Gravitational Collapse and Primordial Black Hole formation

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WORKSHOP DM and Stars

Jussieu (Paris) - 8 June 2016

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THE PRIMORDIAL BLACK HOLE MASS SPECTRUM*

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Received 1975 January 31

ABSTRACT

We examine what mass spectrum of primordial black holes should result if the early universe consisted of small density fluctuations superposed on a Friedmann background. It is shown that only a certain type of fluctuation favors the formation of primordial black holes and that, consequently, their spectrum should always have a particular form. Since both the fluctuations which arise naturally and the fluctuations which are often invoked to explain galaxy formation are of the required type, primordial black holes could have had an important effect on the evolution of the universe. In particular, although primordial black holes are unlikely to have a critical density, big ones could have been sufficiently numerous to act as condensation nuclei for galaxies. Observational limits on the spectrum of primordial black holes place strong constraints on the magnitude of density fluctuations in the early universe and support the assumption that the early universe was nearly Friedmann rather than chaotic. Any model in which the early universe has a soft equation of state for a prolonged period is shown to be suspect, since primordial black holes probably form too prolifically in such a situation to be consistent with observation.

Subject headings: black holes — cosmology — galaxies

I. INTRODUCTION AND SUMMARY

In a previous paper (Carr and Hawking 1974) it was shown that black holes could have formed at very early stages in the history of the universe as a result of initial inhomogeneities. It was also shown that these "primordial" black holes would not have grown very much through accretion and so their masses today should be about the same as when they first formed. Recently, however, Hawking has made the striking prediction (Hawking 1974, 1975) that, because of quantum effects, any black hole should emit particles like a blackbody with a temperature inversely proportional to its mass. Despite the important conceptual change which Hawking's result introduces in the context of black holes in general, probably only a primordial black hole could be sufficiently small for the effect to be important. Hawking's prediction implies that any primordial black holes of less than 10^{15} g should have evaporated by now and raises the question of whether any primordial black holes could still exist.

This motivates a discussion of the expected mass spectrum of primordial black holes. (Henceforth a primordial

Outline

- Schwarzschild black hole
- perfect fluid, spherical symmetry
- Causal horizons: event/apparent horizon, cosmological horizon
- Equation of state (zero rest mass): PBH formation
- Equation of state (Non zero rest mass): virialized structure,
- Critical collapse and self similarity

Introduction

- *Schwarzschild* metric: **static black hole**, location of the event horizon ***R=2M***

$$ds^2 = -\left(1-\frac{2M}{R}\right)dt^2 + \left(1-\frac{2M}{R}\right)^{-1} dR^2 + R^2 d\Omega^2$$

- The most general **spherical symmetric** metric (*Misner-Sharp* approach):

$$ds^2 = -a^2\, dt^2 + b^2\, dr^2 + R^2 d\Omega^2$$

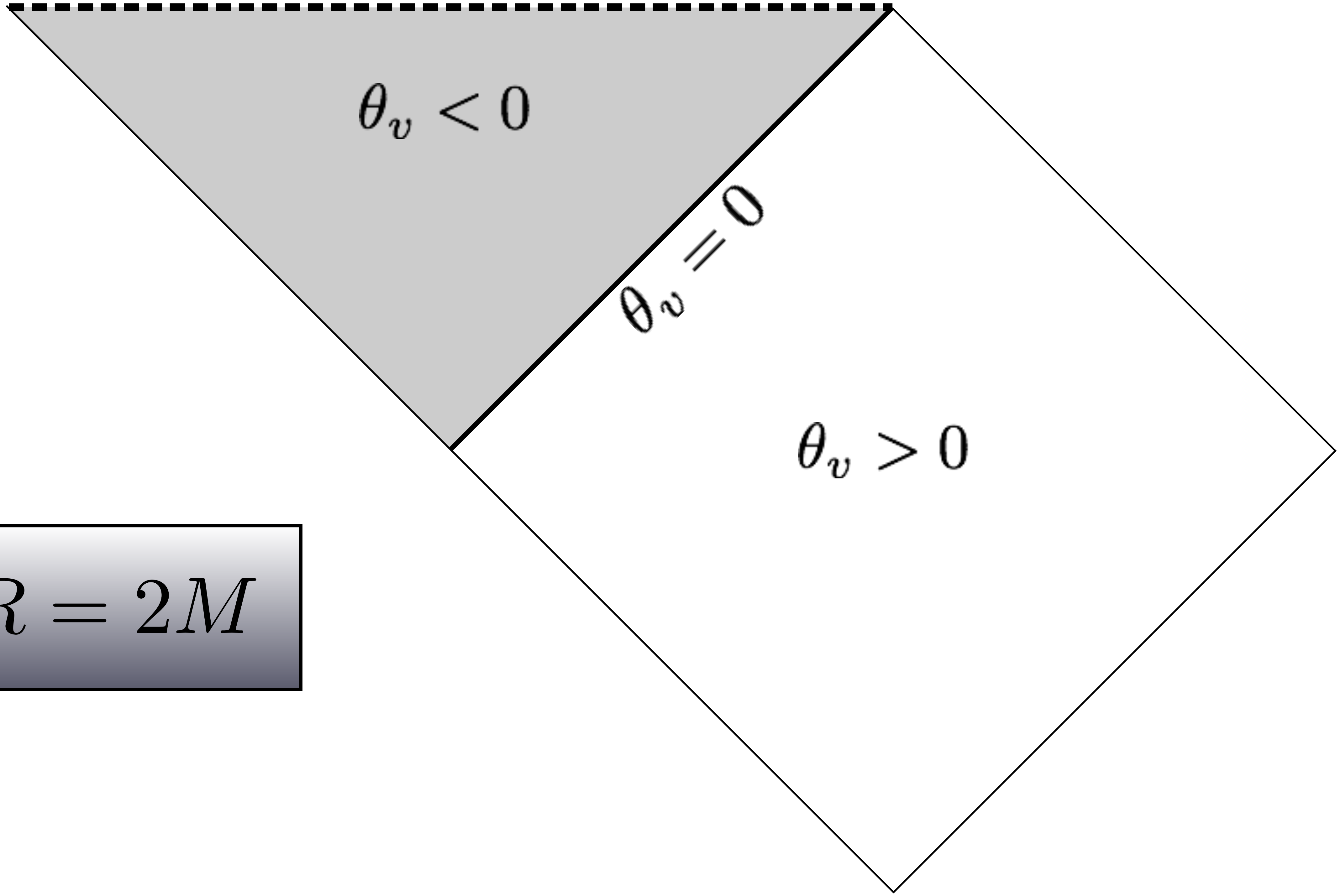
- Metric functions *a*, *b*, and *R* are functions of the comoving coordinate *r* and the so called **cosmic time** *t:* **time slicing** to keep the metric diagonal (gauge choice). The radius *R* is the **circumferential radial coordinate (area radius)**.

- Observer **comoving** with the fluid. $T^{\mu\nu} = (e+p)u^\mu u^\nu + p g^{\mu\nu}$

- **Proper time** and **proper space** operators:

$$D_t \equiv \frac{1}{a}\frac{\partial}{\partial t} \Rightarrow U \equiv D_t R \qquad D_r \equiv \frac{1}{b}\frac{\partial}{\partial r} \Rightarrow \Gamma \equiv D_r R$$

Schwarzschild Black Hole space-time



$$R = 2M$$

Constraint Equation

- For a general perfect fluid, the G_{00} and G_{11} components of the Einstein field equations are

$$(G^0_0) \qquad 4\pi R^2 e R_r = (R + RU^2 - R\Gamma^2)_r/2$$

$$(G^1_1) \qquad 4\pi R^2 a p U = -(R + RU^2 - R\Gamma^2)_t/2$$

- It is convenient to define $M = (R + RU^2 - R\Gamma^2)/2$ where

$$M = \int 4\pi R^2 e dR$$

$$D_t M = -4\pi R^2 p U$$

and

$$\boxed{\Gamma^2 = 1 + U^2 - \frac{2M}{R}}$$

Constraint equation with the Misner-Sharp Mass M measuring the mass inside radius R. The second equation describes the adiabatic expansion or contraction of the fluid.

Trapping Horizons

- **Black Hole horizons :** $\theta_+ = 0 \;\Rightarrow\; \left.\frac{1}{a}\frac{dR}{dt}\right|_+ = (U + \Gamma) = 0$

- **Cosmological horizon :** $\theta_- = 0 \;\Rightarrow\; \left.\frac{1}{a}\frac{dR}{dt}\right|_- = (U - \Gamma) = 0$

$\Gamma^2 = U^2$ inserted into $\Gamma^2 = 1 + U^2 - \frac{2M}{R}$

$$\Downarrow$$

$$\boxed{R = 2M}$$

The horizon condition is independent of the slicing and holds also within a non-vacuum moving medium

- The so-called **apparent horizon** of a black hole (which is a future trapping horizon) is the **outermost trapped surface for outgoing radial null rays** while the **trapping horizon for an expanding universe** (which is a past trapping horizon) **is foliated by the innermost anti-trapped surfaces for ingoing radial null rays**.

Background model & Initial Conditions for PBHs

- The **unperturbed solution,** describing an expanding homogeneous universe**,** is given by the FRW metric: $K = \pm 1, 0$ is the **curvature parameter**, $\tilde{a}(t)$ is the **scale factor**, and $R = \tilde{a}(t)r$ is the **circumferential radial coordinate**.

$$ds^2 = -dt^2 + \tilde{a}(t)[\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2]$$

- In the linear regime of cosmological perturbations, pure growing modes on the super horizon scale can be described by a time independent curvature profile (**quasi-homogeneous / gradient expansion solution**).

$$K(r) \quad \text{or} \quad \zeta(\tilde{r})$$

$$\sqrt{1 - K(r)r^2} = 1 + \tilde{r}\zeta'(\tilde{r}) \qquad\qquad r = \tilde{r}e^{\zeta(\tilde{r})}$$

- In case of BH formation, to study the full evolution of the external fluid

$$f\,du = a\,dt - b\,dr \qquad\qquad ds^2 = -f^2\,du^2 - 2fb\,dr\,du + R^2 d\Omega$$

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \qquad T_{\mu\nu} = (e+p)u_\mu u_\nu - p g_{\mu\nu}$$

COSMIC TIME

$$D_t \equiv \frac{1}{a}\left(\frac{\partial}{\partial t}\right) \qquad D_r \equiv \frac{1}{b}\left(\frac{\partial}{\partial r}\right)$$

$$U \equiv D_t R \qquad \Gamma \equiv D_r R$$

$$D_t U = -\left[\frac{\Gamma}{(e+p)} D_r p + \frac{M}{R^2} + 4\pi R p\right]$$

$$D_t \rho = -\frac{\rho}{\Gamma R^2} D_r (R^2 U)$$

$$D_t e = \frac{e+p}{\rho} D_t \rho$$

$$D_t M = -4\pi R^2 p U$$

$$D_r a = -\frac{a}{e+p} D_r p$$

$$D_r M = 4\pi R^2 \Gamma e$$

$$\Gamma^2 = 1 + U^2 - \frac{2M}{R}$$

NULL TIME

$$D_t \equiv \frac{1}{f}\left(\frac{\partial}{\partial u}\right) \qquad D_k \equiv D_r + D_t$$

$$D_t U = -\frac{1}{1-c_s^2}\left[\frac{\Gamma}{(e+p)} D_k p + \frac{M}{R^2} + 4\pi R p + \right.$$
$$\left. + c_s^2\left(D_k U + \frac{2U\Gamma}{R}\right)\right]$$

$$D_t \rho = \frac{\rho}{\Gamma}\left[D_t U - D_k U - \frac{2U\Gamma}{R}\right]$$

$$D_t e = \left(\frac{e+p}{\rho}\right) D_t \rho$$

$$D_t M = -4\pi R^2 p U$$

$$D_k\left[\frac{(\Gamma + U)}{f}\right] = -4\pi R (e+p) f$$

$$D_k M = 4\pi R^2 [e\Gamma - pU],$$

$$\Gamma = D_k R - U = 1 + U^2 - \frac{2M}{R}$$

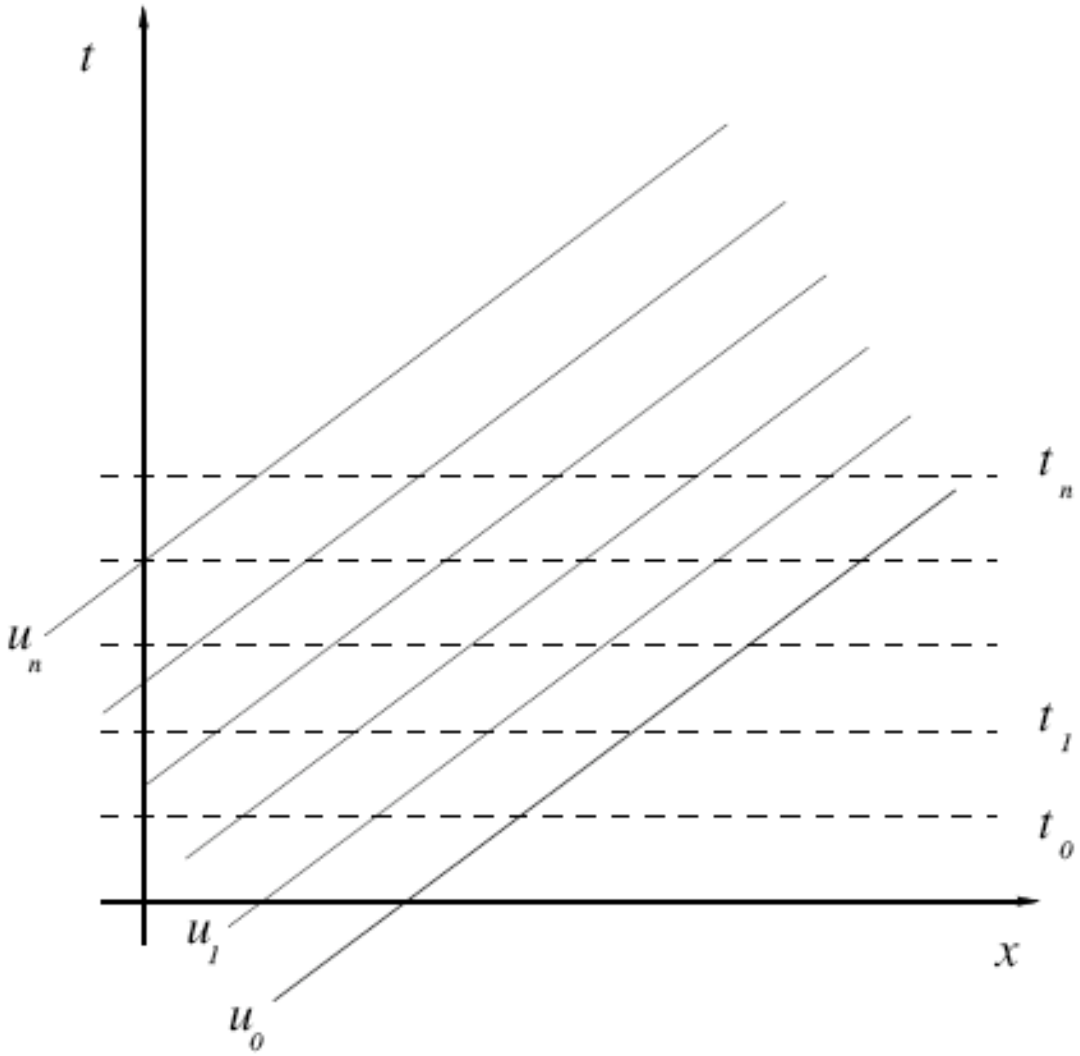
Numerical Results: the method

- Simulations are performed using a **Lagrangian spherically symmetric GR hydro code with an adaptive grid** (AMR).

- We set initial conditions using a **cosmic time coordinate** t.

- We transfer those onto a **null foliation** of the space time, then evolved using an **observer time coordinate** u.

- The **formation of a PBH is seen by a distant external observer** (the singularity is hidden by the asymptotic formation of the apparent horizon).



PBH formation: setting the problem

- Spherical symmetry $$ds^2 = -a^2\,dt^2 + b^2\,dr^2 + R^2 d\Omega^2$$

- Barotropic equation of state: $$p = we$$

- Initial conditions: linear supra-horizon perturbation $(\epsilon << 1)$ of a FRW universe:

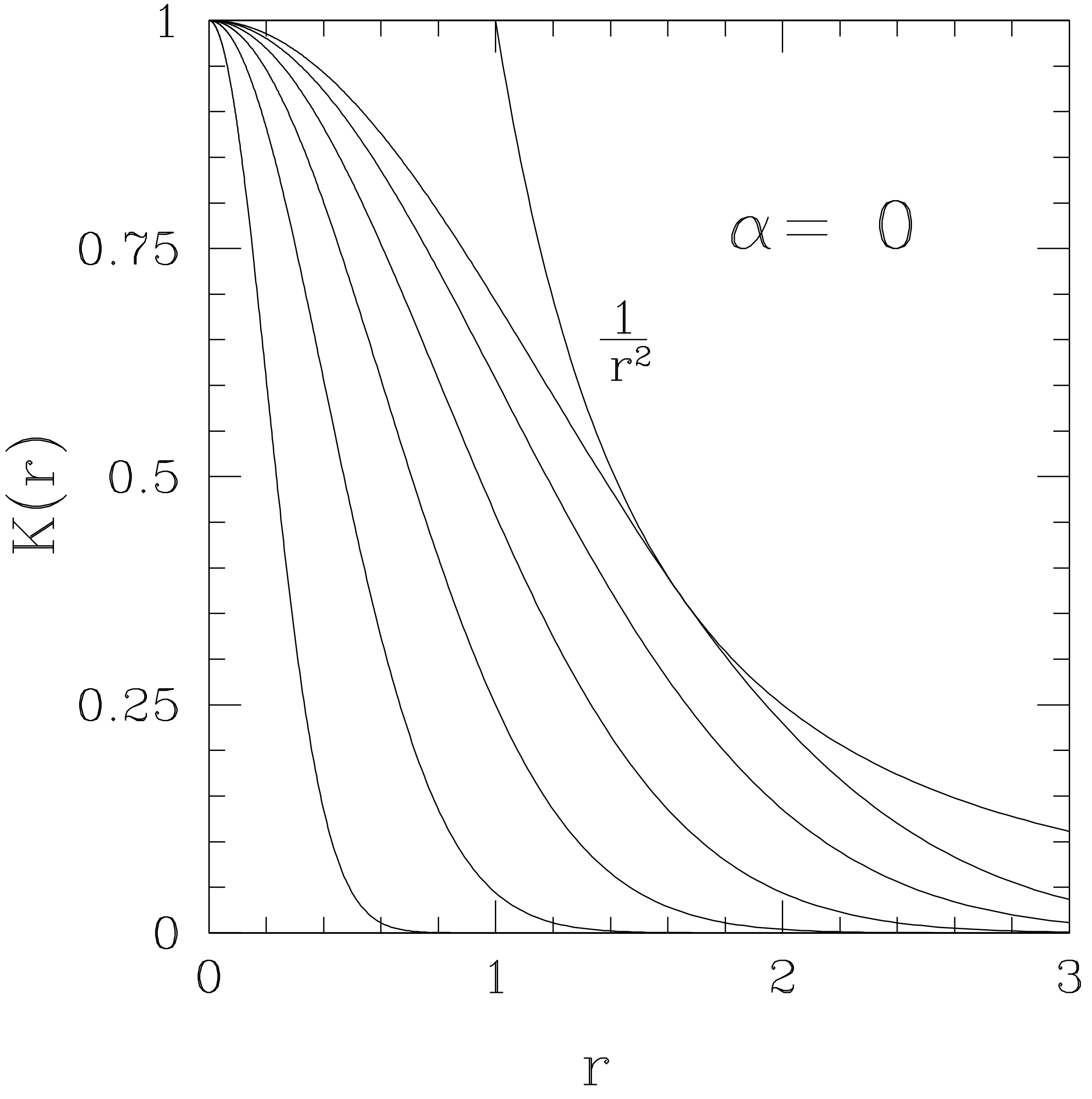
$$b = \frac{\partial_r R}{\sqrt{1 - K(r) r^2}}(1 + \epsilon \tilde{b}) \qquad e = e_b(1 + \epsilon \tilde{e})$$

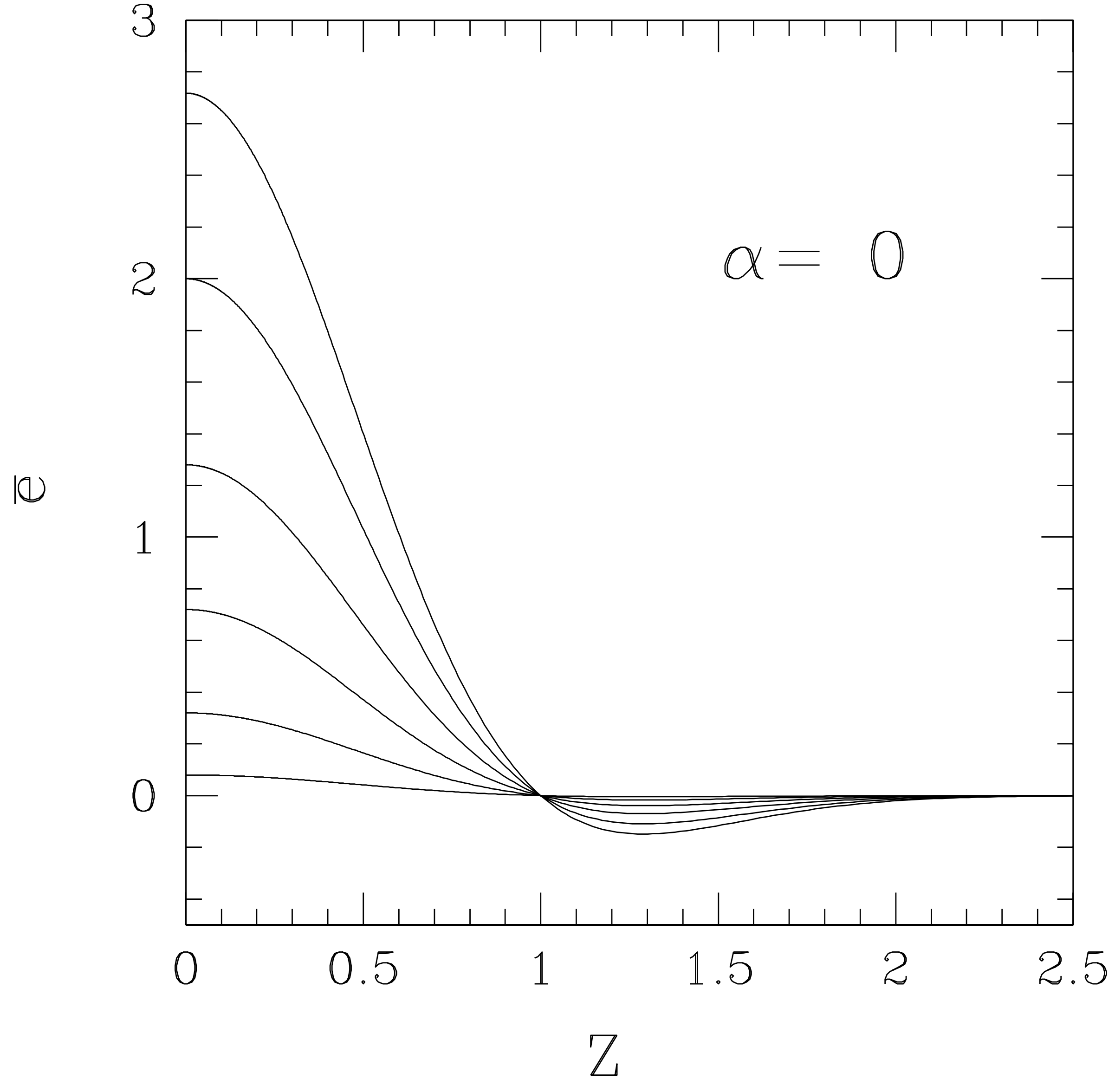
$$\epsilon(t) := \left(\frac{R_H}{R_0}\right)^2 = \epsilon(t_0)\left(\frac{t}{t_0}\right)^{\frac{2(1+3w)}{3(1+w)}} \qquad \tilde{e} = \frac{3(1+w)}{5+3w}\frac{r_0^2}{3r^2}\partial_r\left[K(r)r^3\right]$$

$$K(r) = \exp\left(-\frac{r^2}{2\Delta^2}\right) \qquad \tilde{e} = \frac{3(1+\omega)}{5+3\omega}\Delta^2\left[1 - \left(\frac{R_b}{R_0}\right)^2\right]\exp\left(-\frac{3}{2}\left(\frac{R_b}{R_0}\right)^2\right)$$

$$r_0^2 = 3\Delta^2 \qquad \delta \equiv \left(\frac{4}{3}\pi r_0^3\right)^{-1}\int_0^{r_0} 4\pi \frac{e - e_b}{e_b} r^2 dr = \epsilon(t)\Phi K(r_0) r_0^2$$

- **Gaussian Curvature - Mexican hat energy density perturbation**: the amplitude Δ of the Gaussian profile of $K(r)$ gives a measure of the central peak of the Mexican hat energy density profile $\bar{e}(r)$ that integrated on the 3D spherical volume gives the perturbation amplitude δ.





Equation of State

energy density: $e = \rho(1+\epsilon)$ pressure: $p = (\gamma - 1)\rho\epsilon$

rest mass density

adiabatic index - particle degree of freedom

specific internal energy (velocity dispersion)

- Cosmological fluid (no rest mass density): $p = we$ with $w \in [0,1]$
    - radiation dominated era: $w = 1/3$ RADIATION $(\gamma = 4/3)$
    - matter dominated era: $w = 0$ DUST $(\gamma = 1)$
- Polytropic fluid: $p = K(s)\rho^\gamma$ $(\gamma = 5/3,\ 4/3,\ 2)$
    - If the fluid is adiabatic (no entropy change): $K(s) = K$ (constant)

Structure formation

- Non linear gravitational collapse $\Rightarrow$ structure formation

- Cosmological fluid $p = we$ $\Rightarrow$ **Primordial Black Hole** (PBH) formation. Initial conditions: small super horizon perturbations re-entering the horizon in the radiation dominated era.

- In the matter era the universe is dominated by CDM particles where the pressure is characterised by random motions (**velocity dispersion**) that gives **virialization**.

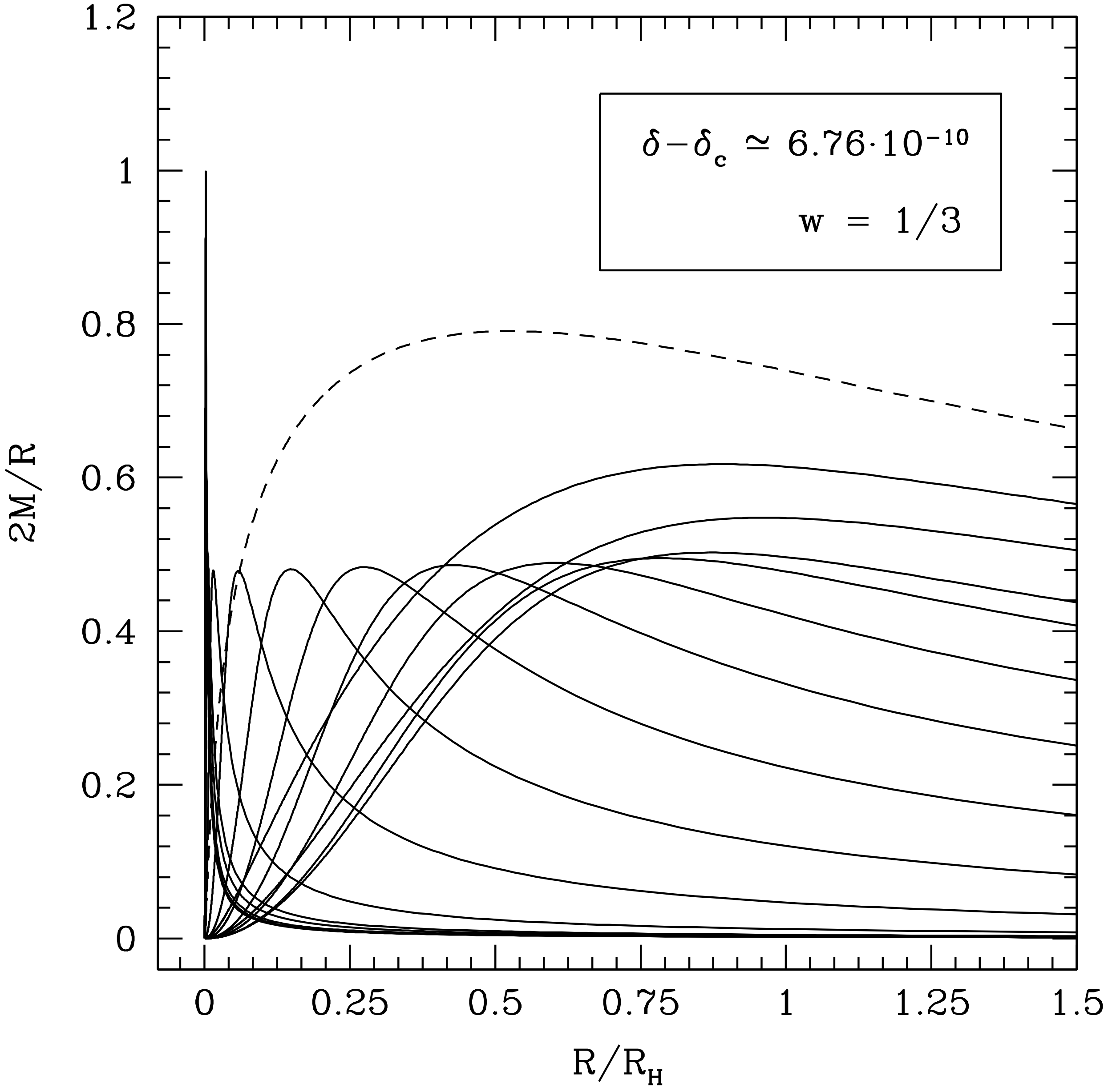
- In a spherically symmetric **Fluid approach** baryoinc matter is characterised by a Mawellian distribution. Non-barotropic equation of state for CDM needs non-Maxwellian velocity dispersion implying non isotropic pressure.

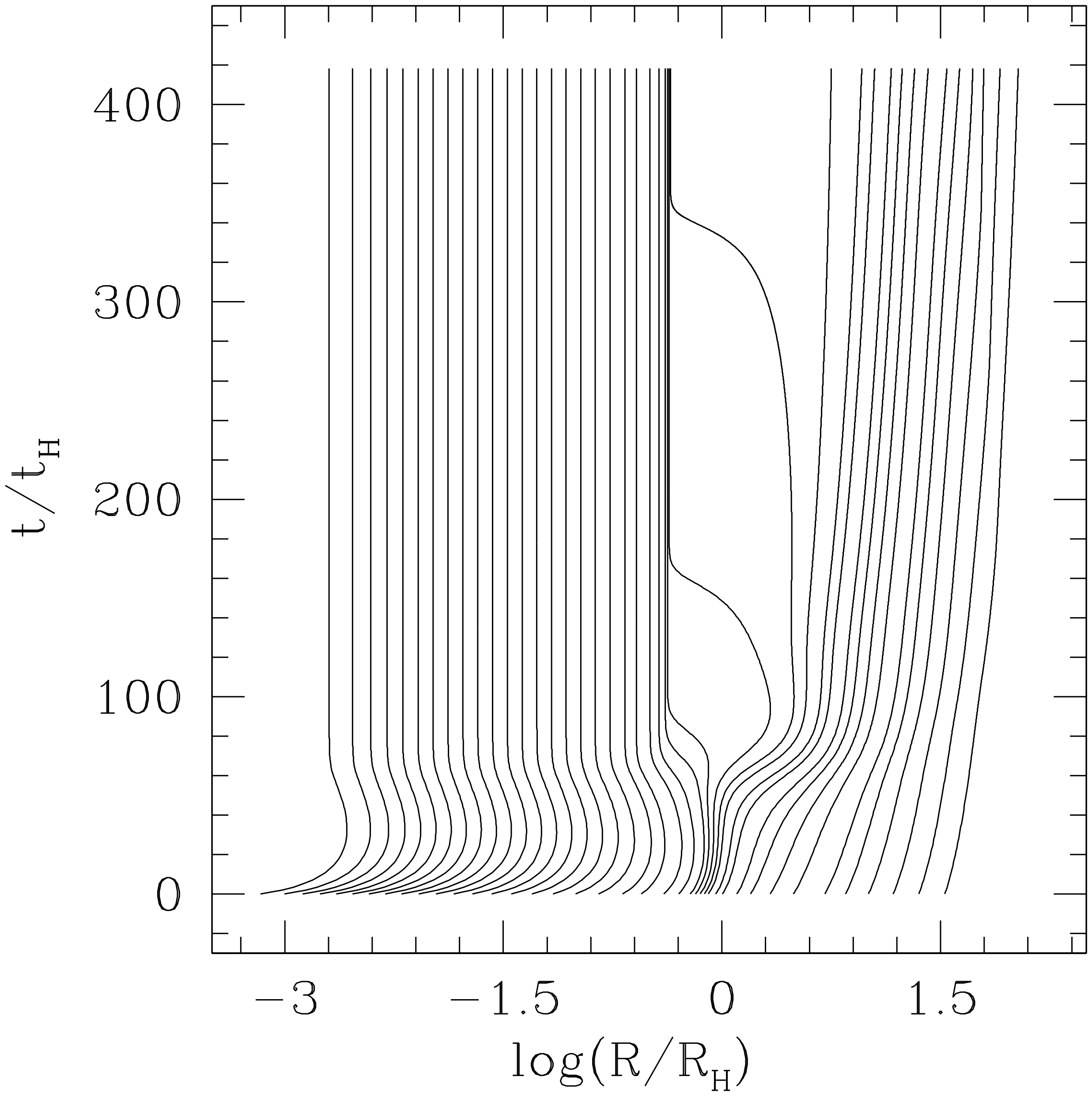
- **Structure formation** is a non relativistic problem that in cosmology is usually studied in 3D using N-body simulations. For relativistic collapse spherical symmetry is very useful. Initial conditions:

  1. Linear cosmological perturbations (early universe).

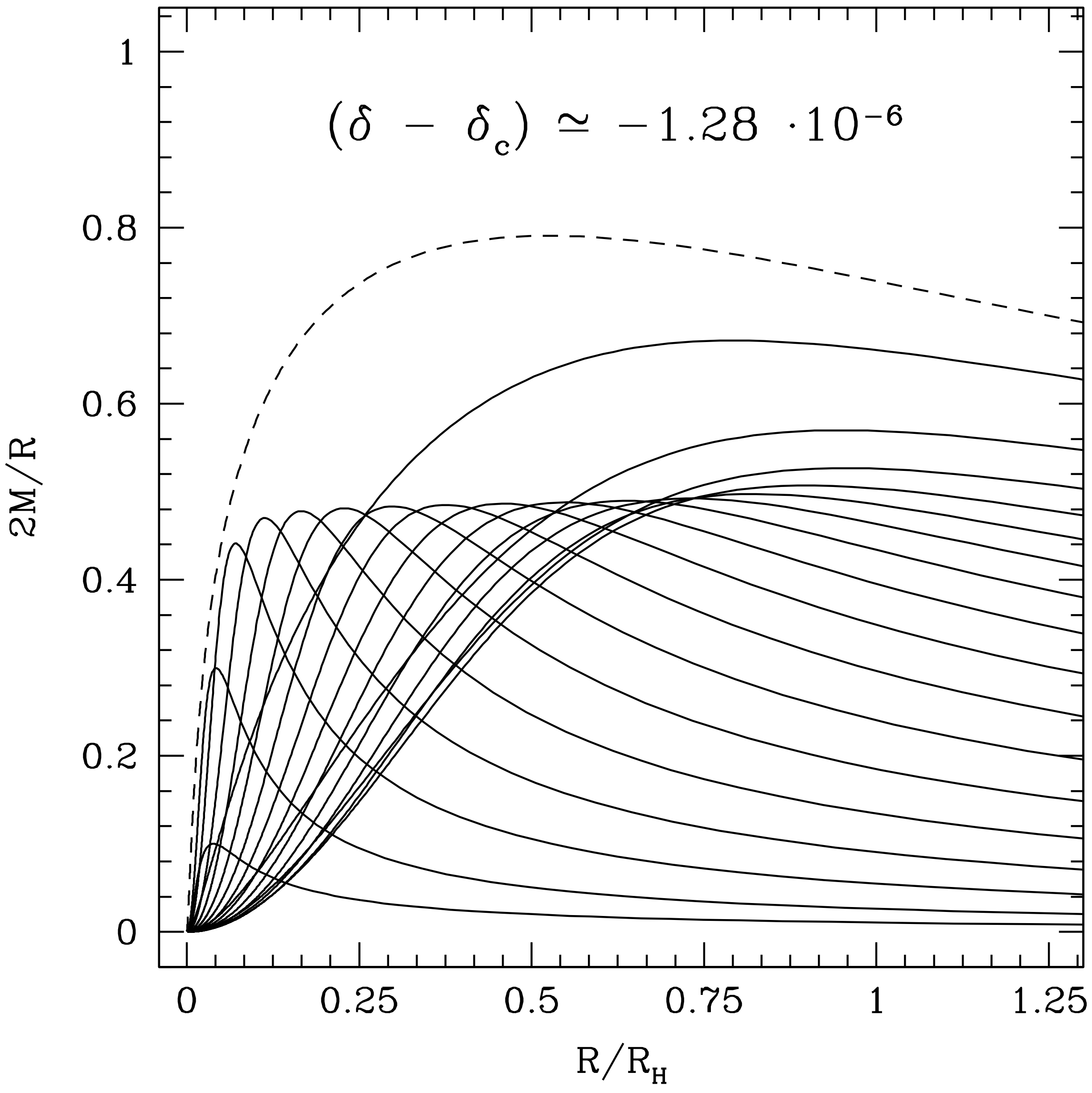
  2. Perturbations of static solutions.

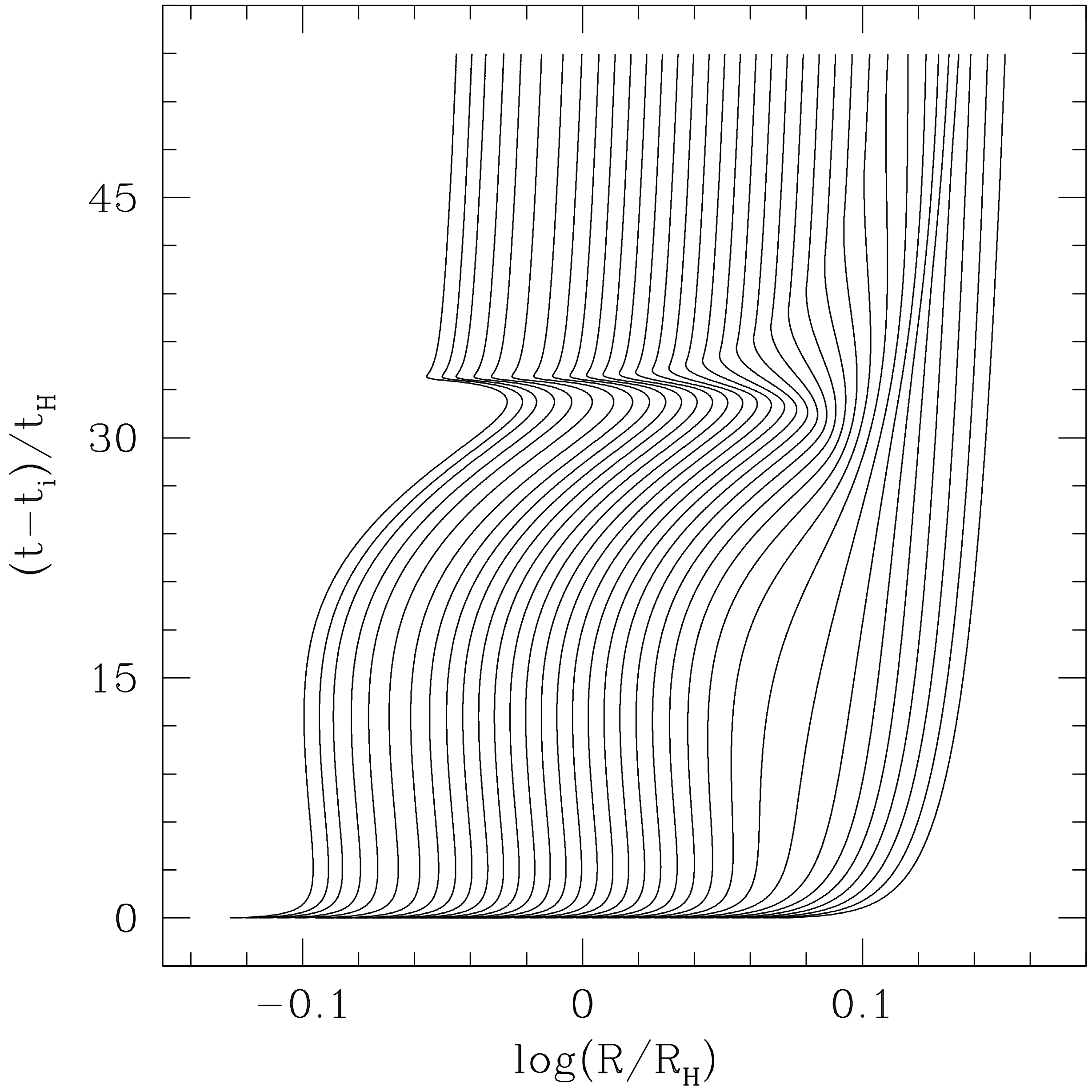
Numerical Results: BH formation



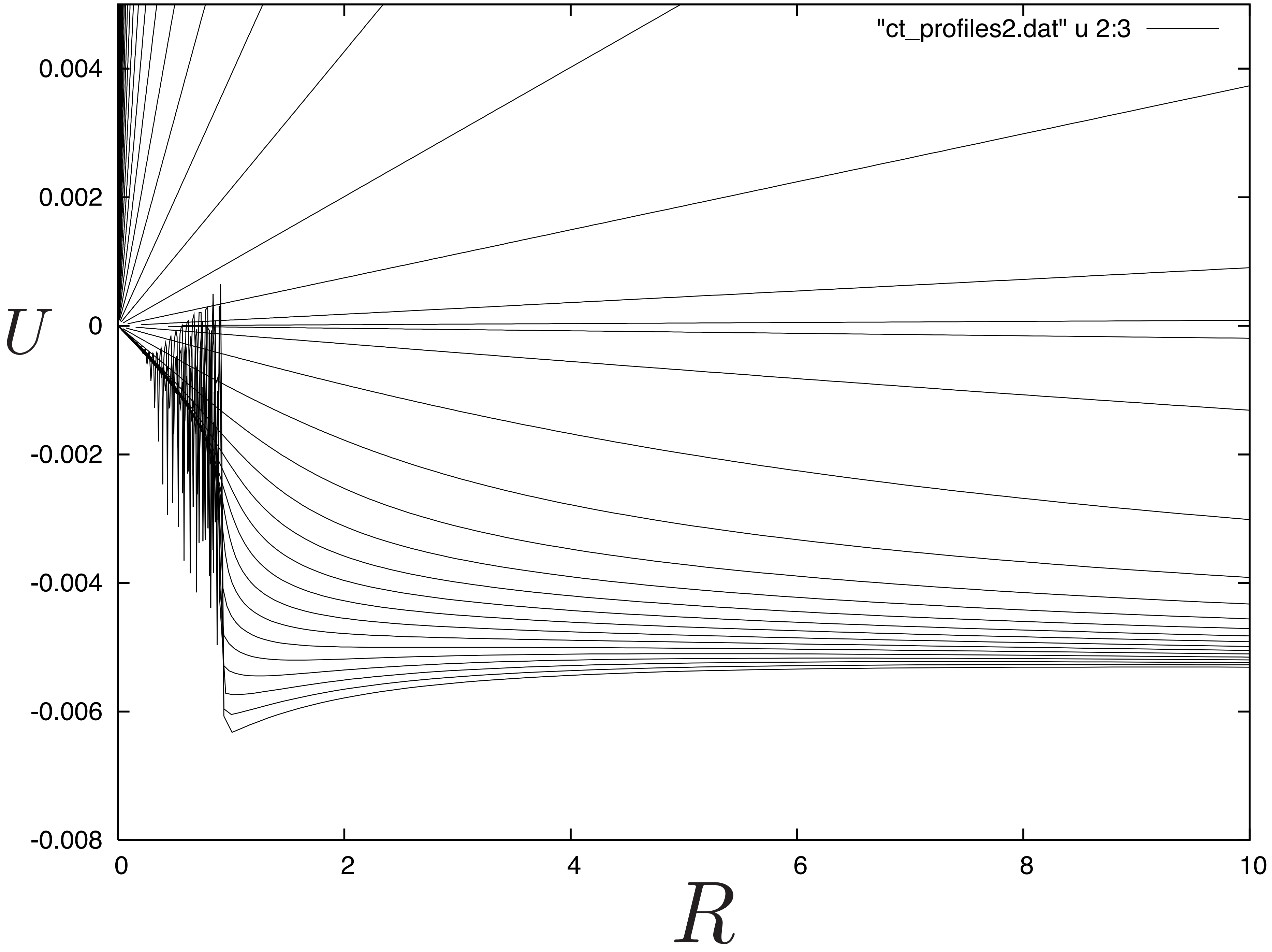


Numerical Results: no-BH formation

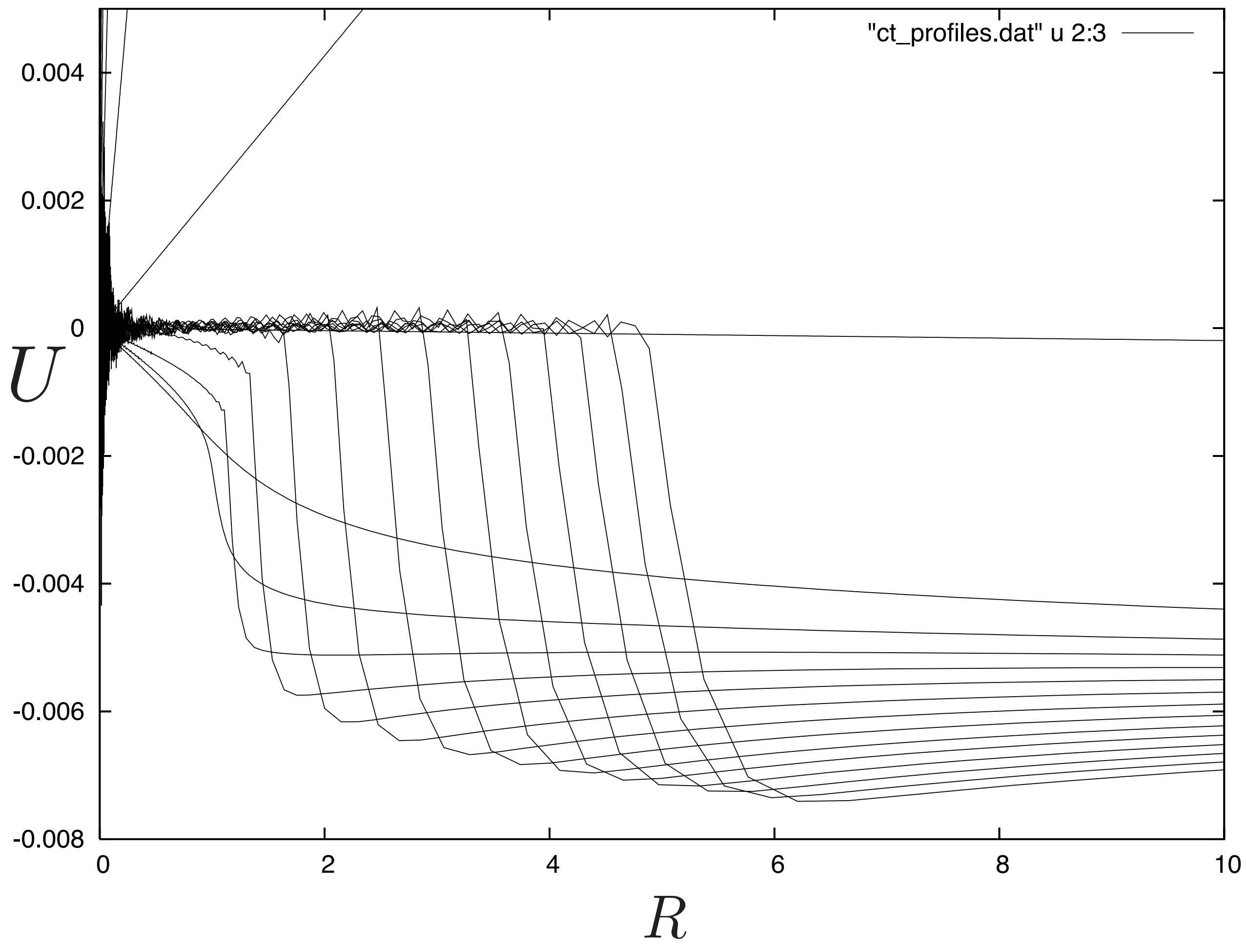


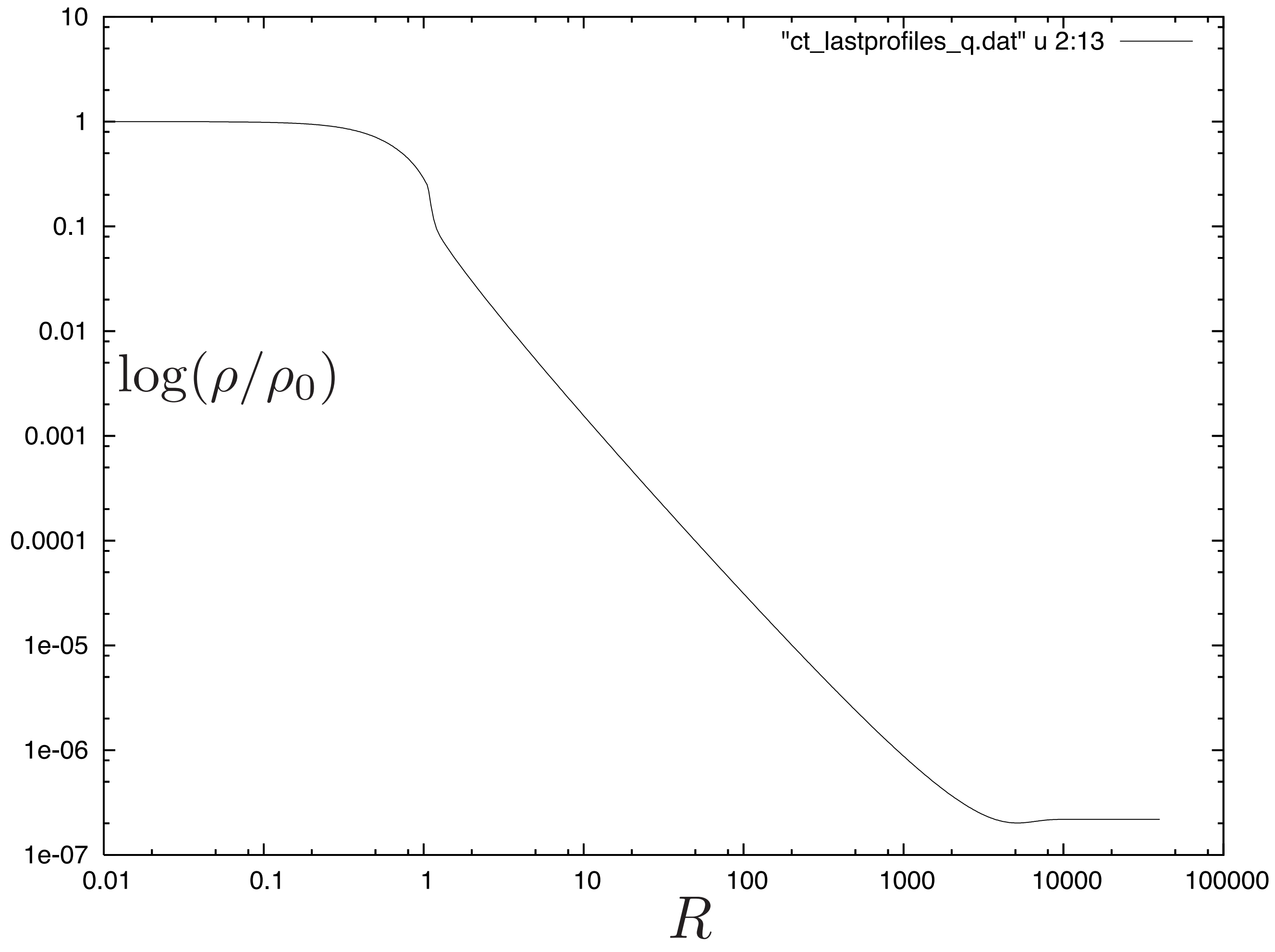


Numerical Results: virialization (no shock treatment)



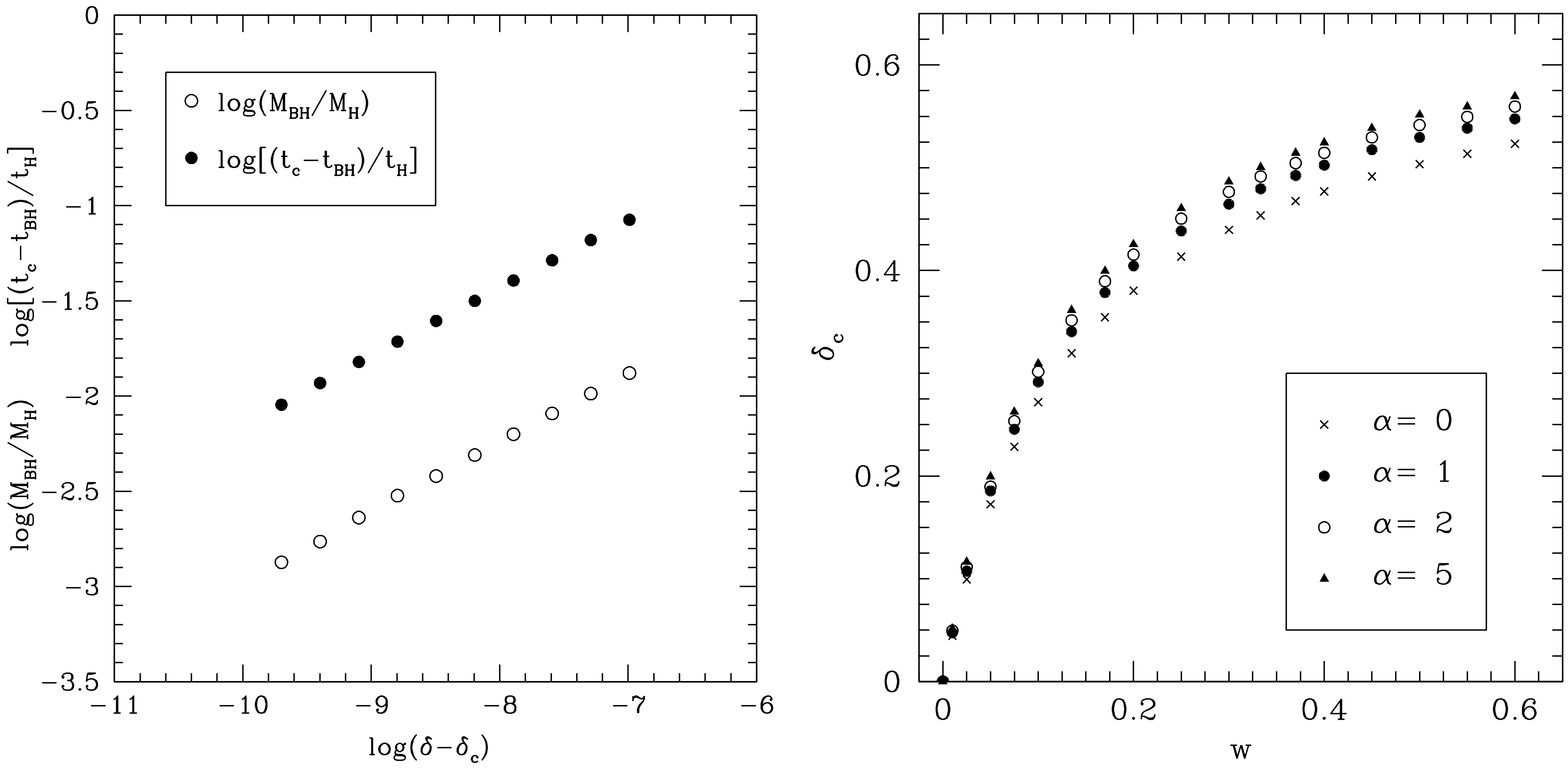
Numerical Results: virialization (artificial viscosity)





Numerical Results: scaling law and threshold

$$M_{BH} = K(\delta - \delta_c)^{\gamma} M_H$$



Self similar equations

The Einstein + fluid equations can be written in a self similar form, using the self similar variables $(U,\ \Omega = 4\pi R^2 e,\ \Phi = M/R,\ \Gamma,\ f)$, function of $\xi = R/(\pm t)$

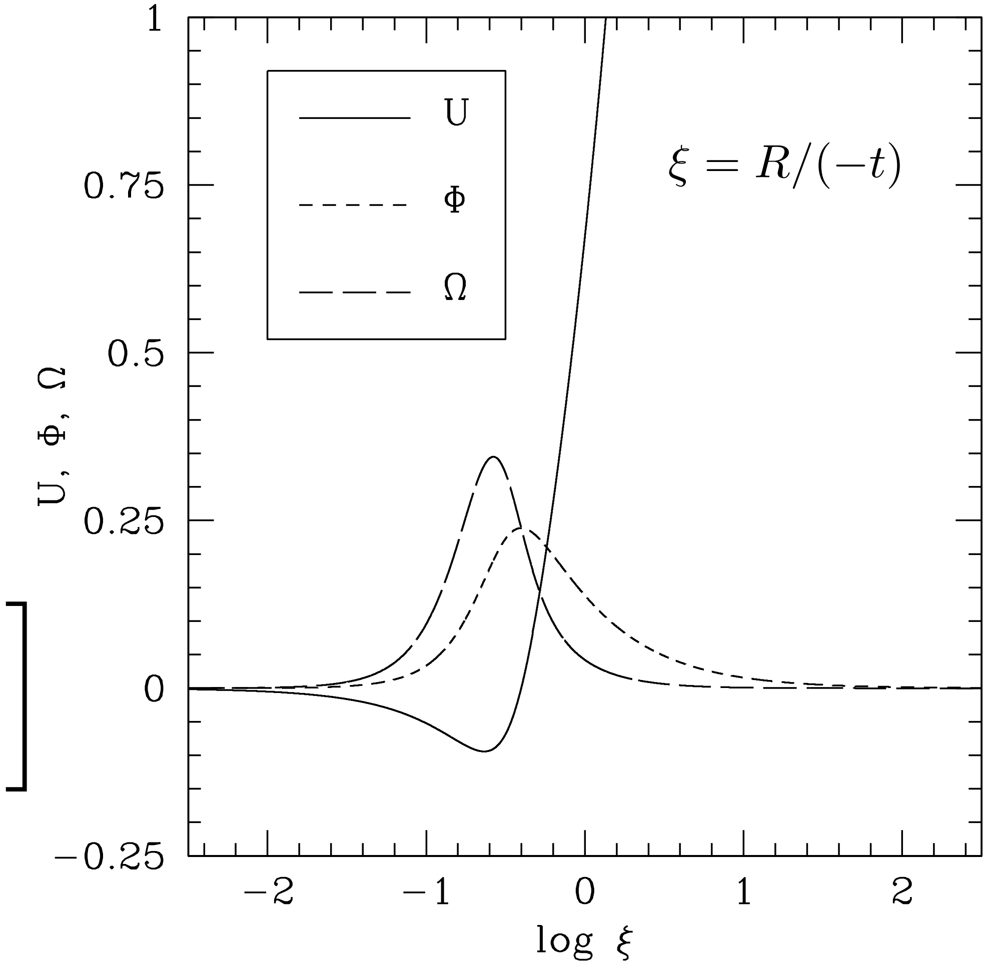
$$\frac{d\ln U}{d\ln\xi} = \left[\frac{(\Phi + w\Omega)^2 - 2w\Gamma^2\Phi}{U^2(\Phi + w\Omega)^2 - w\Gamma^2(\Omega - \Phi)^2}\right]\left[(\Omega - \Phi) - \frac{(1+w)\Omega U}{(\Gamma + U)}\right]$$

$$\frac{d\ln\Omega}{d\ln\xi} = \frac{(1+w)(\Omega - \Phi)}{(\Phi + w\Omega)}\frac{d\ln U}{d\ln\xi} + \frac{2w}{(\Phi + w\Omega)}\left[(\Omega - \Phi) - \frac{(1+w)\Omega U}{(\Gamma + U)}\right]$$

$$\frac{d\ln\Phi}{d\ln\xi} = \frac{1}{\Phi}\left[(\Omega - \Phi) - \frac{(1+w)\Omega U}{(\Gamma + U)}\right]$$

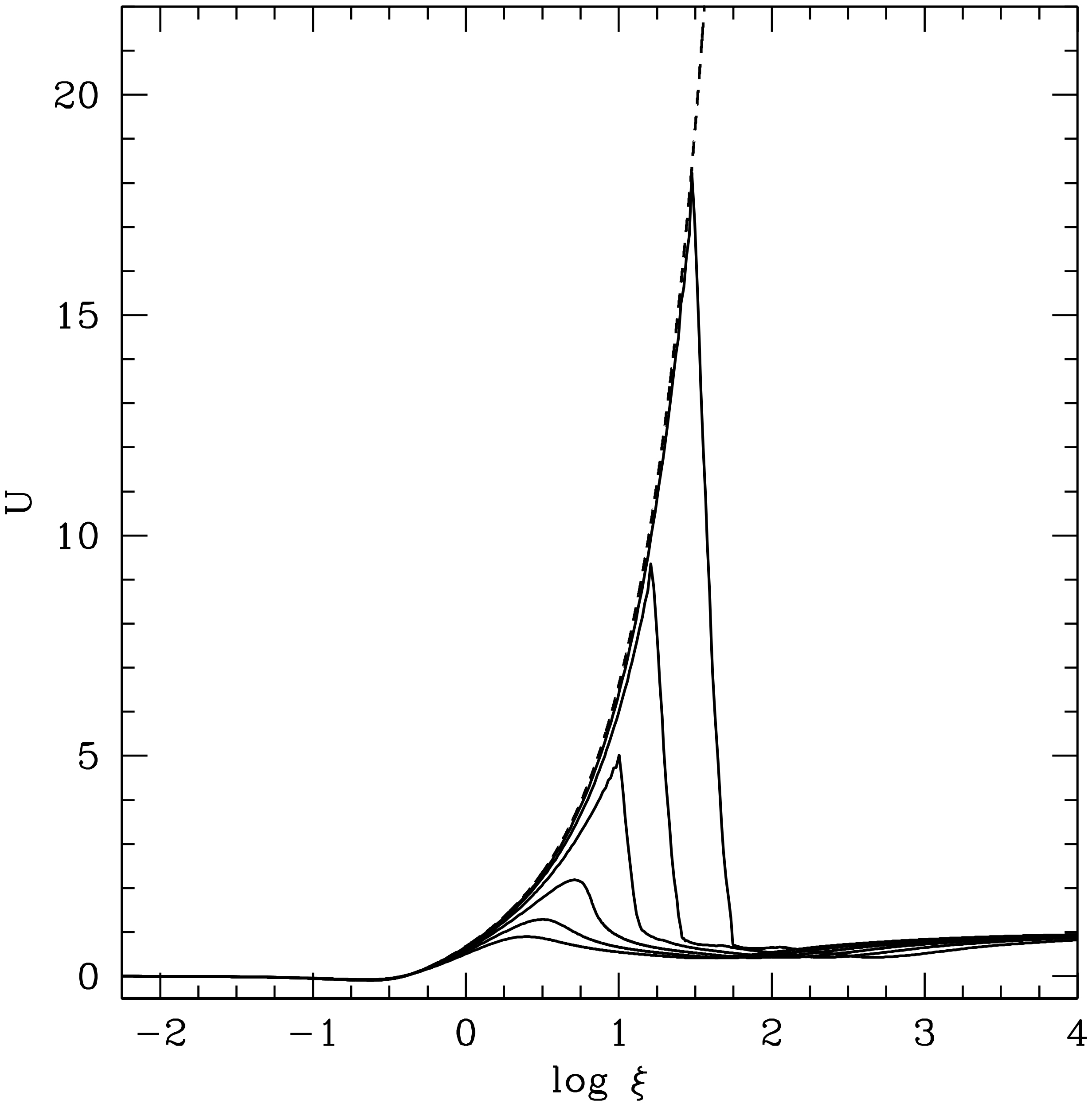
$$\Gamma = 1 + U^2 - 2\Phi$$

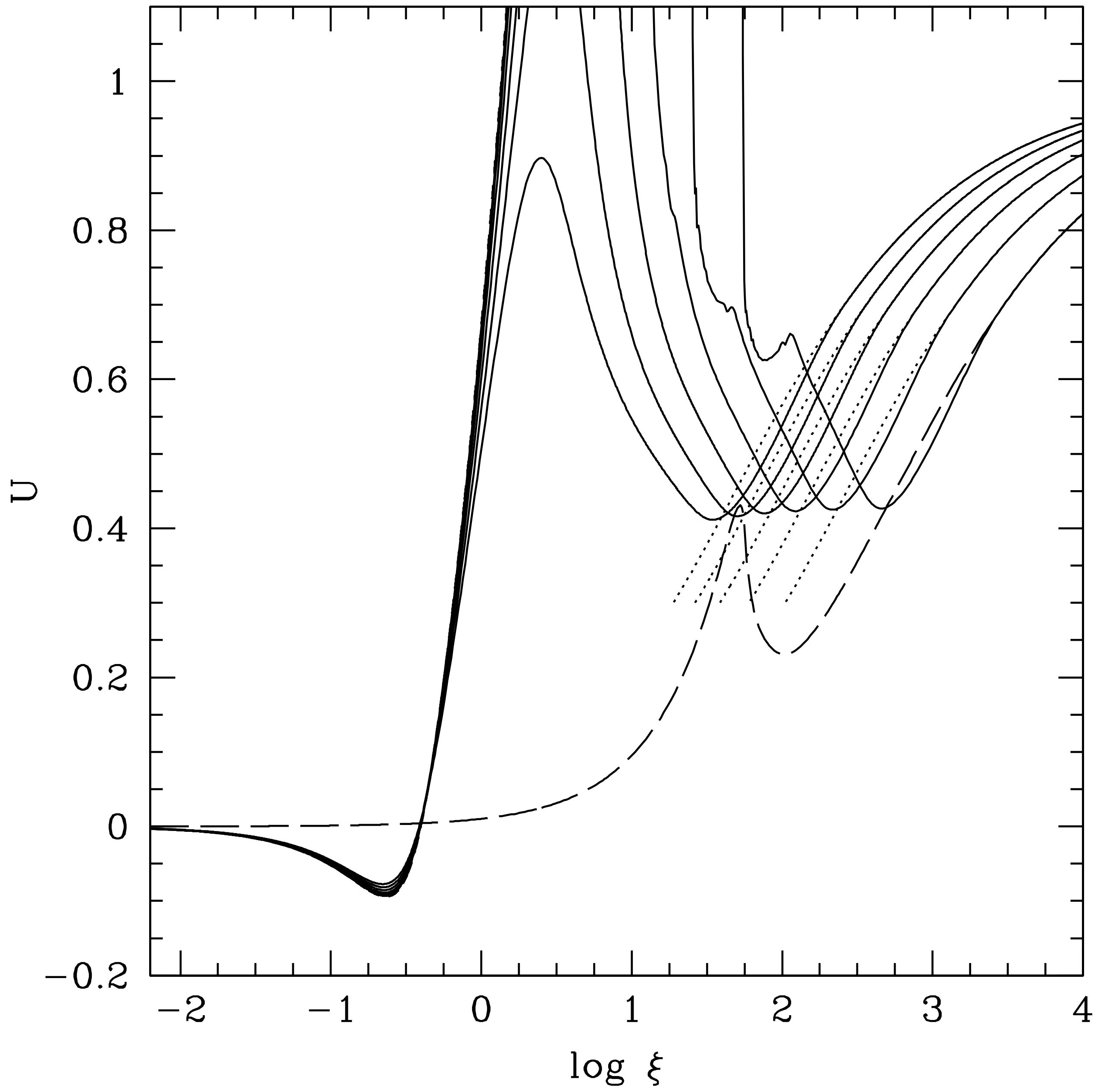
$$f = \pm\frac{\xi}{(1+w)\Omega U}\left[(\Omega - \Phi) - \frac{U}{\Gamma}(\Phi + w\Omega)\right]$$



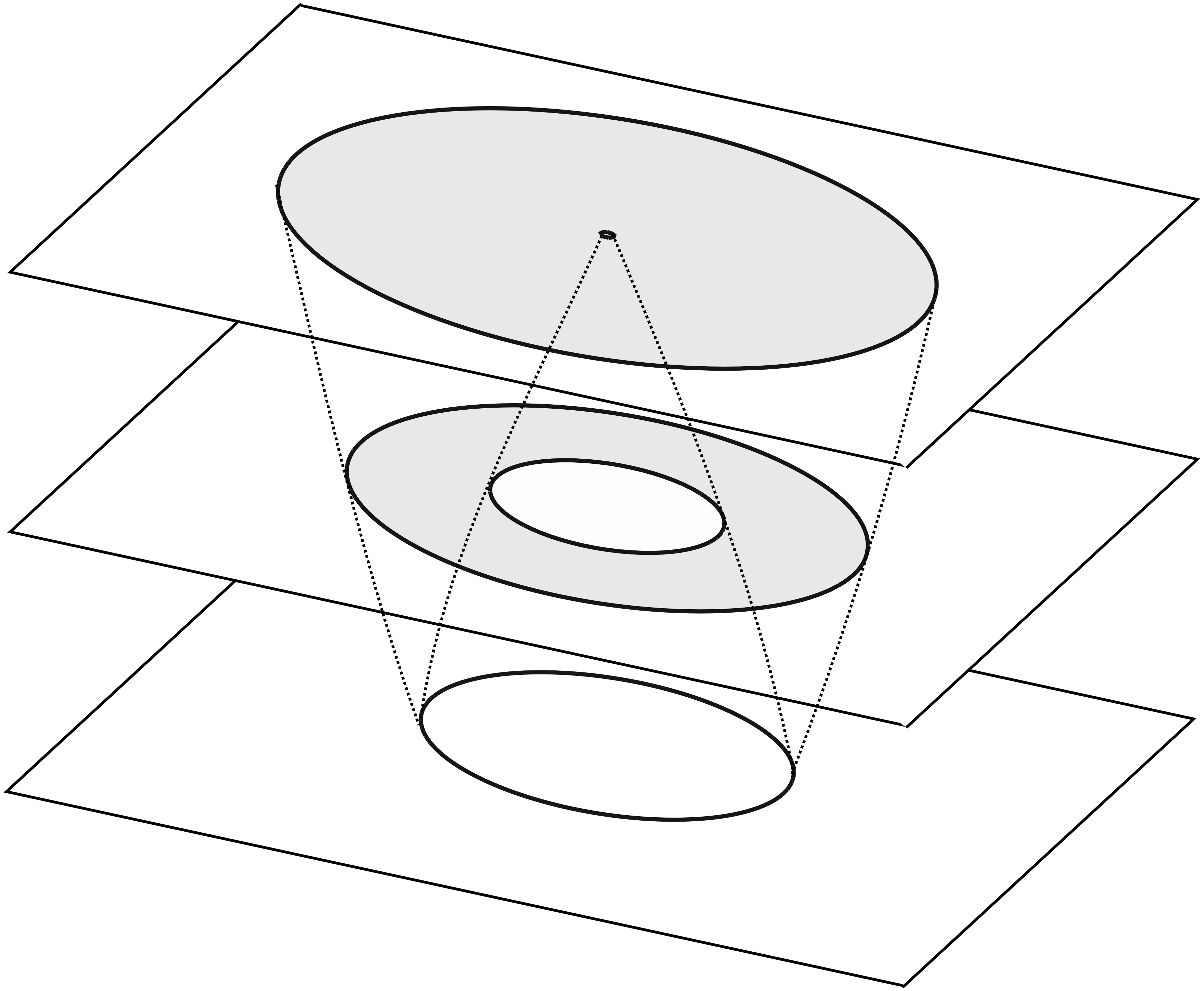
Numerical Results: self similarity (U)

$$\xi = R/(t_c - t)$$



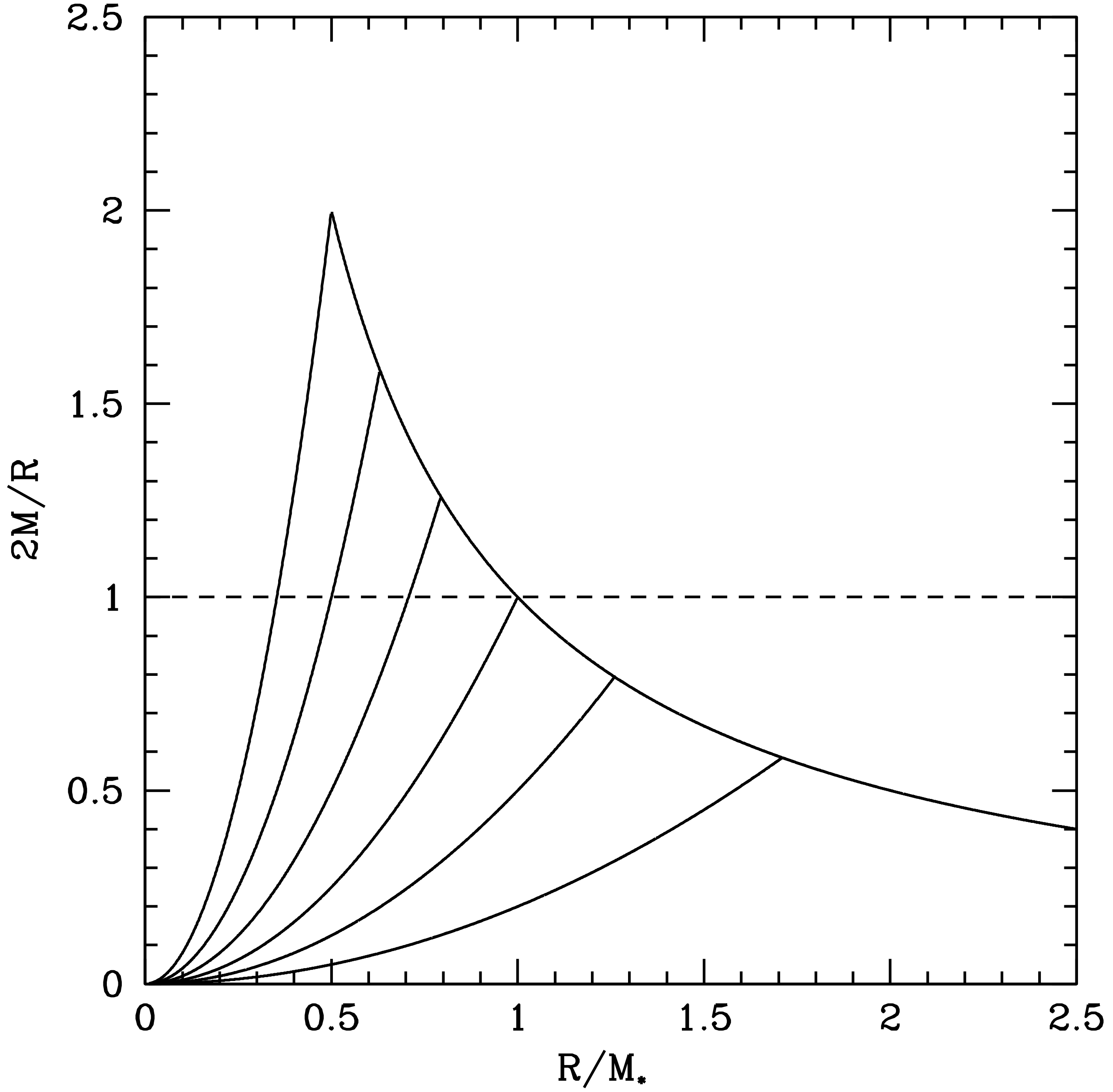


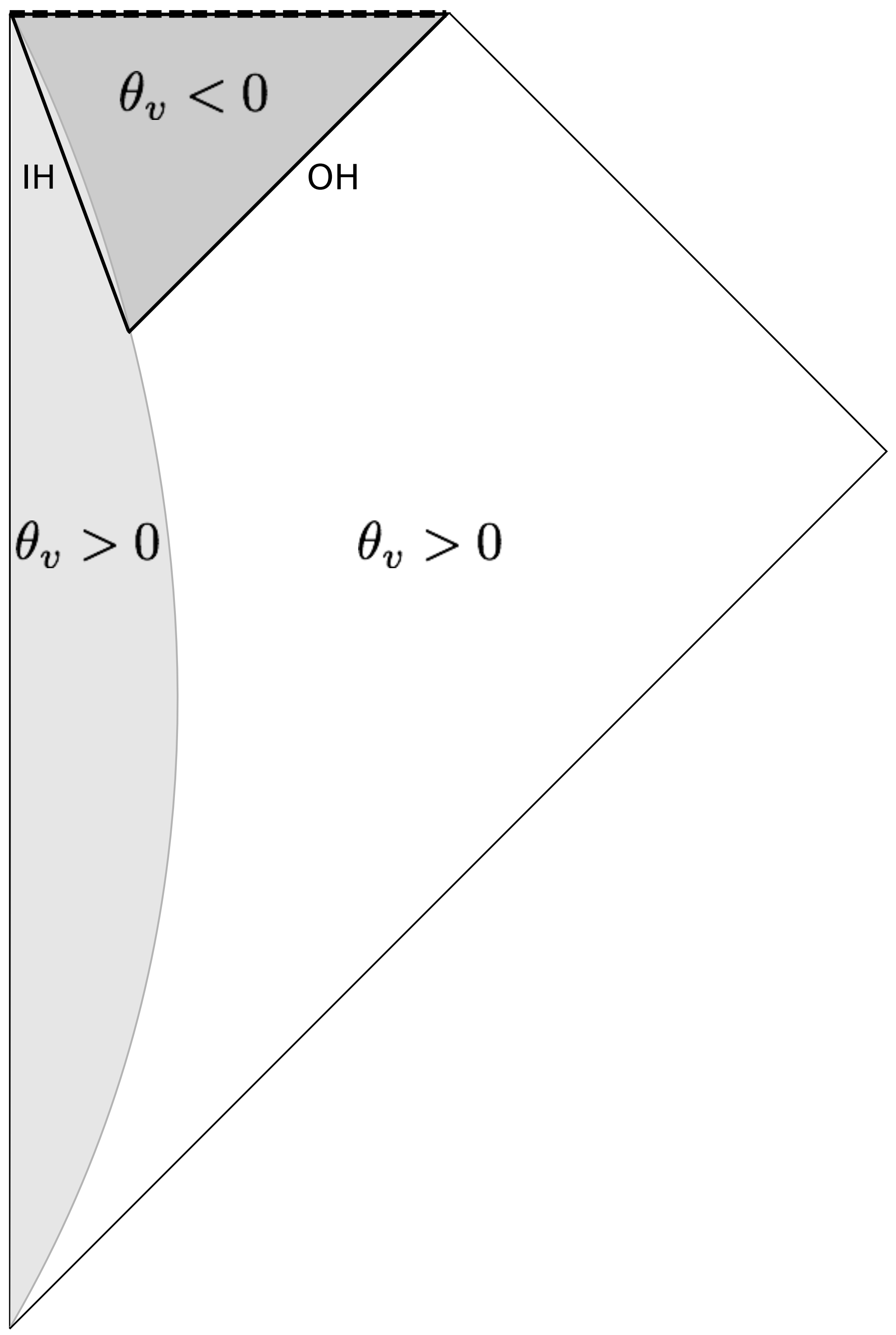
General scheme for in/out-going horizon evolution



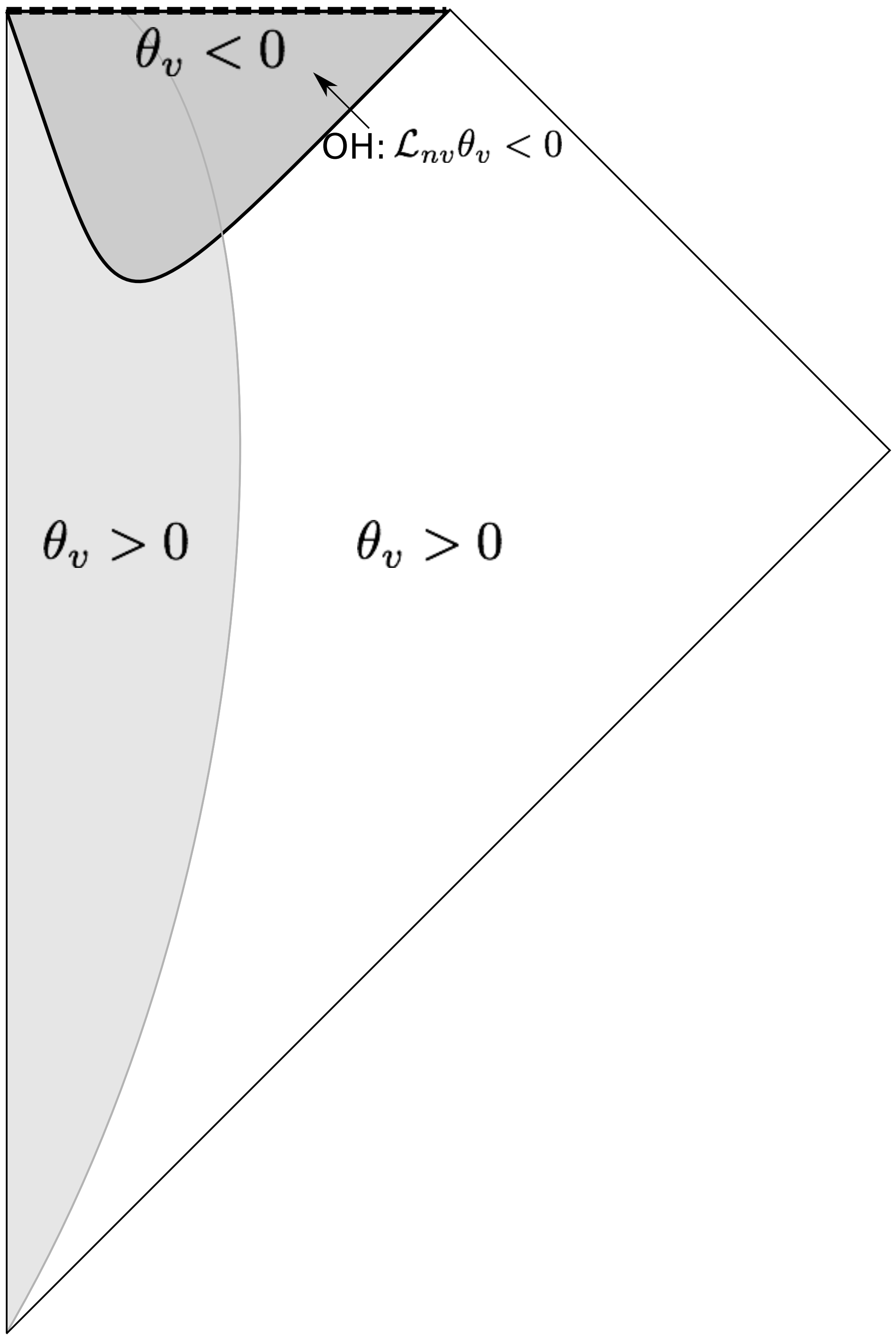
Oppenheimer-Snyder collapse

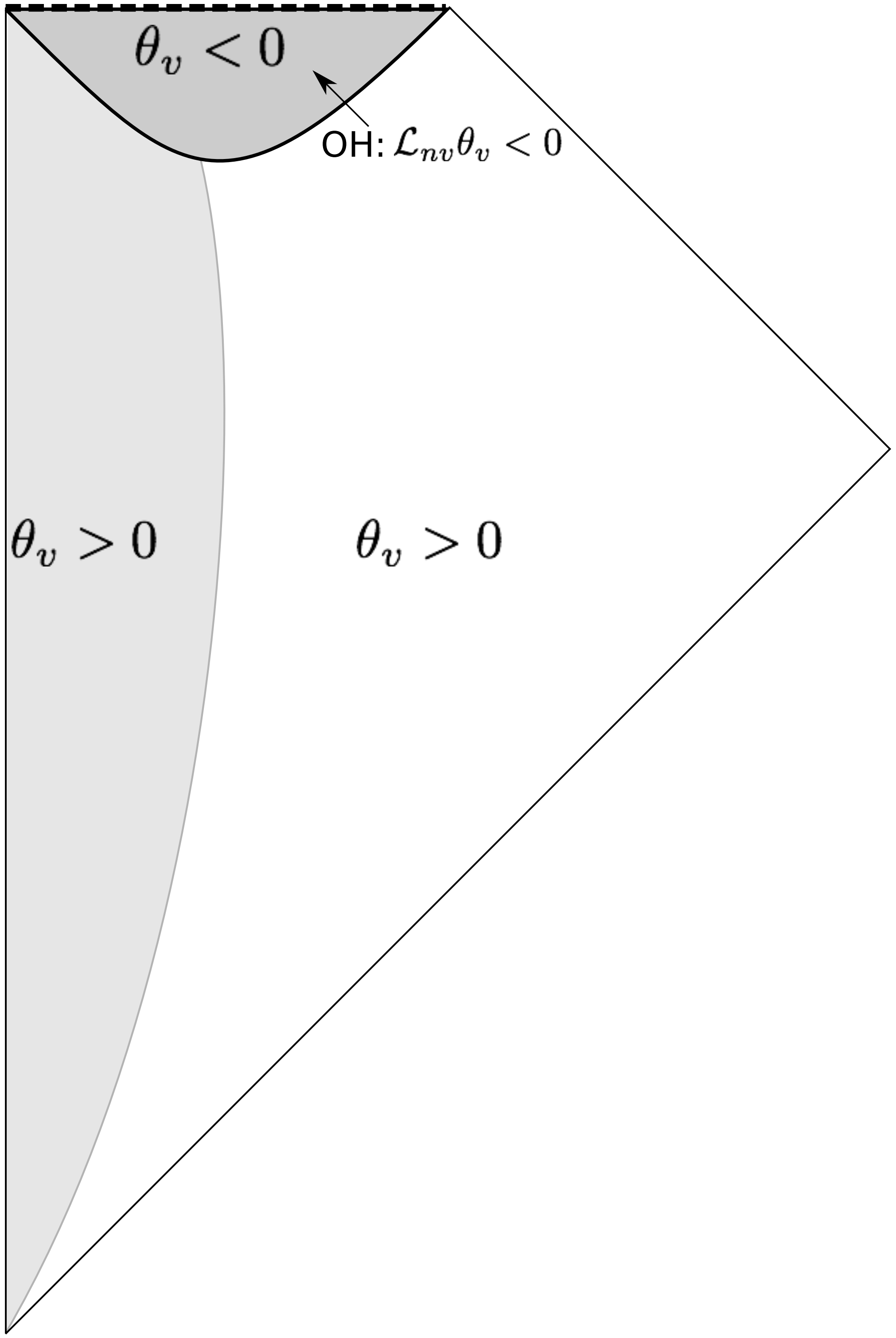
$$\frac{2M}{R} = \frac{8}{3}\pi R^2 e$$



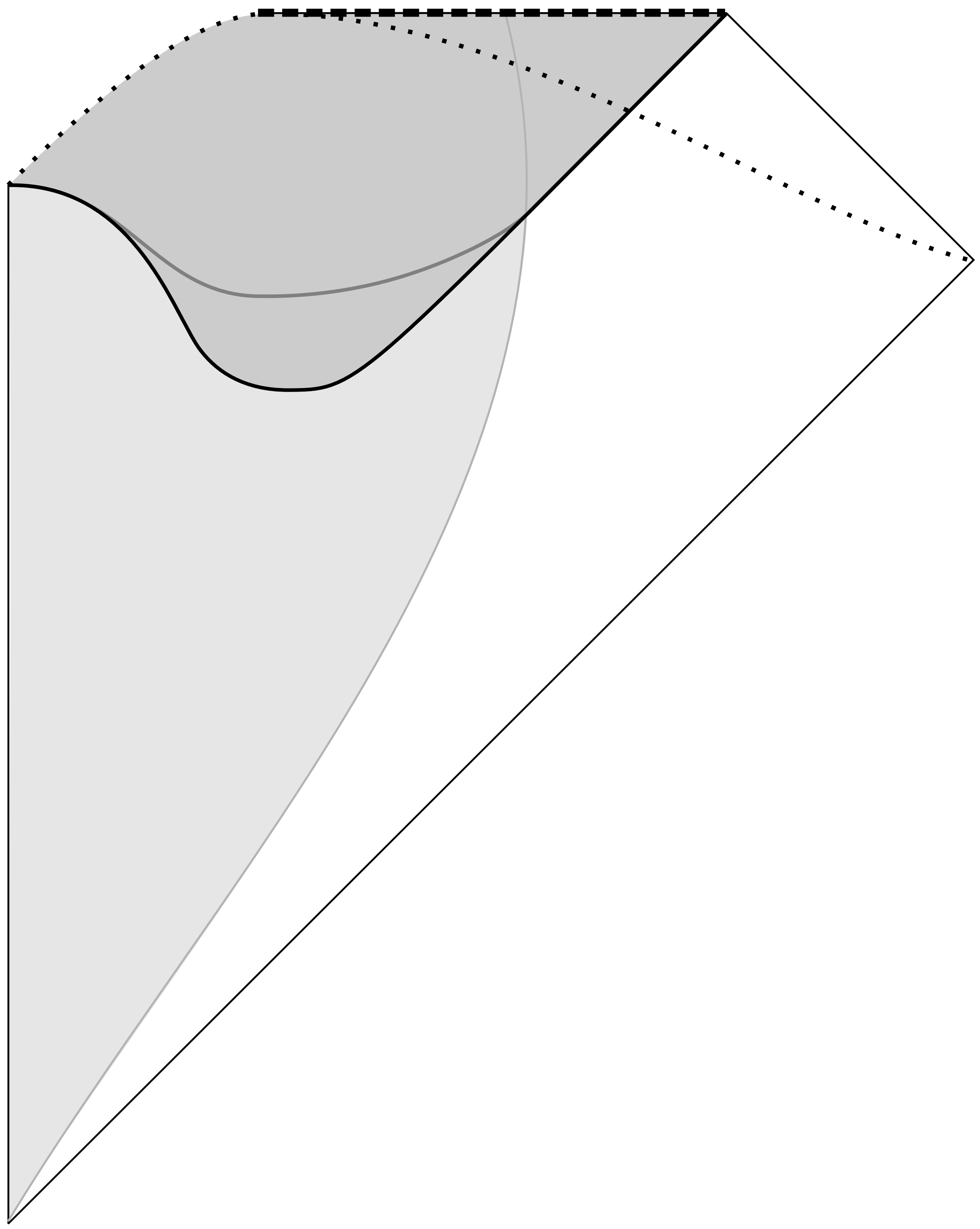


LTB collapse (pressureless, not homogeneous)





Collapse with pressure



Conclusions

- PBHs could have been formed in the Early Universe from non linear cosmological perturbations crossing the horizon if

$$\delta > \delta_c$$

- The threshold for PBH formation is depending on the equation of state, and on the perturbation shape: **can we find a connection between inflation and the statistic of perturbation shapes?**

- Numerical simulations has confirmed that PBH mass spectrum is characterised by a power law because of critical collapse.

$$M_{BH} = K(\delta - \delta_c)^{\gamma} M_H$$