

Microscopic origin of the $c = 1$ universality class

K. K. Kozlowski

CNRS, Laboratoire de Physique, ENS de Lyon.

19th of September 2016

K. K. Kozlowski, J.-M. Maillet "*Microscopic approach to a class of 1D quantum critical models.*"
J. Phys. A: Math. & Theor. 48, 484004 (2015).

Quantum integrable systems, conformal field theories and stochastic processes
Institut d'études scientifiques de Cargèse, Cargèse

Outline

1 Introduction

- Universality and asymptotics

2 The models of interest

- Realisability of the microscopic model
- The microscopic model
- The free boson Hilbert space

3 The correspondence

- Mapping of expectation values at large distances

4 Conclusion

Generalities about lattice models

- ⊗ H_L lattice Hamiltonian in 1D with L sites, periodic bc, on Hilbert space $\mathfrak{h} = \mathfrak{h}_1 \otimes \dots \otimes \mathfrak{h}_L$.
- ⊗ Spaces \mathfrak{h}_ℓ can be finite or infinite dimensional. Often isomorphic $\mathfrak{h}_\ell \simeq \mathfrak{h}_0$.
- ⊗ Basis of operators $O^{(\alpha)}$ on $\mathfrak{h}_0 \rightsquigarrow$ operators $O_\ell^{(\alpha)} = \underbrace{\text{id} \otimes \dots \otimes \text{id}}_{\ell-1 \text{ times}} \otimes O^{(\alpha)} \otimes \underbrace{\text{id} \otimes \dots \otimes \text{id}}_{N-\ell-1}$.
- ⊗ The XXZ spin-1/2 chain on $\mathfrak{h}_{XXZ} = \otimes_{n=1}^L \mathbb{C}^2$, σ^α Pauli matrices

$$H_{XXZ} = \sum_{n=1}^L \left\{ \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y + \Delta \sigma_n^z \sigma_{n+1}^z - h \sigma_n^z \right\} \quad , \quad \sigma_{n+L} \equiv \sigma_n$$

What one would like to know?

- i) Find the Eigenstates and Eigenvectors of $H_L | \Psi_\beta \rangle_L = E_\beta | \Psi_\beta \rangle_L$;
- ii) Characterize the correlation functions in infinite volume

$$\langle O_1^{(\alpha)} O_{1+x}^{(\beta)} \rangle \equiv \lim_{L \rightarrow +\infty} \left\{ \langle G.S. | O_1^{(\alpha)} O_{1+x}^{(\beta)} | G.S. \rangle_L \right\}$$

✓ Well defined for a wide class of models ('60's **Ruelle**)

Large-distance asymptotics

- ⊕ Massive model \equiv spectral gap when $L \rightarrow +\infty$: $E_{\text{ex}} - E_{G.S.} \geq c$

↝ Large- x asymptotics are exponential

$$\langle o_1^{(\alpha)} o_{1+x}^{(\beta)} \rangle \simeq \langle o_1^{(\alpha)} \rangle \langle o_1^{(\beta)} \rangle + C^{(\alpha\beta)} e^{-\frac{x}{\xi}} + \dots$$

- ⊕ Massless model \equiv continuum of excited states above $|G.S.\rangle_L$: $\lim_{L \rightarrow +\infty} \{E_{\text{ex;low}} - E_{G.S.}\} = 0$

↝ Large- x asymptotics are algebraic

$$\langle o_1^{(\alpha)} o_{1+x}^{(\beta)} \rangle \simeq \langle o_1^{(\alpha)} \rangle \langle o_1^{(\beta)} \rangle + \frac{\mathcal{A}_1^{(\alpha\beta)}}{x^{\delta_1^{(\alpha\beta)}}} + \frac{\mathcal{A}_2^{(\alpha\beta)}}{x^{\delta_2^{(\alpha\beta)}}} \cos(x\phi_2) + \dots$$

- ⊕ Critical exponents $\delta_k^{(\alpha\beta)}$ expected to be universal

('66 Fisher, '67 Kadanoff, Götze, Hamblen, Hecht, Lewis, Palciaskas, Rayl, Swift, Aspens, Kane, '70 Griffiths)

Correspondence with a CFT

- ◆ Prediction for the structure of asymptotic expansions in 1D massless models :
 - i) ('70 Polyakov) Conformal invariance of correlation functions in long-distance regime.
 - ii) ('86 Cardy , '86 Affleck) Central charge \rightsquigarrow finite-size corrections to ground state energy.

$$E_{G.S.} = \textcolor{brown}{L}\varepsilon - \textcolor{blue}{c} \frac{\pi v_F}{6\textcolor{brown}{L}} + O\left(\frac{1}{\textcolor{brown}{L}^2}\right) \quad \text{and} \quad E_{\text{ex}} - E_{G.S.} = \frac{2\pi v_F}{\textcolor{brown}{L}} \left(\Delta_k^+ + \Delta_k^- + n_{\text{ex}} \right) + O\left(\frac{1}{\textcolor{brown}{L}^2}\right)$$

- Operator $O_{1+x}^{(\beta)}$ connects $|G.S.\rangle_{\textcolor{brown}{L}}$ and $|\Delta_k^\pm; n_{\text{ex}}\rangle_{\textcolor{brown}{L}}$ for $k \in I^{(\alpha)}$

$$O_{1+x}^{(\beta)} \hookrightarrow \sum_{k \in I^{(\alpha)}} \textcolor{blue}{C}_k^{(\beta)} \cdot \Psi_{\Delta_k^\pm}(z, \bar{z}) \quad z = e^{\frac{2i\pi}{L}x}$$

- ⊗ n -point functions in original model at $x \rightarrow +\infty$ computed from CFT n -point functions.

$$\langle O_1^{(\alpha)} O_{1+x}^{(\beta)} \rangle \simeq \langle O_1^{(\alpha)} \rangle \langle O_1^{(\beta)} \rangle + \sum_{k \in I^{(\alpha)} \cap I^{(\beta)}} \mathcal{A}_k^{(\alpha\beta)} \frac{e^{2ix\ell_k p_F}}{x^{2\Delta_k^+ + 2\Delta_k^-}} \cdot (1 + o(1))$$

- ◆ Large- L behaviour of the amplitudes ('86 Cardy)

$$\mathcal{A}_k^{(\alpha\beta)} = \lim_{L \rightarrow +\infty} \left\{ L^{2\Delta_k^+ + 2\Delta_k^-} \cdot \langle G.S. | O_1^{(\beta)} | \Delta_k^\pm; 0 \rangle_L \cdot \langle \Delta_k^\pm; 0 | O_1^{(\alpha)} | G.S. \rangle_L \right\}$$

Various tests

- ◆ Free fermion models: XX chain, impenetrable bosons,...
'60-'80 (Lieb, Mattis, Schultz, Jimbo, Miwa, Sato, McCoy, Perk, Shrock, Tracy, ...)
- ◆ Test CFT structure of spectrum from Bethe Ansatz
'87-'95 (Batchelor, Destri, DeVega, Klumper, Pearce, Woynarowich, Wehner, Zittartz)
 - ✓ OK for conformal structure of the spectrum at $L \rightarrow +\infty$.
- ◆ Universality of some critical exponents for perturbation of 2D Ising
('00 Penson, Spencer , '04 Mastropietro)
- ◆ *Ab initio* derivation of the large- x asymptotics in XXZ
('08, '12 Kitanine, K., Maillet, Slavnov, Terras)
- ◆ CFT-consistent behaviour of form factors
('90 Slavnov , '09 Kitanine,K,Maillet,Slavnov,Terras , '12 Shashi,Panfil,Caux,Imambekov)
- ◆ Proof of densification properties of Bethe roots in XXZ('15 K.).

The open problems

Questions

- Can one provide an *ab initio* construction of the operator correspondence ?
 - i) Which sector of the Hilbert space maps, at $L \rightarrow +\infty$, to the CFT side ?
 - ii) What makes the correspondence emerge only at large distances?
 - iii) In which sense the correspondence holds?

Answer

Question

- Can one provide an *ab initio* construction of the operator correspondence ?

Answer : YES for $c = 1$

$$\langle 0_{x_1+1}^{(\alpha_1)} \cdots 0_{x_r+1}^{(\alpha_r)} \rangle_{L; \mathfrak{h}_{\text{phys}}} \simeq \langle \mathcal{O}^{(\alpha_1)}(\omega_1) \cdots \mathcal{O}^{(\alpha_r)}(\omega_r) \rangle_{\mathfrak{h}_{\text{eff}}}$$

in the limit $L \gg |x_a - x_b| \gg 1$

Theorem ('15 K., Maillet)

For the XXZ, $|ex>$ low-energy excited state, $0_m^{(\alpha_1)} \in \{\sigma_m^z, \sigma_m^\pm\}$ spin operators,

$$\langle ex_1 | 0_{x_1+1}^{(\alpha_1)} | ex_2 \rangle_{L; \mathfrak{h}_{\text{phys}}} \underset{L \rightarrow +\infty}{\sim} \langle \psi(ex_1) | \mathcal{O}^{(\alpha_1)}(\omega_1) | \psi(ex_2) \rangle_{\mathfrak{h}_{\text{eff}}}$$

A counter example

Universality is encoded in **Matrix elements** and **NOT** in spectrum

- ⊗ H_L with particle-hole spectrum and scaling dimensions Δ^\pm
- ⊗ particle (resp. hole) rapidities density on $]k_F; +\infty[$ (resp. $]-\infty; k_F[$) when $L \rightarrow +\infty$
- ⊗ form factors $\mathcal{F}_x(\mu_h, \mu_p) = \langle \{\mu_p, \mu_h\} | 0(x) | G.S. \rangle$

$$\mathcal{F}_x(\mu_h, \mu_p) \sim \frac{e^{ix[p(\mu_p) - p(\mu_h)]}}{L^2 \rho(\mu_p) \rho(\mu_h)} (\mu_p - k_F)^{\frac{\Delta^+ - 1}{2}} (k_F - \mu_h)^{\frac{\Delta^- - 1}{2}} \mathcal{F}_{\text{reg}}(\mu_h, \mu_p)$$

- ⊗ Form factor series

$$\langle o(x)o(0) \rangle \underset{L \rightarrow +\infty}{=} \langle o(0) \rangle^2 + \int_{-\infty}^{k_F} d\mu_h \int_{k_F}^{+\infty} d\mu_p e^{ix[p(\mu_p) - p(\mu_h)]} (\mu_p - k_F)^{\Delta^+ - 1} (k_F - \mu_h)^{\Delta^- - 1} |\mathcal{F}_{\text{reg}}(\mu_h, \mu_p)|^2 + \dots$$

$$\langle o(x)o(0) \rangle \sim \langle o(0) \rangle^2 + \frac{C}{x^{2\Delta^- + 2\Delta^+}} + \dots$$

Comments on the realisability of the model

◆ Supporting facts

- Works for NLSM, XXZ (particle-hole spectrum) and Calogero-Sutherland ('09-'11 **Kitanine,K.,Maillet,Slavnov,Terras** , '12 **Shashi,Panfil,Caux,Imambekov**)
- Some features checked perturbatively for spinless fermions with pair interactions ('11 **Shashi,Glazman,Caux,Imambekov**)
- Rigorous check of universal relations for exponents for spin systems in perturbation ('09 **Benfatto, Mastropietro** , '13 **Benfatto, Falco, Mastropietro**)
- Plethora of systems argued to be described by non-linear Luttinger liquid (review '11 **Imambekov, Schmidt, Glazman**)
- Non-linear Luttinger Liquid \subset microscopic model

The relative excitation momentum

⊗ Generalities

- ♦ H_L 1D lattice Hamiltonian with conservation of bare-particle number (e.g. S^z)
- ♦ G.S. built of N bare particles, $N/L \rightarrow D$, rapidities which densify on Fermi zone $[-k_F; k_F]$
- ♦ Excited states in sectors with $N' = N + s$ bare-particles, s finite
- ♦ Rapidities densify on $[-k_F; k_F]$ with some holes $\{\mu_{h_a^{(s)}}\}_1^n$ and particles $\{\mu_{p_a^{(s)}}\}_1^n$
- ♦ Excitations on Fermi boundary parametrised by sets of integers $\mathcal{J}_{n;m}^{(s)} = \{\{p_a^{(s)}\}_1^n; \{h_a^{(s)}\}_1^m\}$

$$\mathcal{J}_{n_{p;-}^{(s)};n_{h;-}^{(s)}} \cup \mathcal{J}_{n_{p;+}^{(s)};n_{h;+}^{(s)}} \quad \text{with} \quad \begin{cases} n_{p;+}^{(s)} + n_{p;-}^{(s)} = n_{h;+}^{(s)} + n_{h;-}^{(s)} \\ \ell_s = n_{p;+}^{(s)} - n_{h;+}^{(s)} = n_{h;-}^{(s)} - n_{p;-}^{(s)} \end{cases}$$

⊗ Relative excitation momentum

$$\Delta \mathcal{P}_{\text{ex}} \equiv \mathcal{P}_{\text{ex}} - \mathcal{P}_{\text{GS}} = \sum_{a=1}^n \{p(\mu_{p_a^{(s)}}) - p(\mu_{h_a^{(s)}})\} + O\left(\frac{1}{L}\right)$$

- ♦ Momentum of Fermi-boundary excitations

$$\Delta \mathcal{P}_{\text{ex}} = 2\ell_s p_F + \frac{2\pi}{L} \left\{ \sum_{a=1}^{n_{p;+}^{(s)}} (p_{a;+}^{(s)} - 1) + \sum_{a=1}^{n_{h;+}^{(s)}} h_{a;+}^{(s)} - \sum_{a=1}^{n_{p;-}^{(s)}} p_{a;-}^{(s)} - \sum_{a=1}^{n_{h;-}^{(s)}} (h_{a;-}^{(s)} - 1) \right\} + \dots$$

The form factors I

- Model endowed with basis of local operators $0_{1+x}^{(\alpha)}$ having pure s transitions

$$\langle \text{ex}_{\text{out}}; s_{\text{out}} | 0_{1+x}^{(\alpha)} | \text{ex}_{\text{in}}; s_{\text{in}} \rangle \neq 0 \quad \text{if} \quad s_{\text{in}} - s_{\text{out}} = o_{\alpha}$$

- Form factors on Fermi boundary states in relative $\ell = \ell_{\text{out}} - \ell_{\text{in}}$ class

$$\text{ex}_{\text{out}}; s \rightsquigarrow \mathcal{J}_{m_{p,+}; m_{h,+}} \cup \mathcal{J}_{m_{p,-}; m_{h,-}} \quad \text{and} \quad \text{ex}_{\text{in}}; s + o_{\alpha} \rightsquigarrow \mathcal{J}_{n_{p,+}; n_{h,+}} \cup \mathcal{J}_{n_{p,-}; n_{h,-}}$$

$$\begin{aligned} \langle \text{ex}_{\text{out}}; s | 0_{1+x}^{(\alpha)} | \text{ex}_{\text{in}}; s + o_{\alpha} \rangle &= \left\{ e^{2i\rho_F x} \cdot \omega \right\}^{\ell} \cdot C_{\ell}(v_{\alpha}^+, v_{\alpha}^-) \cdot \mathcal{F}_{\ell}(0^{(\alpha)}) \cdot \left(\frac{2\pi}{L} \right)^{\Delta_{\ell}^{(\alpha)}} \\ &\times \mathcal{F} \left[\mathcal{J}_{m_{p,+}; m_{h,+}}; \mathcal{J}_{n_{p,+}; n_{h,+}} \mid v_{\alpha}^+, \omega \right] \cdot \mathcal{F} \left[\mathcal{J}_{m_{p,-}; m_{h,-}}; \mathcal{J}_{n_{p,-}; n_{h,-}} \mid -v_{\alpha}^-, \omega^{-1} \right] \cdot \left(1 + O \left(\frac{\ln L}{L} \right) \right). \end{aligned}$$

- $v_{\alpha}^{\pm} = \pm v(\ell K + o_{\alpha}/K)$ excited state and operator info \rightsquigarrow fix decay in volume

$$\Delta_{\ell}^{(\alpha)} = \frac{1}{2} \left\{ (v_{\alpha}^+ + \ell)^2 + (v_{\alpha}^- + \ell)^2 \right\} \quad \text{and} \quad \omega = e^{2i\pi \frac{x}{L}}$$

- Lowest energy ℓ class form factor

$$\mathcal{F}_{\ell}(0^{(\alpha)}) = \lim_{L \rightarrow +\infty} \left\{ \left(\frac{L}{2\pi} \right)^{\Delta_{\ell}^{(\alpha)}} \langle \text{ex}_{\text{low}}(\ell); -o_{\alpha} | 0_1^{(\alpha)} | \text{G.S.} \rangle \right\}$$

- Normalisation $C_{\ell}(v, \mu) = G(1+\mu)G(1-v)/G(1+\mu+\ell)G(1-v-\ell)$

The form factors II

$$\mathcal{J}_{n_p; n_h} = \left\{ \{p_a\}_1^{n_p}; \{h_a\}_1^{n_h} \right\} \quad \text{and} \quad \mathcal{J}_{n_k; n_t} = \left\{ \{k_a\}_1^{n_k}; \{t_a\}_1^{n_t} \right\}$$

- Microscopic form factor

$$\mathcal{F}\left(\mathcal{J}_{n_p; n_h}; \mathcal{J}_{n_k; n_t} \mid \nu, \omega\right) = (-1)^{\nu} \left(\frac{\sin[\pi\nu]}{\pi} \right)^{n_p - n_h} \mathcal{D}\left(\mathcal{J}_{n_p; n_h} \mid \nu, \omega\right) \cdot \mathcal{D}\left(\mathcal{J}_{n_k; n_t} \mid -\nu, \frac{1}{\omega}\right) \cdot \varpi\left(\mathcal{J}_{n_p; n_h}; \mathcal{J}_{n_k; n_t} \mid \nu\right)$$

- ★ In/out state building block (Z-measure if $n_p = n_h$)

$$\mathcal{D}\left(\mathcal{J}_{n_p; n_h} \mid \nu; \omega\right) = \left(\frac{\sin[\pi\nu]}{\pi} \right)^{n_h} \frac{\prod_{a>b}^{n_p} (p_a - p_b) \cdot \prod_{a>b}^{n_h} (h_a - h_b)}{\prod_{a=1}^{n_p} \prod_{b=1}^{n_h} (p_a + h_b - 1)} \prod_{a=1}^{n_p} \left\{ \omega^{p_a - 1} \frac{\Gamma(p_a + \nu)}{\Gamma(p_a)} \right\} \prod_{a=1}^{n_h} \left\{ \omega^{h_a} \frac{\Gamma(h_a - \nu)}{\Gamma(h_a)} \right\}$$

- ★ In/out state interaction

$$\varpi\left(\mathcal{J}_{n_p; n_h}; \mathcal{J}_{n_k; n_t} \mid \nu\right) = \prod_{a=1}^{n_h} \left\{ \frac{\prod_{b=1}^{n_k} (1 - k_b - h_a + \nu)}{\prod_{b=1}^{n_t} (t_b - h_a + \nu)} \right\} \cdot \prod_{a=1}^{n_p} \left\{ \frac{\prod_{b=1}^{n_t} (p_a + t_b + \nu - 1)}{\prod_{b=1}^{n_k} (p_a - k_b + \nu)} \right\}$$

The Hilbert space

⊗ Vectors

- Fermionic operators $\{\psi_n\}_{n \in \mathbb{Z}}$ and $*$ -associates $\{\psi_n^*\}_{n \in \mathbb{Z}}$: $\{\psi_n, \psi_m^*\} = \delta_{n,m}$
- Vacuum vector $|0\rangle$: $\psi_n|0\rangle = 0$ for $n < 0$ and $\psi_n^*|0\rangle = 0$ for $n \geq 0$
- s -shifted vacua $|s\rangle = \begin{cases} \psi_{s-1} \cdots \psi_0 |0\rangle & s > 0 \\ \psi_s^* \cdots \psi_{-1}^* |0\rangle & s < 0 \end{cases}$
- Bases labelled by sets of integers $\mathcal{T}_{n_p; n_h} = \left\{ \{p_a\}_1^{n_p}; \{h_a\}_1^{n_h} \right\}$:

$$|\mathcal{T}_{n_p; n_h}\rangle = \psi_{-h_1}^* \cdots \psi_{-h_{n_h}}^* \cdot \psi_{p_{n_p}-1} \cdots \psi_{p_1-1} |0\rangle$$

$$|\mathcal{T}_{n_p; n_h}; s\rangle = \psi_{s-h_1}^* \cdots \psi_{s-h_{n_h}}^* \cdot \psi_{p_{n_p}+s-1} \cdots \psi_{p_1+s-1} |s\rangle$$

⊗ Full Hilbert space

$$\mathfrak{h} = \text{span} \left\{ |\mathcal{T}_{n_p; n_h}\rangle \text{ with } n_p, n_h \in \mathbb{N} \text{ and } \begin{array}{l} 1 \leq p_1 < \cdots < p_{n_p} \\ 1 \leq h_1 < \cdots < h_{n_h} \end{array} \quad p_a, h_a \in \mathbb{N}^* \right\}.$$

The vertex operators

⊗ Operators

- Shift operator $e^P |J_{n,n}; s\rangle = |J_{n,n}, s+1\rangle$;
- Current modes $J_k = \sum_{j \in \mathbb{Z}} \psi_j \psi_{j+k}^*$, $k \neq 0$, $[J_k, J_\ell] = k \delta_{k,-\ell}$
- Current operators $\mathcal{J}_\pm(\nu, \omega) = \mp \nu \sum_{k \geq 1} \frac{1}{k} \omega^{\mp k} \cdot J_{\pm k}$
- r -shifted vertex operators $\mathcal{V}(\nu, r | \omega) = \omega^{(\nu+r)J_0} e^{\mathcal{J}_-(\nu+r, \omega)} \cdot e^{\mathcal{J}_+(\nu+r, \omega)} \cdot e^{(r+\nu)P}$

⊗ Matrix elements ('15 K, Maillet)

$$\langle \mathcal{J}_{n_p; n_h} | e^{-\mu P} \mathcal{V}(\nu, r | \omega) e^{(\mu-\nu)P} | \mathcal{J}_{n_k; n_l} \rangle = \frac{(-1)^{\frac{r(r+1)}{2}} \delta_{n_p - n_h, n_k - n_l + r}}{\omega^{(\nu+r)(n_h - n_p - \mu) + \frac{r(r-1)}{2} + r(n_k - n_l)}} \cdot \frac{G(1-\nu)}{G(1-\nu-r)} \cdot \mathcal{F}(\mathcal{J}_{n_p; n_h}; \mathcal{J}_{n_k; n_l} | \nu, \omega)$$

- Well known result for $\mu, r = 0$ and $n_p = n_h = 0$. (Giambelli rep. Schur functions)
- $r = 0$ and $n_p = n_h = 0$ correspond to a Z-measure on partitions.
- General case interpreted as explicit representation for specific skew Schur functions

The correspondence of matrix elements

- Two copies of \mathfrak{h} : $\mathfrak{h}_{\text{eff}} = \mathfrak{h}_L \otimes \mathfrak{h}_R$
- Distance dependent phase $\omega = e^{2i\pi \frac{x}{L}}$
- Associate to local operator $O_{x+1}^{(\alpha)}$ on $\mathfrak{h}_{\text{phys}}$ the operator on $\mathfrak{h}_{\text{eff}}$:

$$\mathcal{O}^{(\alpha)}(\omega) = \sum_{\ell \in \mathbb{Z}} \mathcal{F}_{\ell}(0^{(\alpha)}) \cdot \left(\frac{2\pi}{L}\right)^{\Delta_{\ell}^{(\alpha)}} \cdot e^{2ip_F \ell x} \cdot \mathcal{V}_L(-v_{\alpha}^-, -\ell | \omega^{-1}) \cdot \mathcal{V}_R(v_{\alpha}^+, \ell - o_{\alpha} | \omega).$$

- Let $\mathcal{J}_{m_{p,\pm}; m_{h,\pm}}^{(s)}$ and $\mathcal{J}_{n_{p,\pm}; n_{h,\pm}}^{(s+o_{\alpha})}$ parametrise low-lying excited states.

$$\langle \text{ex}_{\text{out}}; s | O_{1+x}^{(\alpha)} | \text{ex}_{\text{in}}; s + o_{\alpha} \rangle = \left\{ \langle s; \mathcal{J}_{m_{p,-}; m_{h,-}} | e^{-\mu_- P_L} \otimes \langle s; \mathcal{J}_{m_{p,+}; m_{h,+}} | e^{-\mu_+ P_R} \right\} \mathcal{O}^{(\alpha)}(\omega)$$

$$\left\{ e^{(\mu_- - v_{\alpha}^-) P_L} | \mathcal{J}_{n_{p,-}; n_{h,-}}; s \rangle \otimes e^{(\mu_+ - v_{\alpha}^+) P_R} | \mathcal{J}_{n_{p,+}; n_{h,+}}; s + o_r \rangle \right\} \cdot \left(1 + O\left(\frac{\ln L}{L}\right) \right)$$

Form factor expansion and correspondence

- Heuristic saddle-point like analysis of form factor series $0_1^{(\alpha')} = (0_1^{(\alpha)})^\dagger$

$$\langle G.S. | 0_{1+x}^{(\alpha)} 0_1^{(\alpha')} | G.S. \rangle = \sum_{\text{ex}} e^{ix\Delta\mathcal{P}_{\text{ex}}} \langle G.S. | 0_1^{(\alpha)} | \text{ex}; o_\alpha \rangle \langle \text{ex}; o_\alpha | 0_1^{(\alpha')} | G.S. \rangle$$

Approximations for $x \rightarrow +\infty$ asymptotics

- Stationary points \rightsquigarrow endpoints of Fermi zone \equiv discrete integer spectrum;

$$\begin{aligned} \langle G.S. | 0_{1+x}^{(\alpha)} 0_1^{(\alpha')} | G.S. \rangle &\sim \sum_{\mathcal{T}_{n_p; \pm; n_h; \pm}^{(o_\alpha)}} \langle G.S. | 0_{1+x}^{(\alpha)} | \mathcal{T}_{n_p; \pm; n_h; \pm}^{(o_\alpha)} \rangle \langle \mathcal{T}_{n_p; \pm; n_h; \pm}^{(o_\alpha)} | 0_1^{(\alpha')} | G.S. \rangle \\ &\sim \sum_{\mathcal{T}_{n_p; \pm; n_h; \pm}^{(o_\alpha)}} \langle 0 | \mathcal{O}^{(a)}(\omega) e^{\nu_\alpha^+ p_R - \nu_\alpha^- p_R} | \mathcal{T}_{n_p; -; n_h; -}^{(o_\alpha)} \otimes \mathcal{T}_{n_p; +; n_h; +}^{(o_\alpha)} \rangle \langle \mathcal{T}_{n_p; -; n_h; -}^{(o_\alpha)} \otimes \mathcal{T}_{n_p; +; n_h; +}^{(o_\alpha)} | e^{-\nu_\alpha^+ p_R + \nu_\alpha^- p_R} \mathcal{O}^{(a')}(1) | 0 \rangle \\ &\sim \langle 0 | \mathcal{O}^{(a)}(\omega) \mathcal{O}^{(a')}(1) | 0 \rangle \end{aligned}$$

Conclusion and perspectives

Review of the results

- ✓ Mapping of local operators to operators in free boson model;
- ✓ Can be extended to time dependent case to grasp non-linear Luttinger-liquid structure;
- ✓ Reduces universality to saddle-point calculation;

Further developments

- ✳ Treat the case of $c \neq 1$.
- ✳ Develop a saddle-point based classification of universality classes.