Probing the Atomic H(iggs) force

CD, R. Ozeri, G. Perez, Y. Soreq
hep-ph: 1601.05087 + in progress
CD, Y. Soreq hep-ph:1602.04838
(see also Frugiuele et al. hep-ph:1602.04822)

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15-03-2016 | La Thuile

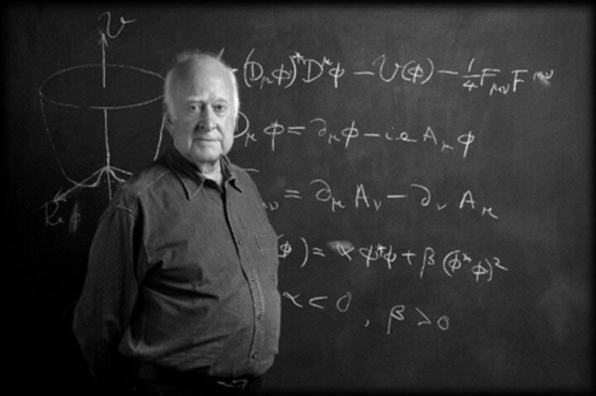
Outline

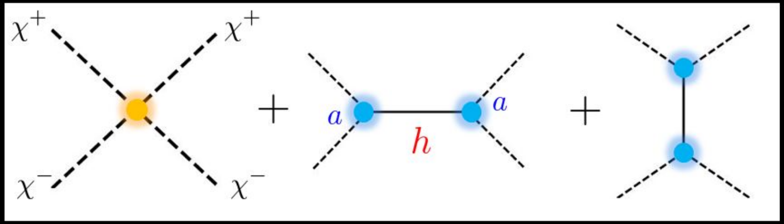
1. The Higgs and the flavor puzzle
2. Higgs force in atoms
3. Probing Higgs couplings with isotope shifts
4. The weak and new physics forces

The BEH mechanism
and the flavor puzzle

The Brout-Englert-Higgs Mechanism

- breaks EW symmetry: $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to U(1)_{\mathrm{QED}}$





$\propto \frac{E^2}{v^2}(1-a^2)$

ATLAS+CMS: $|a-1| \lesssim \mathcal{O}(10\%)$

ATLAS-CONF-2015-044

- provides charged fermion masses:

in the SM: $$m_f = y_f \times v$$

The flavor puzzle

- Charged fermion masses are highly hierarchical:

$$m_t \sim 10^5 m_e$$

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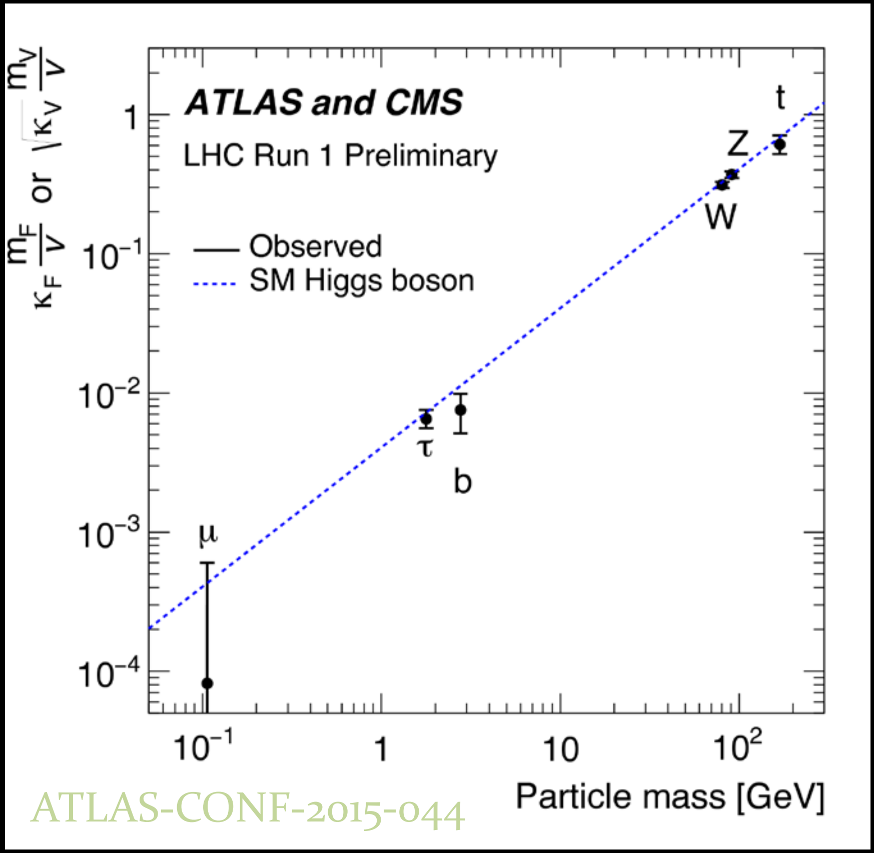
- Within the SM, it is ***assumed*** to originate from hierarchical Higgs-to-fermion couplings:

$$y_f^{\mathrm{SM}} \propto m_f$$

How well can we test?

The flavor puzzle at the LHC

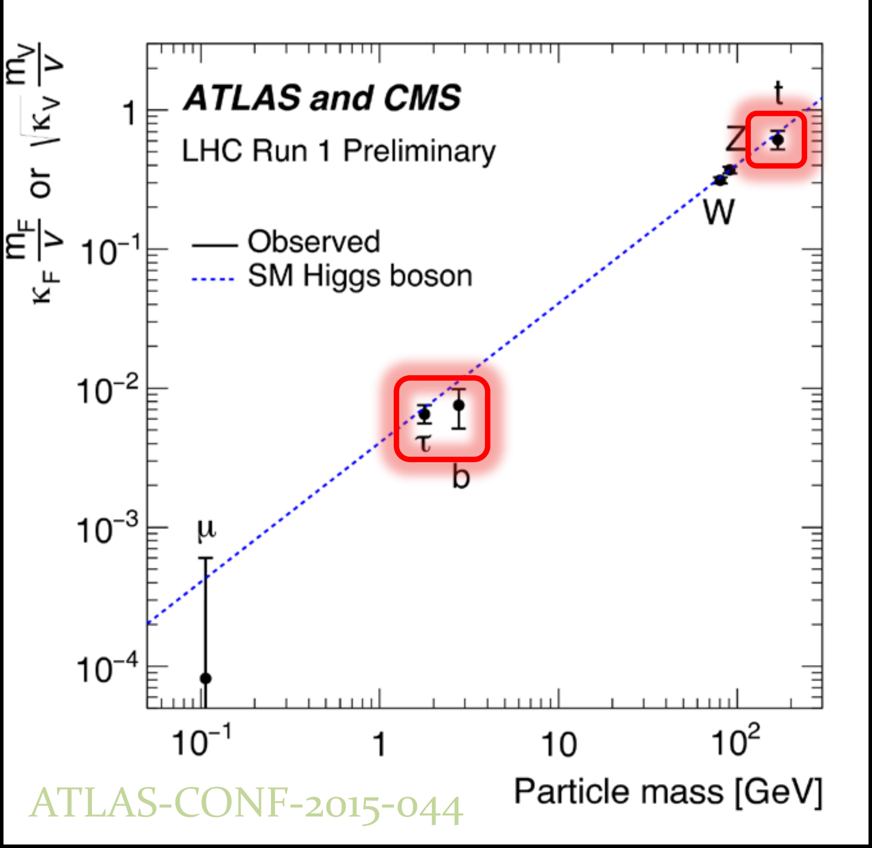


The flavor puzzle at the LHC

ATLAS+CMS
Higgs-signal/SM:

$\mu^{\tau\tau}$	$1.12^{+0.25}_{-0.23}$
μ^{bb}	$0.69^{+0.29}_{-0.27}$

μ_{ttH}	$2.3^{+0.7}_{-0.6}$



→ the Higgs mechanism is likely to be
the dominant source of 3rd generation masses

The flavor puzzle at the LHC

There is an opportunity to probe *c*-coupling directly, thanks to charm-tagging:

in VH production

Perez-Soreq-Stamou-Tobioka '15

in Hc production

Isidori-Goertz '15

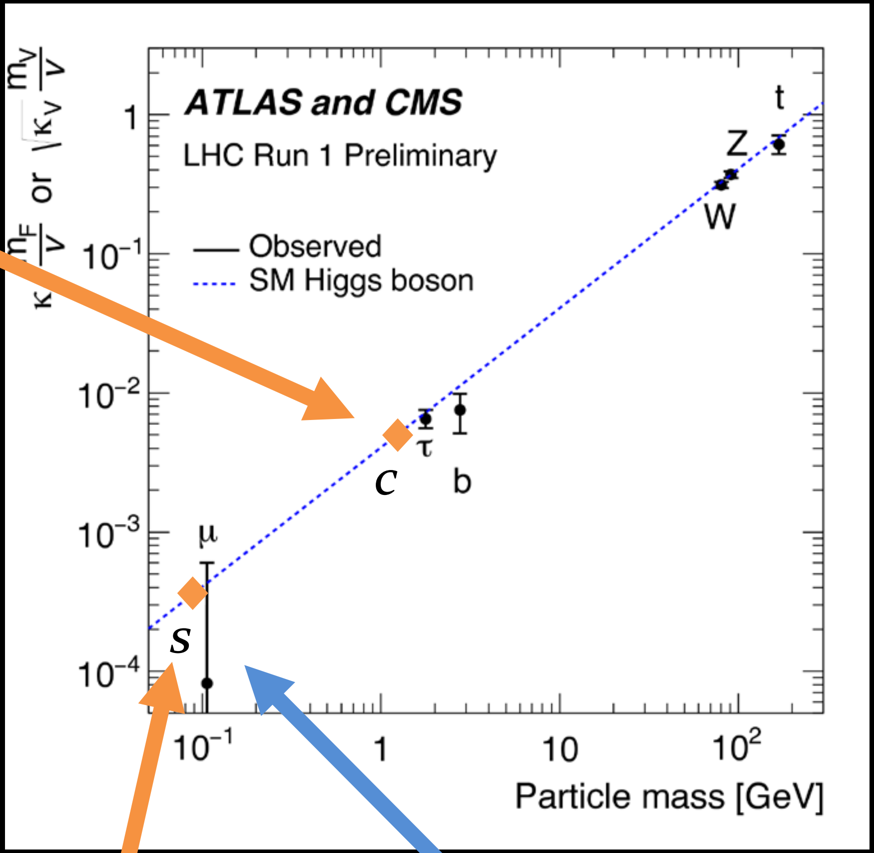
Other probes exist:

- $h \to J/\psi\gamma$

  Perez-Soreq-Stamou-Tobioka '15
- global fits

  CD-Golling-Perez-Soreq '13
- $\Gamma_h \leq 1.7\,\mathrm{GeV}$

  Perez-Soreq-Stamou-Tobioka '15



$h \to \phi\gamma$?

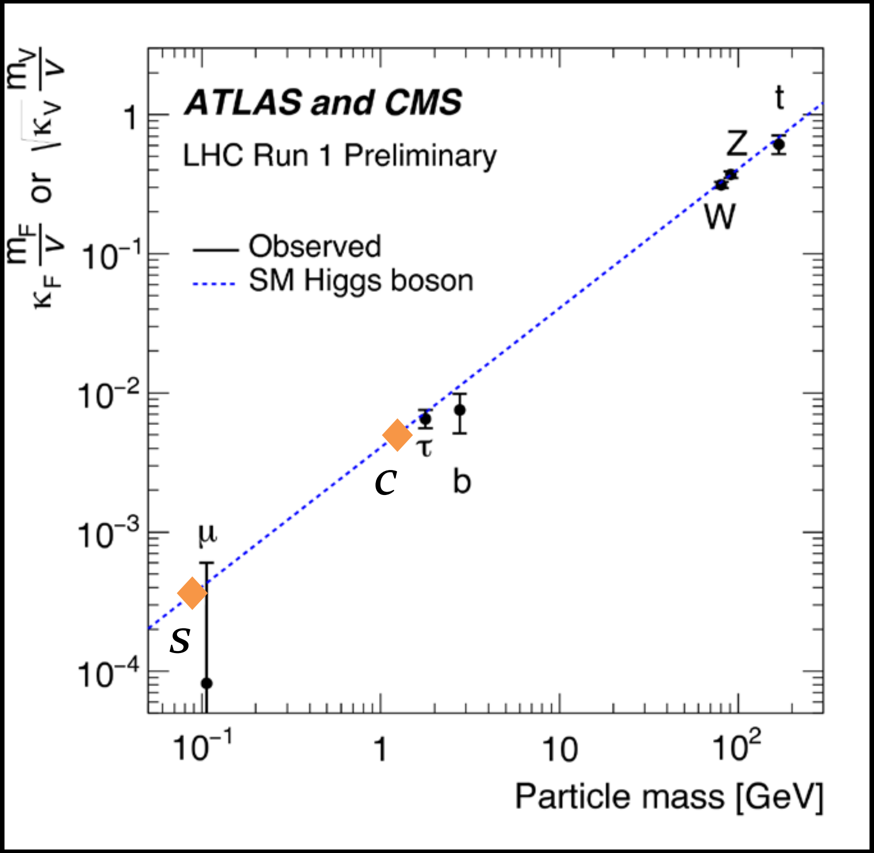
Kagan et al. '14

Sensitivity to muon-coupling, with high-enough luminosity

ATL-PHYS-PUB-2014-016

The flavor puzzle at the LHC

What about e, u, d?



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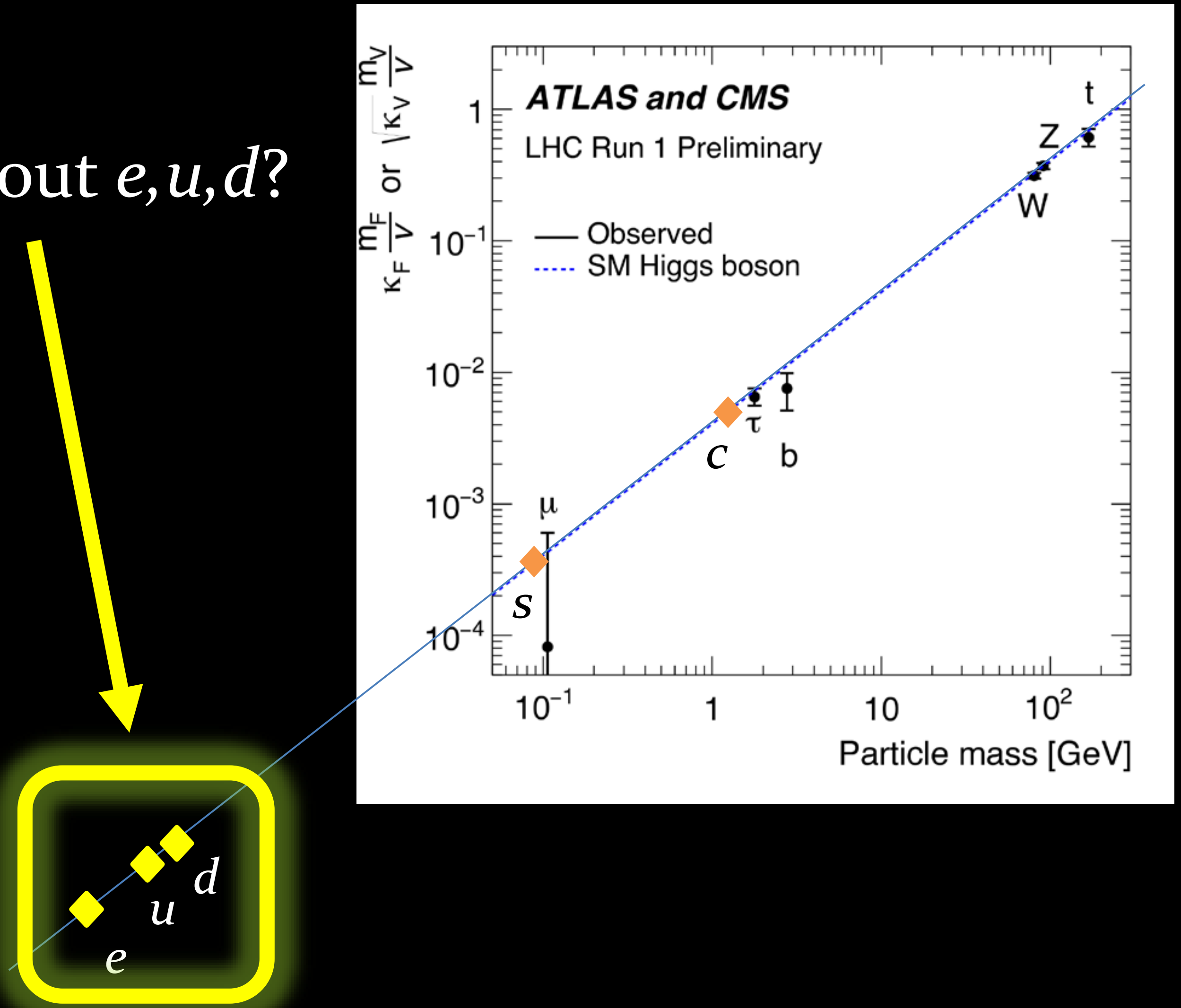

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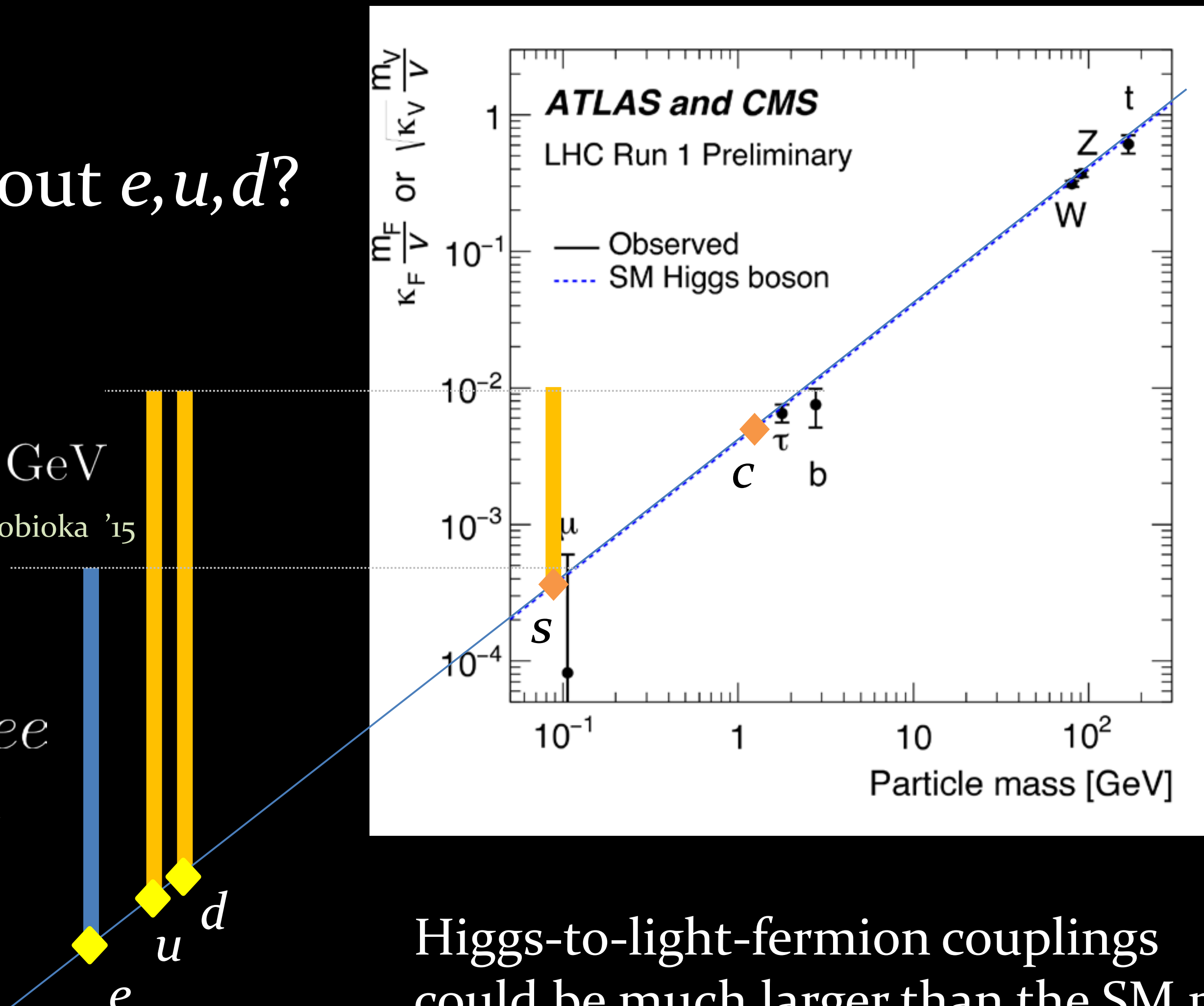
$h \to ee$

Altmannshofer-Brod -Schmaltz '15



Higgs-to-light-fermion couplings
could be much larger than the SM prediction.
LHC is and will remain weak in bounding them.

The atomic Higgs force
and optical clock transitions

Higgs force in atoms

- The Higgs results in an attractive force between nuclei and their bound electrons (à la Yukawa):

$$V_{\text{Higgs}}(r) = -\frac{y_e y_A}{4\pi}\frac{e^{-m_h r}}{r} \approx -\frac{y_e y_A}{4\pi m_h^2}\frac{\delta(r)}{r^2}$$

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- $y_A = Z y_p + (A-Z) y_n$ with:

Shifman-Vainshtein-Zakharov ’78
+ nuclear data, see *e.g.* micrOmegas

$$y_n \approx 7.7 y_u + 9.4 y_d + 0.75 y_s + 2.6\times 10^{-4} c_g$$
$$y_p \approx 11 y_u + 6.5 y_d + 0.75 y_s + 2.6\times 10^{-4} c_g$$

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- c_g constrained by LHC, weaker sensitivity to s-coupling

Atomic Higgs Force Strength

- Under current LHC constraints:

Higgs width (direct)

$$y_{n,p} \lesssim 3 \ (0.2) \qquad \text{and} \qquad y_e \lesssim 1.3 \times 10^{-3}$$

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- Higgs force possibly stronger than SM by $\sim 10^6$!
- This shifts transition frequencies by:

electron-density at the nucleus

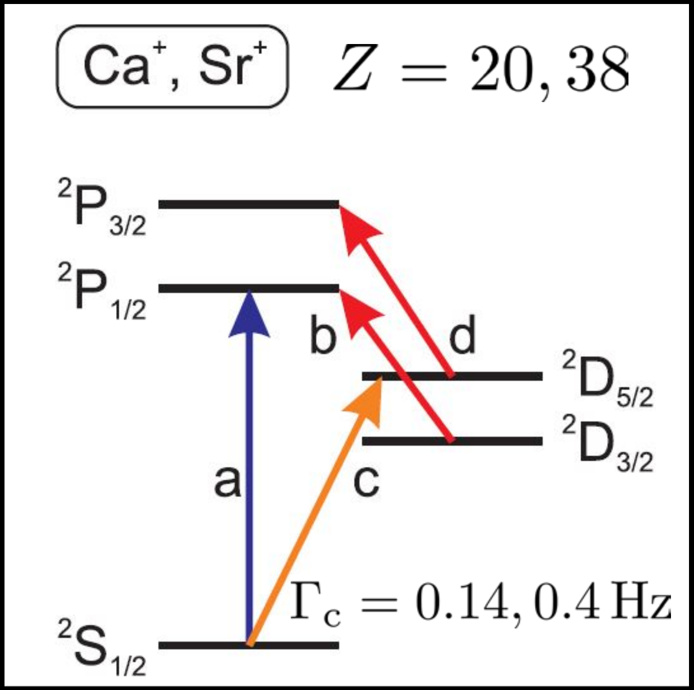
$$\Delta\nu^{\text{Higgs}}_{nS \to n'D,F} \approx 1\,\text{Hz} \times A \frac{y_e y_{n,p}}{0.004} \frac{|\psi(0)|^2}{4n^3 a_0^{-3}}$$

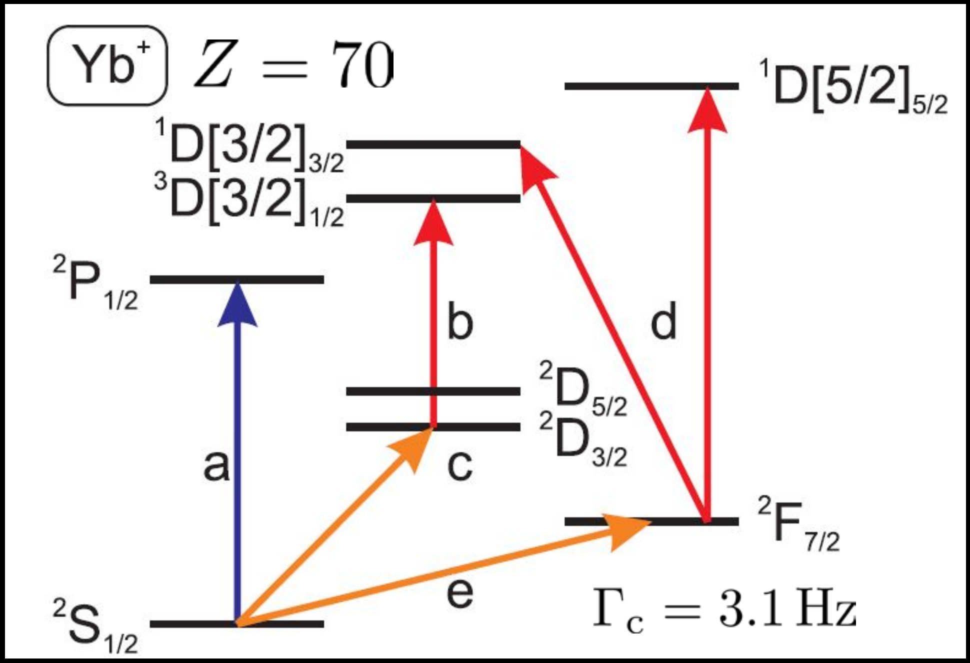
Bohr radius $(\alpha m_r)^{-1}$

Optical Atomic Clocks

- Narrow transitions with S-wave are needed:

Ludlow-Boyd-Ye, Rev. Mod. Phys. 87 (2015)





$\Gamma_e = 10^{-9}\,\mathrm{Hz}$

Frequency comparisons

- Experimental accuracy in $^{40}\mathrm{Ca}^+$, $^{88}\mathrm{Sr}^+$ is $\sim \mathrm{Hz}$

Dube et al., Phys. Rev. A87 (2013)
Chwalla et al., PRL 102 (2009)

$$\nu_{E2}^{\mathrm{Ca}^+} = 411\ 042\ 129\ 776\ 393.2(1.0)\mathrm{Hz}$$
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- Theory side is however much less promissing:

electron-electron correlations, nuclear finite-size,
relativistic corrections, QED...

are not accounted for at the 10^{-15} level...

Isotope shifts and the King plot

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(consider $A' - A = 2, 4, \dots$ *to avoid influence of nuclear spin)*

- The Higgs force however scales like the nuclear mass A, so there is still a net shift between isotopes!

Isotope Shift Theory

- There are yet non-trivial IS from changes in:
  - the reduced mass: $m_r = \dfrac{m_e m_A}{m_e + m_A} \simeq m_e(1 - m_e/m_A)$
  - the nuclear charge distribution: $\langle r^2 \rangle_A / a_0^2$

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- Hence, to leading order, IS for a transition i reads:

$$\delta\nu^i_{AA'} = K_i \mu_{AA'} + F_i \delta\langle r^2 \rangle_{AA'} + H_i (A - A')$$

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- MS/FS effects are typically in the GHz range $\gg$ Hz !

The King Plot

W. H. King, *J. Opt. Soc. Am.* 53, 638 (1963)

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$$F_{21} \equiv F_2/F_1$$
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$$m\delta\nu^2_{AA'} = K_{21} + F_{21}m\delta\nu^1_{AA'} - AA'H_{21}$$

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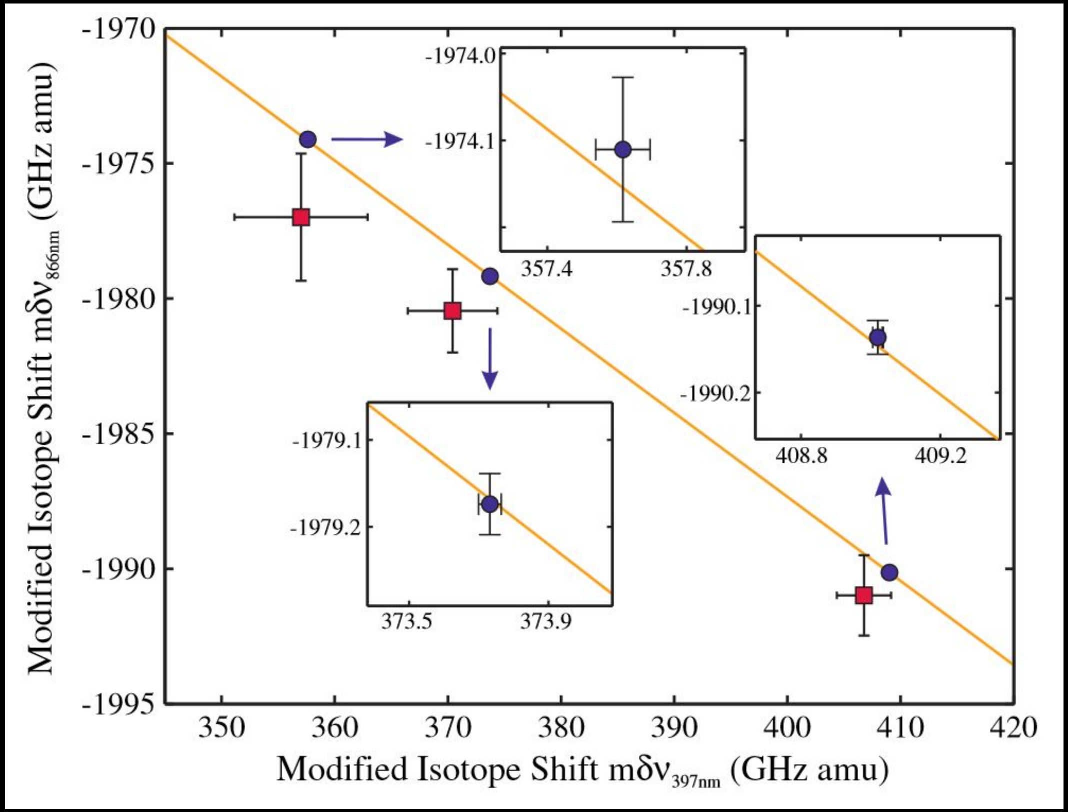
- Plot $m\delta\nu^1_{AA'}$ vs. $m\delta\nu^2_{AA'}$ along the isotopic chain
- as long as linearity is observed, one can bound H_{21} (unless accidentally $m\delta\nu \propto A'$)

Proof of concept with Ca^+ data

Gebert et al. PRL 115 (2015)

$A = 40, A' = 42, 44, 48$

$4S \to 3D_{5/2}$



$4S \to 4P_{1/2}$ (not-clock)

IS ~ 1 GHz

error ~ 100 kHz

$y_e y_n \lesssim 40$

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- This is ~10 times better than (comparable to) LHC8 direct (indirect) bounds, with very good prospect for improvements!

Probing new physics

Few remarks

The weak force

$$V_{\text{weak}}(r) = -\frac{8G_{\text{F}}m_Z^2}{\sqrt{2}}\frac{g_e g_A}{4\pi}\frac{e^{-m_Z r}}{r}$$

- Z-to-electron couplings known at 10^{-3} level

LEPEWWG

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Efrati-Falkowski-Soreq '14

$$[\delta g_L^{Zu}]_{ii} = \begin{pmatrix} -0.8 \pm 3.1 \\ -0.16 \pm 0.36 \\ -0.28 \pm 3.8 \end{pmatrix} \times 10^{-2}, \quad [\delta g_R^{Zu}]_{ii} = \begin{pmatrix} 1.3 \pm 5.1 \\ -0.38 \pm 0.51 \\ \times \end{pmatrix} \times 10^{-2},$$

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- improved bounds from IS: $|\delta g_V^{Zu} + 2\delta g_V^{Zd}| \lesssim 0.03$
- still weaker than APV: $|\delta g_V^{Zu} + 2\delta g_V^{Zd}| \lesssim 10^{-3}$

Generic new physics (EFT)

- Relevant operators at the GeV scale:

$$\mathcal{O}_{eq}^{V} = (\bar{e}\gamma^{\mu}e)(\bar{q}\gamma_{\mu}q) \quad q = u, d$$

$$\mathcal{O}_{eq}^{S} = (\bar{e}e)(\bar{q}q) \quad q = u, d, s, c, b, t$$

$$\mathcal{O}_{eg} = \alpha_s(\bar{e}e)G_{\mu\nu}^2$$

operator $\mathcal{O}_i$	Upper bound on $\lvert c_i \rvert$ ($\Lambda = 1\,\mathrm{TeV}$)	Lower bound on Λ_i [TeV] ($c = 1$)	LEP2
$\mathcal{O}_{eu}^{V}$	2.3×10^{-2}	6.6	$3.4 \rvert 4.5$
$\mathcal{O}_{ed}^{V}$	1.1×10^{-2}	9.3	$3.0 \rvert 2.5$
$\mathcal{O}_{eu}^{S}$	2.6×10^{-3}	20	
$\mathcal{O}_{ed}^{S}$	2.1×10^{-3}	22	$\sim 1 - 2$
$\mathcal{O}_{es}^{S}$	2.7×10^{-2}	6.1	
$\mathcal{O}_{ec}^{S}$	0.20	2.3	
$\mathcal{O}_{eb}^{S}$	0.87	1.1	
$\mathcal{O}_{et}^{S}$	56	0.13	
$\mathcal{O}_{eg}$	9.6	0.47	

CD-Soreq '16

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sensitive to scalar operators up to 20TeV!

CD-Soreq *to appear*

A 750GeV diphoton resonance?

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No IS if pseudoscalar
Assuming scalar:

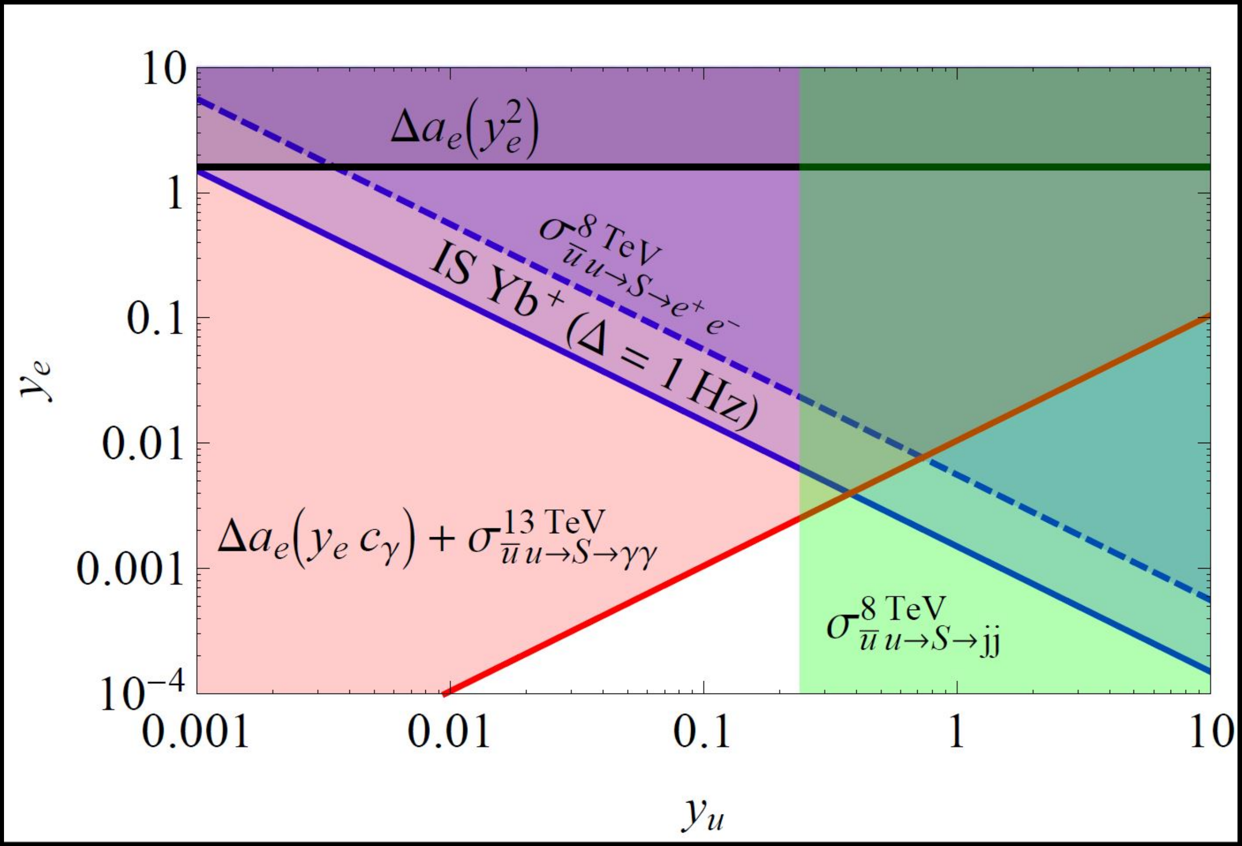
Unless produced
by gluon fusion,
IS has more sensitivity
to its couplings to

$$e \times q = u, d, s, c, b$$

S couplings ($\mu = 750\,\mathrm{GeV}$)	LHC8 bound [20] ($\Gamma_S = 45\,\mathrm{GeV}$)	IS projection ($\Delta = 1\,\mathrm{Hz}$)
$\lvert y_e y_u \rvert$	6.1×10^{-3}	1.5×10^{-3}
$\lvert y_e y_d \rvert$	7.8×10^{-3}	1.2×10^{-3}
$\lvert y_e y_s \rvert$	3.2×10^{-2}	1.5×10^{-2}
$\lvert y_e y_c \rvert$	4.0×10^{-2}	1.1×10^{-2}
$\lvert y_e y_b \rvert$	6.1×10^{-2}	4.9×10^{-2}
$\lvert y_e y_t \rvert$	0.19	3.0
$\lvert y_e c_g \rvert$	0.79	150

A 750GeV diphoton resonance?

- Combining diphoton signal and $g_e - 2$



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  – the weak force
  – BSM, as long as not aligned with QED…

- Measurements in Yb+ are already underway!
- Other possibilities envisaged: Ca/Ca+, Sr/Sr+, Dy

backup

Electron density at the nucleus

- Coulomb potential: $$V(r) = -\frac{Z_{\text{eff}}(r)\alpha}{r}$$

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- Nuclear charge screened by inner electrons:

ion charge

$$Z_{\text{eff}}(r) \sim \begin{cases} Z & r < a_0/Z \\ r/a_0 & a_0/Z < r < a_0/(1+n_e) \\ 1+n_e & r > a_0/(1+n_e) \end{cases}$$

Electron density at the nucleus

- Coulomb potential: $V(r) = -\frac{Z_{\text{eff}}(r)\alpha}{r}$

- Nuclear charge screened by inner electrons:

ion charge

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- Using non-relativistic hydrogenoid wavefunction:

$$|\psi(0)|^2 \simeq \frac{4.2Z}{a_0^3}(1+n_e)^2$$

Higher-orders?

- Need to control King's linearity at least down to:

Higgs force

$$\frac{\mathrm{Hz}}{\mathrm{GHz}} \sim 10^{-9}$$

total IS

- Higher-order corrections are not trivial to compute, many-body, relativistic simulations are needed [in progress]

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total IS

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- Yet, IS are controlled by two small parameters:

$$\varepsilon_\mu = m_e \mu_{AA'} \sim (A - A')10^{-8}$$
$$\varepsilon_r = \delta\langle r^2 \rangle_{AA}/a_0^2 \sim (A - A')10^{-11}$$

- So, we can entertain NDA...

NLO Field Shift

- Perturbation theory: Seltzer '69 Blundell et al. '87

nuclear charge distribution

$$\delta\nu^{\mathrm{FS}}_{AA'} = -e\int d^3r_e|\psi(r_e)|^2\delta V(r_e)\,, \qquad \delta V(r_e) = \frac{Ze}{4\pi}\int d^3r_N\,\frac{\delta\rho(r_N)}{|\vec{r}_e-\vec{r}_N|}$$

electron density

nuclear potential

- LO: $\propto |\psi(0)|^2\delta\langle r^2\rangle_{AA'} \sim \mathcal{O}(\varepsilon_r)$
- NLO/LO: $\sim \mathcal{O}(\varepsilon_\mu^2, \varepsilon_r^2, \varepsilon_\mu\varepsilon_r)/\varepsilon_r \sim 10^{-7}$
- NLO is linear up to overlap with the nucleus $\sim \mathcal{O}(\varepsilon_r)$
- Hence, non-linearities are only of $\mathcal{O}(\varepsilon_\mu^2) \sim 10^{-14}$

NLO Mass Shift

- MS arises from:
  - rescaling Rydberg (normal MS)
  - electron-electron correlation, relativistic... (specific MS)

- at LO, both scale like $m_e \mu_{AA'} \sim \mathcal{O}(\varepsilon_\mu)$
- NLO correction is parametrically: Palmer '87

$$\sim \alpha^2 m_e^2 (m_A^{-2} - m_{A'}^{-2})$$

- Hence, NLO/LO $\sim \mathcal{O}(\alpha^2 \varepsilon_r) \sim 10^{-10}$