MODEL OF DARK MATTER AND DARK ENERGY BASED ON GRAVITATIONAL POLARIZATION

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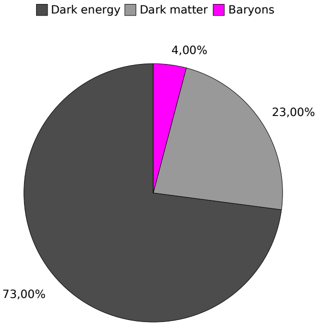
Outline of the talk

1. Phenomenology of dark matter and MOND
2. Relativistic MOND theories
3. Gravitational polarization and MOND
4. Relativistic model of dark matter and dark energy

PHENOMENOLOGY OF DARK MATTER AND MOND

Concordance model in cosmology

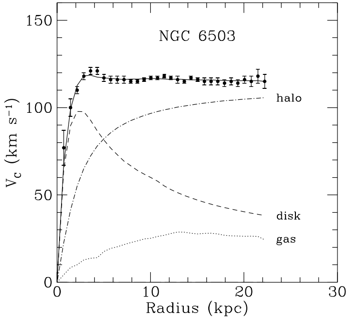


- Einstein field equations with a cosmological constant Λ are assumed to be correct and $\Omega_\Lambda = 73\%$ is measured from the Hubble diagram of supernovas
- Density of baryons $\Omega_\mathrm{b} = 4\%$ is known from Big Bang nucleosynthesis and from CMB anisotropies
- Dark matter is non-baryonic and accounts for the observed dynamical mass of bounded astrophysical systems [Oort 1932, Zwicky 1933]

The dark matter paradigm [Bertone, Hooper & Silk 2004]

1. Dark matter is made of unknown non-baryonic particles, *e.g.*
   - neutrinos (but $\Omega_\nu \lesssim 7\%$)
   - neutralinos (predicted by super-symmetric extensions of the standard model)
   - light scalar [Boehm, Cassé & Fayet 2003]
   - axions
   - $\cdots$
2. It accounts for the observed mass discrepancy between the dynamical and luminous masses of clusters of galaxies
3. It triggers the formation of large-scale structures by gravitational collapse and predicts the scale-dependence of density fluctuations
4. It suggests some universal dark matter density profile around ordinary masses [Navarro, Frenk & White 1995]
5. It has difficulties at explaining naturally the flat rotation curves of galaxies and the Tully-Fisher relation [McGaugh & de Blok 1998, Sanders & McGaugh 2002]

Rotation curves of galaxies are flat

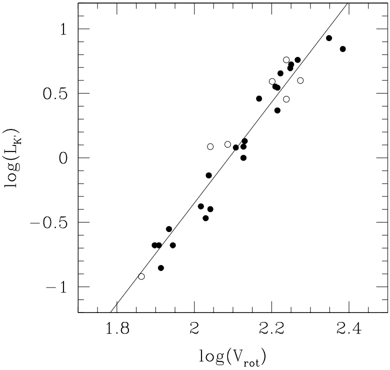


For a circular orbit $v(r) = \sqrt{\frac{G\,M(r)}{r}}$

The fact that $v(r)$ is approximately constant implies that beyond the optical disk

$$M_{\text{halo}}(r) \approx r \qquad \rho_{\text{halo}}(r) \approx \frac{1}{r^2}$$

The Tully-Fisher empirical relation [Tully & Fisher 1977]



The relation between the asymptotic flat velocity and the luminosity of spirals is

$$v_{\text{flat}} \propto L^{1/4}$$

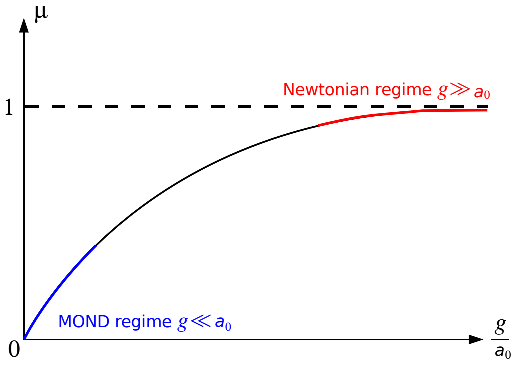
Modified Newtonian dynamics (MOND) [Milgrom 1983]

1. MOND was proposed as an alternative to the dark matter paradigm
2. It states that there is no dark matter and we witness a violation of the fundamental law of gravity
3. MOND is designed to account for the basic phenomenology of the flat rotation curves of galaxies and the Tully-Fisher relation
4. The basic fact about MOND is that the apparent need for dark matter arises in regions of weak gravitational field $g \ll a_0$

The Newtonian gravitational field is modified in an *ad hoc* way

$$\mu\left(\frac{g}{a_0}\right)\boldsymbol{g} = \boldsymbol{g}_{\text{Newton}}$$

The MOND function



In the MOND regime (when $g \ll a_0$) we have [Milgrom 1983]

$$\mu\left(\frac{g}{a_0}\right) \approx \frac{g}{a_0}$$

Recovering flat rotation curves and the Tully-Fisher law

1. For a spherical mass $g_\mathrm{N} = \frac{G\,M}{r^2}$ hence $g \approx \frac{\sqrt{G\,M\,a_0}}{r}$
2. For circular motion $\frac{v^2}{r} = g$ thus v is constant and we get

$$\boxed{v_\text{flat} \approx (G\,M\,a_0)^{1/4}}$$

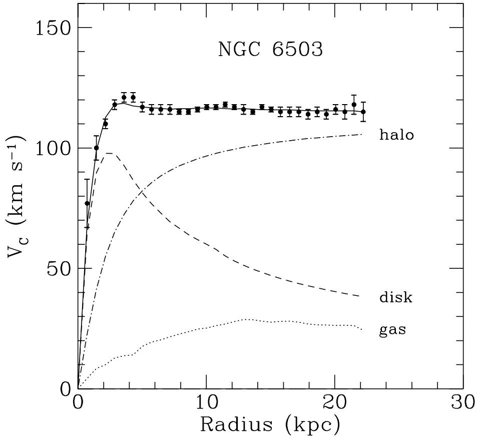
3. Assuming $L/M = \text{const}$ one naturally explains the Tully-Fisher relation
4. The numerical value of the critical acceleration is measured as

$$\boxed{a_0 \approx 1.2\,10^{-10}\,\mathrm{m/s^2}}$$

This value of a_0 is mysteriously close to the acceleration scale associated with the cosmological constant Λ

$$a_0 \approx 1.3\,a_\Lambda \quad \text{where} \quad a_\Lambda = \frac{1}{2\pi}\left(\frac{\Lambda}{3}\right)^{1/2}$$ [Gibbons & Hawking 1977]

The MOND fit of galactic rotation curves



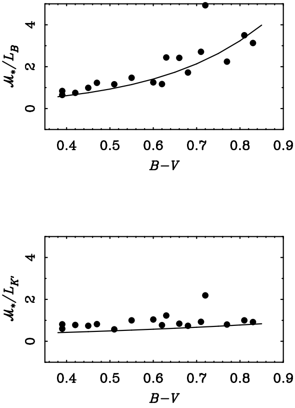
$\Longleftarrow$ the solid line is the MOND fit

Many galaxies are accurately fitted with MOND [Sanders 1996, McGaugh & de Blok 1998]

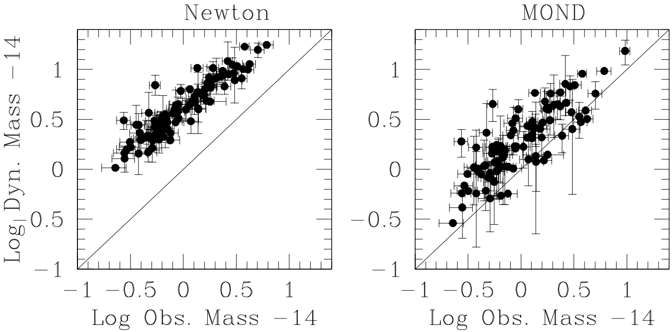
Fit of the mass-to-luminosity ratio

The fit of rotation curves is actually a one-parameter fit. The mass-to-luminosity ratio M/L of each galaxy is adjusted (and is therefore measured by MOND)

M/L shows the same trend with colour as is implied by models of population synthesis [Sanders & Verheijen 1998]



Problem with galaxy clusters [Gerbal, Durret *et al* 1992, Sanders 1999]



The mass discrepancy is $\approx 4-5$ with Newton and ≈ 2 with MOND

RELATIVISTIC MOND THEORIES

Non-relativistic theory for MOND

- The MOND equation can be rewritten as the modified Poisson equation

$$\boldsymbol{\nabla}\cdot\Bigg[\underbrace{\mu\left(\frac{g}{a_0}\right)}_{\text{MOND function}}\,\boldsymbol{g}\Bigg] = -4\pi\,G\,\rho$$

where $\boldsymbol{g} = \boldsymbol{\nabla} U$ is the gravitational field and ρ the density of ordinary matter

- This equation is derivable from the Lagrangian [Bekenstein & Milgrom 1984]

$$L = \frac{a_0^2}{8\pi}\,k\left(\frac{\boldsymbol{\nabla} U^2}{a_0^2}\right) + \rho\,U$$

where k is related to the MOND function by $k'(x) = \mu(\sqrt{x})$.

Relativistic scalar-tensor theory for MOND [Bekenstein & Sanders 1994]

1. The tensor part is the usual Einstein-Hilbert action

$$L_g = \frac{1}{16\pi} R[g, \partial g, \partial^2 g]$$

2. The scalar field is given by an aquadratic kinetic term

$$L_\phi = \frac{a_0^2}{8\pi}\, h \left(\frac{g^{\mu\nu}\partial_\mu \phi \partial_\nu \phi}{a_0^2} \right)$$

3. The matter fields are universally coupled to gravity

$$L_\mathrm{m} = L_\mathrm{m}\Big[\Psi,\, \underbrace{\tilde{g}_{\mu\nu} \equiv e^{2\phi} g_{\mu\nu}}_{\text{physical metric}}\Big]$$

- Light signals do not feel the presence of the scalar field ϕ because the physical metric $\tilde{g}_{\mu\nu}$ is conformally related to the Einstein frame metric $g_{\mu\nu}$
- Since we observe huge amounts of dark matter by gravitational lensing (weak and strong) this theory is not viable

Tensor-vector-scalar theory [Bekenstein 2004, Sanders 2005]

1. The tensor part is the Einstein-Hilbert action L_g
2. The vector field part for $W_{\mu\rho} = \partial_\mu V_\nu - \partial_\nu V_\mu$ is

$$L_{\rm V} = -\frac{1}{32\pi}\Big[K \underbrace{g^{\mu\nu}g^{\rho\sigma}W_{\mu\rho}W_{\nu\sigma}}_{\text{standard spin-1 action}} - 2\lambda \underbrace{\Big(g^{\mu\nu}V_\mu V_\nu + 1\Big)}_{\text{tells that } V^\mu \text{ is time-like and unitary}}\Big]$$

3. The scalar action reads

$$L_\phi = -\overbrace{\frac{1}{2}\Big(g^{\mu\nu} - V^\mu V^\nu\Big)\partial_\mu\phi\partial_\nu\phi}^{\text{dynamical scalar field } \phi} - \overbrace{\frac{\sigma^4}{4\ell^2}F\big(k\,\sigma^2\big)}^{\text{non-dynamical scalar field } \sigma}$$

4. Matter fields are coupled to the non-conformally related physical metric

$$\tilde{g}_{\mu\nu} = e^{-2\phi}(g_{\mu\nu} + V_\mu V_\nu) - e^{2\phi}V_\mu V_\nu$$

This theory has evolved recently toward Einstein-æther like theories [Jacobson & Mattingly 2001; Zlosnik, Ferreira & Starkman 2007]

Particle dark matter versus MOND

The alternative seems to be

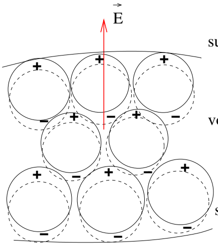
1. Either accept the existence of cold dark matter but
   - made of unknown non-baryonic particles yet to be discovered
   - which fails to reproduce in a natural way the flat rotation curves of galaxies
2. Or postulate a modification of the fundamental law of gravity: MOND and its relativistic extensions (TeVeS and Einstein-æther) but
   - on an *ad hoc* and not yet physically justified basis

Here we shall adopt a different approach

1. Keep the standard law of gravity namely general relativity and its Newtonian limit when $c \to \infty$
2. Use the phenomenology of MOND to guess what could be the (probably unorthodox) nature of dark matter
3. Propose a natural physical mechanism for MOND
4. Develop a relativistic matter model and apply it to cosmology

GRAVITATIONAL POLARIZATION AND MOND

The electric field in a dielectric medium



The atoms in a dielectric are modelled by electric dipole moments

$$\boldsymbol{\pi}_e = q\,\boldsymbol{\xi}$$

The polarization vector is

$$\boldsymbol{\Pi}_e = n\,\boldsymbol{\pi}_e$$

Density of polarization charges: $\rho_e^{\text{polar}} = -\boldsymbol{\nabla}\cdot\boldsymbol{\Pi}_e$

$$\boldsymbol{\nabla}\cdot\boldsymbol{E} = \frac{\rho_e + \rho_e^{\text{polar}}}{\varepsilon_0} \quad\Longleftrightarrow\quad \boldsymbol{\nabla}\cdot\Big(\overbrace{\boldsymbol{E} + \frac{\boldsymbol{\Pi}_e}{\varepsilon_0}}^{\boldsymbol{D}}\Big) = \frac{\rho_e}{\varepsilon_0}$$

The dipoles and polarization vector are aligned with the electric field

$$\frac{\boldsymbol{\Pi}_e}{\varepsilon_0} = \underbrace{\chi_e(E)}_{\text{electric susceptibility}}\;\boldsymbol{E}$$

Interpretation of the MOND equation [Blanchet 2007]

The MOND equation in the form of a modified Poisson equation

$$\nabla \cdot \Big[\underbrace{\mu\left(\frac{g}{a_0}\right)}_{\text{MOND function}} \, \boldsymbol{g} \Big] = -4\pi \, G \, \rho$$

is formally analogous to the equation of electrostatics inside a dielectric. We pose

$$\mu = 1 + \underbrace{\chi(g)}_{\substack{\text{gravitational} \\ \text{susceptibility}}} \quad \text{and} \quad \underbrace{\boldsymbol{\Pi}}_{\substack{\text{gravitational} \\ \text{polarization}}} = -\frac{\chi}{4\pi \, G} \, \boldsymbol{g}$$

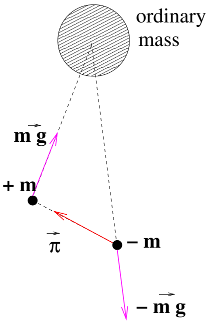
The MOND equation is equivalent to

$$\Delta U = -4\pi \, G \, (\rho + \rho_{\text{polar}})$$

In this interpretation the Newtonian law of gravity is not violated but we are postulating a new form of dark matter consisting of "polarization masses" with density

$$\boxed{\rho_{\text{polar}} = -\nabla \cdot \boldsymbol{\Pi}}$$

Sign of the gravitational susceptibility



The "digravitational" medium is modelled by individual dipole moments $\boldsymbol{\pi}$

$$\boldsymbol{\pi} = m\,\boldsymbol{\xi}$$
$$\boldsymbol{\Pi} = n\,\boldsymbol{\pi}$$

We suppose that the dipoles consist of particles doublets with

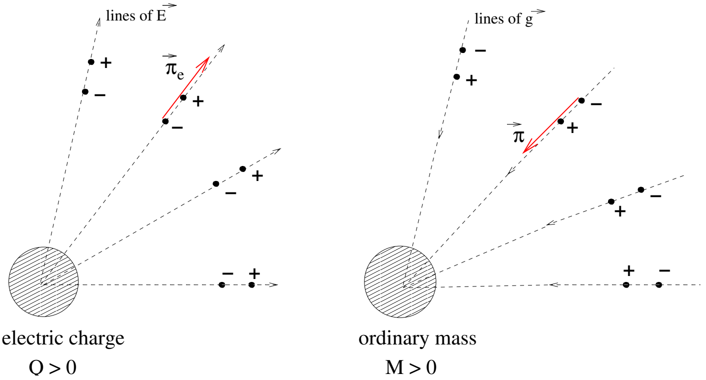
- opposite gravitational masses $m_{\rm g} = \pm m$
- positive inertial masses $m_{\rm i} = m$

1. The gravitational force is governed by a negative Coulomb law [Bondi 1957]
2. The dipoles tend to align in the same direction as the gravitational field thus

$$\boxed{\chi < 0}$$

which is nicely compatible with MOND since $0 < \mu < 1 \Longrightarrow -1 < \chi < 0$

Electric screening versus gravitational anti-screening



Screening by polarization charges

$\chi_e > 0$

Anti-screening by polarization masses

$\chi < 0$

Interpretation of the dark matter medium

1. The dipole moments cannot be stable in a gravitational field and we must invoke some non-gravitational internal force (i.e. a "fifth force") which is attractive between unlike masses
2. The dark matter medium is a gravitational plasma, *i.e.* a medium composed of particles

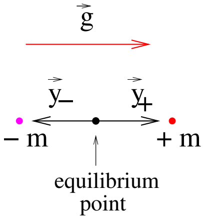
$$(m_\mathrm{i}, m_\mathrm{g}) = (m, \pm m)$$

   in equal numbers (so the plasma is globally neutral) and with local number densities $n_\pm$
3. We find that to obtain MOND we must postulate for the plasma's internal force the Gauss law

$$\boldsymbol{\nabla} \cdot \boldsymbol{f}_\mathrm{int} = -\frac{4\pi\, G\, m}{\chi} \left(n_+ - n_-\right)$$

where $\chi < 0$ is the susceptibility of the dipolar medium

Gravitational plasma oscillations



Let $\boldsymbol{y}_\pm$ be the displacements from the equilibrium position at which the plasma is locally neutral, *i.e.* $n_+ = n_- = n$. The equation of motion is

$$m\,\frac{\mathrm{d}^2\boldsymbol{y}_\pm}{\mathrm{d}t^2} = \pm\, m\left(\boldsymbol{f}_{\text{int}} + \boldsymbol{g}\right)$$

Consider a small departure from equilibrium. The density perturbation reads $n_\pm = n\left(1 - \boldsymbol{\nabla}\cdot\boldsymbol{y}_\pm\right)$ to first order in $\boldsymbol{y}_\pm$

The dipole $\boldsymbol{\xi} = \boldsymbol{y}_+ - \boldsymbol{y}_-$ obeys the harmonic oscillator

$$\frac{\mathrm{d}^2\boldsymbol{\xi}}{\mathrm{d}t^2} + \omega^2\,\boldsymbol{\xi} = 2\boldsymbol{g}$$

in which ω is the usual plasma frequency given here by

$$\omega = \sqrt{-\frac{8\pi\,G\,m\,n}{\chi}}$$

Non viability of the quasi-Newtonian model

The quasi-Newtonian model

- Suggests that the gravitational analogue of the electric polarization is possible
- Yields a simple and natural explanation of the MOND equation
- Requires the existence of a new non-gravitational force
- Interprets the dark matter medium as a polarizable gravitational plasma

BUT THIS MODEL IS NOT VIABLE

- Is not relativistic and not Lagrangian-based
- Involves negative gravitational masses so violates the equivalence principle
- Does not allow to answer questions related to cosmology

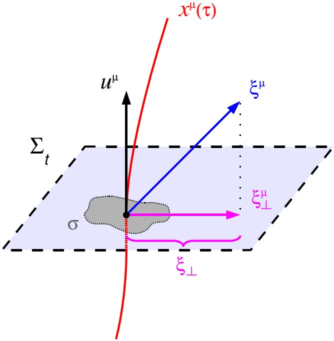
RELATIVISTIC MODEL OF DARK MATTER AND DARK ENERGY

Action principle for dipolar fluid

We propose a matter action in standard general relativity of the type

$$S = \int d^4x \sqrt{-g}\, L\left[J^\mu, \xi^\mu, \dot{\xi}^\mu, g_{\mu\nu}\right]$$

where the current density J^μ and the dipole moment ξ^μ are two independent dynamical variables



- The current density $J^\mu = \sigma u^\mu$ is conserved

$$\nabla_\mu J^\mu = 0$$

- The covariant time derivative is denoted

$$\dot{\xi}^\mu \equiv \frac{\mathrm{D}\xi^\mu}{\mathrm{d}\tau} = u^\nu \nabla_\nu \xi^\mu$$

- Projection perpendicular to the velocity

$$\xi^\mu_\perp \equiv \perp_{\mu\nu} \xi^\nu$$

Dipolar dark matter Lagrangian [Blanchet & Le Tiec 2008]

$$L = \sigma \left[-1 - \sqrt{\left(u_\mu - \dot{\xi}_\mu\right)\left(u^\mu - \dot{\xi}^\mu\right)} + \frac{1}{2}\dot{\xi}_\mu\,\dot{\xi}^\mu \right] - \mathcal{W}(\Pi_\perp)$$

1. The mass term σ represents the inertial mass of the dipole moments (say $\sigma = 2m\,n$)
2. The second term is a potential term inspired by the action of particles with spins in general relativity [Papapetrou 1951, Bailey & Israel 1980]
3. The third term is a kinetic-like term for the dipole moment and will tell how its evolution differs from parallel transport
4. The potential function $\mathcal{W}$ depends on the polarization field

$$\boxed{\Pi_\perp = \sigma\,\xi_\perp}$$

and describes a non-gravitational force $\mathcal{F}^\mu$ internal to the dipole moment

Equations of motion and evolution

Varying the action with respect to the dynamical variables J^μ and ξ^μ we find that one equation can be solved with $\left[\left(u_\mu - \dot{\xi}_\mu\right)\left(u^\mu - \dot{\xi}^\mu\right)\right]^{1/2} = 1$

Equation of motion of dipolar fluid

$$\underbrace{\dot{u}^\mu = -\mathcal{F}^\mu}_{\text{non-geodesic motion}} \qquad \text{where} \qquad \underbrace{\mathcal{F}^\mu \equiv \hat{\xi}^\mu_\perp \frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\Pi_\perp}}_{\text{dipolar internal force}}$$

Equation of evolution of dipole moment

$$\dot{\Omega}^\mu = \frac{1}{\sigma}\nabla^\mu\left(\mathcal{W} - \Pi_\perp \frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\Pi_\perp}\right) - \underbrace{\xi^\nu_\perp R^\mu_{\ \rho\nu\sigma} u^\rho u^\sigma}_{\text{coupling to Riemann curvature}}$$

$$\text{where} \quad \Omega^\mu \equiv \perp^\mu_\nu \dot{\xi}^\nu_\perp + u^\mu\left(1 + \xi_\perp \frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\Pi_\perp}\right)$$

The equations depend only on the (space-like) perpendicular projection $\xi^\mu_\perp = \perp^\mu_\nu \xi^\nu$ which represents the physical dipole moment variable

Stress-energy tensor

Varying the action with respect to the metric we obtain

$$T^{\mu\nu} = \underbrace{r}_{\text{energy density}} u^\mu u^\nu + \underbrace{\mathcal{P}}_{\text{pressure}} \perp^{\mu\nu} + 2 \underbrace{Q^{(\mu}}_{\text{heat flux}} u^{\nu)} + \underbrace{\Sigma^{\mu\nu}}_{\text{anisotropic stresses}}$$

where

$$\begin{aligned} r &= \rho + \mathcal{W} - \Pi_\perp \mathcal{W}' \\ \mathcal{P} &= -\mathcal{W} + \frac{2}{3} \Pi_\perp \mathcal{W}' \\ Q^\mu &= \sigma \dot{\xi}^\mu_\perp + \Pi_\perp \mathcal{W}' u^\mu - \Pi^\lambda_\perp \nabla_\lambda u^\mu \\ \Sigma^{\mu\nu} &= \left(\frac{1}{3} \perp^{\mu\nu} - \hat{\xi}^\mu_\perp \hat{\xi}^\nu_\perp \right) \Pi_\perp \mathcal{W}' \end{aligned}$$

The mass density ρ contains the rest mass σ and a dipolar term $-\nabla_\lambda \Pi^\lambda_\perp$ which appears as a relativistic generalisation of the polarization mass density

$$\boxed{\rho = \sigma - \nabla_\lambda \Pi^\lambda_\perp}$$

Cosmological perturbations at large scales

1. Apply first-order perturbation theory around FLRW background

$$\begin{aligned} g_{\mu\nu} &= \overline{g}_{\mu\nu} + \delta g_{\mu\nu} \\ u^\mu &= \overline{u}^\mu + \delta u^\mu \\ \xi_\perp^\mu &= 0 + \delta\xi_\perp^\mu \\ \Pi_\perp &= 0 + \delta\Pi_\perp \end{aligned}$$

$\Longleftarrow$ dipole moment is perturbative

$\Longleftarrow$ polarization field is perturbative

2. Suppose that the potential takes the form

$$\mathcal{W}\left(\Pi_\perp\right) = \mathcal{W}_0 + \frac{1}{2}\mathcal{W}_2\,\Pi_\perp^2 + \mathcal{O}\left(\Pi_\perp^3\right)$$

3. Apply standard SVT gauge-invariant formalism with $\delta\xi_\perp^\mu = \left(0, D^i y + y^i\right)$

- equations of motion

$$\begin{aligned} V' + \mathcal{H}\,V + \Phi &= -\mathcal{W}_2\,\overline{\sigma}\,a^2\,y \\ V_i' + \mathcal{H}\,V_i &= -\mathcal{W}_2\,\overline{\sigma}\,a^2\,y_i \end{aligned}$$

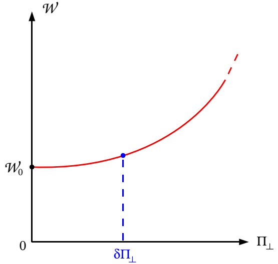
- evolution equations

$$\begin{aligned} y'' + \mathcal{H}\,y' - \mathcal{W}_2\,\overline{\sigma}\,a^2\,y &= 0 \\ y_i'' + \mathcal{H}\,y_i' - \mathcal{W}_2\,\overline{\sigma}\,a^2\,y_i &= 0 \end{aligned}$$

Cosmological expansion of the fundamental potential

We assumed that the potential $\mathcal{W}$ reaches a minimum for the FLRW background

$$\begin{aligned}\mathcal{W}(\Pi_\perp) &= \mathcal{W}_0 + \frac{1}{2}\mathcal{W}_2\,\Pi_\perp^2 + \mathcal{O}(\Pi_\perp^3) \\ \mathcal{F}^\mu &= \underbrace{\mathcal{W}_2\,\Pi_\perp^\mu}_{\text{contributes at first-order cosmological perturbation}} + \mathcal{O}(\Pi_\perp^2)\end{aligned}$$



The minimum of the "fundamental" potential is the cosmological constant

$$\mathcal{W}_0 = \frac{\Lambda}{8\pi}$$

Stress-energy tensor in linear cosmological perturbation

1. To first order in the perturbation, $T^{\mu\nu} = T^{\mu\nu}_{\text{de}} + T^{\mu\nu}_{\text{dm}}$ where

$$T^{\mu\nu}_{\text{de}} = - \underbrace{\mathcal{W}_0}_{\text{cosmological constant}} g^{\mu\nu}$$

$$T^{\mu\nu}_{\text{dm}} = \rho\, u^\mu\, u^\nu + \underbrace{2Q^{(\mu}\, u^{\nu)}}_{\text{heat flux term}}$$

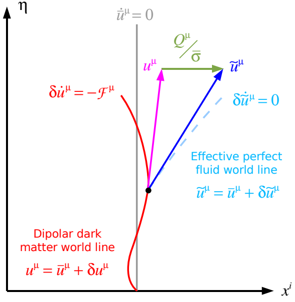
2. Posing $\widetilde{u}^\mu = u^\mu + Q^\mu/\overline{\rho}$ we can recast the dark matter fluid into a perturbed pressureless perfect fluid at linear perturbation order

$$T^{\mu\nu}_{\text{dm}} = \rho\, \widetilde{u}^\mu\, \widetilde{u}^\nu$$

3. The energy density of the dipolar dark matter fluid is

$$\rho = \underbrace{\sigma}_{\text{rest mass energy density}} - \underbrace{D_i \Pi^i_\perp}_{\text{dipolar polarization energy density}}$$

Agreement with the Λ-CDM scenario



The dipolar fluid is undistinguishable from

- standard DE (a cosmological constant)
- standard CDM (a pressureless perfect fluid)

at the level of first-order cosmological perturbations

Adjusting Λ so that $\Omega_{\rm de} \approx 0.73$ and $\overline{\sigma}$ so that $\Omega_{\rm dm} \approx 0.23$ the model is consistent with CMB fluctuations

Non-relativistic limit of the model ($c \to +\infty$)

1. Equation of motion of the dipolar particle

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \underbrace{\boldsymbol{g}}_{\substack{\text{local}\\\text{gravitational field}}} - \underbrace{\boldsymbol{\mathcal{F}}}_{\substack{\text{internal}\\\text{force}}}$$

2. Evolution equation of the dipole moment

$$\frac{\mathrm{d}^2\boldsymbol{\xi}}{\mathrm{d}t^2} = \boldsymbol{\mathcal{F}} + \frac{1}{\sigma}\boldsymbol{\nabla}\left(\mathcal{W} - \Pi\,\mathcal{W}'\right) + \underbrace{(\boldsymbol{\xi}\cdot\boldsymbol{\nabla})\,\boldsymbol{g}}_{\text{tidal effect}}$$

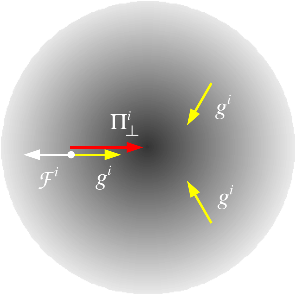
3. Conservation of the number of particles

$$\partial_t\sigma = -\boldsymbol{\nabla}\cdot(\sigma\boldsymbol{v})$$

4. Poisson equation for the gravitational potential

$$\boldsymbol{\nabla}\cdot\boldsymbol{g} = -4\pi\Big(\underbrace{\rho_{\mathrm{b}}}_{\substack{\text{baryonic}\\\text{matter}}} + \sigma - \boldsymbol{\nabla}\cdot\boldsymbol{\Pi}\Big)$$

Dipolar dark matter in a central gravitational field



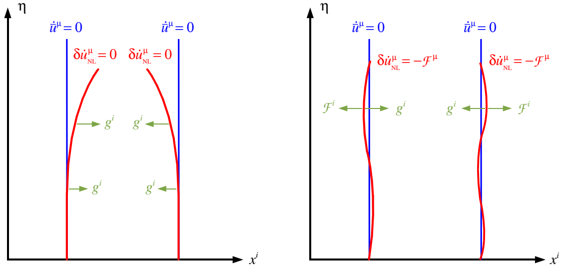
In spherical symmetry we find that there is a solution for which the dipolar fluid is static (i.e. at rest), the dipole moment is aligned with the gravitational field, and varies on a very long time-scale so that it is practically in equilibrium

$$\begin{aligned} \boldsymbol{v} &= \boldsymbol{0} \\ \boldsymbol{\Pi} &= -\Pi(r)\, \boldsymbol{e}_r \end{aligned}$$

In this solution the internal force balances the gravitational field

$$\boldsymbol{\mathcal{F}} = \boldsymbol{g}$$

Weak clustering of dipolar dark matter



Baryonic matter | Dipolar dark matter

- Baryonic matter follows the geodesic equation $\dot{u}^\mu = 0$, therefore collapses in regions of overdensity
- Dipolar dark matter obeys $\dot{u}^\mu = -\mathcal{F}^\mu$, with the internal force $\mathcal{F}^i$ balancing the gravitational field g^i created by an overdensity

We expect that the mass density of dipolar dark matter in a galaxy at low redshift remains close to its mean cosmological value

$$\sigma \approx \overline{\sigma} \ll \rho_{\mathrm{b}} \quad \text{and} \quad \boldsymbol{v} \approx \mathbf{0}$$

Recovering MOND in a galaxy at low red-shift

Using $\boldsymbol{v} \approx \mathbf{0}$ in the equation of motion

$$\boldsymbol{g} = \hat{\mathbf{\Pi}} \frac{d\mathcal{W}}{d\Pi} \quad \Longleftarrow \quad \text{polarization of the dipolar medium}$$

Using $\sigma \approx \overline{\sigma} \ll \rho_\mathsf{b}$ in the gravitational field equation

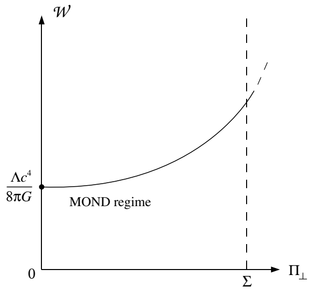
$$\boldsymbol{\nabla} \cdot \Big[\boldsymbol{g} - 4\pi\, \mathbf{\Pi}\Big] = -4\pi\, \rho_\mathsf{b} \quad \Longleftarrow \quad \text{modification of the Poisson equation}$$

1. Since the polarization $\mathbf{\Pi}$ is aligned with the gravitational field $\boldsymbol{g}$ we recover the MOND equation with MOND function $\mu = 1 + \chi$ where

$$\mathbf{\Pi} = -\frac{\chi}{4\pi}\, \boldsymbol{g}$$

2. There is one-to-one correspondence between μ and the potential $\mathcal{W}$

The fundamental potential $\mathcal{W}$



The potential $\mathcal{W}$ is determined through third order

$$\mathcal{W} = \frac{\Lambda}{8\pi} + 2\pi\,\Pi_\perp^2 + \underbrace{\frac{16\pi^2}{3a_0}\,\Pi_\perp^3}_{\text{MOND acceleration appears here}} + \mathcal{O}\left(\Pi_\perp^4\right)$$

Though the model is purely *classical* it is tempting to interpret the cosmological constant Λ as a "vacuum polarization", i.e. the residual polarization that remains when the "classical" part of the polarization $\Pi_\perp \to 0$

Order of magnitude of the cosmological constant

1. Introduce a purely numerical coefficient α such that

$$a_0 = \frac{1}{2\pi\,\alpha}\left(\frac{\Lambda}{3}\right)^{1/2} \qquad \left(\text{i.e.}\quad a_0 = \frac{a_\Lambda}{\alpha}\right)$$

2. Write the fundamental potential as $\mathcal{W} = \frac{3\pi\, a_0^2}{2}\, f\left(\frac{\Pi_\perp}{a_0}\right)$ with

$$f(x) = \underbrace{\alpha^2 + \frac{4}{3}x^2 + \frac{32\pi}{9}x^3 + \mathcal{O}\left(x^4\right)}_{\text{some ``universal'' function of } x \equiv \Pi_\perp/a_0}$$

3. The numerical coefficients in $f(x)$ are expected to be of the order of one, hence the cosmological constant should be of the order of

$$\boxed{\Lambda \sim a_0^2}$$

in good agreement with the observations (which give $\alpha \approx 0.8$)

Conclusions

This model based on gravitational polarization:

1. Offers a natural physical explanation of the phenomenology of MOND
2. Benefits from both the successes of MOND at galactic scales and of Λ-CDM at cosmological scales
3. Yields a nice unification between the dark energy in the form of Λ and the dark matter *à la* MOND
4. But is "phenomenological" and presumably only valid in a regime of weak gravitational fields

More work should be done to:

1. Connect the model to more fundamental (quantum) physics valid at microscopic scale
2. Investigate second-order perturbations in cosmology
3. Numerically compute the non-linear growth of perturbations and compare with large-scale structures
4. Test the model at the intermediate scale of galaxy clusters