Hadronic contributions to the muon $g-2$ from Lattice QCD

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(many thanks to K. Szabo, R. Malak, K. Miura, other Budapest-Marseille-Wuppertal collaboration (BMWc) members and T. Blum)

Introduction

- Muon magnetic moment

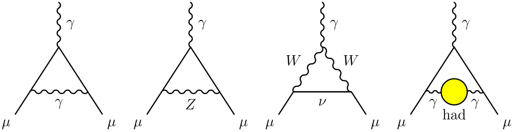
$$\vec{M} = g_\mu \frac{e}{2m_\mu} \vec{S}$$

where $g_\mu = 2$ according to Dirac eq. (i.e. at tree level in SM)

- Measure precession of μ spin in magnetic field and get anomalous magnetic moment

$$a_\mu \equiv \frac{g_\mu - 2}{2} > 0$$

- In SM (beyond tree level)



$$\rightarrow a_\mu^{\rm SM} = \frac{\alpha}{2\pi} + \cdots = 0.0011\ldots + \cdots$$

$$a_\mu^{\rm exp} = a_\mu^{\rm SM} \quad ?$$

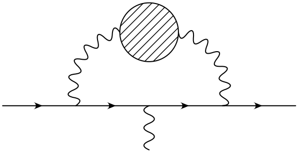
SM prediction and experimental measurements

SM contribution	$a_\mu^{\text{contrib.}} \times 10^{11}$	Ref.
QED [5 loops]	116584718.951 ± 0.080	[Aoyama et al '12]
HVP LO	6923 ± 42	[Davier et al '11]
	6949 ± 43	[Hagiwara et al '11]
	6870 ± 42	[Jegerlehner '16]
HVP NLO	-98.4 ± 0.7	[Hagiwara et al '11]
		[Kurz et al '14, Jegerlehner '16]
HVP NNLO	12.4 ± 0.1	[Kurz et al '14]
		Jegerlehner '16
HLbyL	105 ± 26	[Prades et al '09]
Weak (2 loops)	153.6 ± 1.0	[Gnendiger et al '13]
SM Tot [0.42 ppm]	116591815 ± 49	[w/ Davier et al '11]
[0.43 ppm]	116591841 ± 50	[w/ Hagiwara et al '11]
[0.51 ppm]	116591840 ± 59	[Aoyama et al '12]
[0.42 ppm]	116591760 ± 50	[Jegerlehner '16]
Exp [0.54 ppm]	116592089 ± 63	[Bennett et al '06]
Exp − SM	274 ± 80	[Davier et al '11]
	248 ± 81	[Hagiwara et al '11]
	249 ± 86	[Aoyama et al '12]
	329 ± 80	[Jegerlehner '12]

a_μ present and future

- Fermilab E989 begins in 2017 and aims for 0.14 ppm
- J-PARC E34 to being later in decade and aims for 0.1 ppm
- Today $a_\mu^{\rm exp} - a_\mu^{\rm SM} \simeq 2.9 \div 4.1\,\sigma$
- If both central values stay the same:
  - E989 alone (exp. error $/4$) $\to\sim 5 \div 6\,\sigma$
  - E989 + new HLbyL (w/ error $\sim$ 10%) $\to\sim 5 \div 7\,\sigma$
  - E989 + new HLbyL + new HVP (w/ error $/2$) $\to\sim 9 \div 12\,\sigma$
- Discrepancy is large: $2\times$ electroweak contribution
- Lattice QCD can be used to help reduce the 2 leading theory errors

Hadronic contributions: introduction



LO hadronic vacuum polarization (HVP) $= O(\alpha^2)$

- Obtained from measurement of $e^+e^- \to$ hadrons and $\tau \to \nu_\tau +$ hadrons using dispersion relations
- $\delta a_\mu^{\mathrm{HVP,LO}}$: 0.6% (current) $\to$ 0.3% (in 2-3 years)?

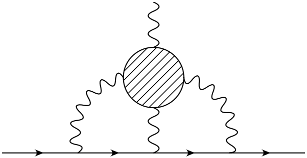
$\Rightarrow$ LQCD not fully competitive in short term . . .
. . . but worth pursuing as cross check and will become competitive in longer term

Hadronic light-by-light (HLbyL) $= O(\alpha^3)$

- Cannot be obtained from experiment
- Currently computed with models (χPT+, MHA, ENJL, AdS-QCD, Schwinger-dyson, . . .)

$\to$ Glasgow concensus: $10.5(2.6) \times 10^{-10}$ [25%]
(Prades et al '09)

- Reasonable, but error is a guesstimate
- LQCD can really help, but very challenging



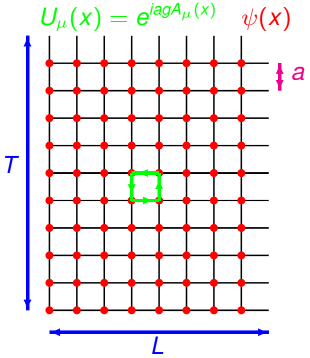
What is lattice QCD (LQCD)?

To describe ordinary matter, QCD requires ≥ 104 numbers at every point of spacetime
$\rightarrow \infty$ number of numbers in our continuous spacetime
$\rightarrow$ must temporarily "simplify" the theory to be able to calculate *(regularization)*
$\Rightarrow$ Lattice gauge theory $\longrightarrow$ mathematically sound definition of NP QCD:

- UV (& IR) cutoff $\rightarrow$ well defined path integral in Euclidean spacetime:

$$\langle O\rangle = \int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{-S_G-\int\bar{\psi}D[M]\psi}\, O[U,\psi,\bar{\psi}]$$
$$= \int \mathcal{D}U\, e^{-S_G}\, \det(D[M])\, O[U]_{\mathrm{Wick}}$$

- $\mathcal{D}Ue^{-S_G}\det(D[M]) \geq 0$ & finite # of dofs $\rightarrow$ evaluate numerically using stochastic methods



LQCD is QCD when $m_q \rightarrow m_q^{\mathrm{ph}}$, $a \rightarrow 0$ (after renormalization), $L \rightarrow \infty$ (and stats $\rightarrow \infty$)

HUGE conceptual and numerical challenge

Challenges of a full lattice calculation

To make contact with experiment need:

- **A valid approximation to the SM**
  - $\rightarrow$ at least u, d, s in the sea w/ $m_u = m_d \ll m_s$ (N_f=2+1)
  - $\rightarrow$ better also include c (N_f=2+1+1) & $m_u \leq m_d$ (N_f=4×1) & EM (N_f=4×1 + QED)
- **u & d w/ masses well w/in $SU(2)$ chiral regime** : $\sigma_\chi \sim (M_\pi/4\pi F_\pi)^2$
  - $\rightarrow$ $M_\pi \sim 135\,\mathrm{MeV}$ or many $M_\pi \leq 400\,\mathrm{MeV}$ w/ $M_\pi^{\mathrm{min}} < 200\,\mathrm{MeV}$ for $M_\pi \to 135\,\mathrm{MeV}$
- **a → 0** : $\sigma_a \sim (a\Lambda_{\mathrm{QCD}})^n, (am_q)^n, (a|\vec{p}|)^n$ w/ $a^{-1} \sim 2 \div 4\,\mathrm{fm}$
  - $\rightarrow$ at least 3 a's $\leq 0.1\,\mathrm{fm}$ for $a \to 0$
- **L → ∞** : $\sigma_L \sim (M_\pi/4\pi F_\pi)^2 \times e^{-LM_\pi}$ for stable hadron pties, $\sim 1/L^n$ for resonances, QED, ...
  - $\rightarrow$ many L w/ $(LM_\pi)^{\mathrm{max}} \gtrsim 4$ for stable hadrons & better otherwise to allow for $L \to \infty$
- These requirements $\Rightarrow O(10^9)$ **dofs** that have to be integrated over
- **Renormalization** : best done nonperturbatively
- **A signal** : $\sigma_{\mathrm{stat}} \sim 1/\sqrt{N_{\mathrm{meas}}}$, reduce w/ $N_{\mathrm{meas}} \to \infty$

Challenges of a full lattice calculation (cont'd)

- If one ingredient is missing, calculation gives only qualitative results

- Difficulty: algorithms loose effectiveness as physical limit approached

- Response: algorithmic and methodological improvements (Sexton et al '92, Hasenbusch '01, Urbach et al '06, Lüscher '04, Del Debbio et al '06, Lüscher '07, BMWc '08, Blum et al '12, Frommer et al '13, ...) and Pflop/s supercomputers

  ⇒ possible to compute simple quantities w/ %-level accuracy

- Still need large # of large simulations over large range of parameters

  ⇒ only a few full QCD calculations of simple quantities exist

HVP from LQCD

Compute directly in Euclidean spacetime

$\Pi_{\mu\nu}(Q) =$

$$= \int d^4x\, e^{iQ\cdot x} \langle J_\mu^{\mathrm{EM}}(x) J_\nu^{\mathrm{EM}}(0)\rangle$$

$$= \left(\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu\right) \Pi(Q^2)$$

Then (Lautrup et al '69, Blum '02)

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2\, w(Q^2/m_\mu^2)\hat{\Pi}(Q^2)$$

w/ $\hat{\Pi}(Q^2) \equiv \left[\Pi(Q^2) - \Pi(0)\right]$ & $w(Q^2/m_\mu^2)$ known fn that heavily weighs $Q^2 \sim (m_\mu/2)^2$

$\Rightarrow$ determine precisely

$\Pi_{\mu\nu}(Q)$ down to below $\sqrt{Q^2} \sim 50\,\mathrm{MeV}$ $\longleftrightarrow$ $\langle J_\mu^{\mathrm{EM}}(x) J_\nu^{\mathrm{EM}}(0)\rangle$ up to above $\sqrt{x^2} \sim 4\,\mathrm{fm}$

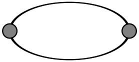
Euclidean $\Pi_{\mu\nu}(Q)$ smooth $\rightarrow$ can extrapolate with sufficient results

down to $\sqrt{Q^2} \sim 70\,\mathrm{MeV}$ $\longleftrightarrow$ up to $\sqrt{x^2} \sim 3\,\mathrm{fm}$

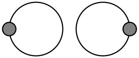
HVP challenges for lattice QCD

- $\pi\pi$ contribution very important $\rightarrow$ must have physically light π
- $\langle J_\mu^{ud}(x)J_\nu^{ud}(0)\rangle$ signal very poor at large $\sqrt{x^2}$ + need high precision result

  $\rightarrow$ very high statistics
- Two contributions



quark-connected (qc)



quark-disconnected (qd)

where qd is $SU(3)_f$ and Zweig suppressed but very challenging

- To control $\langle J_\mu^{\rm EM}(x)J_\nu^{\rm EM}(0)\rangle$ at $\sqrt{x^2} \gtrsim 3\,\mathrm{fm}$

  $\rightarrow$ w/ periodic BCs need L and/or $T \gtrsim 6\,\mathrm{fm}$
- Large finite-volume (FV) effects?

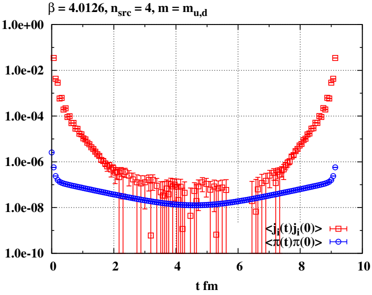
HVP challenges: light pions and statistics

- Physically light pions only being used very recently (BMWc '13 & in progress, HPQCD/MILC '15 & in progress, RBC/UKQCD in progress,)

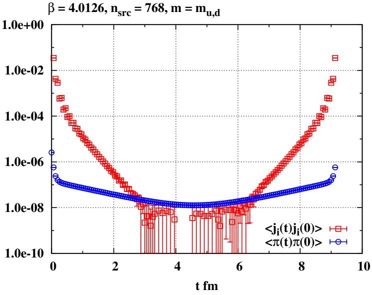
- Errors in $\langle\pi(t)\pi(0)\rangle$ & $\langle J_i^{ud}(t)J_i^{ud}(0)\rangle_{qc}$ as fn of t (BMWc in progress)

$$m_{ud}, m_s, m_c \text{ physical}, \quad a \simeq 0.064\,\mathrm{fm}, \quad L = 96a \simeq 6.1\,\mathrm{fm}, \quad T = 144a \simeq 9.2\,\mathrm{fm}$$

Good stats: 4×441 meas.



For $\delta_{\rm stat} a_\mu^{\rm HVP,LO} \sim 1\%$: 768×441 meas.



$\rightarrow$ need many algorithmic improvements: all-mode-averaging (Blum et al '13), deflation (Lüscher '07), hopping-parameter expansion, . . .

HVP challenges: control small $Q^2 \leftrightarrow$ large distance

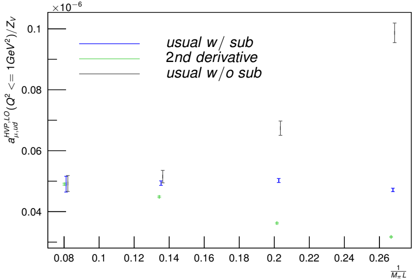
- In L^4, $Q_\mu \Pi_{\mu\nu}(Q) = 0$ does not imply $\Pi_{\mu\nu}(Q=0) = 0$

$$\Pi_{\mu\nu}(Q=0) = \int_\Omega d^4x \langle J_\mu(x) J_\nu(0)\rangle = \int_\Omega d^4x \partial_\rho [x_\mu \langle J_\rho(x) J_\nu(0)\rangle]$$

$$\int_{\partial\Omega} d^3x_\rho [x_\mu \langle J_\rho(x) J_\nu(0)\rangle] \propto L^4 \exp(-EL/2)$$

$\rightarrow$ distortion of $\Pi_{\mu\nu}(Q)/(Q^2\delta_{\mu\nu} - Q_\mu Q_\nu)$ as $Q_\mu \rightarrow 0$

$M_\pi \simeq 290\,\mathrm{MeV}$, $a \simeq 0.104\,\mathrm{fm}$, $L = 24 \div 80a \simeq 2.5 \div 8.3\,\mathrm{fm}$, $T = 48 \div 96a \simeq 5.0 \div 10.\,\mathrm{fm}$



BMWc '13

HVP challenges: control small $Q^2 \leftrightarrow$ large distance

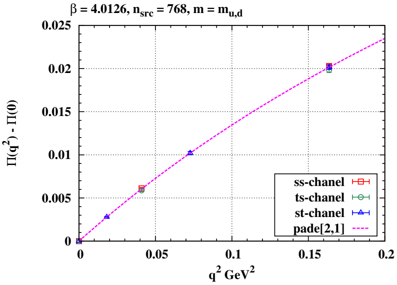
- Also need $\Pi(0)$ to define renormalized $\hat{\Pi}(Q^2) \equiv \Pi(Q^2) - \Pi(0)$

  $\rightarrow$ consider on lattice, e.g. (Bernecker et al '11, BMWc '13, Lehner et al '14, HPQCD '14, ...)

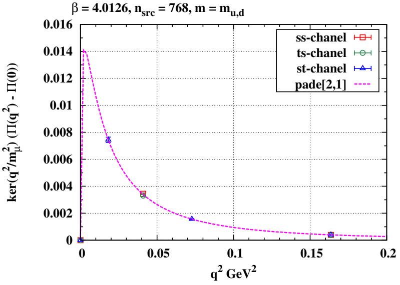
$$\Pi(Q^2) - \Pi(0) \equiv \frac{a^4}{3} \sum_{x_\mu, i} \mathrm{Re}\left(\frac{e^{iQt} - 1}{Q^2} + \frac{t^2}{2}\right) \mathrm{Re}\langle J_i(x) J_i(0)\rangle$$

m_{ud}, m_s, m_c physical, $a \simeq 0.064$ fm, $L = 96a \simeq 6.1$ fm, $T = 144a \simeq 9.2$ fm (BMWc in progress)

$\hat{\Pi}(Q^2)$ vs Q^2



$\omega(Q^2/m_\mu^2)\hat{\Pi}(Q^2)$ vs Q^2



- Need large volume to control important small Q^2 region

HVP challenges: approaches to control low Q^2

- Hi-lo method (Aubin et al '12)

  $> 90\%$ of $a_\mu^{\rm HVP,LO}$ comes from region w/ $Q^2 \le 0.2\,{\rm GeV}^2$

$0 \le Q^2 \le 0.2\ {\rm GeV}^2$	$0.2 \le Q^2/{\rm GeV}^2 \le 3$	$3 \le Q^2/{\rm GeV}^2 \le \infty$
Fit $\hat\Pi(Q^2)$ to $aQ^2/(1+bQ^2)$ or better	$a_\mu^{\rm HVP,LO}$ by numerically integrating results	$a_\mu^{\rm HVP,LO}$ use perturbation theory

  With good $\hat\Pi(Q^2)$ vs Q^2 for $Q^2 \le 0.2\ {\rm GeV}^2$ and in $0.2 \le Q^2/{\rm GeV}^2 \le 3$

  $\rightarrow a_\mu^{\rm HVP,LO}$ w/ $< 1\%$ systematics

- Time moments & Padé (HPQCD '14)

  Defining Π_j such that $\hat\Pi(Q^2) = \sum_{j=1}^{\infty} Q^2 \Pi_j$, compute on lattice

$$\Pi_j = \frac{(-1)^{j+1}}{(2j+2)!}\frac{a^4}{3}\sum_{i,x_\mu} x_0^{2j+2}\langle J_i(x)J_i(0)\rangle$$

  and replace $\hat\Pi(Q^2)$ by $[m,n]$ Padé approximants derived from Π_j (typically up to $[2,2]$)

  Requires good results for $\langle J_\mu(x)J_\nu(0)\rangle$ w/ $\sqrt{x^2}$ up to $\gtrsim 3\,{\rm fm}$

HVP challenges: approaches to control low Q^2

- Moments approximation of $\hat{\Pi}(Q^2)$ based on Mellin transforms (De Rafael '14)
- Sine-cardinal interpolation (Portelli et al '15)

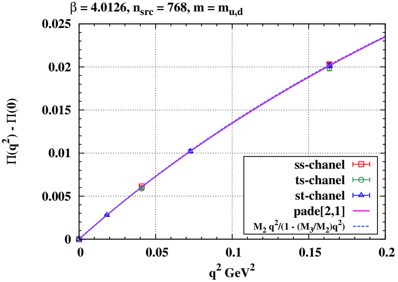
  In $T \times L^3$, define for arbitrary $Q_\mu \in \mathbb{R}^4$,

$$\Pi^{\rm sc}_{\mu\nu}(Q) = \int_{T\times L^3} d^4x\ e^{iQ\cdot x} \langle J_\mu(x) J_\nu(0)\rangle$$

  $\Pi^{\rm sc}_{\mu\nu}(Q)$ converges onto infinite-volume $\Pi^{\rm sc}_{\mu\nu}(Q)$ exponentially fast in L and T

- Both require good results for $\langle J_\mu(x) J_\nu(0)\rangle$ w/ $\sqrt{x^2}$ up to $\gtrsim 3\,\mathrm{fm}$
- Example: hi-lo and moments-Padé for $Q^2 \leq 2\,\mathrm{GeV}^2$ (BMWc in progress)

m_{ud}, m_s, m_c physical, $a \simeq 0.064\,\mathrm{fm}$, $L = 96a \simeq 6.1\,\mathrm{fm}$, $T = 144a \simeq 9.2\,\mathrm{fm}$



HVP challenges: quark-disconnected contributions

- *ud* disc. is -10% of conn. $\pi\pi$ in χPT (Della Morte et al '10)
- Other derivation (Francis et al '14) (take $G(t) = \frac{1}{3}\sum_i \int d^3x \langle J_i^{\rm EM}(x) J_i^{\rm EM}(0)\rangle$)

$$G(t) = G^{I=1}(t) + G^{I=0}(t) = G^{ud}_{qc}(t) + G^{s}_{qc}(t) + G^{c}_{qc}(t) + G_{qd}(t)$$

and

$$G^{ud}_{qc}(t) = \frac{10}{9} G^{I=1}(t) \qquad \text{and} \qquad G(t) \stackrel{t\to+\infty}{\longrightarrow} G^{I=1}(t)$$

$$\Rightarrow \frac{G_{qd}(t)}{G^{ud}_{qc}(t)} = \frac{G(t) - G^{ud}_{qc}(t) - G^{s}_{qc}(t) - G^{c}_{qc}(t)}{G^{ud}_{qc}(t)} \stackrel{t\to+\infty}{\longrightarrow} -\frac{1}{10}$$

- Early estimates gave $a_\mu^{\rm HVP,LO}|_{qd}$ $-4 \div 0\%$ of *ud* conn. contribution (ETMC '11, '15, Mainz '14)
- $-2 \div 0\%$ of *ud* conn. based on phenomenology of ρ–ω properties (HPQCD '15)
- S/N for q-disc. does not simply grow like $L^{3/2}$ as for q-conn.

  $\rightarrow$ need new algorithms and high statistics

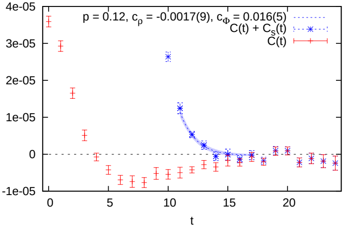
HVP challenges: quark-disconnected contributions

First calculation of $(ud - s)$ q-disc. contribution at physical m_{ud} but single a and $T \times L^3$ (RBC/UKQCD '15)

- $C(t)$ is $(ud - s)$ q-disc. contribution & $C_s(t)$, s q-conn. contribution

$$m_{ud}, m_s \text{ physical}, \quad a \simeq 0.114\,\text{fm}, \quad L = 48a \simeq 5.5\,\text{fm}, \quad T = 96a \simeq 11.\,\text{fm}$$

$$t = 4\,\text{fm} \quad \longleftrightarrow \quad t/a \sim 35$$



- Signal lost at $t/a = 15 \quad \rightarrow \quad t \simeq 1.7\,\text{fm}$
- Gives $(ud - s)$ q-disc. $a_\mu^{\text{HVP,LO}}|_{qd} = -9.6(3.3)(2.3) \times 10^{-10} \sim -1.5\% \times a_\mu^{\text{HVP,LO}}$

HPQCD '15 w/ $M_\pi \sim 390\,\text{MeV}$ find $a_\mu^{\text{HVP,LO}}|_{qd} \sim -0.15\% \times a_\mu^{\text{HVP,LO}}|_{qc}^{ud}$

HVP challenges: finite-volume effects

- NLO χPT describes $\hat{\Pi}(Q^2)$ very poorly even for small Q^2 (Aubin et al '07)
- Aubin et al '16 find LO χPT describes FV errors on $\hat{\Pi}(Q^2)$ much better

  On $T \times L^3$, Euclidian "Lorentz" $\rightarrow$ cubic group

  $\longrightarrow$ project $\Pi_{\mu\nu}(Q)$ onto irreps of cubic group

$$
\begin{aligned}
A1 &: \quad \sum_i \Pi_{ii} \quad \text{and} \quad \Pi_{44} \\
T1 &: \quad \Pi_{4i} = \Pi_{i4} \\
T2 &: \quad \Pi_{i\neq j} = \Pi_{j\neq i} \\
E &: \quad \Pi_{11} - \sum_i \Pi_{ii}/3, \quad \Pi_{22} - \sum_i \Pi_{ii}/3
\end{aligned}
$$

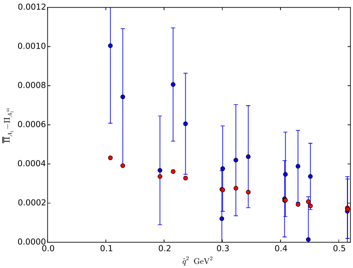
$\rightarrow$ 5 independent scalar fns Π_{A_1} , $\Pi_{A_1^{44}}$, Π_{T_1} , Π_{T_2} , Π_E (also Bernecker et al '11)

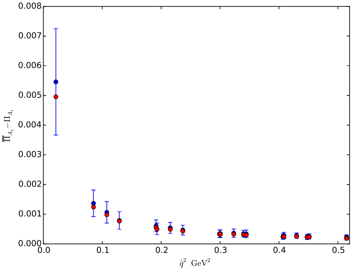
HVP challenges: finite-volume effects

Consider only differences of q-conn. *ud* polarization fns which are FV effects (Aubin et al '16)

$$M_\pi = 220\,\mathrm{MeV},\ \ a \simeq 0.060\,\mathrm{fm},\ \ L = 64a \simeq 3.8\,\mathrm{fm},\ \ T = 144a \simeq 9.2\,\mathrm{fm}$$

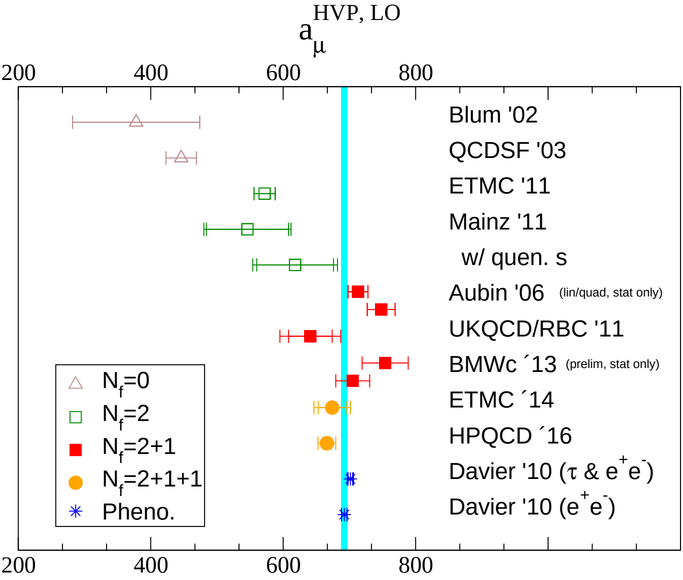
Blue = LQCD   Red = LO staggered χPT



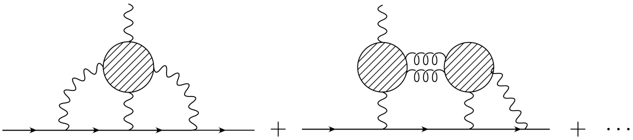


- Conclude $a_\mu^{\mathrm{HVP,LO}}|_{qc}^{ud}$ may suffer $10 \div 15\%$ FV corrections even w/ $L \times M_\pi \gtrsim 4$
- Must be checked with other lattice parameters and, in particular, $M_\pi = 135\,\mathrm{MeV}$

HVP from LQCD: summary

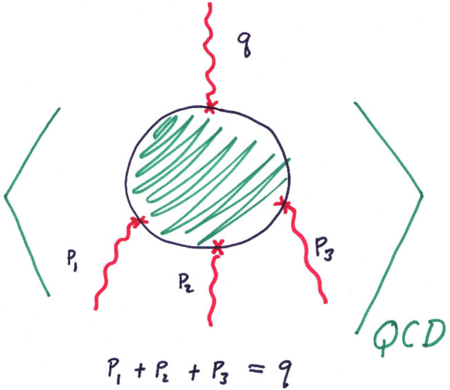


HLbyL from LQCD: introduction



- Models: $(105 \pm 25) \times 10^{-11}$ [Prades et al '09, Benayoun et al '14]
  $(116 \pm 40) \times 10^{-11}$ [Jegerlehner et al '09]
- Dispersive approach difficult, but progress being made [Colangelo et al '14-'15, Pauk et al '14]
- First lattice QED-nonpertubative (npQED) + QCD calculation [Blum et al '15]
- Rapid progress w/ lattice perturbative-QED (pQED) + QCD calculations [Jin et al '15]
- New HLbyL scattering lattice QCD calculation [Mainz '15]
- Hugely more complicated than HVP on lattice but need only 1/20th of the precision

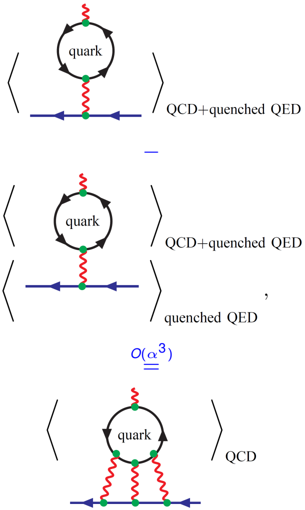
HLbyL from LQCD: conventional approach



- Correlator of 4 EM currents $\Pi^{\mu\nu\rho\sigma}(q, p_1, p_2)$
- 2 loop momenta & q
- Compute for all possible p_1 & p_2 ($O(V_4^2)$) and up to 256 index combinations . . .
- . . . for several q to allow $q \to 0$ . . .
- . . . and for several multiply-disconnected contractions . . .
- . . . and fit and plug into 2-loop QED integrals!!

HLbyL from LQCD: npQED approach

Blum et al '05-'15



- Compute and attach "by hand" $\langle J_\nu^{\rm EM}(y) J_\mu^{\rm EM}(x)\rangle$ to $\langle \mu(p) J_\alpha(z) \mu^\dagger(p')\rangle$ with γ line $D_{\alpha\nu}(z,y)$
- Integrate over y & z and sum over ν & α

$\rightarrow$ q coupled to g and to quenched γ

$\rightarrow$ μ coupled to quenched γ only

- Average over combined gluon & photon configurations
- Subtraction term is product of separate averages of q loop and μ line
- Difference is HLbyL up to $O(\alpha^4)$ corrections
- Gauge configurations identical in both
  $\Rightarrow$ high correlation should allow isolating $O(\alpha^2)$-suppressed difference
- Of course, need quark-disconnected contributions . . .

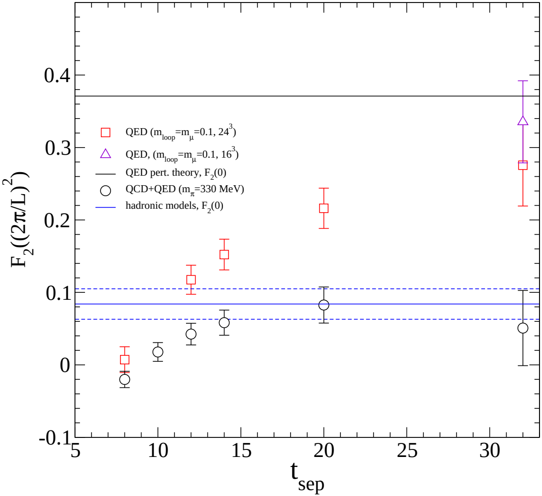
HLbyL from LQCD: npQED results

- Quark-connected part of LbyL in quenched QED & of HLbyL
- $M_\pi = 330\text{MeV}$, $a = 0.11\ \text{fm}$, $24^3 \times 64$, $L = 2.6\ \text{fm}$, $m_\mu = 190\ \text{MeV}$
- $Q = 2\pi/L = 470\ \text{MeV}$ and have to extrapolate to $Q = 0$
- Large excited state contamination
- Signal and right ballpark
- $O(\alpha^2)$ noise & $O(\alpha^4)$ corrections
- Stat. error only
- Large FV effects in presence of QED

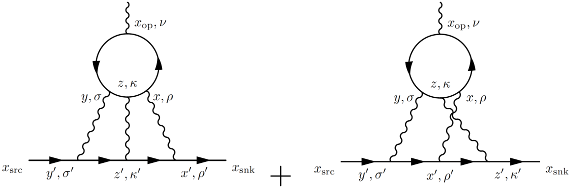
$\rightarrow$ promising, but still a long way to go

Blum et al '15

$a_\mu^{\text{HLbyL}} = F_2(0)$



HLbyL from LQCD: pQED approach [Lin et al ’15]



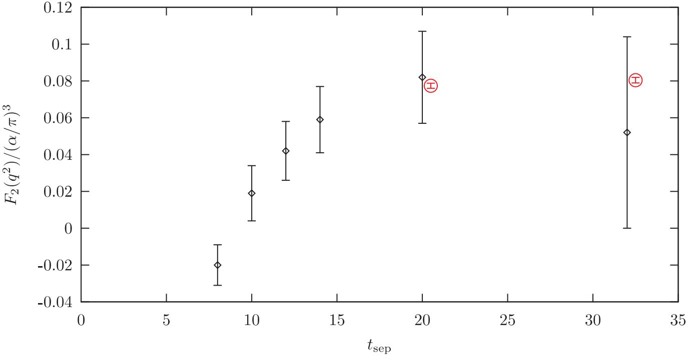
+ 4 more

- Work in position space
- Compute quark loop non-perturbatively
- Photons & muons on lattice but input tree-level propagators “by hand”
- Do two QED loop integrals stochastically: quark loop exponentially suppressed in $|x-y|$ $\rightarrow$ focus on $|x-y| \lesssim \lambda_\pi$ w/ importance sampling
- Use moments to get $a_\mu^{\rm HLbyL} = F_2(0)$ directly
- Large FV effects in presence of QED
- Technically very challenging . . . even before mentioning quark-disconnected contributions

HLbyL from LQCD: pQED approach [Lin et al '15]

Quark-connected HLbyL new pQED vs older npQED results

$M_\pi = 330\text{MeV}$, $a = 0.11\,\text{fm}$, $24^3 \times 64$, $L = 2.6\,\text{fm}$, $m_\mu = 190\,\text{MeV}$, $Q = 2\pi/L = 470\,\text{MeV}$



- Dramatic improvement: statistical error reduced by factor 10
- Strong check on methods

Conclusion

- New $(g-2)_\mu$ experiments are expected to reduce error by 4
- If central values remain the same and $\delta a_\mu^{\rm SM}$ is halved
  $\Rightarrow \sim 8 \div 11\,\sigma$ deviation from the SM!
- Dominant sources of theory uncertainties are $a_\mu^{\rm HVP,LO}$ and $a_\mu^{\rm HLbyL}$
  $\Rightarrow$ LQCD calculations will be very helpful for reducing these uncertainties
- $\delta a_\mu^{\rm HVP,LO}$ using $e^+e^-, \tau \to \text{hadrons}$ are most precise determinations
  $\to$ LQCD can serve as cross-check (and be competitive in long term)
  - Very high precision needed
  - Huge progress in reducing statistical uncertainty
  - Also in understanding and controlling systematics
  - q-connected s and c contributions computed precisely
  - Physical-quark-mass, large-volume calculation producing first results
  - q-disconnected challenging but small $\sim -1.5\% \times a_\mu^{\rm HVP,LO}$
  - Can expect fully controlled $1 \div 2\%$ results soon
    $\to$ valuable cross-check of phenomenological determination
  - To go beyond, need to include isospin breaking corrections, larger volumes, ...
  - ... and improved algorithms to reduce statistical errors

    $\to$ need new ideas and numerical techniques

Conclusion

- It should be possible to reduce $\delta a_\mu^{\mathrm{HLbyL}}$ by 1/2 in $\sim$ 2 years using LQCD, models and experiment for $\pi\gamma^*\gamma$ (KLOE)
  $\to$ LQCD is vital here
  - Precision required here much less great ($\sim$ 10%)
  - First LQCD calculations for connected part very promising ($\sim$ 5% stat. err. for near physical pions)
  - Should be possible to reach precision goal in near future, at least for q-connected part
  - Disconnected contribution even more challenging

$\Rightarrow$ The theory should follow and we all look forward to compare its predictions w/ E989 and J-PARC E34 measurements