

Developing mathematical skills of students by using electronic worksheets

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*WIMS Outil de pédagogies innovantes
Annecy, France, 8 juin 2016*

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A short (personal) history

- **2003/04:** Occasional classroom use of selected WIMS exercises and tools (matrix calculator, etc.)
 - (No Facebook, YouTube, Gmail or Iphones then!)
- **2004/05:** Started using a WIMS virtual classroom regularly in a 1st year **Linear algebra** course for students at Faculty of education (future primary math teachers).
 - Main goal: to motivate the students to work on problems regularly (assign WIMS homework)
 - Large groups in tutorial sessions (up to 50 students), large dropout in the first year

- **2005/06:**
 - Slovene translations of essential WIMS modules
 - A local server set up at wims.pef.uni-lj.si
 - Some promotion on national level
(presentations,
a course for
teachers)

Virtualne učilnice - Mozilla Firefox

Datoteka Urejanje Pogled Zgodovina Zaznamki Orodja Pomoč

Samostojne učilnice

Ustanova	Učilnica	Skrbnik
Gimnazija Ledina	Četrtošolci	Maruša Dobljekar
Gimnazija Ledina	Prvi in drugi	Maruša Dobljekar
Gimnazija Ormož	Matematična učilnica	Sandra Brodnjak
Gimnazija Ormož	Za matematiko	Radovan Milovanović
OSNOVNA ŠOLA DOLENJSKE TOPLICE	4.A	MAJA BOBNAR
Osnovna šola Krmelj	Matematika 7. razred	Boštjan Repovž
Osnovna šola Krmelj	Matematika 8. razred	Boštjan Repovž
Osnovna šola XIV. divizije Senovo	Matematika 9	Marija Hlastan
OŠ BREŽICE	FIZIKA	Saša Silič
OŠ Brežice	8. razred	Metka Pinoza
OŠ Brežice	MAT1	Marija Zore Krivec
OŠ Brežice	Matematika	Metka Pinoza
OŠ Dolenjske Toplice	2.a	Urška Bučar
OŠ Dolenjske Toplice	MATEMATIKA	Antonija Miklavčič-Jenič
OŠ Primož Teubar	MAT	Polona Klun
OŠ Primož Trubar	MATEMATIKA	Polona Klun
OŠ Senovo	Matematika	Marija Hlastan
OŠ Velike Lašče	Matematika za 8. razred	Stanka Dedo Lale
oš Dolenjske Toplice	Matematika za 3. razred	Tatjana Gorenc
Pef Ljubljana	Vzorčna učilnica	Boštjan Kuzman
ZUIM Kamnik	Matematika 9	Maja Poljanšek
ZUIM Kamnik	matematika	Maja Poljanšek
ZUIM Kamnik	Ulomki	Maja Poljanšek
OŠ DOLENJSKE TOPLICE	Naloge	Antonija Miklavčič-Jenič
OŠ DOLENJSKE TOPLICE	LINEARNA FUNKCIJA	Antonija Miklavčič-Jenič

[Področje za učence](#) [Primeri učilnic](#) [Poiasnila](#)

[WIMS začetna stran](#) [O programski enoti](#)

- New materials developed (mostly linear algebra)

WWW interactive multipurpose server - Mozilla Firefox

Datoteka Urejanje Pogled Zgodovina Zaznamki Orodja Pomoč

[Druge učilnice](#)
[Odjava](#)
[Deska sporočil](#)
[Piši skrbniku](#)
[Spremeni nastavitve](#)
[Zamenjaj geslo](#)

Algebra I (letni sem. 06/07)

PeF Ljubljana

Pozdravljeni, Boštjan Kuzman! Dobrodošli v virtualni učilnici Algebra I (letni sem. 06/07), katere skrbnik je [Boštjan Kuzman](#). [Pomoč](#)

Algebra je velikodušna - pogosto nam da več, kot smo jo prosili.
Jean le Rond d'Alembert, 1717-1783

Tukaj so vsebine, pripravljene za to učilnico. [Moji rezultati](#).

Dokument	Rezultati 3. kolokvija 2006/07
Dokument	Gradivo in literatura
Delovni list 1.	Matrika linearne preslikave 1 (zaključeno) (0% opravljeno, kvaliteta 0/10)
Delovni list 2.	Matrika linearne preslikave 2 (zaključeno) (0% opravljeno, kvaliteta 0/10)
Delovni list 3.	Matrika linearne preslikave 3 (zaključeno) (0% opravljeno, kvaliteta 0/10)
Delovni list 4.	Ekvivalenca matrik, sistemi in enačbe (zaključeno) (0% opravljeno, kvaliteta 0/10)
Delovni list 5.	Podobnost matrik, projektorji (zaključeno) (0% opravljeno, kvaliteta 0/10)
Delovni list 6.	Invariantni in lastni podprostori (zaključeno) (0% opravljeno, kvaliteta 0/10)
Delovni list 7.	Diagonalizacija 1 (zaključeno) (0% opravljeno, kvaliteta 0/10)
Delovni list 8.	Diagonalizacija 2 (zaključeno) (0% opravljeno, kvaliteta 0/10)
Delovni list 9.	Jordanova teorija 1 (zaključeno) (0% opravljeno, kvaliteta 0/10)
Delovni list 11.	Dualni prostor. Norma, metrika, skalarni produkt. (zaključeno) (0% opravljeno, kvaliteta 0/10)
Delovni list 12.	Ortogonalnost (zaključeno) (0% opravljeno, kvaliteta 0/10)

Current state of WIMS in Slovenia

- WIMS virtual classrooms are actively used in 2 courses (**Linear algebra, Logic and sets**) for math education students at the Univ. of Ljubljana, Faculty of Education.
- Since 2010/11, we also cover some WIMS related topics (basic classroom maintenance and exercise creation) in **Mathematical Technologies** course for 2nd year students.
- No new institutional users on horizon (alternatives such as Moodle are widely used, some University departments developed their own solutions, commercial solutions for primary and secondary level are available).
 - No translation updates since 3.5 version.
 - Local server `wims.pef.uni-lj.si` also down.

- **Our Linear Algebra worksheets – example 1**

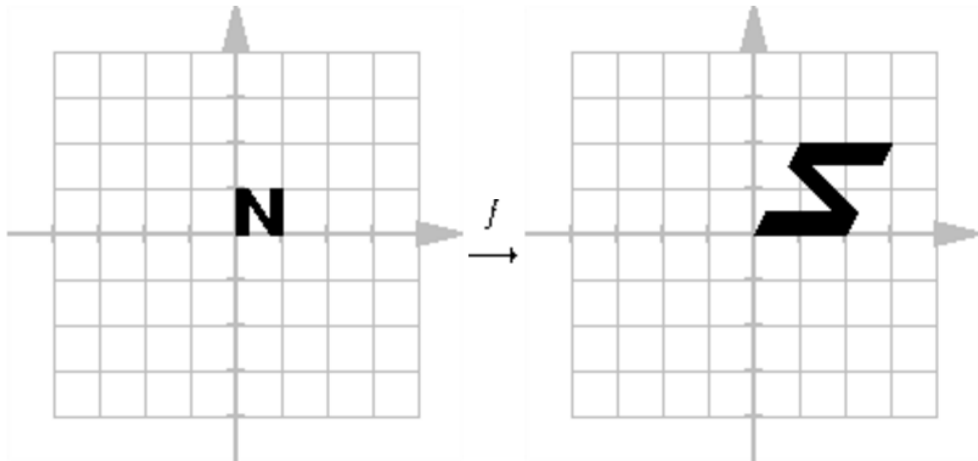
wims.auto.u-psud.fr/wims/ x

wims.auto.u-psud.fr/wims/wims.cgi?session=HU60A5D262.5&+lang=si&+module=

Domov Orodja Delovni list Pomoč O programski enoti Logout

Matrika linearne preslikave

Iz spodnje slike je razvidno učinkovanje neke obrnljive linearne preslikave $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$.

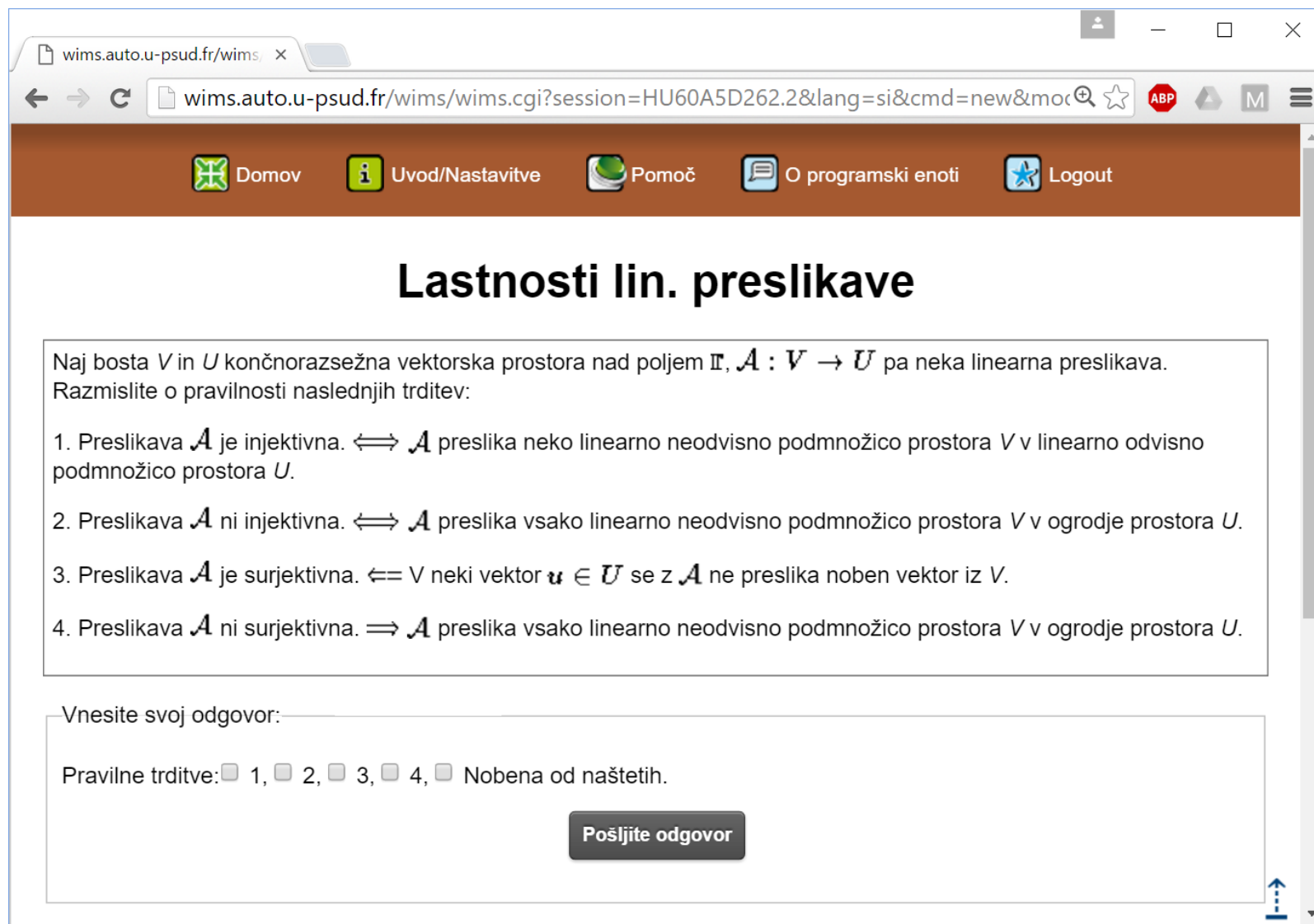


Kakšna je matrika $M_{\Sigma}^{\Sigma}(f)$, ki pripada tej preslikavi glede na običajno urejeno bazo Σ prostora \mathbb{R}^2 ?

Vnesite svoj odgovor:

Linear transformation f is given by above images. Find matrix $M(f)$.

• Our Linear Algebra worksheets – example 2



The screenshot shows a web browser window with the URL `wims.auto.u-psud.fr/wims/wims.cgi?session=HU60A5D262.2&lang=si&cmd=new&moc`. The browser's address bar and tabs are visible at the top. Below the address bar is a navigation bar with five icons and labels: 'Domov' (Home), 'Uvod/Nastavitve' (Introduction/Settings), 'Pomoč' (Help), 'O programski enoti' (About the program unit), and 'Logout'. The main content area has a title 'Lastnosti lin. preslikave' (Properties of linear mappings). Below the title is a text box containing the following text: 'Naj bosta V in U končnorazsežna vektorska prostora nad poljem \mathbb{I} , $\mathcal{A} : V \rightarrow U$ pa neka linearna preslikava. Razmislite o pravilnosti naslednjih trditev:' (Let V and U be finite-dimensional vector spaces over the field \mathbb{I} , and $\mathcal{A} : V \rightarrow U$ be a linear mapping. Consider the validity of the following statements:). Below this text are four numbered statements: 1. 'Preslikava \mathcal{A} je injektivna. $\iff \mathcal{A}$ preslika neko linearno neodvisno podmnožico prostora V v linearno odvisno podmnožico prostora U .' 2. 'Preslikava \mathcal{A} ni injektivna. $\iff \mathcal{A}$ preslika vsako linearno neodvisno podmnožico prostora V v ogrodje prostora U .' 3. 'Preslikava \mathcal{A} je surjektivna. $\iff \forall$ neki vektor $u \in U$ se z \mathcal{A} ne preslika noben vektor iz V .' 4. 'Preslikava \mathcal{A} ni surjektivna. $\implies \mathcal{A}$ preslika vsako linearno neodvisno podmnožico prostora V v ogrodje prostora U .' Below the statements is a text input field with the label 'Vnesite svoj odgovor:' (Enter your answer:). Below the input field is a row of radio buttons for selecting the correct statements: 'Pravilne trditve: ☐ 1, ☐ 2, ☐ 3, ☐ 4, ☐ Nobena od naštetih.' (Correct statements: 1, 2, 3, 4, None of the above). At the bottom right of the input area is a button labeled 'Pošljite odgovor' (Send answer).

Which claims are valid for any linear transformation A ?

Comments

- The students are stimulated to work regularly and actively.

Seznam uporabnikov x

wims.auto.u-psud.fr/wims/wims.cgi?session=9V8942AB2B.3&lang=si&cmd=

Grades computed over a maximum of 10											
Priimek, Ime (75)	Povprečje	Sheet 3	Sheet 4	Sheet 5	Sheet 6	Sheet 7	Sheet 8	Sheet 9	Sheet 10	Sheet 11	Sheet 12
	5.62	10	0.4	8.3	10	10	8.3	8.1	--	--	--
	9.91	10	10	10	10	10	10	10	8.8	10	10
	0.58	--	--	3.3	--	--	--	--	--	--	--
	6.65	6.7	4.4	10	5.7	6.7	8.3	7.3	7.5	--	8.8
	7.56	9.1	5.6	10	10	4.7	10	7.5	--	10	7.5
	4.32	8.6	3.8	10	--	10	--	--	--	--	--
	1.29	--	--	--	--	--	--	7.6	7.5	--	--
	8.58	9.5	3.8	10	5.6	10	10	8.8	10	10	10
	3.26	5.2	1.5	10	1.4	--	7.2	4.1	--	--	--
	9.95	10	10	10	10	10	10	9.4	10	10	10
	7.62	10	4.6	10	8.6	10	9.4	10	6.2	5.7	--
	9.19	10	7.7	10	6.4	8.3	9.4	10	10	10	10
	9.95	10	10	10	10	10	10	9.4	10	10	10

Some doubts

- Q: Does the use of technology influence the contents we teach?
 - A: Certainly.
- Q: Is this good or bad?
 - A: Depends.
- Q: What kind of math do we want to teach?
 - A: The one best suited for technology? (NO!)
- Q: Instant gratification problem: Obtaining scores immediately may defocus the students.

- **Example 1**

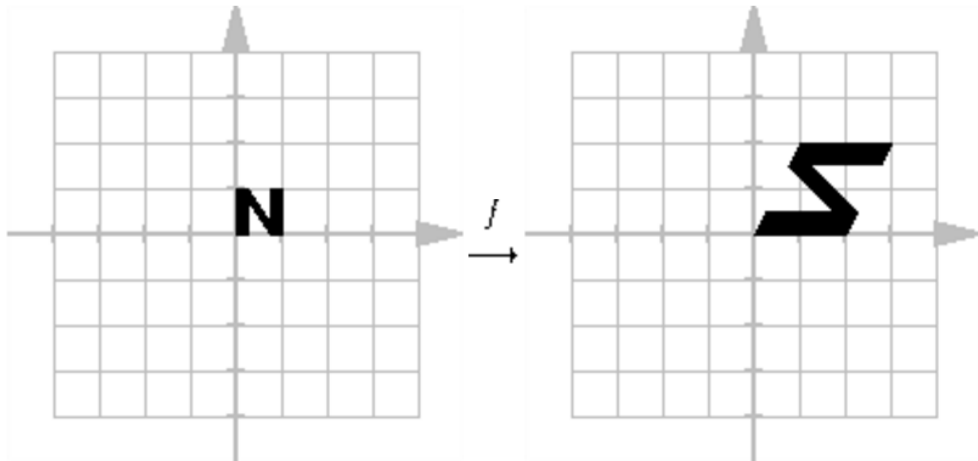
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wims.auto.u-psud.fr/wims/wims.cgi?session=HU60A5D262.5&+lang=si&+module=

Domov Orodja Delovni list Pomoč O programski enoti Logout

Matrika linearne preslikave

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Kakšna je matrika $M_{\Sigma}^{\Sigma}(f)$, ki pripada tej preslikavi glede na običajno urejeno bazo Σ prostora \mathbb{R}^2 ?

Vnesite svoj odgovor:

After a few repetitions, the students just learn the procedure without complete understanding of the problem.

- ## Example 2

wims.auto.u-psud.fr/wims.cgi?session=HU60A5D262.2&lang=si&cmd=new&mo...

Domov Uvod/Nastavitve Pomoč O programski enoti Logout

Lastnosti lin. preslikave

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Vnesite svoj odgovor:

Pravilne trditve: ☐ 1, ☐ 2, ☐ 3, ☐ 4, ☐ Nobena od naštetih.

Pošljite odgovor

Very bad average scores. Many students just try to guess or learn the correct answers by heart. Proper validation missing.

SOLUTION:

Let's not overuse/abuse the system.

*„If [a teacher of mathematics]
fills his allotted time with
drilling his students in routine operations
he kills their interest,
hampers their intellectual development,
and misuses his opportunity.“*

G. Polya, 1948, in *How to Solve It?*

WIMS as part of the Math Technologies course

- Since 2010/11.
- Audience: students in the 2nd year of the university programme for future teachers of mathematics in primary school (11-14) or secondary school (15-18).
- Topics covered this year:
 - math publishing (LaTeX),
 - computer algebra systems (Maxima),
 - interactive geometry (GeoGebra),
 - interactive environments (**WIMS**).
- No use of closed, commercial solutions!

WIMS topics covered in the course

(Note that the students have already met WIMS before in their Linear algebra course.)

- Exploration of tools and other existing materials.
- Classroom creation & basic maintenance.
- Translating examples and creating simple OEF exercises with random parameters via Createxo.
- Worksheet creation and grading.
- **Final project:** students create some exercises, make them available on their worksheets, and write a report.

Main observations

- WIMS architecture embraces different types of software – one has to interconnect.
- The students find the topics very useful in practice. However, software may change before these students actually start their professional teaching carrier.
- The feeling of creativity: modifying existing and creating new exercises to suit one's personal teaching style is often more rewarding than just following some prepared set.
- Regardless to ocassional frustrations (OEF programming), the students actually like the course a lot!
- Creating a good exercise with random parameters can also be a great exercise in mathematical thinking.

Example: Intersection of lines

Line 1: $y=ax+b$

Line 2: $y=cx+d$

Compute the intersection point P!
(if the lines meet)

How to randomize parameters (1)

`a,b,c,d = randint[-5..5]` works, but...

- For beginners, one may require unique intersection and integer coordinates of P.

Solution: Choose random integer coordinates for P and rows of random non-singular 2x2 matrix for direction vectors of your lines (use SLIB).

How to randomize parameters (2)

- At next stage, solutions **MUST** be noninteger!
 - Possible solutions:
 - Use conditionals to fix „bad“ random choices.
 - Apply „nice random transformations“ to some „good“ nonrandom choice of parameters to give fake impression of randomization.

How to randomize parameters (3)

- Intersection should not be (always) unique!
 - But with $a, b, c, d = \text{randint}[-10..10]$, the probability of parallel lines is very small.
 - Possible solution:
 - Force the uniform distribution of parallel / nonparallel case?
 - Assign more repetitions?
 - Etc.

