Beyond the Standard Model

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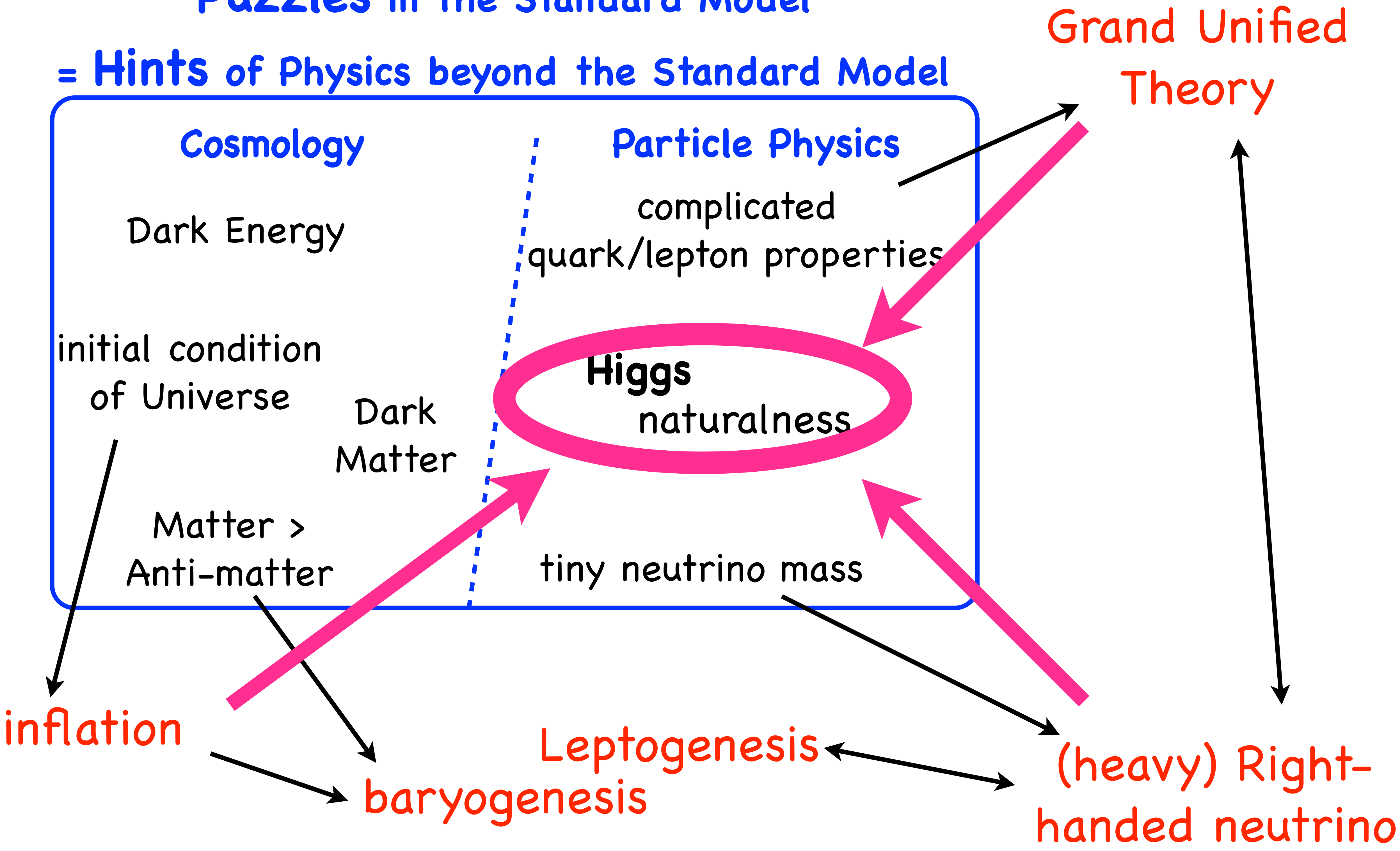
Part 3

(note: part 2 = blackboard lectures)

Puzzles in the Standard Model
= **Hints** of Physics beyond the Standard Model



Cosmology

Dark Energy

initial condition of Universe

Dark Matter

Matter > Anti-matter

Particle Physics

complicated quark/lepton properties

Higgs naturalness

tiny neutrino mass

Grand Unified Theory

inflation

baryogenesis

Leptogenesis

(heavy) Right-handed neutrino

PLAN

3.0. renormalization and naturalness

- (i) renormalization
- (ii) naturalness

cf. the lecture of QCD 5.

§3. SUSY:

3.1. motivations

3.2. supersymmetry

3.3. MSSM (minimal SUSY Standard Model)

3.4. MSSM Lagrangian

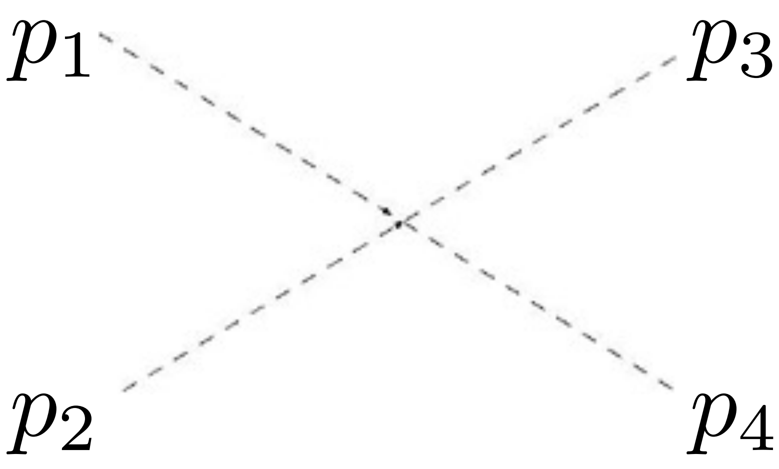
3.5. SUSY after Higgs discovery

(i) renormalization

This part is based on A.Zee's textbook, Chap. III
& a lecture note by R.Kitano (for HEP spring school, May 2013, Biwako, Japan)

Consider a 2-body scattering in a scalar ϕ^4 theory.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$



The **tree level** amplitude is

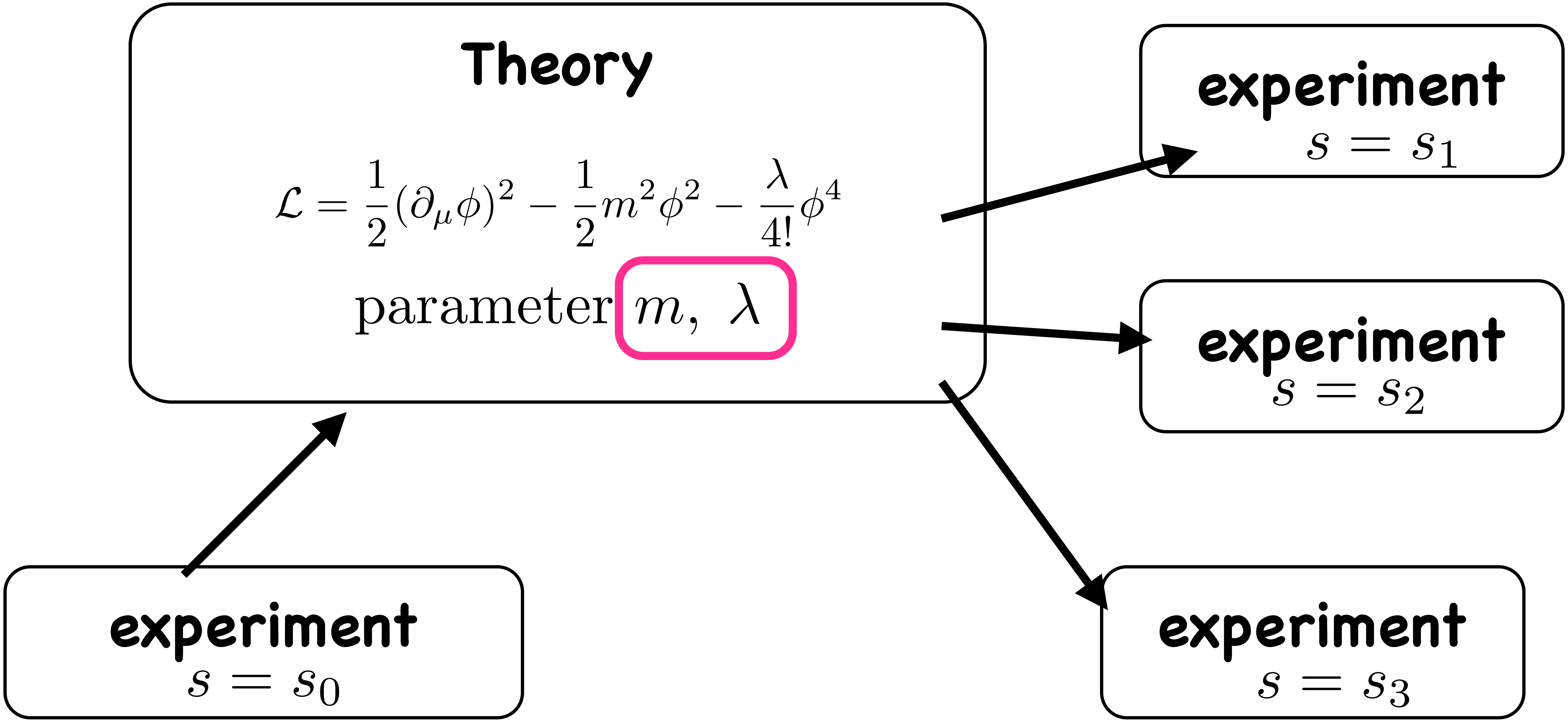
$$\mathcal{M} = -\lambda + \mathcal{O}(\lambda^2)$$

The differential cross section is

$$d\sigma = \frac{1}{16\pi} \cdot \frac{d\Omega}{4\pi} \cdot \frac{1}{s} \underbrace{\cdot |\mathcal{M}|^2,}_{\lambda^2 + \mathcal{O}(\lambda^3)} \qquad s=(p_1+p_2)^2$$

By measuring scattering cross section at e.g., $s = s_0$, we can fix λ.

(i) renormalization



Theory

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

parameter $\boxed{m,\ \lambda}$

experiment $s = s_1$

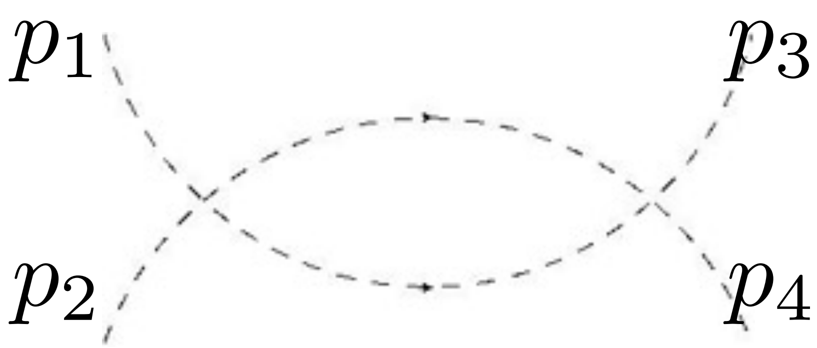
experiment $s = s_2$

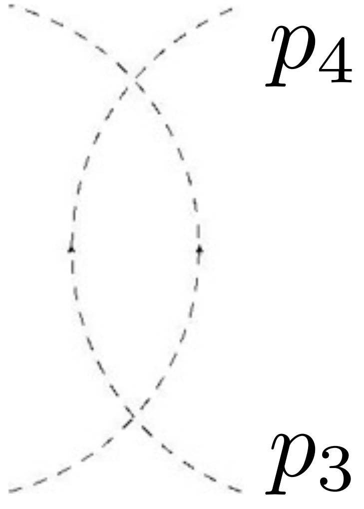
experiment $s = s_0$

experiment $s = s_3$

(i) renormalization

Now let's see the next order in perturbation theory.





The amplitude is

$$\mathcal{M} = -\lambda + \lambda^2 \cdot \frac{-i}{2} \int \frac{d^4k}{(2\pi)^4} \cdot \frac{1}{k^2 - m^2 + i\epsilon} \cdot \frac{1}{(k + p_1 + p_2)^2 - m^2 + i\epsilon} + \mathcal{O}(\lambda^3) + (s \to t, u)$$

Now the integral **diverges**!

$$\mathcal{M} \sim \int^{\infty} \frac{d^4k}{k^4} \sim \int^{\infty} \frac{dk}{k} \sim \log(\infty)$$

But that's OK.

Suppose that the theory is valid only up to a scale Λ,

and cut off the momentum integration.

$$\int^{\infty} dk \longrightarrow \int^{\Lambda} dk$$

(regularization)

(i) renormalization

Then the amplitude becomes (neglecting the mass, for simplicity),

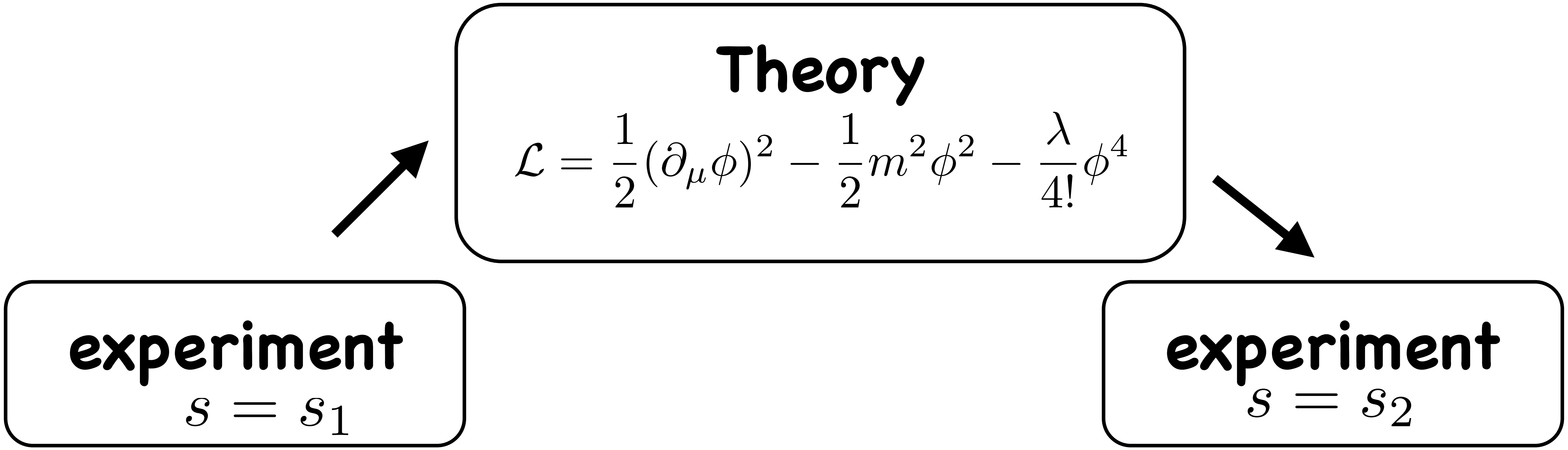
$$\mathcal{M} = -\lambda + C\lambda^2 \left[\log\left(\frac{\Lambda^2}{s}\right) + \log\left(\frac{\Lambda^2}{t}\right) + \log\left(\frac{\Lambda^2}{u}\right) \right] + \mathcal{O}(\lambda^3) \qquad C = 1/32\pi^2$$

LHS can be measured (by scattering at $s = s_0$, for instance).

RHS depends on the artificial cut-off, Λ.

Is that OK? Can this theory still make a prediction?

No problem. We can still compare between experiments.



Theory

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

experiment

$s = s_1$

experiment

$s = s_2$

(i) renormalization

Then the amplitude becomes (neglecting the mass, for simplicity),

$$\mathcal{M} = -\lambda + C\lambda^2 \left[\log\left(\frac{\Lambda^2}{s}\right) + \log\left(\frac{\Lambda^2}{t}\right) + \log\left(\frac{\Lambda^2}{u}\right)\right] + \mathcal{O}(\lambda^3) \qquad C = 1/32\pi^2$$

LHS can be measured (by scattering at $s = s_0$, for instance).

RHS depends on the artificial cut-off, Λ.

Is that OK? Can this theory still make a prediction?

No problem. We can still compare between experiments.

$$\text{exp.1:}\ \mathcal{M}(s_1, t_1, u_1) = -\lambda + C\lambda^2 \left[\log\left(\frac{\Lambda^2}{s_1}\right) + \log\left(\frac{\Lambda^2}{t_1}\right) + \log\left(\frac{\Lambda^2}{u_1}\right)\right] + \mathcal{O}(\lambda^3)$$

$$\text{exp.2:}\ \mathcal{M}(s_2, t_2, u_2) = -\lambda + C\lambda^2 \left[\log\left(\frac{\Lambda^2}{s_2}\right) + \log\left(\frac{\Lambda^2}{t_2}\right) + \log\left(\frac{\Lambda^2}{u_2}\right)\right] + \mathcal{O}(\lambda^3)$$

In each eqs, RHS depends on the artificial cut-off Λ.

But if we subtract,...

(i) renormalization

$$\text{exp.1: } \mathcal{M}(s_1,t_1,u_1) = -\lambda + C\lambda^2\left[\log\left(\frac{\Lambda^2}{s_1}\right) + \log\left(\frac{\Lambda^2}{t_1}\right) + \log\left(\frac{\Lambda^2}{u_1}\right)\right] + \mathcal{O}(\lambda^3)$$

$$\text{exp.2: } \mathcal{M}(s_2,t_2,u_2) = -\lambda + C\lambda^2\left[\log\left(\frac{\Lambda^2}{s_2}\right) + \log\left(\frac{\Lambda^2}{t_2}\right) + \log\left(\frac{\Lambda^2}{u_2}\right)\right] + \mathcal{O}(\lambda^3)$$

In each eqs, RHS depends on the artificial cut-off Λ.

But if we subtract,...

$$\begin{aligned}\mathcal{M}(s_2,t_2,u_2) &= \mathcal{M}(s_1,t_1,u_1) + C\lambda^2\left[\log\left(\frac{s_1}{s_2}\right) + \log\left(\frac{t_1}{t_2}\right) + \log\left(\frac{u_1}{u_2}\right)\right] + \mathcal{O}(\lambda^3)\\ &= \mathcal{M}(s_1,t_1,u_1) + C\mathcal{M}(s_1,t_1,u_1)^2\left[\log\left(\frac{s_1}{s_2}\right) + \log\left(\frac{t_1}{t_2}\right) + \log\left(\frac{u_1}{u_2}\right)\right] + \mathcal{O}(\mathcal{M}(s_1,t_1,u_1)^3)\end{aligned}$$

The exp.2 observable is completely determined by the exp.1 observable. Dependences on the cut-off Λ and λ disappear!

Though the intermediate calculation involves an artificial cut-off Λ, the final relation between exp.1 and exp.2 is independent of Λ.

This is the **"renormalization".**

(i) renormalization

$$\mathcal{M}(s_2,t_2,u_2) = \mathcal{M}(s_1,t_1,u_1) + C\lambda^2\left[\log\left(\frac{s_1}{s_2}\right) + \log\left(\frac{t_1}{t_2}\right) + \log\left(\frac{u_1}{u_2}\right)\right] + \mathcal{O}(\lambda^3)$$

$$= \mathcal{M}(s_1,t_1,u_1) + C\mathcal{M}(s_1,t_1,u_1)^2\left[\log\left(\frac{s_1}{s_2}\right) + \log\left(\frac{t_1}{t_2}\right) + \log\left(\frac{u_1}{u_2}\right)\right] + \mathcal{O}(\mathcal{M}(s_1,t_1,u_1)^3)$$

OK, we can now compare experimentally measurable quantities.
But what is the coupling λ then?

Recall

$$\mathcal{M}(s_0,t_0,u_0) = -\lambda + C\lambda^2\left[\log\left(\frac{\Lambda^2}{s_0}\right) + \log\left(\frac{\Lambda^2}{t_0}\right) + \log\left(\frac{\Lambda^2}{u_0}\right)\right] + \mathcal{O}(\lambda^3)$$

Thus

$$\lambda = -\mathcal{M}(s_0,t_0,u_0) + C\lambda^2\left[\log\left(\frac{\Lambda^2}{s_0}\right) + \log\left(\frac{\Lambda^2}{t_0}\right) + \log\left(\frac{\Lambda^2}{u_0}\right)\right] + \mathcal{O}(\lambda^3)$$

$$= -\mathcal{M}(s_0,t_0,u_0) + C\mathcal{M}(s_0,t_0,u_0)^2\left[\log\left(\frac{\Lambda^2}{s_0}\right) + \log\left(\frac{\Lambda^2}{t_0}\right) + \log\left(\frac{\Lambda^2}{u_0}\right)\right] + \mathcal{O}(\mathcal{M}^3)$$

Fine. λ is now expressed in terms of observable $\mathcal{M}(s_0,t_0,u_0)$.

But it depends on Λ!

$$\boxed{\lambda = \lambda(\Lambda)}$$

(i) renormalization

But it depends on Λ!

$$\boxed{\lambda = \lambda(\Lambda)}$$

No problem. Physics does not depend on Λ.

The combination $\big(\Lambda, \lambda(\Lambda)\big)$ determines the observable.
and the observables are independent of Λ.

By this requirement we can also obtain a differential equation for $\lambda(\Lambda)$.

$$\mathcal{M}(s_0,t_0,u_0) = -\lambda(\Lambda) + C\lambda(\Lambda)^2\left[\log\left(\frac{\Lambda^2}{s_0}\right)+\log\left(\frac{\Lambda^2}{t_0}\right)+\log\left(\frac{\Lambda^2}{u_0}\right)\right]+\mathcal{O}(\lambda^3)$$

$$\to 0 = \frac{d\mathcal{M}(s_0,t_0,u_0)}{d\log\Lambda} = -\frac{d\lambda}{d\log\Lambda} + 6C\lambda^2 + 2C\lambda\frac{d\lambda}{d\log\Lambda}\left[\log\left(\frac{\Lambda^2}{s_0}\right)+\cdots\right]+\mathcal{O}(\lambda^3)$$

$$\to \boxed{\frac{d\lambda}{d\log\Lambda} = 6C\lambda^2+\mathcal{O}(\lambda^3)}$$

Renormalization Group Equation.

As far as this is satisfied, observables are independent of Λ.

(i) renormalization

We regularized the divergent integral by a momentum cut-off Λ.

There are other regularizations.

Dimensional regularization:

We don't discuss what it is, but the basic idea is the same.

- There is an artificial parameter μ with a mass dimension one.
- The combination $\big(\mu, \lambda(\mu)\big)$ determines the observable, and the observables are independent of μ.

- The μ-dependence is given by the Renormalization Group Equation.

$$\frac{d\lambda(\mu)}{d\log\mu} = 6C\lambda^2 + \mathcal{O}(\lambda^3)$$

(i) renormalization

We regularized the divergent integral by a momentum cut-off Λ.

There are other regularizations.

Dimensional regularization:

REMARK: Although the observable is independent of μ,

it is better to use μ close to the energy scale of your interest

when you calculate by perturbation theory.

$$\mathcal{M}(s,\cdots) = -\lambda(\mu) + C\lambda(\mu)^2\left[\log\left(\frac{\mu^2}{s}\right) + \cdots\right] + \mathcal{O}(\lambda^3)$$

When $\mu \sim$ s, this log factor is small

—> better convergence.

e.g., For hard processes at LHC, $\alpha_s(\mu)$ with μ >> 1 GeV is used.

PLAN

3.0. renormalization and naturalness

(i) renormalization

done

(ii) naturalness

§3. SUSY:

3.1. motivations

3.2. supersymmetry

3.3. MSSM (minimal SUSY Standard Model)

3.4. MSSM Lagrangian

3.5. SUSY after Higgs discovery

(ii) naturalness

Now let's discuss the **naturalness** for the Higgs mass parameter.

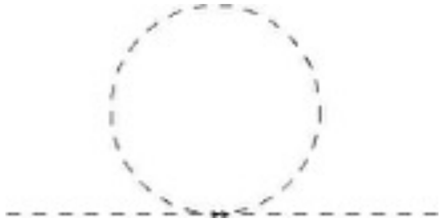
$$V(H) = -m^2|H|^2 + \lambda_H|H|^4.$$

Consider with the **cut-off regularization.**
The correction to the mass parameter is

$$\delta m^2 = -\frac{3}{8\pi^2}\left(y_t^2 - \lambda_H - \frac{3}{8}g^2 - \frac{3}{8}g'^2 + \cdots\right)\Lambda^2$$

NOTE:
quadratic dependence
(not logarithmic)

top loop



Higgs loop

gauge boson loops

(ii) naturalness

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NOTE:
quadratic dependence
(not logarithmic)

Let's consider the largest top contribution.
The corrected mass parameter is then...

$$m^2 = m^2(\Lambda) - \frac{3}{8\pi^2}y_t^2\Lambda^2 + \cdots$$

(ii) naturalness

$$m^2 = m^2(\Lambda) - \frac{3}{8\pi^2} y_t^2 \Lambda^2 + \cdots$$

($\sim$100 GeV)^2

($\sim$100000…… GeV)^2

depending on the cut-off

($\sim$100000…… GeV)^2 + ($\sim$100 GeV)^2

If $m^2(\Lambda)$ is a fundamental parameter,

(for instance, if our space-time becomes somehow latticed at very small scale Λ^{-1})

this is unnatural.

naturalness problem

(ii) naturalness

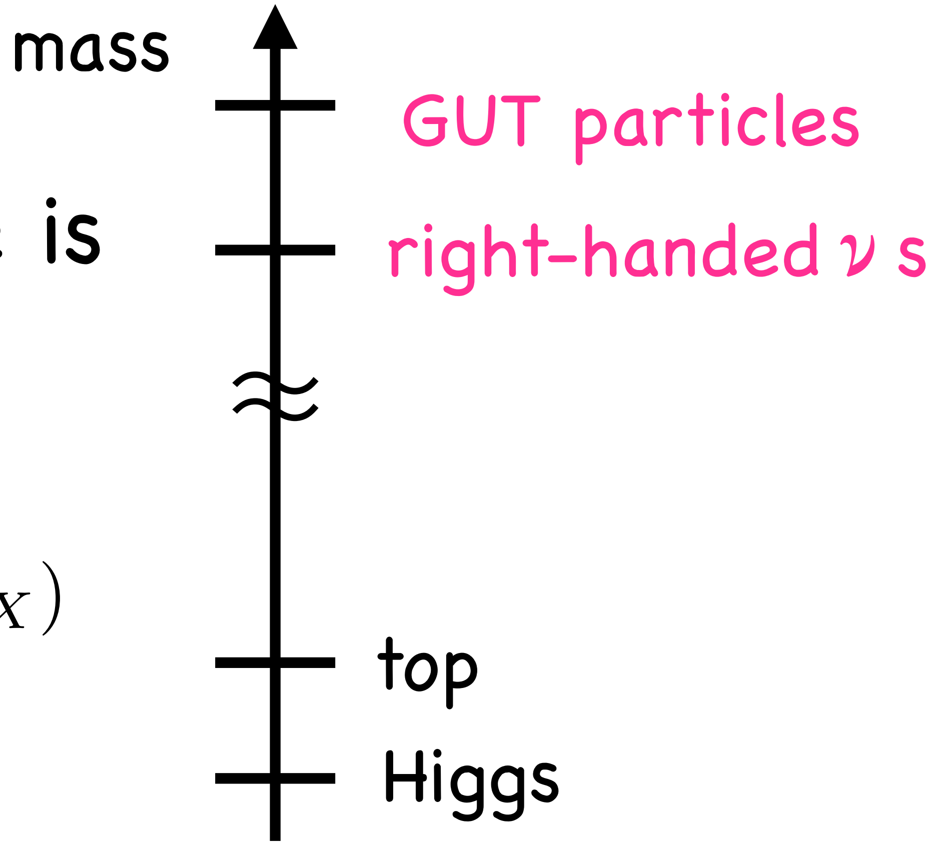
$$m^2 = m^2(\Lambda) - \frac{3}{8\pi^2} y_t^2 \Lambda^2 + \cdots$$

naturalness problem

Remark

Λ^2 term may be an artifact of cut-off regularization.
For instance, it doesn't exist in dimensional regularization.

But even if the Λ^2 term is absent,
a large correction exists as far as there is
a **heavy particles** coupled to Higgs.



$$m^2 = m^2(\mu) + C_X m_X^2 \log\left(\frac{\mu^2}{m_X^2}\right) + \cdots \qquad (\text{for } \mu > m_X)$$

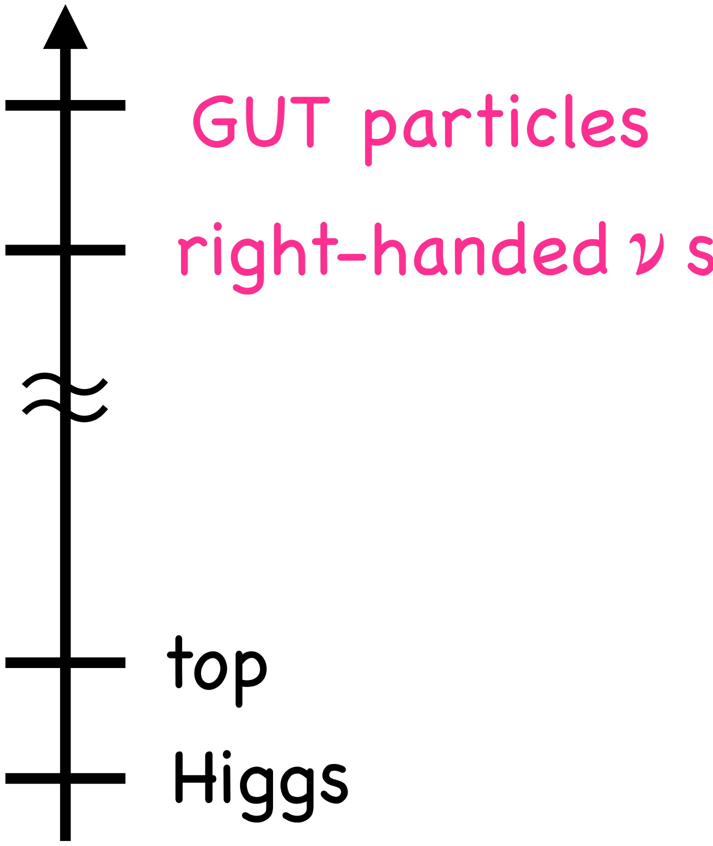
X's coupling to Higgs

X's mass

[See e.g., 1303.7244, 1402.2658]

(ii) naturalness

$$m^2 = m^2(\mu) + C_X m_X^2 \log\left(\frac{\mu^2}{m_X^2}\right) + \cdots \qquad (\text{for } \mu > m_X)$$



naturalness problem

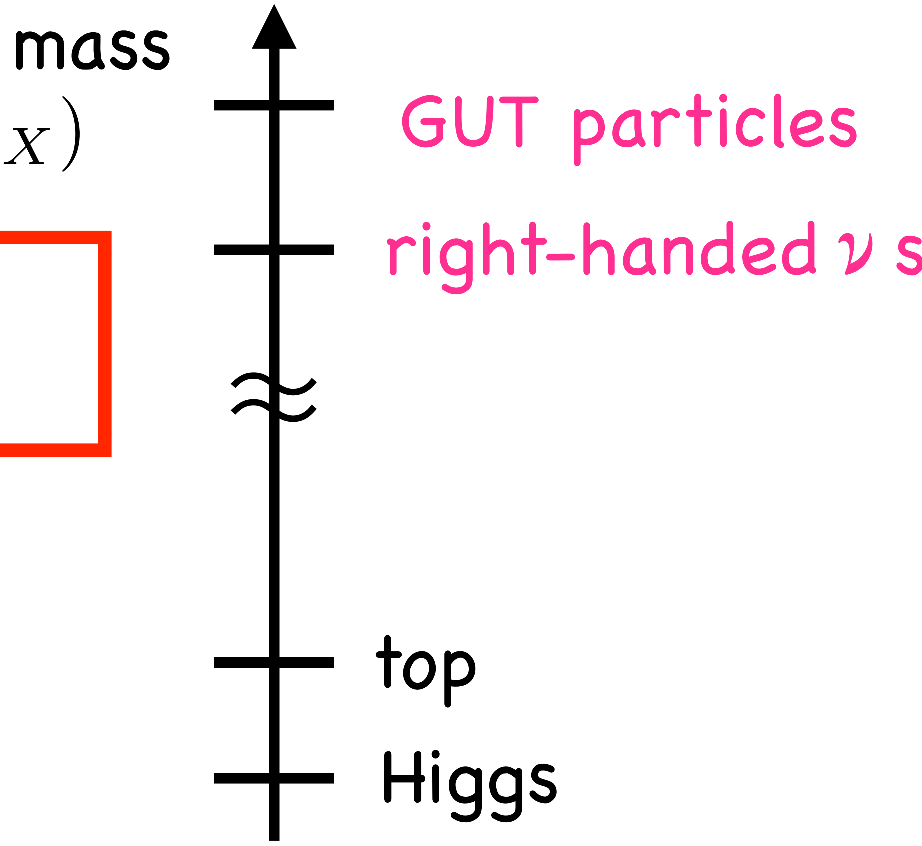
If we think m(μ) at μ > m_X is more fundamental than m(μ) at weak scale μ,

(For instance, in QCD, $m_{u,d}$ at $\mu > \Lambda_{QCD}$ seems more fundamental than the pion masses.....)

this is unnatural.

(ii) naturalness

$$m^2 = m^2(\mu) + C_X m_X^2 \log\left(\frac{\mu^2}{m_X^2}\right) + \cdots \qquad (\text{for } \mu > m_X)$$


mass

GUT particles

right-handed ν s

$\approx$

top

Higgs

naturalness problem

solutions

- Don't mind.

  There is no problem in experimental observables.
  (Don't listen too much to theorists…… 😜.)
- Landscape + anthropic principle

  We live in a fine-tuned vacuum because otherwise we cannot live.
- No such heavy particles

  or 4d perturbative QFT breaks down anyway before that.
- cancellation among loop corrections
  - §3 SUSY
  - little Higgs (top correction canceled.)
  - . . .

PLAN

3.0. renormalization and naturalness

- (i) renormalization
- (ii) naturalness

done

§3. SUSY:

3.1. motivations

3.2. supersymmetry

3.3. MSSM (minimal SUSY Standard Model)

3.4. MSSM Lagrangian

3.5. SUSY after Higgs discovery

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3. Supersymmetry

Fermion <-> Boson

Standard Model		spin		
quark	q	1/2 <-> 0	squark	$\widetilde{q}$
lepton	ℓ	1/2 <-> 0	slepton	$\widetilde{\ell}$
Higgs	H	0 <-> 1/2	higgsino	$\widetilde{h}$
gauge bosons	γ, Z, W, g	1 <-> 1/2	gaugino	$\widetilde{B}, \widetilde{W}, \widetilde{g}$

MSSM (minimal SUSY Standard Model)

PLAN

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3. Supersymmetry

§ 3.1. motivations

(i) naturalness of Higgs boson mass
(ii) coupling unification
(iii) · · ·

3. Supersymmetry

§ 3.1. motivations

(i) naturalness of Higgs boson mass

- In terms of cut-off regularization,

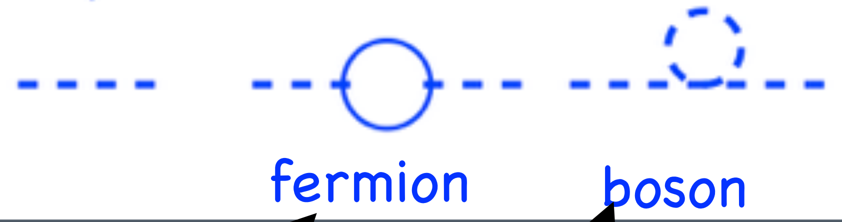
fine-tuning problem

$$m_H^2 = m_{H,0}^2 + \Lambda^2 \qquad (\Lambda \gg m_H)$$

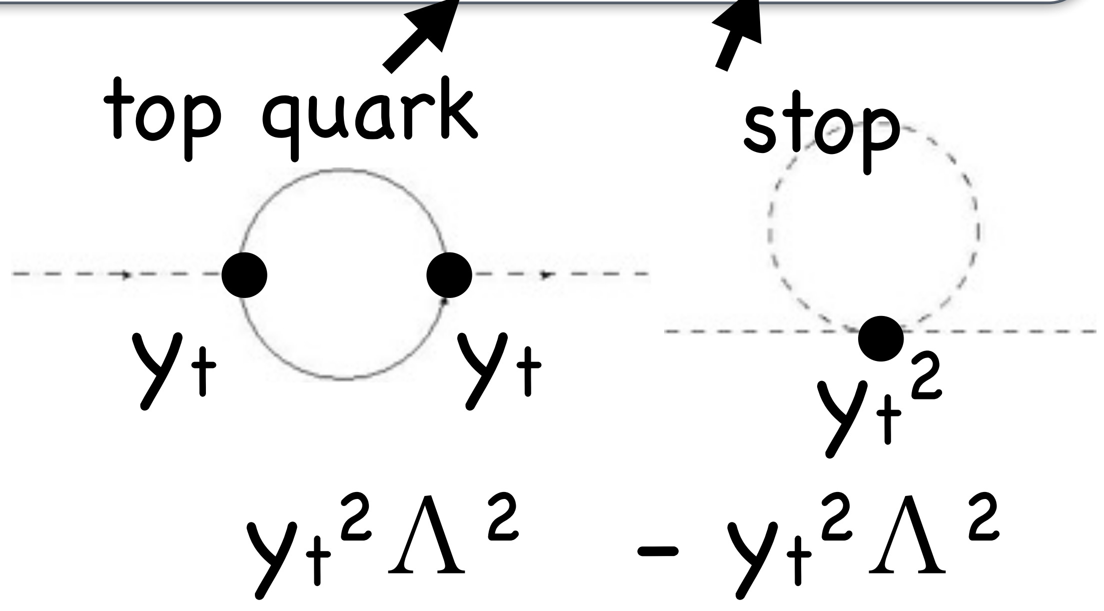
(fine tuning like 1.0000000000000001 - 1)

→ solved by the supersymmetry !

$$m_H^2 = m_{H,0}^2 + (\ \Lambda^2 - \Lambda^2\ )$$



For instance,...



- In terms of dim. reg. + heavy particle X,

$\delta m^2(\mu) \sim (m_X[\text{scalar}]^2 - m_X[\text{fermions}]^2) \log(\mu/m_X)$

3. Supersymmetry

§ 3.1. motivation

(ii) coupling unification

- gauge field kinetic term of GUT (SU(5))

$$\frac{1}{g_{\rm GUT}^2}\sum_{a=1}^{24} F^a_{\mu\nu}F^{a\mu\nu} \qquad \left(F_{\mu\nu}F^{\mu\nu} \to \frac{1}{g^2}F_{\mu\nu}F^{\mu\nu} \text{ by field redefinition } A_\mu \to \frac{1}{g}A_\mu\right)$$

$$= \frac{1}{g_3^2}G^a_{\mu\nu}G^{a\mu\nu} + \frac{1}{g_2^2}W^a_{\mu\nu}W^{a\mu\nu} + \frac{1}{g_1^2}B_{\mu\nu}B^{\mu\nu} + (X, Y \text{ gauge bosons})$$

$$\Longrightarrow \quad g_1 = g_2 = g_3 = g_{\rm GUT} \quad @ \quad \mu = \text{unification scale}$$

(= the scale where GUT is broken)

- running gauge coupling

R.G.eq
$$\frac{d}{d\log\mu}\alpha_i^{-1} = -\frac{1}{2\pi}b_i \qquad \text{(1-loop)}$$

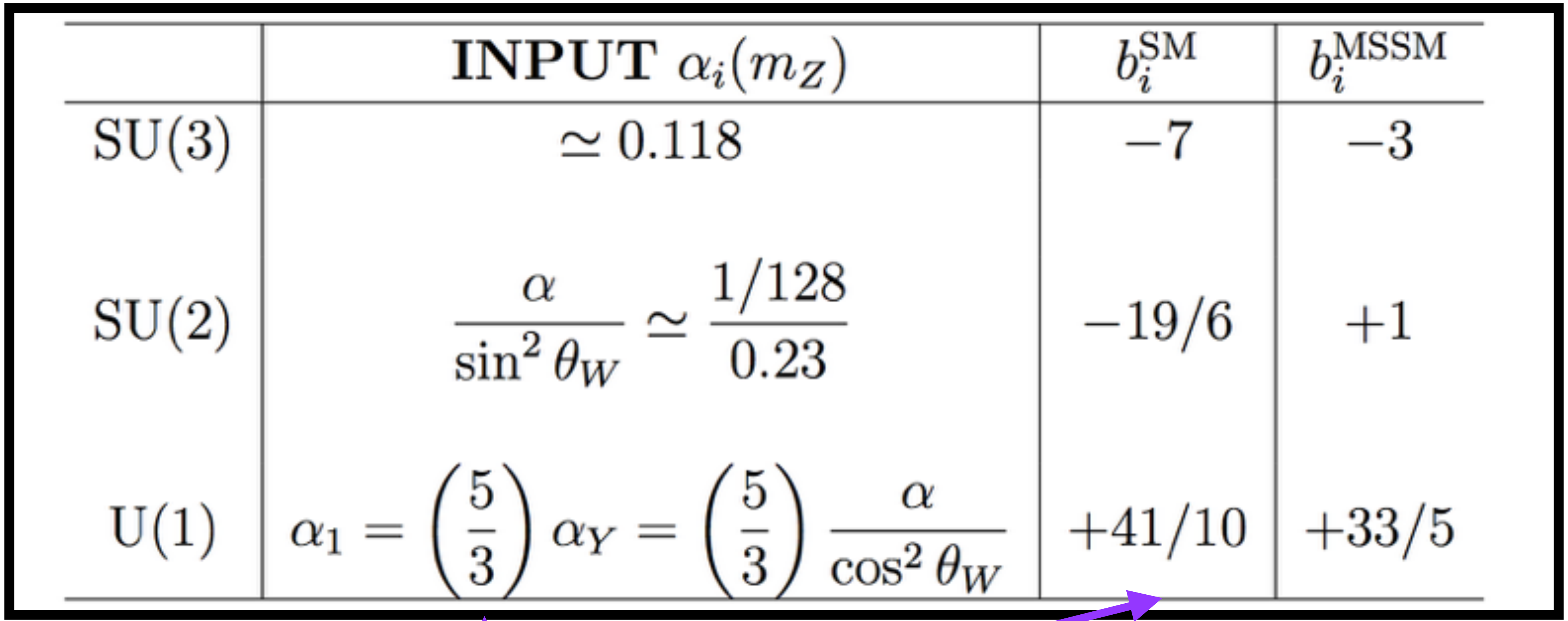
3. Supersymmetry

§ 3.1. motivation

(ii) coupling unification

- running gauge coupling

R.G.eq $$\frac{d}{d\log\mu}\alpha_i^{-1} = -\frac{1}{2\pi}b_i \qquad \text{(1-loop)}$$



	INPUT $\alpha_i(m_Z)$	b_i^{SM}	b_i^{MSSM}
SU(3)	$\simeq 0.118$	-7	-3
SU(2)	$\frac{\alpha}{\sin^2\theta_W} \simeq \frac{1/128}{0.23}$	$-19/6$	$+1$
U(1)	$\alpha_1 = \left(\frac{5}{3}\right)\alpha_Y = \left(\frac{5}{3}\right)\frac{\alpha}{\cos^2\theta_W}$	$+41/10$	$+33/5$

$g_1 = \sqrt{\frac{5}{3}}g_Y, \quad b_1 = \frac{3}{5}b_Y$

3. Supersymmetry

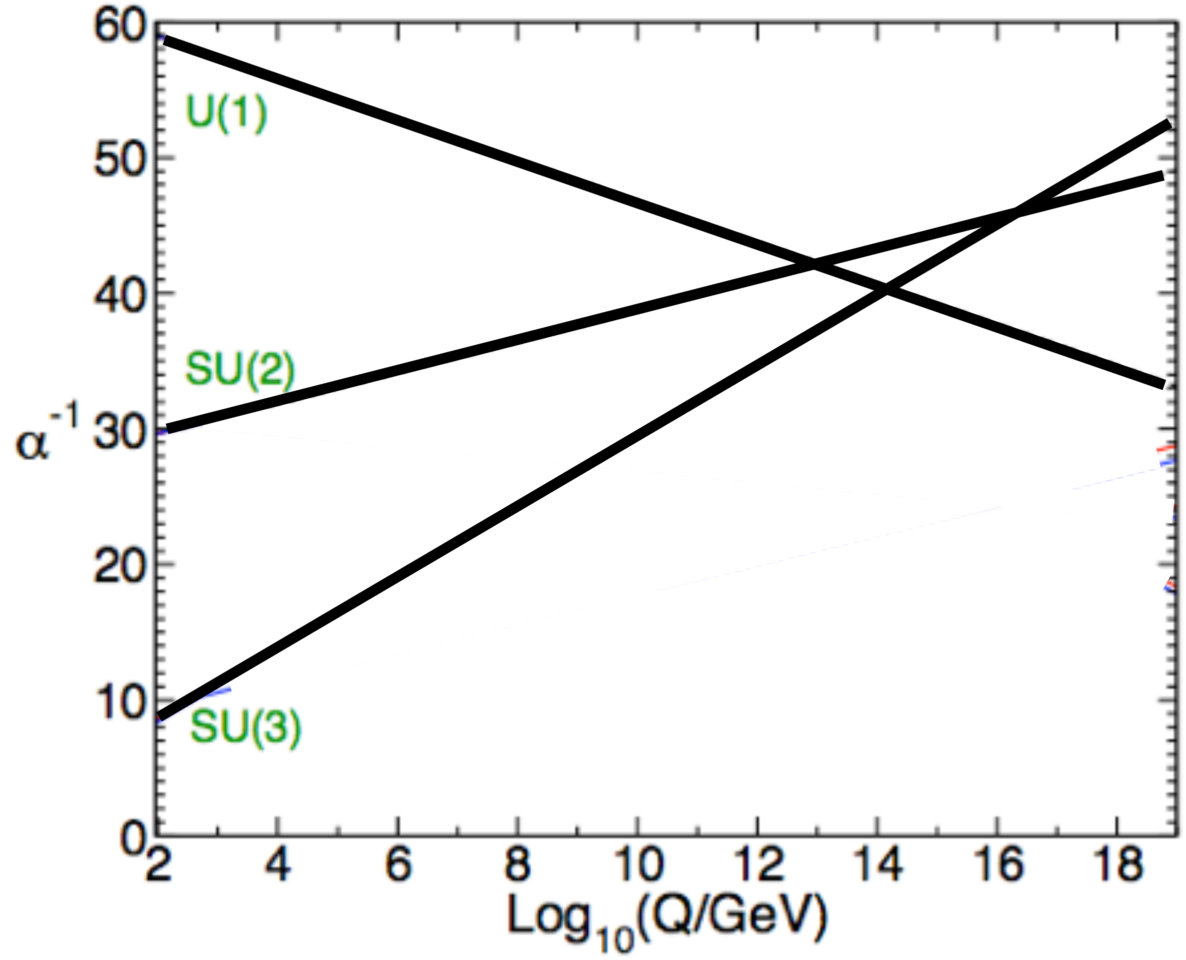
§ 3.1. motivation

(ii) coupling unification

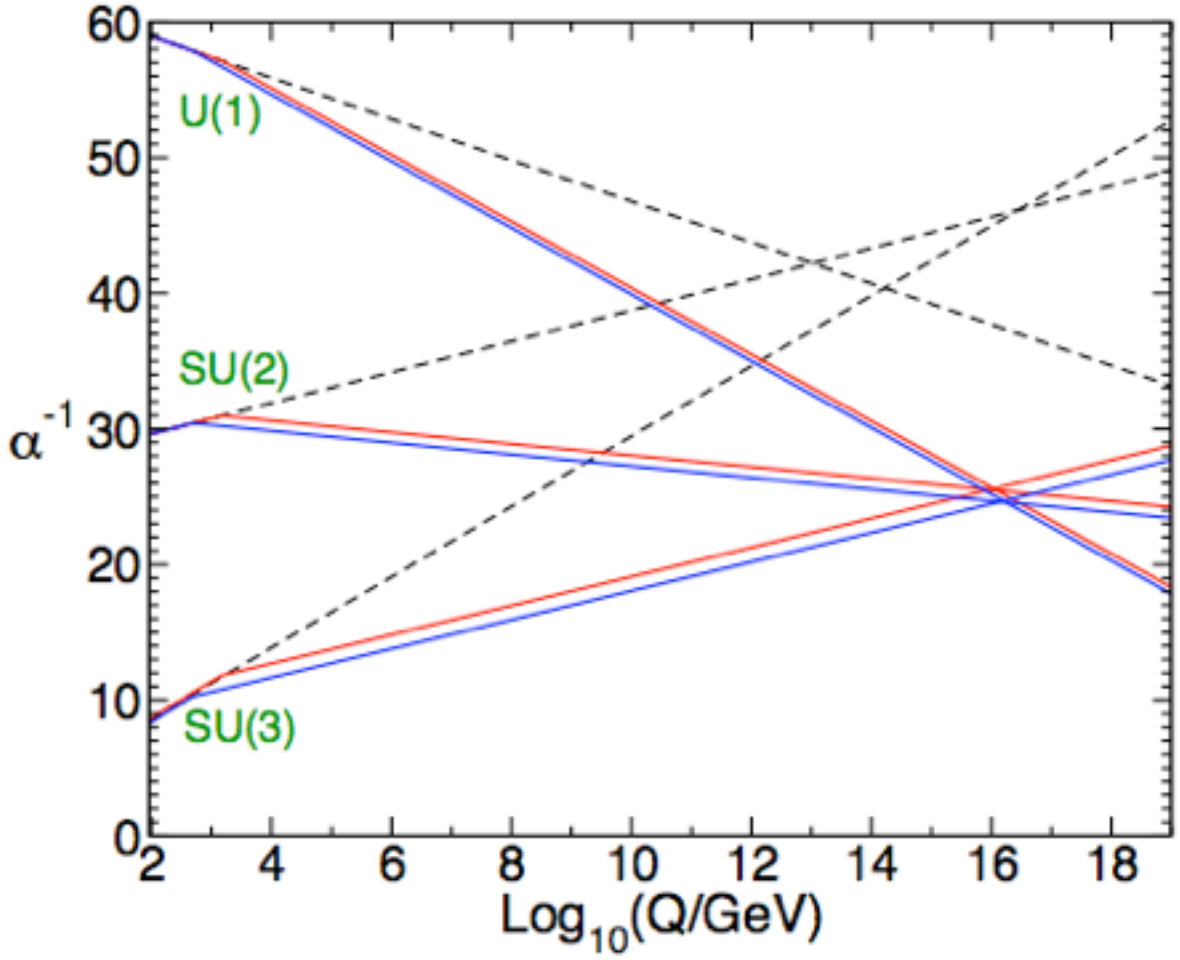
- running gauge coupling

R.G.eq
$$\frac{d}{d\log\mu}\alpha_i^{-1} = -\frac{1}{2\pi}b_i \qquad \text{(1-loop)}$$

Standard Model



Standard Model + SUSY



PLAN

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done

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3. Supersymmetry

§ 3.2. SUSY

(i) simplest model (in 4-dim)

$$\phi \longleftrightarrow \psi$$

complex scalar ⟷ 2-component Weyl fermion

$$\underline{\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi}$$

free massless

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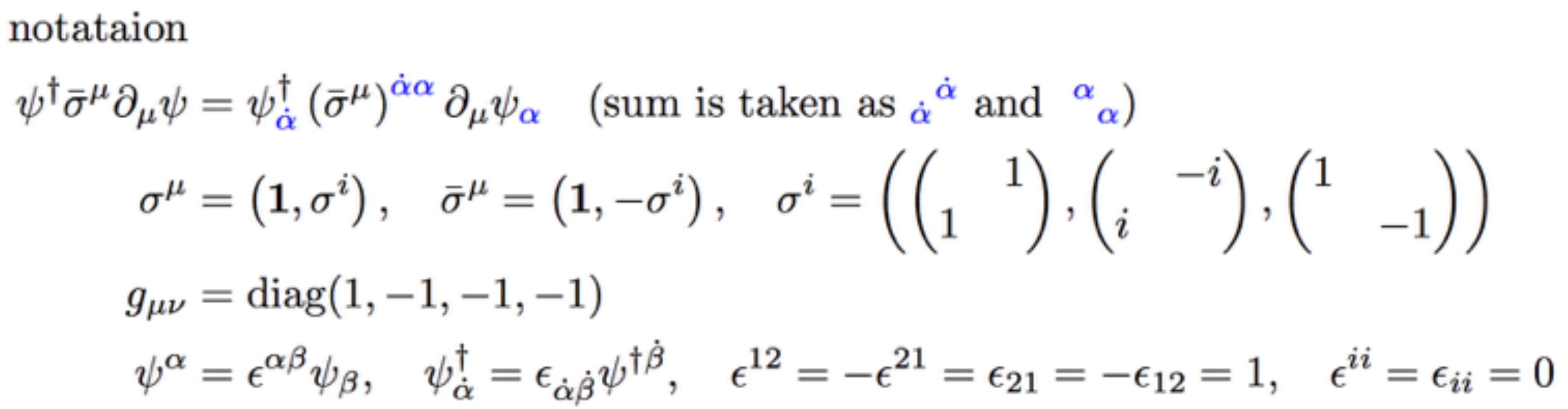


notataion

$$\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi = \psi^\dagger_{\dot{\alpha}} (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} \partial_\mu \psi_\alpha \quad (\text{sum is taken as } {}_{\dot{\alpha}}{}^{\dot{\alpha}} \text{ and } {}^{\alpha}{}_{\alpha})$$

$$\sigma^\mu = (\mathbf{1}, \sigma^i), \quad \bar{\sigma}^\mu = (\mathbf{1}, -\sigma^i), \quad \sigma^i = \left(\begin{pmatrix} & 1 \\ 1 & \end{pmatrix}, \begin{pmatrix} & -i \\ i & \end{pmatrix}, \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \right)$$

$$g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$$

$$\psi^\alpha = \epsilon^{\alpha\beta} \psi_\beta, \quad \psi^\dagger_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}} \psi^{\dagger\dot{\beta}}, \quad \epsilon^{12} = -\epsilon^{21} = \epsilon_{21} = -\epsilon_{12} = 1, \quad \epsilon^{ii} = \epsilon_{ii} = 0$$

)

(i) simplest model (in 4-dim)

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi$$

What kind of symmetry does it have ?

example: U(1) symmetry (scalar sector)

$$\phi \rightarrow e^{i\alpha} \phi$$

or

$$\delta\phi = i\alpha\phi$$

Lagrangian is invariant under this U(1) transformation.

$$\begin{aligned}\delta\mathcal{L} &= \partial_\mu(\delta\phi^*)\partial_\mu\phi + \partial_\mu\phi^*\partial_\mu(\delta\phi) \\ &= \partial_\mu(-i\alpha\phi^*)\partial_\mu\phi + \partial_\mu\phi^*\partial_\mu(i\alpha\phi) \quad = 0\end{aligned}$$

(i) simplest model (in 4-dim)

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi$$

What kind of symmetry does it have ?

SUSY transformation $\boxed{\text{Fermion <-> Boson}}$

(i) simplest model (in 4-dim)

$$\underline{\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi}$$

What kind of symmetry does it have ?

SUSY transformation **Fermion <-> Boson**

$$\begin{cases} \delta\phi &= \chi^\alpha \psi_\alpha \\ \delta\psi_\alpha &= -i(\sigma^\mu \chi^\dagger)_\alpha \partial_\mu \phi \end{cases}$$

χ_α SUSY transformation parameter.

- 2-component
- anti-commuting (fermionic)

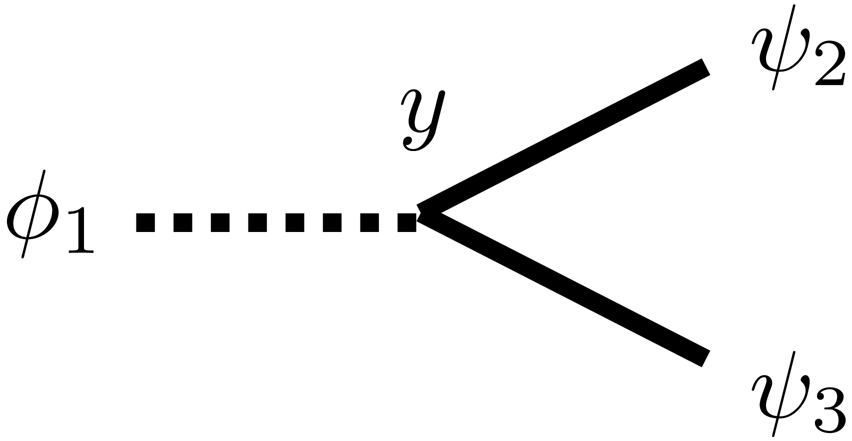
Then,...

$$\begin{aligned} \delta\mathcal{L} &= \delta\left(\partial_\mu \phi^* \partial^\mu \phi\right) && + \delta\left(i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi\right) \\ &= \partial_\mu(\delta\phi^*)\partial^\mu\phi + \partial_\mu\phi^*\partial^\mu(\delta\phi) && + i(\delta\psi^\dagger)\bar{\sigma}^\mu\partial_\mu\psi + i\psi^\dagger\bar{\sigma}^\mu\partial_\mu(\delta\psi) \\ &= \partial_\mu(\chi^\dagger\psi^\dagger)\partial^\mu\phi + \partial_\mu\phi^*\partial^\mu(\chi\psi) + \;.\;\;.\;\;. \end{aligned}$$

$$= 0$$

(ii) interaction

$$\mathcal{L}_{\rm int} = -y \cdot \phi_1 \psi_2 \psi_3$$



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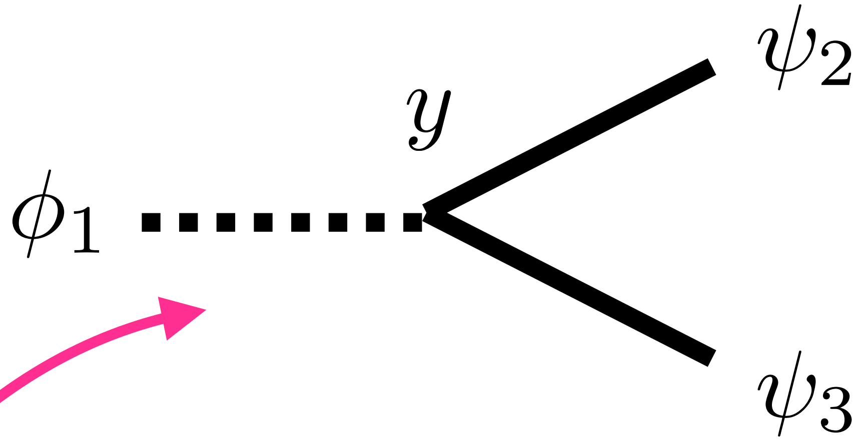
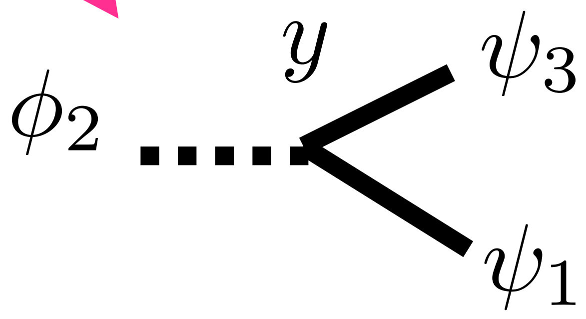
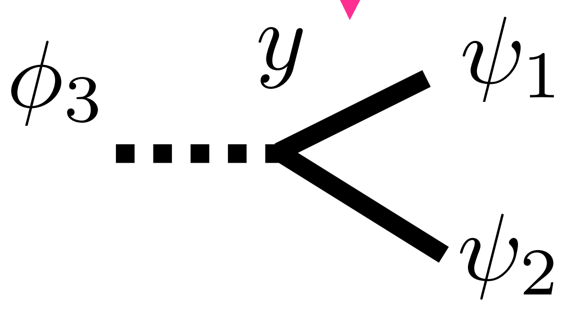


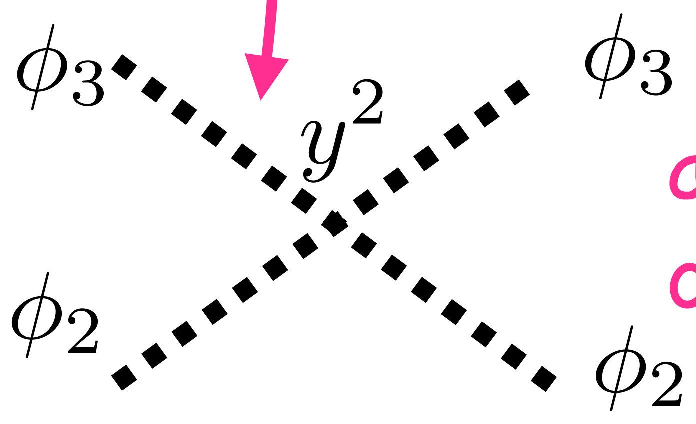
“supersymmetrize”

$$\begin{aligned}\mathcal{L}_{int} = &-y\cdot\phi_1\psi_2\psi_3 &&-|y|^2|\phi_2|^2|\phi_3|^2\\ &-y\cdot\phi_2\psi_3\psi_1 &&-|y|^2|\phi_3|^2|\phi_1|^2\\ &-y\cdot\phi_3\psi_1\psi_2 &&-|y|^2|\phi_1|^2|\phi_2|^2\end{aligned}$$

With all these terms, it is invariant under SUSY transformation.



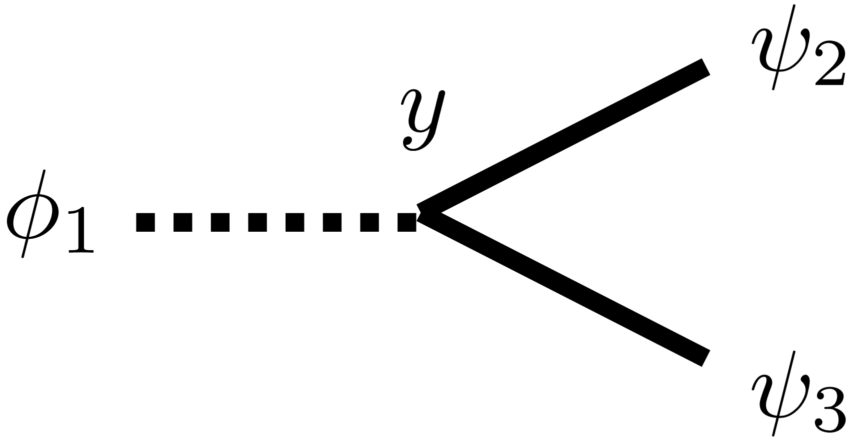




all the same coupling

(ii) interaction

$$\mathcal{L}_{\rm int} = -y\cdot\phi_1\psi_2\psi_3$$



"supersymmetrize"

$$\underbrace{\begin{aligned} \mathcal{L}_{int} = &- y\cdot\phi_1\psi_2\psi_3 && - |y|^2|\phi_2|^2|\phi_3|^2 \\ &- y\cdot\phi_2\psi_3\psi_1 && - |y|^2|\phi_3|^2|\phi_1|^2 \\ &- y\cdot\phi_3\psi_1\psi_2 && - |y|^2|\phi_1|^2|\phi_2|^2 \end{aligned}}$$

With all these terms, it is invariant under SUSY transformation.

$$\boxed{W = y\cdot\Phi_1\Phi_2\Phi_3}$$

superpotential

$$\mathcal{L}_{\rm int} = -\frac{\partial^2 W(\phi)}{\partial\phi_i\partial\phi_j}\psi_i\psi_j - \left|\frac{\partial W}{\partial\phi_i}\right|^2$$

$$\Phi_i \sim (\phi_i, \psi_i)$$

superfield: contains boson-fermion pair

(iii) mass term

$$W = M\Phi_1\Phi_2$$

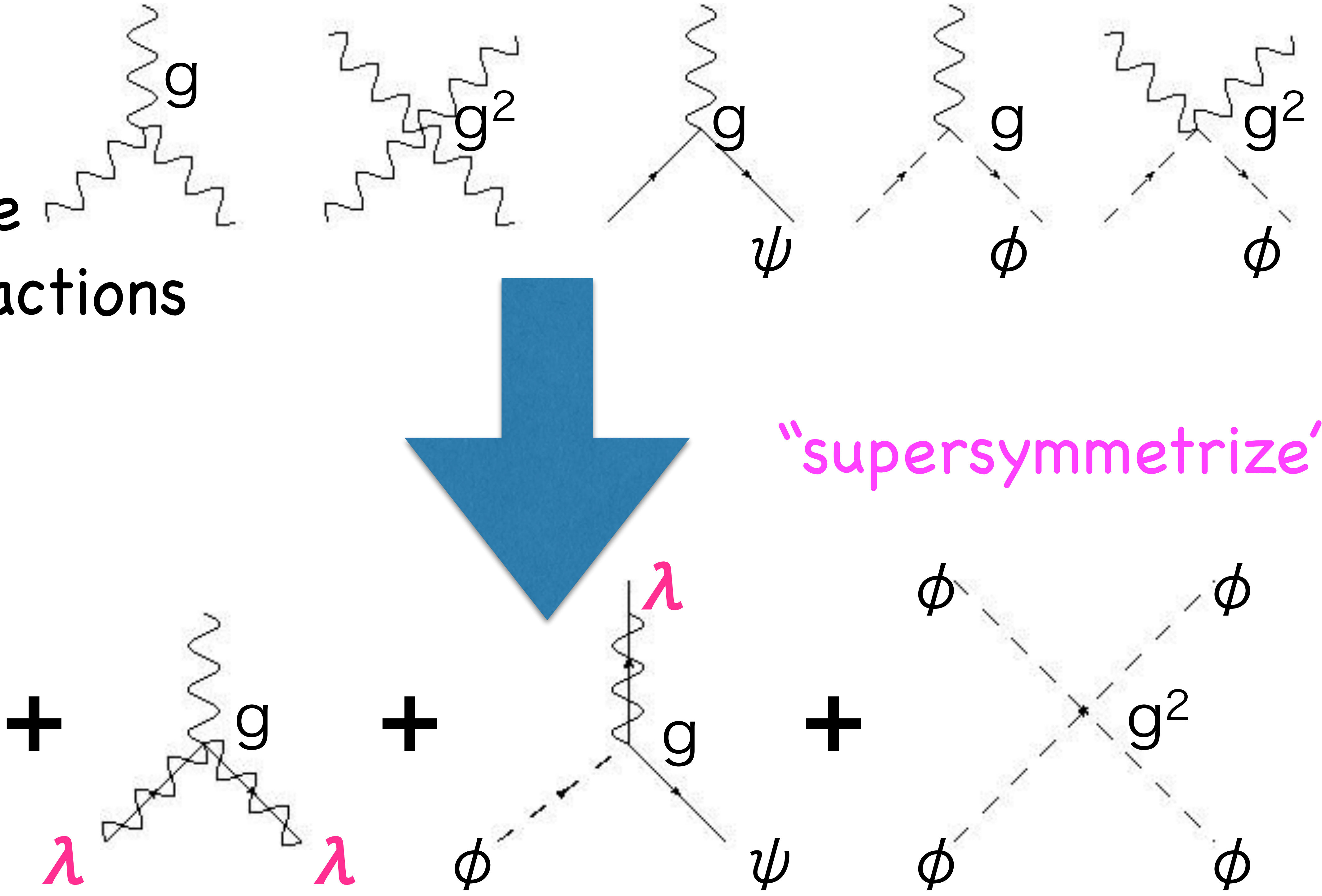
$$\rightarrow \mathcal{L} = -\frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\psi_i\psi_j - \left|\frac{\partial W}{\partial\phi_i}\right|^2$$

$$= -M\psi_1\psi_2 - |M|^2|\phi_1|^2 - |M|^2|\phi_2|^2$$

fermion mass = boson mass

(iv) gauge sector

gauge interactions



“supersymmetrize”

gaugino
(= fermionic partner of gauge boson)

With all these terms, the Lagrangian is invariant under SUSY transformation.

(v) summary

ϕ (scalar) $\longleftrightarrow$ ψ (fermion)

A_μ (gauge) $\longleftrightarrow$ λ (fermion)

They have the same masses,
and the same couplings.

PLAN

3.0. renormalization and naturalness

§3. SUSY:

done

3.1. motivations
3.2. supersymmetry
3.3. MSSM (minimal SUSY Standard Model)
3.4. MSSM Lagrangian
3.5. SUSY after Higgs discovery

§ 3.3. MSSM

Standard Model		spin		
quark	q	1/2 <-> 0	squark	$\widetilde{q}$
lepton	ℓ	1/2 <-> 0	slepton	$\widetilde{\ell}$
Higgs	H	0 <-> 1/2	higgsino	$\widetilde{h}$
gauge bosons	γ, Z, W, g	1 <-> 1/2	gaugino	$\widetilde{B}, \widetilde{W}, \widetilde{g}$

MSSM (minimal SUSY Standard Model)

§ 3.3. MSSM

More precisely..........

Standard Model		spin		
quark	q	1/2 <-> 0	squark	$\widetilde{q}$
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MSSM (minimal SUSY Standard Model)

§ 3.3. MSSM

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MSSM (minimal SUSY Standard Model)

comment 1: Why 2 Higgs ?

reason 1. Yukawa coupling.

$$\mathcal{L}^{\rm SM} = -y_u H^* \bar{u} Q - y_d H \bar{d} Q$$

conjugate

but in SUSY,

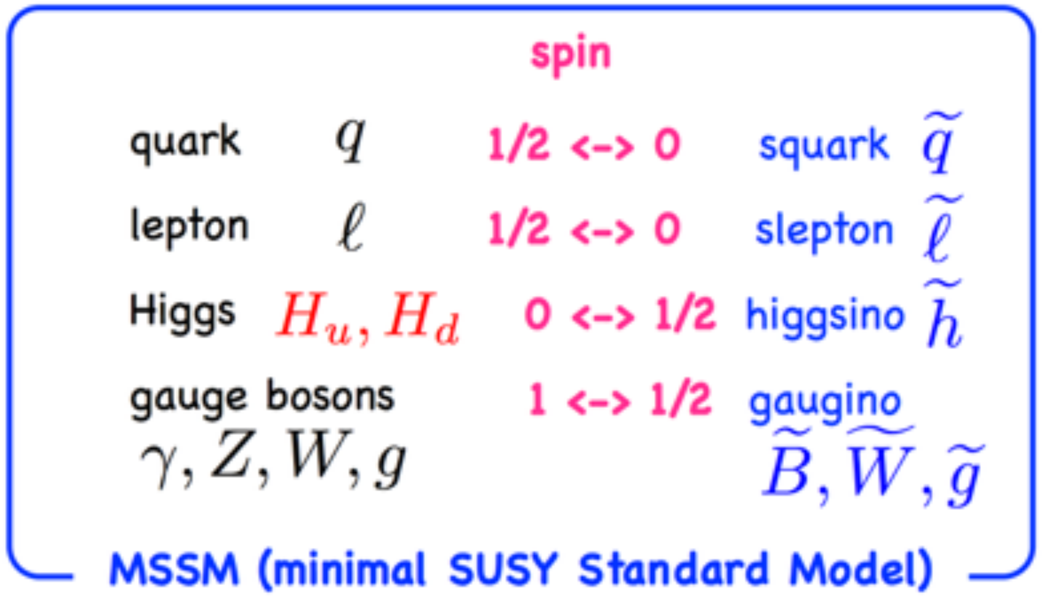
$$W = y_u H^* \xcancel{Q u^c} + y_d H Q d^c$$

$W(\phi)$ must be a function of ϕ, not , ϕ^*

$$W = y_u H_u Q u^c + y_d H_d Q d^c$$

2 Higgs

§ 3.3. MSSM



		spin		
quark	q	1/2 <-> 0	squark	$\widetilde{q}$
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MSSM (minimal SUSY Standard Model)

comment 1: Why 2 Higgs ?

reason 2. Anomaly

$$1 \text{ Higgs}: \quad H \Longleftrightarrow \widetilde{h} = (1,2)_{1/2} \quad \text{gauge anomaly}$$

$$2 \text{ Higgs}: \quad H_u \Longleftrightarrow \widetilde{h_u} = (1,2)_{1/2} \quad \text{anomaly}$$

$$H_d \Longleftrightarrow \widetilde{h_d} = (1,2)_{-1/2} \quad \text{cancellation}$$

§ 3.3. MSSM

		spin		
quark	q	1/2 <-> 0	squark	$\widetilde{q}$
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Higgs	H_u, H_d	0 <-> 1/2	higgsino	$\widetilde{h}$
gauge bosons γ, Z, W, g		1 <-> 1/2	gaugino $\widetilde{B}, \widetilde{W}, \widetilde{g}$	

MSSM (minimal SUSY Standard Model)

comment 2:

mass eigenstates

$$\text{neutral fermions} \quad \widetilde{h_u^0}, \widetilde{h_d^0}, \widetilde{B}, \widetilde{W^0} \Longrightarrow \quad \widetilde{\chi}^0_{1,2,3,4} \quad \text{neutralinos}$$

$$\text{charged fermions} \quad \widetilde{h_u^+}, \widetilde{h_d^-}, \widetilde{W^\pm} \quad \Longrightarrow \quad \widetilde{\chi}^\pm_{1,2} \quad \text{charginos}$$

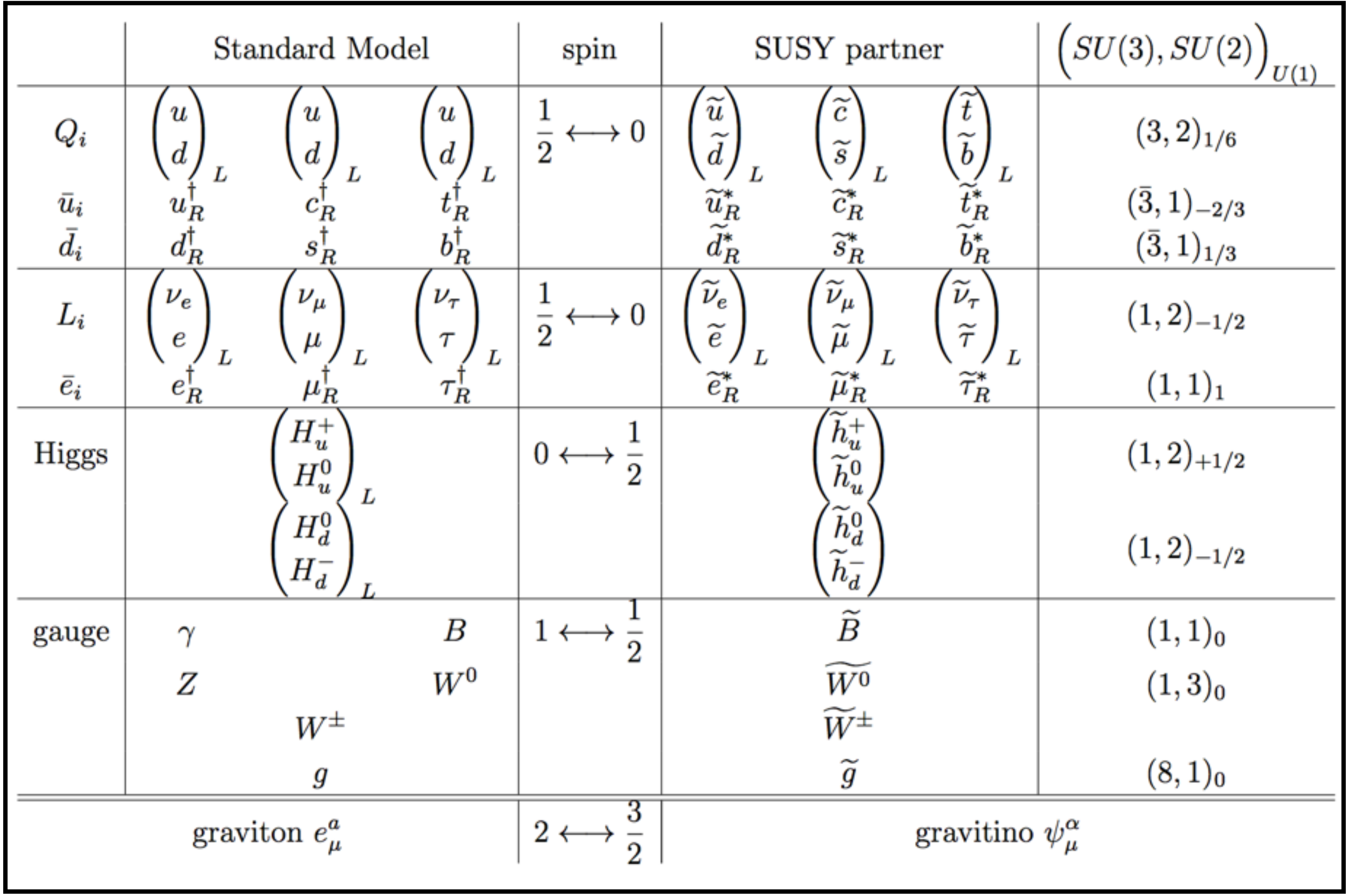
§ 3.3. MSSM

		spin		
quark	q	1/2 <-> 0	squark	$\widetilde{q}$
lepton	ℓ	1/2 <-> 0	slepton	$\widetilde{\ell}$
Higgs	H_u, H_d	0 <-> 1/2	higgsino	$\widetilde{h}$
gauge bosons	γ, Z, W, g	1 <-> 1/2	gaugino	$\widetilde{B}, \widetilde{W}, \widetilde{g}$

MSSM (minimal SUSY Standard Model)

§ 3.3. MSSM

More precisely,…



	Standard Model	spin	SUSY partner	$\left(SU(3), SU(2)\right)_{U(1)}$
Q_i	$\begin{pmatrix} u \\ d \end{pmatrix}_L$ $\begin{pmatrix} u \\ d \end{pmatrix}_L$ $\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\frac{1}{2} \longleftrightarrow 0$	$\begin{pmatrix} \widetilde{u} \\ \widetilde{d} \end{pmatrix}_L$ $\begin{pmatrix} \widetilde{c} \\ \widetilde{s} \end{pmatrix}_L$ $\begin{pmatrix} \widetilde{t} \\ \widetilde{b} \end{pmatrix}_L$	$(3,2)_{1/6}$
$\bar{u}_i$	$u_R^\dagger$ $c_R^\dagger$ $t_R^\dagger$		$\widetilde{u}_R^*$ $\widetilde{c}_R^*$ $\widetilde{t}_R^*$	$(\bar{3},1)_{-2/3}$
$\bar{d}_i$	$d_R^\dagger$ $s_R^\dagger$ $b_R^\dagger$		$\widetilde{d}_R^*$ $\widetilde{s}_R^*$ $\widetilde{b}_R^*$	$(\bar{3},1)_{1/3}$
L_i	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$ $\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$ $\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$\frac{1}{2} \longleftrightarrow 0$	$\begin{pmatrix} \widetilde{\nu}_e \\ \widetilde{e} \end{pmatrix}_L$ $\begin{pmatrix} \widetilde{\nu}_\mu \\ \widetilde{\mu} \end{pmatrix}_L$ $\begin{pmatrix} \widetilde{\nu}_\tau \\ \widetilde{\tau} \end{pmatrix}_L$	$(1,2)_{-1/2}$
$\bar{e}_i$	$e_R^\dagger$ $\mu_R^\dagger$ $\tau_R^\dagger$		$\widetilde{e}_R^*$ $\widetilde{\mu}_R^*$ $\widetilde{\tau}_R^*$	$(1,1)_1$
Higgs	$\begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}_L$	$0 \longleftrightarrow \frac{1}{2}$	$\begin{pmatrix} \widetilde{h}_u^+ \\ \widetilde{h}_u^0 \end{pmatrix}$	$(1,2)_{+1/2}$
	$\begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}_L$		$\begin{pmatrix} \widetilde{h}_d^0 \\ \widetilde{h}_d^- \end{pmatrix}$	$(1,2)_{-1/2}$
gauge	γ B	$1 \longleftrightarrow \frac{1}{2}$	$\widetilde{B}$	$(1,1)_0$
	Z W^0		$\widetilde{W}^0$	$(1,3)_0$
	$W^\pm$		$\widetilde{W}^\pm$	
	g		$\widetilde{g}$	$(8,1)_0$
graviton e_μ^a		$2 \longleftrightarrow \frac{3}{2}$	gravitino ψ_μ^α	

PLAN

3.0. renormalization and naturalness

§3. SUSY:

3.1. motivations

3.2. supersymmetry

3.3. MSSM (minimal SUSY Standard Model)

3.4. MSSM Lagrangian

3.5. SUSY after Higgs discovery

done

§ 3.4. MSSM Lagrangian

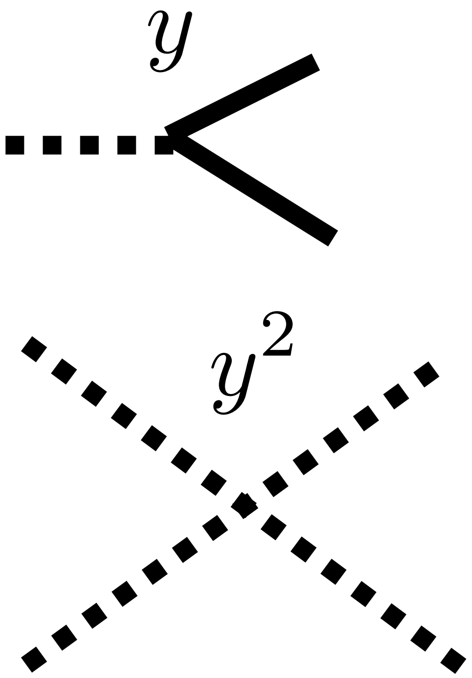
$$\mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{SM}\to\text{SUSY}} + \mathcal{L}_{\text{soft }\cancel{\text{SUSY}}}$$

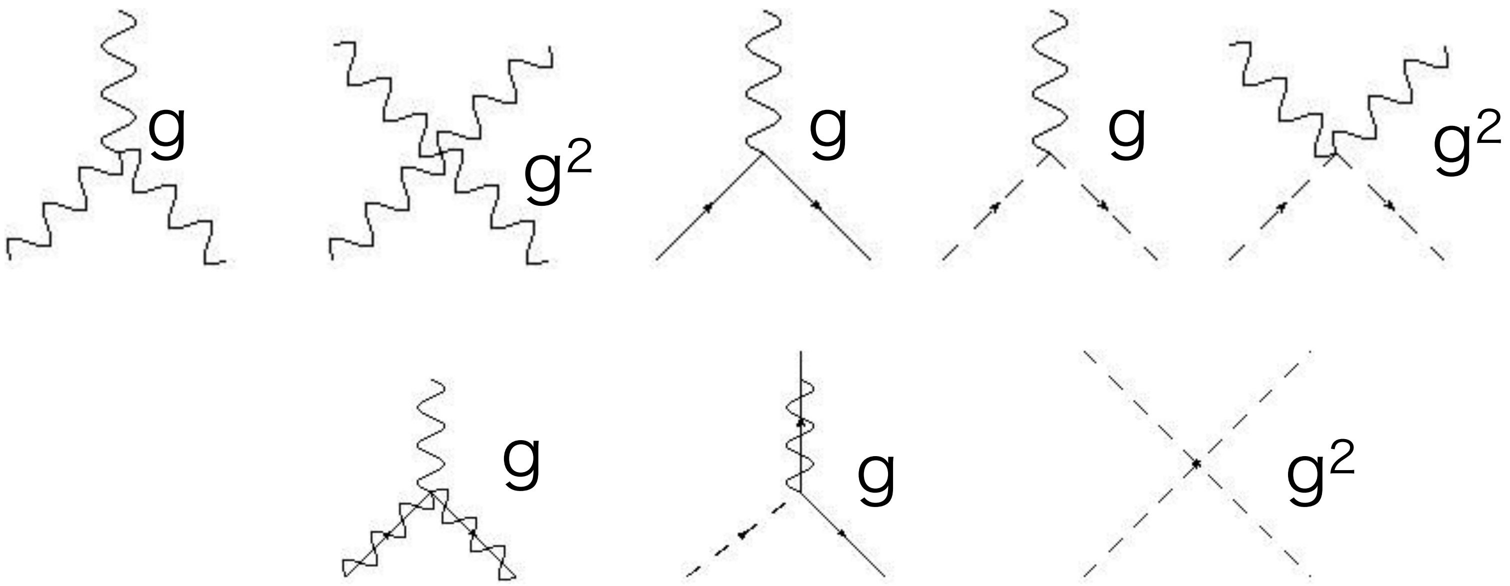
§ 3.4. MSSM Lagrangian

$$\mathcal{L}_{\rm MSSM} = \mathcal{L}_{\rm SM \to SUSY} + \mathcal{L}_{\rm soft\ \cancel{SUSY}}$$

$$\textbf{(i)} \quad \mathcal{L}_{\rm SM \to SUSY}$$

$$= \mathcal{L}_{\rm from\ superpotential} + \mathcal{L}_{\rm from\ gauge\ interactions}$$





no new free parameter
compared to SM.

§ 3.4. MSSM Lagrangian

$$\mathcal{L}_{\mathrm{MSSM}} = \mathcal{L}_{\mathrm{SM}\to\mathrm{SUSY}} + \mathcal{L}_{\mathrm{soft}\ \cancel{\mathrm{SUSY}}}$$

$$\textbf{(i)}\quad \mathcal{L}_{\mathrm{SM}\to\mathrm{SUSY}}$$

$$= \mathcal{L}_{\text{from superpotential}} + \mathcal{L}_{\text{from gauge interactions}}$$

$$W_{\mathrm{MSSM}} = \underbrace{(y_u)_{ij} H_u Q_i u_j^c + (y_d)_{ij} H_d Q_i d_j^c + (y_e)_{ij} H_d Q_i e_i^c}_{\text{corresponding to SM Yukawa}} + \underbrace{\mu H_u H_d}_{\text{“}\mu\text{-term”}}$$

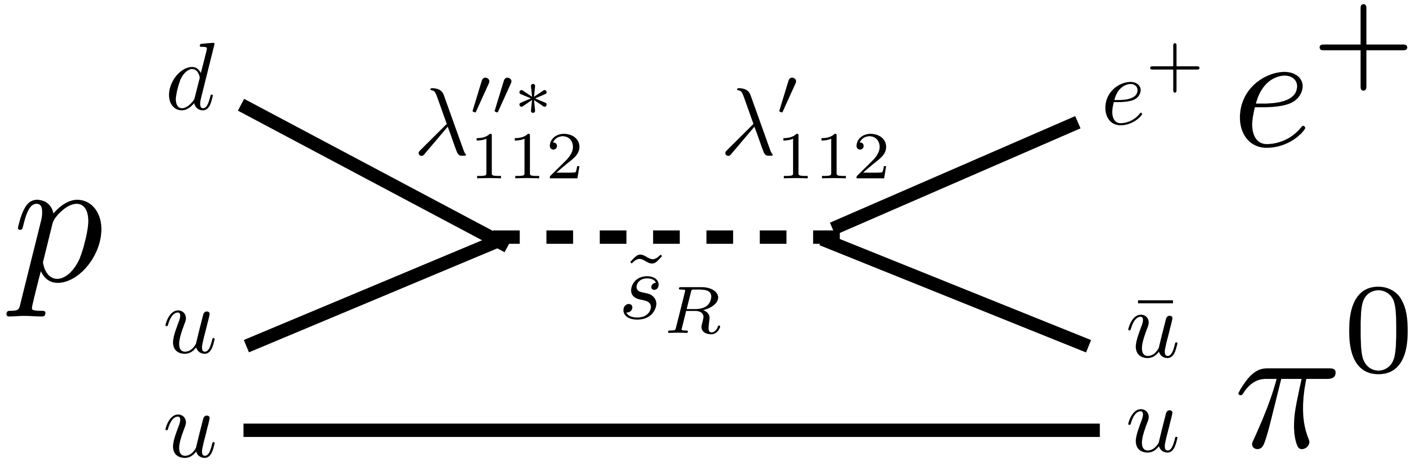
§ 3.4. MSSM Lagrangian

$$W_{\rm MSSM} = \underbrace{(y_u)_{ij} H_u Q_i u^c_j + (y_d)_{ij} H_d Q_i d^c_j + (y_e)_{ij} H_d Q_i e^c_i}_{\text{corresponding to SM Yukawa}} + \underbrace{\mu H_u H_d}_{\text{“}\mu\text{-term”}}$$

NOTE:
There are other renormalizable terms allowed by gauge invariance.

$$W_{\rm RpV} = \underbrace{\frac{1}{2}\lambda^{ijk} L_i L_j e^c_k + \lambda'^{ijk} L_i Q_j d^c_k + \mu'_i L_i H_u}_{L\text{-violating}} + \underbrace{\frac{1}{2}\lambda''^{ijk} u^c_i d^c_j d^c_k}_{B\text{-violating}}$$

But they mediate a very rapid proton decay!



exp. bound (super-K)

$$\tau(p \to e^+\pi^0) > 1.3 \times 10^{34} \text{years}$$

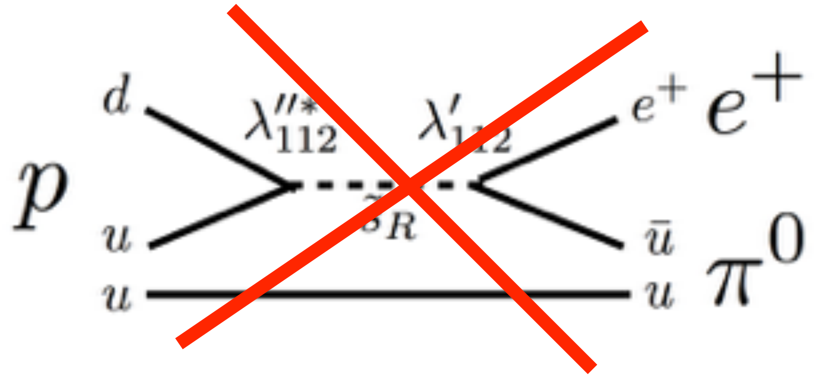
$$\to |\lambda'_{11k}\lambda''_{11k}| \lesssim 10^{-29} \left(\frac{m_{\tilde{d}_i}}{1\ \text{TeV}}\right)^2$$

§ 3.4. MSSM Lagrangian

$$W_{\rm MSSM} = \underbrace{(y_u)_{ij} H_u Q_i u^c_j + (y_d)_{ij} H_d Q_i d^c_j + (y_e)_{ij} H_d Q_i e^c_i}_{\text{corresponding to SM Yukawa}} + \underbrace{\mu H_u H_d}_{\text{“}\mu\text{-term”}}$$

$$\xcancel{W_{\rm RpV} = \underbrace{\frac{1}{2}\lambda^{ijk} L_i L_j e^c_k + \lambda'^{ijk} L_i Q_j d^c_k + \mu'_i L_i H_u}_{L\text{-violating}} + \underbrace{\frac{1}{2}\lambda''^{ijk} u^c_i d^c_j d^c_k}_{B\text{-violating}}}$$

There is a parity symmetry which forbids $W_{\rm RpV}$ and allows $W_{\rm MSSM}$.



R-parity

SM particles —> even (+)

SUSY particles —> odd (-)

rapid proton decay is forbidden.

§ 3.4. MSSM Lagrangian

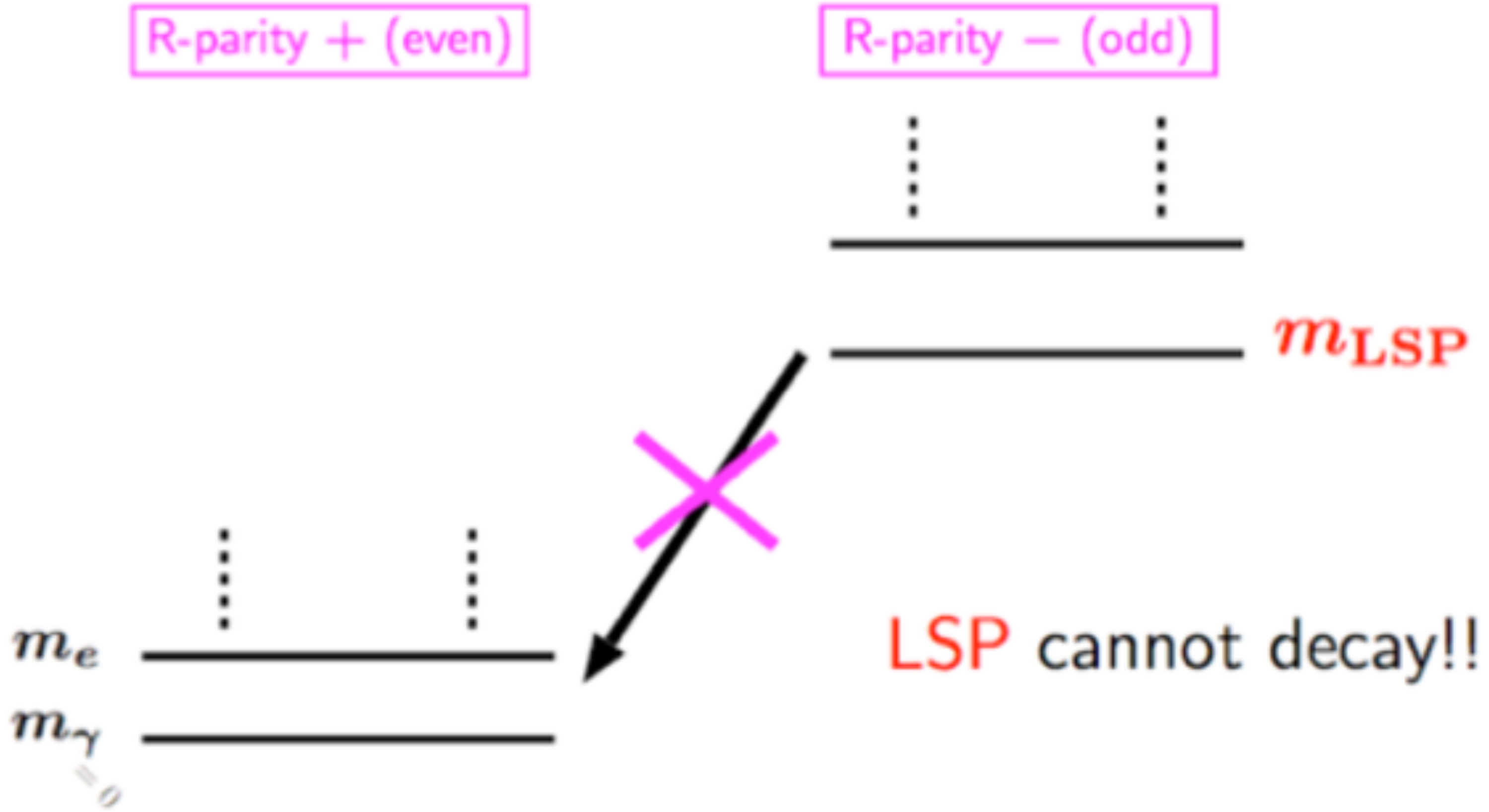
R-parity

SM particles —> even (+)

SUSY particles —> odd (-)

—> Lightest SUSY Particle (**LSP**) becomes stable.

—> **Dark Matter candidate!**



LSP cannot decay!!

§ 3.4. MSSM Lagrangian

R-parity

SM particles —> even (+)

SUSY particles —> odd (-)

—> Lightest SUSY Particle (**LSP**) becomes stable.

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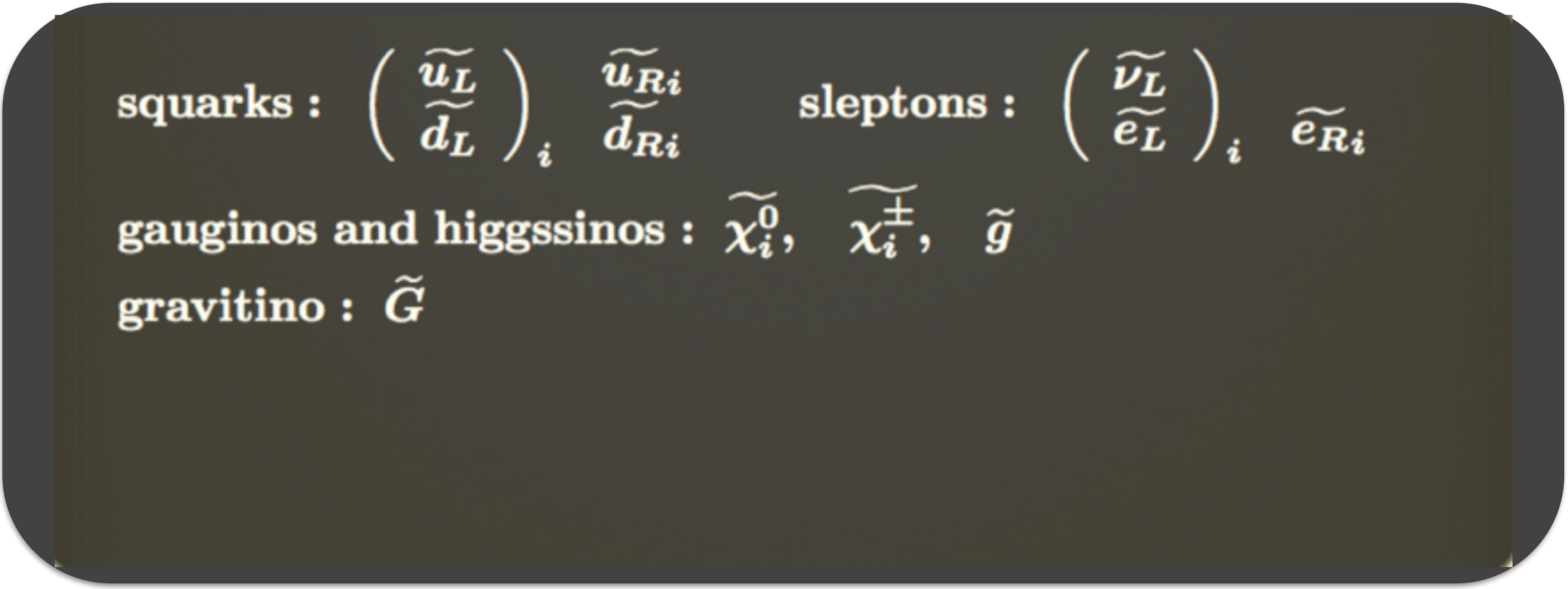

squarks : $\begin{pmatrix} \widetilde{u_L} \\ \widetilde{d_L} \end{pmatrix}_i \quad \begin{matrix} \widetilde{u_{Ri}} \\ \widetilde{d_{Ri}} \end{matrix}$ **sleptons :** $\begin{pmatrix} \widetilde{\nu_L} \\ \widetilde{e_L} \end{pmatrix}_i \quad \widetilde{e_{Ri}}$

gauginos and higgssinos : $\widetilde{\chi_i^0}, \quad \widetilde{\chi_i^\pm}, \quad \tilde{g}$

gravitino : $\tilde{G}$

§ 3.4. MSSM Lagrangian

R-parity

SM particles —> even (+)
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—> Lightest SUSY Particle (**LSP**) becomes stable.

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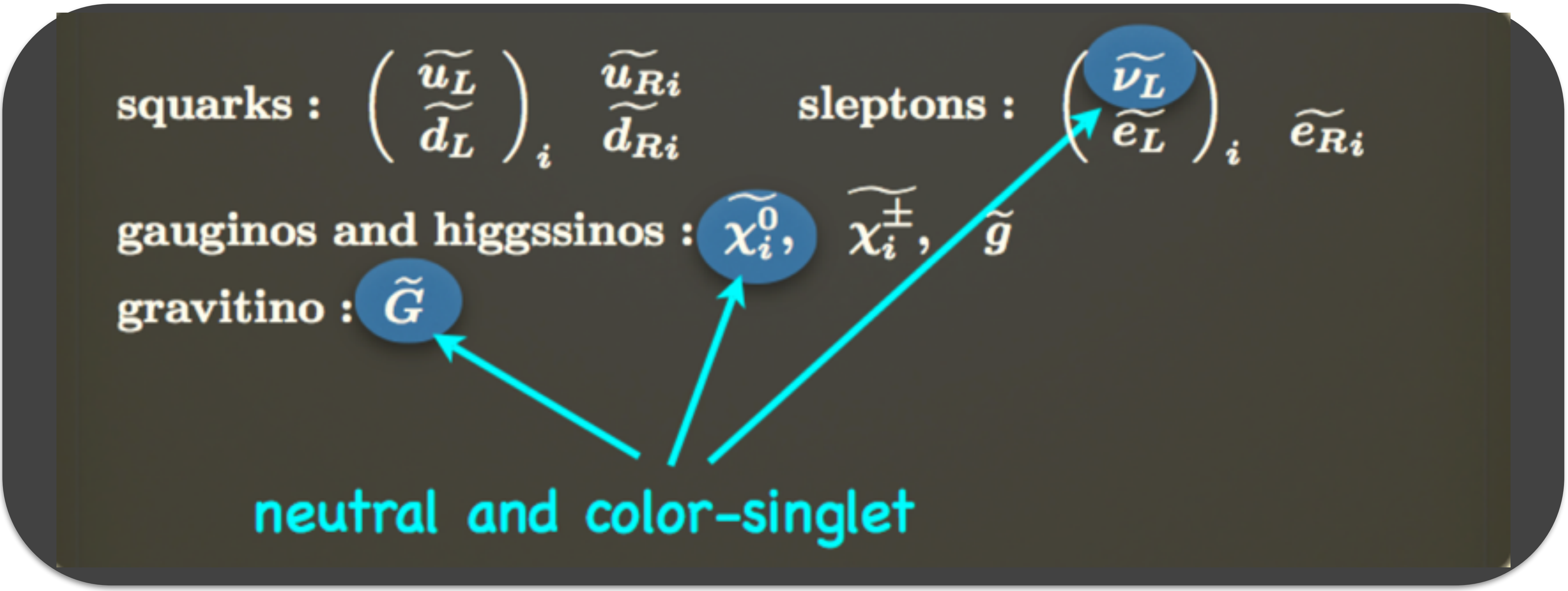


squarks : $\begin{pmatrix}\tilde{u}_L\\ \tilde{d}_L\end{pmatrix}_i$ $\begin{matrix}\tilde{u}_{Ri}\\ \tilde{d}_{Ri}\end{matrix}$ sleptons : $\begin{pmatrix}\tilde{\nu}_L\\ \tilde{e}_L\end{pmatrix}_i$ $\tilde{e}_{Ri}$

gauginos and higgssinos : $\tilde{\chi}_i^0$, $\tilde{\chi}_i^\pm$, $\tilde{g}$

gravitino : $\tilde{G}$

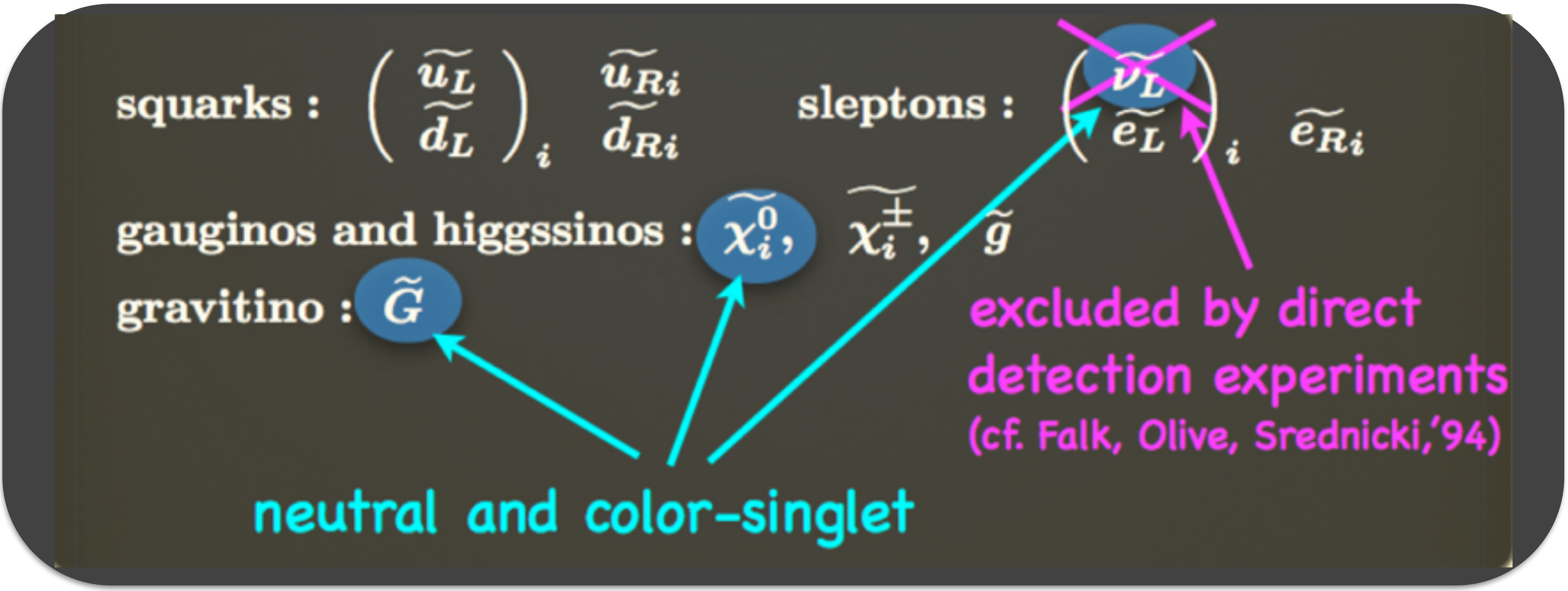
neutral and color-singlet

§ 3.4. MSSM Lagrangian

R-parity

SM particles —> even (+)
SUSY particles —> odd (-)

—> Lightest SUSY Particle (**LSP**) becomes stable.
—> **Dark Matter candidate!**



squarks : $\begin{pmatrix}\tilde{u}_L \\ \tilde{d}_L\end{pmatrix}_i$, $\begin{matrix}\tilde{u}_{Ri} \\ \tilde{d}_{Ri}\end{matrix}$ sleptons : $\begin{pmatrix}\xcancel{\tilde{\nu}_L} \\ \tilde{e}_L\end{pmatrix}_i$ $\tilde{e}_{Ri}$

gauginos and higgssinos : $\tilde{\chi}_i^0$, $\tilde{\chi}_i^{\pm}$, $\tilde{g}$

gravitino : $\tilde{G}$

excluded by direct detection experiments
(cf. Falk, Olive, Srednicki,'94)

neutral and color-singlet

§ 3.4. MSSM Lagrangian

R-parity

SM particles —> even (+)
SUSY particles —> odd (-)

—> Lightest SUSY **P**article (**LSP**) becomes stable.
—> Dark Matter candidate!

LSP DM candidates within MSSM (+ supergravity):

- **neutralino**
- **gravitino**

§ 3.4. MSSM Lagrangian

$$\mathcal{L}_{\rm MSSM} = \mathcal{L}_{\rm SM\to SUSY} + \mathcal{L}_{\text{soft } \cancel{\rm SUSY}}$$

$$\textbf{(ii)} \quad \mathcal{L}_{\text{soft } \cancel{\rm SUSY}}$$

Supersymmetry must be
a (spontaneously) broken symmetry

electron ~~**selectron (scalar)**~~
511 keV <—> ~~**511 keV**~~

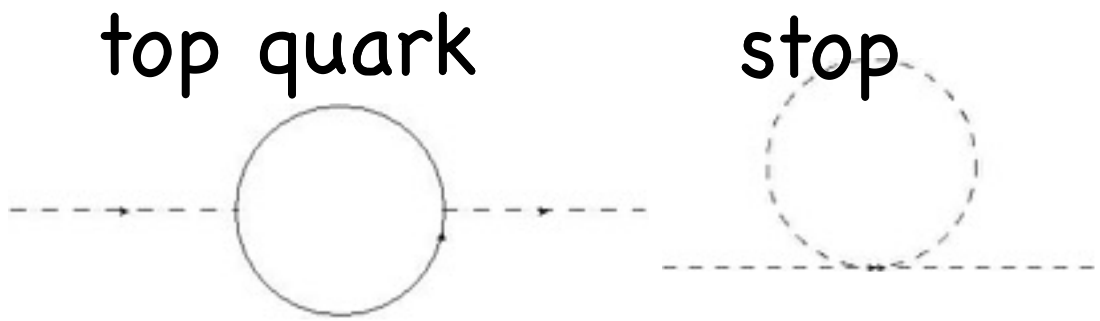
§ 3.4. MSSM Lagrangian

$$\mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{SM}\to\text{SUSY}} + \mathcal{L}_{\text{soft}\ \cancel{\text{SUSY}}}$$

$$\textbf{(ii)}\quad \mathcal{L}_{\boxed{\text{soft}}\ \cancel{\text{SUSY}}}$$

SUSY must broken only by **parameters with mass dim. > 0.**

$\delta m_H{}^2 \sim$



hard breaking $y_{top} \neq y_{stop}$, $\longrightarrow$ $\delta m_H{}^2 \sim (y_{stop}{}^2 - y_{top}{}^2)\Lambda^2$ 😩

soft breaking $m_{top} \neq m_{stop}$, $\longrightarrow$ $\delta m_H{}^2 \sim (m_{stop}{}^2 - m_{top}{}^2) \log\Lambda$ 😐

§ 3.4. MSSM Lagrangian

$$\mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{SM}\to\text{SUSY}} + \mathcal{L}_{\text{soft}\ \cancel{\text{SUSY}}}$$

$$\textbf{(ii)}\quad \mathcal{L}_{\boxed{\text{soft}}\ \cancel{\text{SUSY}}}$$

SUSY must broken only by **parameters with mass dim. > 0.**

$$\begin{aligned}
\mathcal{L}_{\text{soft}}^{\text{MSSM}} = & -\frac{1}{2}\left(M_3\tilde{g}\tilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B} + \text{c.c.}\right) \quad \text{gaugino masses} \\
& -\left(\tilde{\bar{u}}\,\mathbf{a_u}\,\tilde{Q}H_u - \tilde{\bar{d}}\,\mathbf{a_d}\,\tilde{Q}H_d - \tilde{\bar{e}}\,\mathbf{a_e}\,\tilde{L}H_d + \text{c.c.}\right) \quad \text{A-terms (3x3 matrices.)} \\
& -\tilde{Q}^\dagger\,\mathbf{m^2_Q}\,\tilde{Q} - \tilde{L}^\dagger\,\mathbf{m^2_L}\,\tilde{L} - \tilde{\bar{u}}\,\mathbf{m^2_{\bar{u}}}\,\tilde{\bar{u}}^\dagger - \tilde{\bar{d}}\,\mathbf{m^2_{\bar{d}}}\,\tilde{\bar{d}}^\dagger - \tilde{\bar{e}}\,\mathbf{m^2_{\bar{e}}}\,\tilde{\bar{e}}^\dagger \\
& -m^2_{H_u}H_u^*H_u - m^2_{H_d}H_d^*H_d - \left(bH_uH_d + \text{c.c.}\right).
\end{aligned}$$

Higgs soft terms

squark and slepton masses

(3x3 matrices.)

§ 3.4. MSSM Lagrangian

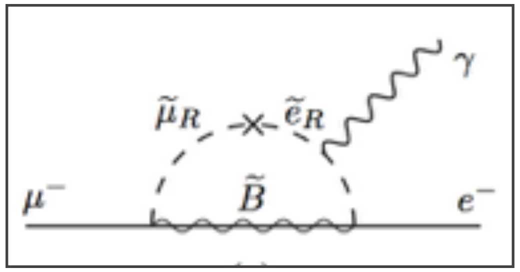
$$\mathcal{L}_{\mathrm{MSSM}} = \mathcal{L}_{\mathrm{SM}\to\mathrm{SUSY}} + \mathcal{L}_{\mathrm{soft}\ \cancel{\mathrm{SUSY}}}$$

(ii) $\mathcal{L}_{\mathrm{soft}\ \cancel{\mathrm{SUSY}}}$

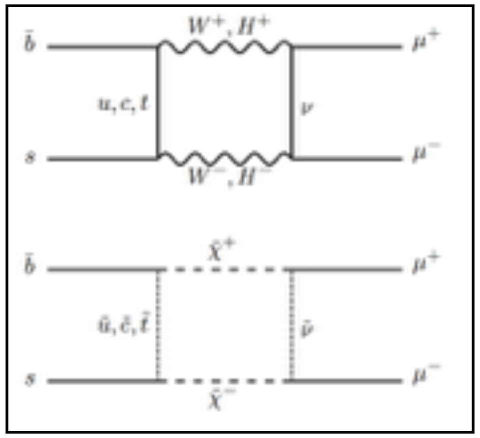
$$\begin{aligned}\mathcal{L}_{\mathrm{soft}}^{\mathrm{MSSM}} \;=\; & -\frac{1}{2}\left(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B} + \mathrm{c.c.}\right) \\ & -\left(\widetilde{\overline{u}}\,\mathbf{a_u}\,\widetilde{Q}H_u - \widetilde{\overline{d}}\,\mathbf{a_d}\,\widetilde{Q}H_d - \widetilde{\overline{e}}\,\mathbf{a_e}\,\widetilde{L}H_d + \mathrm{c.c.}\right) \\ & -\widetilde{Q}^\dagger\,\mathbf{m_Q^2}\,\widetilde{Q} - \widetilde{L}^\dagger\,\mathbf{m_L^2}\,\widetilde{L} - \widetilde{\overline{u}}\,\mathbf{m_{\overline{u}}^2}\,\widetilde{\overline{u}}^\dagger - \widetilde{\overline{d}}\,\mathbf{m_{\overline{d}}^2}\,\widetilde{\overline{d}}^\dagger - \widetilde{\overline{e}}\,\mathbf{m_{\overline{e}}^2}\,\widetilde{\overline{e}}^\dagger \\ & - m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (bH_uH_d + \mathrm{c.c.})\,.\end{aligned}$$

This part contains a variety of interesting SUSY phenomenologies……

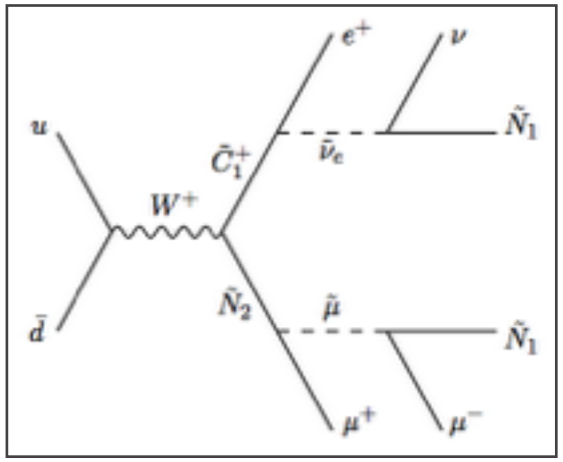
- SUSY particle masses,
- Higgs sector (tree level mass, loop corrections…),
- **Flavor Changing Neutral Current (FCNC) and CP-violation,**
- SUSY breaking mechanism and its mediations, model-building,
  (Gravity mediation, Gauge mediation, Anomaly mediation,……)
- **Collider physics,**
- **Dark Matter**
- ……



$\mu^- \to e^-\gamma$



various B-decays



collider signals…

But I skip the details here.
For a review, see,e.g., hep-ph/9709356 by S.P.Martin.

PLAN

3.0. renormalization and naturalness

§3. SUSY:

3.1. motivations

3.2. supersymmetry

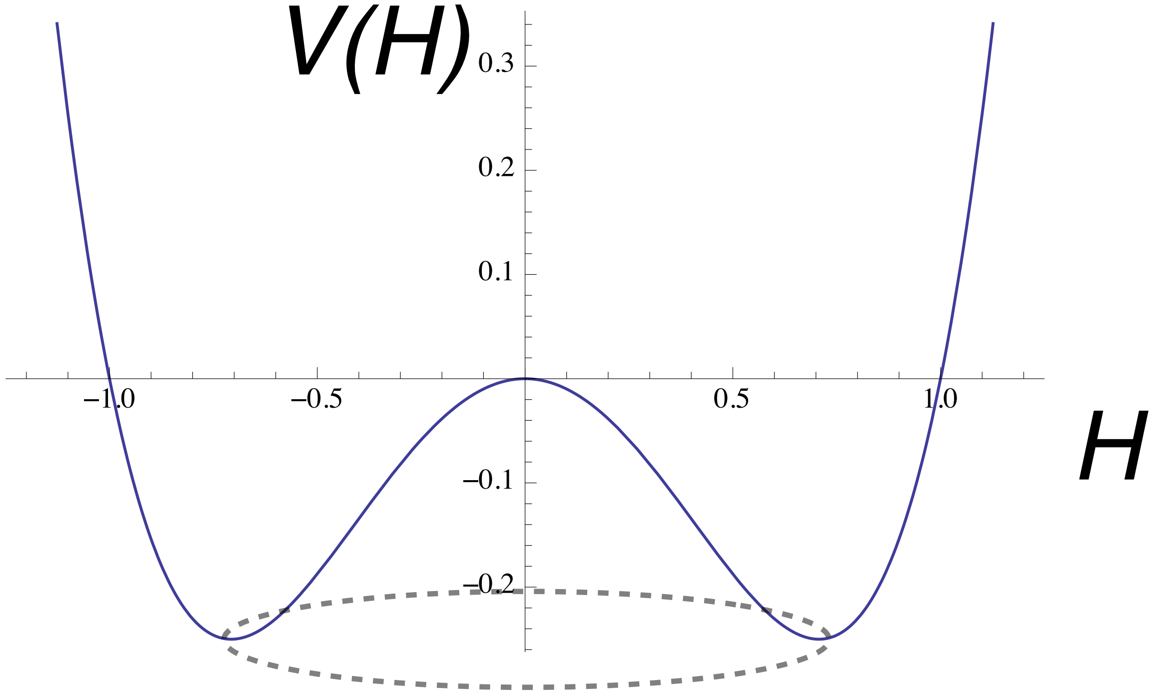
3.3. MSSM (minimal SUSY Standard Model)

3.4. MSSM Lagrangian

3.5. SUSY after Higgs discovery

125 GeV Higgs

$$V(H) = -m^2(H^\dagger H) + \lambda_{\mathrm{H}}(H^\dagger H)^2$$



125 GeV Higgs

$$V(H) = -m^2(H^\dagger H) + \lambda_{\mathrm{H}}(H^\dagger H)^2$$

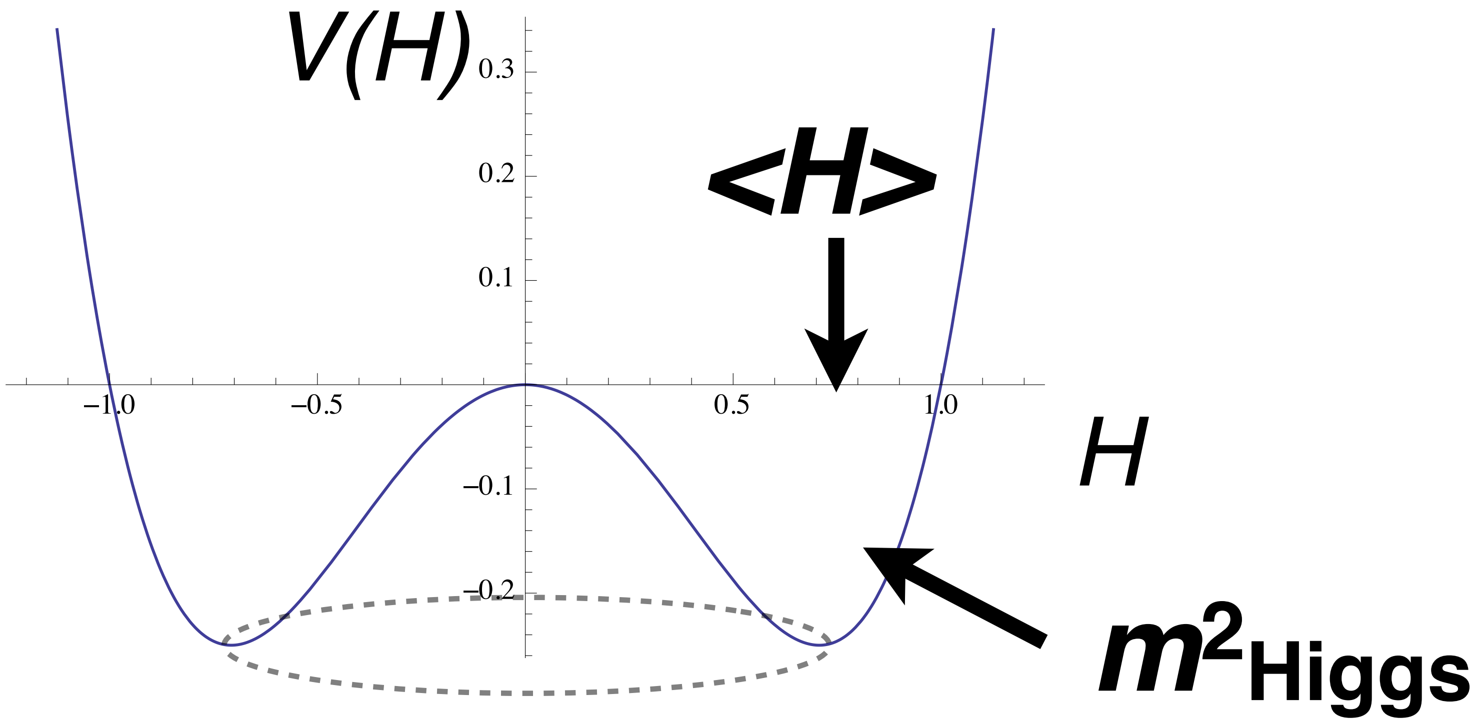
We knew...

Now we also know

$$\to \begin{cases} \langle H \rangle^2 = \dfrac{m^2}{2\,\lambda_{\mathrm{H}}} = \dfrac{1}{2\sqrt{2}\,G_F} \simeq (174\ \mathrm{GeV})^2 \\ m^2_{\mathrm{Higgs}} = 2\,m^2 \simeq \mathbf{(125\ GeV)^2} \end{cases}$$

Fermi constant

$G_F \simeq 1.17 \times 10^{-5}\mathrm{GeV}^{-2}$



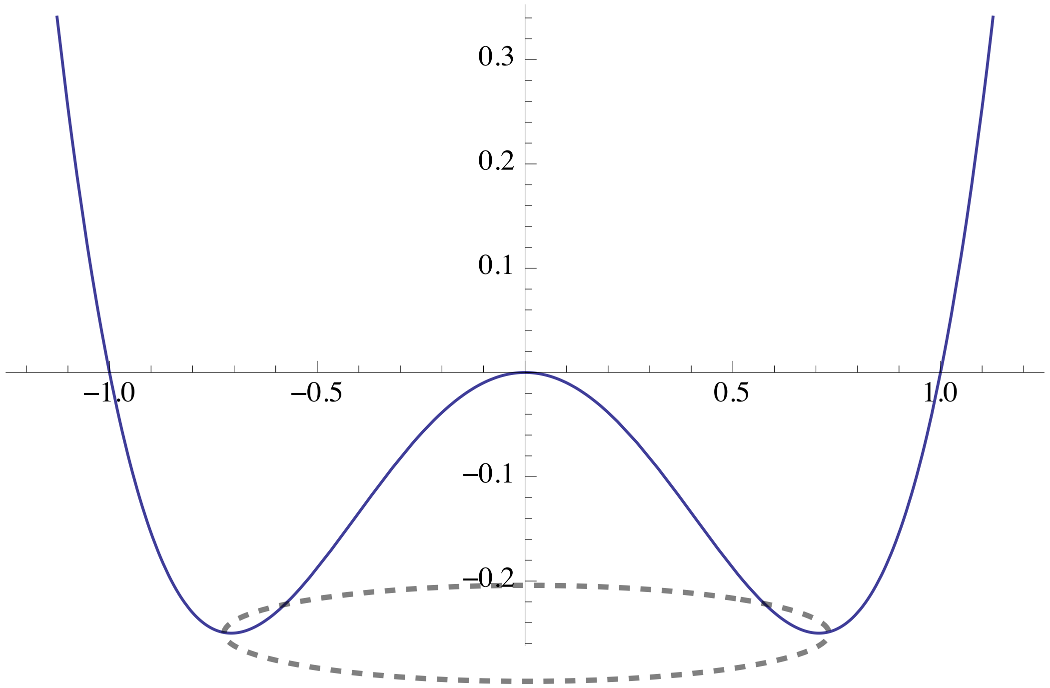
125 GeV Higgs

$$V(H) = -m^2 (H^\dagger H) + \lambda_{\rm H} (H^\dagger H)^2$$

We knew...

$$\rightarrow \begin{cases} \langle H \rangle^2 = \dfrac{m^2}{2\,\lambda_{\rm H}} = \dfrac{1}{2\sqrt{2}G_F} \simeq (174\ \text{GeV})^2 \\ m^2_{\rm Higgs} = 2\,m^2 \simeq \mathbf{(125\ GeV)^2} \end{cases}$$

Now we also know



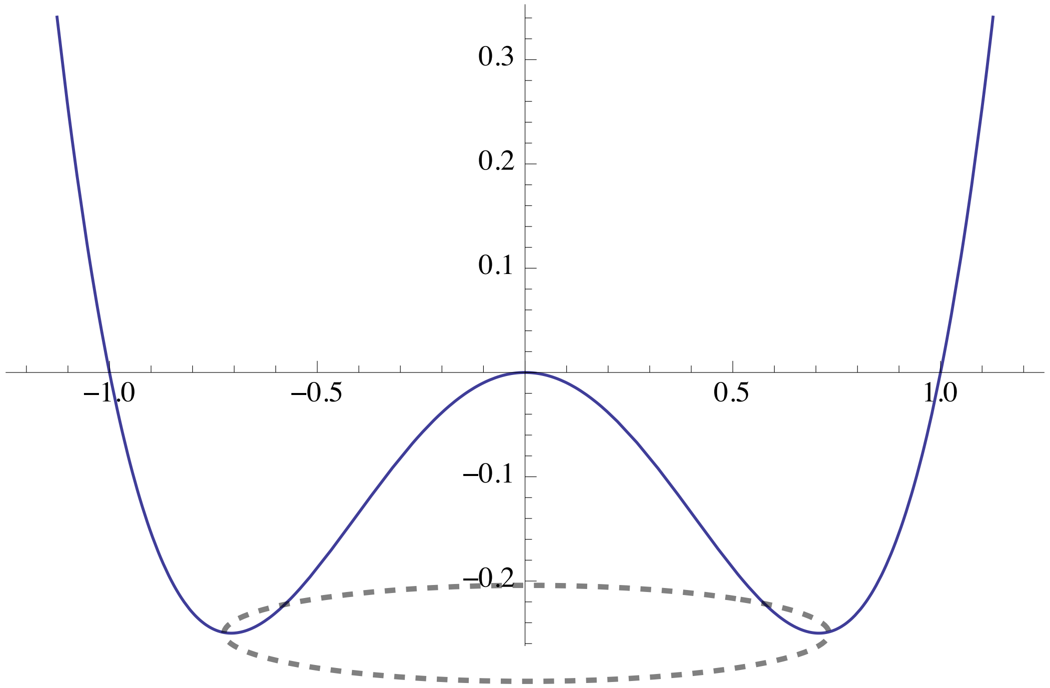
$$\rightarrow \begin{cases} m^2 = \dfrac{m^2_{\rm Higgs}}{2} \simeq \mathbf{(89\ GeV)^2} \\ \lambda_{\rm H} = \dfrac{m^2_{\rm Higgs}}{4\langle H \rangle^2} \simeq \mathbf{0.13} \end{cases}$$

125 GeV Higgs

$$V(H) = -m^2 (H^\dagger H) + \lambda_{\mathrm{H}} (H^\dagger H)^2$$

$$(\mathbf{89\ GeV})^2 \qquad \mathbf{0.13}$$

completely determined !



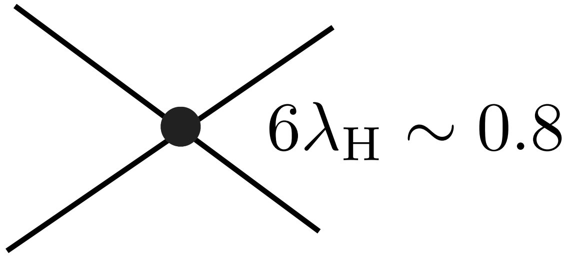
$$\rightarrow \begin{cases} m^2 = \dfrac{m^2_{\mathrm{Higgs}}}{2} \simeq (\mathbf{89\ GeV})^2 \\ \\ \lambda_{\mathrm{H}} = \dfrac{m^2_{\mathrm{Higgs}}}{4\langle H\rangle^2} \simeq \mathbf{0.13} \end{cases}$$

125 GeV Higgs

$$V(H) = -m^2(H^\dagger H) + \lambda_H (H^\dagger H)^2$$

$$(89\ \mathrm{GeV})^2 \qquad 0.13$$

It seems... Higgs sector is also described by weakly coupled, perturbative QFT.
(at least no sign of strong interaction etc, so far...)



Imlications for BSM

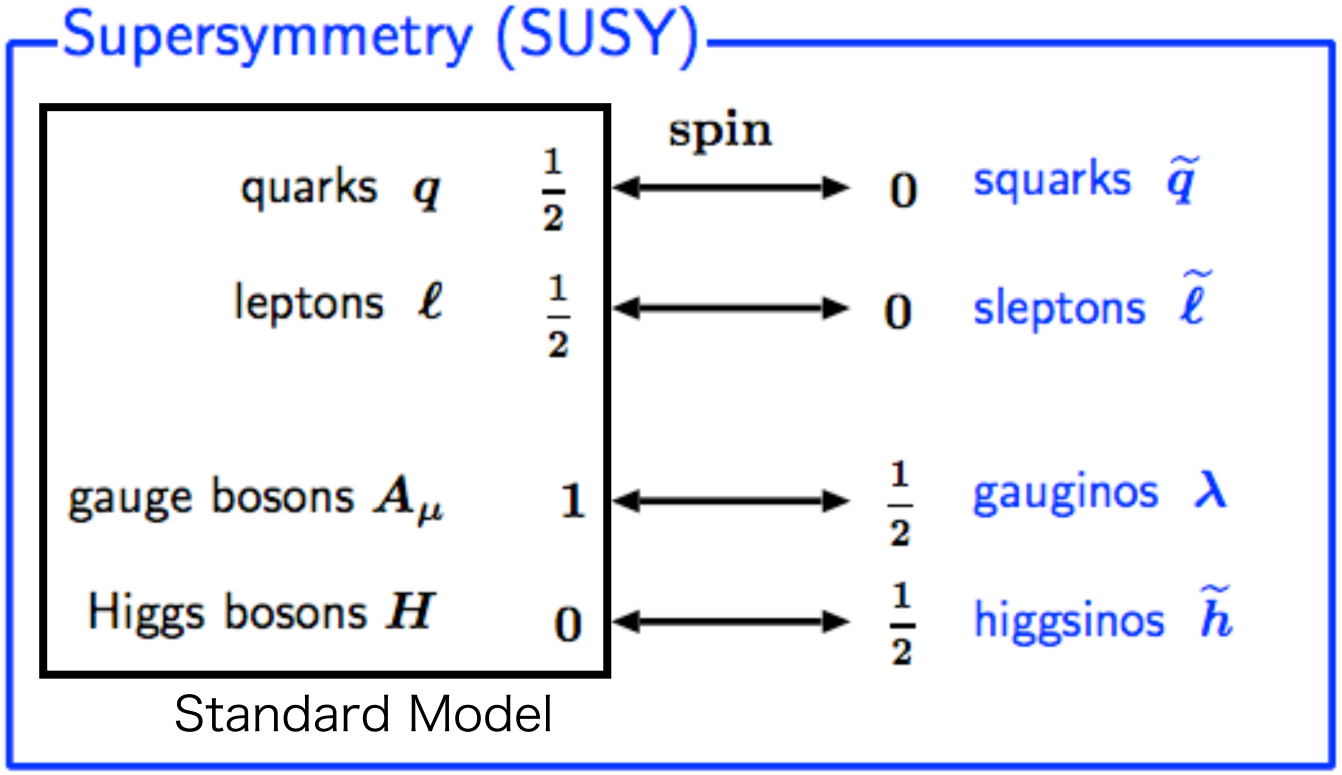
(in my opinion.....)

This is compatible with....

- **GUT and coupling unification** in perturbative QFT. ☞ §2
- **heavy right-handed neutrinos** (Seesaw + Leptogenesis) ☞ §2
- **Supersymmetry** ☞ §3

Supersymmetry

boson ⇔ fermion



Supersymmetry (SUSY)

	spin		spin	
quarks q	$\frac{1}{2}$	⟷	0	squarks $\tilde{q}$
leptons ℓ	$\frac{1}{2}$	⟷	0	sleptons $\tilde{\ell}$
gauge bosons A_μ	1	⟷	$\frac{1}{2}$	gauginos λ
Higgs bosons H	0	⟷	$\frac{1}{2}$	higgsinos $\tilde{h}$

Standard Model

naturalness

fine-tuning problem

$$m_H^2 = m_{H,0}^2 + \Lambda^2 \qquad (\Lambda \gg m_H)$$

(fine tuning like 1.000000000000001 - 1)

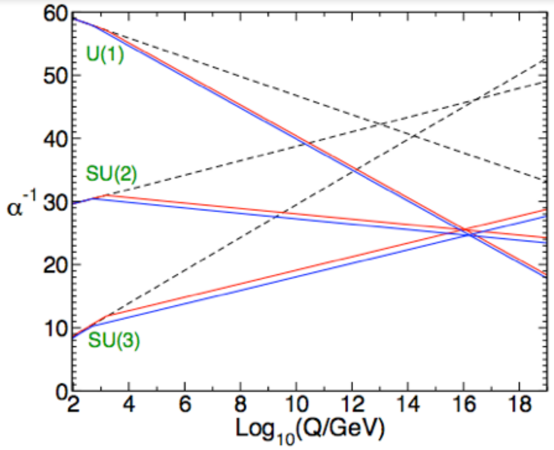
→ solved by the supersymmetry !

$$m_H^2 = m_{H,0}^2 + (\ \Lambda^2 - \Lambda^2\ )$$

fermion boson

coupling unification

Grand Unified Theory



without SUSY

with SUSY

Dark Matter = Lightest SUSY particle

OK, then,....

What's the implications of
125 GeV Higgs for
Supersymmetry (SUSY) ??

125 GeV Higgs and SUSY

$$V(H) = -m^2(H^\dagger H) + \lambda_{\mathrm{H}}(H^\dagger H)^2$$

$(89\ \mathrm{GeV})^2$

0.13

in SUSY...

$$= \lambda_H^{\mathrm{tree}} + \delta\lambda_H^{\mathrm{loop}}$$

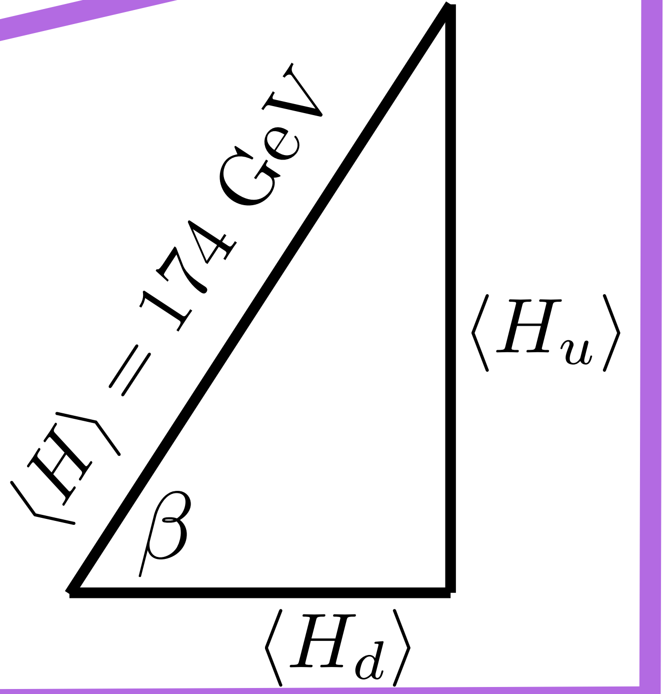
$$\frac{g^2 \cos^2 2\beta}{8\cos^2\theta_W} \simeq \mathbf{0.069}\ \cos^2 2\beta$$

parameters
in Standard Model
(known)

too small...

What is β??

$$\tan\beta = \frac{\langle H_u\rangle}{\langle H_d\rangle}$$



125 GeV Higgs and SUSY

$$V(H) = -m^2 (H^\dagger H) + \lambda_{\mathrm{H}} (H^\dagger H)^2$$

$(89\ \mathrm{GeV})^2$

0.13

in SUSY...

$$= \lambda_H^{\text{tree}} + \delta\lambda_H^{\text{loop}}$$

$$\frac{g^2 \cos^2 2\beta}{8 \cos^2 \theta_W} \simeq \mathbf{0.069} \cos^2 2\beta$$

$$\frac{3 y_t^4}{16\pi^2} \left(\log\left(\frac{m_{\text{stop}}^2}{m_t^2} \right) + \alpha^2 - \frac{\alpha^4}{12} \right) + \cdots$$

for large $\tan\beta$. $(\alpha \simeq A_t/m_{\text{stop}})$

...requires **heavy stop**

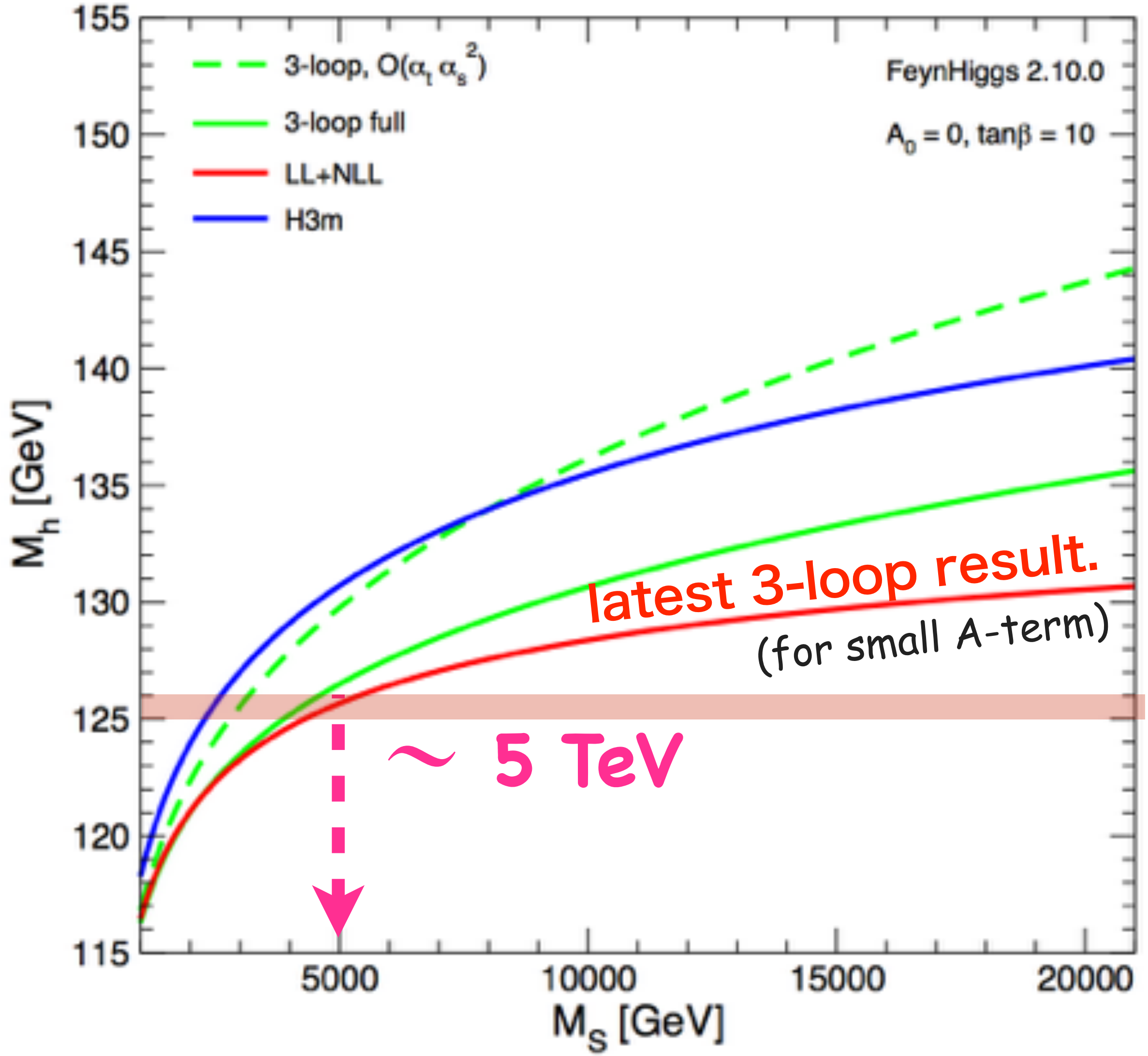
and/or **large A-term**

125 GeV Higgs and SUSY

$$V(H) = -\underset{(89\ \mathrm{GeV})^2}{m^2}(H^\dagger H) + \underset{0.13}{\lambda_{\mathrm{H}}}(H^\dagger H)^2$$

$$= \lambda_H^{\mathrm{tree}} + \delta\lambda_H^{\mathrm{loop}}$$

in SUSY…



latest 3-loop result.

(for small A-term)

$\sim$ 5 TeV

1404.0186 Hahn et.al.

$$\frac{g^2\cos^2 2\beta}{8\cos^2\theta_W} \simeq \mathbf{0.069}\ \cos^2 2\beta$$

$$\frac{3y_t^4}{16\pi^2}\left(\log\left(\frac{m_{\mathrm{stop}}^2}{m_t^2}\right) + \alpha^2 - \frac{\alpha^4}{12}\right) + \cdots$$

for large $\tan\beta$. $(\alpha \simeq A_t/m_{\mathrm{stop}})$

…requires **heavy stop**

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125 GeV Higgs and SUSY

$$V(H) = -m^2 (H^\dagger H) + \lambda_H (H^\dagger H)^2$$

$(89\ \mathrm{GeV})^2$

0.13

$$= \lambda_H^{\text{tree}} + \delta\lambda_H^{\text{loop}}$$

on the other hand

$$-m^2 \simeq |\mu|^2 + m_{H_u}^{2\,(\text{tree})} + \delta m_{H_u}^{2\,(\text{loop})}$$

up to $\mathcal{O}\left(\frac{1}{\tan^2\beta}\right)$

Higgsino mass

soft mass for
up-type Higgs

$$\frac{g^2\cos^2 2\beta}{8\cos^2\theta_W} \simeq 0.069\ \cos^2 2\beta$$

$$\frac{3y_t^4}{16\pi^2}\left(\log\left(\frac{m_{\text{stop}}^2}{m_t^2}\right) + \alpha^2 - \frac{\alpha^4}{12}\right) + \cdots$$

for large $\tan\beta$. $(\alpha \simeq A_t/m_{\text{stop}})$

...requires **heavy stop**

and/or **large A-term**

125 GeV Higgs and SUSY

$$V(H) = -m^2 (H^\dagger H) + \lambda_H (H^\dagger H)^2$$

$(89\ \mathrm{GeV})^2$

0.13

$= \lambda_H^{\mathrm{tree}} + \delta\lambda_H^{\mathrm{loop}}$

on the other hand

$$-m^2 \simeq |\mu|^2 + m_{H_u}^{2\,(\mathrm{tree})} + \delta m_{H_u}^{2\,(\mathrm{loop})}$$

$$\delta m_{H_u}^{2\,(\mathrm{loop})} \sim \frac{-3y_t^2}{8\pi^2}\left(m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 + |A_t|^2\right)\log\left(\frac{M_{\mathrm{mess}}}{m_{\tilde{}}}\right) + \cdot\cdot$$

$$\frac{g^2\cos^2 2\beta}{8\cos^2\theta_W} \simeq \mathbf{0.069}\cos^2 2\beta$$

$$\frac{3y_t^4}{\pi^2}\left(\log\left(\frac{m_{\mathrm{stop}}^2}{m_t^2}\right) + \alpha^2 - \frac{\alpha^4}{12}\right) + \cdots$$

for large $\tan\beta$. $(\alpha \simeq A_t/m_{\mathrm{stop}})$

tension !!

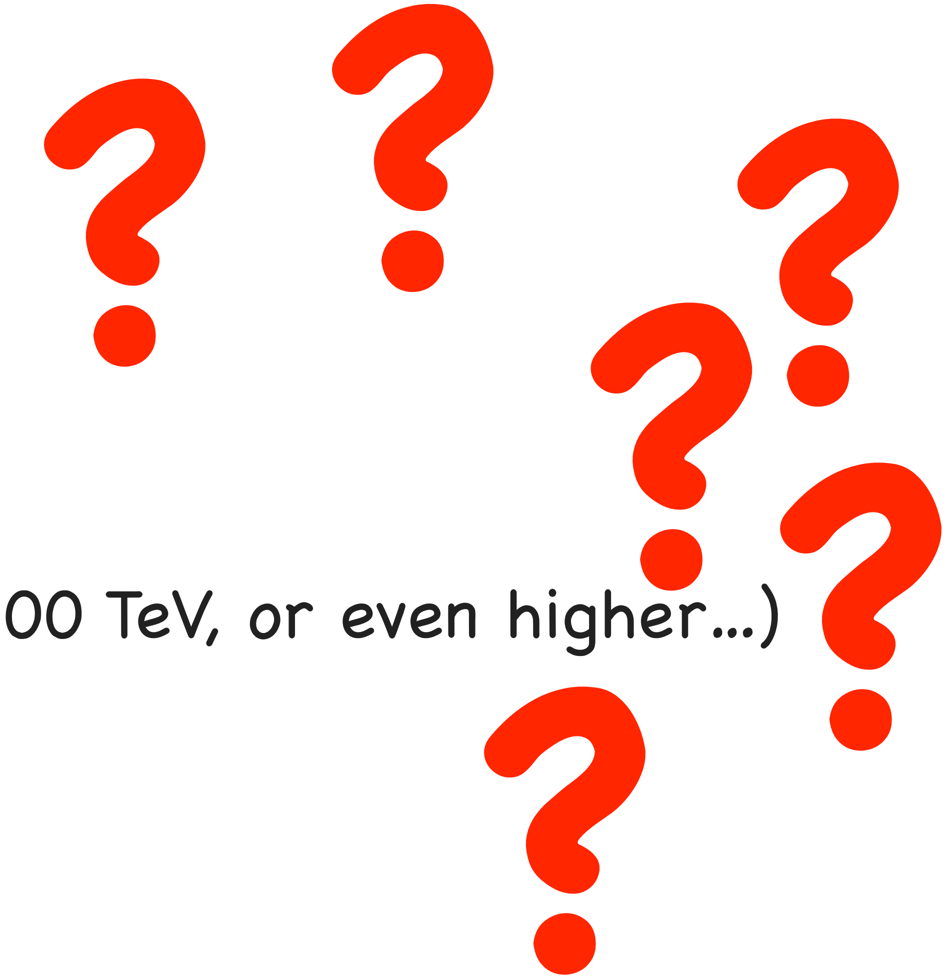
requires **Light stop and small A-term** to avoid a fine-tuning.

...requires **heavy stop** and/or **large A-term**

125 GeV Higgs and SUSY

Fine-tuning worse than 1% seems unavoidable in MSSM.

What does it imply ??



1. No SUSY ?

2. (It's anyway fine-tuned, then....)

Very heavy SUSY ? (10-100 TeV, or even higher...)

3. (still.....)

O(0.1-1) TeV SUSY ? (fine-tuned, but less than 2 and 3...)

125 GeV Higgs and SUSY

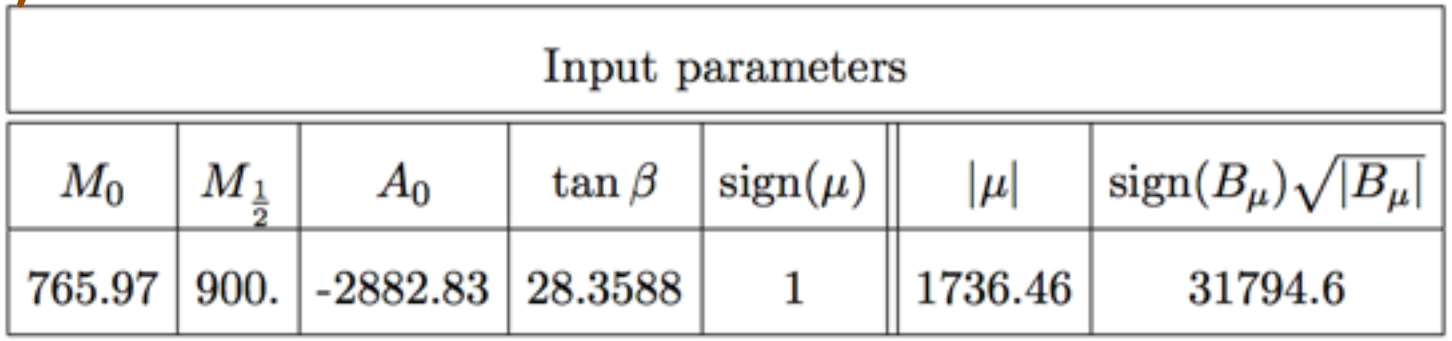
- What happens to neutralino Dark Matter after Higgs discovery?

125 GeV Higgs and SUSY

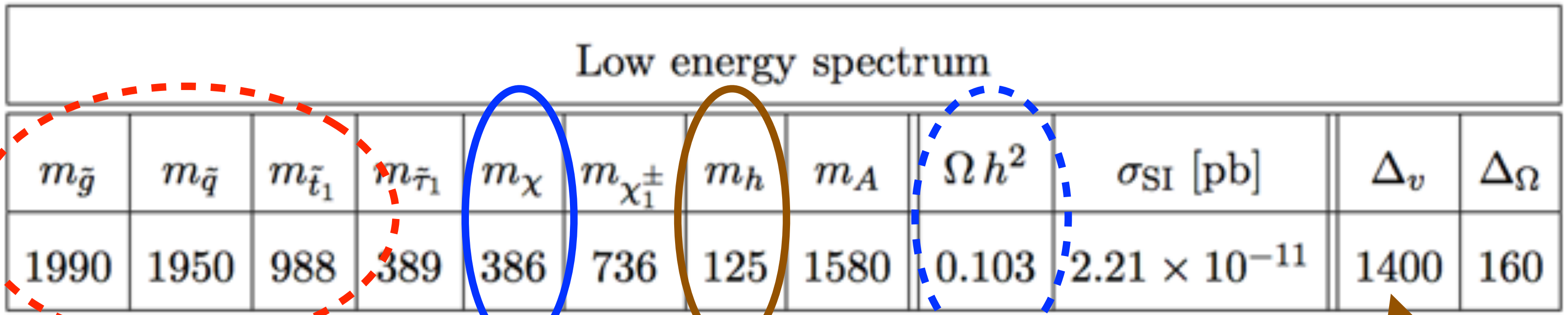
- What happens to neutralino Dark Matter after Higgs discovery?
- Still OK... and SUSY particles may be seen at LHC.

As an example..., benchmark model points in CMSSM/mSUGRA shown in 1305.2914, Cohen Wacker

"Stau coannihilation"



Input parameters						
M_0	$M_{\frac{1}{2}}$	A_0	$\tan\beta$	$\mathrm{sign}(\mu)$	$\lvert\mu\rvert$	$\mathrm{sign}(B_\mu)\sqrt{\lvert B_\mu\rvert}$
765.97	900.	-2882.83	28.3588	1	1736.46	31794.6



Low energy spectrum											
$m_{\tilde g}$	$m_{\tilde q}$	$m_{\tilde t_1}$	$m_{\tilde\tau_1}$	m_χ	$m_{\chi_1^\pm}$	m_h	m_A	Ωh^2	σ_{SI} [pb]	Δ_v	Δ_Ω
1990	1950	988	389	386	736	125	1580	0.103	2.21×10^{-11}	1400	160

gluino
squark
stop

within 13-14 TeV LHC reach

neutralino DM

Higgs

correct DM density

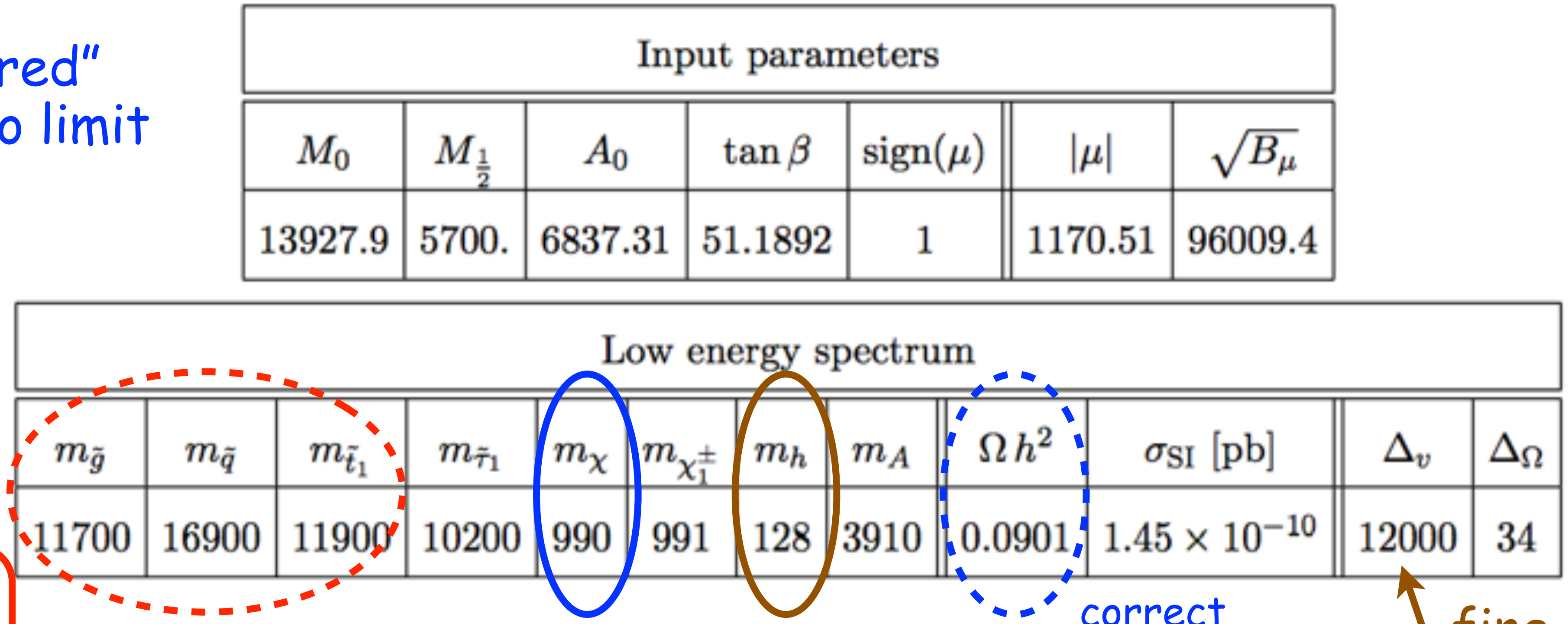
fine-tuning < 0.1 %

125 GeV Higgs and SUSY

- What happens to
  neutralino Dark Matter after Higgs discovery?
- Still OK… and SUSY particles may be seen at LHC,
  …… or may not be seen.

As an example…, benchmark model points in CMSSM/mSUGRA shown in 1305.2914, Cohen Wacker

"Well tempered"
pure Higssino limit



Input parameters						
M_0	$M_{\frac{1}{2}}$	A_0	$\tan\beta$	sign(μ)	$\lvert\mu\rvert$	$\sqrt{B_\mu}$
13927.9	5700.	6837.31	51.1892	1	1170.51	96009.4

Low energy spectrum											
$m_{\tilde{g}}$	$m_{\tilde{q}}$	$m_{\tilde{t}_1}$	$m_{\tilde{\tau}_1}$	m_χ	$m_{\chi_1^\pm}$	m_h	m_A	Ωh^2	$\sigma_{\rm SI}$ [pb]	Δ_v	Δ_Ω
11700	16900	11900	10200	990	991	128	3910	0.0901	1.45×10^{-10}	12000	34

gluino
squark
stop

outside the 13 TeV LHC reach

neutralino DM

Higgs

correct
DM density

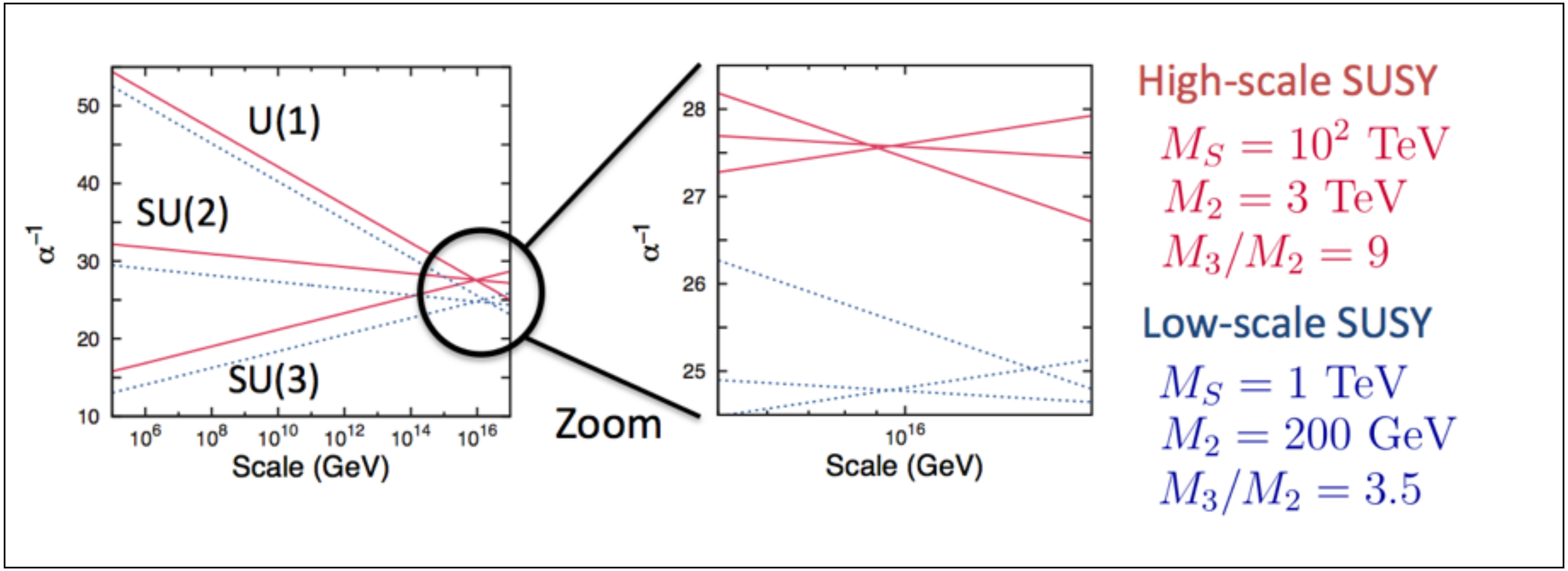
fine-tuning
< 0.01%

125 GeV Higgs and SUSY

- What happens to coupling unification after Higgs discovery?

125 GeV Higgs and SUSY

- **What happens to coupling unification after Higgs discovery?**
- **No problem.**
  O(10-100) TeV SUSY can make it even better……



[talk given by N.Nagata at YITP workshop 2013
Hisano,Kuwahara,Nagata'13, Hisano,Kobayashi,Kuwahara,Nagata'13]

125 GeV Higgs and SUSY

Fine-tuning worse than 1% seems unavoidable in MSSM.

(MSSM =Minimal SUSY Standard Model)

What does it imply ??

1. **No SUSY ?**

2. (It's anyway fine-tuned, then....)

   Very heavy SUSY ? (10-100 TeV, or even higher...)

3. (still.....)

   O(0.1-1) TeV SUSY ? (fine-tuned, but less than 2 and 3...)

125 GeV Higgs and SUSY

(It's anyway fine-tuned, then....)

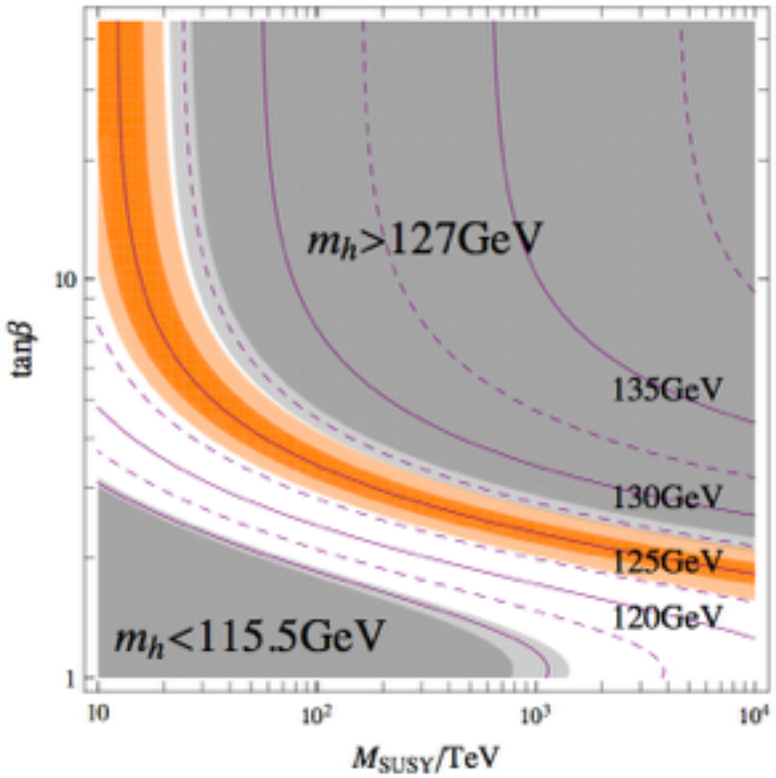
Very heavy SUSY

$$m_H^2 = 4\lambda_H\langle H\rangle^2$$

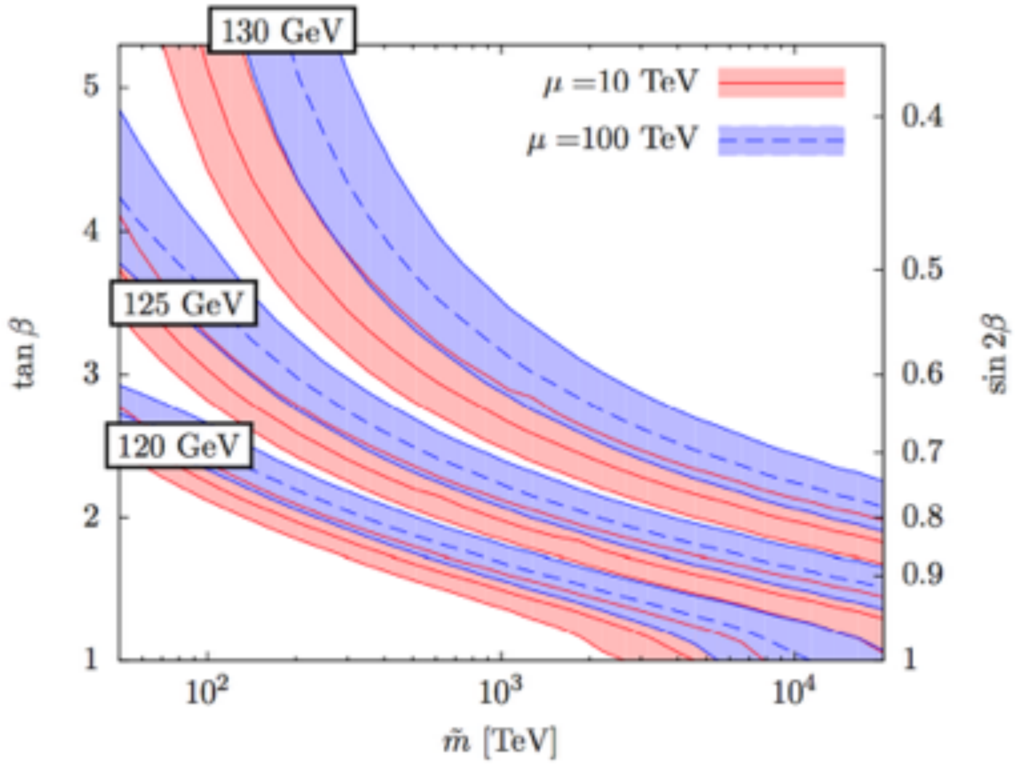
$$\rightarrow \lambda_H \simeq 0.13$$

$$= \underbrace{\lambda_H^{\text{tree}}}_{0.07\cos^2 2\beta} + \underbrace{\delta\lambda_H^{\text{loop}}}_{\sim\log(m_{\text{stop}}^2)}$$

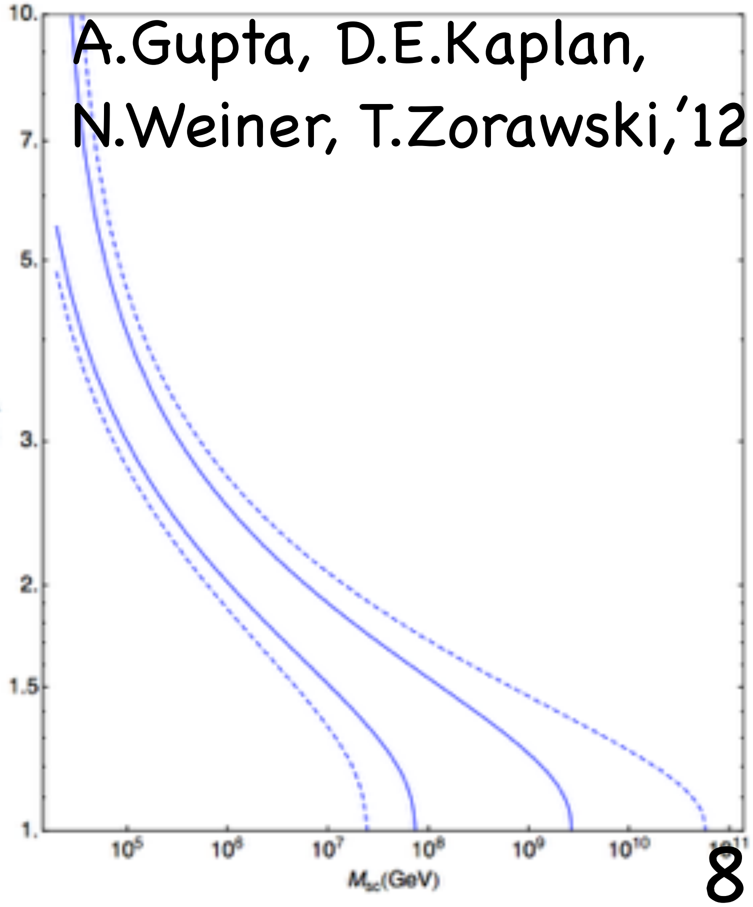
Ibe, Matsumoto, Yanagida,'12



L.Hall, Y.Nomura, S.Shirai '12



N.Arkani-Hamed, A.Gupta, D.E.Kaplan, N.Weiner, T.Zorawski,'12

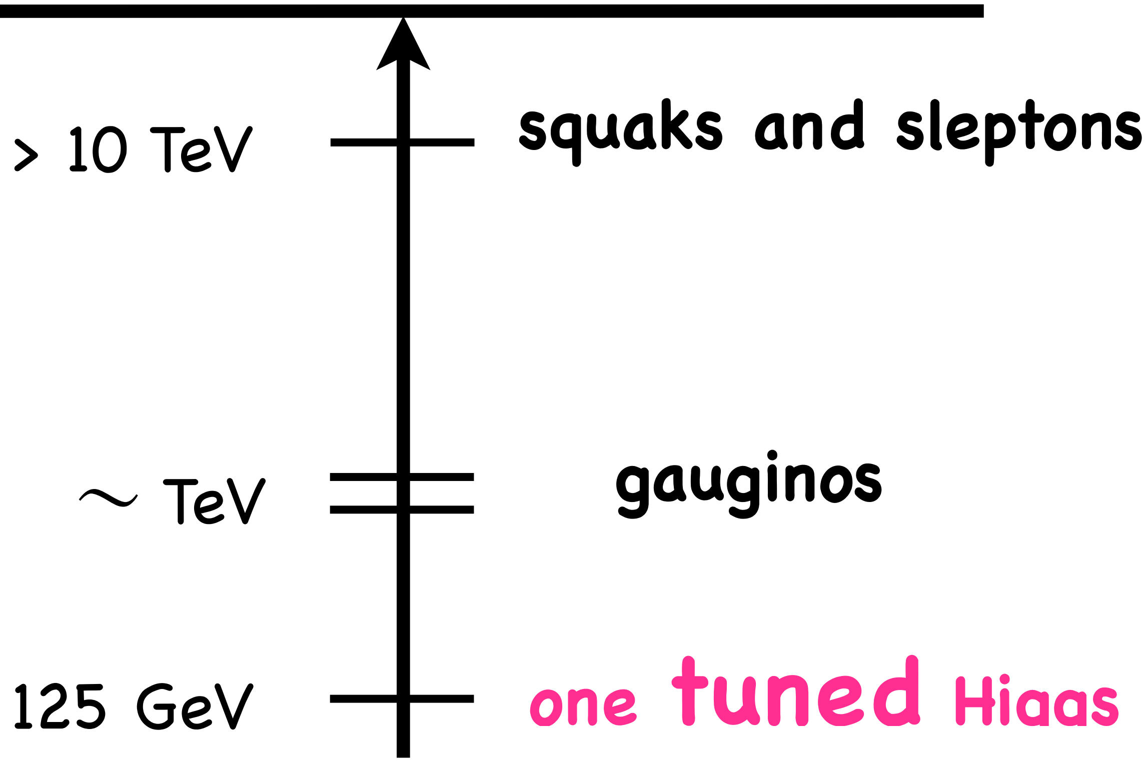


125 GeV Higgs and SUSY

(It's anyway fine-tuned, then....)

Very heavy SUSY

- **consistent with 125 GeV Higgs**
- **No cosmological gravitino problem**
- **Coupling Unification is OK**
- **Dark Matter is also OK**



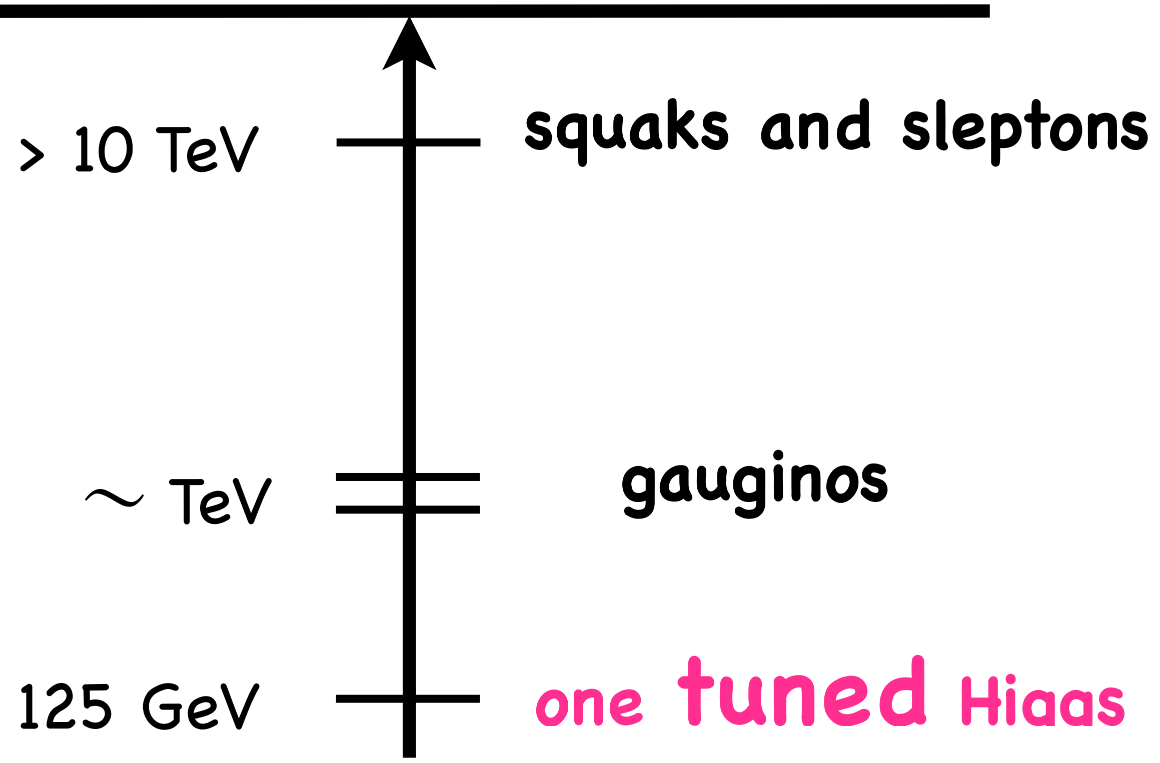
Many many works recently..... (too many to list all...)

Ibe,Yanagida'11, Ibe,Matsumoto,Yanagida'12,
Bhattacherjee,Feldstein,Ibe,Matsumoto,Yanagida'12,
Hall,Nomura'11, Hall,Nomura,Shirai'12,
Giudice,Strumia'11, Arvanitaki,Craig,Dimopoulos,Villadoro'12
Arkani-Hamed,Gupta,Kaplan,Weiner,Zorawski'12, Ibanez,Valenzuela'13,
Jeong,Shimosuka,Yamaguchi'11, Hisano,Ishiwata,Nagata'12, Sato,Shirai,Tobioka'12,
Moroi,Nagai'13, McKeen,Pospelov,Ritz'13,
Hisano,Kuwahara,Nagata'13, Hisano,Kobayashi,Kuwahara,Nagata'13, etc etc.....

125 GeV Higgs and SUSY

(It's anyway fine-tuned, then....)

Very heavy SUSY

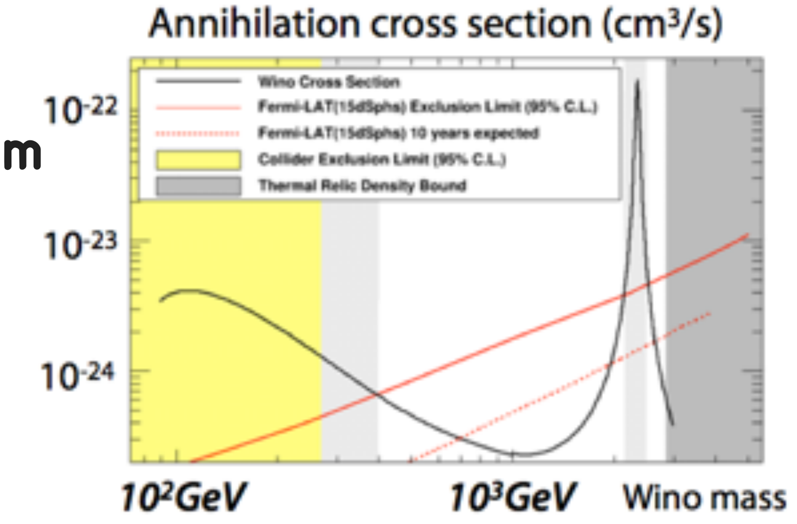


Typical DM = Wino DM

(AMSB)

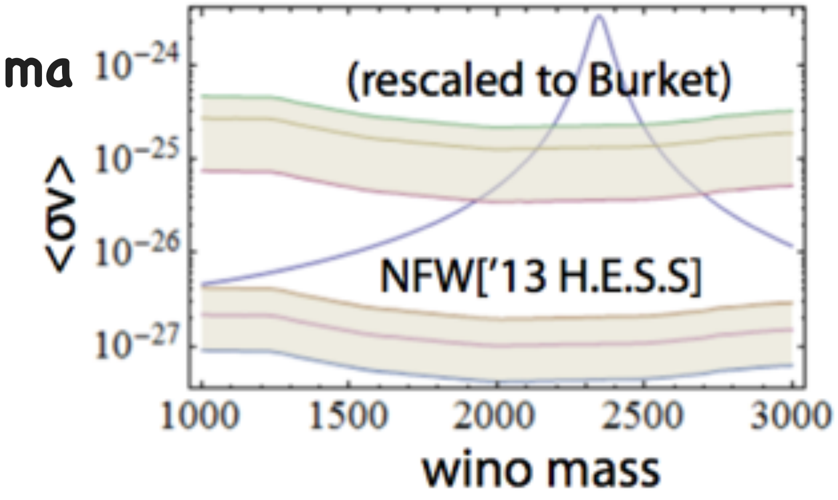
- if thermal relic,... 2.7 TeV
  (>> LHC reach) (Hisano,Matsumoto,Nagai,Saito,Senami'07)
- if non-thermal, it can be lighter.
- indirect DM signal expected !

gamma continuum



[Fermi-LAT:1310.0828 (Translated by K.Ichikawa)]

gamma line



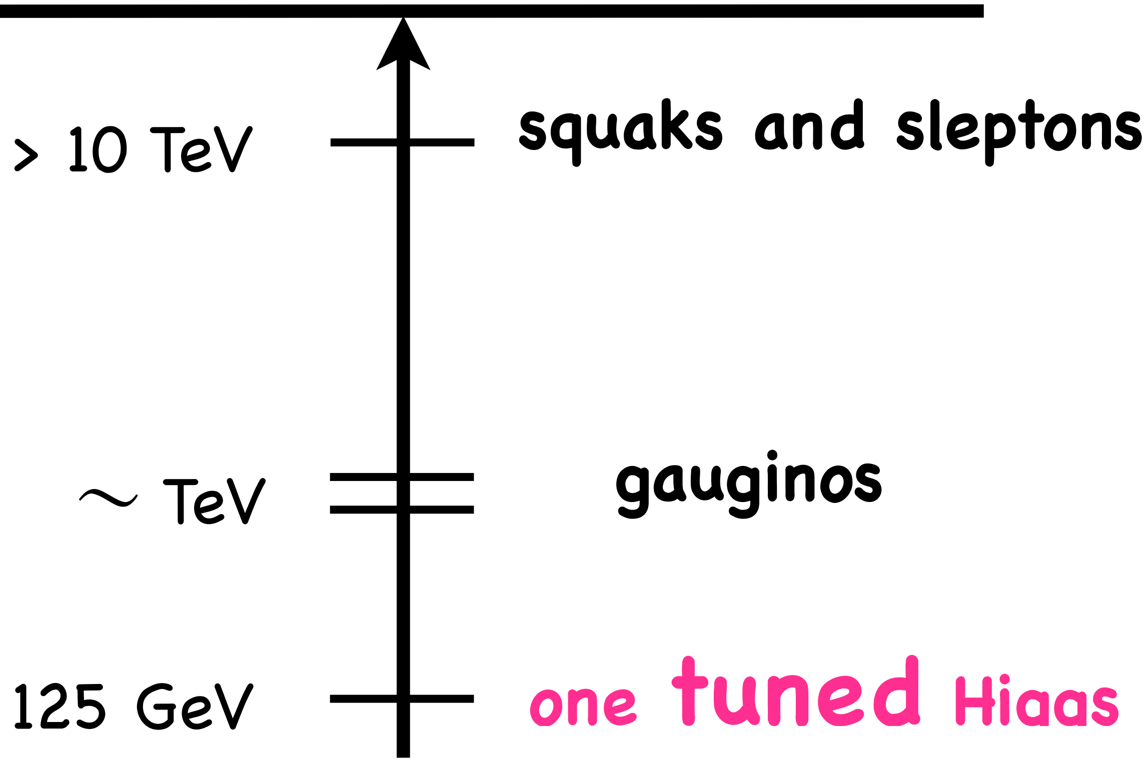
[Figure by S.Matsumoto]

Figures from talk by M.Ibe at KIAS workshop, October 2014.

125 GeV Higgs and SUSY

(It's anyway fine-tuned, then....)

Very heavy SUSY

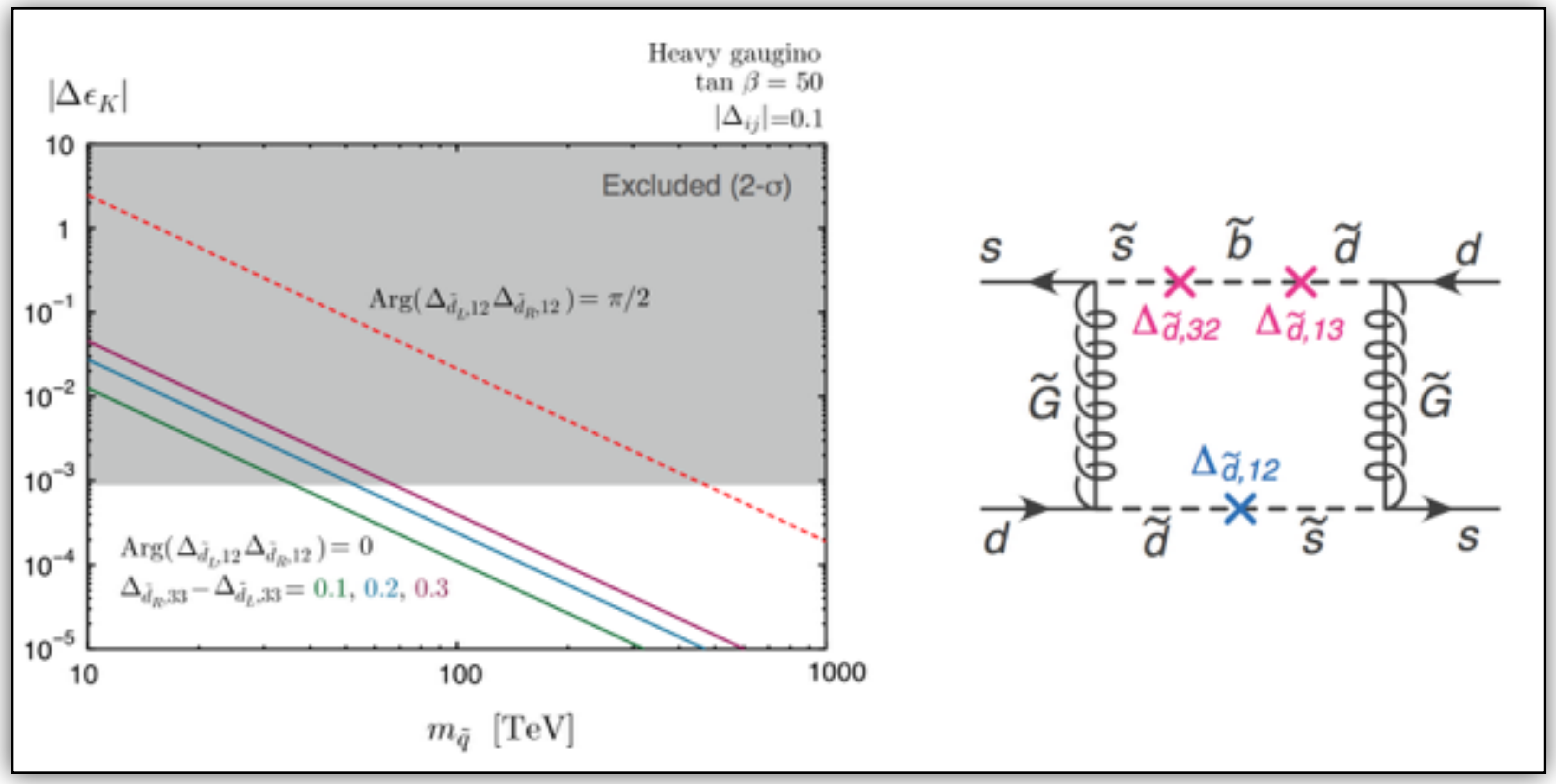


> 10 TeV — squaks and sleptons

~ TeV — gauginos

125 GeV — one tuned Higgs

Flavor Physics can probe it !



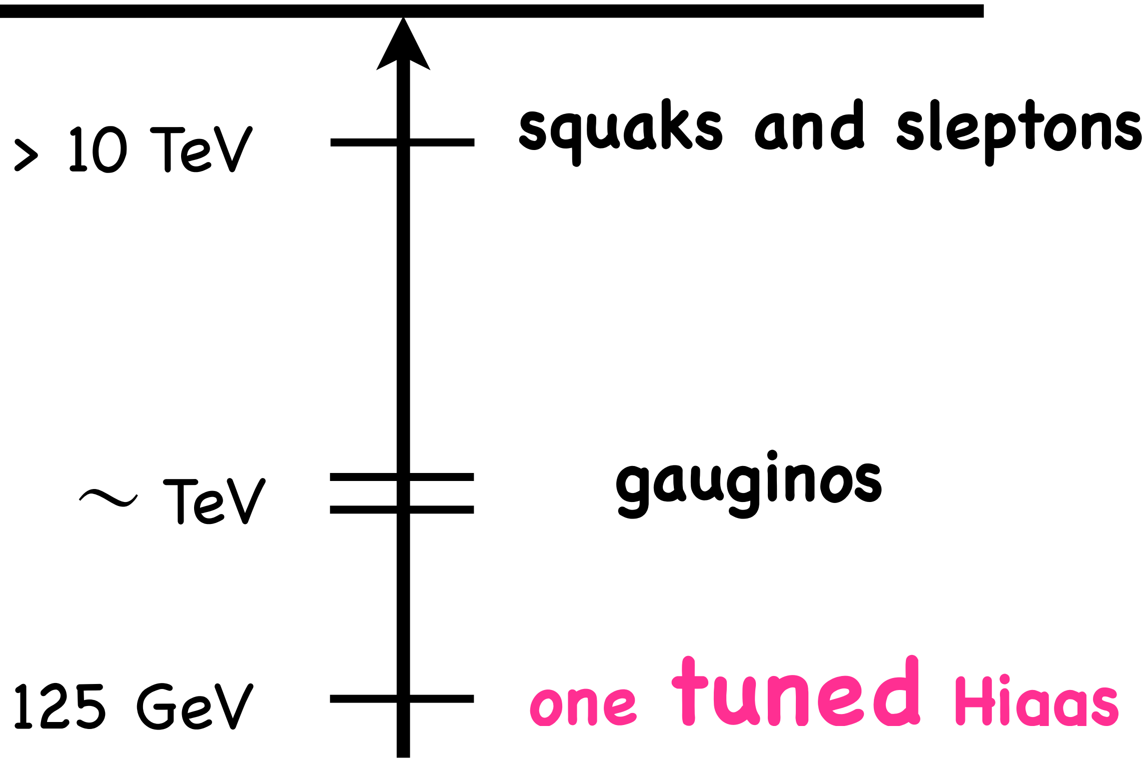
Figures from talk by T.Moroi at KIAS workshop, November 2013.

125 GeV Higgs and SUSY

(It's anyway fine-tuned, then....)

Very heavy SUSY

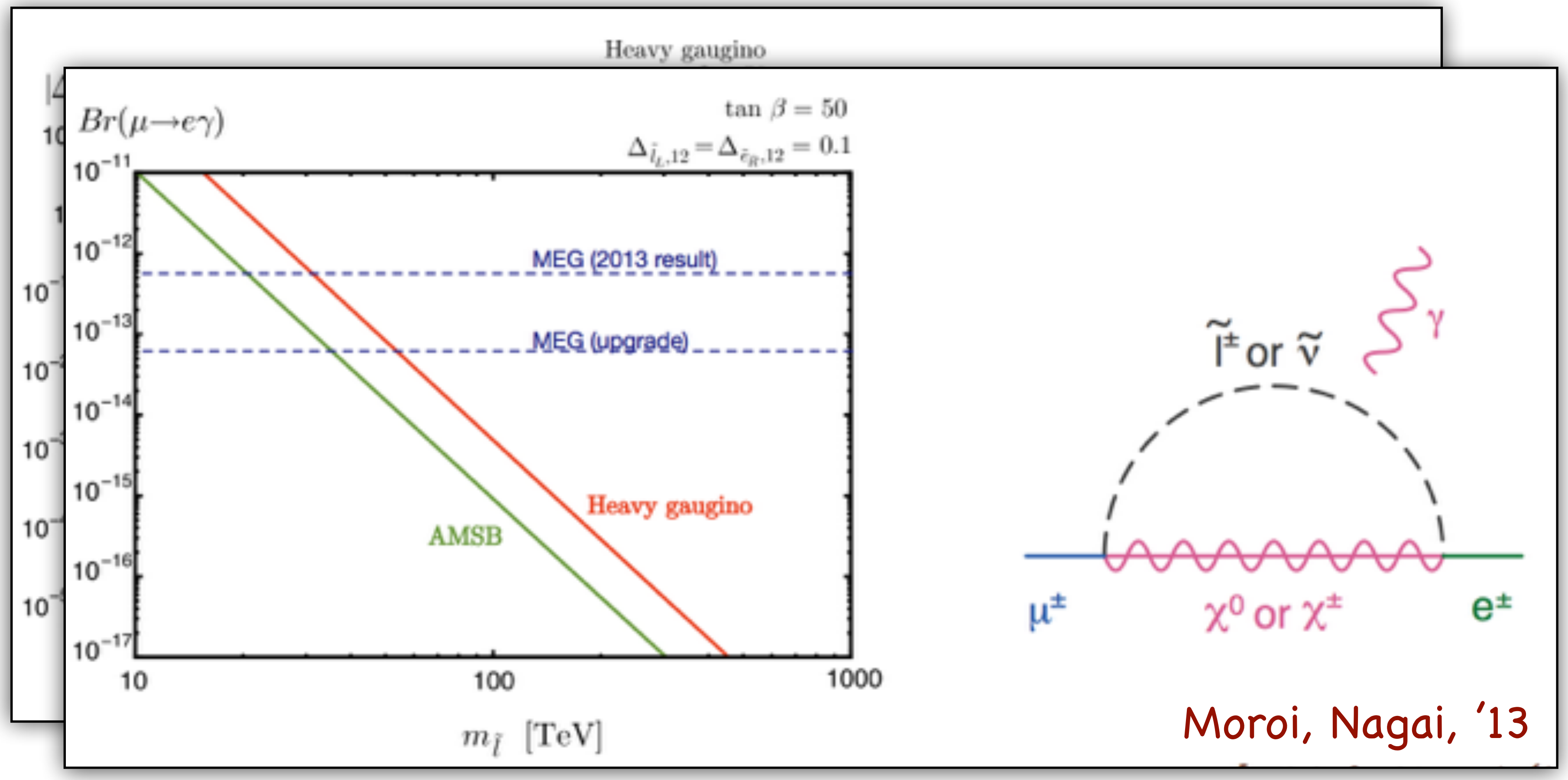
Flavor Physics
can probe it !



> 10 TeV — squaks and sleptons

~ TeV — gauginos

125 GeV — one tuned Higgs



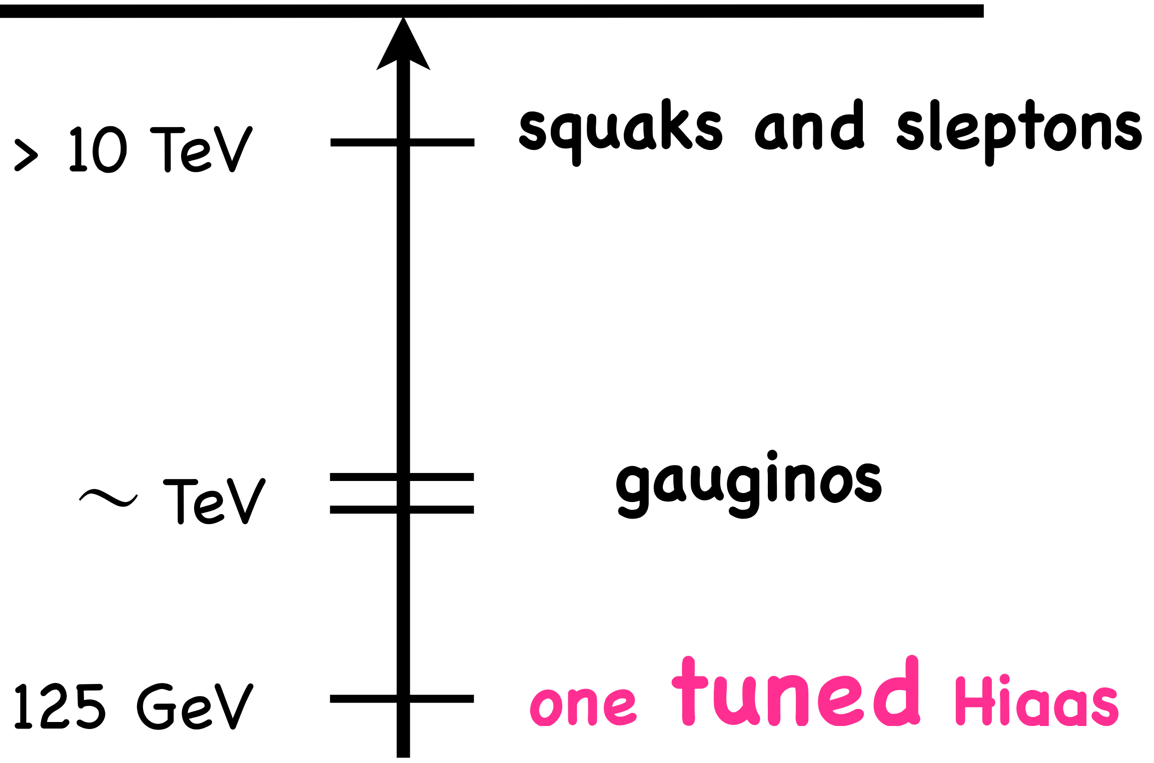
Moroi, Nagai, '13

Figures from talk by
T.Moroi at KIAS workshop,
November 2013.

125 GeV Higgs and SUSY

(It's anyway fine-tuned, then....)

Very heavy SUSY

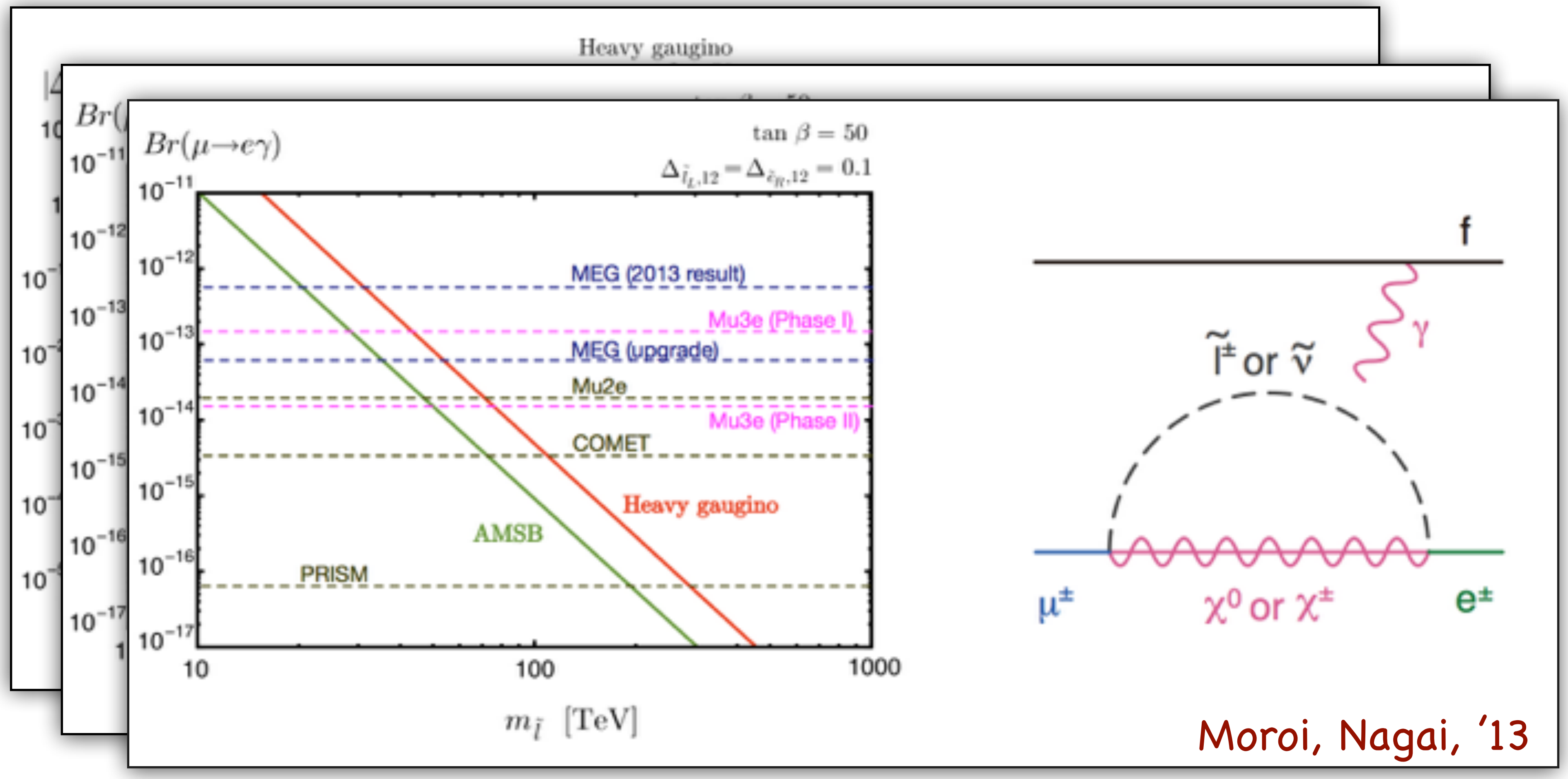


> 10 TeV — **squaks and sleptons**

$\sim$ TeV — **gauginos**

125 GeV — **one tuned Higgs**

Flavor Physics can probe it !



Moroi, Nagai, '13

Figures from talk by T.Moroi at KIAS workshop, November 2013.

125 GeV Higgs and SUSY

(It's anyway fine-tuned, then....)

Very heavy SUSY

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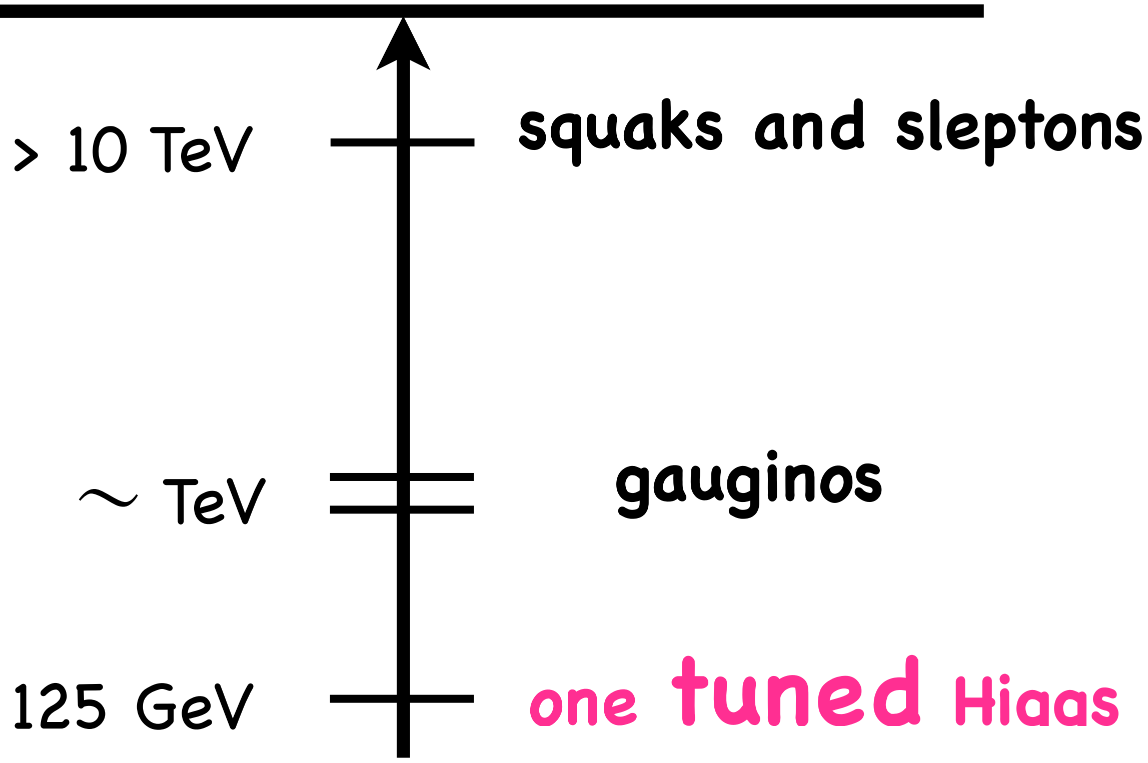

> 10 TeV — **squaks and sleptons**

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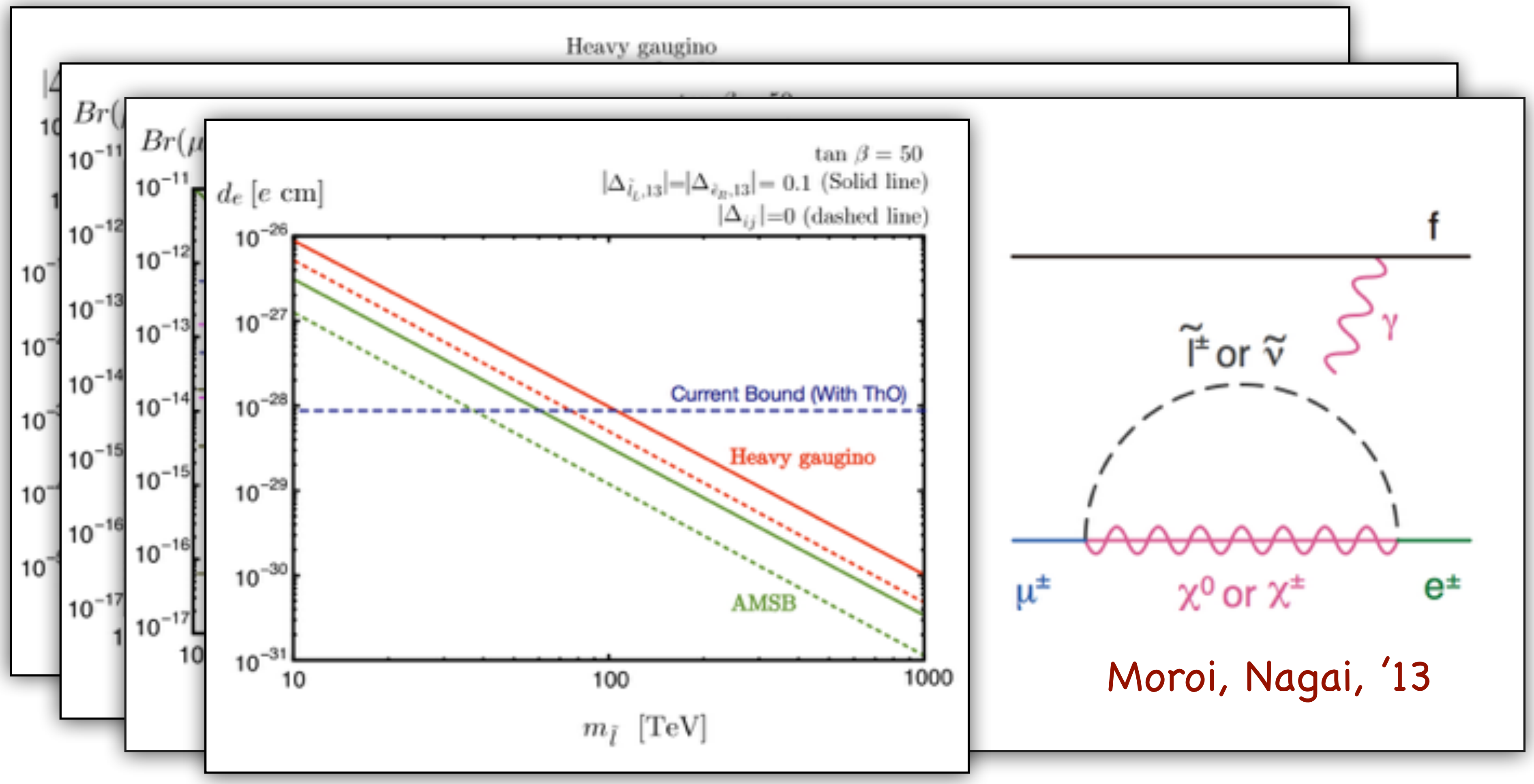


Moroi, Nagai, '13

Figures from talk by T.Moroi at KIAS workshop, November 2013.

125 GeV Higgs and SUSY

Fine-tuning worse than 1% seems unavoidable in MSSM.

(MSSM =Minimal SUSY Standard Model)

What does it imply ??

1. **No SUSY ?**

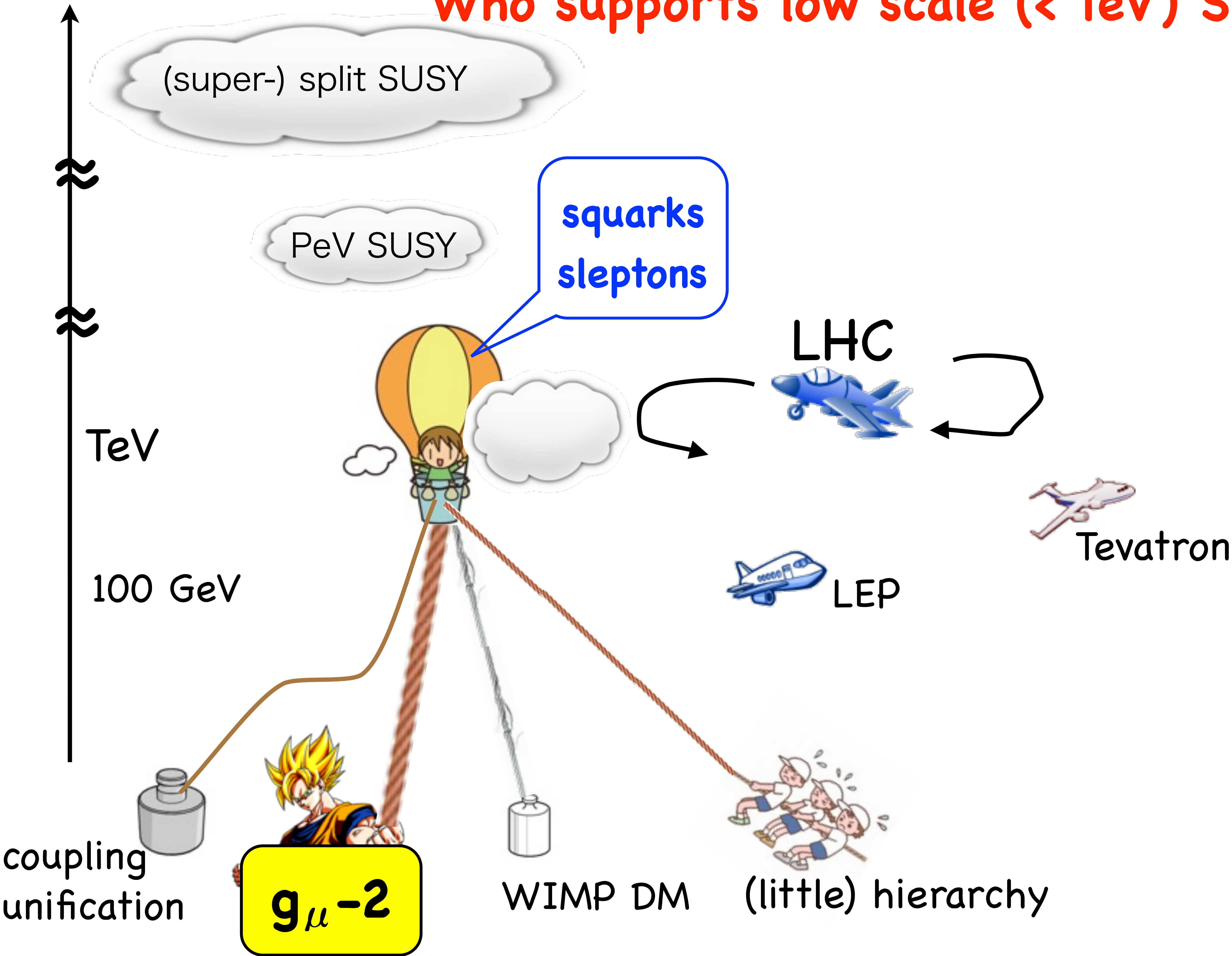
2. (It's anyway fine-tuned, then....)

   Very heavy SUSY ? (10~100 TeV, or even higher...)

3. (still.....)

   (0.1-1) TeV SUSY ? (fine-tuned, but less than 1 and 2...)

Who supports low scale (< TeV) SUSY?



one more motivation for TeV scale SUSY...

muon g-2

$$a_\mu^{\mathrm{EXP}} - a_\mu^{\mathrm{SM}} = (26.1 \pm 8.0) \cdot 10^{-10}$$

> 3 σ deviation !

one more motivation for TeV scale SUSY...

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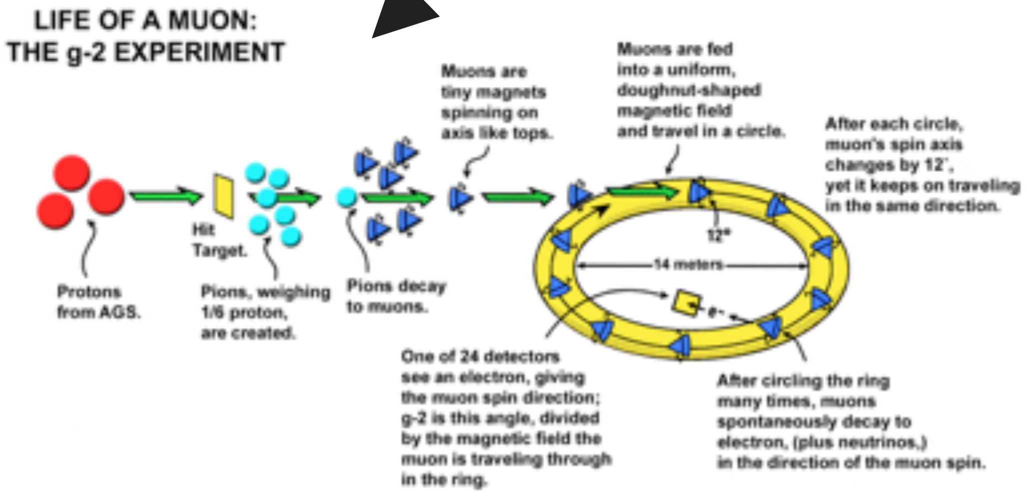


from E821 muon g-2 Home Page

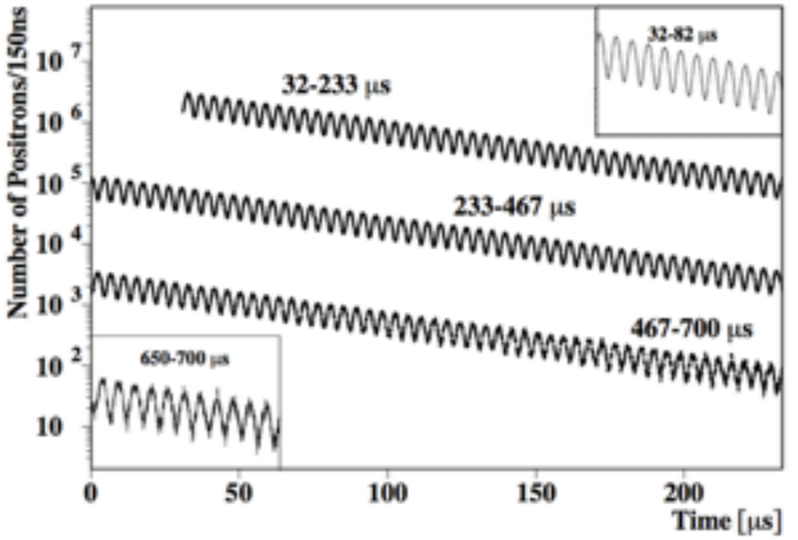
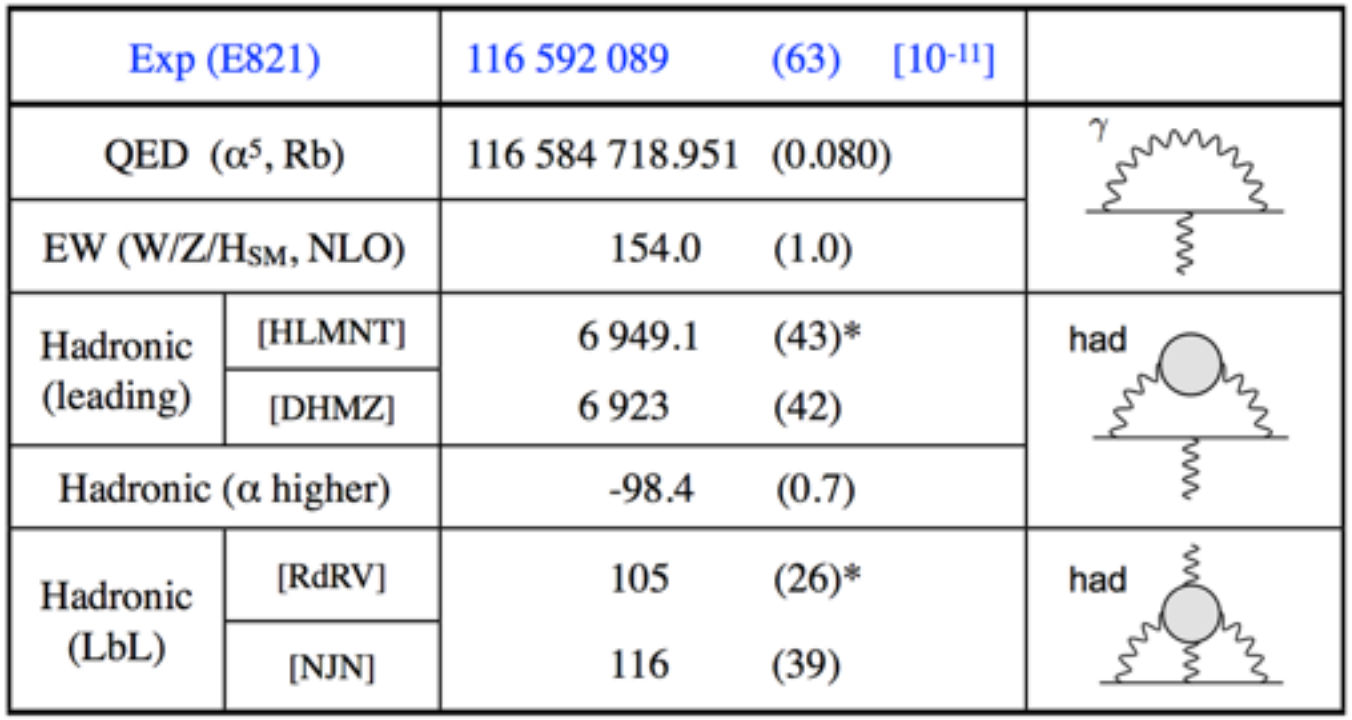


FIG. 3. Positron time spectrum overlaid with the fitted 10 parameter function (χ^2/dof= 3818/3799). The total event sample of 0.95 × 10^9 e^+ with $E \geq 2.0$ GeV is shown.

from hep-ph/0102017

Standard Model Prediction



Exp (E821)		116 592 089	(63) $[10^{-11}]$	
QED (α^5, Rb)		116 584 718.951	(0.080)	γ
EW (W/Z/H_{SM}, NLO)		154.0	(1.0)	
Hadronic (leading)	[HLMNT]	6 949.1	(43)*	had
	[DHMZ]	6 923	(42)	
Hadronic (α higher)		-98.4	(0.7)	
Hadronic (LbL)	[RdRV]	105	(26)*	had
	[NJN]	116	(39)	

from Talk by M.Endo
@Hokkaido Winter School 2013

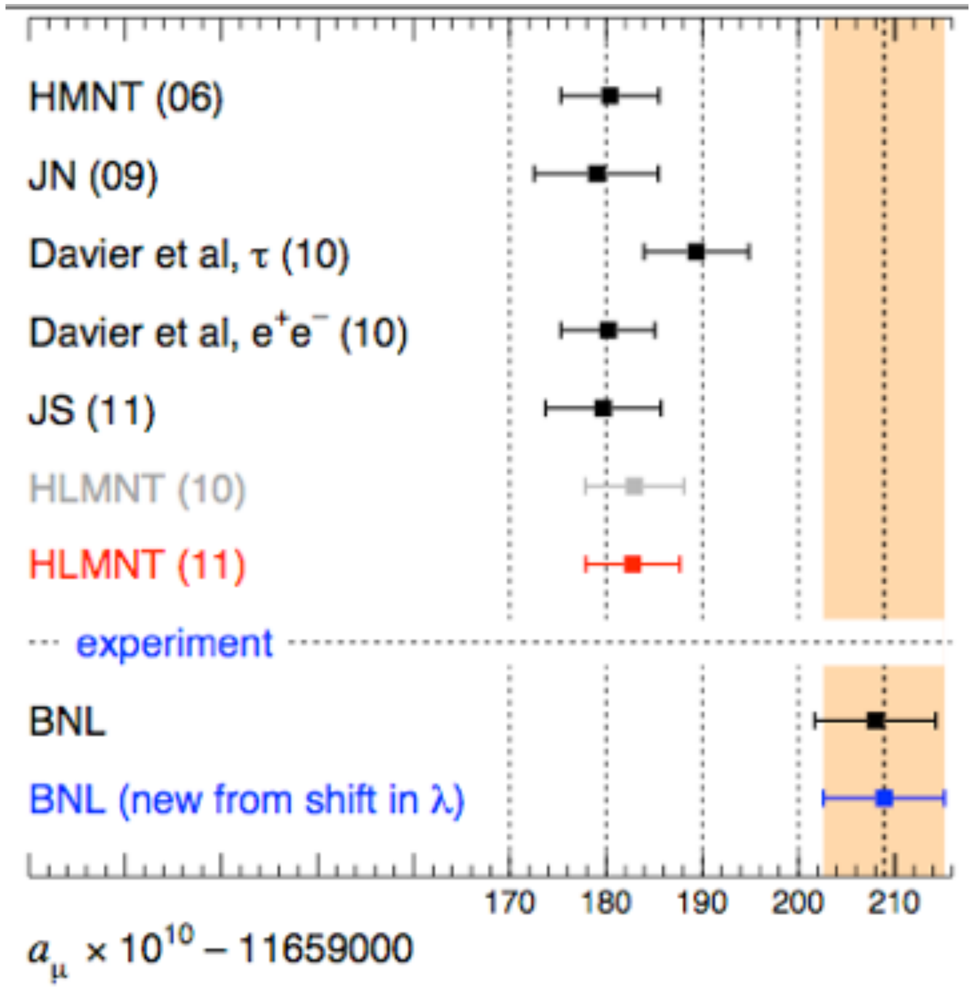
one more motivation for TeV scale SUSY...

muon g-2

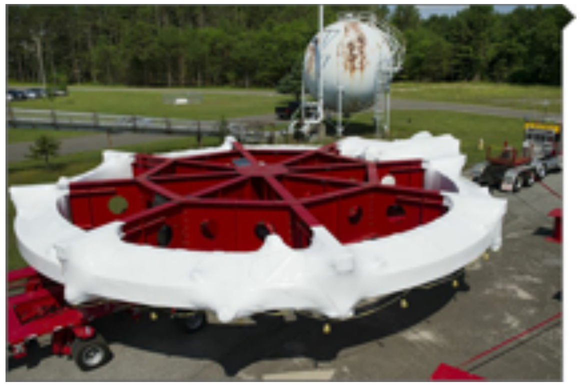
$$a_\mu^{\rm EXP} - a_\mu^{\rm SM} = (26.1 \pm 8.0) \cdot 10^{-10}$$

> 3 σ deviation !

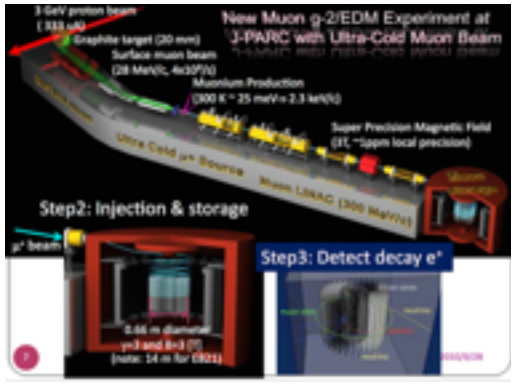
[Hagiwara, Liao, Martin, Nomura, Teubner, arXiv: 1105.3149. See also references therein!]



New experiments also planned.



FermiLab Muon g-2



J-PARC g-2/EDM

one more motivation for TeV scale SUSY...

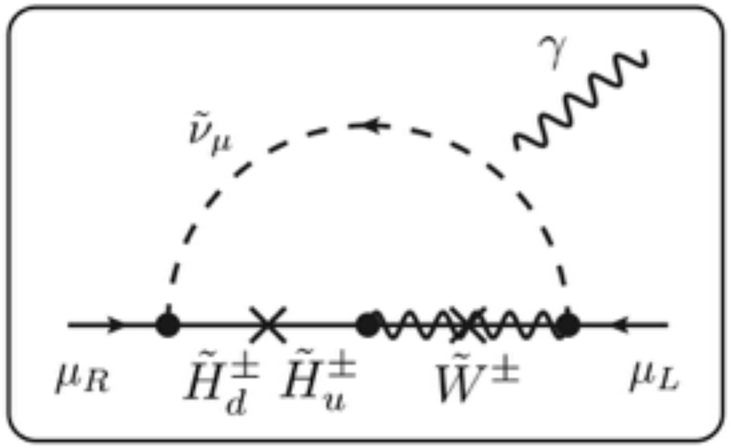
muon g-2

$$a_\mu^{\mathrm{EXP}} - a_\mu^{\mathrm{SM}} = (26.1 \pm 8.0) \cdot 10^{-10}$$

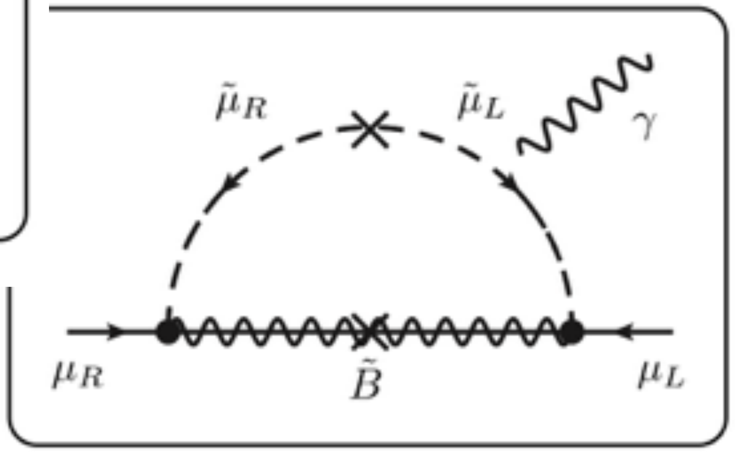
> 3 σ deviation !

...can be explained by SUSY.

chargino



neutralino



... if smuon and chargino/neutralino are O(100 GeV).

125 GeV Higgs + SUSY + g_μ-2

heavy stop

light smuon/ inos

difficult to reconcile in typical models

(mSUGRA/GMSB/AMSB/NMSSM (small tan β) ...)

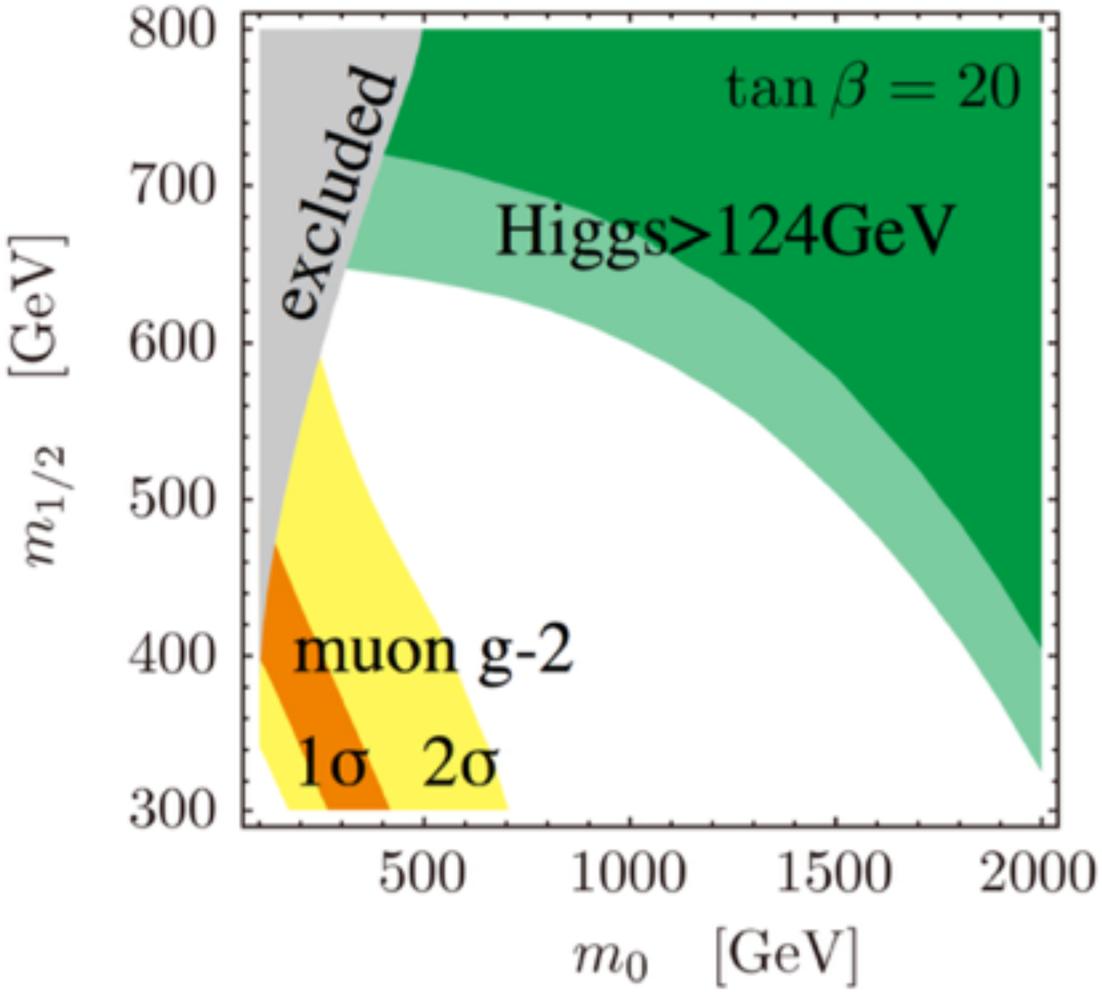
Example in CMSSM/mSUGRA:

Higgs mass is maximized by A-term,

while b -> s γ constraint is satisfied.

(Figure thanks to Motoi Endo.)

[See M.Endo, KH, S.Iwamoto,
K.Nakayama, N.Yokozaki '11]



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2 approaches [from here, I'd like to introduce our works...]

(1) model building

M.Endo, KH, S.Iwamoto, N.Yokozaki, arXiv:1108.3071, 1112.5653, 1202.2751 extra matter

M.Endo, KH, K.Ishikawa, S.Iwamoto, N.Yokozaki, arXiv:1212.3935

M.Endo, KH, S.Iwamoto, K.Nakayama, N.Yokozaki, arXiv:1112.6412 extra gauge

(2) general MSSM

M.Endo, KH, S.Iwamoto, T.Yoshinaga, arXiv:1303.4256 LHC

M.Endo, KH, T.Kitahara, T.Yoshinaga, arXiv:1309.3065 LHC/ILC+flavor+vacuum

M.Endo, KH, S.Iwamoto, T.Kitahara, T.Moroi, arXiv:1310.4496 ILC

125 GeV Higgs + SUSY + g_μ-2

“g-2 motivated” MSSM

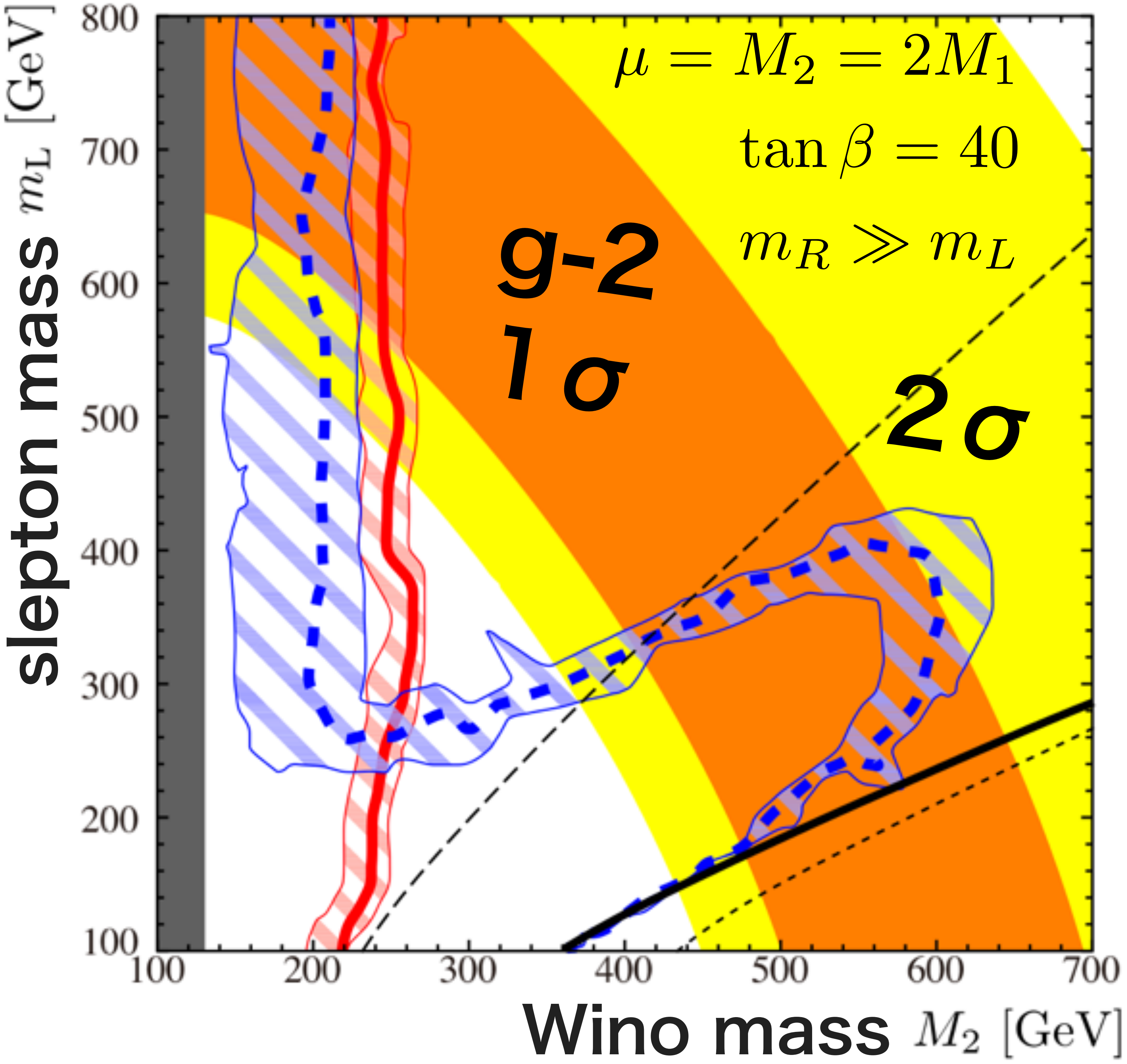
$$m_{\tilde{q}} \gg m_{\tilde{\ell}}, m_{\tilde{\chi}^{\pm}}, m_{\tilde{\chi}^{0}},$$

>> 1 TeV
to explain
Higgs mass

= O(100 GeV)
to explain muon g-2

Can we test it ??

muon g-2 vs LHC in SUSY



M.Endo, KH, S.Iwamoto, T.Yoshinaga [arXiv:1303.4256]

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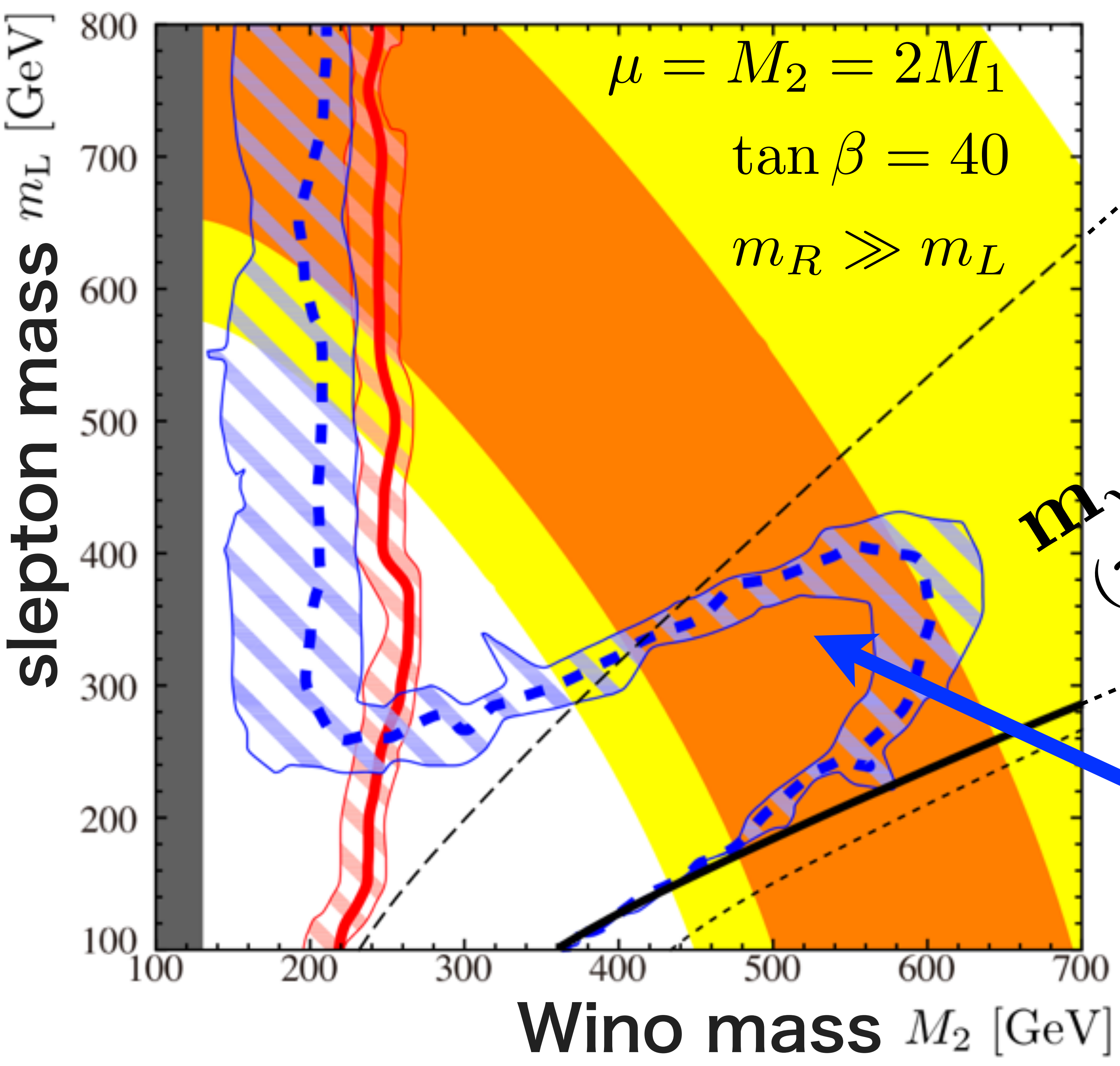


8 TeV LHC

$\mathbf{m}_{\chi_2} > \mathbf{m}_{\tilde{\ell}} > \mathbf{m}_{\chi_1}$

(3-leptons from cascade decay)

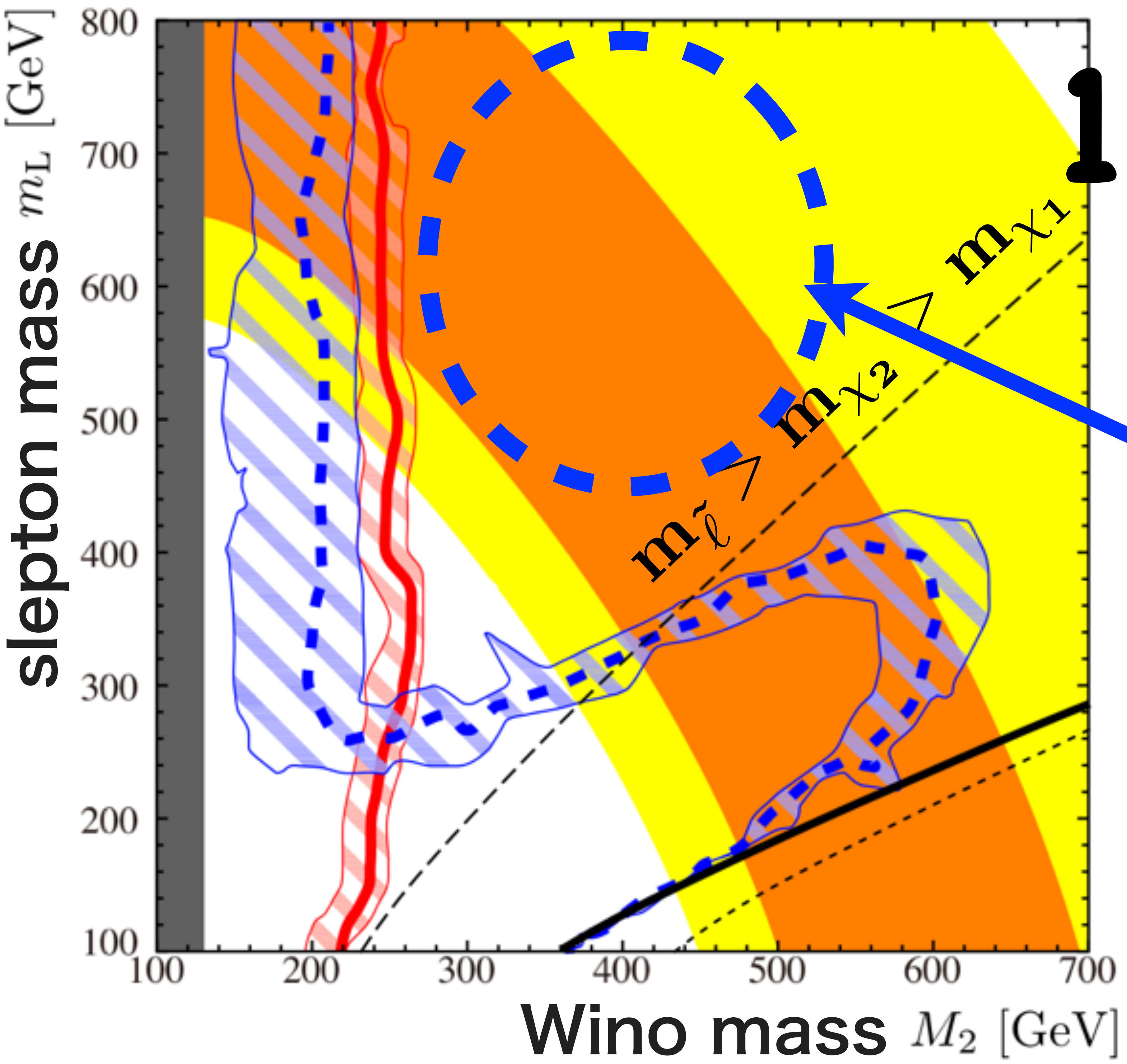
already excluded by 3-lepton search.

* ATLAS 13fb^{-1}@8TeV

(a little stronger bound with 20fb^{-1} data)

M.Endo, KH, S.Iwamoto, T.Yoshinaga [arXiv:1303.4256]

muon g-2 vs LHC in SUSY

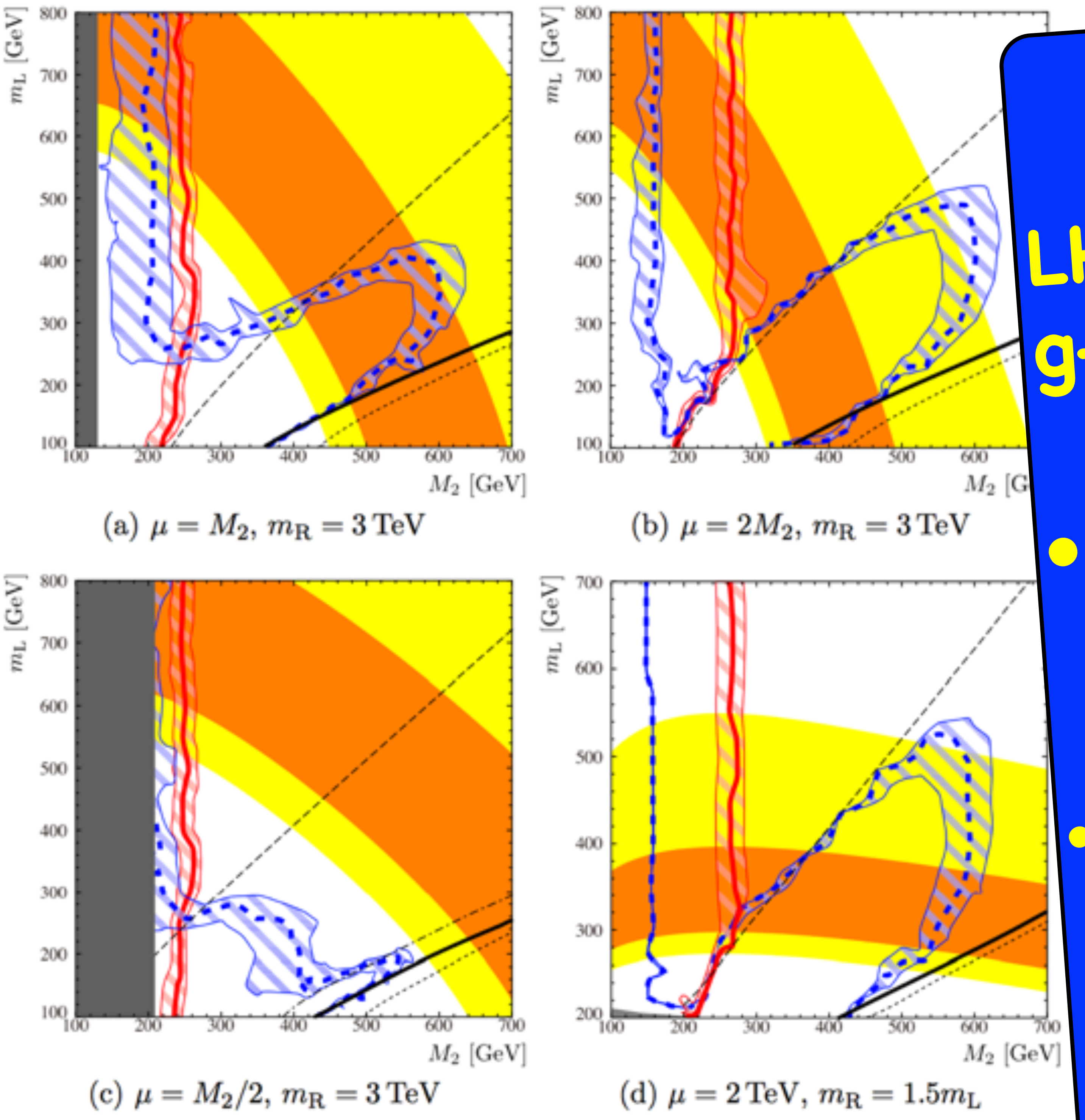


13〜14TeV

New signals like
$\chi_2 \rightarrow \chi_1 + \mathbf{W/Z/h}$
may cover
this region at 13~14 TeV !

M.Endo, KH, S.Iwamoto, T.Yoshinaga [arXiv:1303.4256]

muon g-2 vs LHC in SUSY



(a) $\mu = M_2,\ m_R = 3\,\mathrm{TeV}$

(b) $\mu = 2M_2,\ m_R = 3\,\mathrm{TeV}$

(c) $\mu = M_2/2,\ m_R = 3\,\mathrm{TeV}$

(d) $\mu = 2\,\mathrm{TeV},\ m_R = 1.5 m_L$

M.Endo, KH, S.Iwamoto, T.Yoshinaga [arXiv:13

LHC started exclude g-2 motivated regions !

- 13-14 TeV LHC will test more regions.
- If discovered at LHC, --> further test may be possible at ILC

125 GeV Higgs + SUSY + g_μ-2

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M.Endo, KH, S.Iwamoto, T.Kitahara, T.Moroi, arXiv:1310.4496 ILC

muon g-2 vs ILC

Can we reconstruct the SUSY contributions to the muon g-2 by using **ILC** data ?

Assume one specific (optimistic) model point

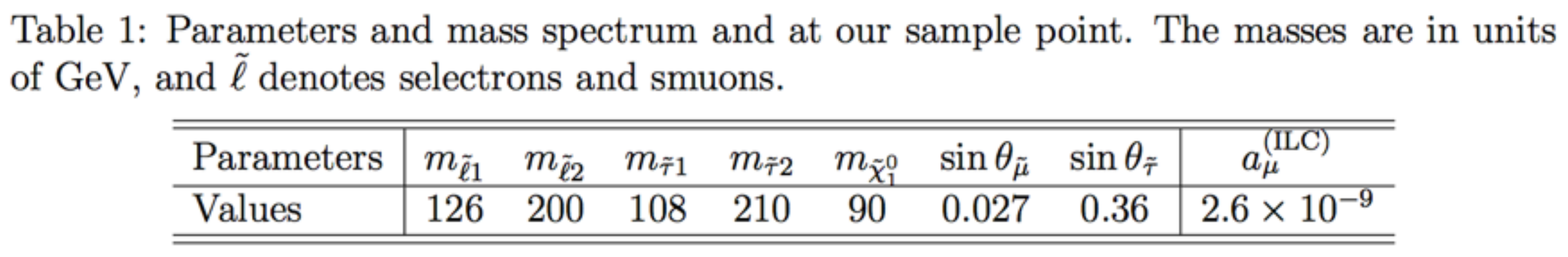
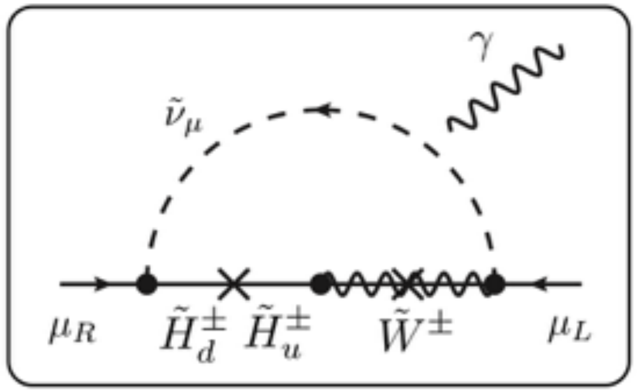


Table 1: Parameters and mass spectrum and at our sample point. The masses are in units of GeV, and $\tilde{\ell}$ denotes selectrons and smuons.

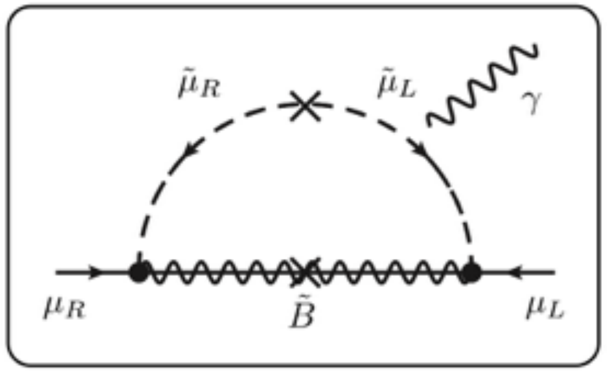
Parameters	$m_{\tilde{\ell}1}$	$m_{\tilde{\ell}2}$	$m_{\tilde{\tau}1}$	$m_{\tilde{\tau}2}$	$m_{\tilde{\chi}_1^0}$	$\sin\theta_{\tilde{\mu}}$	$\sin\theta_{\tilde{\tau}}$	$a_\mu^{(\mathrm{ILC})}$
Values	126	200	108	210	90	0.027	0.36	2.6×10^{-9}

chargino



<<

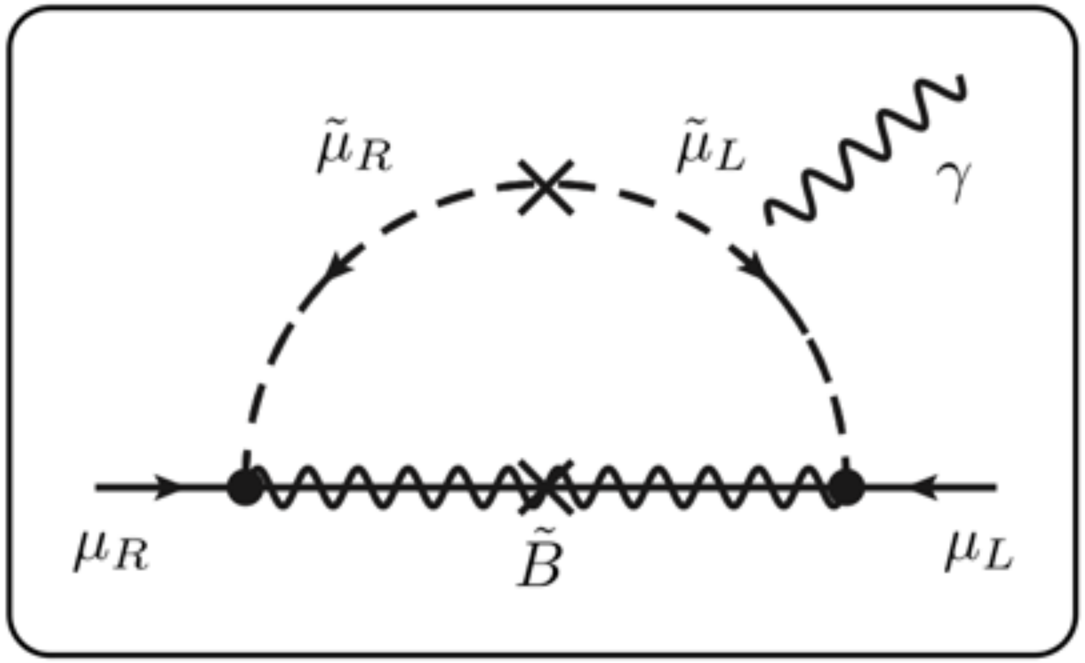
neutralino



this one dominates

muon g-2 vs ILC

neutralino



Can we reconstruct the contribution of this loop-diagram by using ILC measurements?

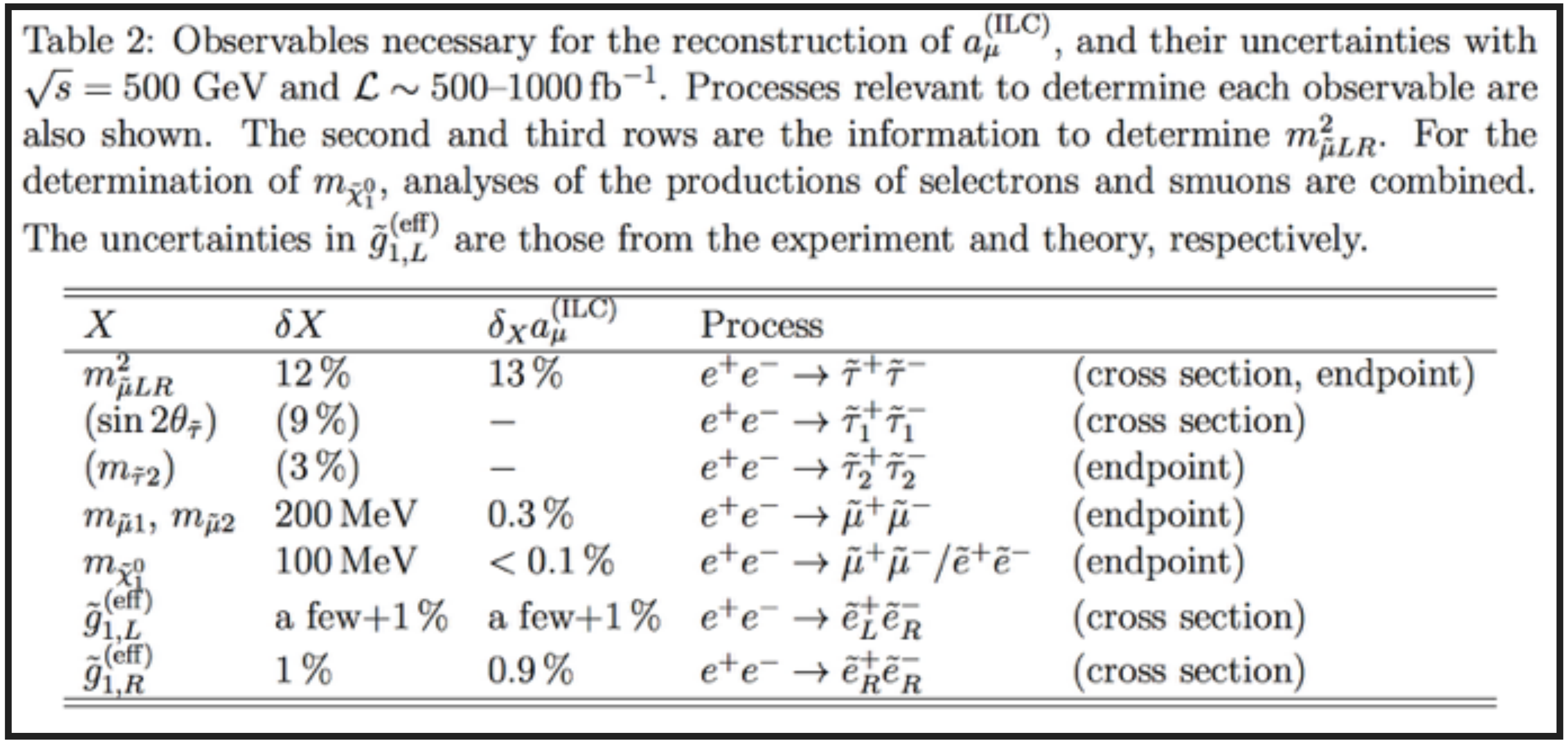
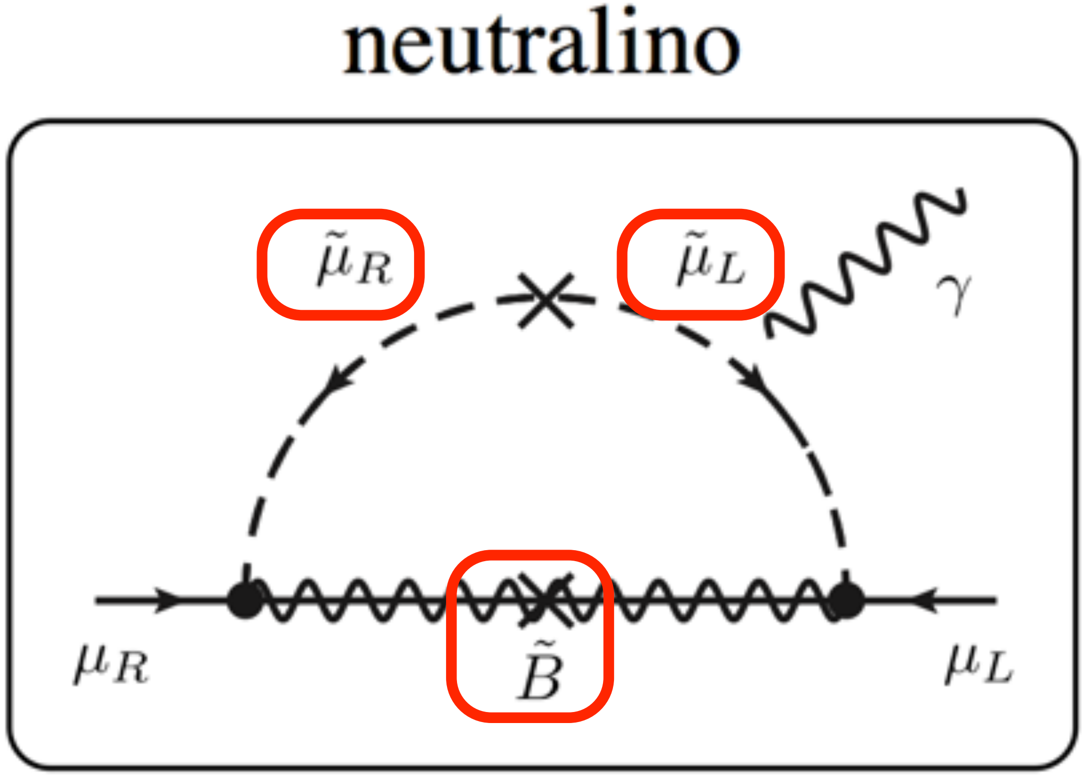


Table 2: Observables necessary for the reconstruction of $a_\mu^{(\mathrm{ILC})}$, and their uncertainties with $\sqrt{s} = 500$ GeV and $\mathcal{L} \sim 500$–$1000\,\mathrm{fb}^{-1}$. Processes relevant to determine each observable are also shown. The second and third rows are the information to determine $m^2_{\tilde{\mu}LR}$. For the determination of $m_{\tilde{\chi}^0_1}$, analyses of the productions of selectrons and smuons are combined. The uncertainties in $\tilde{g}^{(\mathrm{eff})}_{1,L}$ are those from the experiment and theory, respectively.

X	δX	$\delta_X a_\mu^{(\mathrm{ILC})}$	Process	
$m^2_{\tilde{\mu}LR}$	12 %	13 %	$e^+e^- \to \tilde{\tau}^+\tilde{\tau}^-$	(cross section, endpoint)
$(\sin 2\theta_{\tilde{\tau}})$	(9 %)	–	$e^+e^- \to \tilde{\tau}_1^+\tilde{\tau}_1^-$	(cross section)
$(m_{\tilde{\tau}2})$	(3 %)	–	$e^+e^- \to \tilde{\tau}_2^+\tilde{\tau}_2^-$	(endpoint)
$m_{\tilde{\mu}1}$, $m_{\tilde{\mu}2}$	200 MeV	0.3 %	$e^+e^- \to \tilde{\mu}^+\tilde{\mu}^-$	(endpoint)
$m_{\tilde{\chi}^0_1}$	100 MeV	< 0.1 %	$e^+e^- \to \tilde{\mu}^+\tilde{\mu}^-/\tilde{e}^+\tilde{e}^-$	(endpoint)
$\tilde{g}^{(\mathrm{eff})}_{1,L}$	a few+1 %	a few+1 %	$e^+e^- \to \tilde{e}_L^+\tilde{e}_R^-$	(cross section)
$\tilde{g}^{(\mathrm{eff})}_{1,R}$	1 %	0.9 %	$e^+e^- \to \tilde{e}_R^+\tilde{e}_R^-$	(cross section)

M.Endo, KH, S.Iwamoto, T.Kitahara, T.Moroi arXiv:1310.4496

muon g-2 vs ILC



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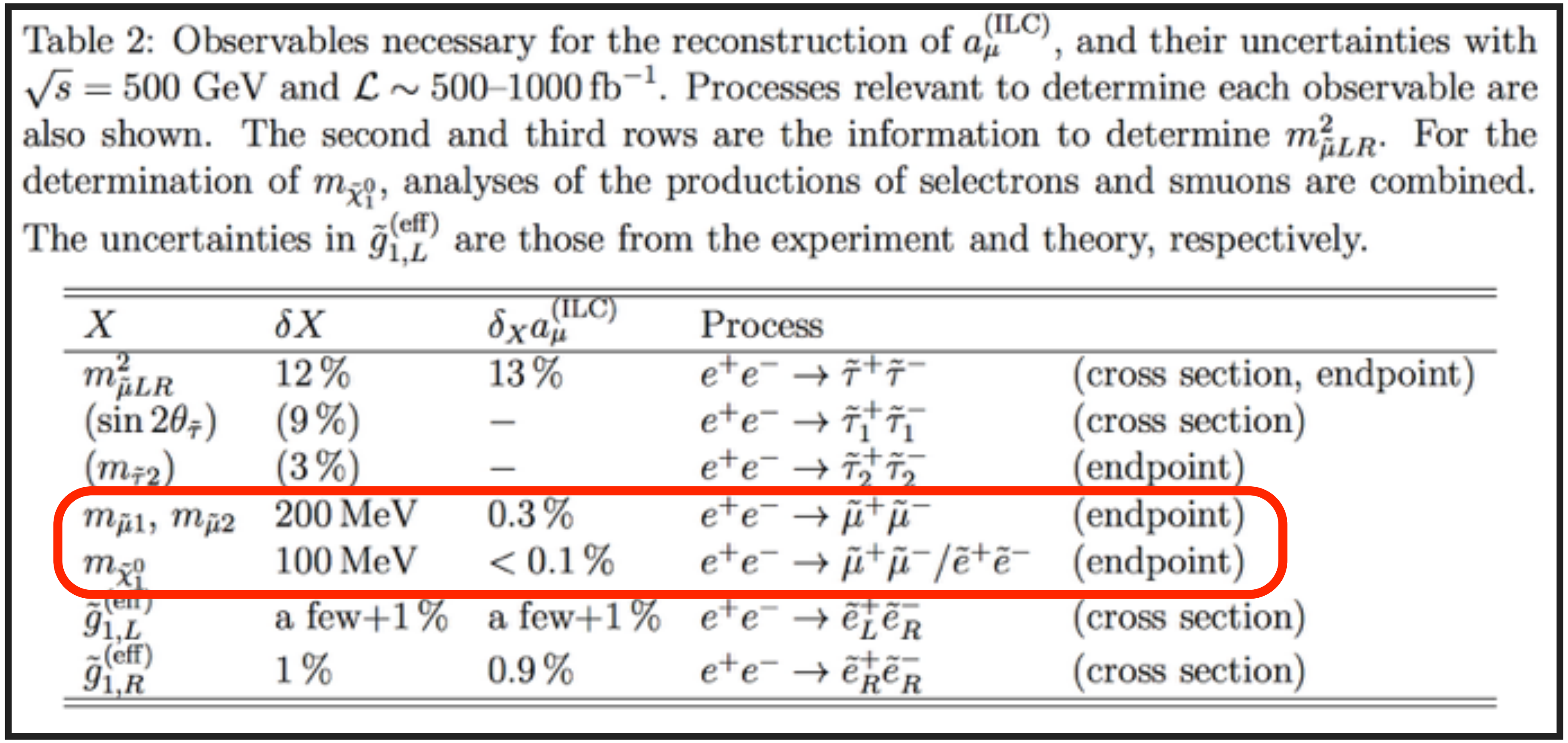

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$m_{\tilde\chi^0_1}$	100 MeV	< 0.1 %	$e^+e^- \to \tilde\mu^+\tilde\mu^- / \tilde e^+\tilde e^-$	(endpoint)
$\tilde g^{(\mathrm{eff})}_{1,L}$	a few+1 %	a few+1 %	$e^+e^- \to \tilde e_L^+\tilde e_R^-$	(cross section)
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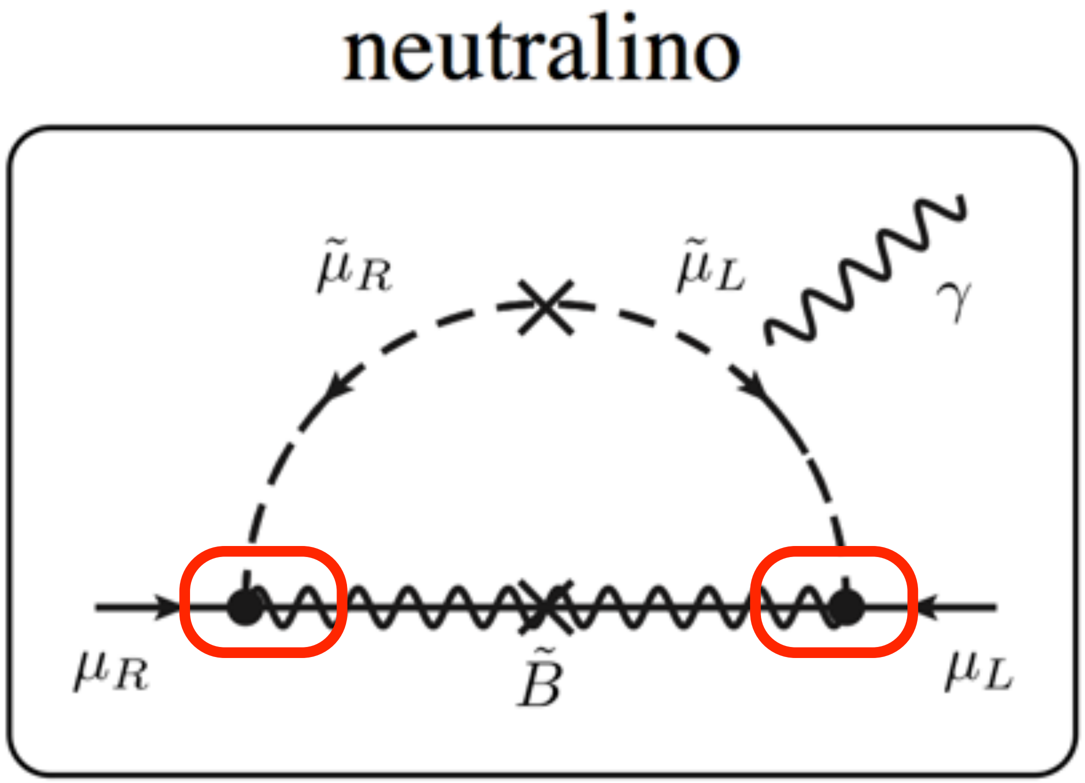
particle masses

M.Endo, KH, S.Iwamoto, T.Kitahara, T.Moroi arXiv:1310.4496

muon g-2 vs ILC



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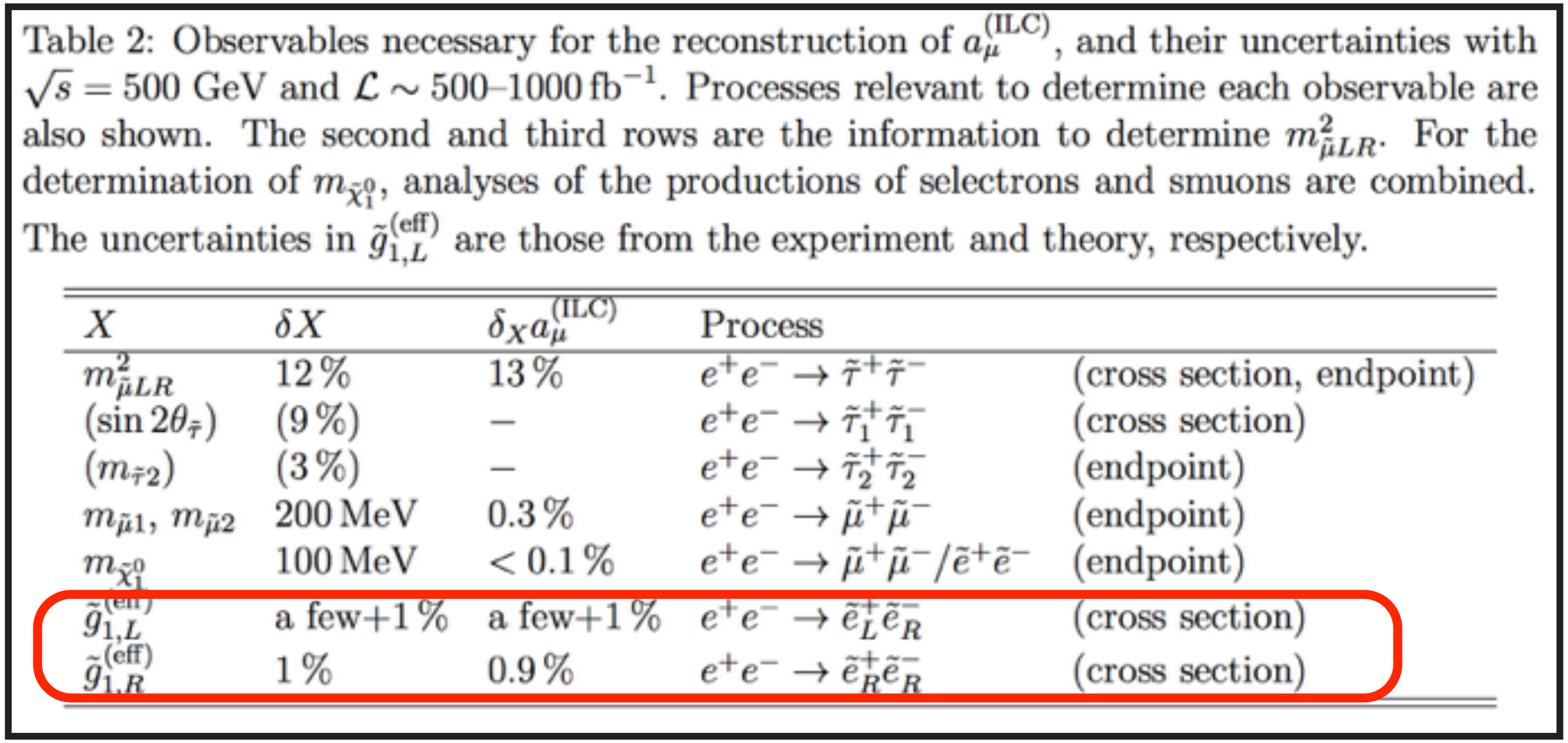

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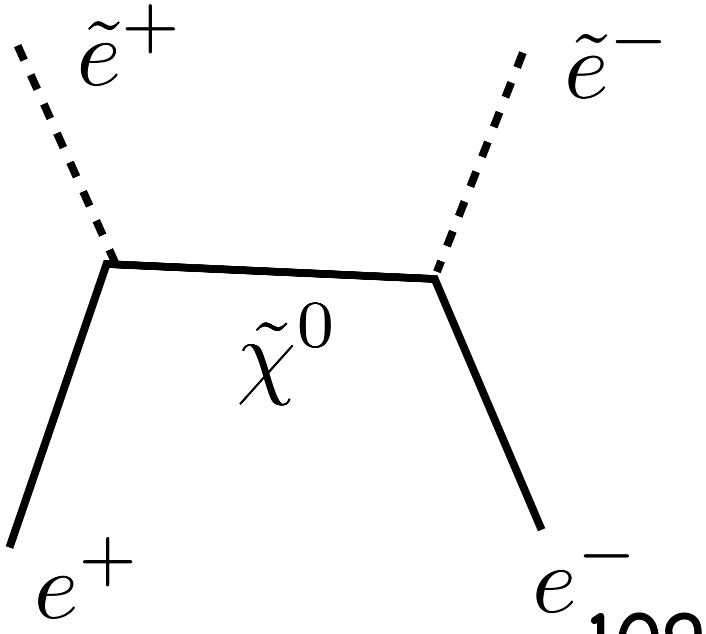
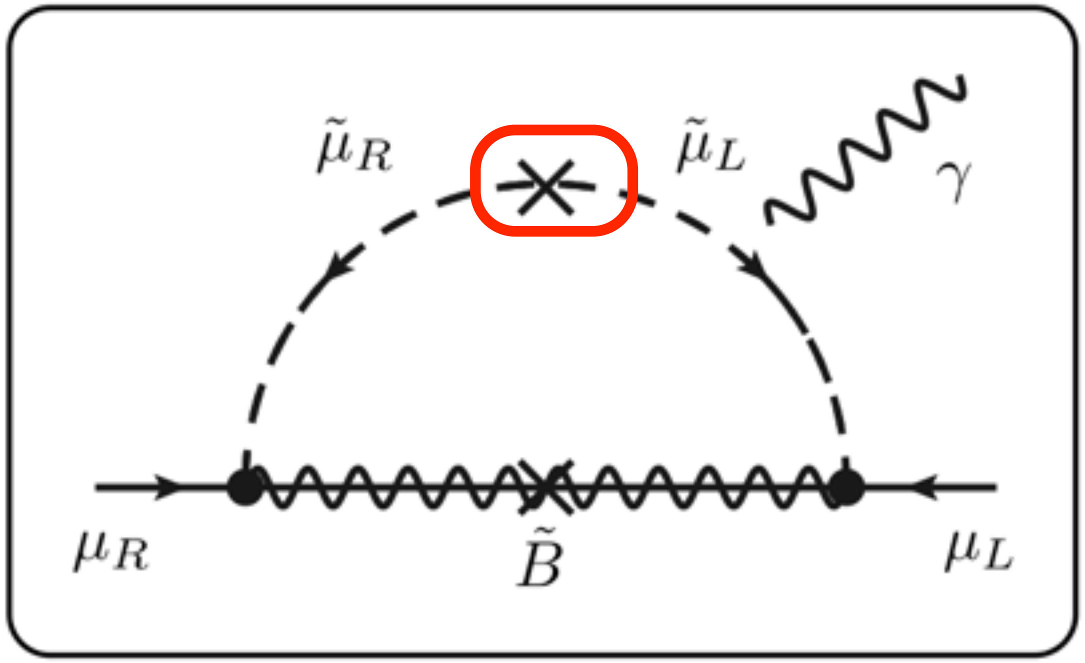
couplings



M.Endo, KH, S.Iwamoto, T.Kitahara, T.Moroi arXiv:1310.4496

muon g-2 vs ILC

neutralino



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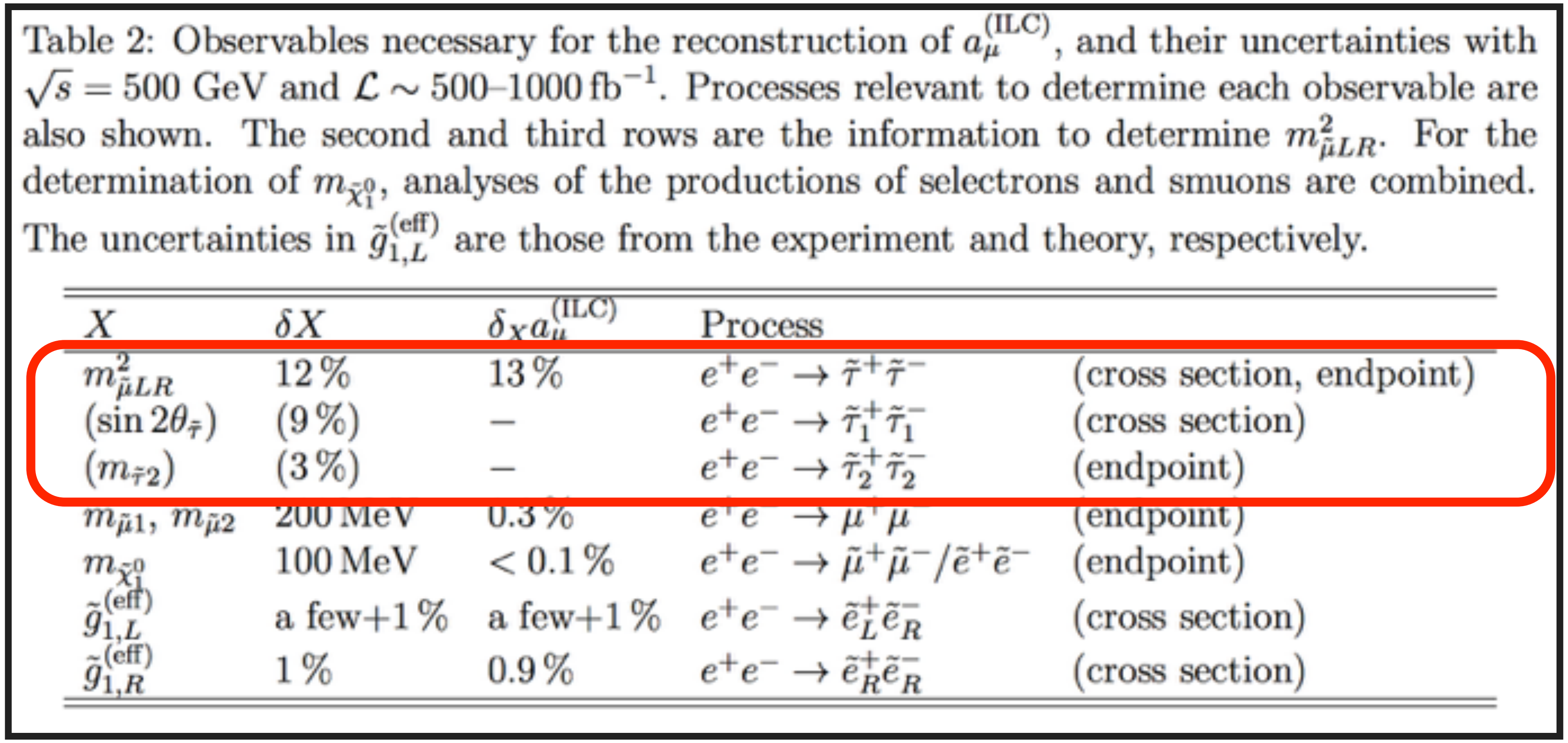

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$\tilde{g}^{(\mathrm{eff})}_{1,R}$	1 %	0.9 %	$e^+e^- \to \tilde{e}_R^+\tilde{e}_R^-$	(cross section)

mixing can also be reconstructed

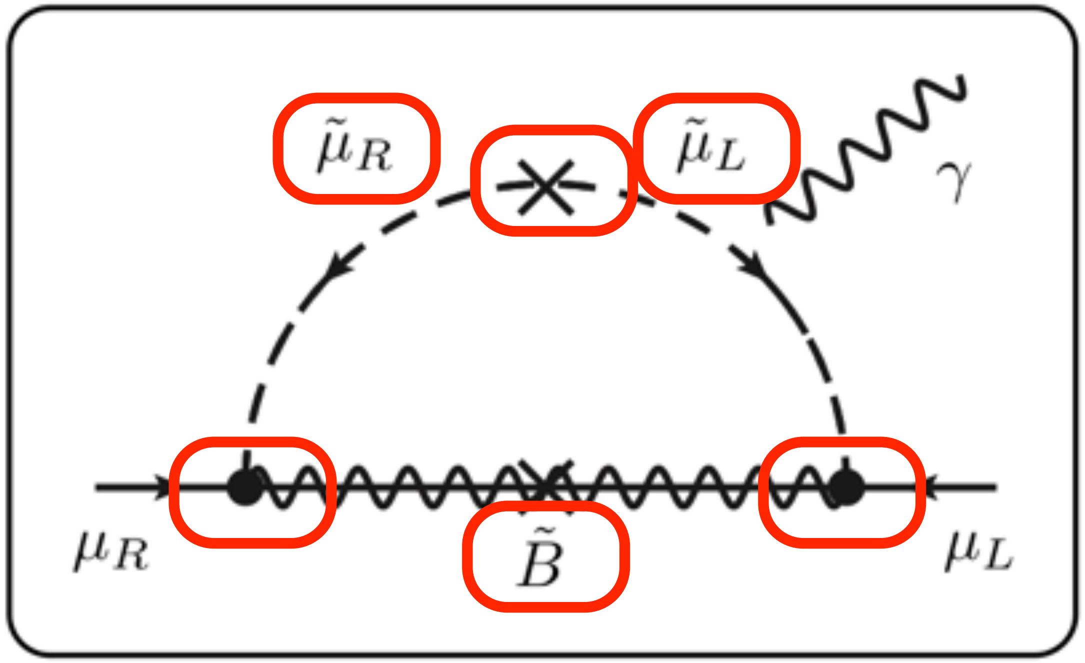
$$m^2_{\tilde{\mu}LR} = \frac{m_\mu}{m_\tau} m^2_{\tilde{\tau}LR}.$$

$$m^2_{\tilde{\ell}LR} = \frac{1}{2}(m^2_{\tilde{\ell}1} - m^2_{\tilde{\ell}2}) \sin 2\theta_{\tilde{\ell}}.$$

M.Endo, KH, S.Iwamoto, T.Kitahara, T.Moroi arXiv:1310.4496

muon g-2 vs ILC

neutralino



Can we reconstruct the contribution of this loop-diagram by using ILC measurements?

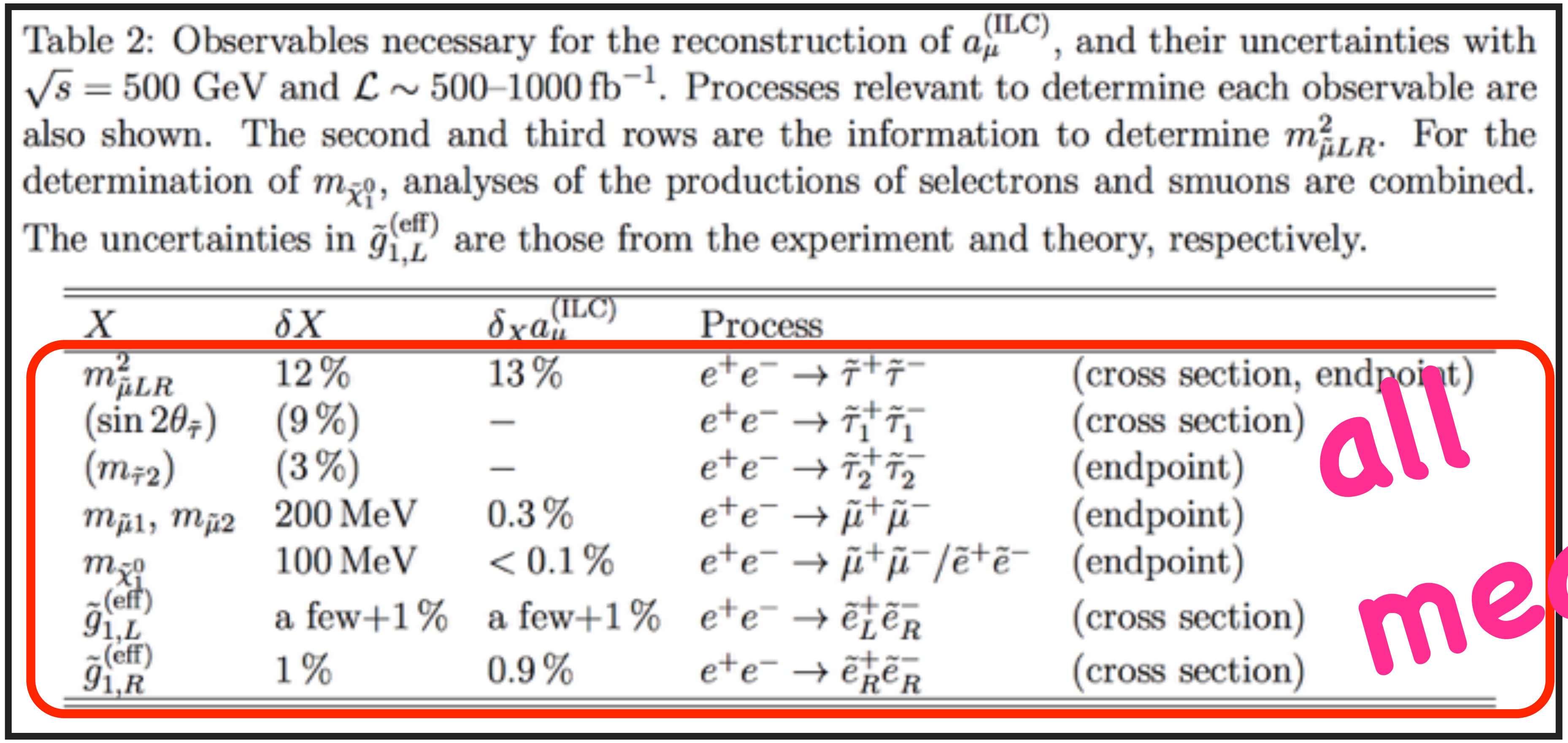


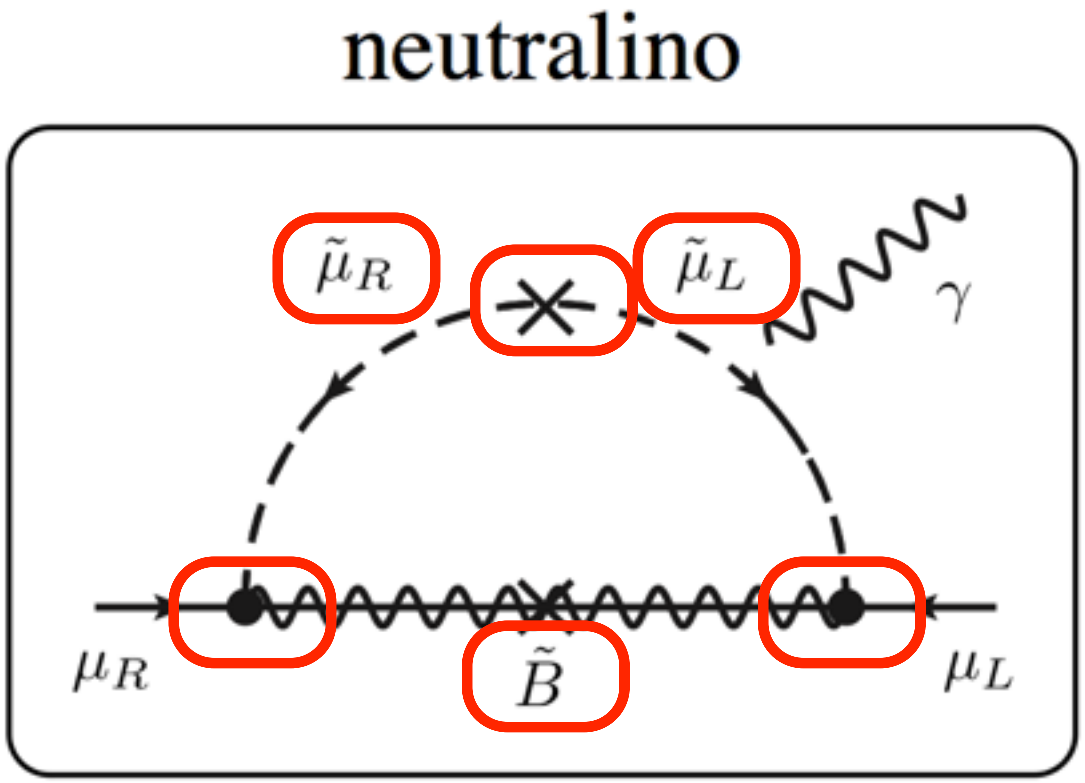
Table 2: Observables necessary for the reconstruction of $a_\mu^{(\mathrm{ILC})}$, and their uncertainties with $\sqrt{s} = 500$ GeV and $\mathcal{L} \sim 500$–$1000\,\mathrm{fb}^{-1}$. Processes relevant to determine each observable are also shown. The second and third rows are the information to determine $m^2_{\tilde{\mu}LR}$. For the determination of $m_{\tilde{\chi}^0_1}$, analyses of the productions of selectrons and smuons are combined. The uncertainties in $\tilde{g}^{(\mathrm{eff})}_{1,L}$ are those from the experiment and theory, respectively.

X	δX	$\delta_X a_\mu^{(\mathrm{ILC})}$	Process	
$m^2_{\tilde{\mu}LR}$	12 %	13 %	$e^+e^- \to \tilde{\tau}^+\tilde{\tau}^-$	(cross section, endpoint)
($\sin 2\theta_{\tilde{\tau}}$)	(9 %)	–	$e^+e^- \to \tilde{\tau}_1^+\tilde{\tau}_1^-$	(cross section)
($m_{\tilde{\tau}2}$)	(3 %)	–	$e^+e^- \to \tilde{\tau}_2^+\tilde{\tau}_2^-$	(endpoint)
$m_{\tilde{\mu}1}$, $m_{\tilde{\mu}2}$	200 MeV	0.3 %	$e^+e^- \to \tilde{\mu}^+\tilde{\mu}^-$	(endpoint)
$m_{\tilde{\chi}^0_1}$	100 MeV	< 0.1 %	$e^+e^- \to \tilde{\mu}^+\tilde{\mu}^- / \tilde{e}^+\tilde{e}^-$	(endpoint)
$\tilde{g}^{(\mathrm{eff})}_{1,L}$	a few+1 %	a few+1 %	$e^+e^- \to \tilde{e}_L^+\tilde{e}_R^-$	(cross section)
$\tilde{g}^{(\mathrm{eff})}_{1,R}$	1 %	0.9 %	$e^+e^- \to \tilde{e}_R^+\tilde{e}_R^-$	(cross section)

all measurable!

M.Endo, KH, S.Iwamoto, T.Kitahara, T.Moroi arXiv:1310.4496

muon g-2 vs ILC



Can we reconstruct the contribution of this loop-diagram by using ILC measurements?

at this model point,

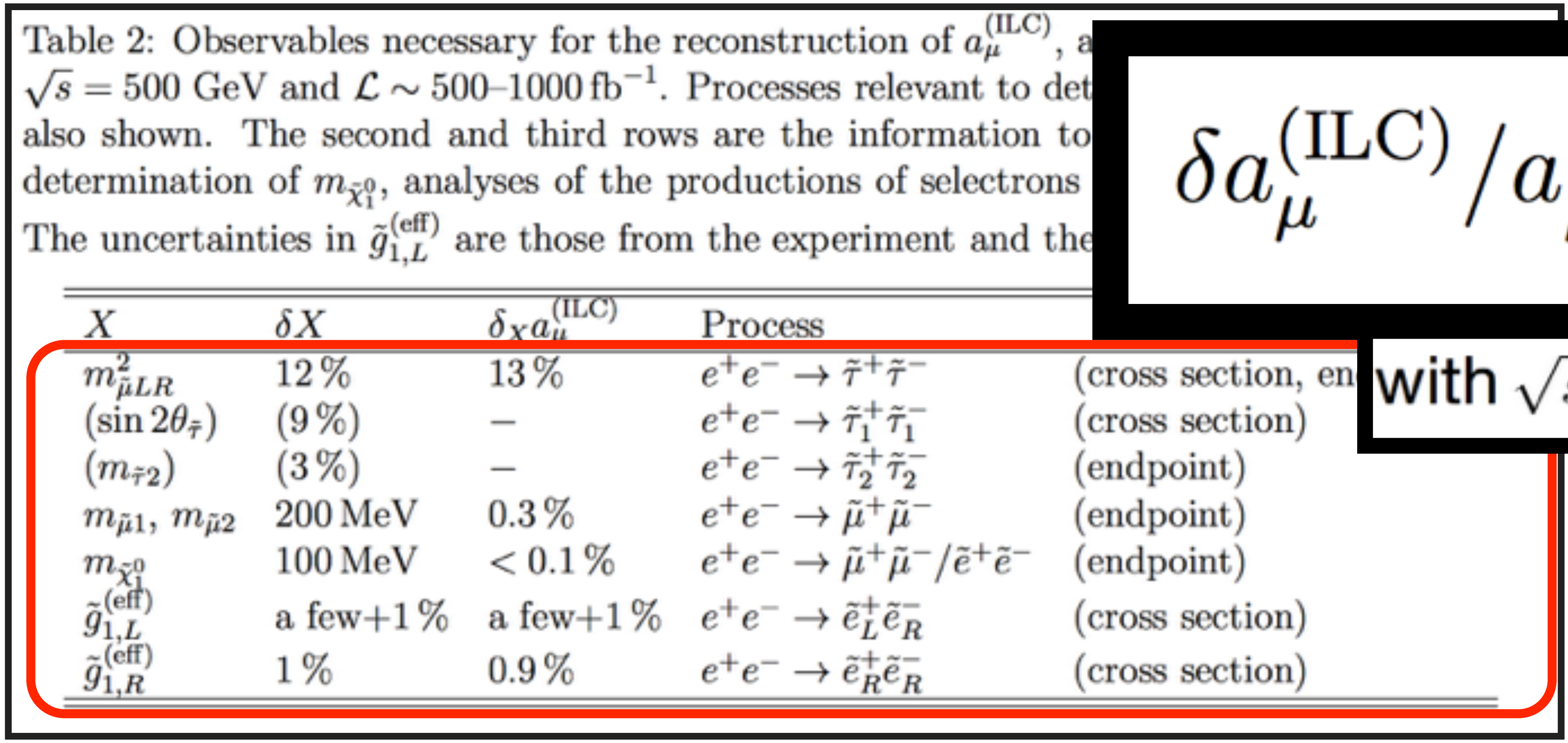


Table 2: Observables necessary for the reconstruction of $a_\mu^{(\mathrm{ILC})}$, a
$\sqrt{s} = 500$ GeV and $\mathcal{L} \sim 500$–$1000\,\mathrm{fb}^{-1}$. Processes relevant to det
also shown. The second and third rows are the information to
determination of $m_{\tilde\chi_1^0}$, analyses of the productions of selectrons
The uncertainties in $\tilde g_{1,L}^{(\mathrm{eff})}$ are those from the experiment and the

X	δX	$\delta_X a_\mu^{(\mathrm{ILC})}$	Process	
$m^2_{\tilde\mu LR}$	12 %	13 %	$e^+e^- \to \tilde\tau^+\tilde\tau^-$	(cross section, en
$(\sin 2\theta_{\tilde\tau})$	(9 %)	–	$e^+e^- \to \tilde\tau_1^+\tilde\tau_1^-$	(cross section)
$(m_{\tilde\tau 2})$	(3 %)	–	$e^+e^- \to \tilde\tau_2^+\tilde\tau_2^-$	(endpoint)
$m_{\tilde\mu 1}$, $m_{\tilde\mu 2}$	200 MeV	0.3 %	$e^+e^- \to \tilde\mu^+\tilde\mu^-$	(endpoint)
$m_{\tilde\chi_1^0}$	100 MeV	<0.1 %	$e^+e^- \to \tilde\mu^+\tilde\mu^-/\tilde e^+\tilde e^-$	(endpoint)
$\tilde g_{1,L}^{(\mathrm{eff})}$	a few+1 %	a few+1 %	$e^+e^- \to \tilde e_L^+\tilde e_R^-$	(cross section)
$\tilde g_{1,R}^{(\mathrm{eff})}$	1 %	0.9 %	$e^+e^- \to \tilde e_R^+\tilde e_R^-$	(cross section)

$$\delta a_\mu^{(\mathrm{ILC})} / a_\mu^{(\mathrm{ILC})} = 13\,\%,$$

with $\sqrt{s} = 500\,\mathrm{GeV},\ \mathcal{L} \sim 500\mathrm{fb}^{-1}$

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125 GeV Higgs and SUSY: SUMMARY

- **Higgs mass 125 GeV has a significant impact on SUSY.**
- at least a "little fine-tuning" seems unavoidable.
- It may imply SUSY particles are (much) heavier than TeV scale…… (but it may still be probed by, .e.g, **Flavor Physics** !)
- Many other possibilities which I didn't discuss:

  NMSSM and other modifications, RpV, light stop, compressed SUSY, etc etc…

- -

- **muon g-2** $a_\mu^{\rm EXP} - a_\mu^{\rm SM} = (26.1 \pm 8.0) \cdot 10^{-10}$ **> 3σ deviation !** may be a BSM signal.

In SUSY, it can be explained if smuon and chargino/neutralino are O(100 GeV).

—> may be tested at **13-14 TeV LHC !** **(and ILC !)**