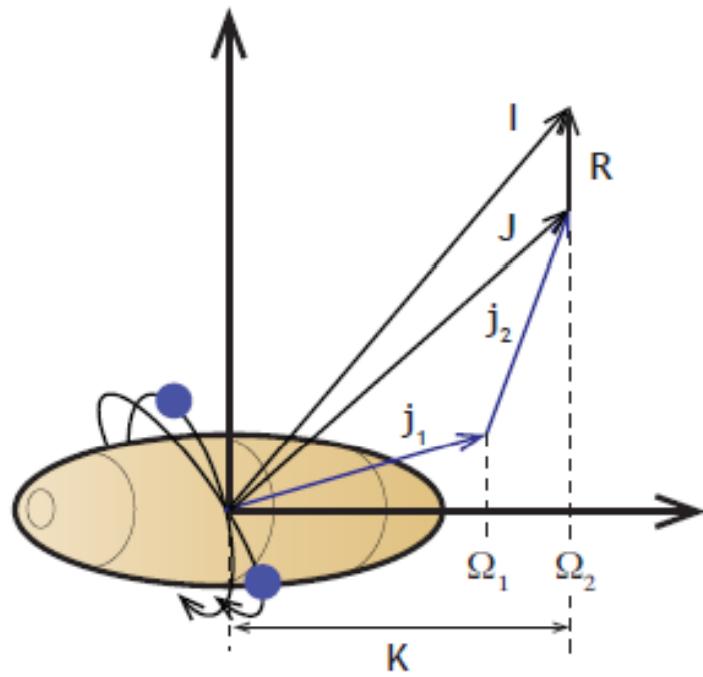
$$\lambda = \frac{2\pi}{\hbar} \left| \int \psi_f^* M(\sigma L) \psi_i d\tau \right|^2 \frac{dN}{dE}$$

Energy traps in atomic nuclei, P. Walker & G.D. Dracoulis. NATURE 339 (1999) 35





# High-K isomers

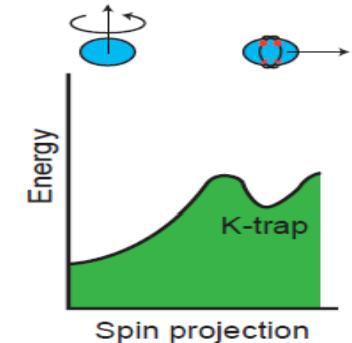


Large changes in angular momentum **direction** between initial and final wave function can seriously **hinder** the decay of a high-K state:  
e.g.  $K^\pi = 16^+$  31-year isomeric state in  $^{178}\text{Hf}$

# The K-Selection Rule

$$K_i \xrightarrow{M(\sigma\lambda)} K_f$$

$$\lambda = \frac{2\pi}{\hbar} \left| \int \psi_f^* M(\sigma L) \psi_i d\tau \right|^2 \frac{dN}{dE}$$



## High-K isomer decay and the K-selection rule

$$\Delta K \leq \lambda$$

*The change in K between the initial and final states must be less than or equal to the transition multipolarity,  $\lambda$*

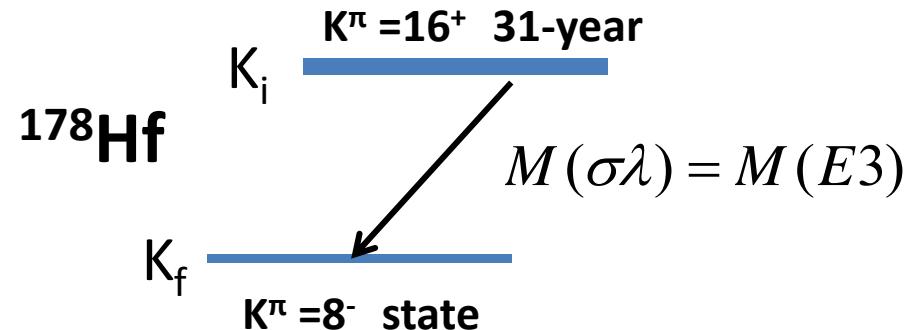
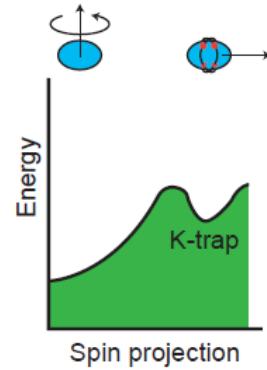
**Löbner** 1968, transitions hindered by a factor of 100 for each degree of K-forbiddenness

$$\nu = \Delta K - \lambda$$

To allow **different** multipolarity isomer decays to be directly compared, define

$$f_\nu = \left( \frac{t_{1/2}^{measured}}{t_{1/2}^{Weisskopf}} \right)^{1/\nu}$$

# The K-Selection Rule



$K^\pi = 16^+$  31-year isomeric state in  $^{178}\text{Hf}$

Isomer decays by an E3 transition:

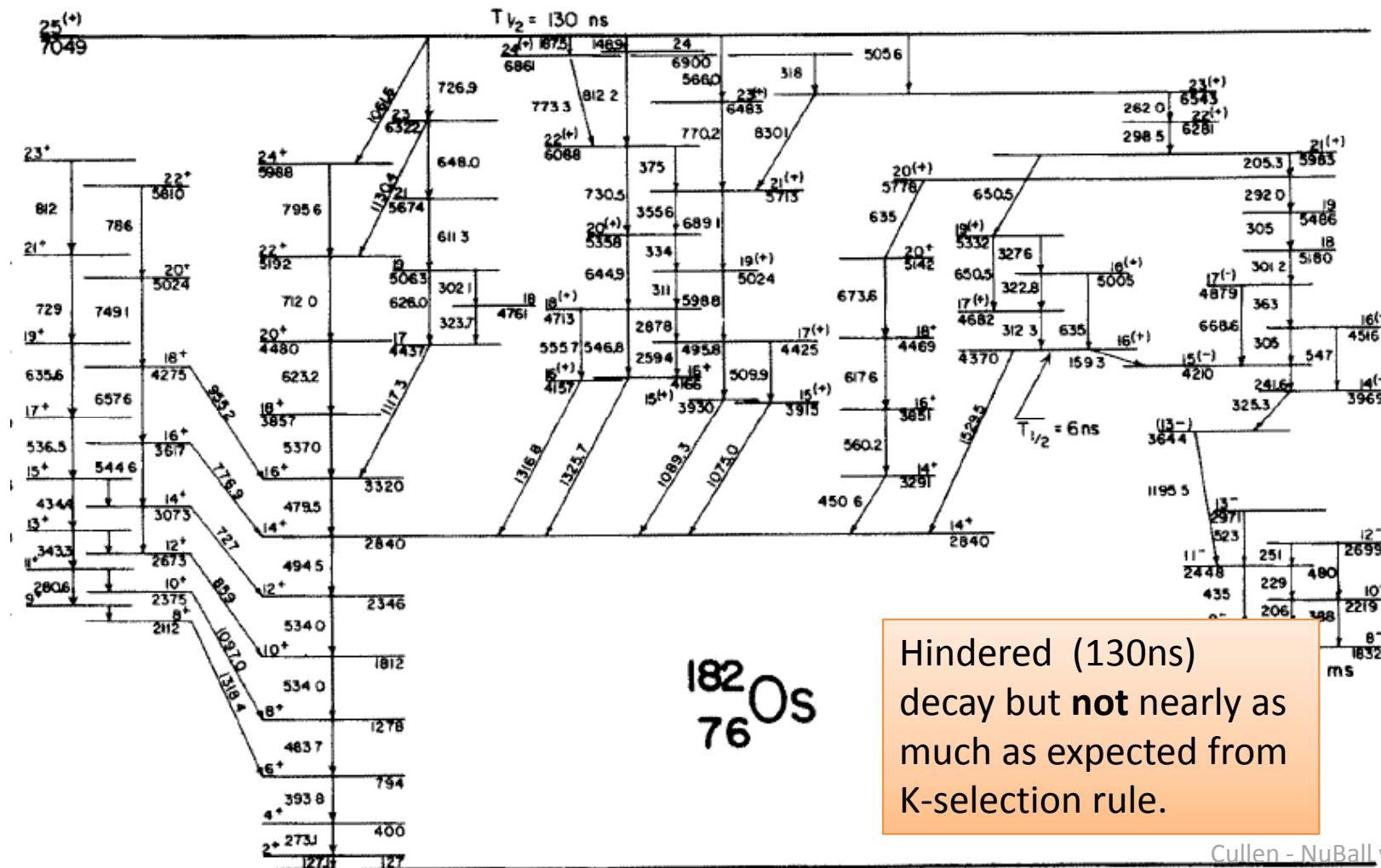
1. Weisskopf (including conversion) gives  $t_{1/2} = 0.1 \text{ s}$
2. Only when factor in Löbner  $100^5$  from the 5 times K-forbidden E3 transition ( $\Delta K=8$ ) do you get  $t_{1/2} \sim 30 \text{ years}$ .

$K^\pi = 16^+$  Isomeric state half-life due to conservation of angular momentum: high-L transfer (spin trap) plus K-selection rule.

# Circumventing the K-Selection Rule

Some notable Isomer decays are known to “ignore” the K-selection rule

$^{182}\text{Os}$   $K^\pi = 25^+$  isomeric state decay to  $K\sim 0$  ground-state band with single  $\Delta K=24$ , M1 transition.



P Chowdhury et al / Decay of high-spin isomers

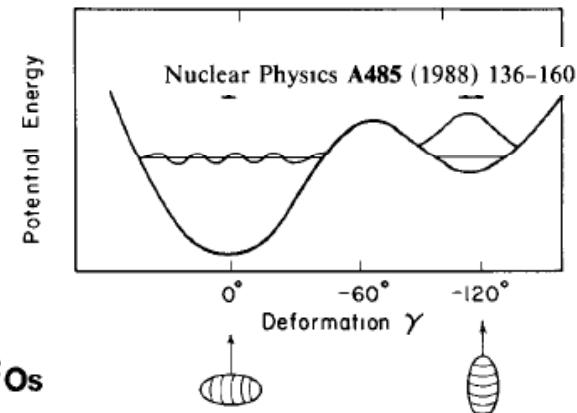
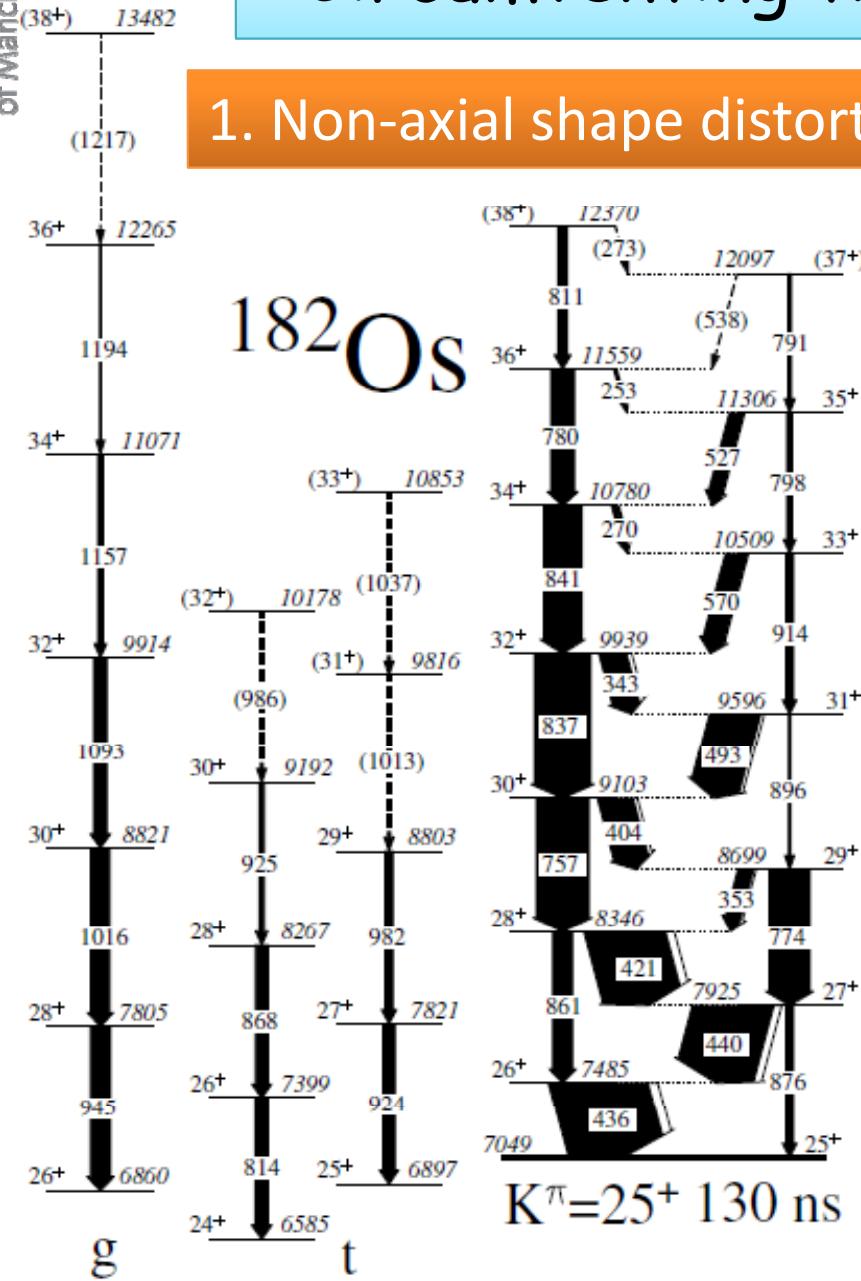
# Circumventing the K-Selection Rule

Analysis of many isomeric states shows that K-selection rule can be circumvented in 3 main ways:

1. The presence of non-axial shape distortions  $\gamma$  softness ( $\beta_{22}$ ) which renders K no longer a good quantum number.
2. Coriolis mixing: In the region of the first backbend, the ground-state band may have non-zero K components mixed into its wave function, making  $\Delta K$  smaller.  $\langle \psi_f^* | \Delta K = \pm 1 | \psi_i \rangle$
3. Increased density of states, enhancing the transition rates, for highly non-yrast isomers. (Statistical mixing of low-K states into the high-K wave function.)  
$$\lambda = \frac{2\pi}{\hbar} \left| \int \psi_f^* M(\sigma L) \psi_i d\tau \right|^2 \frac{dN}{dE}$$

# Circumventing the K-Selection Rule

## 1. Non-axial shape distortions $\gamma$ softness in $^{182}\text{Os}$ (1988).



Tunnelling through the gamma plane.

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PHYSICAL REVIEW LETTERS

week ending  
31 OCTOBER 2000

### Multiphonon Vibrations at High Angular Momentum in $^{182}\text{Os}$

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# Circumventing the K-Selection Rule

## 2. Coriolis mixing: 2-quasi-particle bands $K^\pi = 6^+, 8^-$ bands in $^{172}\text{Hf}$ .

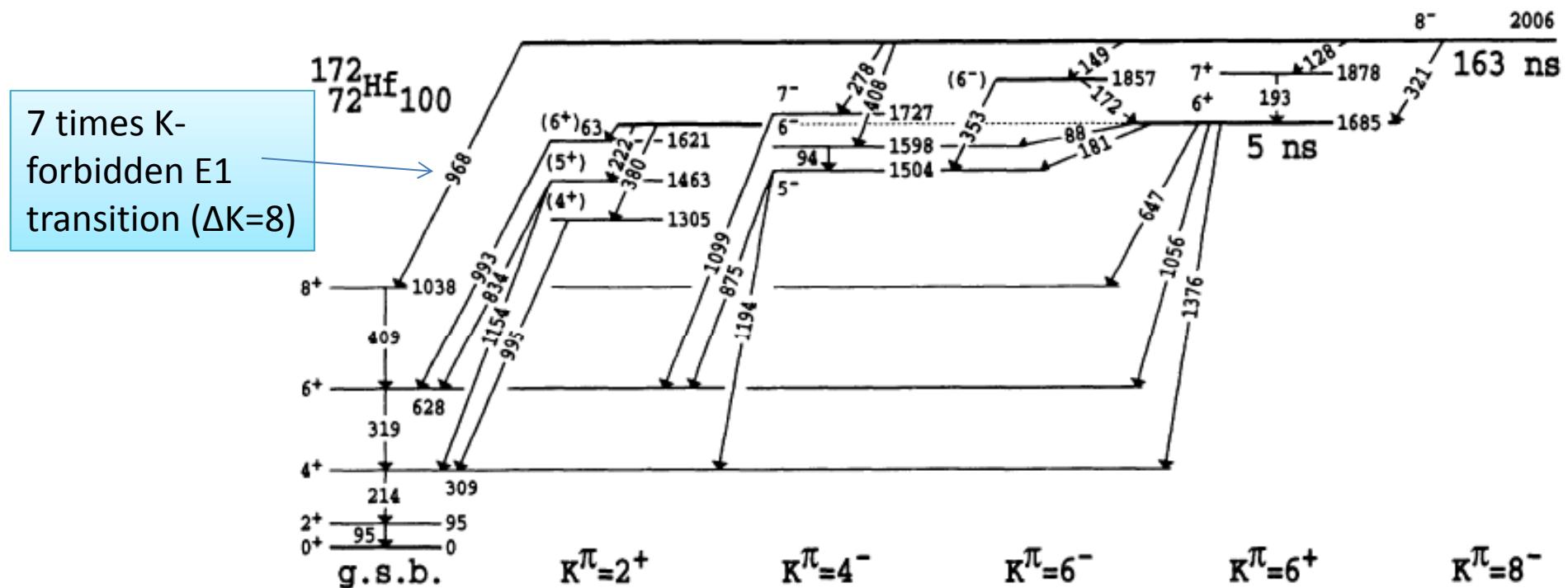
PHYSICAL REVIEW C

VOLUME 49, NUMBER 3

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$K^\pi = 6^+$  and  $8^-$  isomer decays in  $^{172}\text{Hf}$  and  $\Delta K = 8$  E1 transition rates

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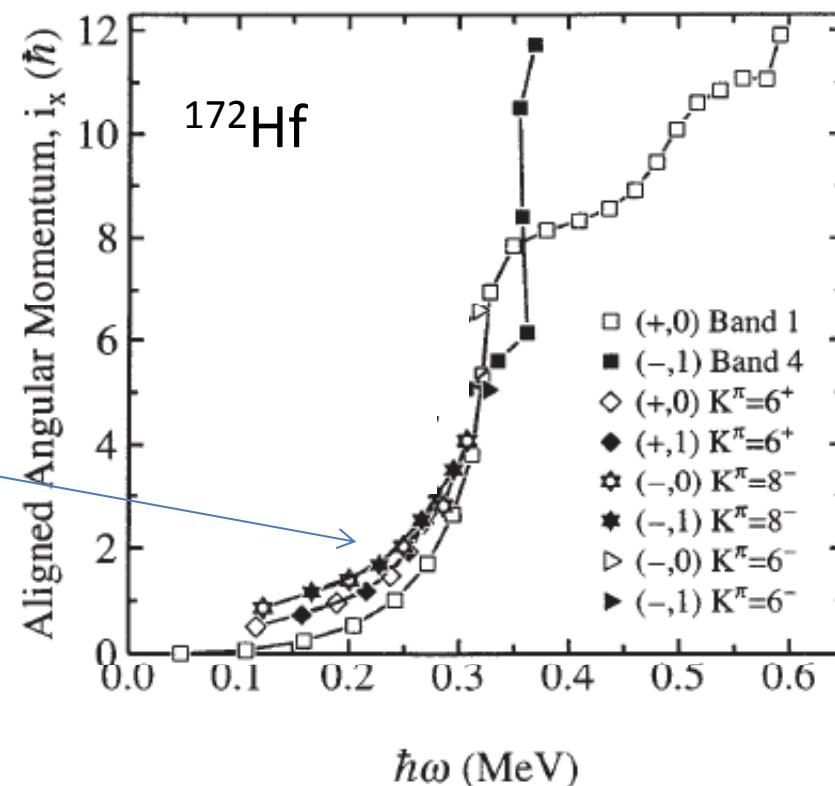
# Circumventing the K-Selection Rule

2. Coriolis mixing: 2 quasi-particle decays occur to ground-state band where it is undergoing first alignment.

D.M. Cullen et al./Nuclear Physics A 638 (1998) 662–700

$$\langle \psi_f^* | \Delta K = \pm 1 | \psi_i \rangle$$

Here, Coriolis force mixes high-K components into ground-state band reducing the effective  $\Delta K$  required by E1, reducing the hindrance factors.



# Circumventing the K-Selection Rule

## 3. Density of States.

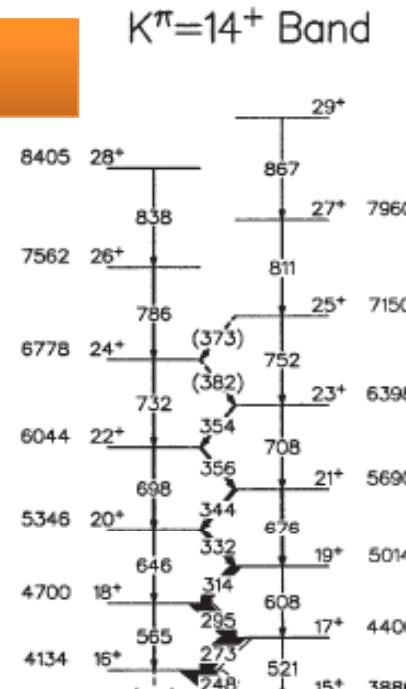


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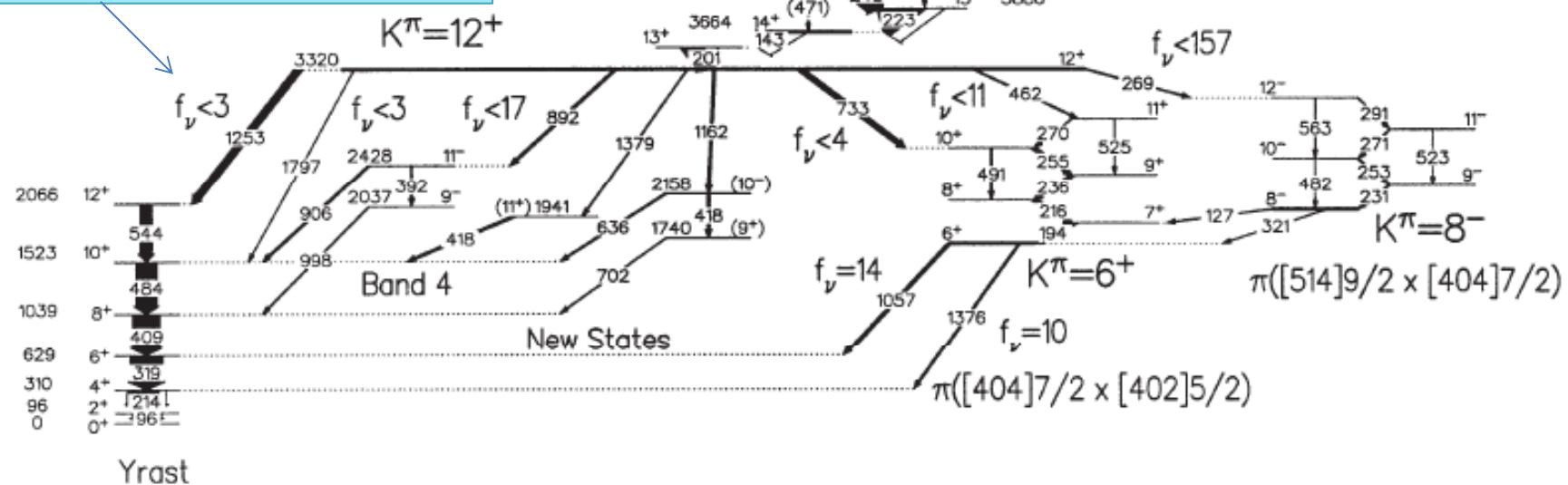
NUCLEAR  
PHYSICS A

High- $K$  decays and lifetime measurements in  $^{172}\text{Hf}$ D.M. Cullen<sup>a</sup>, A.T. Reed<sup>a</sup>, D.E. Appelbe<sup>a</sup>, A.N. Wilson<sup>a,1</sup>, E.S. Paul<sup>a</sup>,

Lifetime and hindrance  
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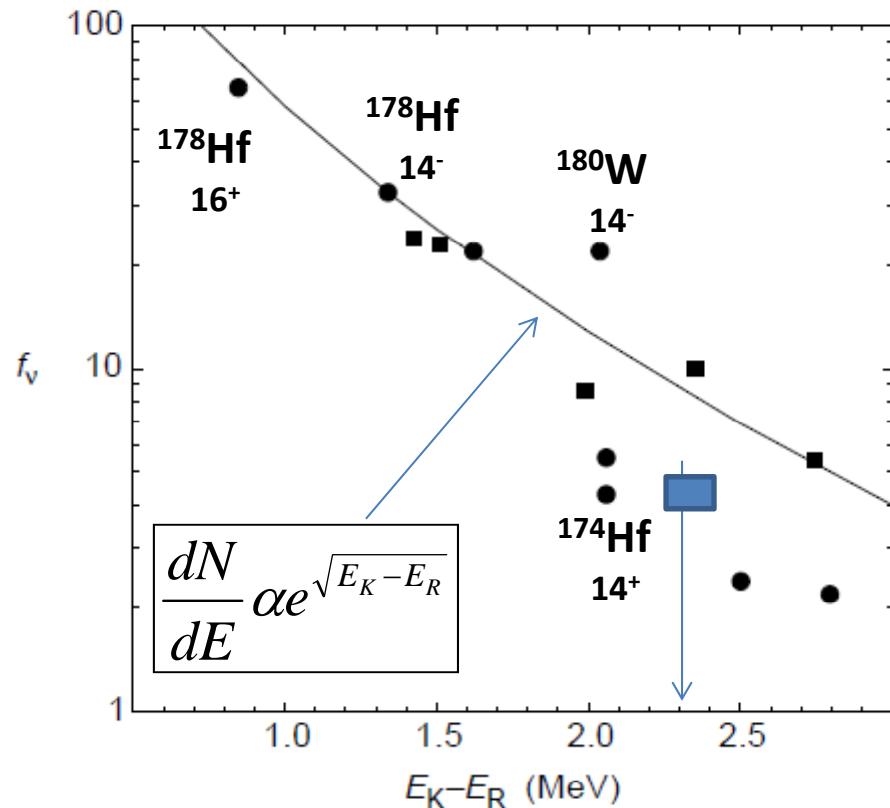


Four quasi-particle states  
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Non-yrast isomers

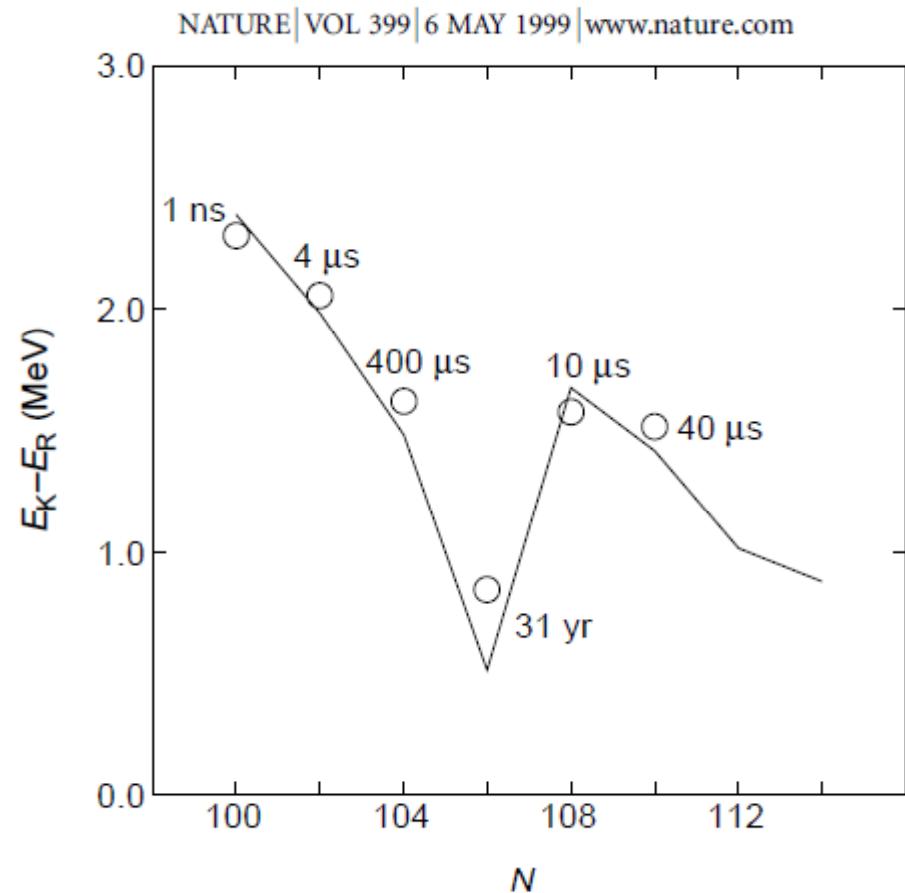


# Circumventing the K-Selection Rule

## 3. Density of States.



**Figure 5** Reduced hindrance as a function of isomer excitation energy, relative to a rotor with the same angular momentum, for selected  $K$ -forbidden transitions. The solid curve represents a statistical-model estimate based on the density of states. The figure is adapted from ref. 45. Circles represent four-quasiparticle isomer decays in even- $N$ , even- $Z$  nuclei; squares represent five-quasiparticle isomer decays in odd-mass nuclei (a pairing energy of 0.9 MeV has been added to the excitation energy above the ground state).



**Figure 6** Excitation energy of even-mass hafnium isomers relative to a rotor as a function of neutron number. The circles, with half-lives indicated, represent the lowest-energy experimentally observed four-quasiparticle isomers. The solid line indicates the corresponding calculated values, which extend to higher neutron numbers than can currently be accessed experimentally. The calculations are based on the method of Jain *et al.* (ref. 49), with fixed pairing forces.

# High-K isomer Experiments

Much of this work relied on:

1. Large arrays of Ge detectors such as *Gammsphere* with backed targets to identify weak decay branches, or
2. State-of-the-art beam *pulsing* and *chopping*, prompt-delayed coincidences (ANU) to select isomers.

However, both methods used Ge or LEPs detectors with relatively poor intrinsic timing resolution (30ns). This creates a problem when the half-life becomes  $< \sim 1\text{ns}$ .

# New Potential Experiments

Nuball offers a unique opportunity to measure lifetimes of *sub-ns* isomers with high precision.

Take advantage of:

1. 36 LaBr<sub>3</sub> detectors to make fast-timing LaBr<sub>3</sub> coincidences.
2. 24 Clovers for added channel selection if necessary.
3. Pulsed tandem beams for isomer selection/cleanliness.

Discuss two representative cases which could form Nuball proposals to better define the role of the density of states on the transition half-life of isomeric states in <sup>172</sup>Hf and <sup>170</sup>Hf.

# New Potential Experiment: $^{172}\text{Hf}$

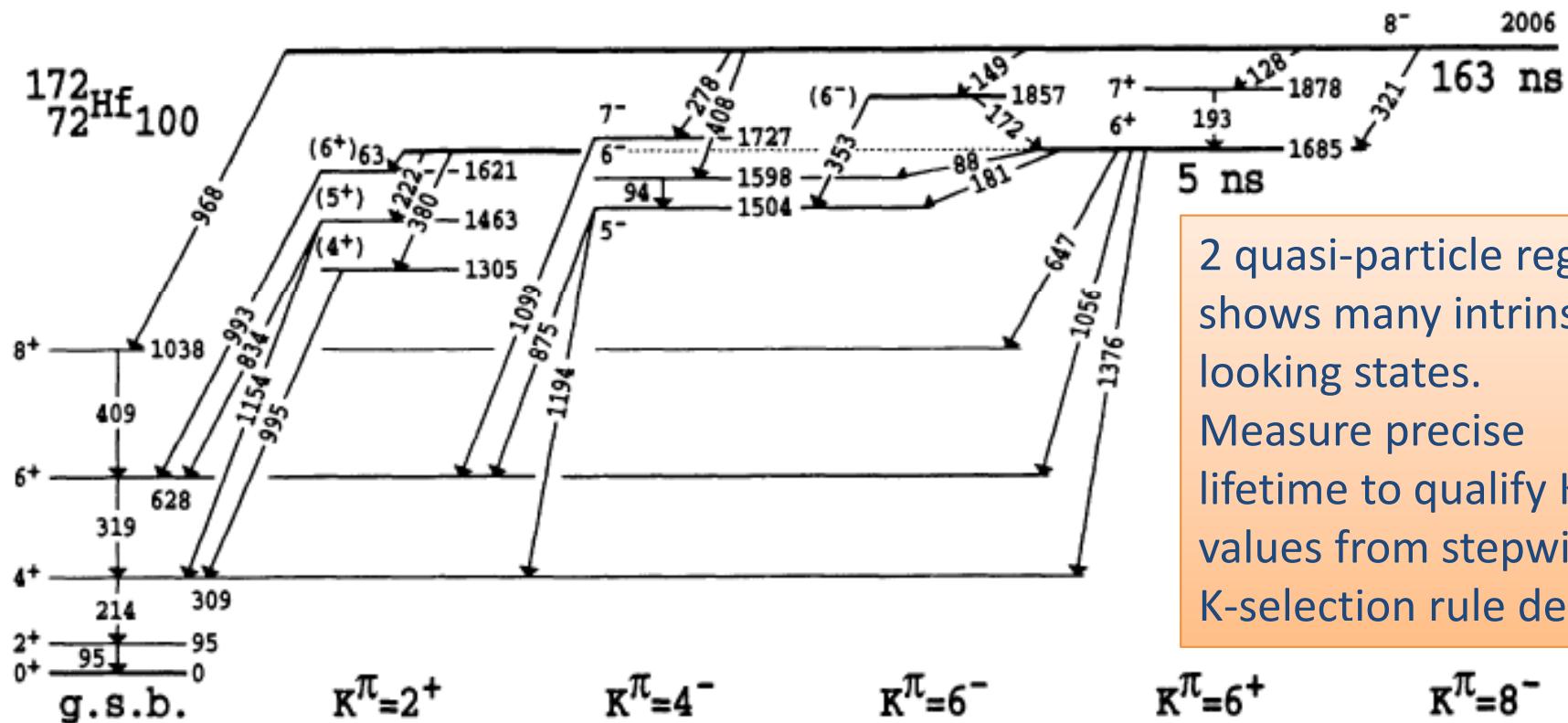
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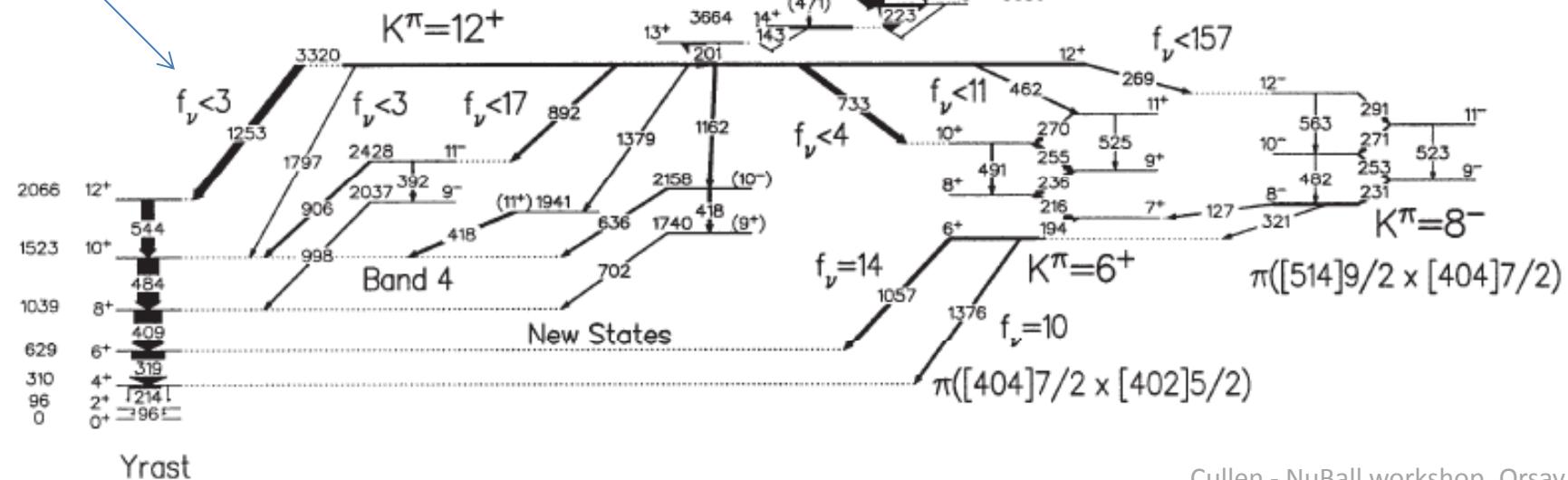


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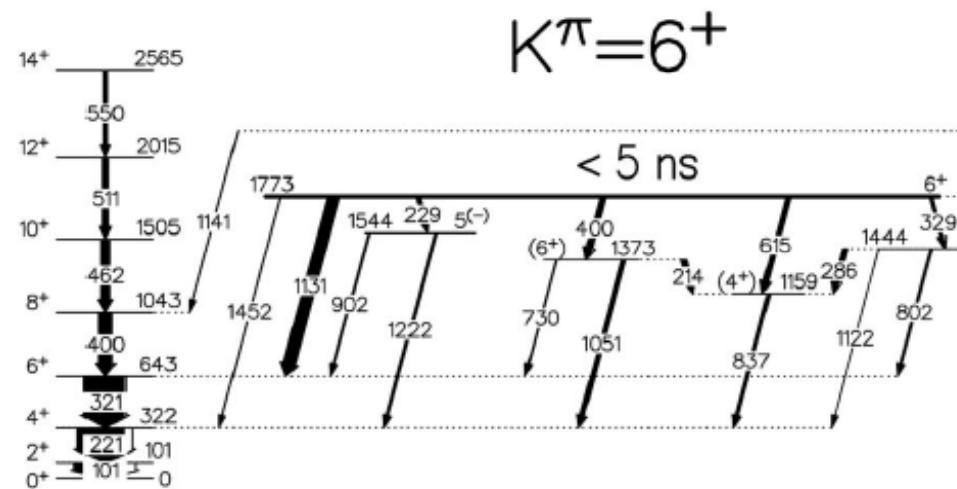
**Four quasi-particle states  
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Non-yrast isomers.**

# New Potential Experiment: $^{170}\text{Hf}$

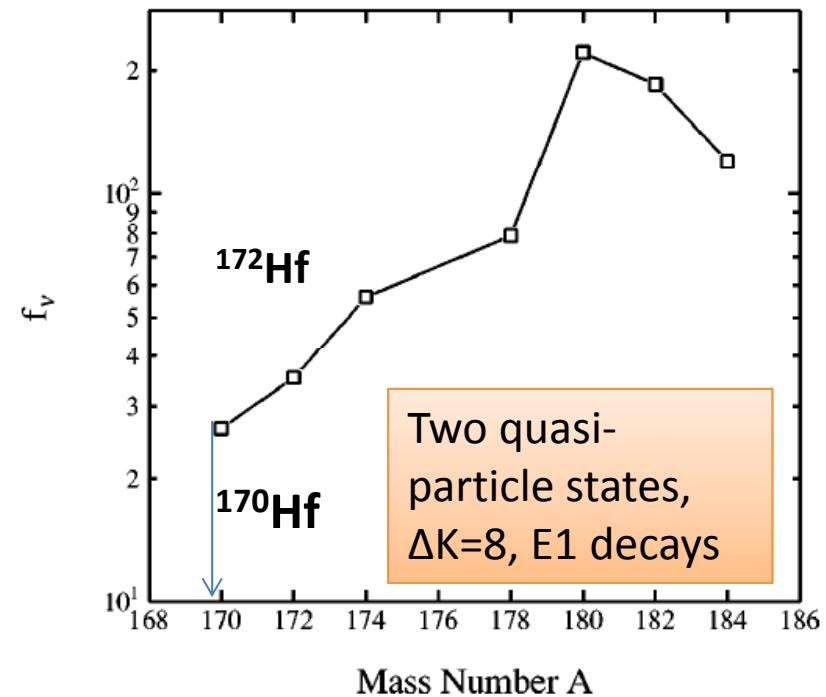
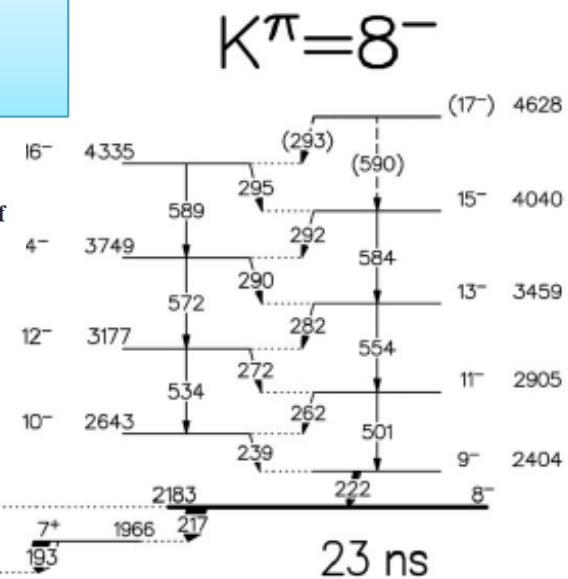
PHYSICAL REVIEW C, VOLUME 60, 057303

 $K^\pi=8^-$  rotational band and its fragmented decay through the  $K^\pi=6^+$  intrinsic state in  $^{170}\text{Hf}$ 

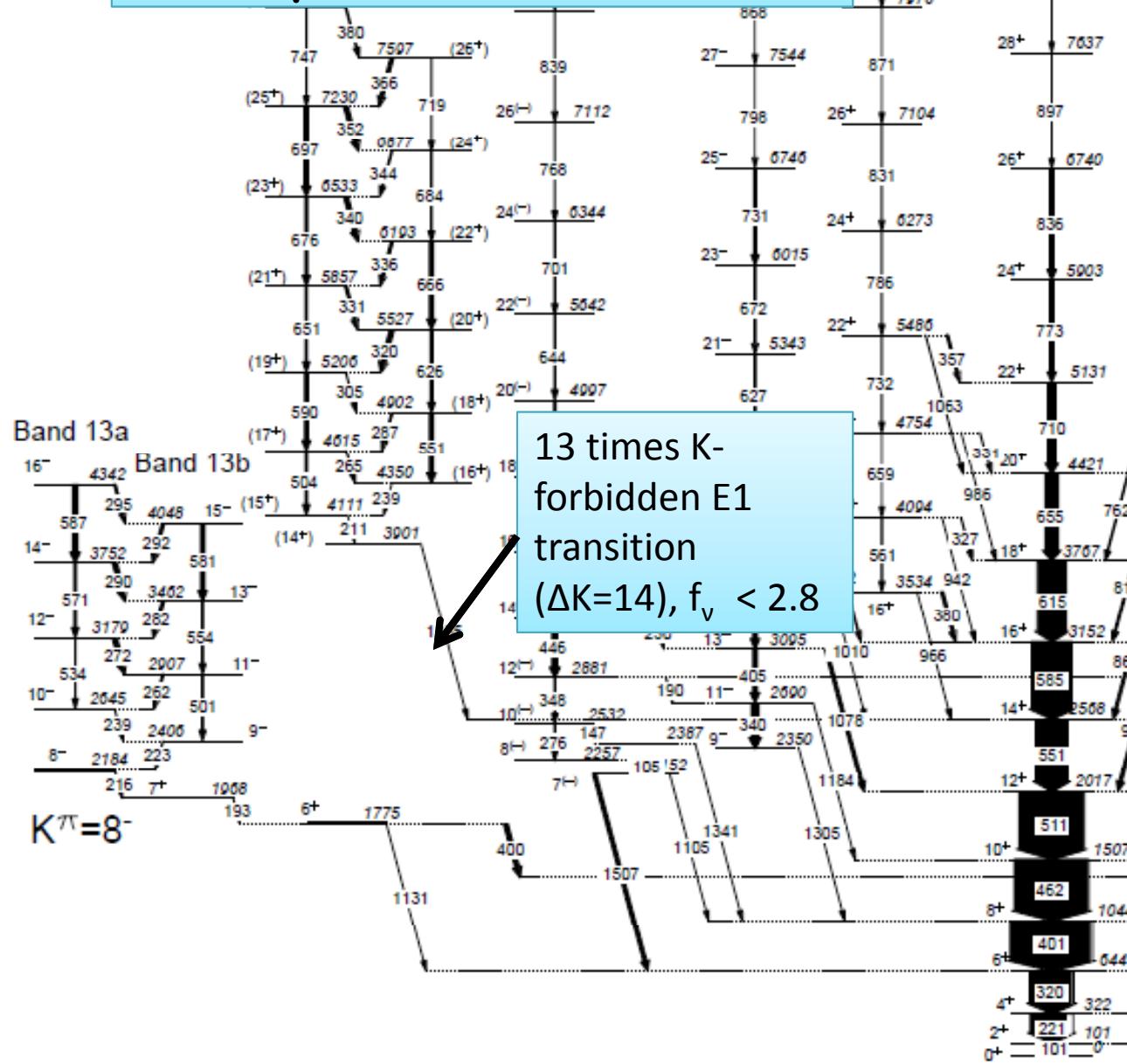
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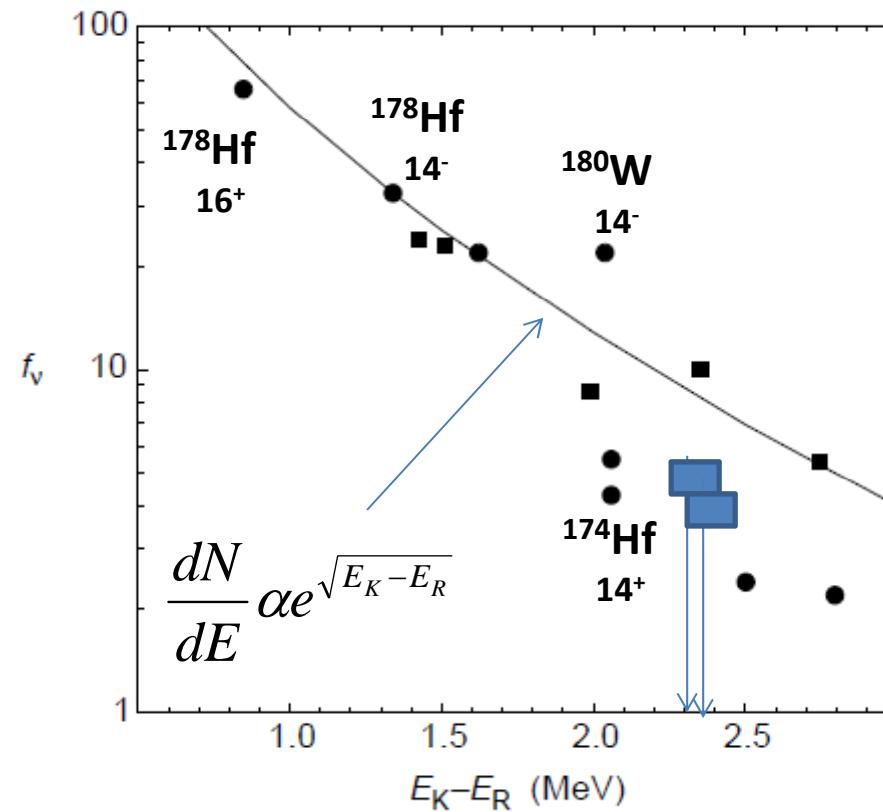


# New Potential Experiment: $^{170}\text{Hf}$



# Better define Hindrance Limits

For the four-quasi-particle states in  $^{170,172}\text{Hf}$  could help characterise the role of density of states in reducing the hindrance.



## Better define Hindrance Limits

***Accurate half-life measurements using LaBr<sub>3</sub>-LaBr<sub>3</sub> coincidence measurements***

***(for short-lived isomeric states,  $t_{1/2} < \sim 2 \text{ ns}$ , which could not be measured with Ge detectors)***

***will help to better define K-hindrances and allow a more detailed study of the mechanisms for the circumvention of the K-selection rule.***

# Thanks to Many People...

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NUCLEAR  
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