Is It Possible to Use Perturbation Theory for Soft Pion Physics?

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1 Introduction and Plan of Talk

If the strength of strong interaction is characterized by a large coupling constant e.g. the pion nucleon interaction $g^2/4\pi = 15$, then it is obvious that perturbation theory makes no sense. Since the effective interaction of pions at low energy is governed by the chiral theory, the corresponding matrix element at the tree level is given by a power series expansion in pion momenta. Using the perturbation technique for this type of interaction is known as the "Chiral Perturbation Theory (ChPT). Although one also deals here with strong interaction, it is not obvious that the perturbation theory cannot be used here. There are now two approaches to ChPT. One advocated by Lehman in 1972 (H. Lehman, Phys. Lett. **41B**, 529 (1972)) for a zero mass pion and the other one by Weinberg seven years later (S. Weinberg, Physica **A 96** 327 (1979)). ChPT was however first proposed prior to the Lehman paper, in a rough form, by L.F. Li and H. Pagels (Phys. Rev. Letters **27**, 27, 1089 (1971)). In the Lehman approach for the pion pion scattering, one does first the one loop perturbation theory, but then one must unitarize the result in order to preserve the elastic unitarity. He called his approach ”Effective Range Theory” which probably escaped attention of most readers. In the Weinberg approach one has to calculate one loop, two loops ...etc. Because one deals here with a non-renormarlizable theory, the number of parameters grows larger with higher order. This program was followed and accepted by many theorists in the last twenty years. Which approach is correct? Or are they both correct, or wrong? Both methods should be tested by experiments.

Because the resulting S-matrix is in general an analytic function, such a power series expansion is only valid outside the cut. The question is how to continue this function on to the cut in the physical region in order to compare with experiments? A related problem which is also of importance is knowing a few terms in a power series expansion of the space like pion form factor (euclidean physics, e.g. lattice gauge pion form factor calculation) can one say something on its time like behavior in the low energy region?

In this talk I wish to convince you that *the existing pion vector form factor experimental data could be used as a laboratory to test these questions.*

The plan of this talk is as follows. I shall first give a review of the "grandpa" physics, the physics of the 50's and 60's of the last century, and then the "papa" physics of a more recent date.

• *Sum Rules based on the Forward Dispersion Relation, Unitarity and the Low Energy Theorems.* All these Sum Rules are based on the Analyticity of the S-matrix and also on the Conservation of the Probability.

-The Drell-Hearn-Gerasimov Compton Sattering Sum Rule

-The Adler-Weisberger Sum Rule for the Axial Coupling Constant

-The Goldberger-Oehme-Miyazawa Sum Rule for the Forward Pion Nucleon Scattering

• *The Non Relativistic Bethe Schwinger Effective Range Theory and its Relativistic Generalisation (another consequence of the analyticity and unitarity). .*

- *Chiral Perturbation Theory (ChPT) and the Unitarized Chiral Perturbation Theory (UChPT)* . The material used here is based on the non perturbative technique for soft pion physics ((T. N. T., Phys. Rev. Lett. **61** 2526, 1988)), and a number of unpublished papers (.arXiv:1002.2519[hep-ph], arXiv:9809476v1[hep-ph], arXiv:9607378v2[hep-ph])

- *"New" Sum Rules for Low Energy Expansion Parameters of the Vector Pion Form and Comparison with ChPT and UChPT*. These "New" sum rules are even more convergent than those cited above, in particular the r.m.s. radius of the pion is calculated in terms of the time like pion form factor data and the measued pion pion P-wave phase shifts; the higher derivatives of the pion form factor are calculated to test the validity of ChPT and UChPT.

- *Can one add the ChPT calculation to the Vector Meson Dominance Amplitude in order to calculate the pion form factor?*

- *Physics at large N_c and the Pade Approximant method. (Power series expansion in momenta is not as good as the Pade Approximant)*

1.1 The Drell Hearn Gerasimov Sum Rule

S.D. Drell and A.C. Hearn, Phys. Rev. Lett. 16 (1966) 908. S.B. Gerasimov, Sov. J. Nucl. Phys. 2 (1966) 430.

Useful sum rules in particle physics deal with the crossing odd forward amplitude. One writes an unsubtracted dispersion relation and using the low energy theorm for this amplitude. The foward Compton amplitude is:

$$f(\nu) = f_1(\nu^2)\vec{e'}^* \cdot \vec{e} + \nu f_2(\nu^2) i\vec{\sigma} \cdot \vec{e'}^* \times \vec{e} \tag{1}$$

DHG sum rule deals with a DR for $\nu f_2(\nu^2)$

$$\mathrm{Re} f_2(\nu^2) = \frac{1}{4\pi^2} \mathbf{P} \int_0^\infty \frac{[\sigma_A(\nu') - \sigma_P(\nu')}{\nu'^2 - \nu^2} d\nu'^2 \tag{2}$$

$\sigma_A(\nu')$ and $\sigma_P(\nu')$ are respectively the total cross sections with proton and photon spins antiparallel and parallel due to the unitarity relation.

The low energy theorem is:

$$f_2(0) = -\frac{1}{2}(\alpha/M_p^2)\kappa_p^2 \tag{3}$$

where α is the fine structure constant, κ_p the anomalous magnetic moment of the nucleon and M_p is the nucleon mass. We then get the DHG sum rule:

$$\int_0^\infty \frac{d\nu}{\nu}[\sigma_P(\nu) - \sigma_A(\nu)] = \frac{2\pi^2\alpha}{M_p^2}\kappa_p^2 \approx 205\,\mu b \tag{4}$$

Using experimental data on $\sigma_P(\nu)$ and $\sigma_A(\nu)$ (for a review see , K. Helbing, Prog. Part. Nucl. Phys. **57**, 405-469 (2006)), we get:

$$\mathrm{LHS} = 212 \pm 6_{\mathrm{stat}} \pm 16_{\mathrm{syst}}\,\mu b \tag{5}$$

57 Hence the sum rule is **satisfied** !

1.2 The Adler-Weisberger Sum Rule for the axial nucleon coupling constant

$$\frac{1}{g_A^2} = 1 + \frac{2m_N^2}{\pi g_{\pi NN}^2} \int_{\nu_0}^{\infty} \frac{d\nu}{\nu} [\sigma_{tot}^{\pi^- p}(\nu) - \sigma_{tot}^{\pi^+ p}(\nu)] \tag{6}$$

This sum rule was derived using the current algebra technique for a zero mass pion. (S. L. Adler, Phys. Rev. **140**, 736 (1965); W. I. Weisberger, Phys. Rev. **143**, 1302 (1966).

Using the experimental data of the pion nucleon total cross sections on the RHS of this equation one gets, $g_A = 1.24$ as compared with the experimental value 1.259 ± 0.017. The AW Sum Rule is therefore satisfied!

Instead of using the sum rule technique, Weinberg pointed out that the AW sum rule is simply the Weinberg pion nucleon scattering length relations (S. Weinberg Phys. Rev. Lett. **17**, 616 (1966)). His result is independent of the high energy behavior of the pion nucleon scattering amplitude.

2 Goldberger-Oehme-Miyazawa Sum Rule

This sum rule is similar to the Adler-Weisberger sum rule and was one of the earliest sum rule in particle physics, dealing with the forward crossing odd pi nucleon scattering, i.e. the difference between the π^+ and π^- forward amplitude; using unitarity, one can evaluate the value for the difference of the S-wave pion nucleon scattering lengths in isospin 1/2 and 3/2 in terms of the difference between the 1/2 and 3/2 total cross sections (M. L. Goldberger, H. Miyazawa and R. Oehme, Phys. Rev. **99**, 986 (1955). The sum rule result is in good agreement with data.

3 The Bethe-Schwinger Effective Range Theory, Its Generalisation and Introduction to Non Perturbative Method for Low Energy Physics

3.1 The Bethe-Schwinger Effective Range Theory

Landau and Smorodinsky (Landau and J. Smorodinsky, J. Phys. U.S.S.R. **8**, 154 (1944)) suggested that an effective range expansion for the the S-wave phase shifts δ :

$$k \cot \delta(k^2) = -\frac{1}{a} + \frac{1}{2} r_0 k^2 + ... \tag{7}$$

where k the relative momenta, a the scattering length and r_0 the effective range. Schwinger (Phys. Rev. **72**, 742A (1947) was the first person to give a general proof of the effective range expansion Eq. (7). His proof is quite complicated. A year later Bethe (Phys. Rev. **76**, 38 (1949)) gave a much simpler proof based on the physical picture of the potential scattering theory of the Schrodinger equation.

Let us consider, for example, the S-wave scattering amplitude $f(\nu)$ and omit the subscripts or superscripts spin and isospin. Setting $\nu = k^2$, the elastic unitarity relation is:

$$Imf(\nu) = \rho(\nu) \mid f(\nu) \mid^2 \tag{8}$$

where $\rho(\nu) = \sqrt{\nu}$ is the non-relativistic phase space factor and ν is the square of the relative momentum k. Eq. (8) implies that:

$$f(\nu) = \frac{e^{i\delta(\nu)} \sin \delta(\nu)}{\rho(\nu)} \tag{9}$$

which is the same as:

$$f(\nu) = \frac{1}{\rho(\nu)(-i + \cot \delta(\nu))} \tag{10}$$

and hence any function representing δ, in particular, for $tan\delta$ or $cot\delta$

used in Eq. (9) or Eq. (10) would give rise to a partial wave amplitude satisfying the unitarity relation.

There is however a restriction on the choice of the appropriate function, namely the analytic property of the constructed partial wave amplitudes which can be proved from general principles. Since $f(\nu)$ is an analytic function, its inverse is also an analytic function. A polynomial is analytic function hence we arrive at the effective range expansion. This argument leads to the use of:

a) The Inverse Amplitude Method (IAM)

b) The N/D method is used in order to satisfy the elastic unitarity relation. It was Noyes and Wong who first pointed out that the effective range is a consequence of unitarity and analyticity in the pole approximation (the pole approximation which is usually done in the phenomenology of the S-matrix theory in the old day, should be interpreted as the Pade approximant for a power series expansion; it has a good high energy behavior as will be seen later in this talk) . I do not have time to discuss these topics, I wish to refer to my review article.

3.2 Padé Approximant Method and Bubble Summation or How to Unitarise a Perturbation Calculation ?

Let us write the partial wave perturbation series as:

$$f^{pert}(\nu) = f^0 + f^1(\nu) + ... \tag{11}$$

where f^0 is the tree amplitude which is assumed here, for simplicity, a constant or a real polynomial (otherwise it has only the left hand cut singularity). $f^1(\nu)$ is the one loop amplitude satisfying the perturbative unitarity, for $\nu > 0$:

$$Im f^1(\nu) = \rho(\nu)(f^0)^2 \tag{12}$$

Let us construct the $[0,1]$ Padé approximant:

$$f^{[0,1]}(\nu) = \frac{f^0}{1 - \frac{f^1(\nu)}{f^0}} \tag{13}$$

This equation gives rise to a geometric series constructed out of f^0 and f^1. Expanding the denominator of Eq.(13) in a power series of f^1/f^0, its first two terms agree with the perturbation expansion, Eq.(11). The presence of the remaining terms is to preserve the elastic unitarity condition because:

$$Imf^{[0,1]}(\nu) = \rho(\nu) \mid f^{[0,1]}(\nu) \mid^2 \tag{14}$$

for $\nu > 0$ and is a generalisation of the relativistic effective range theory.

The Padé approximant method is similar to the bubble summation of the partial wave amplitude.

There are few methods in the relativistic particle physics to treat the non-perturbation problem: the infinite geometric series of the bubble summation used in the study of the Nambu-Jona-Lassinio model, or the infinite ladder summation of the Bethe-Salpeter equation used to treat the bound state problem in Quantum Electrodynamics. These treatments are not as systematic as the perturbation series, but they have successfully been used to treat the non-perturbative phenomena.

I discuss so far the scattering problem. In 1988 without knowing the Lehman paper on ChPT, I proposed that the IAM, N/D and Padé methods should be used for the pion form factor problem to treat the non perturbative effect (T. N. T., Phys. Rev. Lett. **61** 2526, 1988). A few years later with the collaboration of Dobado and Herrero the pion pion scattering problem was studied(A. Dobado, M.J. Herrero and T. N. T., Phys. Lett. **B 235** (1990) 129, 134.

4 "New" Sum Rules for Low Energy Expansion Parameters of the Pion Form Factor

These "New Sum Rules were written in 1998 (T.N.T. arXiv:9809476v1[hep-ph]) but was never published. In the DHG, AW, GOM sum rules, the unitarity relation and experimental data enable us to include all elastic and inelastic effect by measuring the total cross sections. For the pion form factor there is no such optical theorem to be used in the dispersion relation. However, it is an experimental fact that the pion form factor decreases rapidly with experiment at large energy above the ρ resonance , one can saturate the sum rules with the elastic unitarity. This can be tested using the sum rule for the charge. With this fact, one can calculate the r.m.s. radius of the pion, the second, third derivatives... of the pion form factor with the accuracy of 5% or better which is the order of the accuracy of the DHG sum rule (the higher the derivatives the better is the elastic unitarity assumption). And this was done with existing experimental data!

Because the vector pion form factor $V(s)$ is an analytic function with a cut from $4m_\pi^2$ to ∞, the n^{th} times subtracted dispersion relation for $V(s)$ reads:

$$V(s) = a_0 + a_1 s + ...a_{n-1} s^{n-1} + \frac{s^n}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ImV(z)dz}{z^n(z - s - i\epsilon)} \qquad (15)$$

where $n \geq 0$ and, for our purpose, the series around the origin is considered. Because of the real analytic property of $V(s)$, it is real below

$4m_\pi^2$. By taking the real part of this equation, $ReV(s)$ is related to the principal part of the dispersion integral involving the $ImV(s)$ apart from the subtraction constants a_n.

The polynomial on the R.H.S. of Eq. (15) will be referred in the following as the subtraction constants and the last term on the R.H.S. as the dispersion integral (DI). The evaluation of DI as a function of s will be done later. Notice that $a_n = V^n(0)/n!$ is the coefficient of the Taylor series expansion for $V(s)$, where $V^n(0)$ is the nth derivative of $V(s)$ evaluated at the origin. The condition for Eq. (15) to be valid was that, on the real positive s axis, the limit $s^{-n}V(s) \to 0$ as $s \to \infty$. By the Phragmen Lindeloff theorem, this limit would also be true in any direction in the complex s-plane and hence it is straightforward to prove Eq. (15). The coefficient a_{n+m} of the Taylor's series is given by:

$$a_{n+m} = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ImV(z)dz}{z^{(n+m+1)}} \qquad (16)$$

where $m \geq 0$. The meaning of this equation is clear: under the above stated assumption, not only the coefficient a_n can be calculated but all other coefficients a_{n+m} can also be calculated. The larger the value of m, the more sensitive is the value of a_{n+m} to the low energy values of $ImV(s)$. In theoretical work such as in ChPT approach, to be discussed later, the number of subtraction needed is such that to make the dispersion integral converges.

It is important to note that if $ImV(s)$ is taken from experiment, i.e. Unitarity *is satisfied, depending on the high energy behavior of V(z), there are large* cancellation *between the subtraction constants and the Dispersion Integral (in other words, one should not do more subtractions than what is needed). This type of cancellation does not exist if ImV is calculated by a* Perturbation Theory *because* Unitarity *is not respected.*

If $s^{-1}V(s) \to 0$ as $s \to \infty$ which is a weak assumption, the once subtracted dispersion relation for the vector pion form factor is:

$$V(s) = 1 + \frac{s}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ImV(z)}{z(z-s-i\epsilon)}dz \tag{17}$$

The elastic unitarity for the pion form factor states that the phase of the pion form factor in the time like region below the inelastic threshold has the phase of the P-wave pion pion phase shift δ.

$$ImV(z) = \mid V(z) \mid \sin\delta(z) \tag{18}$$

and

$$ReV(z) = \mid V(z) \mid \cos\delta(z) \tag{19}$$

Because $\mid V(z) \mid$ and δ are experimental quantities, ImV and ReV(z) can be determined.

From experiment data, because of the smallnes of the multiple pion production at low energy, it is expected that this relation holds below 1.3 GeV. To have a feeling how good is this assumption, we can assume that the vector form factor vanishes at ∞ ,

We shall first to make a strong assumption that the pion form factor vanishes as $V(s) \to 0$ as $s \to \infty$ on the cut. This is quite plausible as we deal with a bound state quak antiquark. Then there is no contribution from the circle at ∞ to the dispersion relation. Then we can evaluate:

$$a_0 = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ImV(z)dz}{z} \tag{20}$$

Cutting off the integral at 1.7 GeV^2, we get $a_0 = 1.02 \pm .08$ whereas the *Ward Identity requires this value to be exactly 1*. This result shows that the dispersion integral is almost saturated by the elastic unitarity relation. This will be more so for the higher derivatives because of the *larger* power in the denominator of the sum rules. The sensitivity of this integral to the high energy behavior of $ImV(z)$ is the same as in the DHG sum rule. Hence the calculation of the pion r.m.s radius and higher derivatives are even more reliable because of the larger power in the denominator of the sum rule.

The result of the determination of the r.m.s of the pion, $a_2 = c_1$ and $a_3 = c_2$ are shown in Table 1. The calculated value of the r.m.s. radius of the pion agrees very well with that obtained from direct measurement. The value of a's are much more accurate than the values obtained from τ decays and $e^+ + e^-$ experiments.

Figures and Table given here are of circa 1998. An updated version of Table1 and figures is in the process of preparation with the collaboration of Phuoc Ha using better and more recent data. It is possible to get an estimate of the high energy contribution using experimental data using the fact that $ImV(z) \leq \mid F(z) \mid$ in this region.

We have here a very exceptional case of the dominance of the elastic unitarity relation which enables to calculate the low energy parameters. Had we studied the pion form factor using the phase representation, we could not control the high energy contribution beyond the elastic region, because the pion form factor phase in the high energy region was completely unknown.

5 Calculation of the Vector Pion Form Factor

a) Theoretical Consideration: One Loop CPTH Result

The one loop ChPT result can be expressed in the dispersion language:

$$V^{pert.}(s) = 1 + \frac{s}{s_{R_1}} + \frac{1}{96\pi^2 f_\pi^2}((s - 4m_\pi^2)H_{\pi\pi}(s) + \frac{2s}{3}) \tag{21}$$

where the last term behaves as s^2 at low s. The second term on the R.H.S. is the subtraction constant and is the experimental r.m.s. radius of the pion. The last term is the prediction of the one loop ChPT. For 2-loop calculation, one would need one more subtraction which can be determined by fitting data and the prediction of ChPT is the term which behaves at low s as s^3.

b) Non Perturbative Approach: Elastic Unitarity Constraint

If one imposes the constraint of the elastic unitarity in the dispersion integral one would have an integral equation of the Omnes-Muskhelishvili whose solution is well-known. The one-loop ChPT result corresponds to a first iteration of of this integral equation. It has been shown that (TNT Phys. Rev. Lett. **61** (1988) 2526) the solution of the integral equation, to a good accuracy, be approximated by the Pade approximant:

5.1 Can one add ChPT calculation to Vector Meson Dominance amplitude to calculate the pion form factor?

$$V^{(0,1)} = \frac{V^{tree}}{1 - \frac{V^{1-loop}}{V^{tree}}} \tag{22}$$

where V^{tree} refers to the tree amplitude which is equal to unity and V^{1-loop} refers to the one loop amplitude, i.e. the last two terms on the R.H.S. of Eq. (21). The Padé approximant method yields V(s) satisfying the elastic unitarity relation. It is just a bubble summation of the one-loop graph. Fixing the value of s_{R_1} such that the phase of the form factor is 90 degree the r.m.s. radius of the pion is 15 percent too small. This is understandable because we neglect the inelastic and/or higher resonances contribution.

c) Deviation from the Elastic Unitarity Constraint At high energy the contribution of the $\omega\pi$ and possibly the $\rho\pi\pi$ channels etc are experimentally important. The calculation of this contribution to the pion form factor is quite complicated but we can derive a similar equation to the Muskelishvilli-Omnès equation: Instead of the M.O integral equation, we now have:

$$V(s) = 1 + \frac{s}{s_{R_1}} + \frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty} \frac{V(z)f_1^*(z) + \sigma_i(z)}{z^2(z-s-i\epsilon)}dz \qquad (23)$$

where $f_1(s) = \frac{\eta e^{i\delta(s)}-1}{2i}$, η being the inelastic factor, and $\sigma_i(s)$ being the inelastic contribution to the unitarity relation and differing from zero for s above the inelastic threshold s_i. The solution for this equation is

$$V(s) = \frac{1}{D(s)}[1 + \frac{1}{\pi}\int_{s_i}^{\infty} \frac{2D(z)Re(\sigma_i e^{i\delta})dz}{(1+\eta)e^{-i\delta}z(z-s-i\epsilon)}] \qquad (24)$$

where

$$D^{-1}(s) = \exp(\frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\delta_1(z)}{z(z-s-i\epsilon)}dz) \qquad (25)$$

We also have to require that the first derivative of $V(s)$ at s=0 to be s_{R-1}.

Below and sufficiently far from s_i, e.g. the $\omega\pi$ threshold, we can roughly parametrise the contribution of the second term on the R.H.S. of Eq. (24) as a polynomial $P_n(s)$ which is real for $s < s_i$ and is the polynomial ambiguity in the solution of the Muskhelishvilli-Omnès equation. Because of the normalisation $P_n(0) = 1$, the introduction of this factor will not influence the Ward identity, but does influence the value of the r.m.s.value of the pion radius (T. N. Truong, Phys. Rev D30, 1509 (1984)). This type of approximation is reliable for energy below the inelastic threshold s_i, but is wrong at the inelastic threshold and also at higher energy.

(From the radius of convergence of the power series expansion for this effect , we also expect that this parametrisation does not hold for a large space like momentum transfer.

We shall fit the experimental data below with the simple expression $(1 + \alpha s/s_\rho)$, where α is a constant in order to simulate phenomenologically the inelastic effect. The pion form factor can now be written in its final form as:

$$V^f(s) = \frac{1 + \alpha s/s_\rho}{1 - s/s_{R_1} - \frac{1}{96\pi^2 f_\pi^2}\{(s - 4m_\pi^2)H_{\pi\pi}(s) + 2s/3\}} \tag{26}$$

There are parameters in this equation, α and ρ mass. α can be fixed by the magnitude value of the pion form factor at the mass of ρ. The ρ width is calculated and is simply the KSRF relation.

One can unitarized the two-loop ChPT result to get a similar results (Hannah).

The expression for the UChPT $V^f(s)$ is approximately same as the one loop ChPT $V^{pert.}(s)$ if one let the ρ mass becomes much larger than its physical value i.e. much larger than 1.4 GeV.

It is clear that one must obey the unitarity relation.

It should be noticed that the expression for $V^f(s)$ is also useful for lattice gauge calculation of the pion form factor where the pion masses used in the calculation deviate substantially from its experimental value.

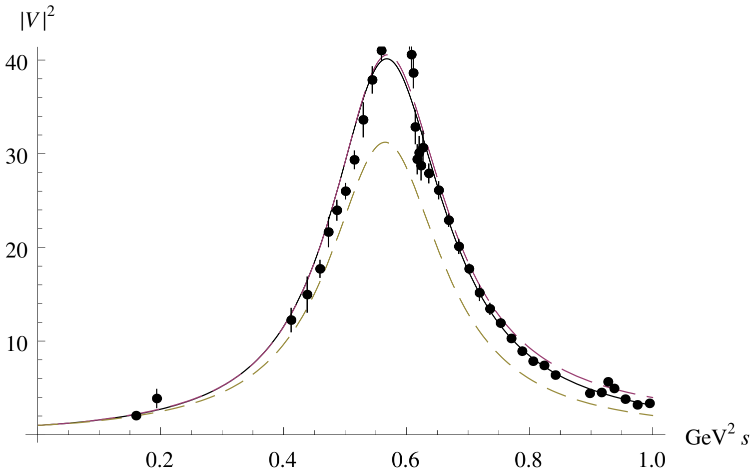

Figure 1: The square of the modulus of the vector pion form factor $V(s)$ as a function of the energy squared s. The experimental data are from e^+e^- data. . The dash line curve is the IAM or Pade result (UChPT) of the one loop ChPT fitted to give the correct experimental P-wave phase shifts which goes through 90 degrees at the ρ mass. It yields however a too low value of the r.m.s. pion radius. The solid curve is the UChPT multiplied by a nomial (1+0.14s/s_ρ) simulating the contribution from higher resonances and /or inelastic effects.

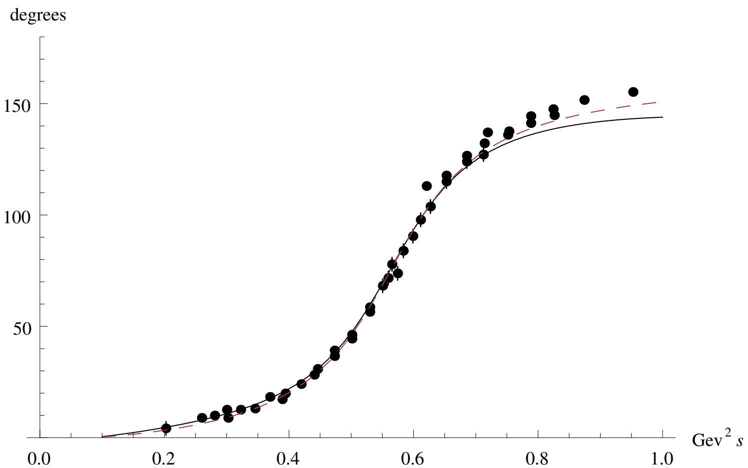

Figure 2: The solid circles are the experimental phase shifts as determined by the pion pion scattering, the solid curve is the calculated pion form factor phase as given by the UChPT calculation.

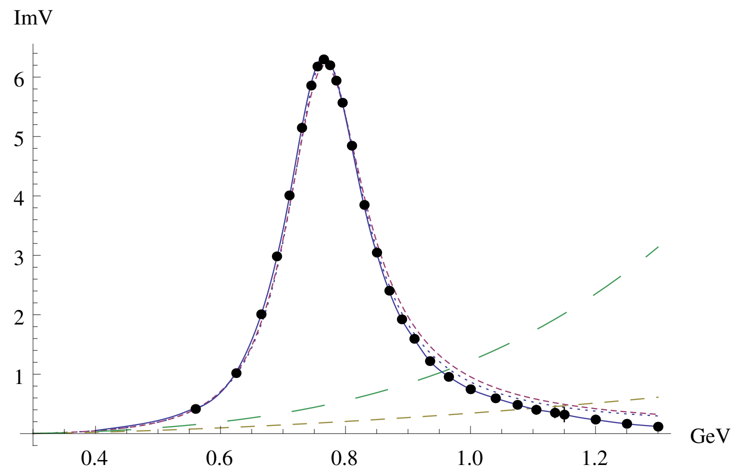

Figure 3: The imaginary part of the vector pion form factor $ImV(s)$ as a function of the energy in the GeV unit. The solid curve is the experimental result as calculated by the elastic unitarity relation using the experimental data; the long-dashed curve is the two-loop ChPT calculation, the medium long-dashed curve is the one-loop ChPT calculation, the short-dashed curve is the UChPT one loop calculation with the inelastic effect and/or higher resonances taken into account.

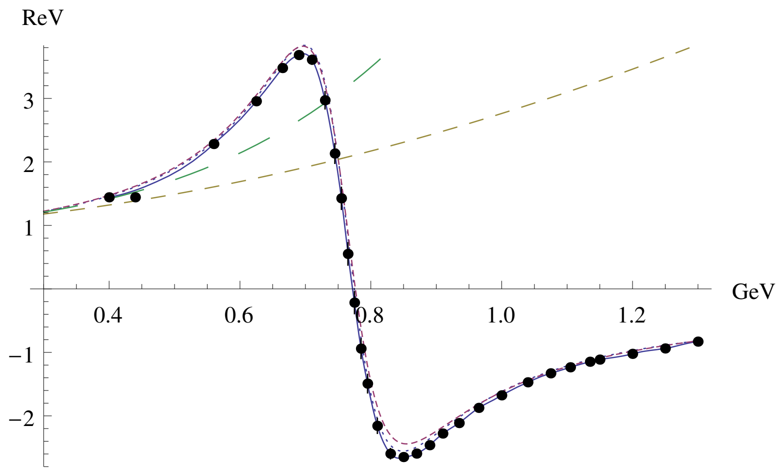

Figure 4: The real part of the vector pion form factor $ImV(s)$ as a function of the energy in the GeV unit. The curves are the same as given in Fig. 3.

	e^+e^-	τ	Sum Rule	ChPT1	ChPT2	UChPT
$\langle r^2\rangle_\pi$ (fm^2)	$(.439 \pm .008)$	$(.439 \pm .008)$	$.45 \pm .015$	$\star$	$\star$	.454
c_1 (GeV^{-4})	(6.8 ± 1.9)	(3.3 ± 1.7)	$3.9 \pm .10$	.626	3.8	3.98
c_2 (GeV^{-6})	(-0.7 ± 6.8)	(13.2 ± 5.7)	$9.7 \pm .4$	2.3	4.1	10.2

Table 1: *Results of calculations of $\langle r^2\rangle_\pi$, c_1, c_2 using Sum Rules, one- loop ChPT1, two-loop ChPT2 and UChPT with inelastic unitarity The result of the one-loop ChPT calculation for c_1 and c_2 is wrong, the two-loop ChPT calculation for c_2 is better but does not agree with the experimental data. The Sum Rules and UChPT calculations of $\langle r^2\rangle_\pi$ are in good agreement with the experimental data; the calculation of c_1 and c_2 by the Sum Rules and UChPT are in good agreement with each other and also with the rough experimental data*

5.2 Can one add ChPT calculation to Vector Meson Dominance amplitude to calculate the pion form factor?

This is the case of double counting the logarithm term. One has:

$$V^{mix}(s) = V^{f}(s) + \frac{1}{96\pi^2 f_\pi^2}((s - 4m_\pi^2)H_{\pi\pi}(s) + \frac{2s}{3}) \tag{27}$$

The second term on the R.H.S. of this equation is the extra term which one adds. Its presence is not visible in the time-like region due to the dominance of $V^{f}(s)$ term, but the space like data almost exclude it.

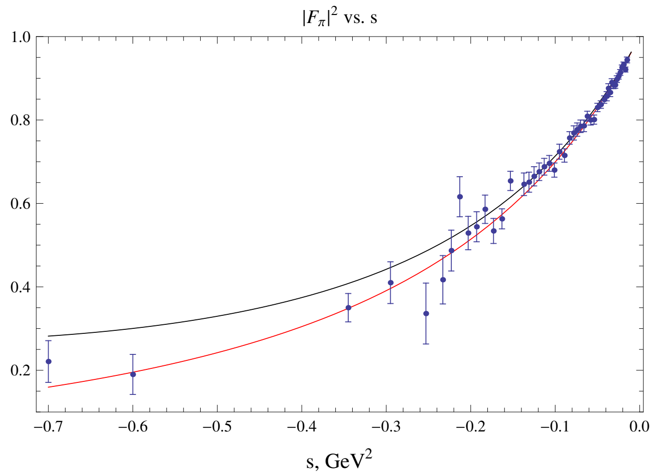

Figure 5: The space like pion form factor. The lower curve is calculated by the one loop UChPT calculation with the inelastic effect and/or higher resonances taken into account ; the upper curve is given by the vector meson dominance with the one loop ChPT result added : the experimental data at high momentum transfer almost excludes this possibility. These two curves are however almost identical (within experimental errors) in the time like region below 1 GeV.

6 Physics at large Nc and the Pade Approximant method. (Power series expansion in momenta is not as good as the Pade Approximant)

Because $f_\pi^2 \to \infty$ as the number of color becomes large, the expression for the vector form factor is just a pole. The form factor is simply the rho resonance.

7 Conclusion

It is shown in this talk that the restriction of unitarity is important for soft pion physics. The IAM, N/D and Pade approximant methods are useful to implement the unitarity condition. Because

ChPT does not satisfy the unitarity constraint, it has no predictive power without making further assumptions. It may however be used to correlate various low energy expansion parameters.

I should like to point out that approximation for an analytic function means that it should be valid over a wide region of energy, not just a small region. In the case of the pion form factor calculation, this region is the elastic region which includes the ρ resonance which is the main feature of the low energy physics. Any calculation which does not include this resonace is bound to fail . This is the meaning of the Dispersion Relation or Hilbert Transform relating the real part of an analytic function to a dispersion integral over its imaginary part over a large energy region or equivalently the mathematical theorem about the equality of two analytic functions. I should like to thank Dick Blankenbecler for having encouraged me to follow this line of physics. When he looked at my unpublished paper 15 years ago, he commented that

”There is not such a thing called a small analytic function” as commented by one of his old colleague at Princeton when he was young and was doing the S-matrix theory in Princeton. The difference of the pion form factor calculations between ChPT and UChPT are extremely small at low energy, but become very large at the ρ resonance!

I would like to apologize to those who have used ChPT; I do not wish to offend them, but would like to suggest that the power series expansion for the S-matrix used in the Effective Chiral Lagrangian is only valid outside the unitarity cut (a fundamental property of an analytic function). Effort should be done in finding suitable non-perturbative methods to continue this amplitude on to the cut which is the same problem that one has to face in the Euclidean lattice gauge calculation. My answer to this problem is not only the analyticity but also the unitarity constraints have to be respected. Thank you for your patience and tolerance.