La Limite des Quarks Lourds de QCD

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Je suis inconsolable à l'idée de ne plus pouvoir voir apparaître la silhouette de Jean-Claude, au fond du couloir, toujours le jour convenu pour notre réunion.

Jean-Claude ne prenait pas soin de lui-même ces dernières années. Mon idée c'est que son monde mental lui suffisait.

C'est comme si l'harmonie, que pour lui ne pouvais être que mathématique, il la laissait pour l'extérieur de lui-même, en produisant des choses exceptionnelles et modestes en même temps.

Jean-Claude était l'attachement aux principes, à la rigueur logique et mathématique, tout en étant un esprit physicien.

Question pertinente pour une personalité comme Jean-Claude : comment caractériser sa philosophie ? Platonicien certes, à cause des mathématiques. Mais je dirais plutôt galiléen :
"la nature est un livre écrit en langage mathématique".

Je me souviens d'une discussion avec lui sur l'art. Il m'a dit : mais qu'est-ce qu'il y a de mieux que l'exactitude mathématique ? C'est une position platonicienne sur les arts plastiques.

Nous avions souvent des discussions de politique ou de philosophie. À propos de platonicien/galiléen, c'est significatif qu'un jour il nous ait fait un petit exposé sur les *Méditations Métaphysiques* de Descartes : "Je doute, donc le monde extérieur existe".

Pour ne pas tomber dans un "culte de la personalité", qui serait drôle chez des anciens gauchistes ("La Bande des Quatre"), il faut dire que chacun d'entre nous a été pour les autres, de façon complémentaire, une fenêtre sur le réel et sur la physique.

Si Jean-Claude nous a apporté ses qualités exceptionnelles de rigueur et de compétence mathématique, par notre curiosité et nos capacités, nous lui avons procuré des éléments d'une vision assez globale des phénomènes en physique des particules.

Le sujet *La Limite des Quarks Lourds de QCD* permet de souligner deux grandes qualités de Jean-Claude comme scientifique :
- ses capacités techniques et mathématiques hors du commun,
- son souci de partir toujours des premiers principes.

Porter le deuil de Jean-Claude est aussi, en même temps, un moment de réflexion sur notre passé, sur les intérêts scientifiques de Jean-Claude et sur l'ensemble du travail de notre groupe.

J'aimerais donc d'abord situer ce sujet dans l'évolution de nos thèmes de recherche, pour ainsi voir leur continuité.

Et, en même temps, rappeler les noms de nos étudiants et collaborateurs, certains présents ici aujourd'hui, qui ont eu la chance d'interagir avec Jean-Claude au cours des années, et qui font ainsi partie de sa mémoire.

ca. **1970**

- Diffractive Production of N^* in the Quark Model
- Electroproduction and Neutrinoproduction of N^*

(with Platon Andreadis, Aristides Baltas, Claude Longuemare)

- Quark Pair Creation Model of Strong Couplings
- Chiral Mixing from Relativistic Effects in the Quark Model
- SU(6)-Strong Breaking and Nucleon Properties
- Non-leptonic Hyperon Decays and the Baryon Spectrum
- Diquonium $qq\overline{qq}$ Decays; Roper Resonance N^* Puzzles

(with Belen Gavela, Sudhir Sood)

- Van Der Waals Forces Between Hadrons

(with Belen Gavela, Sudhir Sood)

ca. **1980**

- Weak Couplings Induced by QCD Corrections (with Belen Gavela)
- Proton Decay Modes in $SU(5)$ and $SO(10)$ in the Quark Model (with Belen Gavela)
- Current Algebra Sum Rules in the Quark Model (with Belen Gavela)
- The Neutron Electric Dipole Moment in the Quark Model (with Belen Gavela, Pham Tri Nang)
- Dynamical Breaking of Chiral Symmetry in the Quark Model (with Ahmed Amer, Seiji Ono)
- Penguin Contribution to $K \to \pi\pi$ (with Belen Gavela)
- CP Conserving and CP Violating Decays in B Mesons (with Belen Gavela, Mohamed Jarfi, Ouafae Lazrak)

ca. **1980 – 1990**

• $B - \overline{B}$ Transitions in Supergravity

(with Belen Gavela, Mohamed Jarfi, Ouafae Lazrak)

• Hamitonian Lattice QCD with Wilson Fermions in Strong Coupling at Zero and Finite Temperature and Chemical Potential

(with Mohamed Jarfi, Ouafae Lazrak)

• Hybrid $q\overline{q}g$ and Diquonium $qq\overline{qq}$ Decays

(with Farida Iddir, Seiji Ono)

• Hadron Transitions in the Quark Model (Eds. *Gordon and Breach*)

• Form Factors $B \to D^{(*)}$; B_s Decays at LEP

(with Eugene Golowich, Farida Iddir)

• Strongly Interacting Higgs sector (with Farida Iddir)

ca. **1990 – 2000**

- Dynamical Generation of Majorana Masses (with Asmâa Abada)

- K, B and $NEDM$ Phenomenology in the Left-Right model
(with Jean-Marie Frère, Julien Galand)

- Strong CP and $NEDM$ (with Asmâa Abada, Julien Galand)

- Herbst Hamiltonian (with André Martin)

- Gauge Invariance, Dimensional Regularization in FCNC Processes
(with Ruggero Ferrari)

- Theory and Phenomenology of B decays
(with Roy Aleksan, Franco Buccella, Marco Zito)

- The BaBar Book (with Jérôme Charles)

ca. **1990 – 2000**

• Bakamjian-Thomas Relativistic Quark Model of Form Factors in the Heavy Quark Limit $\overline{B} \to D^{(*)}, D^{**}$ (with Vincent Morénas)

• CP Violation in D mesons and FSI; CP Violation in B Decays (with Jérôme Charles)

• Heavy to Light Form Factors in the Heavy Mass and Large Energy Limit of QCD (with Jérôme Charles)

• A Bjorken-like Sum Rule at $O(1/m_Q)$ (with Dmitri Melikhov, Vincent Morénas)

• Factorization vs. Duality in Nonleptonic Decays; Duality in Semileptonic Inclusive B Decays (with Dmitri Melikhov, Vincent Morénas)

ca. **2000 – 2015**

• Light Cone QCD Sum Rule Calculation of $g_{DD^*\pi}$

(with Damir Bećirević, Jérôme Charles)

• Spatial Distributions in Static Heavy-Light Mesons in the Quark Model and Lattice QCD (with Damir Bećirević, Emmanuel Chang)

• The D^{**} Puzzle in Semileptonic B Decays

(with Ikaros Bigi, Benoît Blossier, Arantza Oyanguren, Patrick Roudeau)

• Proposal to Study $\overline{B}_s \to D_{sJ}$ Transitions at LHC

(with Damir Bećirević, Patrick Roudeau, Justine Serrano)

• Phenomenological Analysis of *Belle* Results on $\overline{B} \to D^{**}\pi$

(with Frédéric Jugeau)

ca. **2000 – 2015**

• Sum Rules in the Heavy Quark Limit of QCD

• Bounds on the Derivatives of the Isgur-Wise Function

• Bounds on the Derivatives of the Isgur-Wise Function in the Non-Relativistic Limit (with Frédéric Jugeau)

• Subleading Form Factors of Current and Lagrangian type (with Frédéric Jugeau)

• The Isgur-Wise Function in the BPS Limit (with Frédéric Jugeau)

• Bjorken-Uraltsev Sum Rules and the Lorentz Group

• The Bakamjian-Thomas Relativistic Quark Model Satisfies all the Sum Rules of the Bjorken-Uraltsev Type

Contributions to the Heavy Quark Limit of QCD

Elastic meson transitions $\overline{B}_d \to D\ell\nu$ $\overline{B}_d \to D^*\ell\nu$

Light cloud transitions $\frac{1}{2}^- \to \frac{1}{2}^-$

By spin-flavor Heavy Quark Symmetry $SU(2N_f)$ (N_f heavy flavors)

six form factors $\overline{B} \to D(D^*)\ell\nu \to$ the Isgur-Wise function $\boxed{\xi(w)}$

Excited meson transitions $\overline{B}_d \to D^{**}\ell\nu$

D^{**} with $L = 1$, $P = +$ states $0^+_{1/2}, 1^+_{1/2}, 1^+_{3/2}, 2^+_{3/2}$

Light cloud transitions $\frac{1}{2}^- \to \frac{1}{2}^+$ and $\frac{1}{2}^- \to \frac{3}{2}^+$

two IW functions $\boxed{\tau_{1/2}(w), \tau_{3/2}(w)}$

Bjorken and Uraltsev Sum Rules

Bjorken SR $$-\xi'(1) = \frac{1}{4} + \sum_n \left[|\tau_{1/2}^{(n)}(1)|^2 + 2|\tau_{3/2}^{(n)}(1)|^2 \right] \geqslant \frac{1}{4}$$

Uraltsev SR $$\sum_n \left[|\tau_{3/2}^{(n)}(1)|^2 - |\tau_{1/2}^{(n)}(1)|^2 \right] = \frac{1}{4}$$

Bjorken (1990-1991) + Uraltsev (2001) $\rightarrow$ $\boxed{-\xi'(1) \geqslant \frac{3}{4}}$

Bound already obtained in Bakamjian-Thomas quark models (Le Yaouanc et al. 1996)

- covariant for $m_Q \rightarrow \infty$
- explicit Isgur-Wise scaling
- satisfying Bjorken and Uraltsev SR

Isgur-Wise functions and Sum Rules in HQET

(Bjorken; Isgur and Wise; Uraltsev; Le Yaouanc et al.)

Consider the non-forward amplitude

$$\overline{B}(v_i) \to D^{(n)}(v') \to \overline{B}(v_f)$$

Sum Rule

$$\sum_{D^{(n)}} < \overline{B}_f(v_f)|\Gamma_f|D^{(n)}(v') >< \overline{D}^{(n)}(v')|\Gamma_i|B_i(v_i) > \xi^{(n)}(w_i)\xi^{(n)}(w_f)$$

$$+ \ \textit{Other excited states} = -2\xi(w_{if}) \ < \overline{B}_f(v_f)|\Gamma_f P'_+ \Gamma_i|B_i(v_i) >$$

$P'_+ = \frac{1+\not{v}'}{2}$: positive energy projector on the intermediate c

$$w_i = v_i \cdot v', w_f = v_f \cdot v', w_{if} = v_i \cdot v_f$$

in a domain $D(w_i, w_f, w_{if})$

To get the Sum Rules in covariant form one needs the object

$$\boxed{S_J(w_i, w_f, w_{if}) = v_{f\nu_1} \ldots v_{f\nu_J} T^{v_{f\nu_1} \ldots v_{f\nu_J}, v_{i\mu_1} \ldots v_{i\mu_J}} v_{i\mu_1} \ldots v_{i\mu_J}}$$

Projector on polarization tensor of integer spin J

$T^{v_{f\nu_1} \ldots v_{f\nu_J}, v_{i\mu_1} \ldots v_{i\mu_J}} = \sum_\lambda \epsilon'^{(\lambda)*\nu_1 \ldots \nu_J} \epsilon'^{(\lambda)\mu_1 \ldots \mu_J}$ (depends on v')

$T^{\mu\nu} = -g^{\mu\nu} + v'^\mu v'^\nu \quad (J = 1), \qquad T^{\mu\nu,\rho\sigma} \quad (J = 2), \ldots$

Jean-Claude, in the general case

$$S_J(w_i, w_f, w_{if}) = \sum_{0 \leq k \leq J/2} C_{J,k} (w_i^2 - 1)^k (w_f^2 - 1)^k (w_i w_f - w_{if})^{J-2k}$$

$$C_{J,k} = (-1)^k \frac{(J!)^2}{(2J)!} \frac{(2J-2k)!}{k!(J-k)!(J-2k)!}$$

Light cloud angular momentum j and bound state spin J

$\overline{B}$: pseudoscalar ground state $\left(\frac{1}{2}^-, 0^-\right)$

$D^{(n)}$: tower of states (j^P, J^P) (Falk, 1992)

Heavy quark currents : $\bar{h}_{v'}\Gamma_i h_{v_i}$ $\bar{h}_{v_f}\Gamma_f h_{v'}$

Sum Rules $\boxed{L_{Hadrons}(w_i, w_f, w_{if}) = R_{OPE}(w_i, w_f, w_{if})}$

Differentiating within the domain $D(w_i, w_f, w_{if})$

$$\boxed{\left[\frac{d^{p+q+r}(L_{Hadrons} - L_{OPE})}{dw_{if}^p dw_i^q dw_i^r}\right]_{w_{if}=w_i=w_f=1} = 0}$$

and varying the currents Γ_i, Γ_f

$\rightarrow$ Bjorken-Uraltsev Sum Rules

Inequalities for derivatives

Inelastic IW functions $\tau_{L\pm 1/2}^{(L)(n)}(w)$

Elastic IW function $\xi(w) = \tau_{1/2}^{(0)(n)}(w)$

Slope $-\xi'(1) = \frac{3}{4}[1 + [\tau_{1/2}^{(1)(n)}(1)]^2] \quad \rightarrow \quad \boxed{-\xi'(1) \geqslant \frac{3}{4}}$

L-th derivative $\boxed{(-1)^L \xi^{(L)}(1) \geqslant \frac{(2L+1)!!}{2^{2L}}}$

Improved bound on the curvature $\xi''(1)$

$-\frac{4}{3}\xi'(1) + \xi'(1)^2 - \frac{5}{3}\xi''(1)^2 + \sum_{n\neq 0}[\xi'^{(n)}(1)]^2 = 0$ (only $\frac{1}{2}^-$ states)

$$\boxed{\xi''(1) \geqslant \frac{1}{5}\left[-4\xi'(1) + 3\xi'(1)^2\right] \geqslant \frac{15}{16}}$$

$\frac{3}{5}\xi'(1)^2$: dominant term in non-relativistic limit for the light quark

The so-called BPS limit of HQET

If the slope reaches the lower bound $-\xi'(1) = \frac{3}{4}$ (Uraltsev, 2001)

using the Sum Rules, by induction $\rightarrow$ $(-1)^L \xi^{(L)}(1) = \frac{(2L+1)!!}{2^{2L}}$

implies the defined limit of HQET

$$\boxed{\xi(w) = \left(\frac{2}{w+1}\right)^{3/2}}$$

This limit has a simple group theoretical interpretation

Isgur-Wise functions and the Lorentz group

Factorization into a heavy quark current matrix element and a light cloud overlap, the long distance physics (A. Falk, 1991)

$$< H'(v')|J^{Q'Q}|H(v) > = < Q'(v')|J^{Q'Q}(q)|Q(v) >< v'|v >$$

The light cloud follows the heavy quark with the same four-velocity

The Isgur-Wise functions are the light cloud overlaps

$$\boxed{\xi(v.v') = < v'|v > = < U(\Lambda_{v'})\phi|U(\Lambda_v)\phi > = < \phi|U(\Lambda)\phi >}$$

Sensible hypothesis : light cloud states form a Hilbert space on which acts a unitary representation of the Lorentz group $U(\Lambda)$

Integral formula for the Isgur-Wise function

The unitary representation $U(\Lambda)$ is in general reducible, decomposes into irreducible representations $U_\rho(\Lambda)$

Irreducible Isgur-Wise function $\quad \xi_\rho(w) = < \phi_\rho | U_\rho(\Lambda) \phi_\rho >$

$U_\rho(\Lambda)$ restricted to irreducible representations of $SL(2, C)$ containing a $SU(2)$ subspace of spin j for the light cloud

Isgur-Wise function $\quad \boxed{\xi(w) = \int \xi_\rho(w) \; d\nu(\rho)}$

positive measure $d\nu(\rho)$

Irreducible IW function $\xi_\rho(w)$ when $\nu(\rho) = \delta(\rho - \rho_0)$

Bounds from the positivity of the measure

Integral representation of the ground state IW function ($j = \frac{1}{2}$)

$$\xi(w) = \frac{1}{1+\cosh(\tau)} \frac{1}{\sinh(\tau)}$$

$$\times \int \frac{4}{4\rho^2+1} \left[\sinh\left(\frac{\tau}{2}\right)\cos(\rho\tau) + 2\rho\cosh\left(\frac{\tau}{2}\right)\sin(\rho\tau)\right] \ d\nu(\rho)$$

$(w = \cosh(\tau))$

$\rightarrow$ moments of a variable with positive values, implying

$$-\xi'(1) \geq \frac{3}{4}$$
$$\xi''(1) \geq \frac{1}{5}\left[-4\xi'(1) + 3\xi'(1)^2\right]$$
...

Same results as with Sum Rule approach

A consistency test for any Ansatz of the Isgur-Wise function

Inverting the integral representation

$$\frac{d\nu(\rho)}{d\rho} = \frac{1}{2\pi} \int e^{i\tau\rho} \, d\tau \, \frac{1}{2\cosh\left(\frac{\tau}{2}\right)} \frac{d}{d\tau} \left[(\cosh(\tau) + 1) \sinh(\tau) \xi(\cosh(\tau)) \right]$$

For a given model of $\xi(w)$ $\rightarrow$ $\frac{d\nu(\rho)}{d\rho}$ must be positive

Phenomenological one-parameter examples

Linear form $\xi(w) = 1 - c(w-1)$

Does not satisfy the integral representation

Exponential form $\xi(w) = exp[-c(w-1)]$

Does not satisfy the integral representation

"Dipole" form $\xi(w) = \left(\frac{2}{1+w}\right)^{2c}$

Satisfies the integral representation if slope $c \geq \frac{3}{4}$

The BPS form $\xi(w) = \left(\frac{2}{1+w}\right)^{3/2} \qquad (c = \frac{3}{4})$

corresponds to the irreducible Isgur-Wise function $\nu(\rho) = \delta(\rho)$

Conclusions

• Non-forward amplitude in heavy quark limit $\rightarrow$ Bjorken-Uraltsev Sum Rules give bounds on the derivatives of Isgur-Wise function

• Decomposing into irreducible representations a general unitary representation of the Lorentz group $\rightarrow$ integral formula for the Isgur-Wise function with a positive measure

• Positivity of the measure $\rightarrow$ inequalities between the derivatives, same as obtained from Bjorken-Uraltsev Sum Rules

• Consistency test for any Ansatz of the IW function

• Applications to phenomenological examples

• Completeness in the Hilbert space $\rightarrow$ Bjorken-Uraltsev Sum Rules

Caminante, son tus huellas
el camino y nada más;
Caminante, no hay camino,
se hace camino al andar.
Al andar se hace el camino,
y al volver la vista atrás
se ve la senda que nunca
se ha de volver a pisar.
Caminante no hay camino
sino estelas en la mar.

Marchant, tes empreintes sont
le chemin, et c'est tout;
Marchant, il n'y a pas de chemin,
On fait le chemin en marchant.
En marchant on fait le chemin,
et en regardant en arrière,
on voit le sentier que jamais
l'on n'aura à refouler.
Marchant, il n'y a pas de chemin
mais des sillages dans la mer.

Antonio Machado

(Sevilla, 26 Juillet 1875 - Collioure, 22 Février 1939)