Synopsis

1 Context
   - V-A theory
   - Weak interaction universality
   - $a_{\beta \nu}$ kinematics
   - Shake-Off (SO) effect

2 Experiment: LPCTrap
   - Beam preparation
   - Trapping & Detection
   - Results

3 *New simulation
   - Trapped ion dynamic: CLOUDA
   - $\beta$ decay generator
   - Tracking: Bayeux

4 Conclusions & Perspectives
Overview: simulation package

Simulation: initial event

BOLTZMANN -
Initial state of trapped ions

CLOUDA -
Trapped ions dynamic

BETA DECAY GENERATOR -
Kinematic distribution of Q-value

RI tracking (BAYEUX)

In-trap decay event

Beta tracking (BAYEUX)

detection data

- Reconstructed ToF
- DSSSD profiles
- Scintillator E
- MCPPSD position
- ...

detection data
Overview: simulation package

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- ...
Boltzmann initial state

Simulation: initial event

Buffer gas injected in the trap
\[ \sim 10^{-5} \text{ mbar (He or H}_2\text{)} \]

\[ \Downarrow \]

Thermalization process kills all prior information

\[ \Downarrow \]

No need to simulate before trap

\[ \Downarrow \]

Any "reasonable" initial state will do

Maxwell-Boltz. Distribution

\[ T = 1000 \text{ K}, \bar{r}_{\text{init.}} = (0, 0, 0) \]
Overview: simulation package

Simulation: initial event

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Ph.D. Defense  Wednesday, December 16, 2015
*Trapped ion dynamic: CLOUDA

Simulation: initial event

Trapped ions are subject to:
- Collisions with buffer gas
- Electromagnetic trapping field
- Coulomb N-body repulsion ($O(N^2)$)

CLOUDA was thus developed:
- GPU based: massive parallelization
  Classic CPU: $4 \times 2$ cores, GTX TITAN Black: 2880 CUDA cores
- Steppers: 4th order Runge-Kutta, Leapfrog, Euler (tracking in parallel)
- Timestep $\Delta t = 1$ ns
- Generic: usable for other trap experiments (Paul or Penning)

Objective

Provide a realistic trapped cloud profile $\Rightarrow$ Set $\beta$ decay in lab. frame.
**CLOUDA – Buffer gas: overview**

Simulation: initial event

Issue: simulate buffer gas cooling microscopically ⇒ ion-atom collision

Realistic models: collaboration with CELIA (Bordeaux) & Madrid, ion-atom potentials effect

Four models:
- Classical: Hard Spheres (HS), *Classical*
- Quantum: *Cold gas*, *Full*

Global logic applied for any model:
1. Check if collision happens ⇒ Compute mean free path \( \lambda \) (model dependent)

\[
p_{\text{collision}} = 1 - \exp \left( \frac{v_{\text{ion}} \cdot \Delta t}{\lambda} \right)
\]

2. If collision happens, randomize \( \vec{v}_{\text{atom}} \) and compute scattering angle \( \theta \) (model dependent) in center-of-mass frame
**CLOUDA – Buffer gas : Classical models**

Simulation : initial event

Hard Spheres

\[ b < b_{\text{max}} = r_{\text{atom}} + r_{\text{ion}} \]

Classical

\[ b < b_{\text{max}} = \text{numerical cut} \]
**CLOUDA – Buffer gas: Classical models**

Simulation: initial event

**Hard Spheres**

\[ b < b_{\text{max}} = r_{\text{atom}} + r_{\text{ion}} \]

**Classical**

\[ b < b_{\text{max}} = \text{numerical cut} \]

Mean free path \( \lambda = \frac{v_{\text{ion}}}{\nu} \) (\( \nu \): collision frequency):

\[
\nu(v_{\text{ion}}) = \rho_{\text{gas}} \pi b_{\text{max}}^2 v_{mp} \left[ \left( \frac{v_{\text{ion}}}{v_{mp}} + \frac{v_{mp}}{2v_{\text{ion}}} \right) \text{erf} \left( \frac{v_{\text{ion}}}{v_{mp}} \right) + \frac{1}{\sqrt{\pi}} \exp \left( - \left( \frac{v_{\text{ion}}}{v_{mp}} \right)^2 \right) \right]
\]

\( (\pi b_{\text{max}}^2) : \text{constant cross section} \)
*CLOUDA – Buffer gas : Classical models

Simulation : initial event

Hard Spheres

\[ \theta = \pi - 2 \arcsin \left( \frac{b}{b_{\text{max}}} \right) \]

Classical

\[ \theta = f(E_{\text{rel}}, b) \]
**CLOUDA – Buffer gas: Quantum models**

Simulation: initial event

Mean free path $\lambda = \nu_{\text{ion}} / \nu$ ($\nu$: collision frequency)

- **Cold gas**: $\nu(\nu_{\text{ion}}) = \rho_{\text{gas}} \sigma(\nu_{\text{ion}}) \nu_{\text{ion}}$

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*Cross sections in $\text{Ar}^+ + \text{He}$*

- **Quantum cross section**
- $\pi b_{\text{max}}^2$ with $b_{\text{max}} = 20$ a.u.
**CLOUDA – Buffer gas : Quantum models**

Simulation : initial event

Mean free path $\lambda = \frac{v_{\text{ion}}}{\nu}$ ($\nu$ : collision frequency)

- **Cold gas** : $\nu(v_{\text{ion}}) = \rho_{\text{gas}} \sigma(v_{\text{ion}}) v_{\text{ion}}$
- **Full** :

\[
\nu(v_{\text{ion}}) = \frac{\rho_{\text{gas}}}{\sqrt{\pi} v_{\text{mp}} v_{\text{ion}}} \int_{0}^{+\infty} dv_{\text{rel}} \sigma(v_{\text{rel}}) v_{\text{rel}}^2 \left\{ \exp \left[ - \left( \frac{v_{\text{rel}} - v_{\text{ion}}}{v_{\text{mp}}} \right)^2 \right] - \exp \left[ - \left( \frac{v_{\text{rel}} + v_{\text{ion}}}{v_{\text{mp}}} \right)^2 \right] \right\}
\]

---

**Cross sections in Ar$^+$ + He**

- **Quantum cross section**
- $\pi b_{\text{max}}^2$ with $b_{\text{max}} = 20$ a.u.

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*Precision measurements in the W interaction framework: dev. of real. sim. for the LPCTrap device installed at GANIL.*

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*CLOUDA – Buffer gas : Quantum models*

Simulation : initial event

(If collision) Scattering angle : \( \theta = f(E_{\text{rel.}}, r) \)

\[
\frac{\int_{0}^{\theta} \frac{d\sigma(v, \theta)}{d\omega} \sin(\theta) d\theta}{\int_{0}^{\pi} \frac{d\sigma(v, \theta)}{d\omega} \sin(\theta) d\theta} = r
\]
*CLOUDA – Buffer gas : Models assessment

Simulation : initial event

- Constant, uniform, infinite $\vec{E}_x$
*CLOUDA – Buffer gas: Models assessment*

Simulation: initial event

- Constant, uniform, infinite $E_x$

- 2 experimental observables for ion-atom collisions:
  - Drift velocities $v_d$
    - $^{40}\text{Ar}^+ + ^4\text{He}$ & $^7\text{Li}^+ + ^4\text{He}$

Data will be function of Townsend: $(T_d) \sim$ Electric field
Buffer gas density
Ranging in:
- $5 – 130$ Td for $^{40}\text{Ar}^+ + ^4\text{He}$
- $2 – 200$ Td for $^7\text{Li}^+ + ^4\text{He}$
**CLOUDA – Buffer gas : Models assessment**

Simulation : initial event

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- 2 experimental observables for ion-atom collisions :
  - Drift velocities $\mathbf{v}_d$ ($^{40}\text{Ar}^+ + ^4\text{He} & ^7\text{Li}^+ + ^4\text{He}$)
  - Diffusion coefficient $D_L, D_T$ ($^7\text{Li}^+ + ^4\text{He}$ only)

  \[
  \sigma_x^2 = \frac{D_L}{2} \times t + b \\
  \sigma_y^2, z = \frac{D_T}{2} \times t + b
  \]
**CLOUDA – Buffer gas: Models assessment**

Simulation: initial event

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\sigma_{y,z}^2 = \frac{D_T}{2} \times t + b
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**CLOUDA – Buffer gas : Models assessment**

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\sigma_{y,z}^2 = \frac{D_T}{2} \times t + b
\]

- Data will be function of Townsend :

\[
(Td) \sim \frac{\text{Electric field}}{\text{Buffer gas density}}
\]

Ranging in :

- 5 – 130 Td for $^{40}\text{Ar}^+ + ^4\text{He}$
- 2 – 200 Td for $^7\text{Li}^+ + ^4\text{He}$
*CLOUDA – Buffer gas : Models assessment

Simulation : initial event
Drift velocities $v_d - ^{40}\text{Ar}^+ + ^4\text{He}$

$\Rightarrow$ Good agreement, except for Hard Spheres
**CLOUDA – Buffer gas: Models assessment**

Simulation: initial event

Drift velocities $v_d - ^7\text{Li}^+ + ^4\text{He}$

$\Rightarrow$ Good agreement, except for Hard Spheres
*CLOUDA – Buffer gas : Models assessment

Simulation : initial event

Transverse diffusion coefficient $D_T - ^7\text{Li}^+ + ^4\text{He}$

⇒ Good agreement
*CLOUDA – Buffer gas: Models assessment

Simulation: initial event
Longitudinal diffusion coefficient $D_L - ^7\text{Li}^+ + ^4\text{He}$

⇒ Good agreement until 80 Td, then discrepancies (esp. *full* ...)
**CLOUDA – Buffer gas : Models assessment**

Simulation : initial event

Longitudinal diffusion coefficient $D_L - ^7\text{Li}^+ + ^4\text{He}$
*CLOUDA – Buffer gas: Models assessment*

Simulation: initial event

Longitudinal diffusion coefficient $D_L - ^7\text{Li}^+ + ^4\text{He}$

\[
\nu(v_{\text{ion}}) = \frac{\rho_{\text{gas}}}{\sqrt{\pi} v_{mp} v_{\text{ion}}} \int_0^{+\infty} dv_{\text{rel}} \sigma(v_{\text{rel}}) \left( \frac{v_{\text{rel}}}{v_{mp}} \right)^2 \left\{ \exp \left[ -\left( \frac{v_{\text{rel}} - v_{\text{ion}}}{v_{mp}} \right)^2 \right] - \exp \left[ -\left( \frac{v_{\text{rel}} + v_{\text{ion}}}{v_{mp}} \right)^2 \right] \right\}
\]
*CLOUDA – Buffer gas : Models assessment

Simulation : initial event

Longitudinal diffusion coefficient $D_L$ – $^7\text{Li}^+ + ^4\text{He}$

- Discrepancies source (probably) found
- Computations with wider tables on the way...
**CLOUDA – EM fields description**

Simulation: initial event

Implemented fields:

- Penning trap: Ideal
- Paul trap: Ideal
  - $\vec{E} = -\frac{2}{r_0^2} (\Phi_0 + \Phi_1 \cos(2\pi f_{RF} t)) (\vec{x} + \vec{y} - 2\vec{z})$
  - "Radio-Frequency Phase (RFP)" = $(2\pi f_{RF} t) \mod (2\pi)$
- Paul trap: Realistic
  - Include: real geometry, real potential applied on electrodes
  - Use a Boundary Element Method (BEM) to compute the field in all space
  - Parametrize realistic field using harmonic synthesis:

$$\Phi(r, \theta, \varphi) = \sum_{\ell=0}^{\infty} \left( \frac{r}{r_0} \right) ^\ell \sum_{m=0}^{\ell} \sqrt{\frac{2\ell+1}{4\pi}} \frac{(\ell-m)!}{(\ell+m)!} \\ \times P^m_\ell (\cos \theta) (A_{\ell m} \cos(m\varphi) - B_{\ell m} \sin(m\varphi))$$

Typically, $\ell_{max} = 12$
*CLOUDA – N-body space charge

Issue: compute the $N^2$ Coulomb repulsions at each timestep

Multiple algorithms possible:

- Exact computation ($O(N^2)$):
  - Tile Calculation
  - Chamomile Scheme

- Faster methods (approximate long range interactions):
  - Barnes-Hut $\Rightarrow O(N \log N)$
  - Fast Multipole Methods (FMM), e.g. ExaFMM $\Rightarrow O(N)$
*Trapped ion dynamic : CLOUDA – Summary

We have:

- **Buffer gas collisions:**
  - Hard Spheres (HS)
  - Classical
  - Full
  - Cold gas

- **EM fields:**
  - Ideal Paul trap
  - Realistic Paul trap (Harmonic Synthesis)

- **N-body : Tile Calculation. Either On or Off.**

**Cloud profile**

⇒ What effect all models have on the final cloud?
*CLOUDA – Buffer gas effect

Simulation: initial event

Energy profiles

- HS
- Classical
- Full
- Cold gas

> Ideal Field
> No N-body
*CLOUDA – Buffer gas effect*

Simulation: initial event

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![Graphs showing the effect of buffer gas on particle behavior in a W interaction framework. The graphs compare different buffer gas models against ideal and no N-body field scenarios.](image-url)

- **X**
  - $\sigma_x$ vs. RFP (rad)
  - Models: HS, Classical, Full, Cold Gas
- **Z**
  - $\sigma_z$ vs. RFP (rad)
  - > Ideal Field
  - > No N-body

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*CLOUDA – Buffer gas effect*

Simulation: initial event

![Graphs showing the impact of different buffer gas models on precision measurements in the W interaction framework.](image)

⇒ Chosen buffer gas model has an important impact
*CLOUDA – EM fields effect

Simulation: initial event

Energy profile

- HF
- IF

> Hard spheres
> No N-body
**CLOUDA – EM fields effect**

Simulation: initial event

- Hard spheres
- No N-body

### Diagrams

- **X**
- **Y**
- **Z**

- **VX**
- **VY**
- **VZ**
*CLOUDA – EM fields effect

Simulation: initial event

- Hard spheres
- No N-body
Precision measurements in the W interaction framework: dev. of real. sim. for the LPCTrap device installed at GANIL.

*CLOUDA – EM fields effect*

Simulation: initial event

⇒ Trapping EM field must be properly described
*CLOUDA – N-body effect

Simulation: initial event

Cloud profile at thermal equilibrium

- No N-body
- N-Body (N=1024)
- N-Body (N=2048)
- N-Body (N=4096)
- N-Body (N=8192)
- N-Body (N=16384)
- N-Body (N=32768)
- N-Body (N=65536)

> Ideal field
> Hard spheres

LPCTrap: up to $10^5$ trapped ions
*CLOUDA — N-body effect

Simulation: initial event

**Cloud profile at thermal equilibrium**

<table>
<thead>
<tr>
<th>Mean energy (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.16</td>
</tr>
<tr>
<td>0.14</td>
</tr>
<tr>
<td>0.12</td>
</tr>
<tr>
<td>0.10</td>
</tr>
<tr>
<td>0.08</td>
</tr>
<tr>
<td>0.06</td>
</tr>
<tr>
<td>0.04</td>
</tr>
</tbody>
</table>

**Relative difference (%)**

- Ideal field
- Hard spheres

LPCTrap: up to $10^5$ trapped ions

⇒ N-body has an important impact
Overview: simulation package

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  - DSSSD profiles
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  - ...
- Beta tracking (BAYEUX)
- RI tracking (BAYEUX)
- Detection data
- Detection data
β decay generator

Simulation: initial event

- Distribution of Q-value between decay products

\[ \begin{align*}
\frac{6}{2}\text{He} & \quad 0^+ \\
Q &= 3507.8 \text{ keV} \\
T(1/2) &= 806.7 \text{ ms} \\
\frac{100}{1+}\frac{2.9059}{3}\text{Li} &
\end{align*} \]
β decay generator

Simulation: initial event

- Distribution of Q-value between decay products
- Take $a_{\beta\nu}$ into account
β decay generator

Simulation: initial event

- Distribution of Q-value between decay products
- Take $a_{\beta\nu}$ into account
- γ de-excitation

![Emission spectrum](image)

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Intensity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1763</td>
<td>0.249</td>
</tr>
<tr>
<td>1219.3</td>
<td>1.23</td>
</tr>
</tbody>
</table>

$\frac{35}{17}$Ar

$Q = 5966.1$ keV

$T(1/2) = 1.7756$ s
β decay generator

Simulation: initial event

- Distribution of Q-value between decay products
- Take $a_{\beta\nu}$ into account
- γ de-excitation
- Fermi function
- Other corrections to consider: radiative, recoil, . . .
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Detection data:
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Bayeux

Simulation : tracking

- $\beta$ scattering : important systematic effect
- RI and $\beta$ tracking requires:
  - Fine geometry description
  - Realistic EM fields
    (Paul trap, free flight tube)
  - Data filtering tools
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Bayeux

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$\Rightarrow$ Done using BAYEUX :

- Geomtools to control geometry
- Wraps GEANT4 for $\beta$ scattering
- Able to inject field maps in the Monte Carlo engine

Thanks to Ph. Velten!
Bayeux

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