Our contributions to the development of Relativistic Quark Models

Alain Le Yaouanc

Laboratoire de Physique Théorique, Orsay

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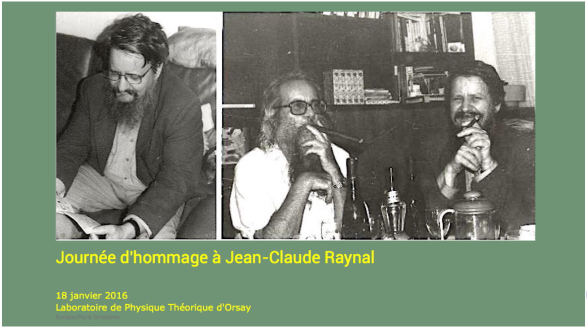

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Plan

- Motivations of the quark model after the advent of QCD
- Weaknesses of the Non-Relativistic Quark Model (NRQM)
- The problems encountered in going beyond NRQM
    - The phenomenological problem
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- A short overview of our efforts
    - The potential model with spontaneous chiral symmetry breaking
    - The Bakamjian-Thomas approach
- Phenomenological successes and perspectives in hadron-hadron transitions

Motivations of the quark model after the advent of QCD

- QCD – at last the true theory of strong interaction
- Why still a need for Quark Models?
- Lagrangian simple but the answer for non-perturbative phenomena notoriously complex (spectroscopy, hadronic transitions): Lattice QCD gives the answer, but it needs time – especially for excited states (tenths of years! $\approx$ time of our career)
- Quark Models still necessary 1) for processes where Lattice QCD cannot answer yet and 2) crucially, to understand the numerical results provided by lattices. But it was requiring improvement

Weaknesses of the Non-Relativistic Quark Model

Indeed, except for heavy quarkonia, NRQM is not quantitatively predictive

- The pion: Why its mass is so low? Spontaneous $\chi - S$ breaking
- The large internal quark velocities within hadrons, manifested through $\Delta E \simeq m_q$, $\hat{g} = 0.5$ vs 1 ($NRQM$)
- The large external momenta (velocities) of hadrons
- The large changes of masses in hadron transitions:
  - strong interaction decays ($q\bar{q} \to q\bar{q}q\bar{q}$, ${}^3P_0$)
  - heavy-to-light quark weak transitions (e.g. $B \to \pi\ell\nu$)

The problems encountered in going beyond NRQM

[A “relativistic” QM is simply trying to do better than NRQM somewhere]

- No common solution to all the problems of NRQM, except QCD. Not “the” RQM but RQM’s.
- Principle :Maintaining the Spirit of Quark Models, i.e.
  Quantum Field Theory (too many degrees of freedom)
  $\longrightarrow$ Quantum Mechanics (*constituent picture, 3 dimensional*) with relativistic ingredients like $\frac{p^2}{2m_q} \to \sqrt{p^2 + m_q^2} \qquad \chi_s \to u_s$
  Hadron c.o.m. motion much more difficult to handle.
- Little can be retained from QCD: linear confining potential and short distance physics (OGE)
- Why then believe that RQM’s exist ? No reason *a priori*. The proof is just
  agreement with experiment and with numerical simulations of QCD!

The phenomenological problem

→Importance to compare with experiment and with Lattice QCD

- Look for favorable conditions to compare RQM's with experiment
- Heavy-light mesons favorable:
  - phenomenologically appealing [weak interaction, CP-violation]
  - fertile ground for testing ideas about strong interaction

  However, very few optimal cases for an accurate comparison in practice
- Lattice QCD offers a complementary laboratory to compare RQM's with QCD instead of experiment, with more favorable situations. Economical ! See last transparent

About the multiplicity of quark models

Proliferation of models → one must make distinctions :

- Different models may correspond to either
    - different formalisms [Bethe-Salpeter, Dirac, Bakamjian-Thomas] or
    - different potentials [Godfrey-Isgur, Veseli-Dunietz,...]
- Certain approaches simply miss basic principles like closure (unitarity, sum rules)
- Some models may be more adapted to a specific situation than the others
  e.g. NRQM to Υ's, Dirac to light quark currents with heavy source, Bakamjian-Thomas well adapted to both heavy and light currents in heavy-light systems
- The potential parameters should be fixed from spectroscopy only, while agreement should be required for a large set of similar processes ⟶ Finally, very few models pass the "exam"

A short overview of our efforts

[Selected topics]

- We have made many contributions to the analysis of relativistic effects: Lorentz contraction of wave functions, Wigner rotations etc. [e.g. explaining $SU(3) \otimes SU(3)$ configuration mixing of Franco]
- Two main sets of works, in which Jean-Claude has played a particularly important role:
  - Potential model with spontaneous chiral symmetry breaking [Ono]
  - Bakamjian-Thomas formalism [Morénas]

The potential model with spontaneous χS breaking

- Problem of accommodating the lightness of pion in the usual potential quark models: adding hyperfine force helps but calculability compromised (no viable prediction)

- Correct explanation of the lightness of pion is known since Nambu and Jona-Lasinio: spontaneous χS breaking. Independent of the hyperfine force. Bethe-Salpeter equation

- Is it possible to combine the constituent quark picture with the Nambu-Jona-Lasinio mechanism (NJL)? The answer is yes – in principle: substitute the local four-fermion interaction of NJL by the three-dimensional potential V; If V is χ-symmetric, then a massless bound state is automatically generated at zero quark mass

- Main obstacle: the chiral $\gamma_\mu \otimes \gamma_\mu$ (with $\mu = 0$) generates too large a spin-orbit splitting

- Moreover: instantaneous potential excludes covariance $\Rightarrow$ model OK for spectroscopy but not for the hadronic transitions

The Bakamjian-Thomas approach (BT)

- General BT framework stemming from the research by Foldy. Nuclear physicists! The null-plane formalism by Terent'ev is similar
- Starting from the standard one-particle variables, one defines two sets:
  - global variables $\vec{P}$, $\vec{R}$, $\vec{S}$, describing the state as a whole
  - internal variables, $\vec{k}_i$ ($\sum_i \vec{k}_i = \vec{0}$), $\vec{s}_i$
- Generators of the group of Poincaré are the same as for the free particle, except that the free mass is replaced by the bound state mass operator M_{op} (must be a rotationally invariant function of internal variables)

$$\boxed{\vec{J} = \vec{R} \times \vec{P} + \vec{S} \qquad H = \sqrt{M_{op}^2 + \vec{P}^2} \qquad \vec{K} = -\frac{1}{2}\{H, \vec{R}\} - \frac{\vec{P} \times \vec{S}}{H + M_{op}}}$$

- The internal w.f. $\phi_n(\vec{k}_i)$ are the eigenstates of the mass operator M_{op}

Matrix elements of currents

[remarkable properties in the $m_Q \to \infty$ limit]

- Current assumed to be a sum of one-body j_μ operators for each quark (additivity principle)
- Main problem is the lack of covariance and of the conservation of vector current (CVC)
- However, in the $m_Q \to \infty$ limit of the heavy-light systems, for the heavy→heavy currents, the miracle happens: both covariance and CVC recovered. Moreover, a large set of general HQET properties satisfied: Isgur-Wise scaling, Bjorken and Uraltsev sum rules. In addition, **a whole series of new HQET sum rules** (→ cf. Lluis)
- LEET limit ($m_Q \to \infty$, $q^2 \to 0$) : BT also covariant for heavy-to-light currents $\oplus$ Only three independent form factors for $B \to P(V)$ weak transitions, i.e. the general result in the large energy limit of the light meson [Charles]

Choice of the mass operator M_{op}

- To fix entirely the model one needs the mass operator M_{op}. A natural structure is

$$M_{op} = \sum_i \sqrt{\vec{k}_i^2 + m_i^2} + V$$

- For V one can choose a standard potential: a very good choice is the one by Godfrey and Isgur, tested on the spectroscopy, for a very large set of hadronic states

"Phenomenological" successes and perspectives in hadron-hadron transitions

- One main application is Isgur-Wise functions (heavy-light hadron transitions through the heavy-quark current). For transitions to excited states of mesons, the best testing ground is the non-leptonic decays such as $B \to D^{**}\pi$
- Very encouraging success for $L = 0 \to L = 1$ transitions. The striking expectation for rates "$j_{3/2} \gg j_{1/2}$" is now well verified (Belle, LHCb) after twenty years of doubt.
  Also, agreement with lattice QCD
- Another successful application are the transition matrix elements via the light quark currents, in the static limit. A remarkable agreement with lattice QCD has been found in the elastic case for the current densities as function of the distance r from the static quark: one finds a zero of $\rho_A(r)$ for $L = 1$, a purely relativistic effect
- Very encouraging prospects for light quark transitions to radial excitations with similar densities

Illustration of validity of the BT approach
Dirac models also relevant!

- Using Godfrey-Isgur w.f., the BT approach reproduces remarkably well the lattice QCD results for radial distribution of various current densities in the $m_Q \to \infty$ limit. Dirac does well also. No adjusted parameter. The zero below would be absent in the NRQM → **there are RQM's**

- Example of the density $\rho_A(r)$: $\langle B_1 | \bar{u} \vec{\gamma} \gamma_5 d | B_0^* \rangle = \int_0^\infty \rho_A(r) d\vec{r}$

B_1, B_0^* being $(1/2)^+$-states