H Effective Theory: Signs of Deviations from the Standard Model

Aneesh Manohar

CERN
&
University of California, San Diego

16 Mar 2016
Moriond Electroweak

Outline

- SM and the geometry of the scalar sector
- Standard Model EFT (SMEFT) and Higgs EFT (HEFT)
- Curvature of the scalar manifold
- Experimental probes

Rodrigo Alonso, Elizabeth Jenkins, AM

arXiv:1511.00724, arXiv:1602.00706

Experimental Input

- An $SU(2) \times U(1)$ gauge theory spontaneously broken to $U(1)_{\rm em}$ at a scale $v = 246$ GeV.
- A particle has been seen with a mass $M_h \sim 126$ GeV; 0^+ quantum numbers favored
- No evidence for any new particles in the few hundred GeV range.

Many theoretical ideas on how the electroweak symmetry is spontaneously broken.

Need precision tests of the EW breaking mechanism —

Higgs-Gauge (i.e. scalar) sector $W_L, Z_L \sim \varphi$

The gauge sector is already well tested.

Standard Model – A fundamental scalar doublet

A field H which transforms as $\mathbf{2}_{1/2}$

$$H = \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix} \qquad L = \partial_\mu H^\dagger \partial^\mu H - \lambda (H^\dagger H - v^2/2)^2$$

Expanding about the minimum,

$$H = \begin{bmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + \mathrm{h} + i\eta) \end{bmatrix}$$

h is the neutral scalar with mass

$$m_h^2 = 2\lambda v^2$$

of order the electroweak scale. ϕ^+ and η are the eaten Goldstone bosons that give mass to the W and Z.

Standard Model – Custodial Symmetry

$$H = \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} i\phi_1 + \phi_2 \\ \phi_4 - i\phi_3 \end{bmatrix}$$

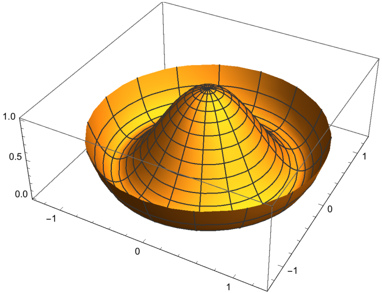
$$\boldsymbol{\phi} = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix} \qquad H^\dagger H = \frac{1}{2}\boldsymbol{\phi} \cdot \boldsymbol{\phi} = \frac{1}{2}\left(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2\right)$$

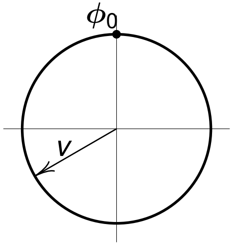
$$L = \frac{1}{2}\partial_\mu \boldsymbol{\phi} \cdot \partial^\mu \boldsymbol{\phi} - \frac{\lambda}{4}\left(\boldsymbol{\phi} \cdot \boldsymbol{\phi} - v^2\right)^2$$

Assume custodial symmetry: $O(4) \sim SU(2) \times SU(2) \supset SU(2) \times U(1)$.

Scalar Potential

$$V = -\frac{\lambda}{4}\left(\phi\cdot\phi - v^2\right)^2 \qquad \phi_0 = (0,0,0,v)^T$$





Pick ϕ_0 to be the North pole of S^3. $\mathcal{H} = O(3)$ leaves ϕ_0 invariant.

$\mathcal{G} = O(4)$ maps ϕ_0 to points on S^3.

Vacuum manifold is $\mathcal{G}/\mathcal{H} = O(4)/O(3) = S^3$.

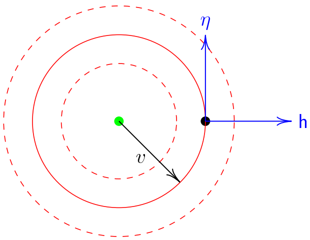
Goldstone bosons are fluctuations on S^3.

Scalar Manifold $\mathcal{M}$

SM the manifold is flat, with fields $(\phi_1, \phi_2, \phi_3, \phi_4) \in \mathbb{R}^4$.

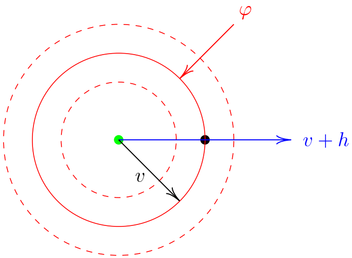
Cartesian coordinates

h, η, ϕ^+



Polar coordinates

h, φ^a



All that is need for EW symmetry breaking is the sphere S^3 of radius v.

SM in Polar Coordinates

Kinetic energy

$$L = \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}(v+h)^2(\partial_\mu \mathbf{n})^2, \qquad F(h) = 1 + \frac{h}{v},$$

where

$$\phi = h\,\boldsymbol{n} \qquad \boldsymbol{n}\cdot\boldsymbol{n} = 1$$

Just like spherical polar coordinates in $\mathbb{R}^3$

$$\mathrm{d}s^2 = \mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2) = \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

$$r \to v + h \qquad \mathrm{d}\Omega^2 \to (\partial_\mu \mathbf{n})^2$$

$$(\theta,\phi) \to \mathbf{n} = (\cos\theta, \sin\theta\cos\phi, \sin\theta\sin\phi)$$

We have one additional angular direction — 3 angles instead of 2.

SM Linear or Non-linear?

$$L_1 = \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}(v+h)^2(\partial_\mu \mathbf{n})^2, \qquad \boldsymbol{n}\cdot\boldsymbol{n} = 1$$

Non-linear form because

$$\boldsymbol{n} = \left(n_1, n_2, n_3, \sqrt{1-n_1^2-n_2^2-n_3^2}\right)$$

Linear form

$$L_2 = \frac{1}{2}\partial_\mu \phi \cdot \partial^\mu \phi - \frac{\lambda}{4}\left(\phi\cdot\phi - v^2\right)^2$$

Both forms are a perfectly fine renormalizable theory, and identical.
Changed variables in an integral.

Extensions of the SM

Some dynamics at a scale $f \gg v$.

Rather than worry about details about the high energy theory, try and characterize what aspects of the theory can be measured at low (i.e. LHC) energies.

SMEFT and HEFT

SMEFT

An effective theory with the same field content as the SM — a field H that transforms as $\mathbf{2}_{1/2}$.

$$\mathcal{L} = \partial_\mu H^\dagger \partial^\mu H - \frac{c_\Box}{\Lambda^2} \partial_\mu \left(H^\dagger H \right) \partial^\mu \left(H^\dagger H \right) + \dots$$

Will use the custodial $SU(2)$ invariant form

$$\mathcal{L} = \frac{1}{2} f(\boldsymbol{\phi} \cdot \boldsymbol{\phi}) \, \partial_\mu \boldsymbol{\phi} \cdot \partial^\mu \boldsymbol{\phi} + \frac{1}{2} h(\boldsymbol{\phi} \cdot \boldsymbol{\phi}) \, (\boldsymbol{\phi} \cdot \partial_\mu \boldsymbol{\phi})(\boldsymbol{\phi} \cdot \partial^\mu \boldsymbol{\phi}) + \mathcal{O}(p^4)$$

and

$$f(\boldsymbol{\phi} \cdot \boldsymbol{\phi}) = 1 + \frac{f_1}{\Lambda^2} \boldsymbol{\phi} \cdot \boldsymbol{\phi} + \dots \qquad h(\boldsymbol{\phi} \cdot \boldsymbol{\phi}) = \frac{h_1}{\Lambda^2} + \frac{h_2}{\Lambda^4} \boldsymbol{\phi} \cdot \boldsymbol{\phi} + \dots$$

Switch to polar coordinates:

$$\boldsymbol{\phi} = (v + h) \, \boldsymbol{n}$$

SMEFT (Standard Model Effective Field Theory)

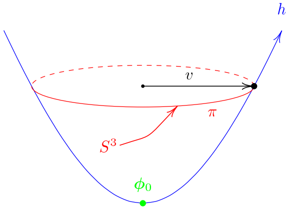
Kinetic energy

$$L = \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}v^2\, F(h)^2\, (\partial_\mu \mathbf{n})^{\mathbf{2}} \qquad\qquad F(0) = 1$$

where $F(h)$ can be computed in terms of $f(\phi^2)$ and $g(\phi^2)$.
Spherical polar coordinates:

$$\mathrm{d}s^2 = \mathrm{d}r^2 + F(r)^2\, \mathrm{d}\Omega^2$$

is no longer a flat manifold.



We have an unbroken symmetry point

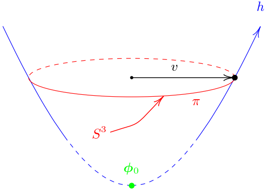
$$F(h_*) = 0$$

invariant under $O(4) \supset SU(2) \times U(1)$

$v + h = 0$, i.e. $\phi = 0$, $H = 0$.

HEFT (Higgs Effective Field Theory)

$$L = \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2} v^2\, F(h)^2\, (\partial_\mu \mathbf{n})^{\mathbf{2}}, \qquad\qquad F(0) = 1$$



h may have no relation to **n**

Higgs as a dilaton — Goldberger, Grinstein, Skiba

Looks like the SMEFT picture, except that we do not require a place where $F(h_*) = 0$. the green dot which is $O(4)$ invariant

Q: Is h part of an $SU(2)$ doublet H and linearly related to the eaten Goldstone bosons φ?

A: Depends on whether ϕ_0 exists. This is a non-perturbative question — we are stuck near the black dot.

Observables

In a QFT, the *S* matrix is unchanged under field redefinitions.

H.D. Politzer, NPB172 (1980) 349

$$\phi = f(\phi') \qquad\qquad L'(\phi') = L(f(\phi'))$$

Highly non-trivial result which follows from the LSZ reduction formula.

- Green's functions change
- Types of interactions can change, so the diagrams look different

$$\lambda\phi^4 \to \lambda(\phi + a\,\phi^3)^4$$

  e.g. has ϕ^{12} interaction
- *S*-matrix elements are unchanged

The geometrical properties of a manifold are independent of the coordinate parameterization.

Geometry

Observables only depend on the geometry of $\mathcal{M}$:

$$S \text{ matrix} \longleftrightarrow \text{Geometry}$$

$$L_{\text{linear}} = \frac{1}{2} D_\mu \phi \cdot D^\mu \phi - \frac{\lambda}{4}\left(\phi \cdot \phi - v^2\right)^2$$

$$L_{\text{nonlinear}} = \frac{1}{2}\left(\partial_\mu h\right)^2 + \frac{1}{2}(v+h)^2\, D_\mu \boldsymbol{n} \cdot D^\mu \boldsymbol{n} - \frac{\lambda}{4}\left(2vh + h^2\right)^2$$

SM in linear sigma model and nonlinear sigma model form are completely equivalent.

Experimentally measured quantities are given by the curvature of $\mathcal{M}$.

Scalar Metric

$$L = \frac{1}{2} g_{ij}(\phi)\, \partial_\mu \phi^i \partial^\mu \phi^j$$

defines the metric tensor on $\mathcal{M}$.

$$L = \frac{1}{2} (\partial_\mu h)^2 + \frac{1}{2} v^2 F(h)^2\, D_\mu \boldsymbol{n} \cdot D^\mu \boldsymbol{n}$$

$\phi = \{\boldsymbol{\pi}, h\}$ with $(n_1, n_2, n_3) = \boldsymbol{\pi}$, $n_4 = \sqrt{1 - \boldsymbol{\pi} \cdot \boldsymbol{\pi}}$.

$$g = \begin{bmatrix} v^2 F(h)^2 g_{ab}(\pi) & 0 \\ 0 & 1 \end{bmatrix}$$

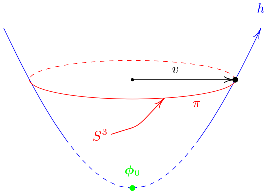
$g_{ab}(\boldsymbol{\pi})$ is the metric on a sphere of radius v.

$$\mathrm{d}s^2 = d\chi^2 + \sin^2\chi \left(\mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\phi^2\right)$$

Curvature of S^3

$$\widehat{R}_{abcd}(\varphi) = \frac{1}{v^2}\left[\, g_{ac}(\varphi)\, g_{bd}(\varphi) - g_{ad}(\varphi)\, g_{bc}(\varphi)\,\right],$$

since S^3 is a maximally symmetric space.



Sectional curvature:

$$K(X,Y) = R_{abcd} X^a Y^b X^c Y^d, \quad X \perp Y, \quad X \cdot X = 1, \quad Y \cdot Y = 1$$

$$K = \frac{1}{v^2}$$

Curvature of $\mathcal{M}$

Now look at the full manifold $\mathcal{M}$. Indices $\in \{a, h\}$

$$R_{abcd}(h) = \left[\frac{1}{v^2} - (F'(h))^2\right] F(h)^2 \left(g_{ac}g_{bd} - g_{ad}g_{bc}\right),$$
$$R_{ahbh}(h) = -F(h)F''(h)g_{ab},$$

$$\mathfrak{R}_4(h) = K(X_\pi, Y_\pi) = \left[1 - v^2\,(F'(h))^2\right] F(h)^2,$$
$$\mathfrak{R}_{2h}(h) = K(X_\pi, Y_h) = -\frac{1}{2}v^2\,F(h)\,F''(h),$$

Note that the curvature of $\mathcal{M}$ and S^3 are not the same. You can have a curved sphere imbedded in flat space.

For the SM:

$$F(h) = 1 + \frac{h}{v}\,.$$

even though S^3 is curved, $\mathcal{M}$ is flat, and all the curvatures vanish.

In HEFT, one considers a general radial function

$$F(h) = 1 + c_1 \left(\frac{h}{v}\right) + \frac{1}{2} c_2 \left(\frac{h}{v}\right)^2 + \ldots,$$

In this case,

$$\mathfrak{r}_4 \equiv \mathfrak{R}_4(0) = 1 - c_1^2, \qquad \mathfrak{r}_{2h} \equiv \mathfrak{R}_{2h}(0) = -\frac{1}{2} c_2$$

For composite Higgs theories where H is generated by symmetry breaking $G \to H$ at a scale $f \gg v$, all sectional curvatures are positive.

Experimental Consequences

Scalar observables depend on $\mathfrak{r}_{4,2h}$

analogous to S, T, U for the Higgs sector.

$$L = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{v^2}{8} F(h)^2 \left[2g^2 W^+_\mu W^{-\mu} + (g^2 + g'^2) Z_\mu Z^\mu\right]$$

$v \sim 246\,\text{GeV}$ is fixed by the gauge boson masses
Expect that

$$\mathfrak{r} \sim \frac{1}{f^2}$$

f is the scale of new physics.

S parameter:

$$\Delta S = \frac{1}{12\pi}\mathfrak{r}_4 \log\left(\frac{\Lambda^2}{M_Z^2}\right) .$$

Experimental Consequences

Barbieri, Bellazzini, Rychkov, Varagnolo

The scattering amplitude at high energies of longitudinal W-bosons W_L depends on the curvature:

$$\mathcal{A}\,(W_L W_L \to W_L W_L) = \frac{s+t}{v^2}\mathfrak{r}_4\,, \qquad K(X_\pi, Y_\pi)$$

$$\mathcal{A}\,(W_L W_L \to hh) = -\frac{2s}{v^2}\mathfrak{r}_{2h}\,. \qquad K(X_\pi, Y_h)$$

The scale of new physics governing the mass of these resonances is $\Lambda \sim 4\pi v/\sqrt{\mathfrak{r}}$

Note that this result is in accordance with the scenario of the Higgs boson as a Goldstone boson (Georgi, Kaplan) where resonances are expected at $\Lambda \sim 4\pi f$.

In composite Higgs models, the sign is fixed; $\mathfrak{r}_{4,2h} \geq 0$

Conclusions

- The S matrix depends on the geometric properties of $\mathcal{M}$.
- Can measure this experimentally in a model-independent way
- Geometrical method provides an efficient way to compute radiative corrections
- Measure the parameters $\mathfrak{r}_4$ and $\mathfrak{r}_{2h}$, and determine whether $\mathcal{M}$ is curved or flat. (SM $\implies$ $\mathcal{M}$ is flat)