PCAC Condition in the Quark Model

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1 Introduction

• The success of the nonrelativistic quark model in describing hadron spectroscopy shows that quarks confined in hadrons acquire an effective mass, about 1/3 of the nucleon mass (the so-called constituent quark mass) and that their motion is nonrelatiavistic to a good approximation.

• An important consequence of the quark model is that a new degree of freedom is needed to preserve the anti-symmetry of the wave function of a fermion bound state system(the generalized Pauli principle). This is the discovery of the color, in addition to the spin, as a new degree of freedom. This is a great success of the quark model, for which the Orsay Quark Model Group has made many important contributions, to mention a few : Quark pair creation model, Heavy to light Form factor in the heavy mass and large energy limit and many other important contributions.

• The dynamic of quarks is governed by Quantum Chromodynamics(QCD), with gluons as massless gauge vector boson of an exact $SU(3)$ color-symmetry. It has also an approximate $SU(3) \times SU(3)$ flavor symmetry, with $SU(2) \times SU(2)$ almost an exact chiral symmetry, due to the small pion mass compared with other low-lying pseudo scalar meson mass. This gives PCAC and soft pion and current algebra technique used in the past for pion physics.

• PCAC with the quark model allows us to compute hadron-pion coupling constants using the quark model in a very simple manner. This is the subject of this talk.

• Consider now the divergence of the isovector axial vector current in $SU(2) \times SU(2)$:

$$\partial_\mu(\bar{u}\,\gamma_\mu\gamma_5\,d) = -i\,(m_u + m_d)\bar{u}\,\gamma_5 d \tag{1}$$

which vanishes in the limit of massless current quark mass. This is an operator relation and one should get a relation between the form factor (at $q^2 = 0$) for the matrix element of the axial-vector current element and the pion-hadron coupling constants given by the pion pole term. This is the generalized Goldberger-Treiman(GT) relation, the most familiar such relation is naturally the GT relation for G_A/G_V in neutron β decay.

• Thus the non-vanishing of G_A/G_V, or the non-vanishing of the nucleon mass implies the existence of the pion pole term. • Since the axial vector current matrix elements can be computed in a simple manner in the quark model, it would be possible to obtain the pion-hadron coupling constants with chiral symmetry constraints imposed. This is PCAC in the Quark Model.

2 PCAC in the Quark Model

• Let $g_{\pi qq}$ be the coupling constant of pion with quarks, since the motion of quarks in the QCD potential assumed to be nonrelativistic, in the limit $q^2 \to 0$ and for charged pion, we have:

$$2\,m_q < \gamma_5 > + \frac{g_{\pi\,qq}}{f} < \gamma_5 >= 0, \quad f^{-1} = f_\pi \tag{2}$$

where $< \gamma_5 >= \bar{q}\gamma_5 q >$, and $g_{\pi qq}$, the $\pi^+ qq$ vertex function when the quark momenta are on the mass shell ($\not{p}_1 = \not{p}_1 = m_q$), m_q being constituent quark mass, Eq. (2) then gives:

$$g_{\pi qq} = -2\,m_q\,f \tag{3}$$

which is the GT relation at the quark level.

• In the presence of chiral symmetry breaking, the octet pseudo scalar mesons acquire masses which are related to the current quark mass, and for $q^2 \rightarrow 0$, we have

$$f_\pi \, g_{\pi qq} + (m_u + m_q) = 0$$
$$f_K \, g_{Kqq} + (m_u + m_s) = 0 \qquad (4)$$

• Assuming $SU(3)$ symmetry relation between the dimensionless meson-quark coupling constants:

$$g_{\pi^+ qq} = g_{K^+ qq} \qquad (5)$$

we get

$$\frac{f_K}{f_\pi} = \frac{m_u + m_d}{m_u + m_s} \qquad (6)$$

With $m_u = m_d = 350\,\mathrm{MeV}$, this gives:

$$\frac{f_K}{f_\pi} = 1.2 \qquad (7)$$

which agrees well with the experimental value of 1.28.

• Eq. (2) , when applied to the matrix element for the axial vector current between nucleon states, gives the GT relation for G_A/G_V:

$$g_{\pi^+pn}\langle\frac{\tau^+\vec{q}\cdot\vec{\sigma}}{2\,m_N}\rangle = g_{\pi^+qq}\left\langle\frac{\sum_i\tau_i^+\vec{q}\cdot\vec{\sigma_i}}{2\,m_q}\right\rangle \quad (8)$$

where $\frac{1}{2}\tau^+\vec{\sigma}$ and $\frac{1}{2}\tau_i^+\vec{\sigma_i}$ are the nucleon and quark Gamow-Teller transition operators, respectively.

- Since

$$\left\langle\frac{1}{2}\sum_i\tau_i^+\vec{\sigma_i}\right\rangle = \frac{G_A}{G_V}\langle\tau^+\vec{\sigma}\rangle \quad (9)$$

we have

$$g_{\pi^+pn} = -\frac{G_A}{G_V}\frac{2m_N}{f_\pi} \quad (10)$$

- This is the GT relation for G_A/G_V in terms of the π^+pn coupling constant.

- Using the nonrelativistic quark model [SU(6)] value for G_A/G_V, we get

$$\frac{g^2_{\pi^+pn}}{4\pi} = 42. \quad (11)$$

which is bigger than the measured value by 40%

• This discrepancy could be attributed to a renormalization due to QCD interactions for G_A/G_V of confined quarks and/or possible configuration mixing of the nucleon wave function as discussed in the literature.

• Similar relations can be obtained for any bound states with nonvanishing Gamow-Teller matrix elements. In particular, the matrix elements of the axial-vector current between vector-meson states is another example in which the dynamics can be described by a nonrelativistic constituent quark model which is successful to some extent in describing radiative transitions between low-lying meson states.

• Consider now the axial vector current matrix elements between $\omega(p, \epsilon^\omega)$ and $\rho^0(p', \epsilon^\rho)$ defined as

$$\langle \omega(p) | A_{3\mu}(0) | \rho^0(p') \rangle = i\, \epsilon_{\mu\nu\rho\sigma}(p_\nu + p'_\nu)\epsilon^\omega_\rho \epsilon^\rho_\sigma F_A(q^2). \tag{12}$$

• The $\omega\rho^0\pi^0$ vertex is

$$\langle \omega(p) | j_3(0) | \rho^0(p') \rangle = i\, \epsilon_{\mu\nu\rho\sigma} q_\mu p_\nu \epsilon^\omega_\rho \epsilon^\rho_\sigma g_{\omega\rho^0\pi^0}(q^2). \tag{13}$$

($q = p - p'$ being the momentum carried by the neutral pion and $j_3(0)$, the source of the pion field.

• The GT relation for vector meson can be obtained either from PCAC or from the analog of Eq. (8) applied to 1^- states. For $q^2 \to 0$, we have:

$$2F_A(0) + \frac{f_\pi}{\sqrt{2}} g_{\omega\rho^0\pi^0} = 0. \tag{14}$$

• In the nonrelativistic quark model, $F_A(0)$ is given by the Gamow-Teller transition matrix element for a $1^- \to 1^-$ transition with the axial vector current matrix element:

$$\langle \omega(p) | A_{3\mu}(0) | \rho^0(p') \rangle = 2p_0 \langle \chi_\omega^\dagger | \sum_a \frac{\tau_a^3}{2} \sigma_{ai} | \chi_\rho \rangle \tag{15}$$

• Since ω and ρ meson are in a triplet state according to the $q\bar{q}$ bound-state picture of low-lying mesons, comparing Eq. (12) with Eq. (15), we get $F_A(0) = 1$. This gives

$$g_{\omega\rho^0\pi^0} = -\frac{2\sqrt{2}}{f_\pi} \tag{16}$$

• This corresponds to a value of $0.6/m_\pi^2$ for $g^2_{\omega\rho^0\pi^0}/4\pi$ which is only 30% larger than the experimental value of $(0.41 \pm 0.09)/m_\pi^2$ obtained from $\omega \rightarrow 3\pi$ decay.

• To produce an $\omega\rho\pi$ coupling closed to the measuresd value, as with the case of nucleon G_A/G_V, one needs an effective $F_A(0)$ lower than the nonrelativistic quark model value by 20%. Configuration mixing of the qq wave function and renormalization of G_A/G_V for bound quarks may be responsible for this reduction. Since the axial vector currents for current quarks are found not to be renormalized in deep-inelastic neutrino hadron interactions, renormalization of the axial vector current for constituent quarks would probably come from confinement effects associated with transverse motion of quarks in a QCD potential.

• Standard quark-model and MIT-bag-model calculations seem to obtain such a reduction of the right order of magnitude. We believe that this reduction of the single-quark matrix element $< \bar{q}\gamma_\mu\gamma_5 q >$ is more important than configuration mixing since it reduces also the $SU(6)$ value for the nucleon G_A/G_V by the same amount.

• Assuming that $< \bar{q}\gamma_\mu\gamma_5 q >$ for a single bound quark is renormalized with $g_A = 3/4$, obtained from the measured nucleom G_A/G_V, we have

$$g_{\omega\rho^0\pi^0} = -\left(\frac{2\sqrt{2}}{f_\pi})g_A\right) \tag{17}$$

which gives,

$$\frac{g^2_{\omega\rho^0\pi^0}}{4\pi} = \frac{0.36}{m_\pi^2} \tag{18}$$

in good agreement with the measured value of $(0.41 \pm 0.09)/m_\pi^2$ mentioned above.

• For the axial vector current matrix elements between D^{*+} and D^{+}:

$$\langle D^{*+}(p)|A_{3\mu}(0)|D^{+}(p')\rangle >= \epsilon_{\mu}G_{1} + \epsilon\cdot q[(p+p')_{\mu}G_{+} + q_{\mu}G_{-}] \qquad (19)$$

in terms of $G_{1}(q^{2}), G_{+}(q^{2}), G_{-}(q^{2})$ form factors, similar PCAC condition gives,

$$g_{D^{*+}D^{+}\pi^{0}} = -(\frac{\sqrt{2}}{2})\left(\frac{M_{D^{*+}}}{f_{\pi}}\right)g_{A}. \qquad (20)$$

for the $g_{D^{*+}D^{+}\pi^{0}}$ coupling constant in $D^{*+}\rightarrow D^{+}\pi^{0}$ decay.

• With $g_{A} = 3/4$, we get

$$\Gamma(D^{*+}\rightarrow D^{+}\pi^{0}) = 42\,\mathrm{keV}, \qquad \Gamma(\mathrm{D}^{*0}\rightarrow \mathrm{D}^{0}\pi^{0}) = 86\,\mathrm{keV}, \qquad (21)$$

This value for $D^{*+}\rightarrow D^{+}\pi^{0})$ is somewhat larger than the measured value by 30%. This indicates that there could be some small contribution from the neglected $G_{+}(0)(p^{2}-p'^{2})$ form factor terms,

• It is doubtful whether Eq. (20) can be applied to $K^{*}\rightarrow K\pi$ and $\rho\rightarrow\pi\pi$ decays, since the contribution from $G_{+}(0)$ cannot in general be neglected. However if one ignores $G_{+}(q^{2})$ as usually done in nonrelativistic calculations, then Eq. (20) can be used to obtain $g_{K^*K\pi}$ and a decay rate

$$\Gamma(K^{*+} \to K^+\pi^0) = \frac{g_A^2}{12\pi}\left(\frac{p_{\rm c.m.}}{f_\pi}\right)^2 p_{\rm c.m.} \tag{22}$$

calculated to be 15 MeV, in good agreement with the measured value of 17 MeV, indicating that $G_+(q^2)$ is negligible. It follows then that $G_+(q^2)$ for $\rho^+ \to \pi^+$ transitions is also small and $g_{\rho\pi\pi}$ can now be obtained in terms of $G_1(0)$ only and is given by

$$\frac{g_{\rho\pi\pi}^2}{4\pi} = \frac{g_A^2}{4\pi}\left(\frac{2m_\rho^2}{f_\pi^2}\right) = 2.68. \tag{23}$$

in good agreement with the value of 2.86 obtained from the ρ meson width.

3 Conclusion

The suppression of the axial vector current for single quark in nucleon by a factor of 3/4 relative to the $SU(6)$ value occurs also in the axial vector current for single quark in processes with vector mesons and pions with the same suppression factor of 3/4. This enables us in a simple manner, to obtain a quark model calculation of the pion-hadron coupling constants. The coupling constants we obtained are in good agreement with experiments.

• A final note. While preparing this talk, I learned that the suppression factor 3/4 relative to $SU(6)$ value for nucleon G_A/G_V has also been invoked by H. Lipkin in 1992, 10 years after this work (e.g. H. Lipkin, Paradoxes in Strange Quark Contributions to Hyperon Spins, WIS-92/105/Dec-Ph), hep-ph/9212316, which contains the following statement: "$G_A/G_V = 1.259 \pm 0.004$ for the neutron β decay is in strong disagreement with the $SU(6)$ prediction $G_A/G_V = 5/3$ implies $(g_A/g_V)_{d\to u} \approx 3/4$"

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