

Analytical description of the beta spectrum shape

Leendert Hayen

Weak Interactions Group

Instituut voor Kern- en Stralingsfyica
KU Leuven

LPC-IKS Meeting, December 15 2015

- 1 Introduction
- 2 Beta spectrum shape
- 3 State of the code
- 4 Beyond Standard Model
- 5 Conclusion & Outlook

Introduction

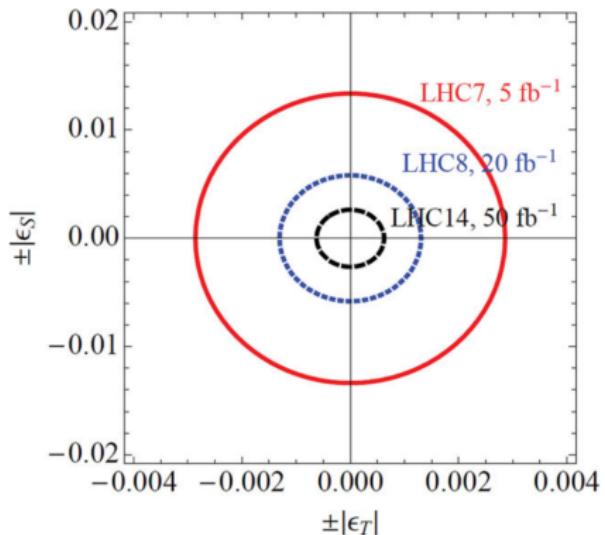
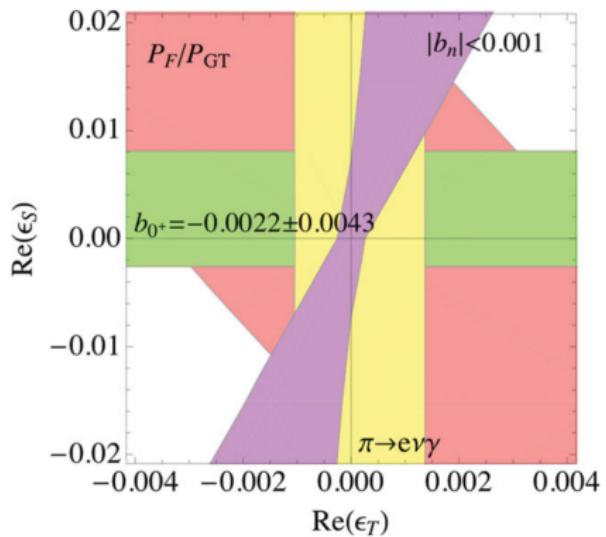
General Hamiltonian

$$\mathcal{H} = \sum_{i=V,A,S,P,T} \langle p | \mathcal{O}_i | n \rangle \langle e | \mathcal{O}_i [C_i + C'_i \gamma_5] | \nu \rangle + h.c.$$

Why only V-A in Standard Model (or even Nature)? No one knows!

⇒ Others exist, or fundamental reason why only V-A exists. Several Beyond SM theories predict other contributions

Current limits on Exotic Currents

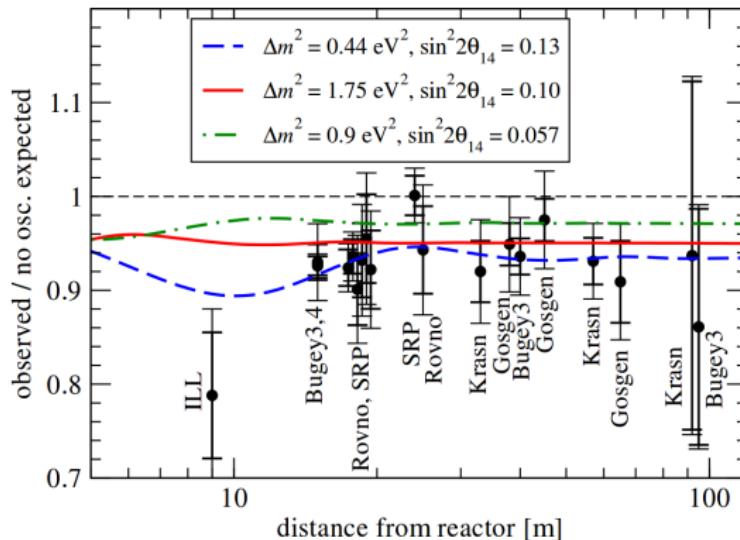


Low and High energy physics are **competitive**

O. Naviliat-Cuncic and M. Gonzalez-Alonso, Ann. Phys. 525 (2013) 600

Neutrino asymmetry

Current deficiency in neutrino countrate at 94% (3σ)



Poor knowledge of forbidden transitions and its weak magnetism requires 4% error on 6% deficiency

A. Hayes et al., PRL 112, 202501 (2014) & J. Kopp et al., JHEP 05 (2013) 050

1 Introduction

2 Beta spectrum shape

3 State of the code

4 Beyond Standard Model

5 Conclusion & Outlook

Requirements

Exploring the Standard Model and Beyond via the β spectrum shape:

$$\frac{dN}{dE_e} \propto 1 + b_{\text{Fierz}} \gamma \frac{m_e}{E_e} + b_{\text{WM}} E_e$$

b_{Fierz} : Proportional to scalar (Fermi) and tensor (Gamow-Teller) couplings

b_{WM} : QCD renormalisation effect, poorly known for $A > 40$, forbidden decays (neutrino reactor anomaly)

Requirements

Exploring the Standard Model and Beyond via the β spectrum shape:

$$\frac{dN}{dE_e} \propto 1 + b_{\text{Fierz}} \gamma \frac{m_e}{E_e} + b_{\text{WM}} E_e$$

b_{Fierz} : Proportional to scalar (Fermi) and tensor (Gamow-Teller) couplings

b_{WM} : QCD renormalisation effect, poorly known for $A > 40$, forbidden decays (neutrino reactor anomaly)

This requires knowledge of the theoretical spectrum shape to $\leq 10^{-3}$ level!

General description

General matrix element: Combination of \mathcal{H}_β and Coulomb effects.

$$M_{fi} = -2\pi i \delta(E_f - E_i) \langle f | T \left[\exp \left(-i \int_0^\infty dt \mathcal{H}^Z(t) \right) \right] \\ \times \mathcal{H}_\beta(0) T \left[\exp \left(-i \int_{-\infty}^0 dt \mathcal{H}^{Z'}(t) \right) \right] |i\rangle$$

Immediately two main parts:

- ① Electromagnetic corrections
- ② Nuclear & recoil corrections

Specifically...

Expanding slightly:

- ① Electromagnetic corrections
 - Fermi function
 - Radiative corrections
 - Atomic effects
- ② Nuclear & recoil corrections
 - Finite nuclear size & mass
 - Nuclear structure

And then there's Beyond Standard Model

- Fierz searches
- CKM Unitarity
- ...

Specifically...

Expanding slightly:

① Electromagnetic corrections

- Fermi function ✓
- Radiative corrections ✓
- **Atomic effects**

② Nuclear & recoil corrections

- Finite nuclear size & mass ✓
- Nuclear structure ✓

And then there's Beyond Standard Model

- Fierz searches
- CKM Unitarity
- ...

Quick overview

Equal parts physics and archaeology. Different formalisms:

- ① Holstein *et al.*
- ② Behrens & Bühring *et al.*
- ③ Wilkinson
- ④ Effective Field Theories (Bhattacharya & Cirigliano)

Quick overview

Equal parts physics and archaeology. Different formalisms:

- ① Holstein *et al.*
- ② Behrens & Bühring *et al.*
- ③ Wilkinson
- ④ Effective Field Theories (Bhattacharya & Cirigliano)

Our goal:

Fully analytical description → Wilkinson + Holstein + other contributions

Crosscheck between all formalisms

Quick overview

Result:

$$\begin{aligned} N(W)dW &= \frac{G_V^2 \cos^2 \theta_C}{2\pi^3} F_0(Z, W) L_0(Z, W) R_N(W, W_0, M) U(Z, W) \\ &\quad \times C(Z, W) Q(Z, W, M) R(W, W_0) Sc(Z, W) X(Z, W) \\ &\quad \times pW(W_0 - W)^2 h'_1(W) dW \\ &\equiv \frac{G_V^2 \cos^2 \theta_C}{2\pi^3} K(Z, W, W_0, M) pW(W_0 - W)^2 h'_1(W) dW \end{aligned}$$

Where K considers electromagnetic and kinematic effects, and h'_1 contains nuclear structure info.

Fermi function

Influence on the spectrum because of Coulomb field of daughter nucleus.
Generally

$$F(Z, W) = \lim_{r \rightarrow 0} \frac{f_1^2(r) + g_{-1}^2(r)}{2p^2} .$$

with

$$\Psi_\kappa(\hat{r}) = \begin{pmatrix} g_\kappa(r) \sum_\mu \chi_\kappa^\mu \\ i f_\kappa(r) \sum_\mu \chi_{-\kappa}^\mu \end{pmatrix}$$

solution of the radial Dirac equation with some Coulomb field.

Fermi function

Influence on the spectrum because of Coulomb field of daughter nucleus.
Generally

$$F(Z, W) = \lim_{r \rightarrow 0} \frac{f_1^2(r) + g_{-1}^2(r)}{2p^2} .$$

with

$$\Psi_\kappa(\hat{r}) = \begin{pmatrix} g_\kappa(r) \sum_\mu \chi_\kappa^\mu \\ i f_\kappa(r) \sum_\mu \chi_{-\kappa}^\mu \end{pmatrix}$$

solution of the radial Dirac equation with some Coulomb field.

Problem: Only solvable for point charge \rightarrow split into

$$F(Z, W) = F_0 L_0$$

with L_0 calculated numerically (tabulated by Wilkinson).

D. H. Wilkinson, Nucl. Instr. & Meth. A 335, 203 (1990)

Radiative corrections

Seminal work by Sirlin, Zucchini & Marciano. Split into *inner* (nucleus-independent) and *outer* (nucleus-dependent). Focus on *outer*:

$$R(W, W_0) = 1 + \delta_1 + \delta_2 + \delta_3$$

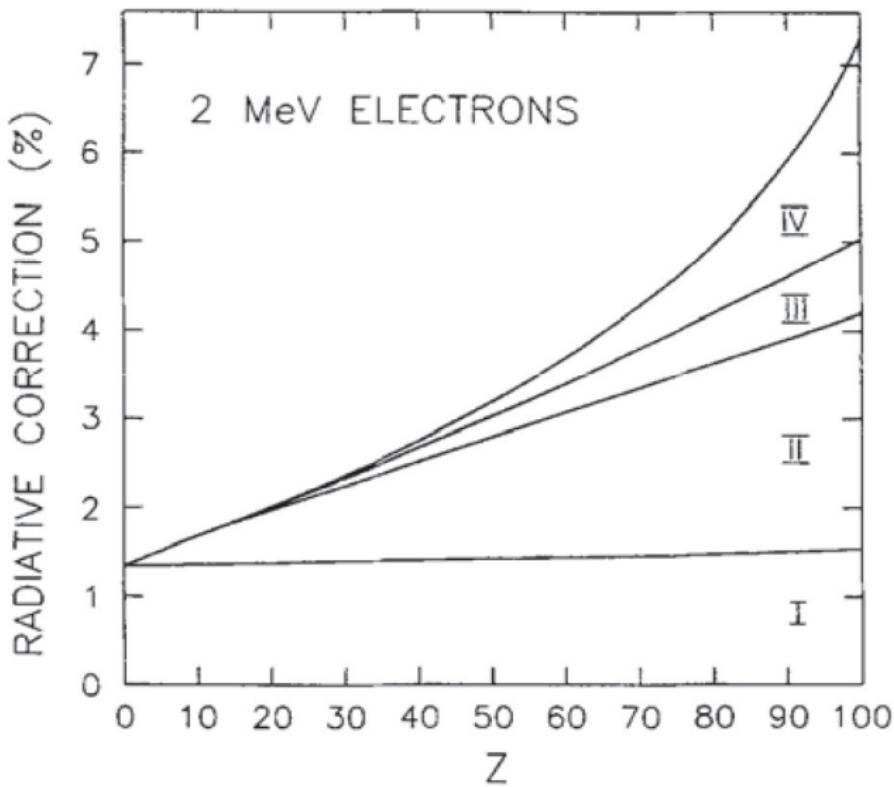
Higher order corrections estimate

$$\delta_{higher} \approx \sum_{n=3}^{n=\infty} \delta_{Z^n \alpha^{n+1}} = \delta_{Z^3 \alpha^4} / (1 - Z\alpha) .$$

D. H. Wilkinson, Nucl. Instr. & Methods A **365**, 497 (1995) & **401**, 275 (1997)

I. S. Towner and J. C. Hardy, Phys. Rev. C **77**, 012501 (2014)

Radiative Corrections



Atomic Screening Corrections

Final state interactions with atomic electrons.

Atomic Screening Corrections

Final state interactions with atomic electrons.

Change free lepton spinors by Dirac spinors in Coulomb field in matrix element \mathcal{H}_β

$$\int d^3r \bar{\Psi}_e(\mathbf{r}, \mathbf{p}) \gamma_\mu (1 + \gamma_5) v(I) \int \frac{d^3k}{2\pi^3} e^{i\mathbf{r}\cdot\mathbf{k}} \langle f | V^\mu + A^\mu | i \rangle$$

Ψ_e not analytically solvable for anything but pure Coulomb $\frac{\alpha Z}{r}$

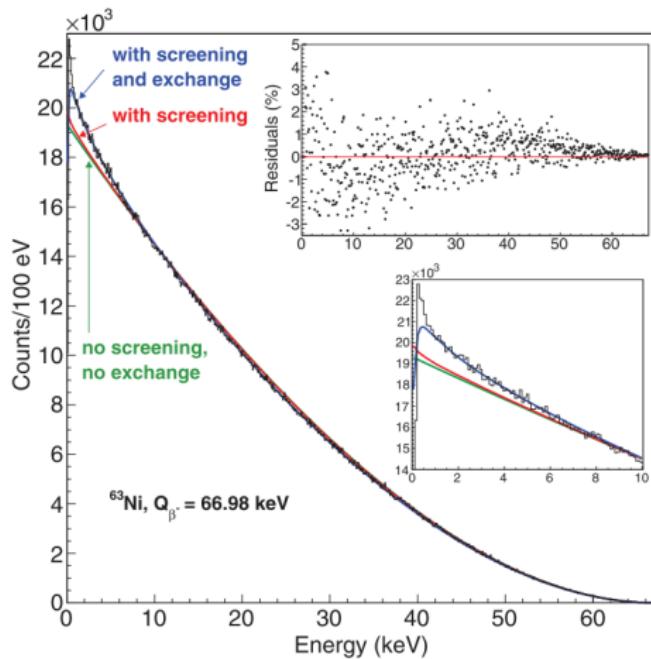
Atomic Screening Corrections

Look at modification of Fermi function, only **local** change.

Atomic Screening Corrections

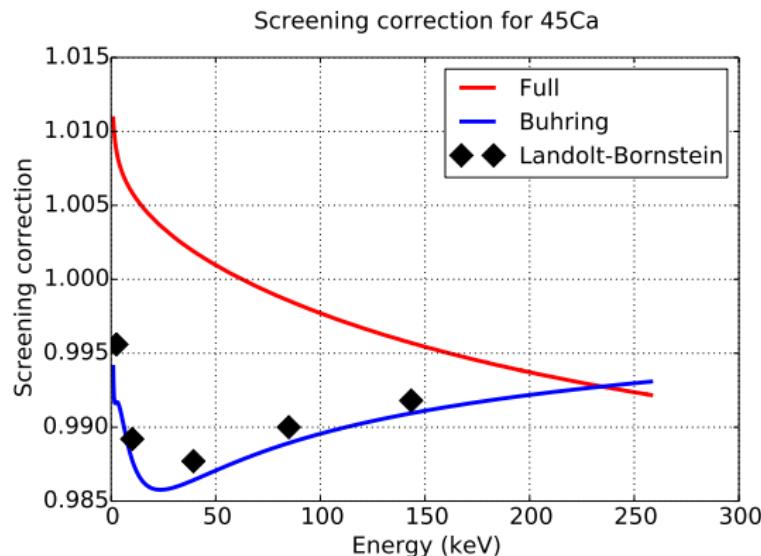
Look at modification of Fermi function, only **local** change.

Analytically by Behrens & Bühring, however experimental disagreement?



Atomic Screening Corrections

Influence of atomic screening on β spectrum shape via Fermi function



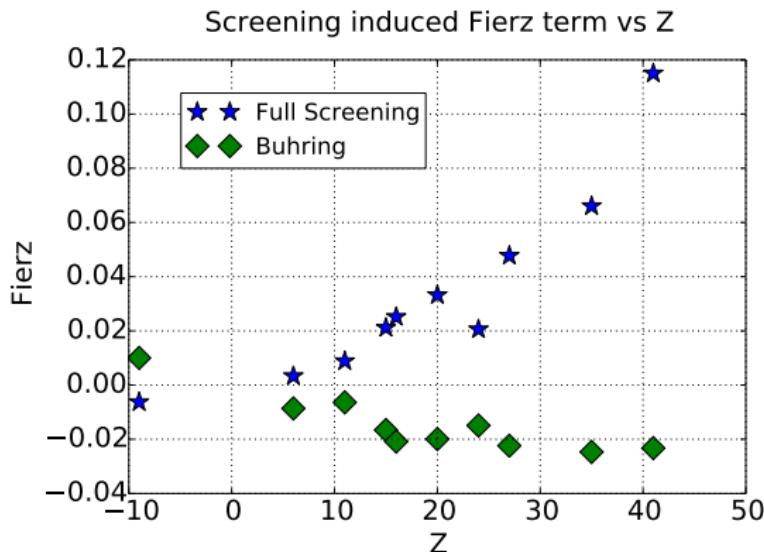
'Full': X. Mousseot and C. Bisch, PRA 90, 012501 (2014)

'Buhring': W. Bühring, Nucl. Phys. A 430 (1984) 1-20

'Landolt-Bornstein': Behrens and Jänecke, Landolt-Bornstein Tables, Springer (1969)

Atomic screening corrections

Look at the 'false' Fierz signal caused by neglecting atomic screening effects



Its understanding is **crucial**, but still theoretical issues to overcome!

Atomic exchange and shake-off

- **Exchange:** Probability of decaying into bound state with emission of bound e^-
 - Again, no full analytical solution, but reasonably well-described
 - Important at low E (≤ 50 keV)

- **Shakeoff:** Emission of bound e^- because of non-orthogonality of wavefunctions
 - Changes screening and exchange corrections
 - Small correction on already small corrections → neglected

Additional finite nuclear size and mass corrections

Descending order of importance:

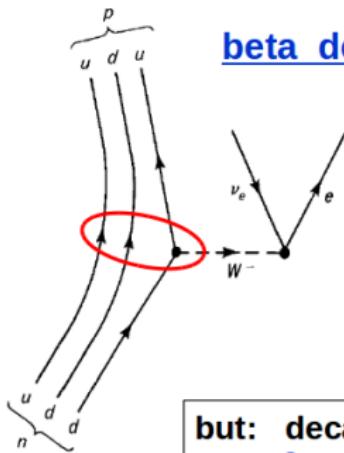
- ① $C(Z, W)$: Convolution of nuclear and leptonic wavefunctions in nucleus: **up to 1%**
- ② $U(Z, W)$: Diffuse nuclear surface replacing spherical charge distribution: **up to 0.1%**
- ③ $R_N(W, W_0, M)$: Kinematical effect because of nuclear recoil: $\leq 0.1\%$
- ④ $Q(Z, W, M)$: Change in Coulomb field of nucleus because of recoil: **negligible**

Nuclear and recoil corrections

Concerns **nuclear** part of β decay matrix element:

- Nucleon is not single quark, QCD influences → form factors
- Recoil with momentum transfer $\mathbf{q} = -(\mathbf{p}_e + \mathbf{p}_\nu)$ → allows expansion in terms of q/M

Weak Magnetism Basics



beta decay: $H = G_F \langle \psi_f | V_\mu(0) + A_\mu(0) | \psi_i \rangle l^\mu$

with $l^\mu = \bar{e}(p)\gamma^\mu(1 + \gamma_5)v(k)$

free quark:

$$V_\mu(q^2) = \bar{u}[g_V(q^2)\gamma_\mu]d ,$$

$$A_\mu(q^2) = \bar{u}[g_A(q^2)\gamma_\mu\gamma_5]d$$

but: decaying quark is not free but bound in a nucleon
→ extra terms induced by strong interaction

neutron decay:

weak magnetism

$$V_\mu(q^2) = \bar{p}[g_V(q^2)\gamma_\mu + \boxed{g_M(q^2)\sigma_{\mu\nu}\frac{q_\nu}{2M} + ig_S(q^2)\frac{q_\mu}{m_e}}]n$$

$$A_\mu(q^2) = \bar{p}[g_A(q^2)\gamma_\mu\gamma_5 + g_T(q^2)\sigma_{\mu\nu}\gamma_5\frac{q_\nu}{2M} + ig_P(q^2)\frac{q_\mu}{m_e}\gamma_5]n$$

Full Nuclear corrections

For a nuclear GT β decay

$$h_1(E) = \tilde{h}_1(E) + \frac{2}{9} c_1 c_2 \left[11m_e^2 + 20EE_0 - 20E^2 - \frac{2m_e^2 E_0}{E} \right] ,$$

with

$$\begin{aligned} \tilde{h}_1(E) = & c_1^2 - \frac{2}{3} \frac{E_0}{M} c_1 (c_1 + d \pm b) + \frac{2}{3} \frac{E}{M} c_1 (5c_1 \pm 2b) \\ & - \frac{m_e^2}{3ME} \left[2c_1^2 + c_1 (d \pm 2b) - c_1 h \frac{E_0 - E}{2M} \right] \end{aligned}$$

Full Nuclear corrections

For a nuclear GT β decay

$$h_1(E) = \tilde{h}_1(E) + \frac{2}{9} c_1 c_2 \left[11m_e^2 + 20EE_0 - 20E^2 - \frac{2m_e^2 E_0}{E} \right] ,$$

with

$$\begin{aligned} \tilde{h}_1(E) = & c_1^2 - \frac{2}{3} \frac{E_0}{M} c_1 (c_1 + d \pm b) + \frac{2}{3} \frac{E}{M} c_1 (5c_1 \pm 2b) \\ & - \frac{m_e^2}{3ME} \left[2c_1^2 + c_1 (d \pm 2b) - c_1 h \frac{E_0 - E}{2M} \right] \end{aligned}$$

Lots of overlap in electromagnetic corrections for Holstein and Wilkinson formalism, however

$$\delta h_1(E) \approx \frac{\sqrt{10}}{6} \frac{\alpha Z}{MR} c_1 (2b + d'' \pm d' \pm c_1)$$

Weak Magnetism: β decay

nuclear β decay:

form factor	formula Imp. App.
Vector type	
a	$a \cong g_V M_F$
e	$e \cong g_V (M_F \pm A g_S)$
b	$b \cong A(g_M M_{GT} + g_V M_L)$
f	$f \cong g_V \sqrt{\frac{2}{3}} M \frac{\Delta}{\hbar c^2} M_Q$
g	$g \cong -\frac{4}{3} M^2 g_V \frac{M_Q}{\hbar c^2}$
Axial vector type	
c	$c \cong g_A M_{GT}$
d	$d \cong A(g_A M_{\sigma L} \pm g_{II} M_{GT})$
h	$h \cong \frac{-2}{\sqrt{10}} M^2 g_A \frac{M_{1y}}{\hbar c^2} - A^2 g_P M_{GT}$
j_2	$j_2 \cong \frac{-2}{3} M^2 g_A M_{2y}$
j_3	$j_3 \cong \frac{-2}{3} M^2 g_A M_{3y}$

Matrix element	Operator form
M_F	$\langle \beta \Sigma \tau_i^\pm \alpha \rangle$
M_{GT}	$\langle \beta \Sigma \tau_i^\pm \vec{\sigma}_i \alpha \rangle$
M_L	$\langle \beta \Sigma \tau_i^\pm \vec{l}_i \alpha \rangle$
$M_{\sigma r^2}$	$\langle \beta \Sigma \tau_i^\pm \vec{\sigma}_i r_i^2 \alpha \rangle$
$M_{\sigma L}$	$\langle \beta \Sigma \tau_i^\pm i \vec{\sigma}_i \times \vec{l}_i \alpha \rangle$
M_Q	$(\frac{4\pi}{5})^{\frac{1}{2}} \langle \beta \Sigma \tau_i^\pm r_i^2 Y_2(\hat{r}_i) \alpha \rangle$
M_{ky}	$(\frac{16\pi}{5})^{\frac{1}{2}} \langle \beta \Sigma \tau_i^\pm \sigma_i^2 C_{12k}^{nn'k} \sigma_{in} Y_2^{n'}(\hat{r}_i) \alpha \rangle$

B. R. Holstein, Rev. Mod. Phys. 46 (1974) 789
 F.P. Calaprice et al., Phys. Rev. C 15 (1977) 2178

Weak Magnetism: β decay

for a pure GT transition,
and neglecting terms $\propto 1/M^2$ and $\propto m_e^2/E$:

$$H_0(E) = c^2 - \frac{2}{3} \frac{E_0}{M} c(c + d \pm b) + \frac{2}{3} \frac{E}{M} c(5c \pm 2b)$$

$$\rightarrow H_0(E) = f_1 + f_2 E$$

$$\rightarrow S(E) \equiv \frac{H_0(E)}{H_0(E=0)}$$

$$S(E) \approx 1 + \frac{2}{3M} \left(5 \pm 2 \frac{b}{c} \right) E_e$$

1 Introduction

2 Beta spectrum shape

3 State of the code

4 Beyond Standard Model

5 Conclusion & Outlook

State of the code

Effect	Comment	Formula
Phase space factor	✓	$pW(W_0 - W)^2$
Neutrino mass	negligible	
Recoiling nucleus	✓	R_N
Forbidden decays	✗	
Traditional Fermi function	✓	F_0
Coulomb screening	TBI	S_c
Atomic exchange	TBI	X
Finite size of the nucleus	✓	L_0
Radiative corrections	✓	R
Nuclear convolution	✓	C_0
Nucleonic decay in convolution	✗	C_I
Diffuse nuclear surface	✗	U
Distorted Coulomb potential due to recoil	✗	Q
Nuclear structure	✓	h_1
Coulomb corrections on induced terms	✗	δh_1

Implementation

C++ code allowing inclusion/exclusion of all effects

Previously only for GT decays, formula's are being completed to allow F (and mixed?) transitions as well.

Planned to be distributed together with publication

1 Introduction

2 Beta spectrum shape

3 State of the code

4 Beyond Standard Model

5 Conclusion & Outlook

Effective Lagrangian for β decay

$$\begin{aligned}\mathcal{L}_{\text{eff}} = & -\frac{G_F V_{ud}}{\sqrt{2}} [1 + \text{Re}(\epsilon_L + \epsilon_R)] \\ & \times \left\{ \bar{e} \gamma_\mu (1 - \gamma_5) v_e \cdot \bar{u} \gamma^\mu [1 - (1 - 2\epsilon_R) \gamma_5] d \right. \\ & + \epsilon_S \bar{e} (1 - \gamma_5) v_e \cdot \bar{u} d \\ & \left. + \epsilon_T \bar{e} \sigma_{\mu\nu} (1 - \gamma_5) v_e \cdot \bar{u} \sigma^{\mu\nu} (1 - \gamma_5) d \right\} + \text{h.c.}\end{aligned}$$

Neglecting pseudoscalar contributions + right-handed neutrino's

T. Bhattacharya et al., PRD 85, 054512 (2012)

V. Cirigliano et al., JHEP 1302, 046 (2013)

Current status

Currently crosschecking results of older formalisms with EFT: Recoil corrections defined (grouped) differently. Make sure all terms match up.

Current status

Currently crosschecking results of older formalisms with EFT: Recoil corrections defined (grouped) differently. Make sure all terms match up.

Example: Typically Fierz = 0 in SM, but Bhattacharya *et al.* introduce

$$b^{\text{SM}} = -\frac{m_e}{M_N} \frac{1 + 2\mu_V \lambda + \lambda^2}{1 + 2\lambda^2}$$

for neutron decay from recoil corrections. μ_V points to weak magnetism contribution

However correction goes like

$$\frac{m_e}{E_e} b^{\text{SM}}$$

→ same factor as in $h'_1(E)$!

1 Introduction

2 Beta spectrum shape

3 State of the code

4 Beyond Standard Model

5 Conclusion & Outlook

Conclusion & Outlook

Conclusion:

- Crosschecks mostly completed, errors mostly traced
- Code is functional, awaiting complete crosscheck and expansion
- Atomic corrections are important and not yet verified

Outlook:

- Finalise crosschecks with EFT
- Verify screening corrections → implications for RNA
- Verify analytical atomic exchange corrections
- Experimental measurements

Atomic Screening Correction

Formula used by X. Mugeot based upon exchange results by Pyper & Harston

$$S(E) = \frac{\int f_c^2(r) dr d\Omega}{\int (f_c^2(r) + g_c^2(r)) dr d\Omega} \quad (1)$$

Important: Spatial average \Leftrightarrow amplitude at origin

Not theoretically sound! Currently working on *ab initio* approach based on Behrens & Bühring approach

Bhattacharya formalism

The full correction goes like

$$\frac{dN}{dt} \propto 1 + c_0 + c_1 \frac{E_e}{M_N} + \frac{m_e}{E_e} \bar{b}$$

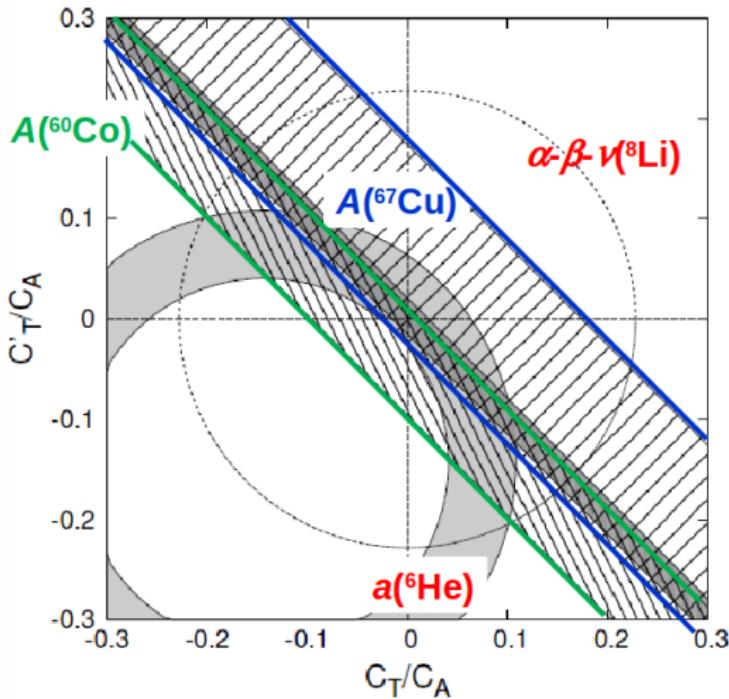
where

$$c_0 = -\frac{2\lambda(\lambda + \mu_V)}{1 + 3\lambda^2} \frac{E_0}{M_N}$$

$$c_1 = \frac{3 + 4\lambda\mu_V + 9\lambda^2}{1 + 3\lambda^2}$$

$$b = -\frac{m_e}{M_N} \frac{1 + 2\mu_V\lambda + \lambda^2}{1 + 3\lambda^2}$$

Tensor constraints



$a(^6\text{He})$

C. Johnston et al.,
PR 132 (1963) 1149

$A(^{60}\text{Co})$

F. Wauters, N.S. et al.,
PR C 82 (2010) 055502

$\alpha\beta\nu(^8\text{Li})$

G.Li, G.Savard et al.,
PRL 110 (2013) 082502

$A(^{67}\text{Cu})$

G. Soti, N.S. et al., (2013) submitted

black band: P_F/P_{GT}

A.S. Carnoy et al.,
PR C 43 (1991) 2825

Atomic Screening

