HLS improved data input for Lattice QCD evaluations of $(g-2)_\mu$

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Refs. M. Benayoun *et al.* EPJ C75 (2015) 613
M. Benayoun *et al.* ArXiv : 1605.04474

OUTLINE

- HLS Model & Breaking : BKY & (ρ,ω,ϕ) mixing
- τ +PDG predictions for $F_\pi(s)$ & $e^+e^- \rightarrow \pi^+ \pi^-$ samples
- Global Fits

$[\tau^\pm \rightarrow \pi^\pm \pi^0 \nu_\tau // e^+e^- \rightarrow \pi^+\pi^- / K^+K^- / K_L K_S / \pi \gamma / \eta \gamma / \pi \pi \pi]$

- Estimating $(g-2)_\mu$ from (B)HLS & Data
- Mellin-Barnes Moment Analysis
- Sharing the various pieces in a_μ^{had} : I=1, I=0, IB,γ
- Conclusions

The HLS Model & Breaking

The HLS Lagrangian + FKTUY (anomalous) pieces

M. Harada & K. Yamawaki Phys. Rep. 381 (2003) 1

- **Practical use of HLS implies breaking schemes :**
- **BKY mechanism :**

M.Bando *et al*. Nucl. Phys. B259 (1985) 493

M.Hashimoto Phys. Rev. D54 (1996) 5611

M.Benayoun *et al*. Phys. Rev. D58 (1998) 074006

- **Vector meson mixing :**

M.Benayoun *et al*. EPJ C55 (2008) 199

M.Benayoun *et al*. EPJ C65 (2010) 211

- **Latest Model Status :**

M.Benayoun *et al*. EPJ C72 (2012) 1848

The HLS Model

- Define

M.Bando *et al.* Phys. Rep. 164 (1988) 217

M. Harada & K. Yamawaki Phys. Rep. 381 (2003) 1

$$\xi_{L/R} = e^{[\mp i P / f_\pi]}$$

PS field matrix

- The covariant derivative

V Field Matrix

$$D_\mu \xi_{L/R} = \partial_\mu \xi_{L/R} - igV_\mu \xi_{L/R} + i\xi_{L/R} G_{L/R}$$

- with

$$G_R = eQA_\mu , \quad G_L = eQA_\mu + \frac{g_2}{\sqrt{2}}\left(W_\mu^+ T_+ + W_\mu^- T_-\right)$$

The HLS Model & BKY Breaking

M. Harada & K. Yamawaki Phys. Rep. 381 (2003) 1

- **The L/R matrices ->** $L/R = (D_\mu \xi_{L/R})\xi^\dagger_{L/R}$
- **HLS :** $L_{A/V} = -\frac{f_\pi^2}{4} Tr[L \mp R]^2 \rightarrow$ $L_{HLS} = L_A + aL_V$
- **BKY Breaking :** $\rightarrow$ $L_{A/V} = -\frac{f_\pi^2}{4} Tr\{[L \mp R] X_{A/V}\}^2$
- With $X_{A/V}$ = Diag $\{q_{A/V}, y_{A/V}, z_{A/V}\}$

$$q_{A/V} / y_{A/V} = 1 + (\Sigma \pm \Delta)_{A/V} / 2$$

Isospin Breaking : Vector Field Mixing

- At one loop, the HLS Lagrangian piece (**no IB**)

$$\left(\rho_{R1}+\omega_{R1}-\sqrt{2}\,z_V\,\varphi_{R1}\right)K^-\overleftrightarrow{\partial}K^+ + \left(\rho_{R1}-\omega_{R1}+\sqrt{2}\,z_V\,\varphi_{R1}\right)K^0\overleftrightarrow{\partial}\bar{K}^0$$

→ **s-dependent** transitions among vector fields

VVP Lagrangian : K*$\overline{\text{K}}$ loops // Yang-Mills term : K*$\overline{\text{K}}$*loops

→ **ideal fields no longer mass eigenstates**

PS mesons :: isospin symmetry breaking : $m_{K^\pm} \neq m_{K^0}$

The Mass Matrix Eigen System

At one loop (no IB):

$$M^2(s) = \begin{pmatrix} m^2 + \Pi_{\pi\pi}(s) + \varepsilon_2(s) & \varepsilon_1(s) & -\sqrt{2}\, z_V \varepsilon_1(s) \\ \varepsilon_1(s) & m^2 + \varepsilon_2(s) & -\sqrt{2}\, z_V \varepsilon_2(s) \\ -\sqrt{2}\, z_V \varepsilon_1(s) & -\sqrt{2}\, z_V \varepsilon_2(s) & z_V [m^2 + 2 z_V \varepsilon_2(s)] \end{pmatrix}$$

As :

$$\left(m^2, \Pi_{\pi\pi}(s)\right) >> \varepsilon_2(s) >> \varepsilon_1(s)$$

Solve perturbatively

$$M^2(s) = M_0^2(s) + \delta M^2(s)$$

$\varepsilon_2(s), \varepsilon_1(s)$ **: (kaon loop) Sum , Difference**

From **Ideal** To **Physical** Fields

$$\begin{pmatrix} \rho^0_{R1} \\ \omega_{R1} \\ \varphi_{R1} \end{pmatrix} = \begin{bmatrix} 1 & -\alpha & \beta \\ \alpha & 1 & \gamma \\ -\beta & -\gamma & 1 \end{bmatrix} (s) \begin{pmatrix} \rho^0 \\ \omega \\ \varphi \end{pmatrix}$$

$$R(s+i\varepsilon)\tilde{R}(s+i\varepsilon) = 1$$

Physical Fields

BKY renorm. Fields

At leading order in

$\varepsilon_1(s)$, $\varepsilon_2(s)$

$$\begin{bmatrix} \alpha(s) \\ \beta(s) \\ \gamma(s) \end{bmatrix} = \begin{bmatrix} \dfrac{\varepsilon_1(s)}{\left[m_\rho^{HK}\right]^2 + \Pi^\rho_{\pi\pi}(s) - \left[m_\omega^{HK}\right]^2} \\ \dfrac{\sqrt{2}\, z_V\, \varepsilon_1(s)}{\left[m_\rho^{HK}\right]^2 + \Pi^\rho_{\pi\pi}(s) - \left[m_\varphi^{HK}\right]^2} \\ \dfrac{\sqrt{2}\, z_V\, \varepsilon_2(s)}{\left[m_\omega^{HK}\right]^2 - \left[m_\varphi^{HK}\right]^2} \end{bmatrix}$$

Vector Couplings to π π :: I=1 in ω/ϕ

$$\frac{iag}{2}\rho_I\ \pi^- \overleftrightarrow{\partial} \pi^+ \Rightarrow$$

BKY

$$\frac{iag(1+\Sigma_V)}{2}\left[\rho^0 + \left[(1-h_V)\Delta_V - \alpha(s)\right]\omega + \beta(s)\,\varphi\right]\pi^- \overleftrightarrow{\partial} \pi^+$$

Vector Mixing→I=1

*ρ term unchanged : Charged ≡ neutral (I=1 part)

*s-dependent ω and ϕ couplings to ππ (I=1 part)

Vector couplings to γ : Γ (V→e^+e^-)

- **(γ) V transitions become s-dependent via IB:**

$$f_\rho^\gamma = agf_\pi^2 \left[1 + \Sigma_V + \frac{h_V \Delta_V}{3} + \frac{\alpha(s)}{3} + \frac{\sqrt{2}\, z_V}{3} \beta(s) \right]$$

$$f_\omega^\gamma = \frac{agf_\pi^2}{3} \left[1 + \Sigma_V + 3(1 - h_V)\Delta_V - 3\alpha(s) + \sqrt{2}\, z_V \, \gamma(s) \right]$$

$$f_\varphi^\gamma = \frac{agf_\pi^2}{3} \left[-\sqrt{2}\, z_V + 3\beta(s) + \gamma(s) \right]$$

Γ (V→e^+e^-)

s-dependent

ρ couplings to γ/W

- **(γ/W) V transitions become s-dependent :**

$$\frac{f_\rho^\gamma}{f_\rho^W} = \left[1 + \frac{h_V \Delta_V}{3} + \frac{\alpha(s)}{3} + \frac{\sqrt{2}\, z_V}{3}\beta(s)\right] \Leftarrow \left[f_\rho^W = a g f_\pi (1+\Sigma_V)\right]$$

Γ (ρ→e⁺e⁻)

IB

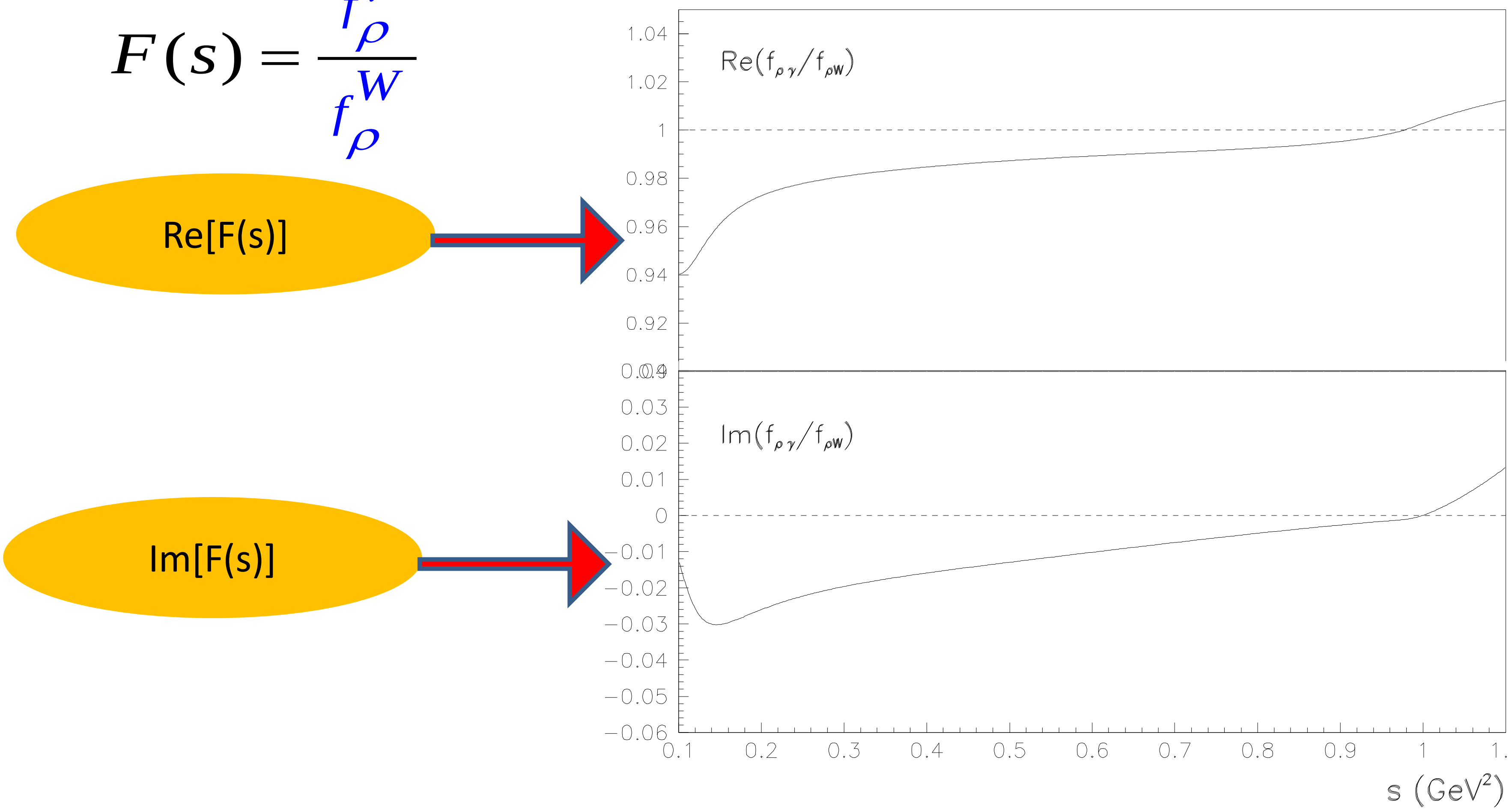
IB

→ $e^+ e^-$ & τ data : Full agreement

→ IB in PS sector →→ Γ (V→e⁺e⁻)

ρ couplings to γ/W

$$F(s) = \frac{f_\rho^\gamma}{f_\rho^W}$$



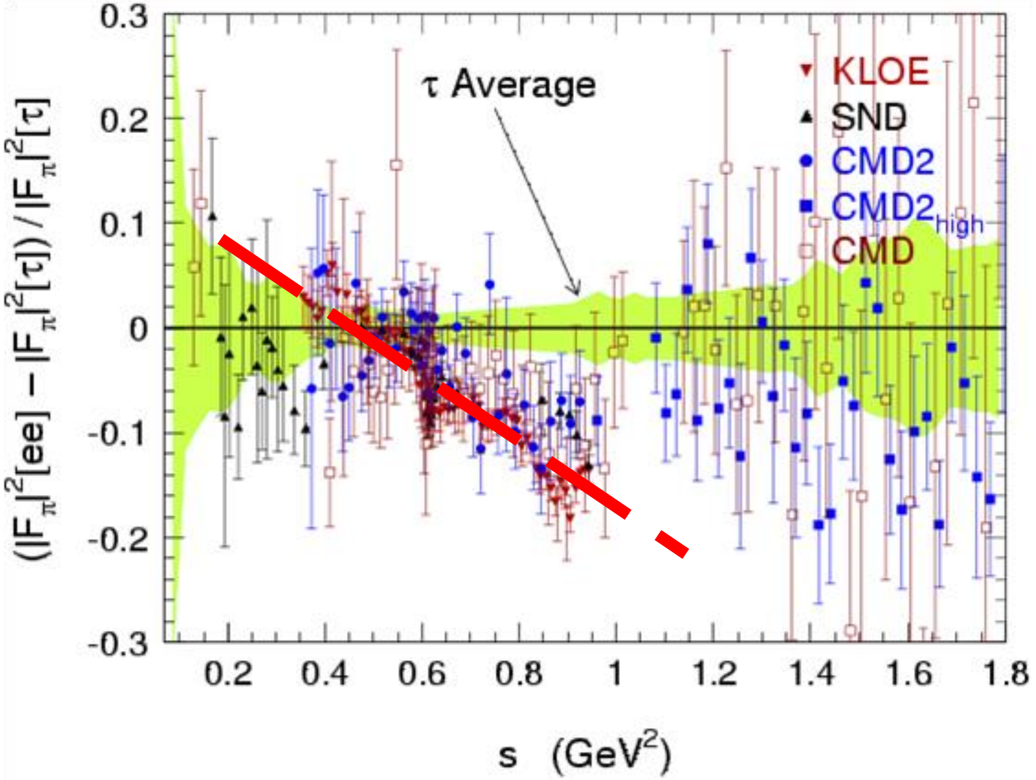
Dipion Mass Spectrum

•

M. Davier NP Proc. Supp. 169 (2007) 288

- **s-dependent (missing) effect !**

- **Motivated V mixing**



M.Benayoun *et al*. EPJ C55 (2008) 199

Isospin Breaking in the τ Sector

Short Range & Long Range IB corrections

W. Marciano & A. Sirlin, PRL 71 (1993) 3629.

$$H(s) \equiv B_{\pi\pi} \frac{1}{N}\frac{dN}{ds} = \frac{1}{\Gamma_\tau}\frac{d\Gamma_{\pi\pi}(s)}{ds} S_{EW} G_{EM}(s)$$

V. Cirigliano *et al.* PL B 513 (2001) 361
& JHEP 08 (2002) 002

- **$\pi^\pm\pi^0$ Phase space factor**

- **≈ No sensitivity to $\rho^\pm - \rho^0$ mass/width diff.**

→ **IB in τ & IB in e^+e^- :: unrelated**
but both involved in global fits

Broken HLS : A Global VMD Model

- The (Broken) HLS model (BHLS) is :
  - a unified VMD framework for :

$e^+e^- \to \pi^+\pi^- / K^+K^- / K_L K_S / \pi^0\gamma / \eta^0\gamma / \pi^+\pi^-\pi^0$

& $\tau^\pm \to \pi^\pm\pi^0\nu_\tau$ & PVγ, P$\gamma\gamma$ decays & $\eta/\eta' \to \gamma\pi^+\pi^-$

  - VALID UP TO THE ≈ Φ MASS REGION (1.05 GEV)
  - a nearly empty shell [$\alpha_{em}, G_F, f_\pi, V_{ud}, V_{us}$] fed via :
  - a Global Fit (≈ 50 data samples simultaneously)

$\pi^+ \pi^-$ Spectra : NSK, KLOE, BaBar, BES

- Several recent measurements of the $\pi^+ \pi^-$ FF :

i. CMD2, SND

CMD2: Phys. Lett. B648 (2007) 28, JETP Lett. 84 (2006) 413
SND: JETP 103 (2006) 380

ii. KLOE

KLOE08 : AIP Conf. Proc. 1182 (2009) 665 *
KLOE10: Phys. Lett. B700 (2011) 102
KLOE12: Phys. Lett. B720 (2013)336

iii. BaBar

BaBar : Phys. Rev. Lett. 103 (2009) 231801 *
Phys. Rev . D86 (2012) 032013

iv. BES III

BES III : M. Ablikim *et al* Phys. Lett. B753 (2016) 629

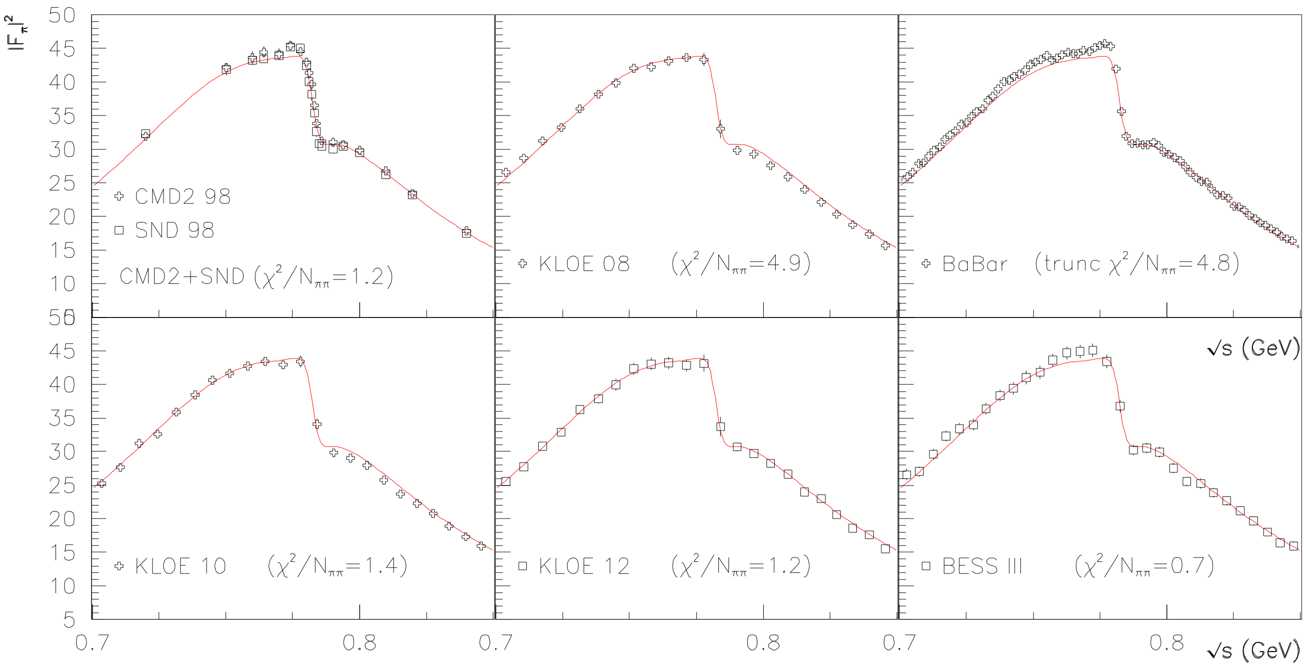
Conflicting behaviors within the BHLS framework

M. Benayoun *et al* EPJ C73 (2013)2453

M. Benayoun *et al* EPJ C75 (2015)613

A «Democratic» Criterium : τ+PDG

- **BHLS predicts $F_\pi(s)$ from τ spectrum +(5) PDG**
- **→ compare each** $F_\pi^{exp}(s)$ **to** $F_\pi^{\tau+PDG}(s)$



M. Benayoun *et al* EPJ C75 (2015)613

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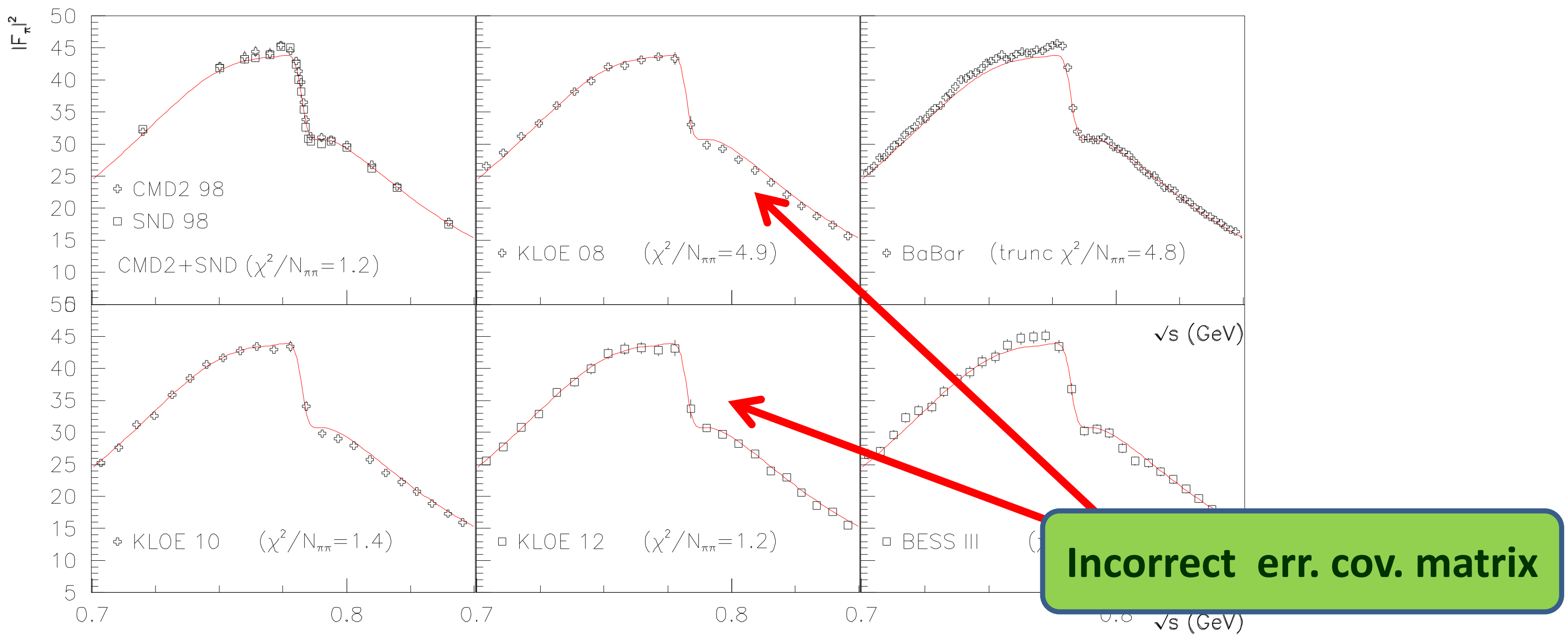

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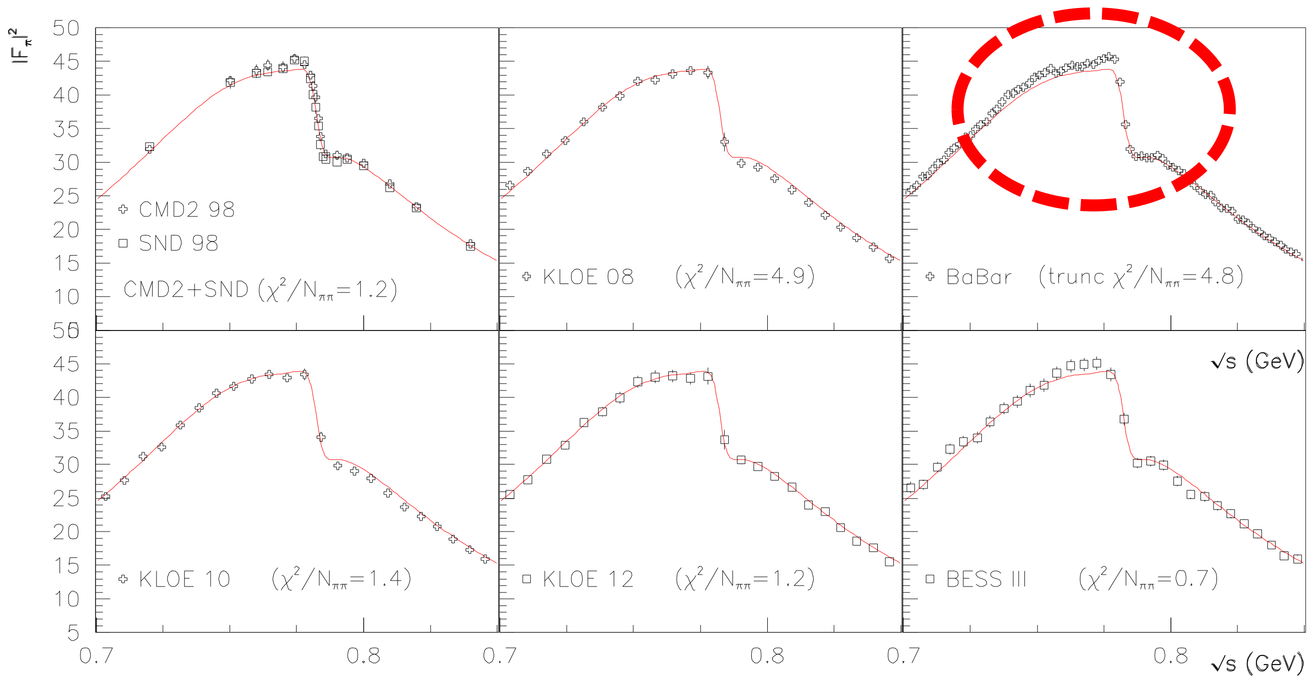

Fitting $\pi^+\pi^-$ Data using τ Samples

Fit Cond. ($\chi^2/N_{\pi+\pi-}$)	KLOE08 (60)	KLOE10 (75)	KLOE12 (60)	NSK (127/209)	BES III (60)	BaBar trunc (250)
Single ($\chi^2/N_{\pi+\pi-}$)	1.64 59 %	0.96 97%	1.02 97 %	0.96 [0.83] 97 %[99%]	0.56 99%	1.15 74%
Comb 1 χ^2/N : 1.28 (11%)		1.02	1.48	1.18[0.96]		1.35
Comb 2 χ^2/N: 1.21 (22%)		1.01	1.54	1.18[0.96]	0.56	1.36
Comb A χ^2/N: 1.06 (97%)		1.02	1.05	1.10[0.89]		
Comb B χ^2/N : 0.98 (99%)		1.00	1.05	1.11[0.90]	0.61	

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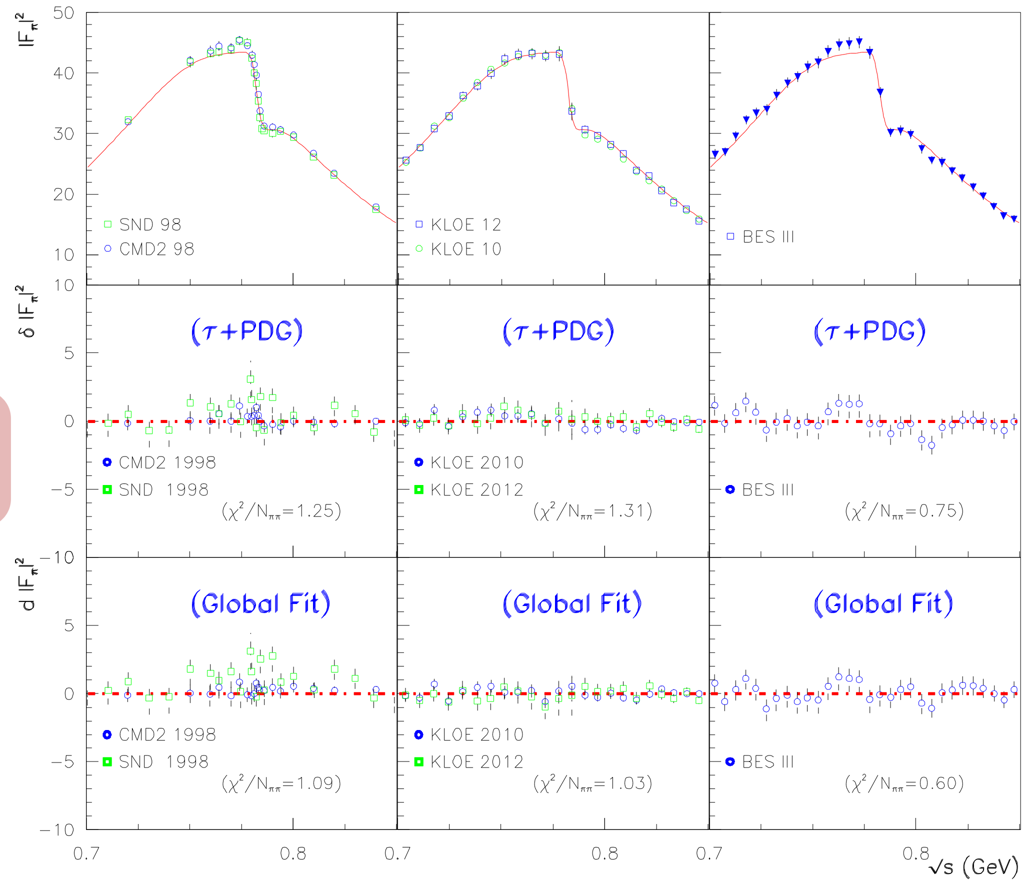
$\pi^+ \pi^-$ Spectra : A τ vs e^+e^- Puzzle?

Average χ2 distance to τ+PDG prediction & to Global Fit (conf A)

Residuals τ + PDG prediction

Residuals BHLS Global FIT



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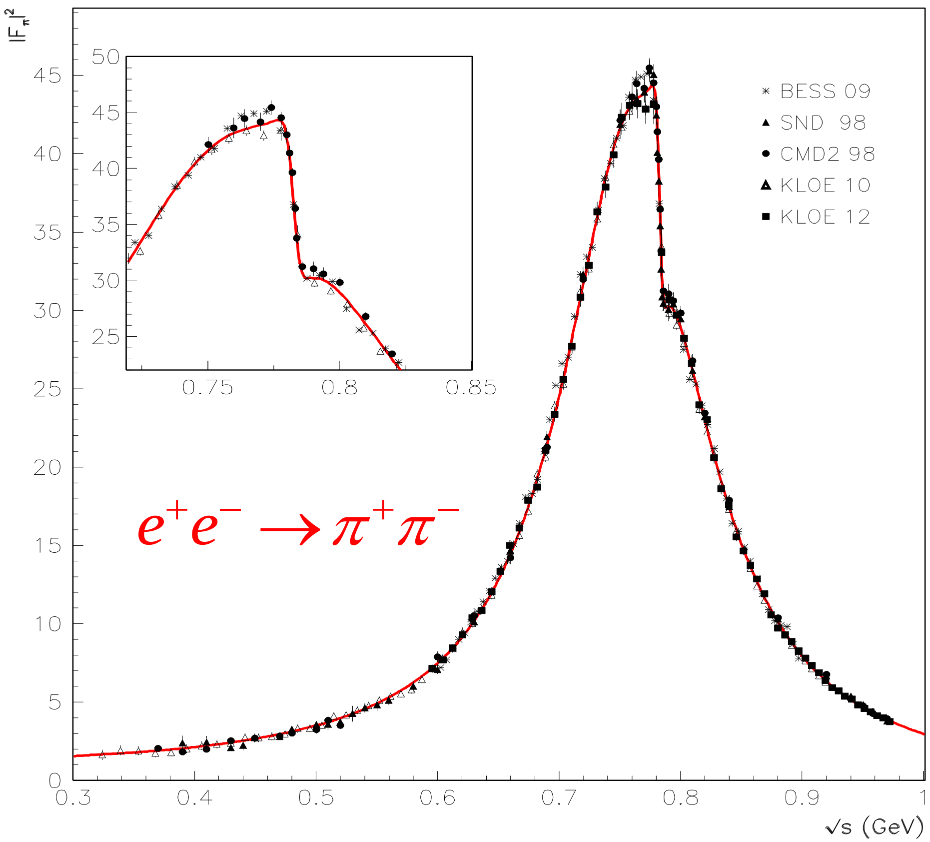
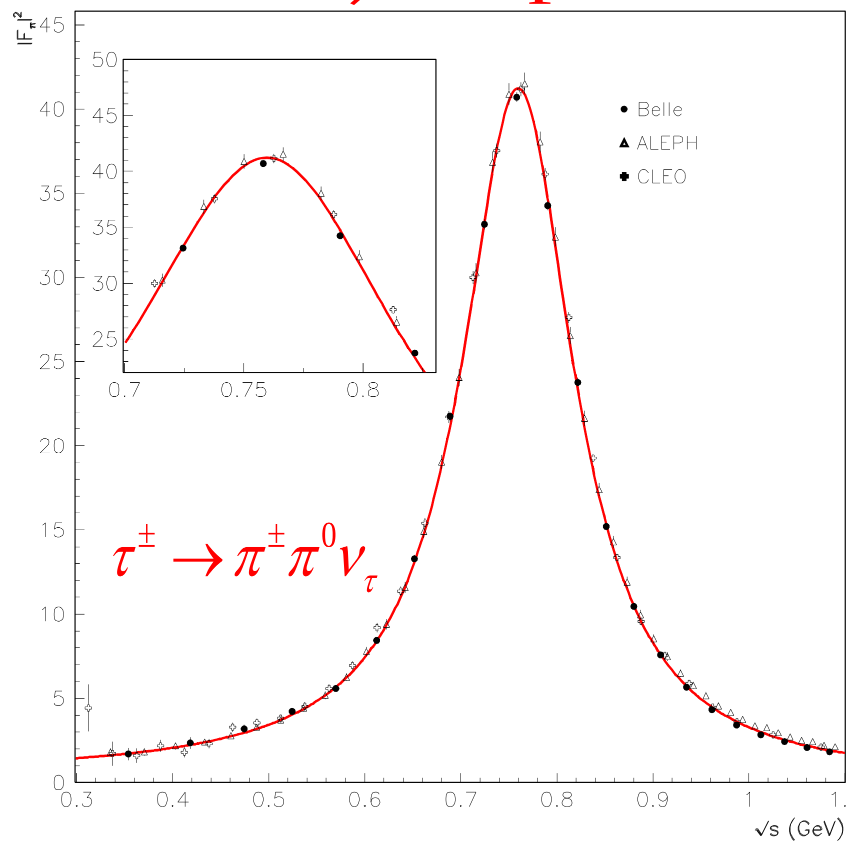
Dipion Spectra (τ & NSK+KLOE+BES)

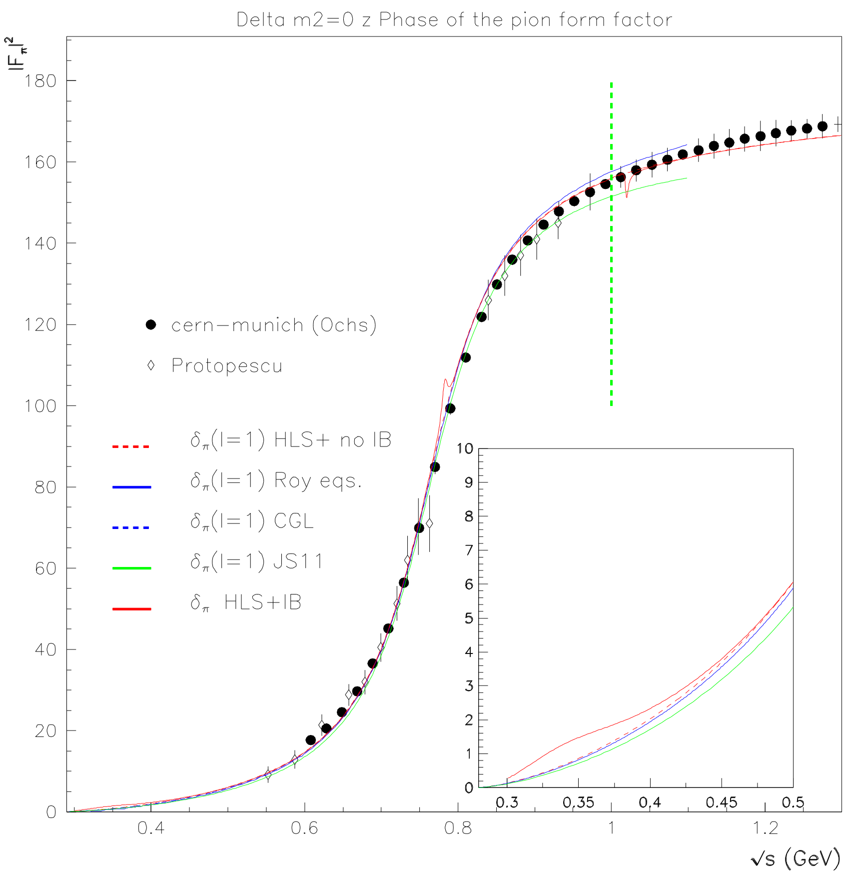
$\chi^2/N_p = 361.5/404$ $(Prob \approx 99\%)$ $\chi^2/N_p = 86.8/85$

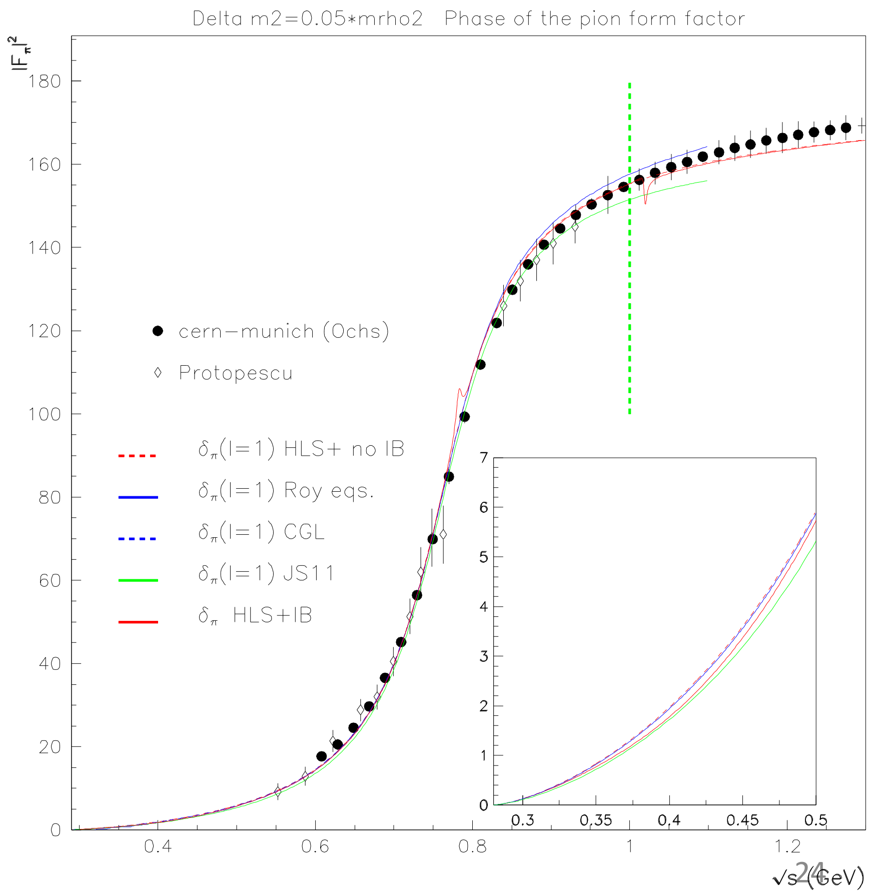




A Test : The ππ phase shift

HLS Predictions superimposed





g-2 & Global Models/Fits

- **NP Hadronic VP contributions to g-2**

$$a_\mu(H_i) \approx \int_{s_{th}}^{s_{cut}} ds\, K(s)\, \sigma(e^+e^- \to H_i, s)$$ ← **WA Measured** Xsection

- **VMD : Underlying physics correlations absorbed -> HLS cross-sections derived through a global fit (param. values & error covariance matrix) :**

Measured Xsection → **Model Xsection (ϕ)**

- **→ Complement for √s ≥ 1.05 GeV with WA Xsection**

HVP contributions to a_μ (E≤ 1.05 GeV)

- new samples : SND($\pi^0\gamma$) & CMD3($K_L K_S$) || $[\pi\pi\pi]_\phi$
- σ improved by x 4 (for √s≤1.05 GeV) !

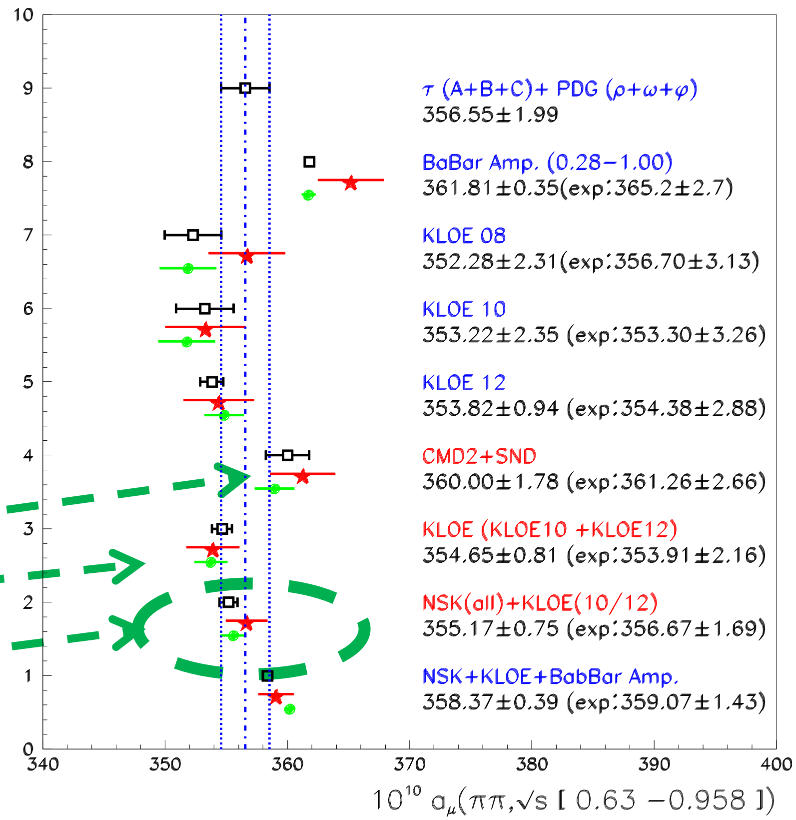
Channel	$A=M_0$ (incl. τ)	Direct Estimate
$\pi^+\pi^-$	493.46 ± 0.69	(492.98 ± 3. 38)
$\pi^0\gamma$	4.42 ± 0.02	3.67 ± 0.11
$\eta\gamma$	0.64 ± 0.004	0.56 ± 0.02
$\pi^+\pi^-\pi^0$	40.86 ± 0.51	43.54 ± 1.29
$K_L K_S$	11.67 ± 0.07	12.21 ± 0.33
K^+K^-	17.14 ± 0.16	17.72 ± 0.52
Total < 1.05 GeV	568.20 ± 0.89	(570.68 ± 3.67)

$a_\mu (\pi^+\pi^- + \tau, \sqrt{s}=[0.630,0.958]$ GeV)

Data and BHLS estimates

- **Green** : $A = \sigma_{exp}$ (it=0)
- **Black** : $A = \sigma_{iter}$ (it=1)
- **Red** : Exp. Values

It=0 ≈ It=1



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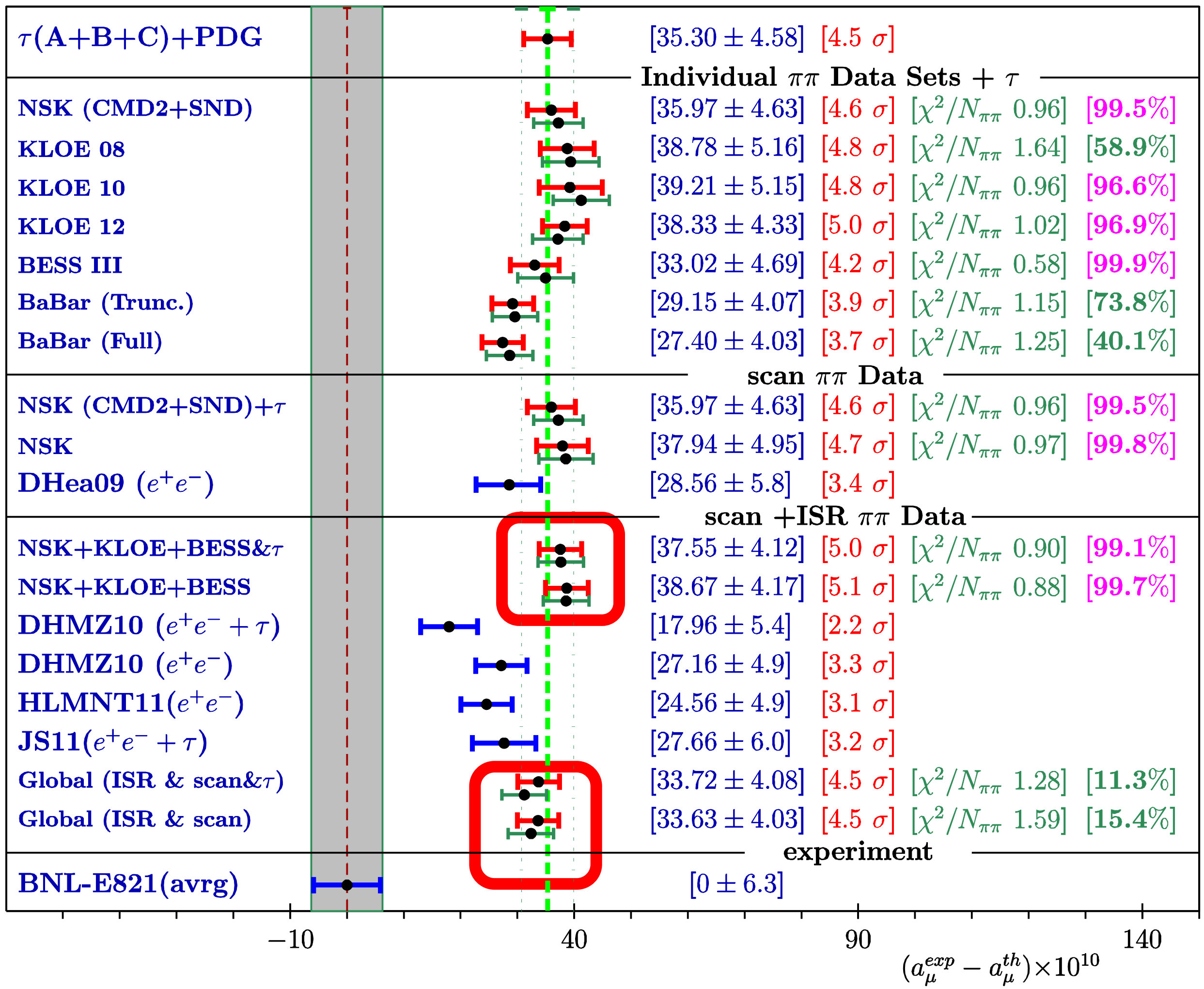
g-2 Estimates & Discrepancy



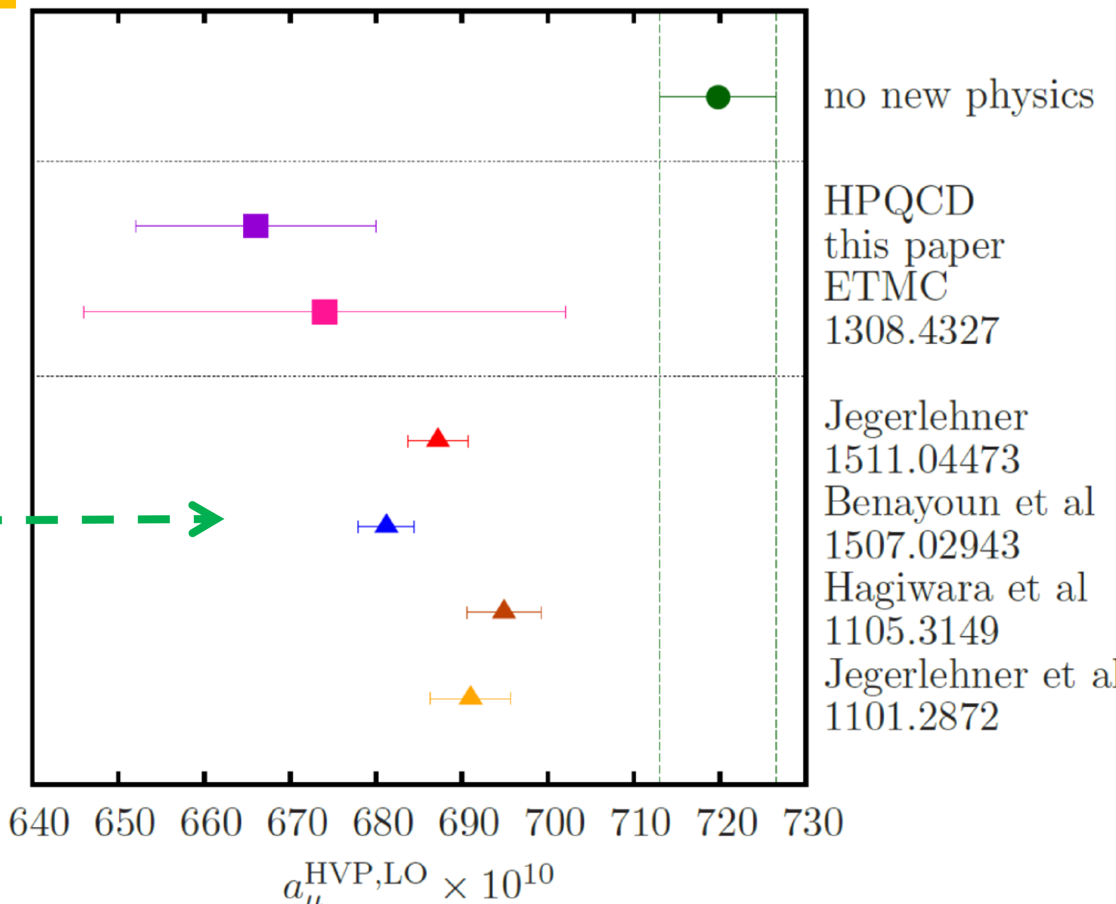
Muon HVP : Phenomenology vs LQCD

B. Chakraborty ArXiv:1601.03071

(HPQCD group)

LQCD

BHLS



HVP Evaluations: DR vs MBM

Dispersion Relation on R(s)

$$s_0 = m^2_{\pi^0} \left(or \; 4m^2_\pi \right)$$

F. Jegerlehner & A.Nyffeler, ArXiv:0902.3360

$$a_\mu^{had} = \left[\frac{\alpha m_\mu}{3\pi} \right]^2 \int_{s_0}^{\infty} \frac{ds}{s^2} \hat{K}(s) R_{hadr}(s)$$

Mellin Transform of R(s)

E de Rafael Phys. Lett B736(2014) 522

$$a_\mu^{had} = \left[\frac{\alpha}{\pi} \right] \frac{1}{2i\pi} \int_{c-i\infty}^{c+i\infty} ds \, \hat{F}(s) M(s)$$

$$\hat{F}(s) = -\Gamma(3-2s)\Gamma(-3+s)\Gamma(1+s)$$

HVP Evaluations: DR vs MBM

Mellin Transform of R(s)

E de Rafael Phys. Lett B736(2014) 522

$$a_\mu^{had} = \left[\frac{\alpha}{\pi}\right] \frac{1}{2i\pi} \int_{c-i\infty}^{c+i\infty} ds\, \hat{F}(s) M(s)$$

$$s_0 = m_{\pi^0}^2 \left(or\ 4m_\pi^2 \right)$$

$$M(u) = \frac{\alpha}{3\pi} \int_{s_0}^{\infty} \frac{dt}{t} \left(\frac{m_\mu^2}{t} \right)^{(1-u)} R_{hadr}(t) \qquad (>0)$$

$$\tilde{M}(u) = \frac{\alpha}{3\pi} \int_{s_0}^{\infty} \frac{dt}{t} \left(\frac{m_\mu^2}{t} \right)^{(1-u)} \ln\left(\frac{m_\mu^2}{t} \right) R_{hadr}(t) = -\frac{dM(u)}{du} \qquad (<0)$$

Muon HVP & Mellin Moments

HVP :: fastly convergent series of Mellin Moments

E de Rafael Phys. Lett B736(2014) 522

$$\rightarrow a_{\mu}^{had}(0) = \left[\frac{\alpha}{\pi}\right]\left[\frac{1}{3}M(0)\right]$$

$$\rightarrow a_{\mu}^{had}(1) = a_{\mu}^{had}(0) + \left[\frac{\alpha}{\pi}\right]\left[\frac{25}{12}M(-1) + M(-1)\right]$$

$$\rightarrow a_{\mu}^{had}(2) = a_{\mu}^{had}(1) + \left[\frac{\alpha}{\pi}\right]\left[\frac{97}{10}M(-2) + 6M(-2)\right]$$

....

Mellin Moments from Data & HLS

Moment	Evaluation from Data only	Evaluation from HLS (&Data)
M(0)	10.307 ± 0.0745	10.1073 ± 0.0575
M(-1)	(2.3507 ± 0.0185) 10^{-1}	(2.3760 ± 0.0038) 10^{-1}
M(-2)	(0.8702 ± 0.0115) 10^{-2}	(0.9007 ± 0.0011) 10^{-2}
M(-3)	(0.4852 ± 0.0093) 10^{-3}	(0.5149 ± 0.00064) 10^{-3}
M(-4)	(0.3676 ± 0.0083) 10^{-4}	(0.3956 ± 0.0005) 10^{-4}
~M(-1)	-(8.2592 ± 0.0516) 10^{-1}	-(8.3035 ± 0.0168) 10^{-1}
~M(-2)	-(2.6808 ± 0.0294) 10^{-2}	-(2.7503 ± 0.0035) 10^{-2}
~M(-3)	-(1.3160 ± 0.0228) 10^{-3}	-(1.3858 ± 0.0017) 10^{-3}
~M(-4)	-(0.9064 ± 0.0199) 10^{-4}	-(0.9726 ± 0.0012) 10^{-4}

M. Benayoun *et al. ArXiv:1605.04474*

n≥2 : Dominance of {√s < 1.05 GeV}

Moment	Evaluation HLS (<1.05 GeV)	Evaluation HLS & Data (all E)
M(0)	8.6041 ± 0.0130	10.1073 ± 0.0575
M(-1)	(2.3197 ± 0.0031) 10^{-1}	(2.3760 ± 0.0038) 10^{-1}
M(-2)	(0.8974 ± 0.0011) 10^{-2}	(0.9007 ± 0.0011) 10^{-2}
M(-3)	(0.5147 ± 0.00064) 10^{-3}	(0.5149 ± 0.00064) 10^{-3}
M(-4)	(0.3956 ± 0.0005) 10^{-4}	(0.3956 ± 0.0005) 10^{-4}
~M(-1)	-(8.0054 ± 0.0113) 10^{-1}	-(8.3035 ± 0.0168) 10^{-1}
~M(-2)	-(2.7338 ± 0.0035) 10^{-2}	-(2.7503 ± 0.0035) 10^{-2}
~M(-3)	-(1.3847 ± 0.0017) 10^{-3}	-(1.3858 ± 0.0017) 10^{-3}
~M(-4)	-(0.9725 ± 0.0012) 10^{-4}	-(0.9726 ± 0.0012) 10^{-4}

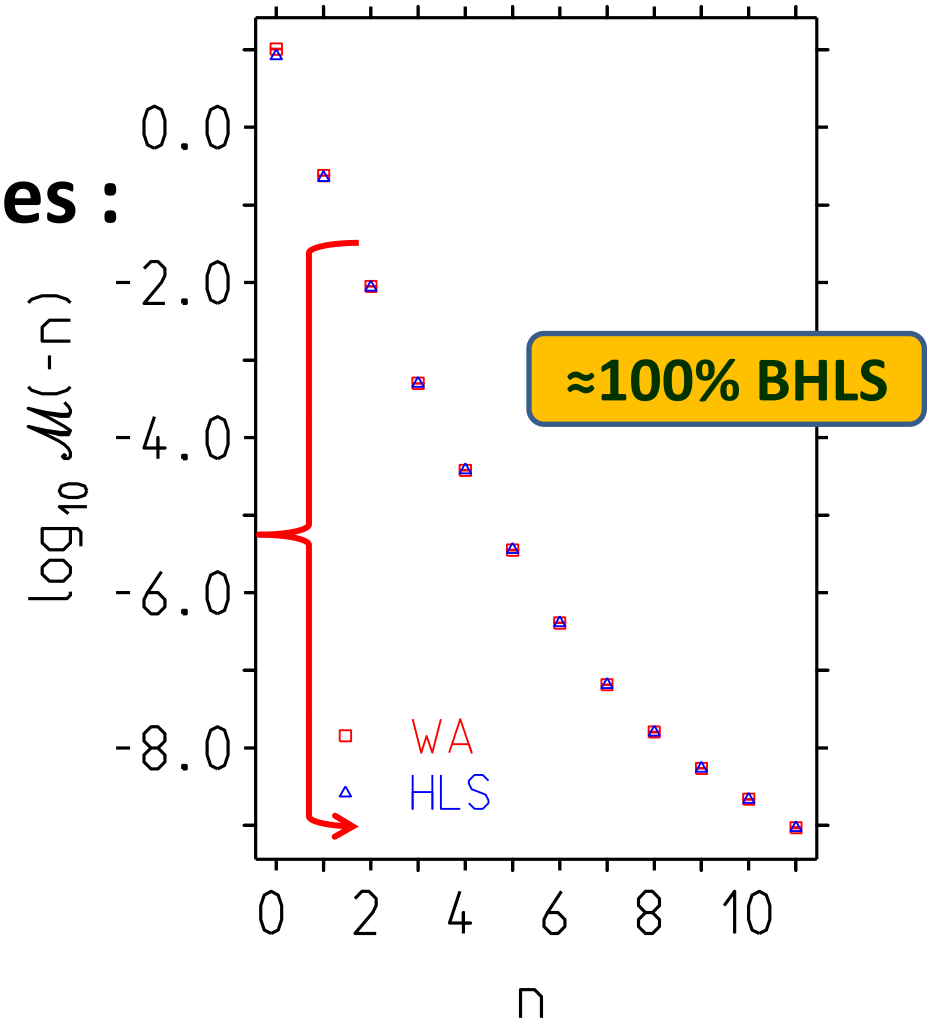
HLS Dominated

Mellin Moments : BHLS vs Data

- **Log M(-n) vs n :**

{√s>1.05 GeV} contributes :

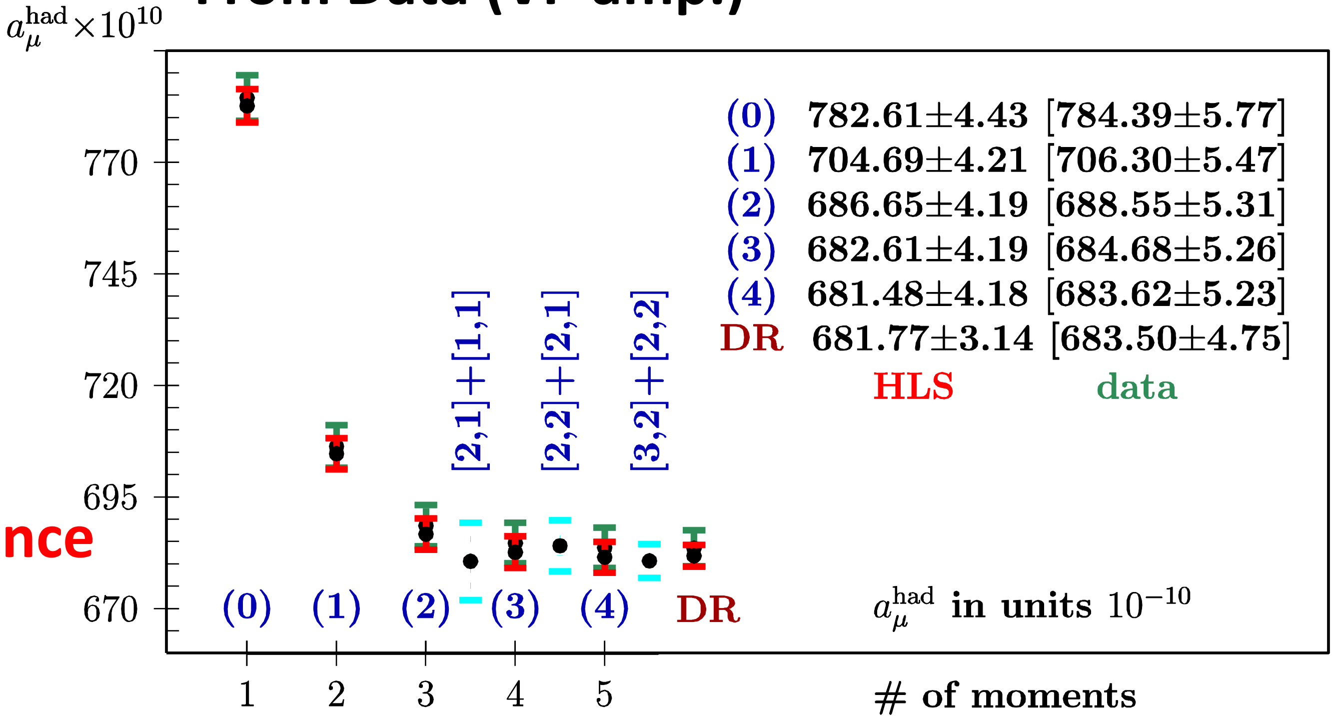
- ➢ **M(0) : 15 % (σ :: 75%)**
- ➢ **M(-1) : 2.4%**
- ➢ **M(-n), n≥2 : ≈0%**



Muon HVP from Mellin Moments

Estimates from different methods :

- ✓ **Disp. Rel.** — **From Data alone / BHLS**
- ✓ **Mellin-Barnes Mom.** — **From Data alone/ BHLS** (up to 4th order)
- ✓ **Padé App.** — **From Data (VP amp.)**



✓ **Good convergence**

Effects of the tilde Moments on a_μ^{had}

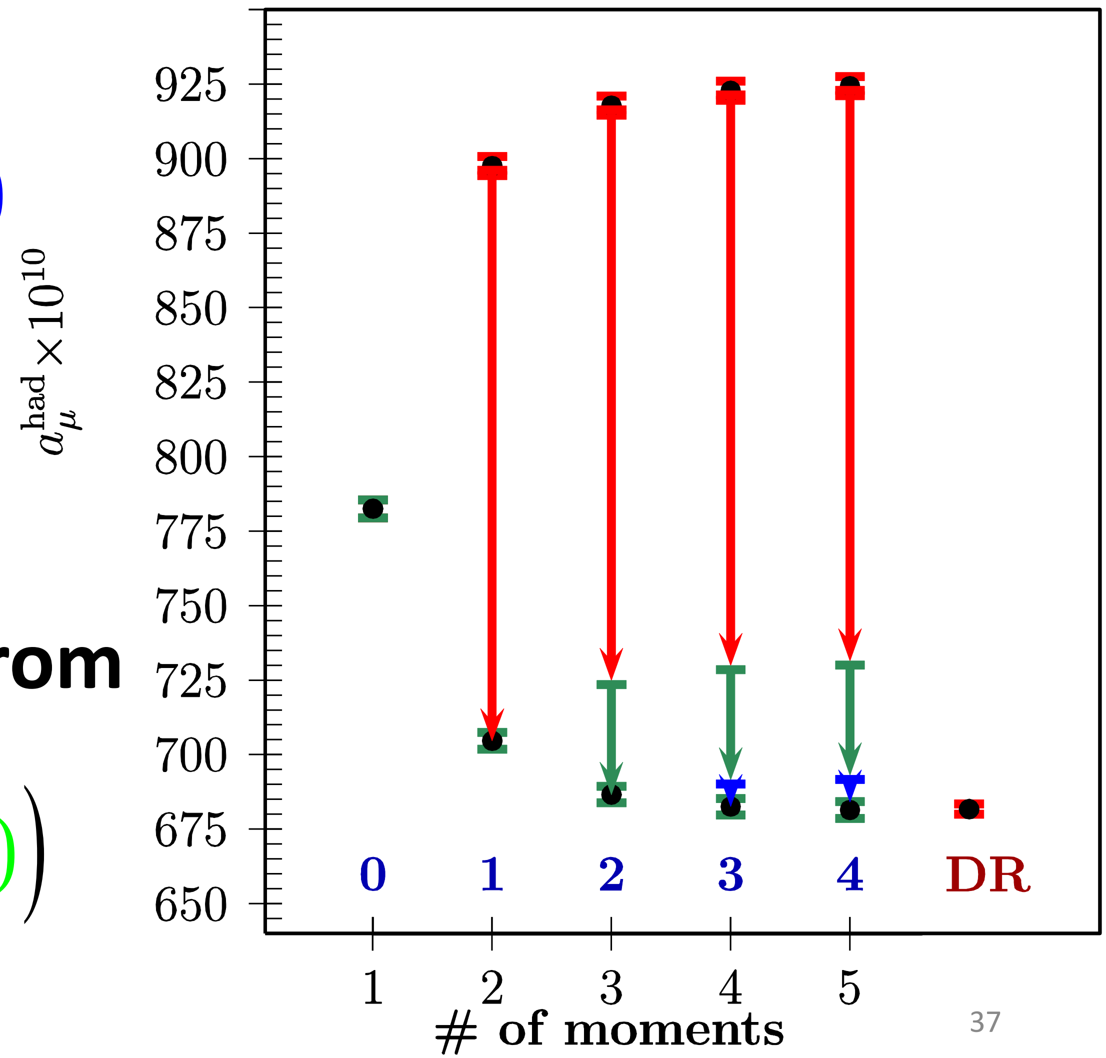
- **Effects of**

$\tilde{M}(-1)\,,\tilde{M}(-2)\,,\tilde{M}(-3)$

($\tilde{M}(-4)$ not shown)

- **Dramatic effects from**

$\tilde{M}(-1)\left(\&\ \tilde{M}(-2)\right)$



Remarks on Mellin Moments I

- Dispersion Integral limits : $(m_{\pi 0})^2 \rightarrow \infty$
- BHLS integral limits : $(m_{\pi 0})^2 \rightarrow (1.05)^2$ GeV^2

HOWEVER

- Moments $M(-n) \,\&\, \tilde{M}(-n) \quad (n \geq 2)$:
  almost 100% dominated by BHLS (m,σ)

- Error on $a_\mu^{had}(n)$ driven by error on $M(0)$:
  almost 80% coming from $s \in \left[(1.05)^2 GeV \rightarrow \infty \right]$

Remarks on Mellin Moments II

- **Numerical analysis of $a_\mu^{had}(n)$ implies that :**

R(s ≥ $(1.05)^2$ GeV^2) essentially goes into $M(0)$

R(s ≥ $(1.05)^2$ GeV^2) ≈ 100% in $M(0)\,, M(-1)\,\&\,\tilde{M}(-1)$

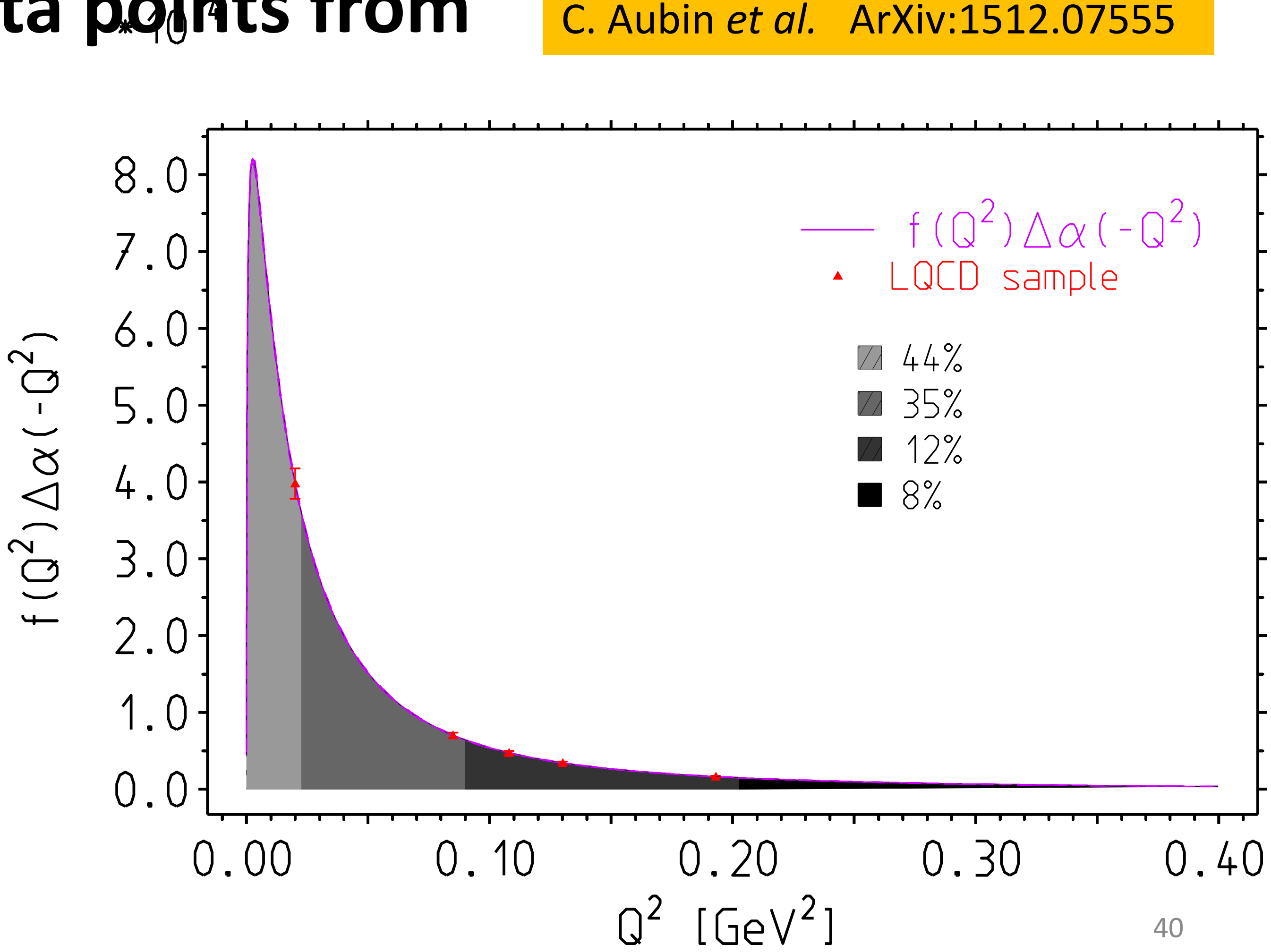
Reliable predictions for $M(0)\,, M(-1)\,\&\,\tilde{M}(-1)$ may replace $R_{hadr}(\sqrt{s} > m_\phi)$

LQCD Predictions and Data

- **4 LQCD data points from**

C. Aubin *et al.* ArXiv:1512.07555

Zon limits (GeV)
0.00<Q<0.15
0.15<Q<0.30
0.30<Q<0.45
0.45<Q<1.00



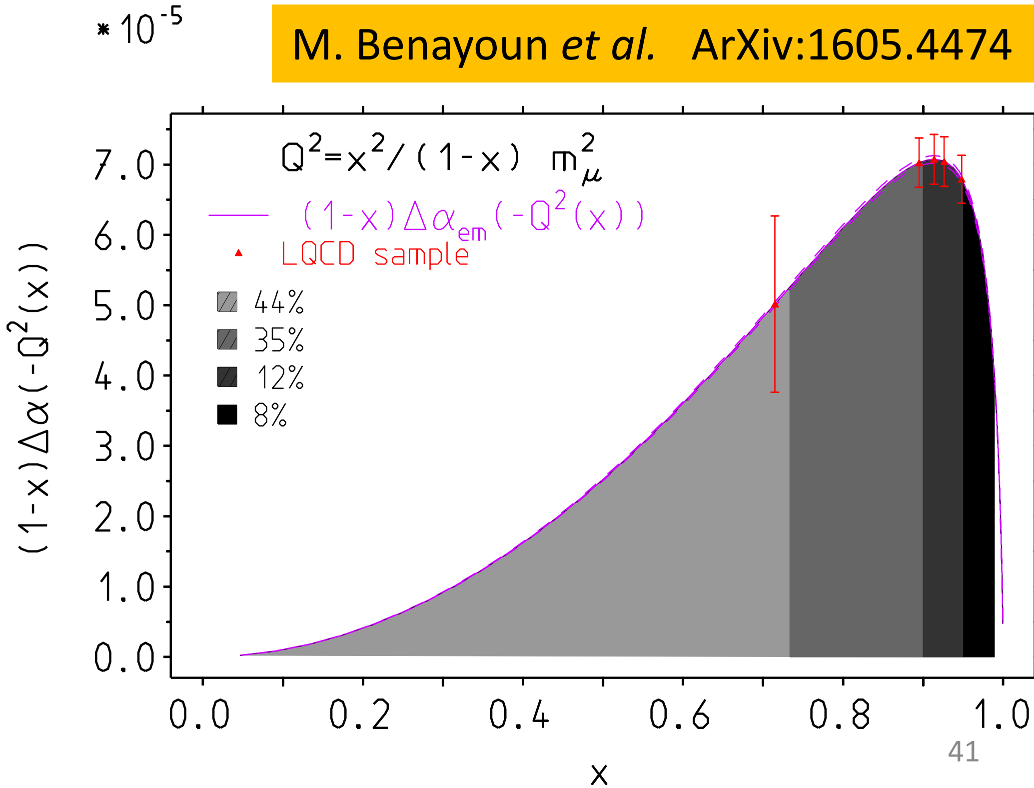
LQCD Predictions and Data

- ***x representation :***

→ ***crucial role of extrapolation for*** $a_{\mu}^{had}(n)$

Zon limits (GeV)
0.00<Q<0.15
0.15<Q<0.30
0.30<Q<0.45
0.45<Q<1.00

M. Benayoun *et al.* ArXiv:1605.4474



Going the closest to the LQCD scope?

- Data contain I=0, I=1 components & IB & γ:
- Test of consistency {(Data/BHLS) vs LQCD} easier :

➢ Single out I=1 component in data

➢ → Isospin Breaking Effects

➢ → I=0 component

➢ → Photon coupling to hadrons

Not easy with data → fit functions

Extract I=1 from Data via BHLS

- The BHLS model :: ≈ perfect fit to data

Extraction method from phenomenology :

- Cancel out I=0, IB (& γ?) parameters in fit functions

- I=0 ? :: cancel out ω(782) & ϕ(1020) terms

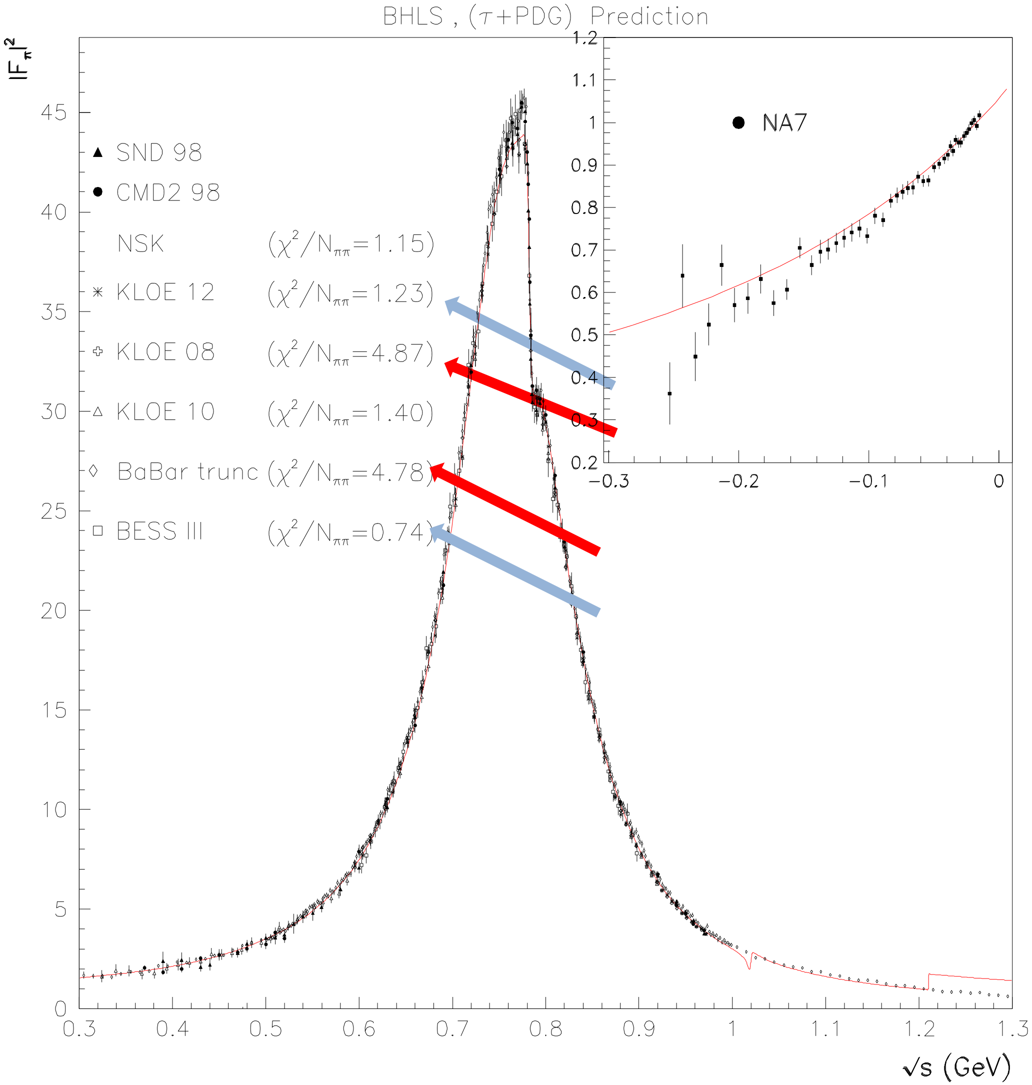
- IB identified via «τ +PDG» method : switched off

The « τ + PDG » Method

- Perform a global fit **within BHLS** with:
  - $\tau^{\pm} \rightarrow \pi^{\pm}\pi^{0}\nu_{\tau}$ (Aleph, CLEO, Belle)
  - $e^+e^- \rightarrow K^+K^-/K_LK_S/\pi^0\gamma/\eta\gamma/\pi^+\pi^-\pi^0$ (scan data from NSK)
  - $e^+e^- \rightarrow \pi^+\pi^-$ samples : **ALL DISCARDED**
- **&** Isospin Breaking information : **from RPP**
  - $\Gamma(\rho \rightarrow e^+e^-)$
  - $\Gamma(\omega \rightarrow \pi^+\pi^-)$ & ORSAY phase (~104°)
  - $\Gamma(\phi \rightarrow \pi^+\pi^-)$ & $\Gamma(\phi \rightarrow \pi^+\pi^-) \times \Gamma(\phi \rightarrow e^+e^-)$
- → **Derive** $F_{\pi}^{\tau}(s) \rightarrow F_{\pi}^{ee}(s)$

M. Benayoun *et al.* EPJ C73 (2013) 2453

$\pi^+ \pi^-$ Spectra : τ + PDG Predictions



τ + PDG predictions
In spacelike & timelike regions

π π samples NOT FITTED
Average χ2 distance to
τ + PDG prediction displayed
(conf. B)

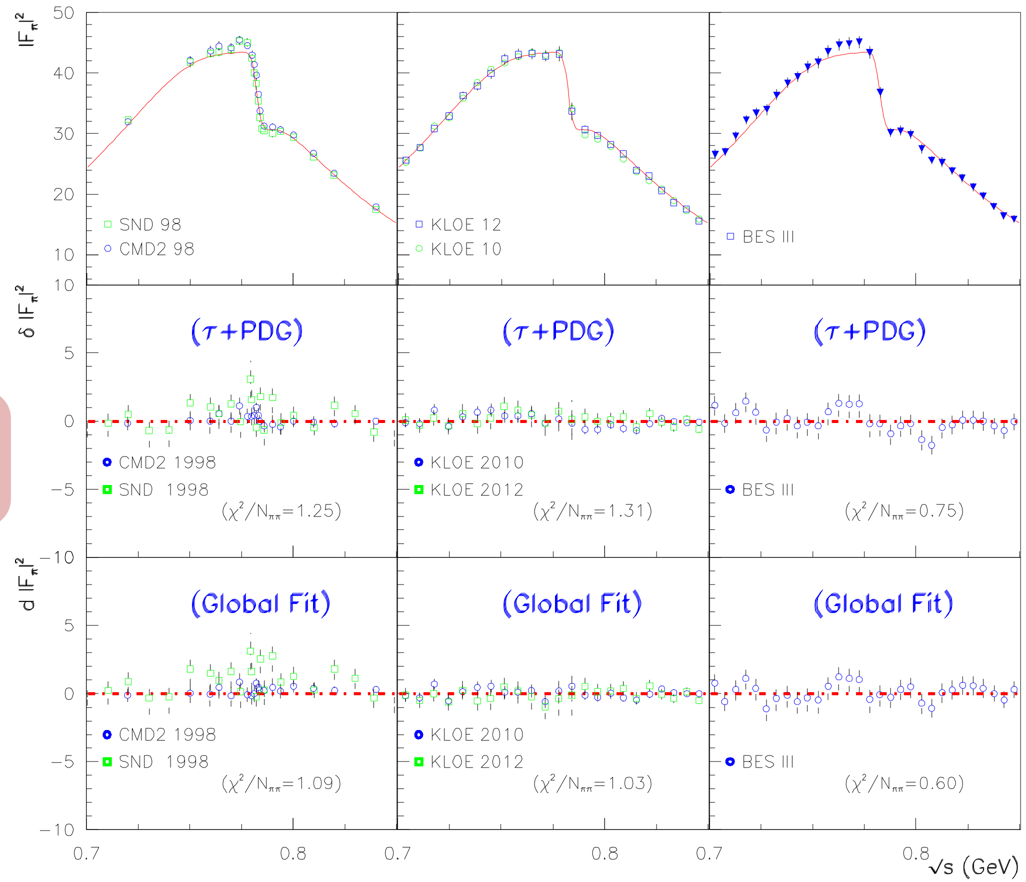
M. Benayoun *et al* EPJ C73 (2013)2453

$\pi^+ \pi^-$ Spectra : τ Prediction & FIT

Average χ2 distance to τ+PDG prediction & to Global Fit (conf A)

Residuals τ + PDG prediction

Residuals BHLS Global FIT



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τ Form Factor as I=1 Reference

$$\frac{iag}{2}\rho_I\, \pi^- \overleftrightarrow{\partial} \pi^+ \Longrightarrow$$

Loop($K^+ K^-$) – Loop($K_L K_S$)

$$\frac{iag(1+\Sigma_V)}{2}\left[\rho^0 + \left[(1-h_V)\Delta_V - \alpha(s)\right]\omega + \beta(s)\,\varphi\right]\pi^- \overleftrightarrow{\partial} \pi^+$$

$$\frac{f_\rho^\gamma}{f_\rho^W} = \left[1 + \frac{h_V \Delta_V}{3} + \frac{\alpha(s)}{3} + \frac{\sqrt{2}\, z_V}{3}\beta(s)\right] \Longleftarrow \left[f_\rho^W = ag\, f_\pi (1+\Sigma_V)\right]$$

IB

IB

Vector meson couplings to γ

- **(γ) V transitions become s-dependent via IB:**

$$f_\rho^\gamma = a g f_\pi^2 \left[1 + \Sigma_V + \frac{h_V \Delta_V}{3} + \frac{\alpha(s)}{3} + \frac{\sqrt{2}\, z_V}{3} \beta(s) \right]$$

$$f_\omega^\gamma = \frac{a g f_\pi^2}{3} \left[1 + \Sigma_V + 3(1 - h_V)\Delta_V - 3\alpha(s) + \sqrt{2}\, z_V\, \gamma(s) \right]$$

$$f_\varphi^\gamma = \frac{a g f_\pi^2}{3} \left[-\sqrt{2}\, z_V + 3\beta(s) + \gamma(s) \right]$$

ω-ϕ mixing

Vector meson couplings to γ

- **(γ) V transitions become s-dependent via IB:**

$$f_\rho^\gamma = agf_\pi^2\left[1+\Sigma_V+\frac{h_V\Delta_V}{3}+\frac{\alpha(s)}{3}+\frac{\sqrt{2}\,z_V}{3}\beta(s)\right]$$

$$f_\omega^\gamma = \frac{agf_\pi^2}{3}\left[1+\Sigma_V+3(1-h_V)\Delta_V-3\alpha(s)+\sqrt{2}\,z_V\,\gamma(s)\right]$$

$$f_\varphi^\gamma = \frac{agf_\pi^2}{3}\left[-\sqrt{2}\,z_V+3\beta(s)+\gamma(s)\right]$$

The γ issue in Effective Lagrangians

- γ coupling to hadrons :: Lagrangian dependent
- γ coupling to π π :

- HLS Model : $g_{\gamma\pi\pi} = (1 - a/2)\, e$ (a ≈ 2.5)
- Gounaris–Sakurai : $g_{\gamma\pi\pi} = 0$ (a ≡ 2.0)
- scalarQED : $g_{\gamma\pi\pi} = e$

As all give a good description of $F_\pi(s)$

- switching off γ : highly model-dependent

a_μ^{hadr} components from BHLS

10^{10} *	Exp Data ($\sqrt{s} \leq 1.05$ GeV)	HLS Standard Fit	HLS Fit (no IB) γ & I=0 & I=1	HLS Fit (no IB) γ & I=1
$a_\mu^{hadr}(0)$	668.00 ± 3.83	666.22 ± 1.01	631.95 ± 0.95	552.15 ± 0.75
$a_\mu^{hadr}(1)$	594.11± 3.56	592.50 ± 0.90	562.08 ± 0.84	489.66 ± 0.67
$a_\mu^{hadr}(2)$	576.51± 3.41	574.64 ± 0.88	545.08 ± 0.83	473.49 ± 0.65
$a_\mu^{hadr}(3)$	572.65 ± 3.35	570.58 ± 0.88	541.25 ± 0.82	469.70 ± 0.64
$a_\mu^{hadr}(4)$	571.59 ± 3.33	569.41 ± 0.87	540.16 ± 0.82	468.62 ± 0.64
$a_\mu^{hadr}(\infty)$	570.68 ± 3.67	568.20 ± 0.89	539.56 ± 0.81	468.03 ± 0.65

BHLS dispatching of a_μ^{hadr} (I=1)

HLS Fit (no IB)	γ & I=1 ALL channels	γ & I=1 ππ only	γ & I=1 ππ + (π/η) γ	I=1 & NO γ ππ only
$a_\mu^{hadr}(0)$	552.15 ± 0.75	551.81 ± 0.75	552.14 ± 0.75	562.45 ±0.78
$a_\mu^{hadr}(1)$	489.66 ± 0.67	489.37 ± 0.67	489.65 ± 0.67	498.55 ± 0.68
$a_\mu^{hadr}(2)$	473.49 ± 0.65	473.20 ± 0.64	473.48 ± 0.65	481.60 ± 0.67
$a_\mu^{hadr}(3)$	469.70 ± 0.64	469.42 ± 0.64	469.70 ± 0.63	477.51 ± 0.66
$a_\mu^{hadr}(4)$	468.62 ± 0.64	468.34 ± 0.64	468.61 ± 0.64	476.35 ± 0.66
$a_\mu^{hadr}(\infty)$	468.03 ± 0.65	467.75 ± 0.64	468.02 ± 0.64	475.70 ± 0.66

Comments about I=1 & a_μ^{hadr} (I=1)

- Evaluating the various effects (units 10^{-10}) :

✓ Isospin Breaking :: ≈30/568 (≈ 5.2 %)

✓ I=0 (ω & ϕ) :: ≈ 72/568 (≈ 12.6 %)

✓ I=1 & γ :: ≈ 468/568 (≈ 82.2 %)

- I=1 & γ :: ≈ 100% ππ

[(π/η) γ → 0.2 10^{-10}, (K^+K^-/K_LK_S/πππ → ≈ 0.01 10^{-10}]

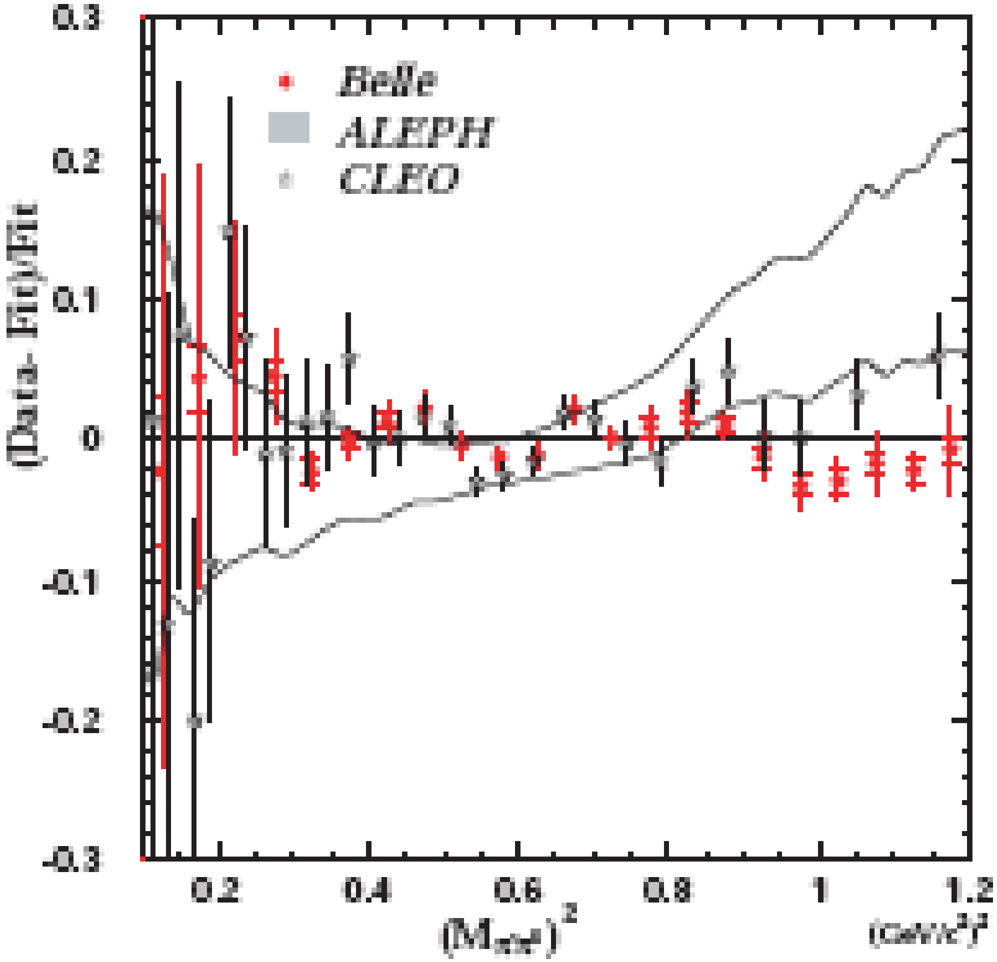
- Within BHLS :: γ contributes ≈ -7 10^{-10} !

CONCLUSIONS

- Effective Lagrangians (BHLS) :: Global Fits
- optimum to absorb physics correlations
- merge e+e- data & decays (incl. τ/η’) consistently
  - → increase (effective) statistics vs e+e- data only
- detect doubtful samples by consistency checks
- Moment information : cross checks with LQCD?
- specific LQCD predict. vs measurements/models ::
- removing IB, I=0 from data ≈ a priori possible
- however, an issue with γ

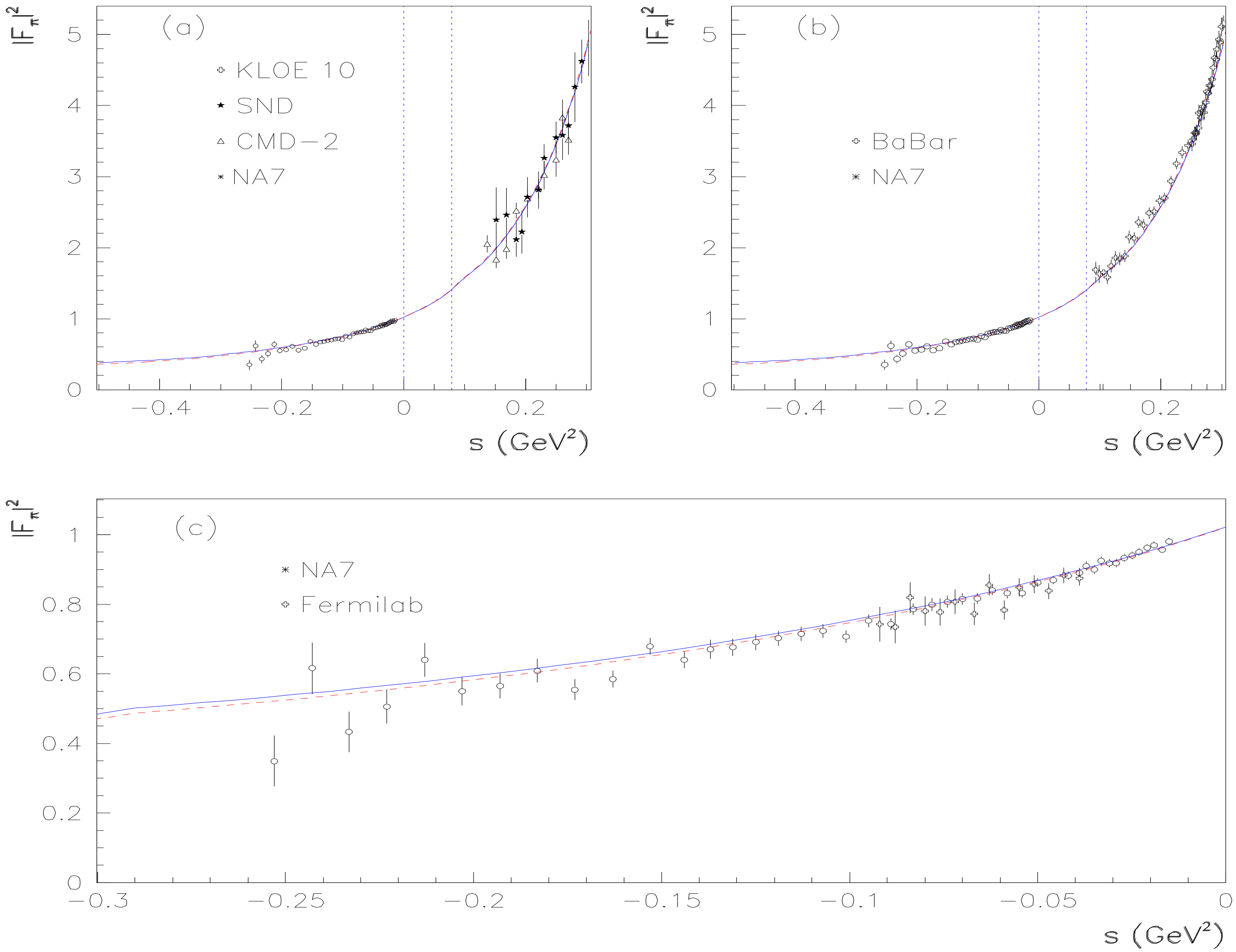
Backup Slides

From Belle



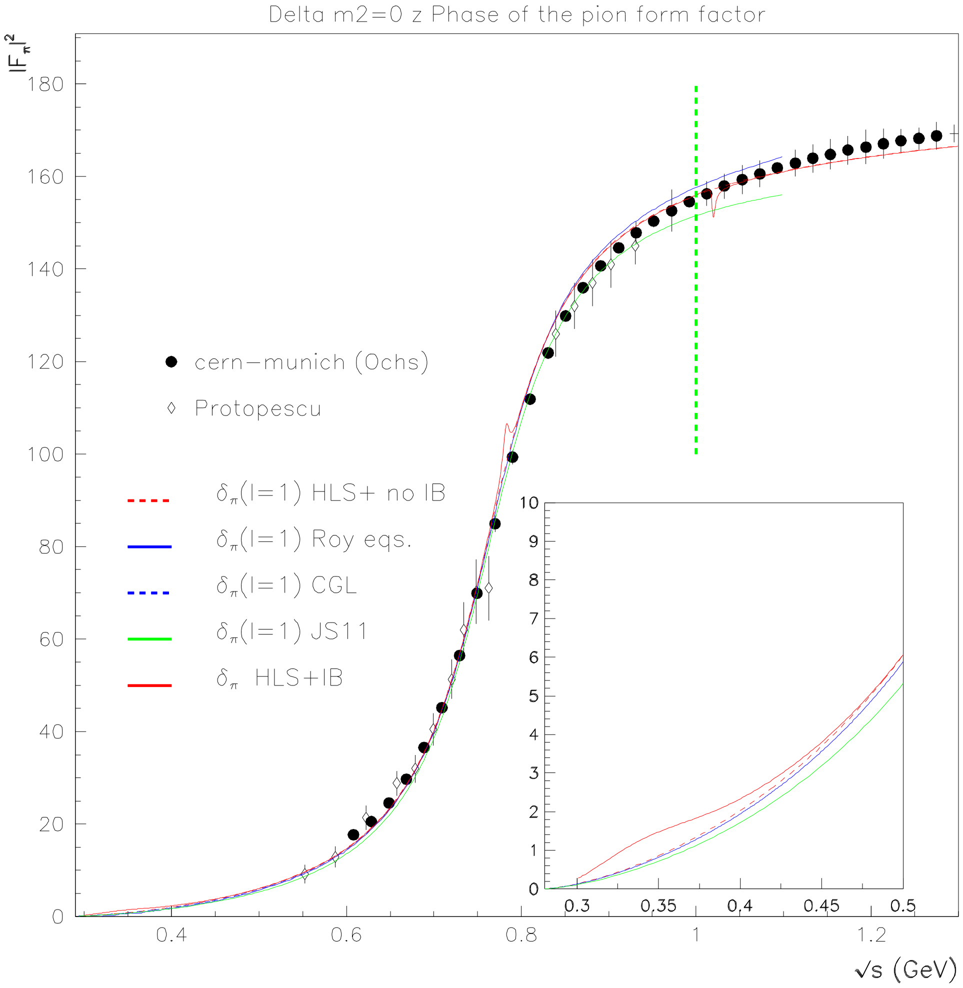
Belle
ALEPH
CLEO
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
(Data- Fit)/Fit
0.2
0.4
0.6
0.8
1
1.2
$(M_{\pi\pi^0})^2$
$(GeV/c^2)^2$

The Spacelike & threshold Regions



Extrapolation
to s<0 perfect

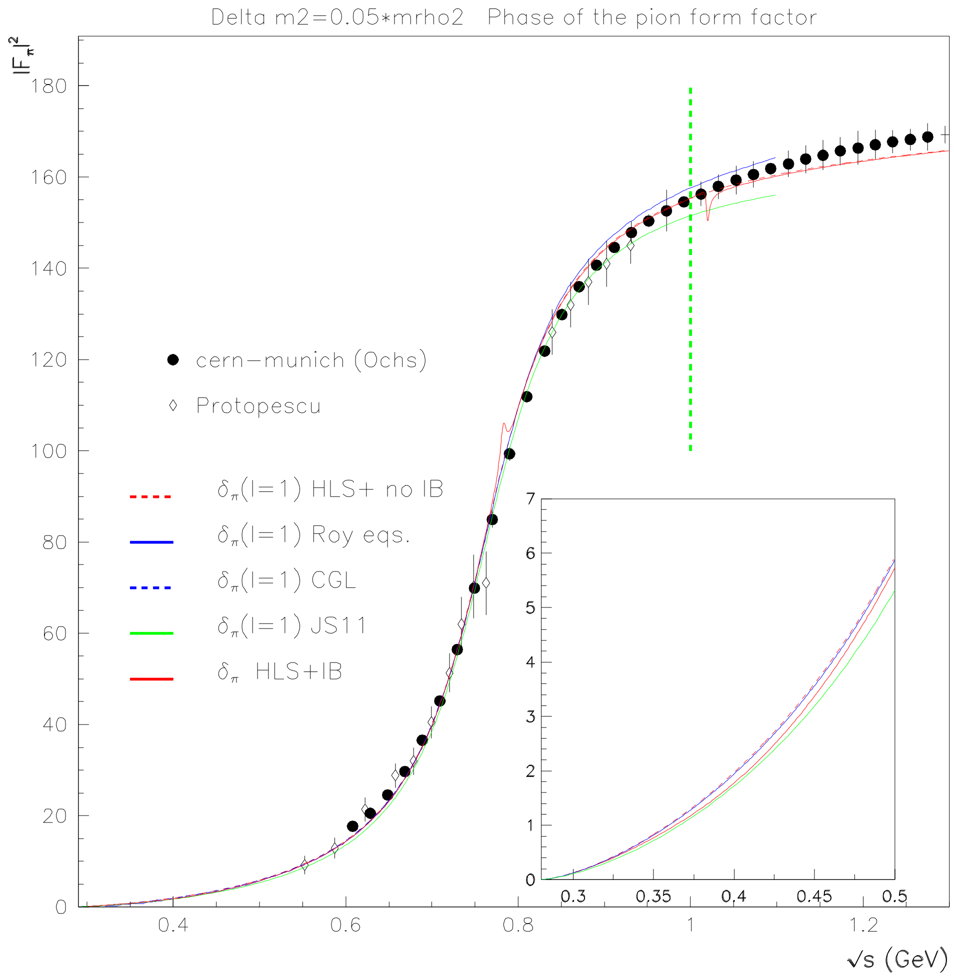
Predicted Phase shift (I)



$$\left[\frac{m_\omega^{HK}}{m_\rho^{HK}}\right]^2 = 1.00$$

$$\alpha(s) = \frac{\varepsilon_1(s)}{\left[m_\rho^{HK}\right]^2 + \Pi_{\pi\pi}^{\rho}(s) - \left[m_\omega^{HK}\right]^2}$$

Predicted Phase shift (II)



$$\left[\frac{m_\omega^{HK}}{m_\rho^{HK}}\right]^2 = 1.05$$

$$\alpha(s) = \frac{\varepsilon_1(s)}{\left[m_\rho^{HK}\right]^2 + \Pi_{\pi\pi}^{\rho}(s) - \left[m_\omega^{HK}\right]^2}$$

Final Results + Additional Systematics

NSK+KLOE

$$10^{10}\times\left[a_\mu^{\exp}-a_\mu^{th}\right]=38.34+\begin{bmatrix}+0.6\\-1.3\end{bmatrix}_\phi+\begin{bmatrix}+0.9\\-0.0\end{bmatrix}_\tau+\begin{bmatrix}+0.0\\-1.4\end{bmatrix}_{s=0}\pm 4.19_{th}\pm 6.30_{\exp}$$

$$10^{10}\times\left[a_\mu^{\exp}-a_\mu^{th}\right]=37.55+\begin{bmatrix}+0.6\\-1.3\end{bmatrix}_\phi+\begin{bmatrix}+1.1\\-0.0\end{bmatrix}_\tau+\begin{bmatrix}+0.0\\-1.4\end{bmatrix}_{s=0}\pm 4.12_{th}\pm 6.30_{\exp}$$

NSK+KLOE+BES

$$10^{10}\times\left[a_\mu^{\exp}-a_\mu^{th}\right]=33.72+\begin{bmatrix}+0.6\\-1.3\end{bmatrix}_\phi+\begin{bmatrix}+0.9\\-0.0\end{bmatrix}_\tau+\begin{bmatrix}+0.0\\-1.4\end{bmatrix}_{s=0}\pm 4.08_{th}\pm 6.30_{\exp}$$

NSK+KLOE+ BaBar

$$10^{10}\times\left[a_\mu^{\exp}-a_\mu^{th}\right]=33.16+\begin{bmatrix}+0.6\\-1.3\end{bmatrix}_\phi+\begin{bmatrix}+1.1\\-0.0\end{bmatrix}_\tau+\begin{bmatrix}+0.0\\-1.4\end{bmatrix}_{s=0}\pm 4.03_{th}\pm 6.30_{\exp}$$

NSK+KLOE+BESS+ BaBar

significance for $\Delta a_\mu > 4.6/4.1\,\sigma$

Conclusions

1/ broken HLS model → good simultaneous fit of ≈ all data

2/ IB → NO signal of a (e^+e^- vs τ) puzzle within BHLS

3/ Relative inconsistencies of π π data sets substantiated

4/ Consistent π π data sets: CMD2, SND, KLOE 10&12, BES

5/ The discrepancy with BNL g-2 value is ≈ 4.6/4.1 σ

6/ Error on LO-HVP dominated by [1.05, 2] GeV data

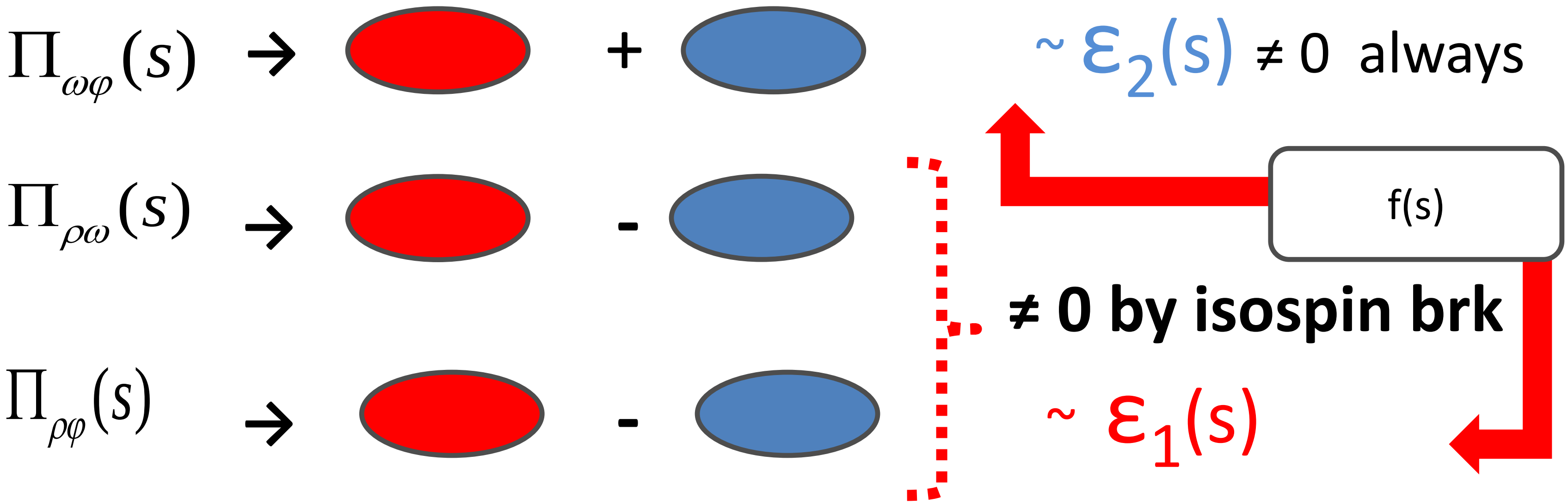
7/ Biasing Effects of normalization uncertainties

Backup Slides

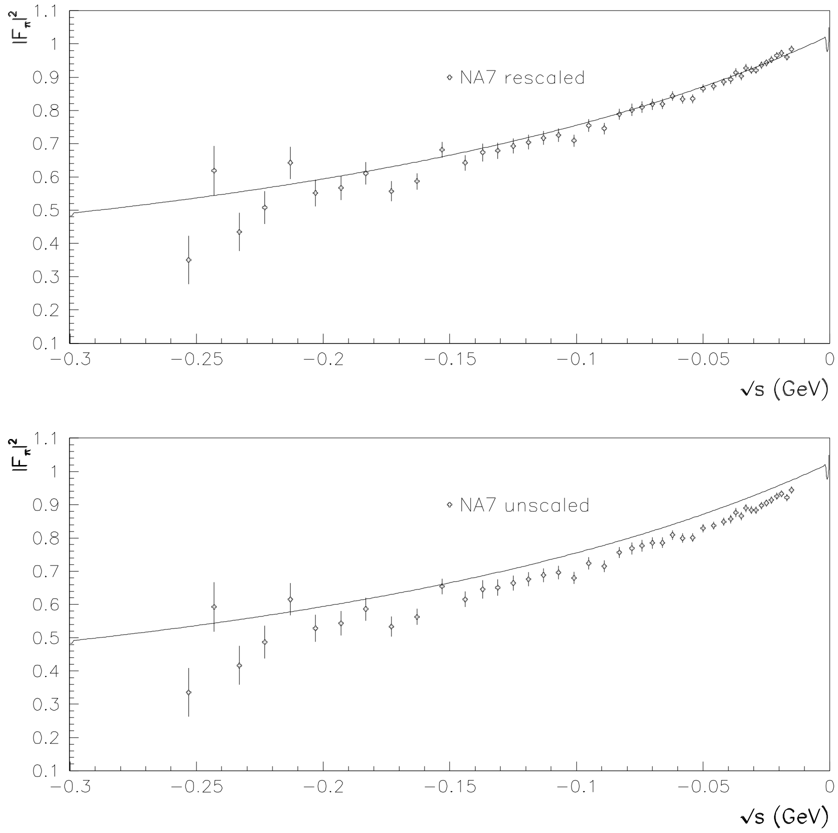
Transitions at one loop

- Define the loops :

Then, beside self-masses :

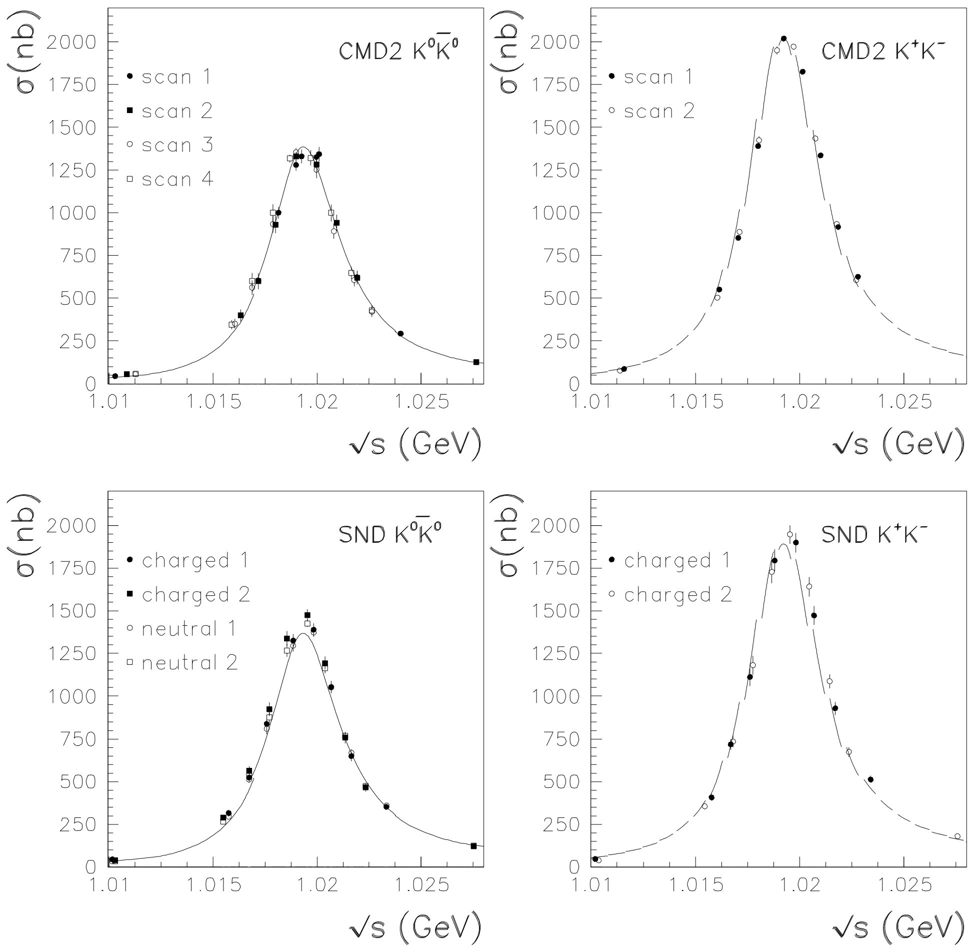


NA7 Residuals ($\chi^2/N \approx 90/60$)!



Backup Slides II

$e^+e^- \rightarrow$ K Kbar Data



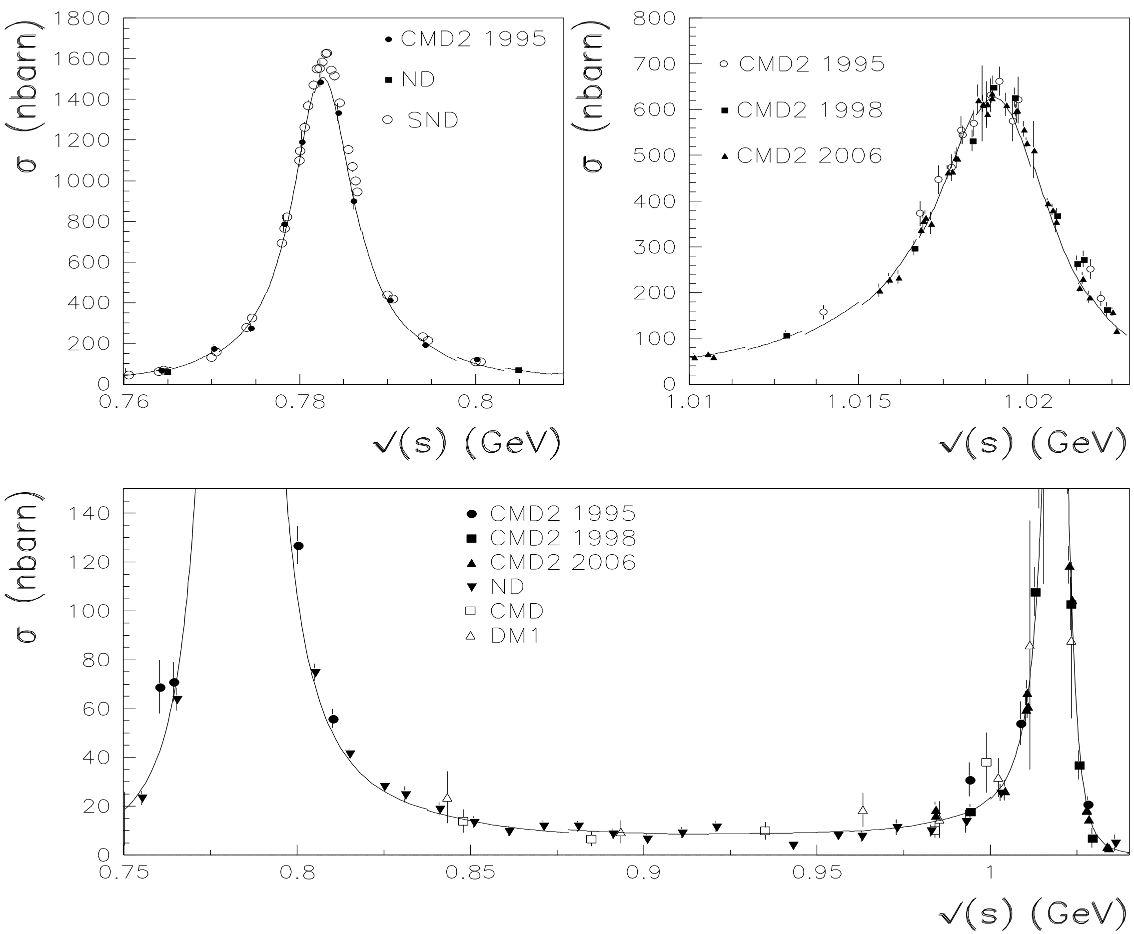
$K^+ K^-$

$$\chi^2 / N_p = 30/36$$

$K_L K_S$

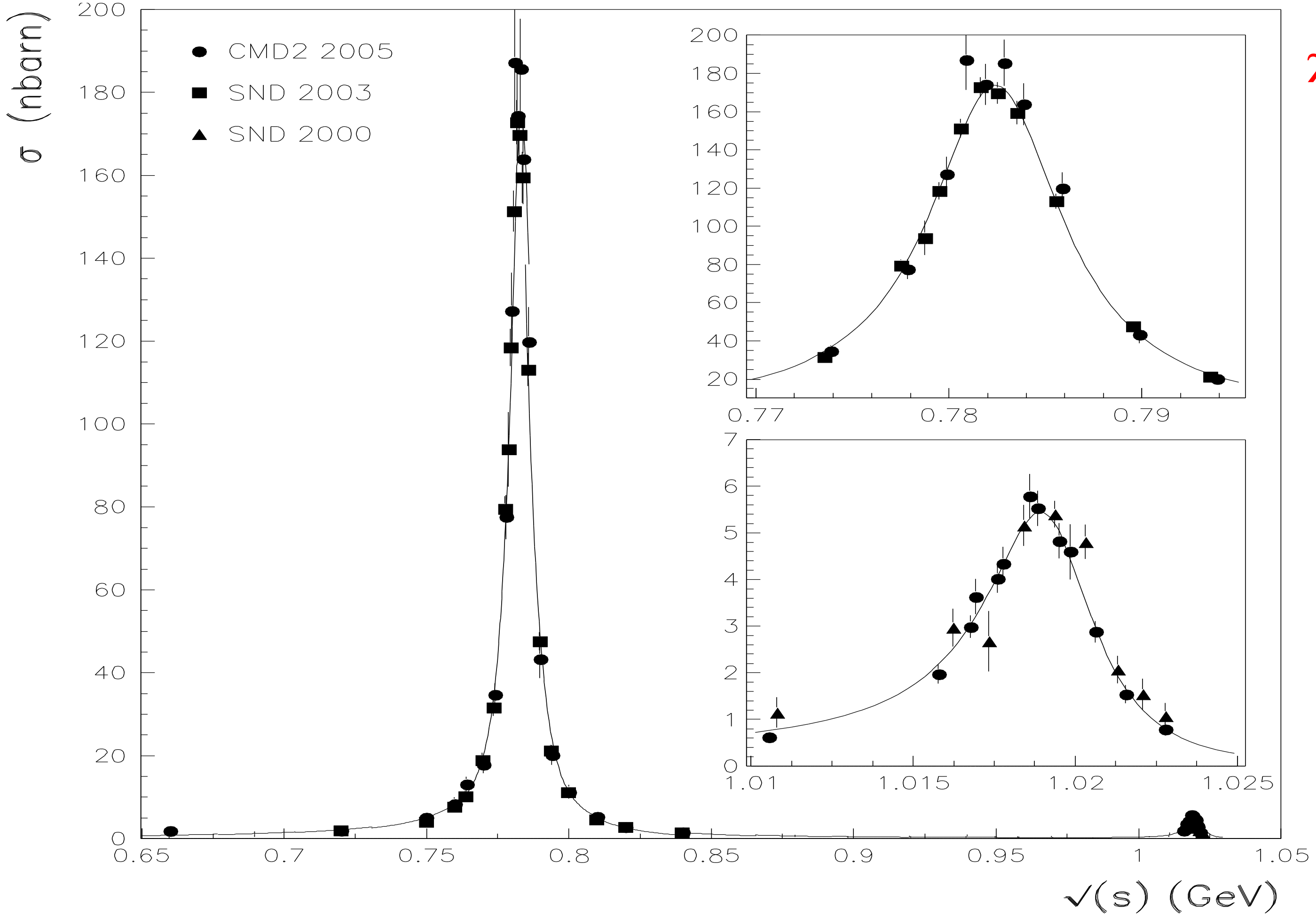
$$\chi^2 / N_p = 123/119$$

3-pion Data



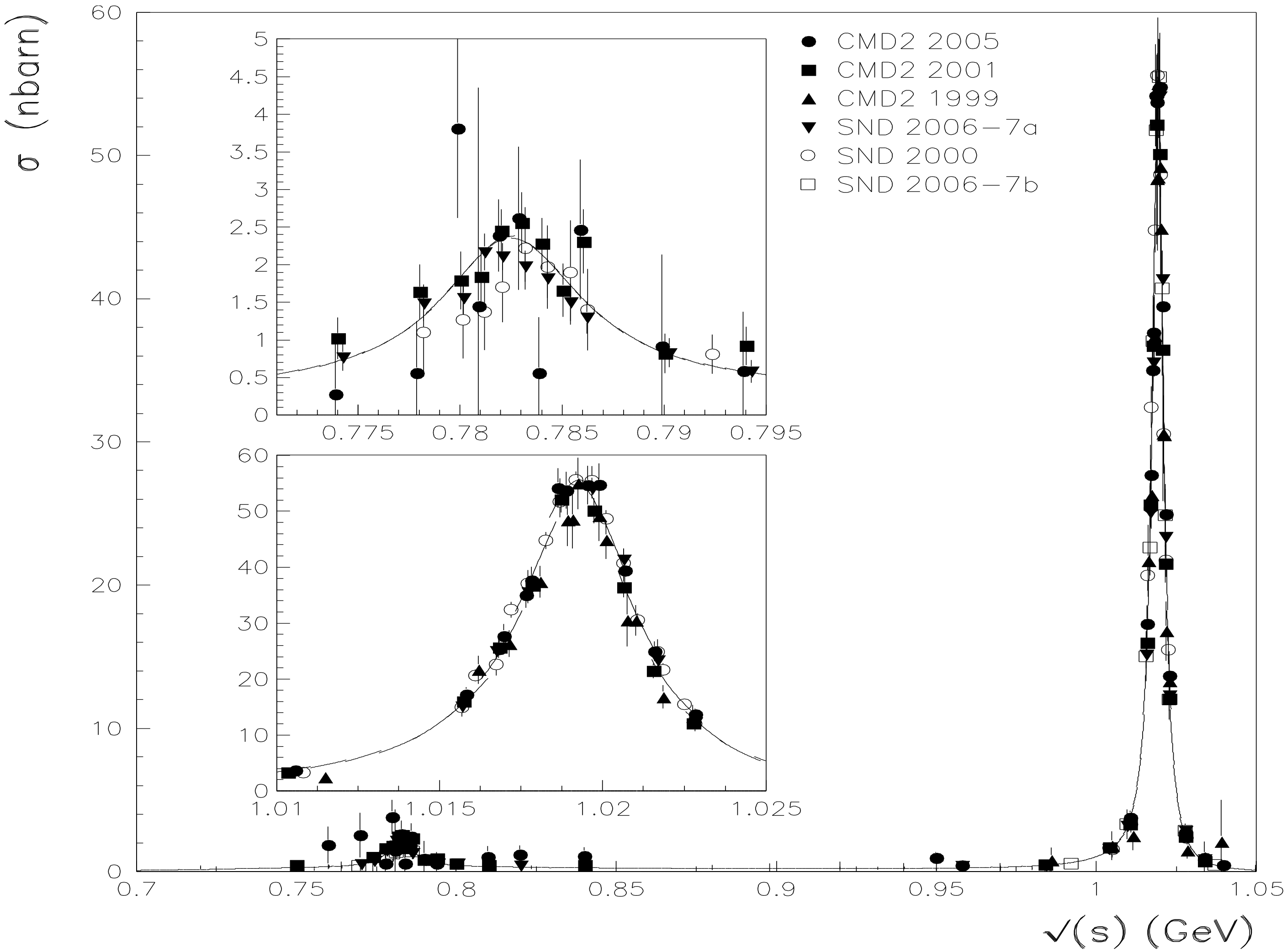
$$\chi^2 / N_p = 229 / 179$$

$e^+e^- \rightarrow \pi^0 \gamma$



$$\chi^2 / N_p = 67/86$$

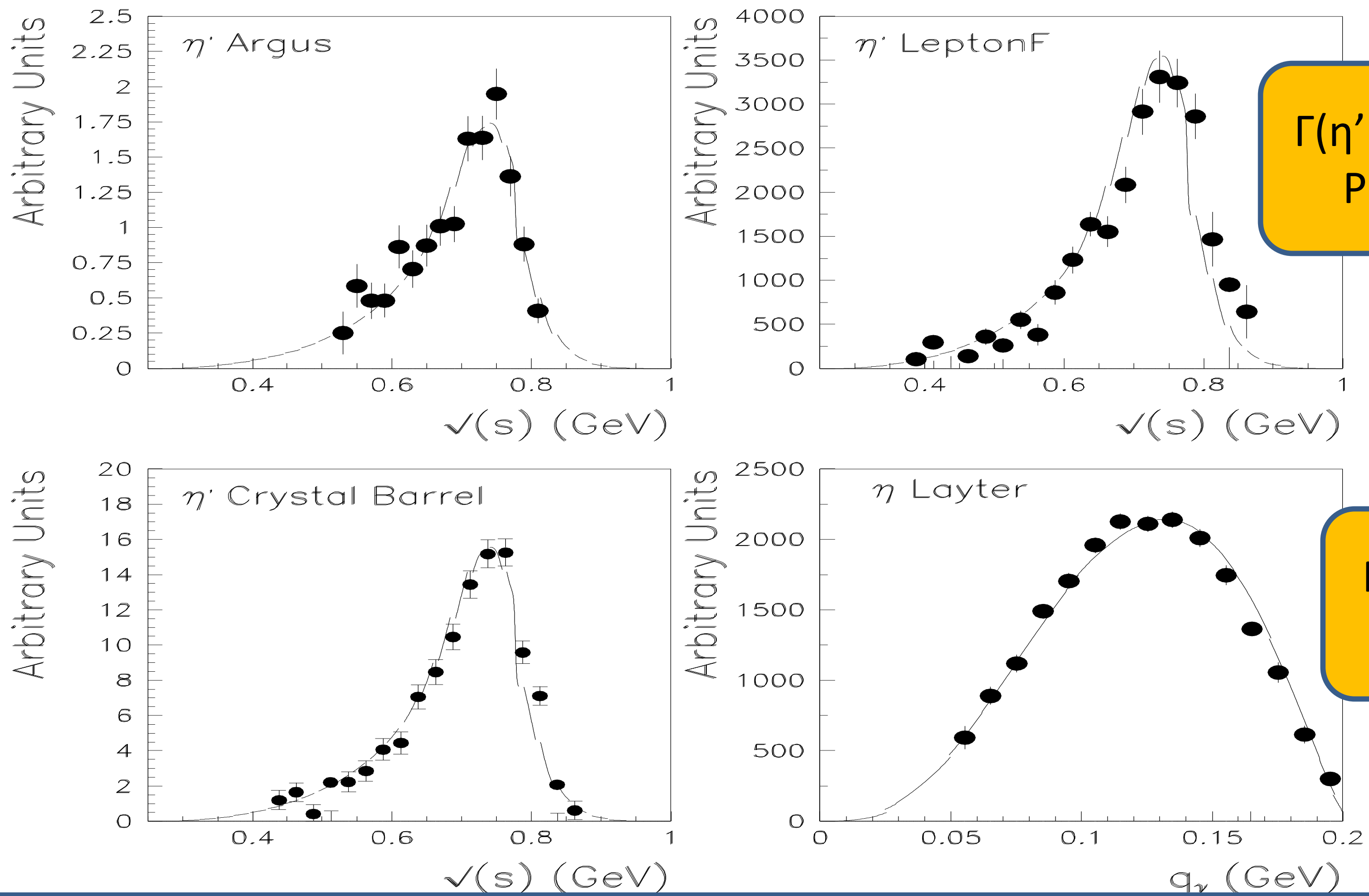
$e^+e^- \rightarrow \eta\gamma$ Data



$$\chi^2 / N_p = 121/182$$

Former : Prediction for $\eta/\eta' \to \pi\pi\gamma$

M. Benayoun *et al.* ArXiv 0907.4047

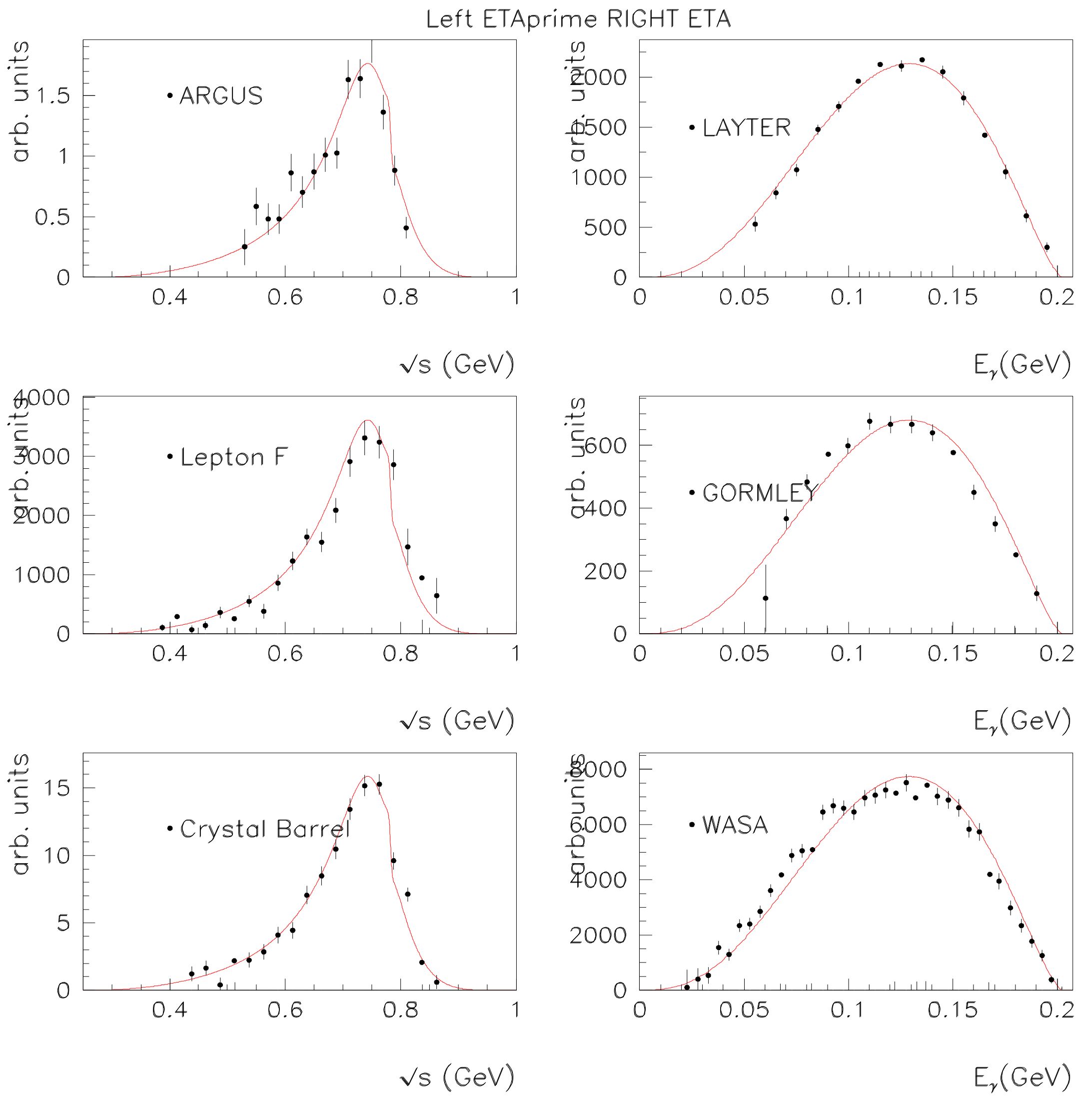


$\Gamma(\eta' \to \pi\pi\gamma) = 53.11 \pm 1.47$
PDG : 58.3 ± 2.8 keV

$\Gamma(\eta \to \pi\pi\gamma) = 55.82 \pm 0.83$
PDG : 60 ± 4 eV

Lineshapes and yields : **tests** for **g** value and ρ^0 **lineshape**

Former : Prediction for $\eta/\eta' \to \pi\pi\gamma$



Global Fit & Scale Uncertainty

M. Benayoun *et al* EPJ C73 (2013)2453

- Minimize :

$$\chi^2 = \left[\sigma_{\exp} - \sigma_{th} - \lambda A \right]^T V^{-1} \left[\sigma_{\exp} - \sigma_{th} - \lambda A \right] + \lambda^2 / \eta^2$$

•Solve for λ :

$$\chi^2 \equiv \left[\sigma_{\exp} - \sigma_{th} \right]^T \left[V + \eta^2 A A^T \right]^{-1} \left[\sigma_{\exp} - \sigma_{th} \right]$$

•Choice of A : σ_{exp} (bias) , [$\sigma_{th} \approx \sigma_{iter}$](unbiased)

S. Chiba & D. Smith, ANL/NDM-121 (1991)

G. D'Agostini NIM A346 (1994)306

M. Benayoun *et al* , arXiv:1507.02943

V. Blobel eConf C030908 MOET002 (2003)

R.D.Ball *et al* JHEP 1005 (2010)075// **Nucl. Phys. B838** (2010) 136