Basic concepts – part 2

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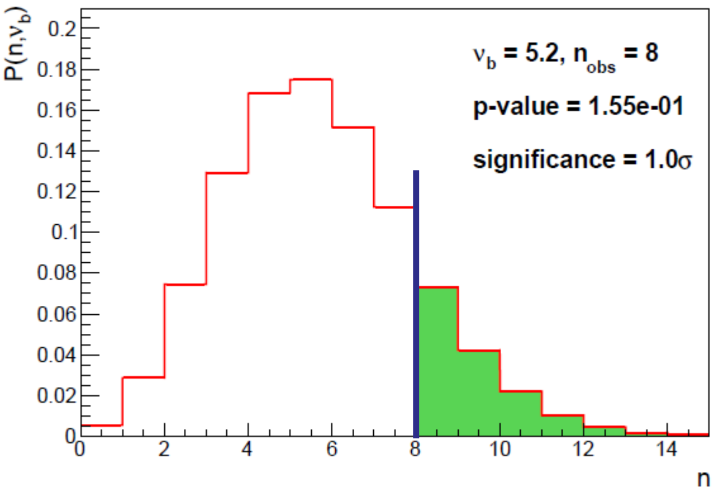
Compatibility test – cont'd



Example: BumpHunter algorithm

Software used to search for excess or deficit in a spectrum.

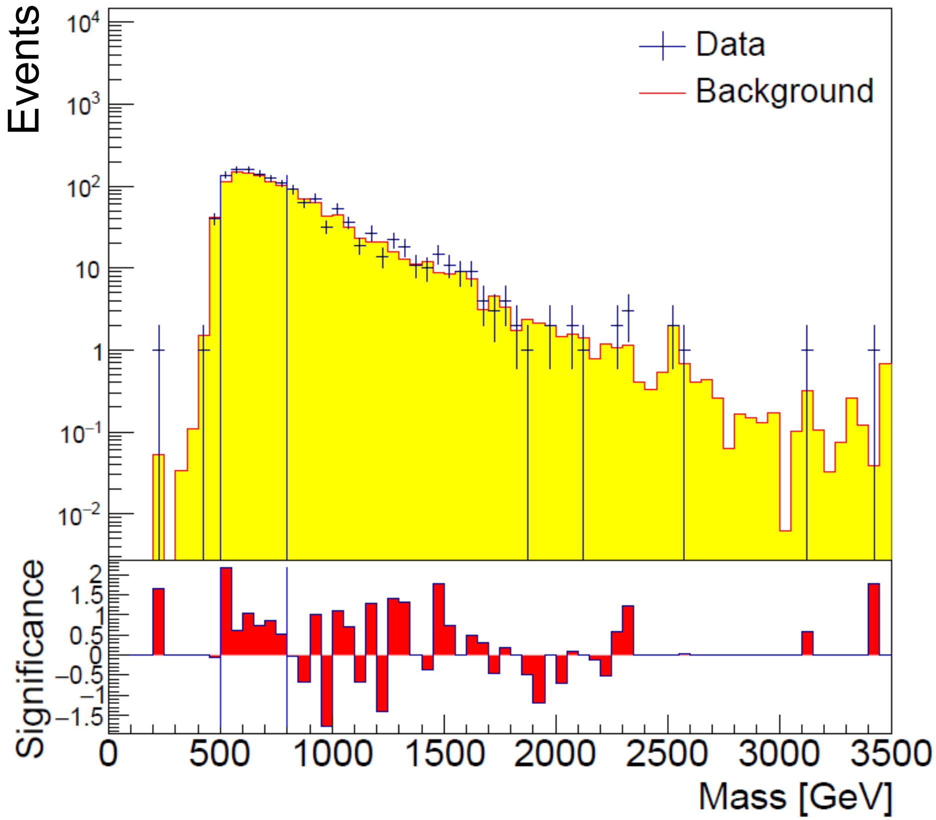
G. Choudalakis
1101.0390

- **No assumptions** are made on the signal shape or yield
- Just test data against **background-only hypothesis**

➔ Compute the p-value for all possible intervals.

➔ Select the interval with smallest p-value.

This gives the local p-value: $P_{\text{min}}^{\text{local}}$

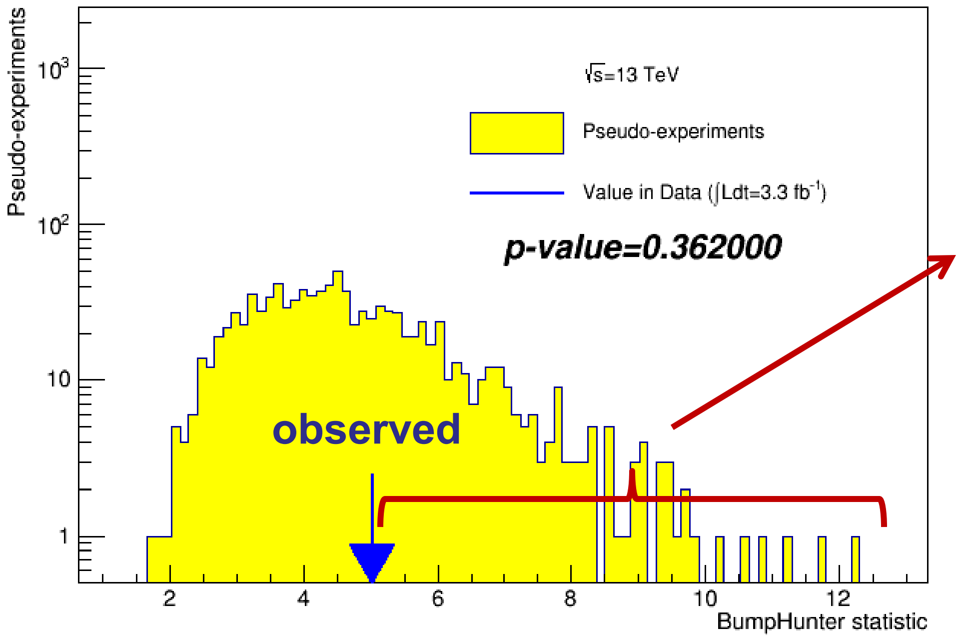


Example: BumpHunter algorithm

Since many intervals are considered there is a increasing probability that an excess is found due to statistical fluctuations

- This is the (in)famous (and misnamed) **Look Elsewhere Effect: LEE**
- To cope for this effect a **global p-value** is calculated

➔ The global p-value is extracted by comparing $-\log(P_{\min}^{\text{local}})$ to a set of $-\log\left(P_{\min}^{\text{local}}\right)$ generated using background-only pseudo-experiments



p^{global}: **fraction of PE** that gives a result higher than the one observed

$$p^{\text{global}} = \text{fraction of } (P_{\min}^{\text{PE}} > P_{\min}^{\text{obs}})$$

Pearson's χ^2 test: estimate global compatibility between data and a model

- The data is regrouped in an **histogram** of N bins
- A **goodness-of-fit test** K^2 is computed as follows

$$\boxed{K^2 = \sum_{i=1}^{N} \frac{(n_i - \nu_i)^2}{\nu_i}}$$

n_i: number of observed events in bin i

ν_i: expected number of events in bin i

If the data n_i are **Poisson** distributed with mean values ν_i and $\boldsymbol{n_i} > \sim\mathbf{5}$ then: K^2 is a random variable following a **χ^2 distribution** with **N** degrees of freedom.

A variant of this test statistics is the **Neyman's χ^2**

$$\boxed{K^2 = \sum_{i=1}^{N} \frac{(n_i - \nu_i)^2}{n_i}}$$

Easier to code (in particular for fits)

Asymptotically equivalent to Pearson's χ^2

Follows χ^2 with **N-1** degrees of freedom

Probability density function

k degrees of freedom, x>0

$$\chi^2(x;k) = \frac{x^{\frac{k}{2}-1} e^{-\frac{x}{2}}}{2^{\frac{k}{2}} \Gamma\left(\frac{k}{2}\right)}$$

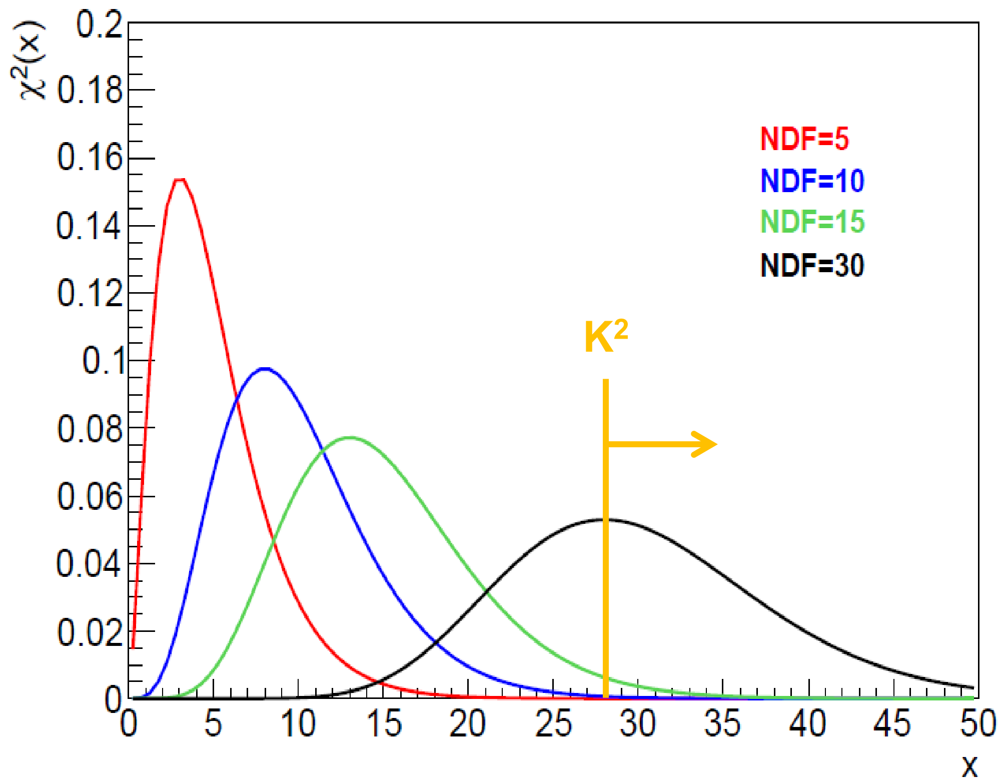
Cumulative distribution

$$F(x;k) = \frac{\gamma\left(\frac{k}{2}, \frac{x}{2}\right)}{\Gamma\left(\frac{k}{2}\right)}$$

Mean: k Variance: 2k

With: $\gamma(s,x) = \int_0^x t^{s-1} e^{-t} dt$

$$\Gamma(s) = \int_0^{+\infty} t^{s-1} e^{-t} dt$$

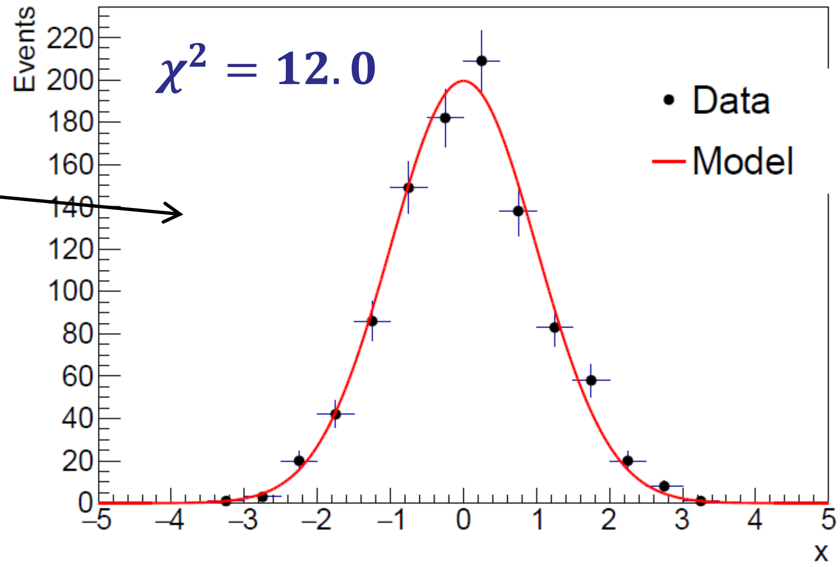


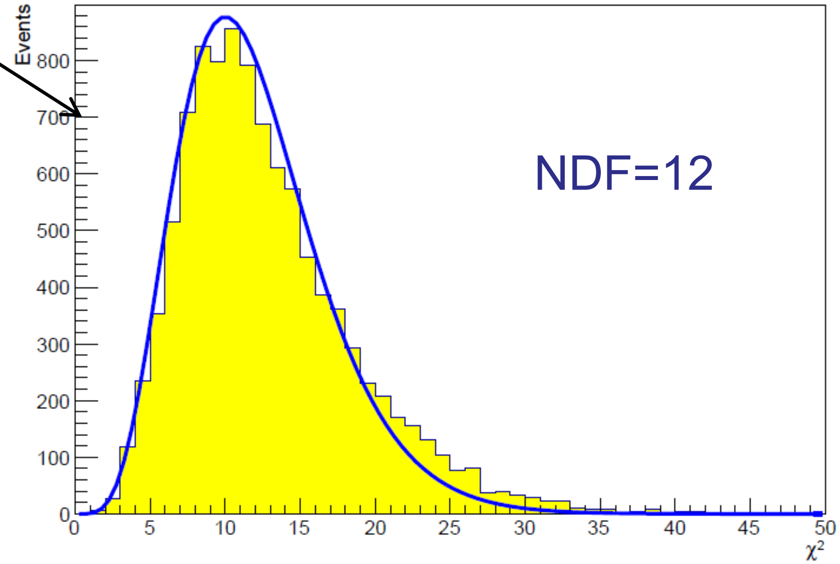
The p-value of a χ^2 test is obtained by integrating the χ^2 distribution above the measured K^2 value.

$$\text{p-value} = \int_{K^2}^{+\infty} \chi^2(x;k)\, dx$$

Procedure

- Generate events following a Gaussian distribution
- Calculate (Neyman's) K^2
- Repeat 10k time and plot the distribution of K^2
- Compare to χ^2 distribution





Note:

K^2 is calculated only with non-empty bins

NDF is the number of non-empty bins - 1

Kolmogorov-Smirnov test

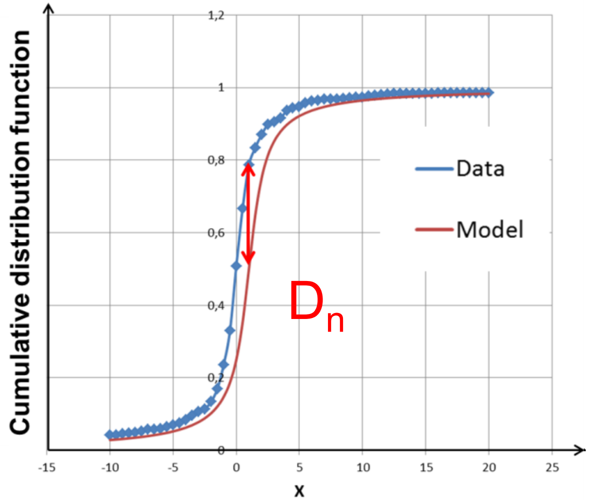
The KS test is an **unbinned** method that uses **all the measured values** of variable **x** to test the compatibility of the data to a model.

- The M measured values x_i are first sorted in ascending order: $x_1 < x_2 < \ldots < x_M$
- The sample cumulative distribution is calculated as:

$$F_{\text{data}}(x) = \begin{cases} 0 \text{ if } x \le x_1 \\ \text{i/M if } x_i \le x < x_{i+1} \\ 1 \; if \; x \ge x_M \end{cases}$$

The test compares **cumulative distribution** of the sample to that of the model. The **maximum distance D_n** between the two is the test statistics:

$$D_n = \sup_x |F_{\text{model}}(x) - F_{\text{data}}(x)|$$



The **p-value** of the KS test is given (for large M) by:

$$\text{p-value} = 2 \sum_{r=1}^{+\infty} (-1)^{r-1} e^{-2Mr^2 D_n^2}$$

Example

Exponential p.d.f

$$f(x;\lambda) = \lambda e^{-\lambda x}, x > 0$$

- Data: λ=0.4 (500 events)
- Model: λ=0.35

$$F_{\text{data}}(x) = \begin{cases} 0 & x \le x_1 \\ \text{i/n} & x_i \le x < x_{i+1} \\ 1 & x \ge x_M \end{cases}$$

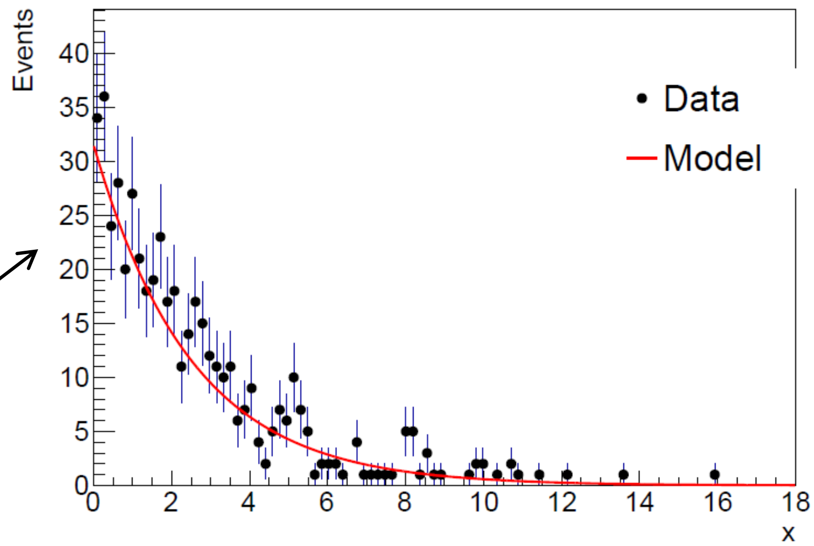
$$F_{\text{model}}(x) = 1 - e^{-\lambda x}$$

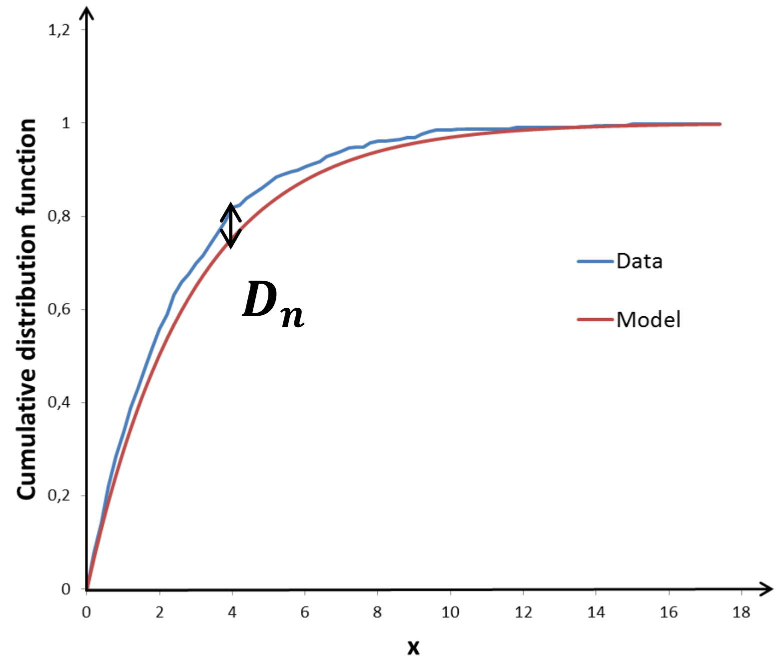
Max distance between cumulative distributions:

$D_n = 0{,}0646$

➔ p-value = 0,03

x_i
0,011401647
0,017623018
0,018095279
0,020447056
0,02616019
0,026849926
0,029898988
0,044689801
0,045548065
0,048410584
0,058308293
0,062655827
0,065376242
…
9,312545995
9,335461119
9,378006281
9,40176752
9,450497283
10,04570365
11,78017539
13,57118477
15,80234274





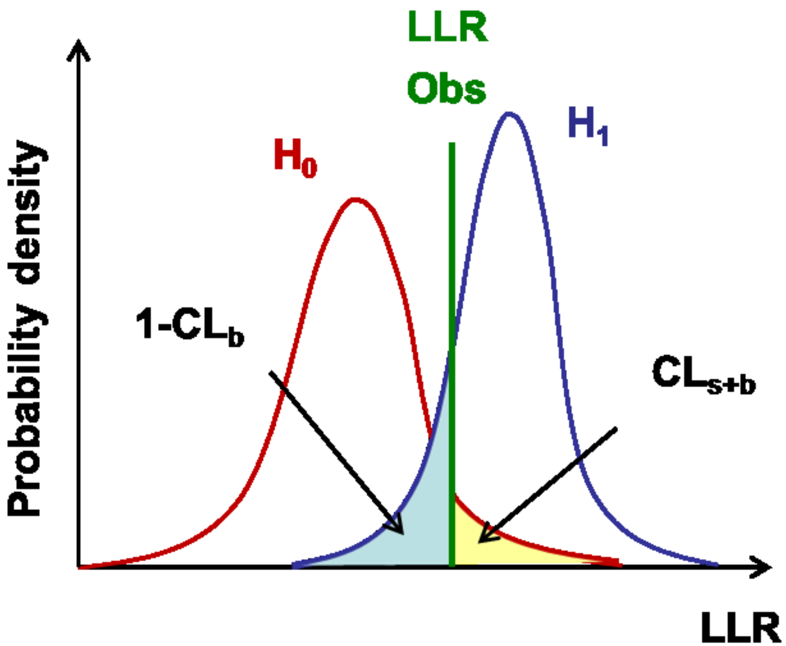
Hypothesis test: CLs method

Test of two hypothesis $\mathbf{H_0}$ and $\mathbf{H_1}$ using data

- Likelihood of data given an hypothesis: $L(\text{data}|H_0)$ or $L(\text{data}|H_1)$

Neyman-Pearson lemma: optimal **test statistics** for hypothesis testing is given by (log) **likelihood ratio**

$$\boxed{\text{LLR} = -2\log\frac{L(\text{data}|H_0)}{L(\text{data}|H_1)}}$$



$$\int_{LLR_{obs}}^{\infty} f(t|H_0)dt = \text{CL}_{s+b}$$

$$\int_{-\infty}^{LLR_{obs}} f(t|H_1)dt = 1 - \text{CL}_b$$

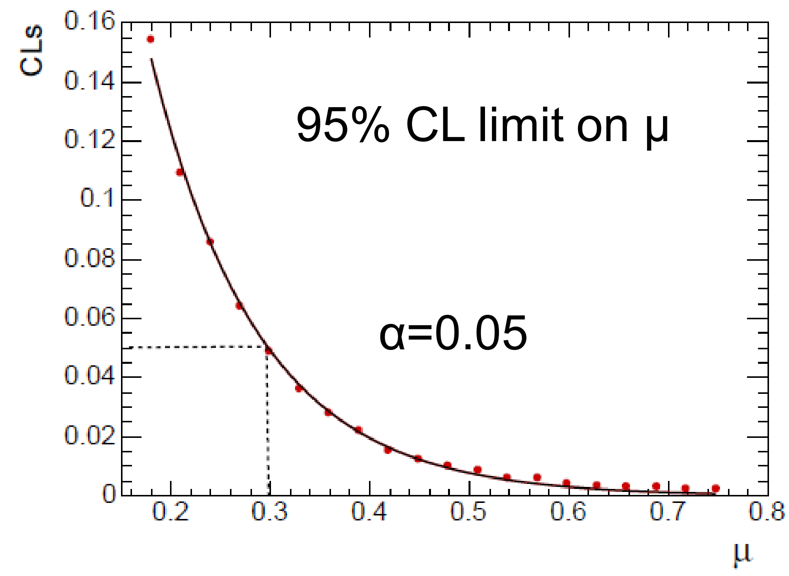
H_0 rejected at (1-α) **confidence level** if $\text{CL}_{s+b} < \alpha$

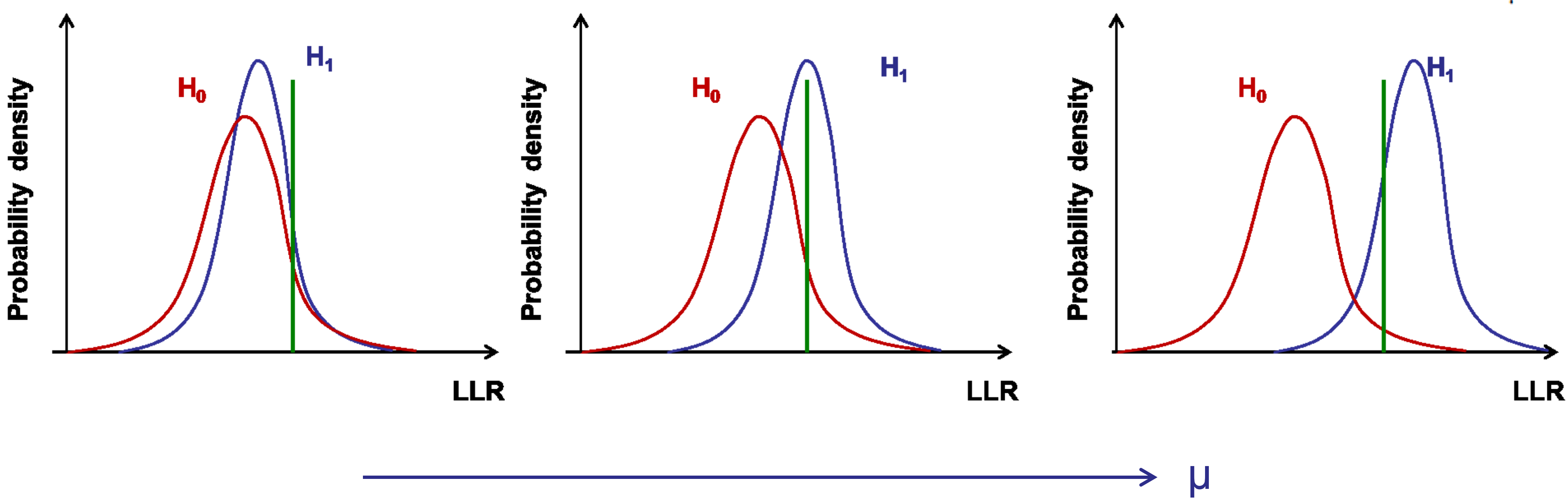
More robust test $\boxed{\text{CL}_s = \frac{\text{CL}_{s+b}}{\text{CL}_b} < \alpha}$

Hypothesis test: CLs method

Testing signal strenght (μ):

- Express number of event of signal as $s = \mu \times s_{nominal}$
- CLs test can be performed for increasing values of μ
- Exclusion limit on μ when CLs<α





Samples and parameter estimation

A random variable X can be described by its p.d.f $f(x)$

f depends of (generally unknown) **parameters** $\vec{\theta} = \{\theta_1, \dots, \theta_P\}$: $\mathbf{f}(\mathbf{x}; \vec{\theta})$

An **experiment** measuring X provides a **sample** of values $\vec{x} = \{x_1, \dots, x_N\}$

One can construct a function of $\vec{x}$ to **infer** the properties of the p.d.f

- This function is called an **estimator**
- The estimator for a parameter θ is often written: $\hat{\theta}$
- **Parameter fitting:** estimate θ using estimator $\hat{\theta}$ and data $\vec{x}$
- $\hat{\theta}(\vec{x})$ is itself a random variable following a p.d.f $g(\hat{\theta}; \theta)$

A **good estimator** should be

Consistent: $\hat{\theta}$ converges to θ for infinite sample ($N \to +\infty$)

Unbiased: average of $\hat{\theta}$ for infinite number of measurements is θ

→ that is: $E[\hat{\theta}(\vec{x})] - \theta = b = 0$

Basic estimators

Consider a **sample** of size N of a random variable X: $\vec{\boldsymbol{x}} = \{\boldsymbol{x_1}, \dots, \boldsymbol{x_N}\}$

X follows a p.d.f $f(x)$ of truth **mean $\boldsymbol{\mu}$ and variance $\boldsymbol{\sigma^2}$**

A simple estimator is the **arithmetic mean** of values x_i: $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$

$$E[\bar{x}] = \frac{1}{N}\sum_{i=1}^{N} E[x_i] = \mu$$ ➔ Unbiased estimator of μ

$$V[\bar{x}] = \boldsymbol{E}\left[\bar{x}^{\boldsymbol{2}}\right] - \boldsymbol{E}[\bar{x}]^{\boldsymbol{2}} = \frac{\boldsymbol{\sigma^2}}{\boldsymbol{N}}$$

This implies that the uncertainty on the sample mean $\bar{x}$ is: $\boldsymbol{\sigma/\sqrt{N}}$

Estimator of the variance: $v = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2 = \overline{x^2} - \bar{x}^2$

Expected value of the estimator: $E[v] = \sigma^2 - \frac{\sigma^2}{N} = \frac{N-1}{N}\sigma^2$

➔ Biased estimator of σ^2 !

Basic estimators

Consider a **sample** of size N of a random variable X: $\vec{\boldsymbol{x}} = \{\boldsymbol{x_1}, \dots, \boldsymbol{x_N}\}$

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Estimator of the variance: $v = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2 = \frac{N}{N-1}\left(\overline{x^2} - \bar{x}^2\right)$

Expected value of the estimator: $E[v] = \sigma^2$

➔ Unbiased estimator of σ^2

Maximum Likelihood estimator (ML)

Suppose a random variable **X** distributed according to a p.d.f $\boldsymbol{f}(\boldsymbol{x}; \vec{\boldsymbol{\theta}})$

- The form of f being know but not the parameters $\vec{\theta} = \{\theta_1, \dots, \theta_P\}$
- Consider a **sample** of X of N values: $\vec{\boldsymbol{x}} = \{\boldsymbol{x}_1, \dots, \boldsymbol{x}_N\}$

The method of ML is a technique to estimate $\vec{\boldsymbol{\theta}}$ given data $\vec{\boldsymbol{x}}$

Joint **likelihood function**
(the x_i are fixed here)

$$\boldsymbol{L}(\vec{\boldsymbol{\theta}}) = \prod_{i=1}^{N} \boldsymbol{f}(\boldsymbol{x}_i; \vec{\boldsymbol{\theta}})$$

The **estimators** $\widehat{\theta}_i$ are given by: $\dfrac{\partial \boldsymbol{L}}{\partial \boldsymbol{\theta}_i} = \boldsymbol{0}, \boldsymbol{i} = \boldsymbol{1} \dots \boldsymbol{P}$

Notes:

- maximizing the likelihood provides and estimate of parameters θ
- In practice the log of L (log likelihoood) is often used
- The likelihood is not a p.d.f !
- Bayesian do transform the likelihood in a p.d.f

Exponential distribution $f(x;\tau) = \frac{1}{\tau} e^{-\frac{x}{\tau}}$

Likelihood: $L(\tau) = \prod_{i=1}^{N} \frac{1}{\tau} e^{-\frac{x_i}{\tau}}$

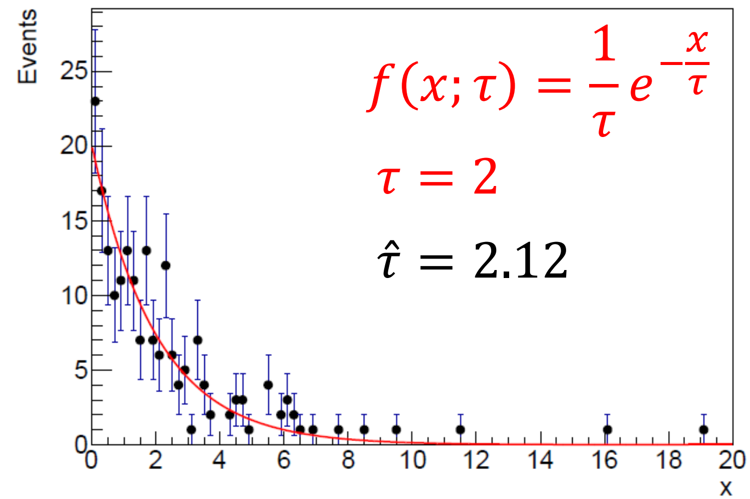
Log-likelihood:

$$\log L(\tau) = \sum_{i=1}^{N} \log f(x_i;\tau) = -N \log \tau - \sum_{i=1}^{N} \frac{x_i}{\tau}$$

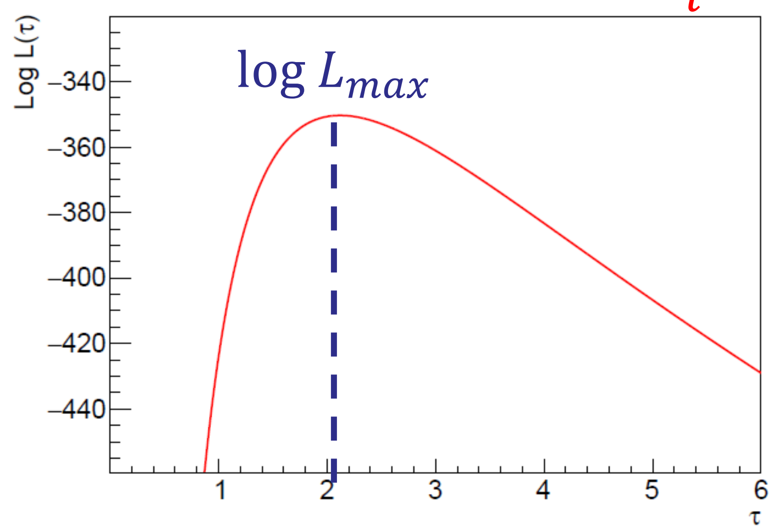
Estimator: $\frac{d\log L}{d\tau} = 0 \Leftrightarrow \boldsymbol{\tau} = \boldsymbol{\hat{\tau}} = \frac{\mathbf{1}}{\boldsymbol{N}} \sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{N}} \boldsymbol{x_i}$

$E[\hat{\tau}] = \tau$ (unbiased estimator)

$N = 200$



$\log L(\tau) = -N \log \tau - N \frac{\hat{\tau}}{\tau}$



Simple examples

Gaussian distribution $f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$, $\log L(\vec{\theta}) = \sum_{i=1}^{N} \log f(x_i;\mu,\sigma)$

Estimators:

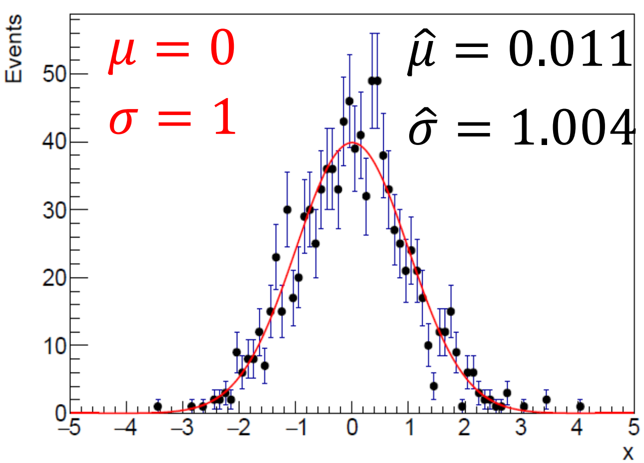
$$\begin{cases} \dfrac{\partial \log L}{\partial \mu} = 0 \Leftrightarrow \boldsymbol{\hat{\mu}} = \dfrac{\mathbf{1}}{\boldsymbol{N}}\sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{N}} \boldsymbol{x_i} \\ \dfrac{\partial \log L}{\partial \sigma^2} = 0 \Leftrightarrow \boldsymbol{\widehat{\sigma^2}} = \dfrac{\mathbf{1}}{\boldsymbol{N}}\sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{N}} (\boldsymbol{x_i}-\boldsymbol{\hat{\mu}})^2 \end{cases}$$

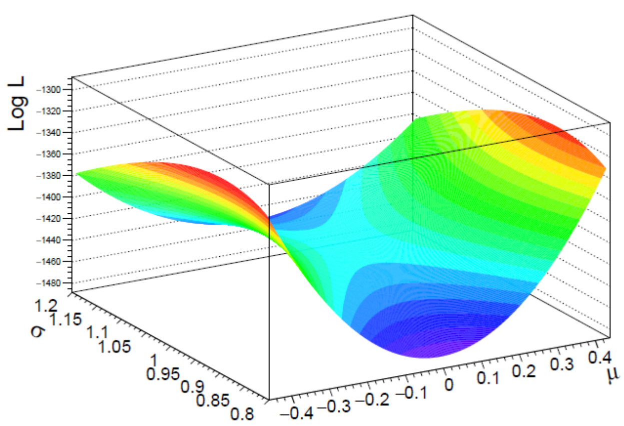
$E[\hat{\mu}] = \mu$ (unbiased)

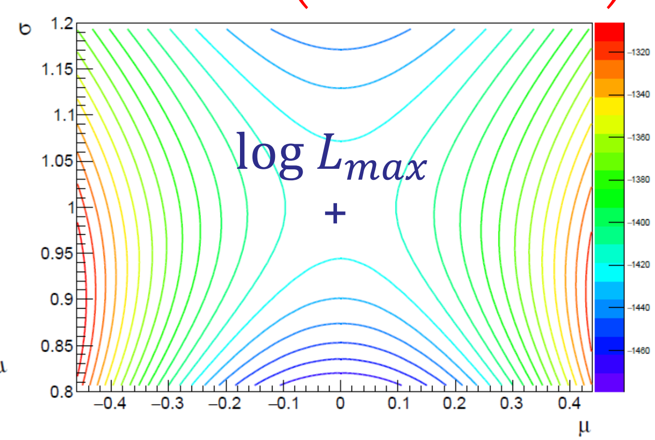
$E\left[\widehat{\sigma^2}\right] = \frac{N-1}{N}\sigma^2$ (biased)

$N = 1000$

$\log L(\mu,\sigma) = -N\log\left(\sqrt{2\pi}\sigma\right) - \frac{1}{2\sigma^2}\left(\sum x_i^2 - N\mu^2\right)$







Uncertainty of ML estimator

Variance of estimator, $V[\hat{\tau}]$, can be tricky to estimate. Several methods exist:

1) Analytical method

For example for the previous exponential distribution

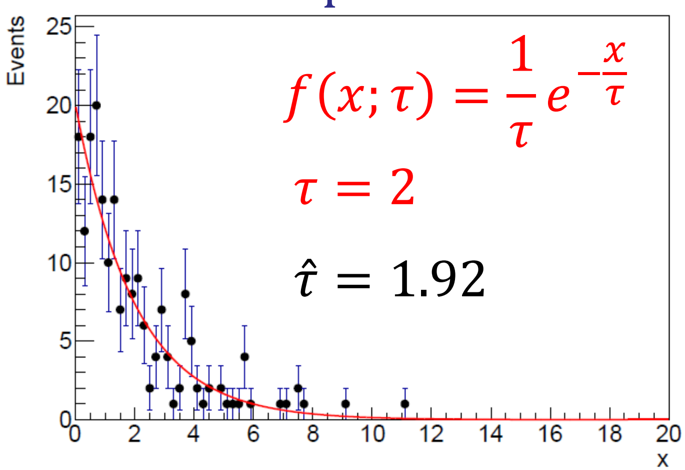
$$\hat{\tau} = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad \text{and} \qquad V[\hat{\tau}] = (\dots) = \frac{\tau^2}{N}$$

2) Monte-Carlo method

Very useful for complex cases (multiparameters, systematic uncertainties)

Ex: generate samples distributed exponentially

$N_{\text{sample}} = 200$

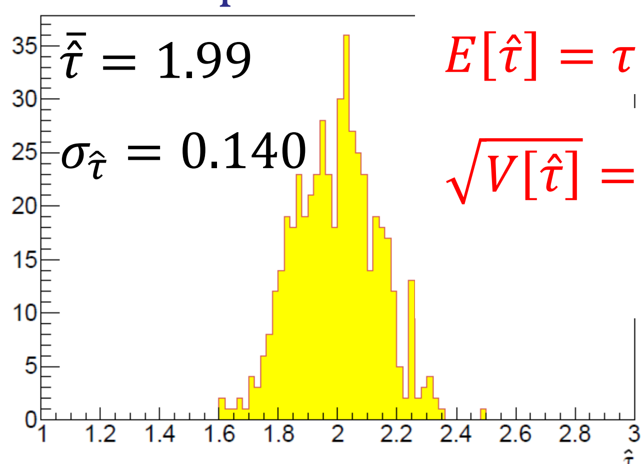


$$f(x;\tau) = \frac{1}{\tau} e^{-\frac{x}{\tau}}$$

$$\tau = 2$$

$$\hat{\tau} = 1.92$$

$N_{\text{experiments}} = 500$



$$\bar{\hat{\tau}} = 1.99$$

$$\sigma_{\hat{\tau}} = 0.140$$

$$E[\hat{\tau}] = \tau = 2$$

$$\sqrt{V[\hat{\tau}]} = \frac{\tau}{\sqrt{N_{\text{sample}}}} = 0.141$$

3) Cramér-Rao bound

Gives a lower bound on any estimator variance (not only ML)

$$\boxed{V[\theta] \geq \frac{\left(1 + \frac{\partial b}{\partial \theta}\right)^2}{E\left[-\frac{\partial^2 \log L}{\partial \theta^2}\right]}, (b\text{: bias})}$$

Equality: estimator is **efficient**

ML are asymptotically efficient

For multiple parameters $\vec{\theta} = \{\theta_1, \dots, \theta_P\}$: $\quad (V^{-1})_{ij} = E\left[-\frac{\partial^2 \log L}{\partial \theta_i \partial \theta_j}\right]$

(and assuming efficiency and b=0)

For large samples: an estimate of the inverse covariant matrix V^{-1} is: $\quad \left(\widehat{V^{-1}}\right)_{ij} = -\frac{\partial^2 \log L}{\partial \theta_i \partial \theta_j}(\theta = \hat{\theta})$

1 parameter: $\quad \boxed{\widehat{\sigma^2} = \frac{-1}{\frac{\partial^2 \log L}{\partial \theta^2}(\hat{\theta})}}$

4) Graphical method

Taylor expansion of log L on estimate $\hat{\theta}$:

$$\log L(\theta) = \log L(\hat{\theta}) + (\theta - \hat{\theta})\frac{\partial \log L}{\partial \theta}(\hat{\theta}) + \frac{1}{2}(\theta - \hat{\theta})^2 \frac{\partial^2 \log L}{\partial \theta^2}(\hat{\theta})$$

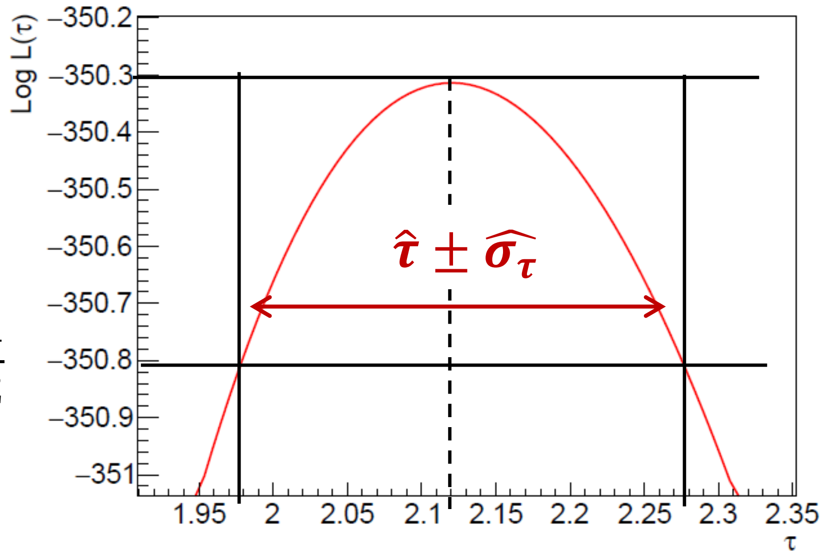
$$= \log L_{\max} - \frac{1}{2\widehat{\sigma^2}}(\theta - \hat{\theta})^2$$

$$\boxed{\Rightarrow \log L(\hat{\theta} \pm \hat{\sigma}) = \log L_{\max} - \frac{1}{2}}$$

$\hat{\tau} \pm \widehat{\sigma_\tau}$ **corresponds to a 68.3% confidence interval**

$\log L_{\max}$

$\log L_{\max} - \frac{1}{2}$

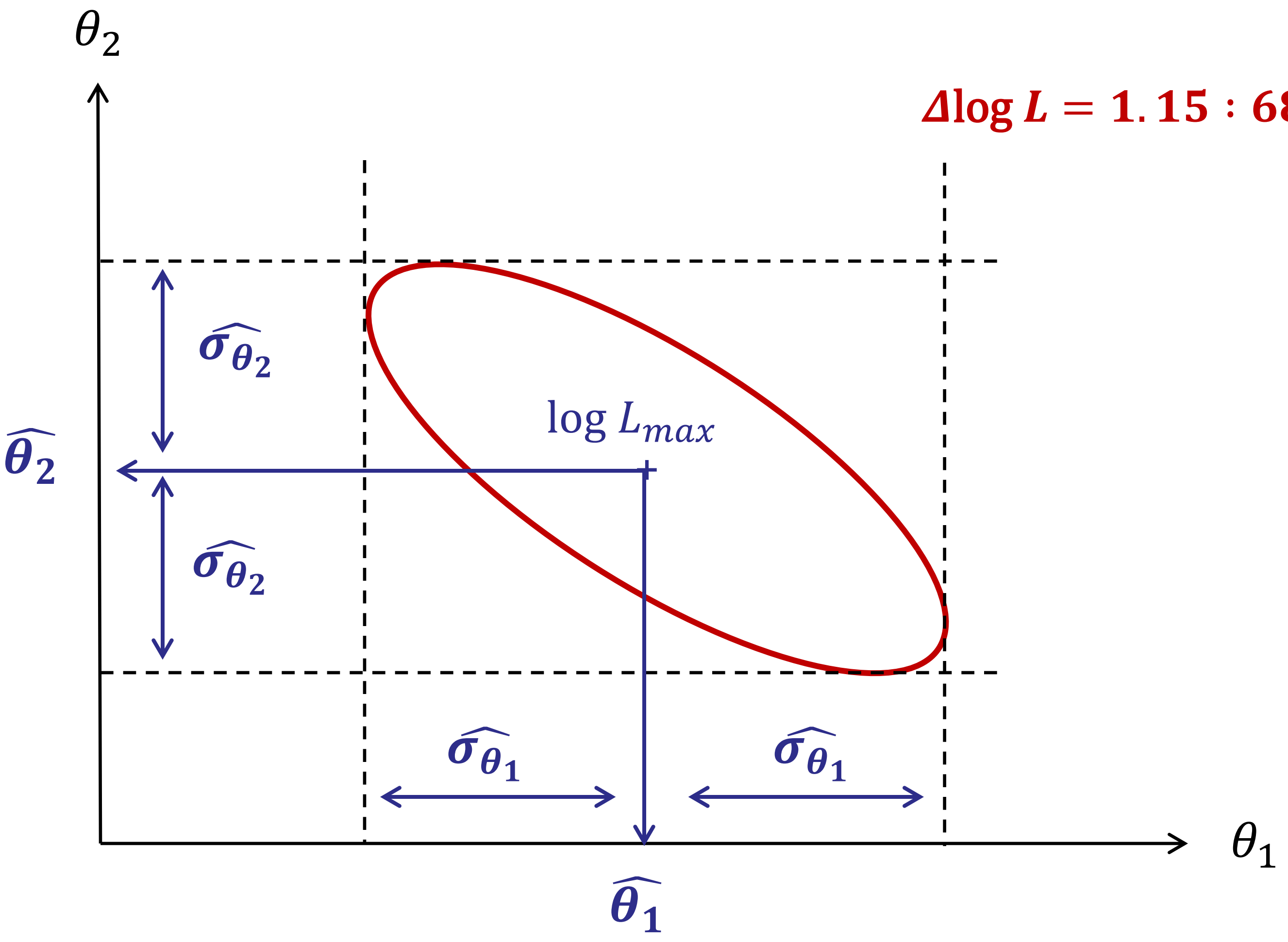


$\Delta \log L = 0.5$: 68.3% CI

$\Delta \log L = 2$: 95.4% CI

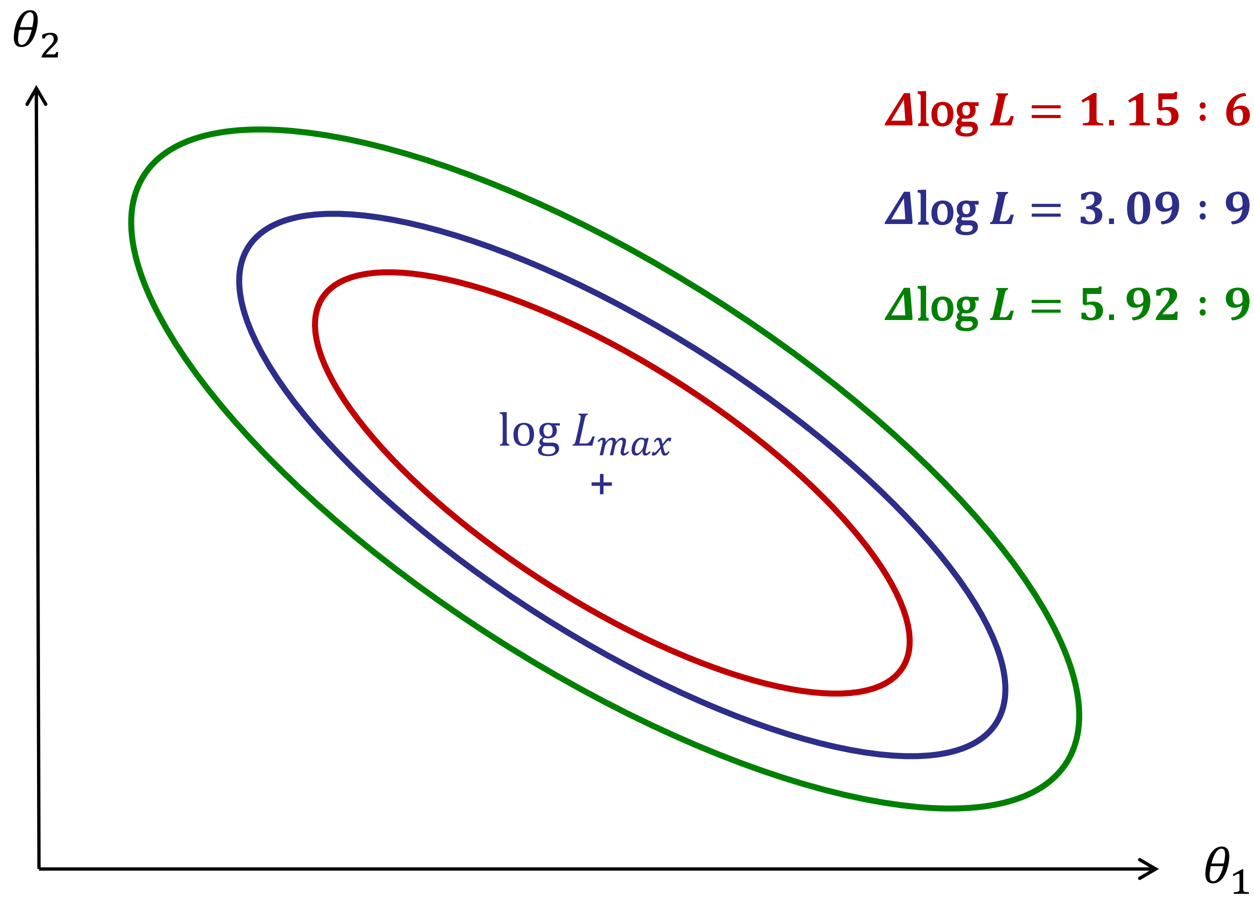
$\Delta \log L = 4.5$: 99.7% CI

Case for 2 parameter $\boldsymbol{\theta_1}$ and $\boldsymbol{\theta_2}$:



Error ellipse

Case for 2 parameter θ_1 and θ_2:



$\Delta\log L = 1.15$: 68.3% CI

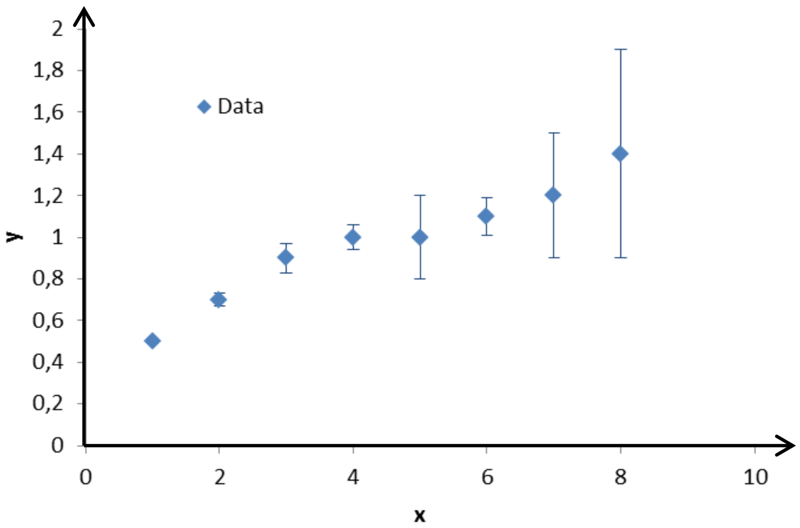
$\Delta\log L = 3.09$: 95.4% CI

$\Delta\log L = 5.92$: 99.7% CI

Chi-square method

Consider N independent variables $\mathbf{y_i}$ function of a another variable $\mathbf{x_i}$

- The y_i are **Gaussian** distributed of mean $\boldsymbol{\mu_i}$ and (known) std $\boldsymbol{\sigma_i}$
- Suppose that $\boldsymbol{\mu} = \boldsymbol{f}(\boldsymbol{x}; \vec{\boldsymbol{\theta}})$ with unknow parameters $\vec{\boldsymbol{\theta}}$



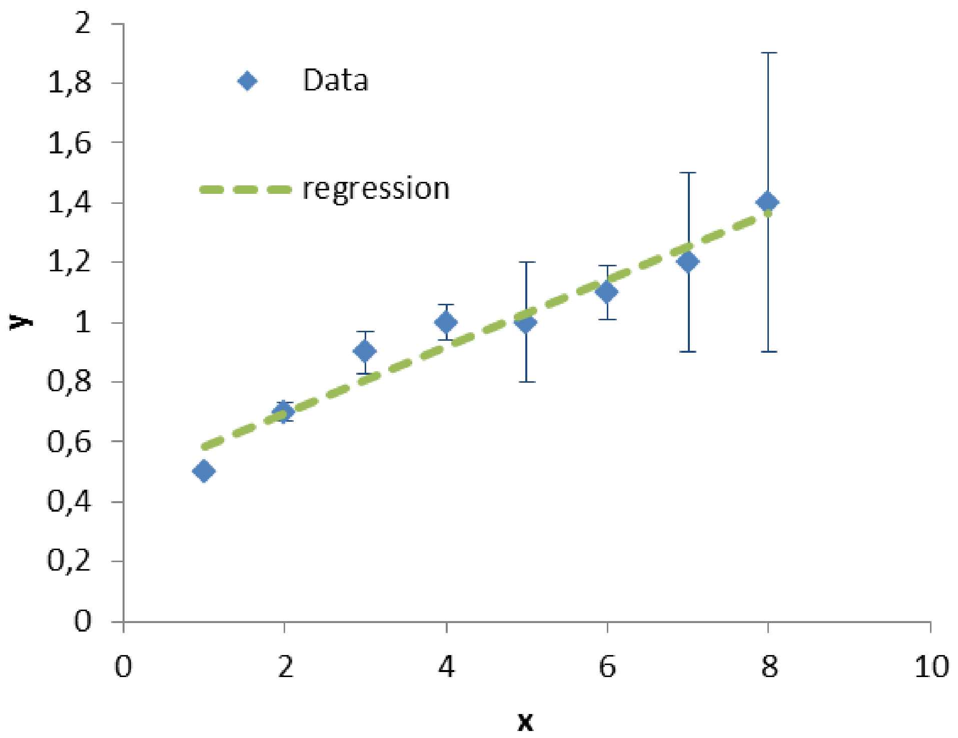
Likelihood: $$L(\vec{\theta}) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{1}{2}\left(\frac{y_i - f(x_i;\vec{\theta})}{\sigma_i}\right)^2}$$

Maximizing $\log L(\vec{\theta})$ to estimate parameters $\vec{\theta}$ is equivalent to **minimize**:

$$\boxed{\chi^2(\vec{\theta}) = \sum_{i=1}^{N} \left(\frac{y_i - f(x_i;\vec{\theta})}{\sigma_i}\right)^2}$$

Simple example

Fit data with a line $f(x; a, b) = ax + b$



Simple **linear regression**: minimize the variance of $y_i - f(x_i; a, b)$

$$w(a,b) = \sqrt{\frac{1}{n}\sum_i (y_i - (ax_i + b))^2}$$

$$\begin{cases} \dfrac{\partial w(a,b)}{\partial a} & = 0 \\ \dfrac{\partial w(a,b)}{\partial b} & = 0 \end{cases}$$

$$\begin{cases} a & = \dfrac{\mathrm{cov}(x,y)}{\mathrm{var}(x)} = r\dfrac{\sigma(y)}{\sigma(x)} \\ b & = \bar{y} - r\dfrac{\sigma(y)}{\sigma(x)}\bar{x} \end{cases}$$

(r: correlation factor between x and y)

Simple example

Fit data with a line $f(x;a,b) = ax + b$

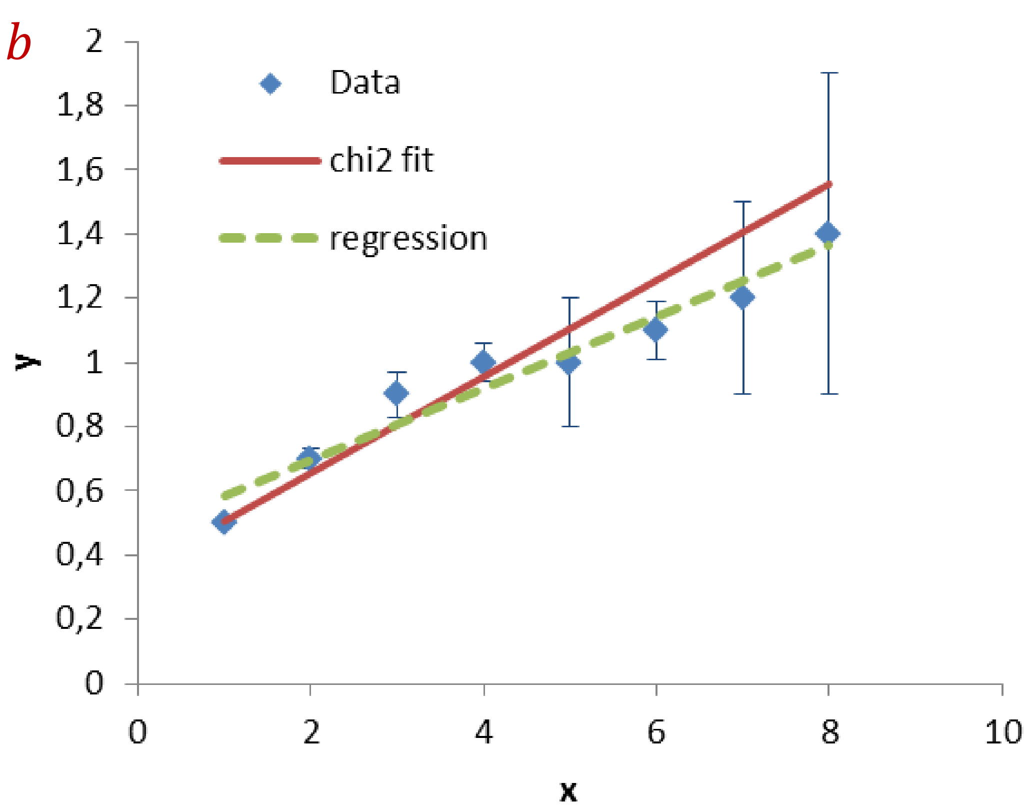
Chi-square fit: minimize $\chi^2(a,b)$

$$\chi^2(a,b) = \sum_{i=1}^{N} \left(\frac{y_i - f(x_i;a,b)}{\sigma_i} \right)^2$$

$$\frac{\partial \chi^2}{\partial a} = 0 \qquad \frac{\partial \chi^2}{\partial b} = 0$$

$$a = \frac{AE - DC}{BE - C^2} \quad b = \frac{DB - AC}{BE - C^2}$$

$$A = \sum_i \frac{x_i y_i}{(\Delta y_i)^2}, \quad B = \sum_i \frac{x_i^2}{(\Delta y_i)^2}, \quad C = \sum_i \frac{x_i}{(\Delta y_i)^2}, \quad D = \sum_i \frac{y_i}{(\Delta y_i)^2}, \quad E = \sum_i \frac{1}{(\Delta y_i)^2}$$



Chi-square: generalization

If $\mathbf{y}_i$ measurements are not independent but related by their cov. matrix V_{ij}

$$\log L(\vec{\theta}) = -\frac{1}{2}\sum_{i,j=1}^{N}(y_i - f(x_i;\vec{\theta}))(V^{-1})_{ij}(y_j - f(x_j;\vec{\theta})) + \text{additive terms}$$

$\log L(\vec{\theta})$ is maximized by minimizing:

$$\chi^2(\vec{\theta}) = \sum_{i,j=1}^{N}(y_i - f(x_i;\vec{\theta}))(V^{-1})_{ij}(y_j - f(x_j;\vec{\theta})$$

Written in matrix notation: $\boxed{\chi^2(\vec{\theta}) = (\vec{y} - \vec{f})^T V^{-1}(\vec{y} - \vec{f})}$

If $f(x_i;\vec{\theta})$ is linear in the parameters $\vec{\theta}$: 1-σ uncertainty contour given by:

$$\boxed{\chi^2(\vec{\theta}) = \chi^2\left(\hat{\vec{\theta}}\right) + 1 = \chi^2_{min} + q}$$

N param.	1	2	3
q	1.00	2.30	3.53

Best Linear Unbiased Estimator: L.Lyons et al. NIM A270 (1988) 110

- Find linear (unbiased) combination of results: $x = \Sigma\, w_i x_i$ with weights w_i that give minimum possible variance σ_x^2
- Account properly of correlations between measurements
- For Gaussian errors: method equivalent to χ^2 minimization

- Two measurements: $x_1 \pm \sigma_1$, $x_2 \pm \sigma_2$ with correlation ρ
- The weights that minimize the χ^2:

Cov. matrix

$$\chi^2 = \begin{pmatrix} x_1 - x & x_2 - x \end{pmatrix} \boxed{\begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}}^{-1} \begin{pmatrix} x_1 - x \\ x_2 - x \end{pmatrix}$$

are:

$$w_1 = \frac{\sigma_2^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2} \qquad w_2 = \frac{\sigma_1^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2} \qquad (w_1 + w_2 = 1)$$

BLUE method

Best Linear Unbiased Estimator: L.Lyons et al. NIM A270 (1988) 110

- Find linear (unbiased) combination of results: $x = \Sigma\, w_i x_i$
  with weights w_i that give minimum possible variance σ_x^2
- Account properly of correlations between measurements
- For Gaussian errors: method equivalent to χ^2 minimization

- Two measurements: $x_1 \pm \sigma_1$, $x_2 \pm \sigma_2$ with correlation ρ
- The combined result is: $x = w_1 x_1 + w_1 x_2$
- And the uncertainty on the combined measurement is:

$$\sigma_x = \sqrt{\frac{\sigma_1^2 \sigma_2^2 (1-\rho^2)}{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2}}$$

Iterative method

- Biases could appear when uncertainties depend on central value of each measurement (L. Lyons et al., Phys. Rev. D41 (1990) 982985)
- Reduced if covariance matrix determined as if the central value is the one obtained from combination
  - Rescale uncertainties to combined value

    ex: for measurement 1, and category i: $\sigma_{i,1}^{rescaled} = \sigma_{i,1} . x_1/x_{blue}$
  - Iterate until central value converges to stable value

Single-top t-channel 8 TeV results

ATLAS [ATLAS-CONF-2012-132, 5.8 fb^{-1}]:

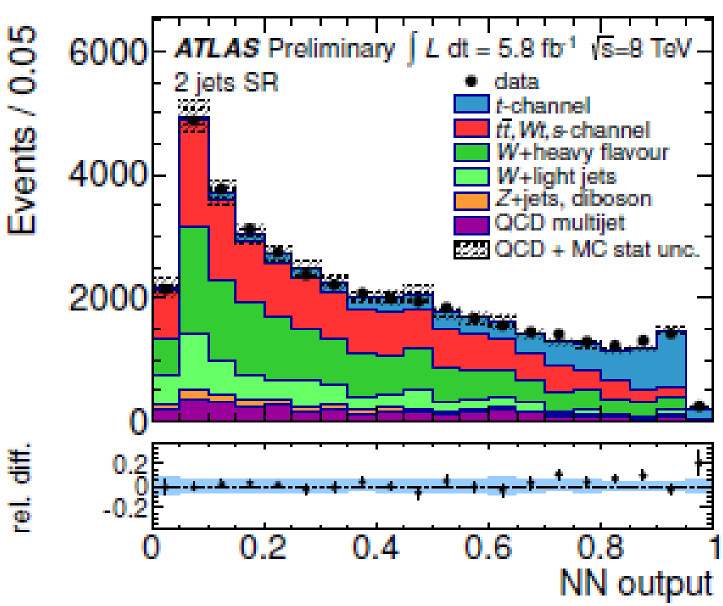
σ_t(t-ch.) = 95 ± 2 (stat.) ± 18 (syst.) pb = 95 ± 18 pb

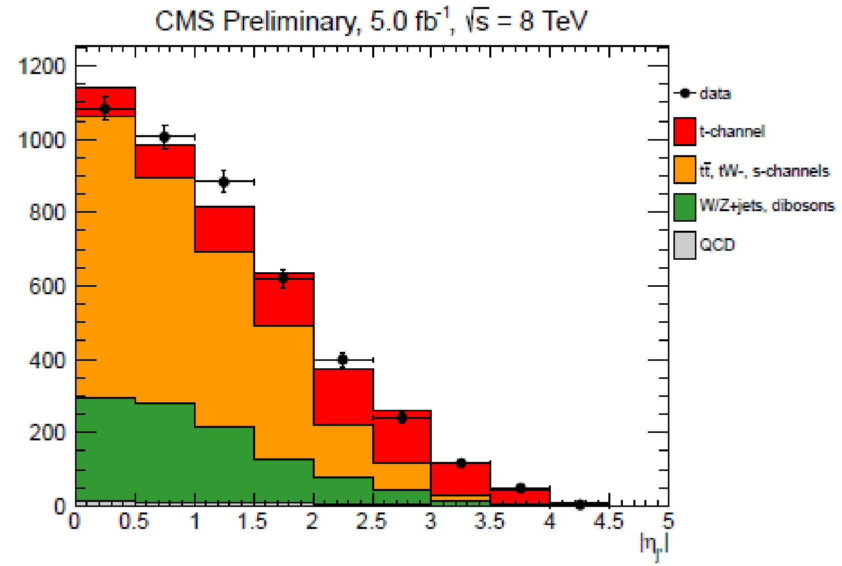
- Multivariate analysis with limited assumptions on simulations
- Fit of **NN distribution** in the data in **e/μ+2/3 jet events, with 1-btag**

CMS [CMS PAS TOP-12-011, 5.0 fb^{-1}]:

σ_t(t-ch.) = 80.1 ± 5.7(stat.) ± 11.0(syst.) ± 4.0(lumi.) pb = 80.1 ± 12.8 pb

- Cut-based analysis, data-driven background estimates (shapes, rates)
- Fit **|η| distribution of forward jet** in **μ+2 jet events, with 1-btag**





Uncertainties categories and correlations

6 categories of uncertainties. Correlation factor between ATLAS/CMS estimated for each.

Category	ATLAS		CMS		ρ
Statistics	Stat. data	2.4%	Stat. data	7.1%	0
	Stat. sim.	2.9%	Stat. sim.	2.2%	0
Total		**3.8%**		**7.5%**	**0**
Luminosity	Calibration	3.0%	Calibration	4.1%	1
	Long-term stability	2.0%	Long-term stability	1.6%	0
Total		**3.6%**		**4.4%**	**0.78**
Simulation and modelling	ISR/FSR	9.1%	Q^2 scale	3.1%	1
	PDF	2.8%	PDF	4.6%	1
	t-ch. generator	7.1%	t-ch. generator	5.5%	1
	$t\bar{t}$ generator	3.3%			0
	Parton shower/had.	0.8%			0
Total		**12.3%**		**7.8%**	**0.83**
Jets	JES	7.7%	JES	6.8%	0
	Jet res. & reco.	3.0%	Jet res.	0.7%	0
Total		**8.3%**		**6.8%**	**0**
Backgrounds	Norm. to theory	1.6%	Norm. to theory	2.1%	1
	Multijet (data-driven)	3.1%	Multijet (data-driven)	0.9%	0
			W+jets, $t\bar{t}$ (data-driven)	4.5%	0
Total		**3.5%**		**5.0%**	**0.19**
Detector modelling	b-tagging	8.5%	b-tagging	4.6%	0.5
	E_T^{miss}	2.3%	Unclustered E_T^{miss}	1.0%	0
	Jet Vertex fraction	1.6%			0
			pile up	0.5%	0
	lepton eff.	4.1%			0
			μ trigger + reco.	5.1%	0
	lepton res.	2.2%			0
	lepton scale	2.1%			0
Total		**10.3%**		**6.9%**	**0.27**
Total uncert.		**19.2%**		**16.0%**	**0.38**

Combined t-channel single-top cross section

Sum covariance matrices in each category to obtain total covariance matrix.

$$\mathbf{C} = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$$

$$\Big\downarrow \ \Sigma$$

$$\mathbf{C} = \begin{pmatrix} 269 & 84 \\ 84 & 182 \end{pmatrix} \mathrm{pb}^2$$

Source	Uncertainty (pb)
Statistics	4.1
Luminosity	3.4
Simulation and modelling	7.7
Jets	4.5
Backgrounds	3.2
Detector modelling	5.5
Total systematics (excl. lumi)	11.0
Total systematics (incl. lumi)	11.5
Total uncertainty	12.2

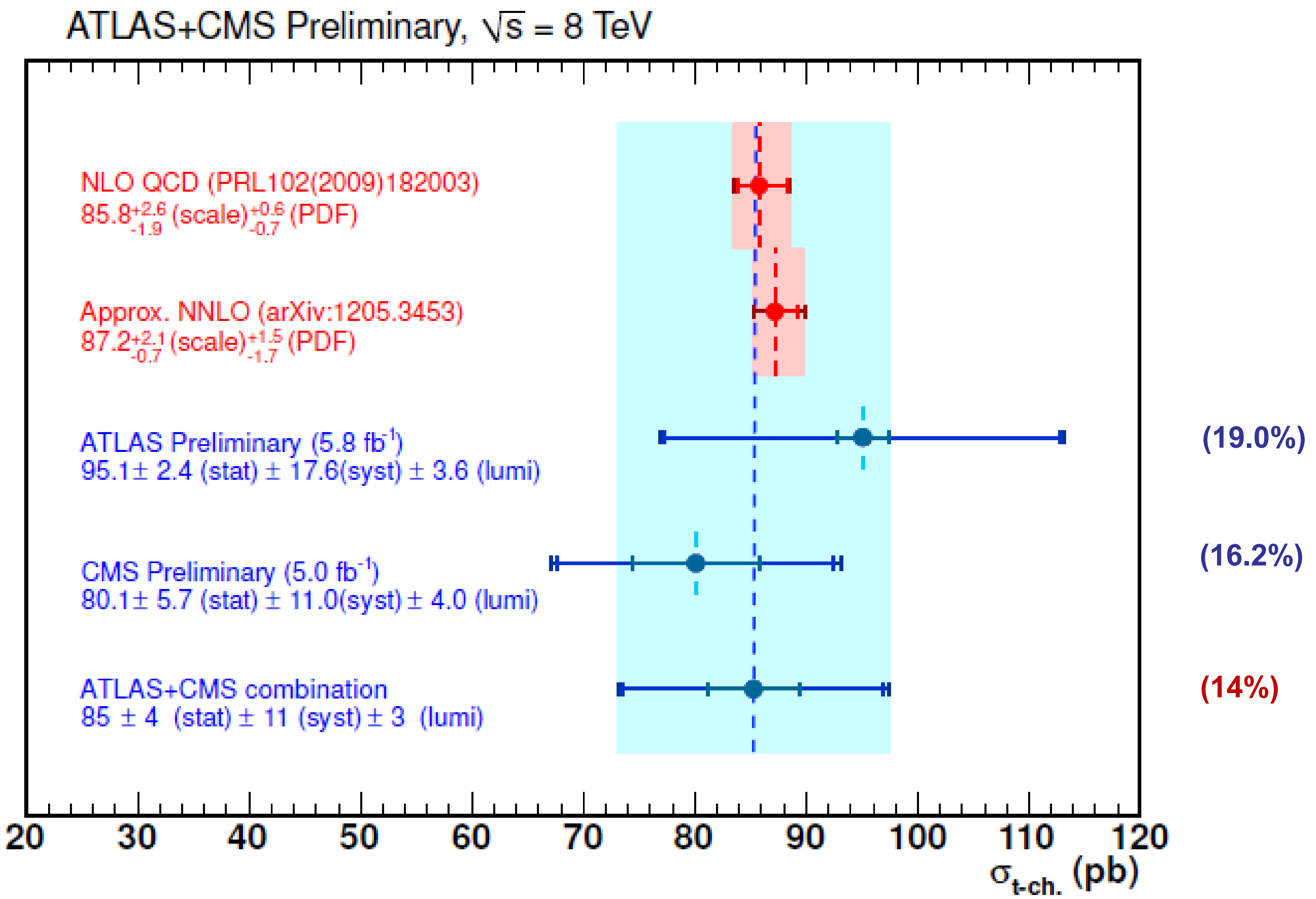
Breakdown of uncertainties

$$\sigma_i^2 = w_1^2\sigma_{i,1}^2 + 2w_1w_2\rho_i\sigma_{i,1}\sigma_{i,2} + w_2^2\sigma_{i,2}^2$$

$$\boxed{\sigma_{\text{t-ch.}} = 85.3 \pm 4.1\,(\text{stat.}) \pm 11.0\,(\text{syst.}) \pm 3.4\,(\text{lumi.})\,\text{pb} = 85.3 \pm 12.2\,\text{pb}}$$

With $w_{ATLAS} = 0.35$ and $w_{CMS} = 0.65$, $\chi^2 = 0.79/1$

Overall correlation of measurements is $\rho_{tot} = 0.38$.


ATLAS+CMS Preliminary, √s = 8 TeV
NLO QCD (PRL102(2009)182003)
85.8+2.6 -1.9 (scale)+0.6 -0.7 (PDF)
Approx. NNLO (arXiv:1205.3453)
87.2+2.1 -0.7 (scale)+1.5 -1.7 (PDF)
ATLAS Preliminary (5.8 fb-1)
95.1± 2.4 (stat) ± 17.6(syst) ± 3.6 (lumi)
CMS Preliminary (5.0 fb-1)
80.1± 5.7 (stat) ± 11.0(syst) ± 4.0 (lumi)
ATLAS+CMS combination
85 ± 4 (stat) ± 11 (syst) ± 3 (lumi)
20
30
40
50
60
70
80
90
100
110
120
σt-ch. (pb)
(19.0%)
(16.2%)
(14%)