Cargèse school on *Quantum integrable systems, conformal field theories and stochastic processes*
Short talks and posters

Organizers: Jérémie Bouttier, Ivan Corwin, Rémi Rhodes and Vincent Vargas

12-23 September 2016

Short talks (15 min each)

Wednesday 14 September (18:40-18:55)

- **Amol Aggarwal** (Harvard University)
  Current Fluctuations for the Stationary ASEP and Stochastic Six-Vertex Model

  A long-standing conjecture within the framework of Kardar-Parisi-Zhang (KPZ) universality has been to show that the long-time current fluctuations of the stationary asymmetric simple exclusion process (ASEP) along the characteristic line converge to the long-time height fluctuations of the stationary KPZ equation. In this talk, we survey a recent proof of this conjecture. Interestingly, this proof uses a mapping from a ferromagnetic six-vertex model to the ASEP; in particular, many of the results proven for the stationary ASEP can be "lifted" to this ferromagnetic six-vertex model, in a sense to also be explained in this presentation.

Thursday 15 September (18:40-18:55)

- **Zhipeng Liu** (New York University)
  TASEP on a ring

  We consider the totally asymmetric simple exclusion process on a ring with flat and step initial conditions. We assume that the size of the ring and the number of particles tend to infinity proportionally and evaluate the fluctuations of tagged particles and currents as the time tends to infinity. The crossover from the KPZ dynamics to the equilibrium dynamics occurs when the time is proportional to the $3/2$ power of the ring size. We compute the limiting distributions in this relaxation time scale. The analysis is based on an explicit formula of the finite-time one-point distribution obtained from the coordinate Bethe ansatz method.

Friday 16 September (17:40-18:40)

- **Michael Wheeler** (University of Melbourne)
  A generalization of Macdonald polynomials

  Recently, a new class of symmetric polynomials was discovered, mutually generalizing Macdonald polynomials and a family of "spin" Hall–Littlewood polynomials introduced by

Borodin. We will discuss the construction of this family, using an integrable multi-species bosonic model.

- **Olya Mandelshtam** (UC Berkeley)
  Combinatorics of the asymmetric simple exclusion process

  The asymmetric simple exclusion process (ASEP) is an important and well-studied non-equilibrium model in which particles hop left and right on a finite one-dimensional lattice with open boundaries, with parameters α, β, and q describing the hopping probabilities. The two-species ASEP is a generalization in which there are two types of particles, one heavy and one light. We introduce the "rhombic alternative tableaux", whose weight generating functions provide combinatorial formulae for the steady state probabilities of the two-species ASEP. We also introduce "k-rhombic alternative tableaux" with analogous combinatorial formulae for a more general k-species ASEP. Finally, we introduce "rhombic staircase tableaux," which provide combinatorial formulae for the probabilities of the more general 5-parameter two-species ASEP. Using this result, we additionally obtain combinatorial formulae for moments of Koornwinder polynomials (also known as type BC Macdonald polynomials).

- **Alisa Knizel** (Massachusetts Institute of Technology)
  Asymptotics of random domino tilings of rectangular aztec diamonds

  We consider asymptotics of a domino tiling model on a class of domains which we call rectangular Aztec diamonds. We prove the Law of Large Numbers for the corresponding height functions and provide explicit formulas for the limit. Moreover, we establish the convergence of the fluctuations to the Gaussian Free Field in appropriate coordinates.

- **Andrei Prokhorov** (Indiana University-Purdue University Indianapolis)
  Asymptotic analysis of tau-function of Painlevé equations

  Solutions of Painlevé equations are called sometimes nonlinear special functions. They admit representation through Riemann-Hilbert problem, which allows to conduct effective asymptotic analysis. Tau-function is the integral of some polynomial of Painlevé function and its derivatives. Tau function for some particular solutions of Painlevé equations arises in random matrix theory, Ising model and other places. We perform asymptotic analysis of tau-function, paying attention to the constant term. Expression for the last one does not follow from asymptotics of Painlevé functions and requires additional work.

Monday 19 September (11:20-12:05)

- **Yvain Bruned** (University of Warwick)
  Hopf algebras in regularity structures

- **Peter Nejjar** (École normale supérieure)
  From flat TASEP to shocks: transition of fluctuations

  We consider the totally asymmetric, simple exclusion process (TASEP) with periodic initial data. When all particles have the same speed, the exponent of the correlation length is $2/3$, and the appearing limit law is the GOE Tracy-Widom distribution. By introducing a second speed, the periodic TASEP can develop a shock, and together with Ferrari we showed that at the shock the exponent of the correlation length degenerates to $1/3$, and the limit law is a product of two GOEs. Here we consider a critical scaling between these two statistical regimes and show that there is a smooth transition from one to the other.

- **Ananth Sridhar** (UC Berkeley)
  Limit shapes in the stochastic six vertex model

  The six vertex model can be reformulated as a theory of random stepped surfaces called height functions. In the thermodynamic limit, the random height function of the model typically converge to a deterministic limit shape. We study the limit shapes of the six vertex model with "stochastic" weights for which the transfer matrix is the transition matrix of a Markov process. We show that the limit shapes are determined by a conservation law type PDE, that can be solved by the method of characteristics.

Wednesday 21 September (11:20-12:05)

- **Jimmy Hutasoit** (Leiden University)
  Chiral conformal field theory and electrons in a quantum Hall edge

  The edge of a quantum Hall state can be described by a chiral conformal field theory. An ubiquitous feature of such an effective edge theory is that it exhibits emergent symmetries that were not parts of the underlying Hamiltonian. As a result, electron operators constructed out of the effective degrees of freedom form multiplets transforming under the emergent symmetry. I will describe two ways of getting rid of this unwanted symmetry, either by considering deformations to the chiral CFT, which will break the emergent symmetry, or by explicitly constructing the theory such that it has exactly one electron operator. The former can be related to the so-called warped CFT while the latter is related to an open problem in classifying non-trivial simple current with trivial tensor structure.

- **Yoshiki Fukusumi** (Institute for Solid State Physics, University of Tokyo)
  Deformations of conformal field theories and Schramm-Loewner evolutions

  We introduce some (integrable) deformations of conformal field theories and corresponding Schramm-Loewner evolutions. The fundamental concept is SLE/CFT correspondence based on operator martingale. In this short talk I briefly discuss various forms of SLE and CFT. And then, we introduce our recent results of multiple Schramm-Loewner evolutions on coset Wess-Zumino-Witten models [1].

  [1] Y. Fukusumi, K. Sakai, in preparation.

- **Oleksandr Gamayun** (Leiden University)
  Topological aspects of the time-dependent generalized sine kernel Fredholm determinants

  We consider Fredholm determinants with the so-called time-dependent generalized sine kernel introduced in [arXiv:1011.5897]. These determinants are appropriate for a description of two-point functions in a wide class of integrable models. The long-distance/long-time asymptotic behaviour of these objects has been analysed by means of Riemann-Hilbert problem (RHP) [arXiv:1011.5897]. We re-derive this asymptotic by means of the summations of microscoptic form-factors (similar to Refs. [arXiv:1206.2630,1501.07711]). This allows us to bypass restrictions on the kernel needed for the RHP analysis. In particular, we consider the possibility for certain periodic functions in a kernel to have a topological phase-slip. We study how these phase-slips affect the asymptotic behaviour and demonstrate how they appear in a specific physical models.

Posters

A poster session is planned on Tuesday 20 September 11:20-12:10 but, weather permitting, we will try to keep the posters displayed in the patio all the time during the school.

- **Ian Alevy** (Brown University)
  Building surfaces with polygons

  A *polydron* is a graph embedded on a surface together with a continuous map into Euclidean 3-space which maps faces to regular polygons. When the underlying surface is homeomorphic to the sphere and all faces have degree five we prove that the image of the polydron can be decomposed into dodecahedra. Using different techniques we show that the image of a genus zero polydron whose faces are of degree four and eight can be decomposed into cubes and octagonal prisms. Finally, we introduce a model for constructing random polydrons.

- **Dan Betea** (Université Paris-Diderot)
  Simple growth diagram rules for all sorts of geometric and tropical Robinson–Schensted–Knuth correspondences, last passage percolation and discrete polymers

  Based on recent work of Borodin–Petrov and Petrov–Matveev and some older results, we present a $(\max, +)$–algebra realization of the row insertion and column insertion RSK correspondences. Based on Borodin–Petrov, we can mix the two and get a plethora of other in–between bijective correspondences. Well–known connections to diagonal (North–West paths; for row insertion) and anti–diagonal (South–West paths; for column insertion) last passage percolation models via Green's theorem will be discussed. One feature of the rules is they detropicalize (change to $(+, \times)$–algebra) easily. We will present these simple versions of what is called geometric RSK that lead to partition functions of diagonal or anti–diagonal polymers via simple geometric growth diagram rules equivalent to the corresponding descriptions of O'Connell et al.

- **Xiangyu Cao** (Université Paris-Sud Orsay)
  One-step replica symmetry breaking and extreme value statistics of 2d Gaussian Free Field

  I will explain the replica approach to the problem of extreme extreme value statistics of 2d Gaussian Free Field. I will recall, with old and new examples, how this approach turns exactly solved Coulomb gas integrals into analytic predictions, e.g., of minimum distributions of a 2d Gaussian Free Field restricted on a circle. I will clarify the close relation between the replica symmetry breaking involved and the freezing phenomenon. Then, I will extend the approach to higher order minima, which are believed to be described by a randomly shifted decorated Gumbel Poisson point process. I will give a physicist's derivation, with a new and general description of the decoration process. Finally high-precision numerical tests of the theory will be presented.

- **Brad Elwood and Dražen Petrović** (Indiana University-Purdue University Indianapolis)
  Pfaffian sign problem for the dimer model on a triangular lattice

  Joint work with Dr Pavel Bleher.

- **Boris Hanin** (Massachussets Institute of Technology)
  Scaling of harmonic oscillator eigenfunctions and their nodal sets around the caustic

  Random eigenfunctions of energy E for the isotropic harmonic oscillator in R^d have a $U(d)$ symmetry and are in some ways analogous to random spherical harmonics of fixed degree on S^d, whose nodal sets have been the subject of many recent studies. However, there is a fundamentally new aspect to this ensemble, namely the existence of allowed and forbidden regions. In the allowed region, the Hermite functions behave like spherical harmonics, while in the forbidden region, Hermite functions are exponentially decaying and it is unclear to what extent they oscillate and have zeros.

The purpose of this talk is to present several results about the expected volume of the zero set of a random Hermite function in both the allowed and forbidden regions as well as around the caustic. These results are based on a new explicit formula for the scaling limit around the caustic of the fixed energy spectral projector for the isotropic harmonic oscillator. This is joint work with Steve Zelditch and Peng Zhou.

- **Alexander Moll** (Institut des Hautes Études Scientifiques)
  Correspondence principle for the quantum Hopf-Burgers equation

  After Fourier series, the quantum Hopf-Burgers equation $v_t + vv_x = 0$ with periodic boundary conditions is equivalent to a system of coupled quantum harmonic oscillators. Preparing these oscillators in *coherent states* as initial conditions, if we send the displacement of each oscillator to infinity at the same rate, we (1) confirm and (2) determine corrections to the quantum-classical *correspondence principle*. Diagonalizing the quantum Hamiltonian with Schur functions, this is equivalent to proving (1) the concentration of profiles of Young diagrams around a limit shape and (2) their global Gaussian fluctuations for Schur measures with symbol $v : T- > R$ on the unit circle T. We identify the emergent objects as the *push-forward* along v of (1) the uniform measure on T and (2) $H^{1/2}$ noise on T.

- **Mihai Nica** (New York University)
  Intermediate disorder limits for multi-layer polymers

  For a simple symmetric random walk in a disordered environment, Alberts, Khanin and Quastel (2014) proved that under intermediate disorder scaling (in which time and space are scaled diffusively, and the strength of the environment is scaled to zero in a critical manner) the polymer partition function converges to the solution to the stochastic heat equation with multiplicative white noise. We present an analogous result for multi-layer polymers started and ended grouped together. The limiting object now is the multi-layer extension of the stochastic heat equation introduced by O'Connell and Warren (2015).

- **Yi Sun** (Columbia University)
  Laguerre and Jacobi analogues of the Warren process

  We define Laguerre and Jacobi analogues of the Warren process. That is, we construct local dynamics on a triangular array of particles so that (1) the projections to each level recover the Laguerre and Jacobi eigenvalue processes of Konig-O'Connell and Doumerc and (2) the fixed time distributions recover the Laguerre and Jacobi corners processes. Our techniques extend and generalize the framework of intertwining diffusions developed by Pal-Shkolnikov. One consequence is a construction of a particle system with local interactions whose fixed time distributions recover the hard edge of random matrix theory.

- **Hugo Tschirhart** (Université de Lorraine and Coventry University)
  Numerical results using a determinant representation of the domain-wall boundary condition partition

  In this work we present our numerical results for two different physical quantities, the magnetization along the z-axis and the bosonic occupation when the times goes to infinity for rational (XXX) Richardson-Gaudin models which contains N spins 1/2 and M bosons. These results are valid regardless the Hamiltonian we chose as long as it is a linear combination of the conserved quantities of the Richardson-Gaudin models.

- **Sascha Wald** (Université de Lorraine)
  Lindblad dynamics of a quantum spherical spin

  The coherent quantum dynamics of a single bosonic spin variable, subject to a constraint derived from the quantum spherical model of a ferromagnet, and coupled to an external heat bath, is studied through the Lindblad equation for the reduced density matrix. Closed systems of equations of motion for several quantum observables are derived and solved exactly. The relationship to the single-mode Dicke model from quantum optics is discussed. The analysis of the interplay of the quantum fluctuation and the dissipation and their influence on the relaxation of the time-dependent magnetisation leads to the distinction of qualitatively different regimes of weak and strong quantum couplings. Considering the model's behaviour in an external field as a simple mean-field approximation of the dynamics of a quantum spherical ferromagnet, the magnetic phase diagram appears to be re-entrant and presents a quantum analogue of well-established classical examples of fluctuation-induced order.

  Reference: J. Phys. A: Math. Theor. 49 (2016) 125001 IOPSelect