Quantum integrable systems, conformal field theories and stochastic processes



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From zeros of random analytic functions to random conformal metrics

On any Riemann surface X with a conformal class \kcal of metrics, and for each positive integer N, there is a space B_N of "algebraic metrics of degree N" known as Bergman metrics. They are induced by holomorphic embeddings of X into projective space PC^N. Any smooth metric can be approximated as N -> infty by degree N Bergman metrics, very much like the Bernstein polynomial approximation to any continous function. But B_N \simeq P_N: = GL(N,C)/U(N) is a symmetric space equivalent to positive NxN Hermitian matrices. Each such matrix P determines a Bergman metric g_P. Using this matrix-metric correspondence we can endow B_N with many natural probability measures Prob_N, such as heat kernel measure on P_N. As N -> infinity the spaces B_N fill out the entire infinite dimensional space of metrics and we get an asymptotic notion of a random metric in a conformal class. They automatically have fixed area.

The holomorphic embeddings are defined by N-tuples of holomorphic functions (better, sections) (f_1, \dots, f_N) and the metric is the Hessian of log \sum_k $|f_k|^2$. If one used only the first section, the same procedure gives the sum of point masses at the zeros of f_1. In fact, such point mass measures are singular metrics which form the boundary of B_N. Thus, the theory above of random Kahler metrics is a generalization of the well-developed theory of zeros of random analytic functions.

My talks will start with the (simpler) theory of random analytic functions and their zeros, then proceed to metrics defined by embeddings by several analytic functions, then proceed to random Bergman metrics and their large N limits.

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