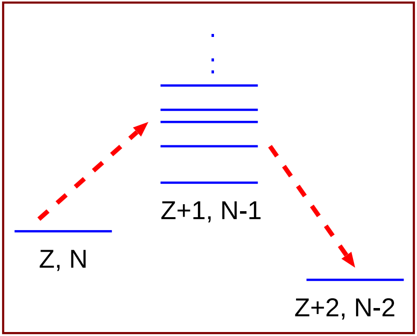
Nuclear Matrix Elements for Neutrinoless Double-Beta Decay

J. Engel

University of North Carolina

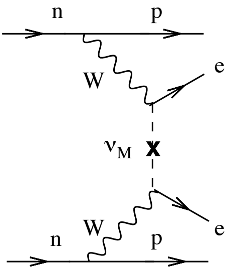
$0\nu\ \beta\beta$ Decay

If energetics are right (ordinary beta decay forbidden)...

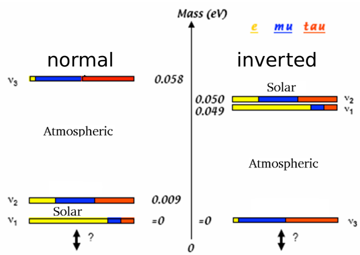


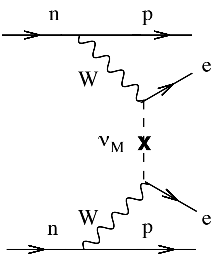
and neutrinos are their own antiparticles...

can observe two neutrons turning into protons, emitting two electrons and nothing else, e.g. via



Significance



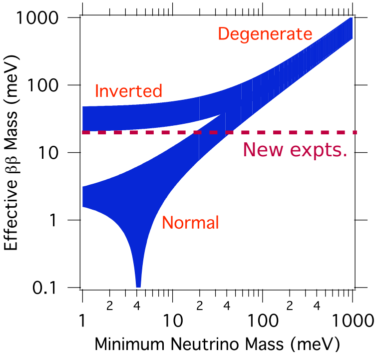


In usual scenario, rate depends on effective neutrino mass:

$$m_{\rm eff} \equiv \sum_i m_i U_{ei}^2$$

If lightest neutrino is light:

- $m_{\rm eff} \propto \sqrt{\Delta m^2_{\rm sol}}$ normal
- $m_{\rm eff} \propto \sqrt{\Delta m^2_{\rm atm}}$ inverted



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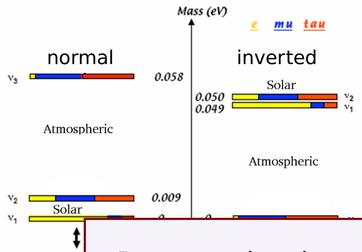

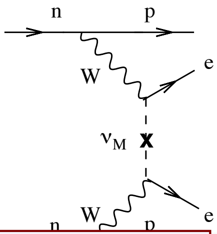

But rate also depends on a nuclear matrix element.

In us…
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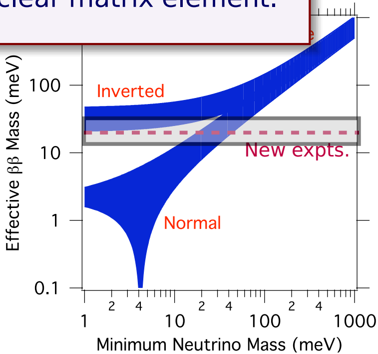
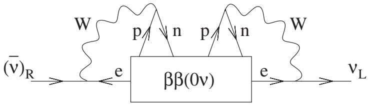
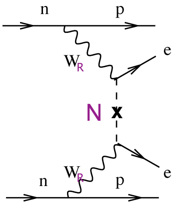


Other Mechanisms Can Contribute

If neutrinoless decay occurs then
ν's are Majorana, no matter what:



but light neutrinos may not drive the decay:



Exchange of heavy right-handed neutrino
in left-right symmetric model.

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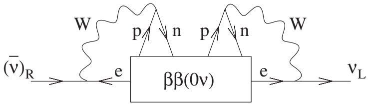

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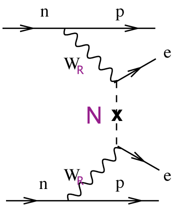

Exchange of heavy right-handed neutrino
in left-right symmetric model.

Amplitude of exotic mechanism:

$$\frac{Z_{0\nu}^{\text{heavy}}}{Z_{0\nu}^{\text{light}}} \approx \left(\frac{M_{W_L}}{M_{W_R}}\right)^4 \left(\frac{\langle q^2 \rangle}{m_{\text{eff}}\, m_N}\right)$$

$\langle q^2 \rangle \approx 10^4\ \text{MeV}^2$

$$\approx \mathbf{1} \quad \text{if} \quad m_N \approx 1\ \text{TeV} \quad \text{and} \quad m_{\text{eff}} \approx \sqrt{\Delta m^2_{\text{atm}}}$$

So exotic stuff can occur with roughly the same rate as light-ν exchange. Untangling would seem to require several expts and accurate nuclear matrix elements for all processes.

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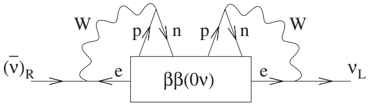

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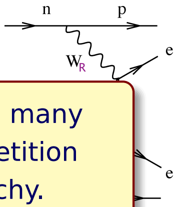

But apparently, LHC should either see many such things or rule them out as competition to light-ν exchange in inverted hierarchy.

Amplit

$$\frac{Z_{0\nu}}{Z_{0\nu}^{\text{light}}} \approx \left(\frac{M_{W_L}}{M_{W_R}}\right) \left(\frac{\langle q^2 \rangle}{m_{\text{eff}}\, m_N}\right)$$

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Light-ν-Exchange Matrix Element

$$M_{0\nu} = M_{0\nu}^{GT} - \frac{g_V^2}{g_A^2} M_{0\nu}^{F} + \ldots$$

with

$$M_{0\nu}^{GT} = \langle F | \, | \sum_{i,j} H(r_{ij}) \, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \; \tau_i^+ \tau_j^+ \; | I \rangle + \ldots$$

$$M_{0\nu}^{F} = \langle F | \sum_{i,j} H(r_{ij}) \; \tau_i^+ \tau_j^+ \; | I \rangle + \ldots$$

$$H(r) \approx \frac{2R}{\pi r} \int_0^\infty dq \frac{\sin qr}{q + \overline{E} - (E_i + E_f)/2} \quad \text{roughly} \propto 1/r$$

Contribution to integral peaks at $q \approx 200$ MeV inside nucleus. Corrections are from “forbidden” terms, weak nucleon form factors, many-body currents . . .

Nuclear-Structure Methods in One Slide

- **Density Functional Theory & Related Techniques:** Mean-field-like theory plus relatively simple corrections in very large single-particle space with phenomenological (perhaps density-dependent) interaction.

- **Shell Model:** Partly phenomenological interaction in a small single-particle space — a few orbitals near nuclear Fermi surface — but with arbitrarily complex correlations.

- **Ab Initio Calculations:** Start from a well justified two-nucleon + three-nucleon Hamiltonian, then solve full many-body Schrödinger equation to good accuracy in space large enough to include all important correlations. At present, works pretty well in systems near closed shells up to $A \approx 50$.

New!

- **Interacting Boson Model:** Model for collective states (as bosonic excitations).

$\vdots$

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New!

- **Inter**… (as boson…

Has potential to combine and ground virtues of shell model and density functional theory.

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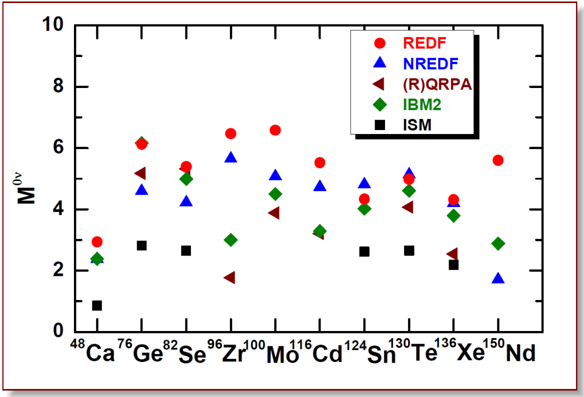
Level of Agreement So Far

Significant spread. And all the models could be missing important physics.

Uncertainty hard to quantify.



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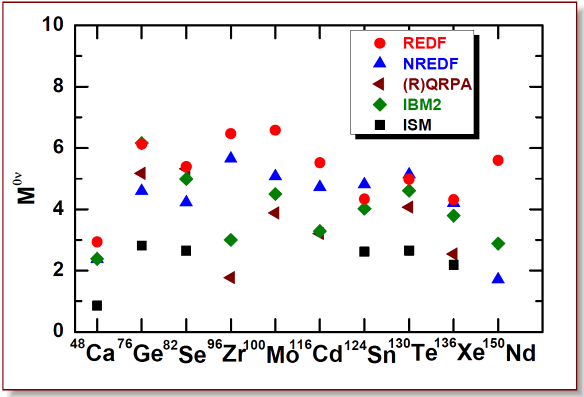
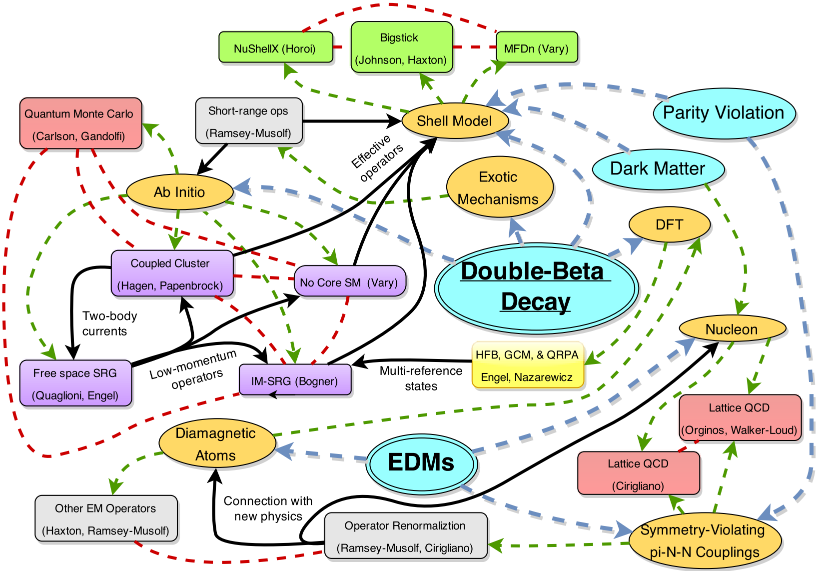


More computing power and new many-body methods responsible for major recent progress in ab initio theory.

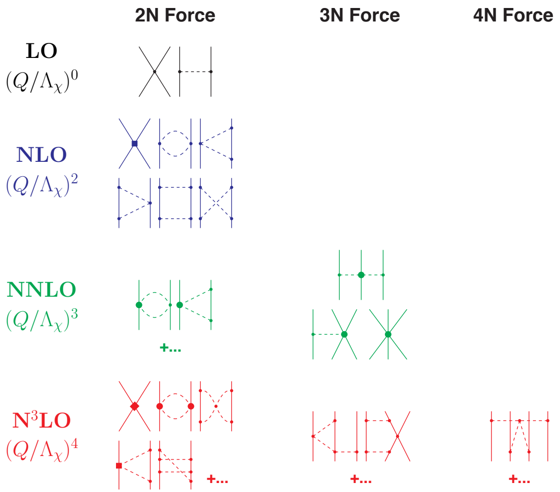
Theorists are organizing; should be able to improve all the models above and connect them to ab initio work, reducing and quantifying uncertainty.

ββ and Fund. Symmetries Topical DOE Collaboration



Ab Initio Nuclear Structure in Heavy Nuclei

Typically starts with chiral effective field theory; degrees of freedom are nucleons and pions below the chiral-symmetry breaking scale.



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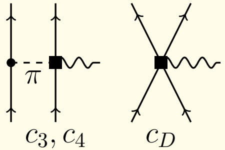
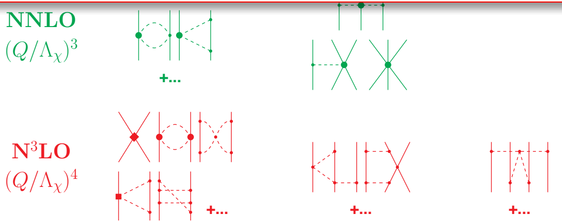
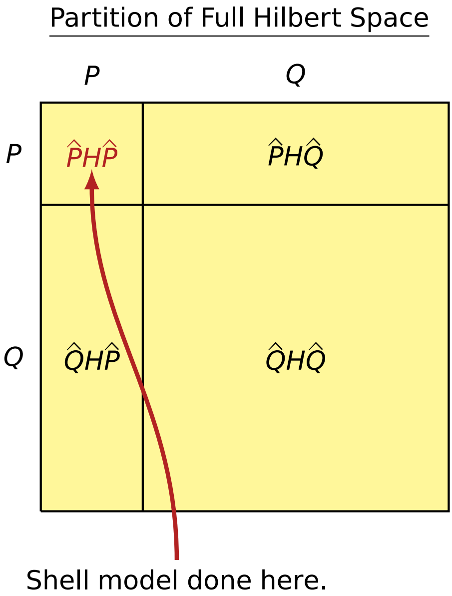


And comes with consistent weak current.



Ab Initio Shell Model

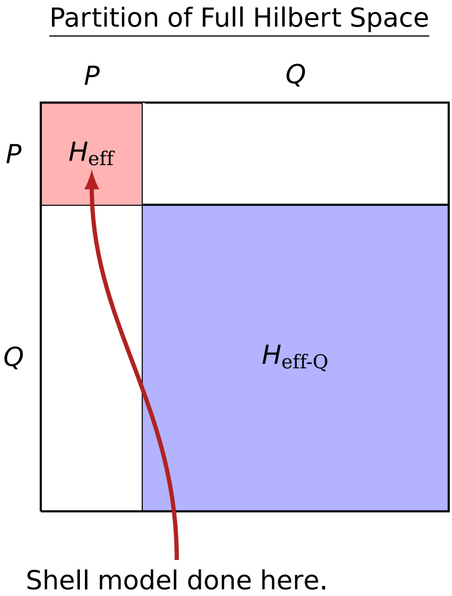


P = valence space

Q = the rest

Task: Find unitary transformation to make H block-diagonal in P and Q, with $H_{\rm eff}$ in P reproducing d most important eigenvalues.

Ab Initio Shell Model



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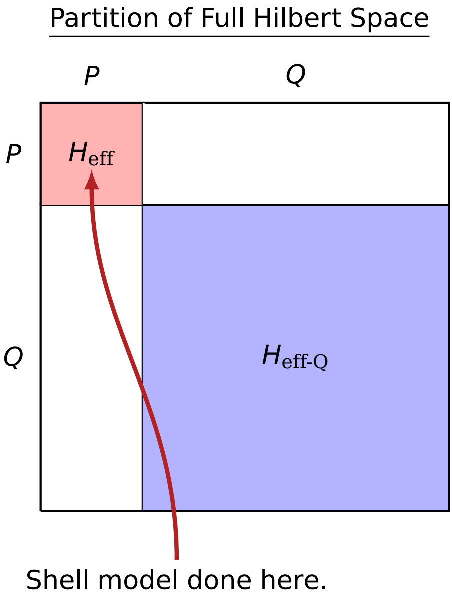

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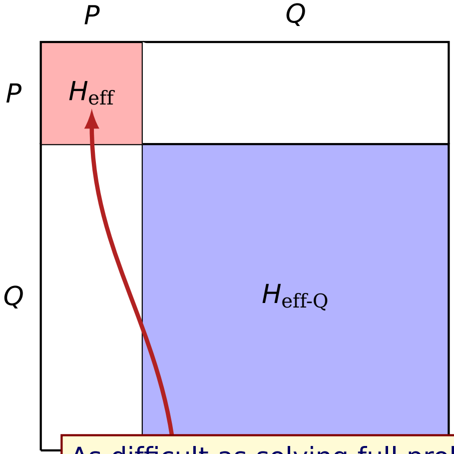
For transition operator $\hat{M}$, must apply same transformation to get $\hat{M}_{\text{eff}}$.

Ab Initio Shell Model

Partition of Full Hilbert Space



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As difficult as solving full problem. But idea is that N-body effective operators may not be important for N > 2 or 3.

Shell model done here.

Method 1: Coupled-Cluster Theory

Ground state in closed-shell nucleus:

$$|\Psi_0\rangle = e^T |\varphi_0\rangle \qquad T = \sum_{i,m} t_i^m a_m^\dagger a_i + \sum_{ij,mn} \frac{1}{4} t_{ij}^{mn} a_m^\dagger a_n^\dagger a_i a_j + \ldots$$

Slater determinant

$m,n > F \quad i,j < F$

States in closed-shell + a few constructed in similar way.

Construction of Unitary Transformation to Shell Model:

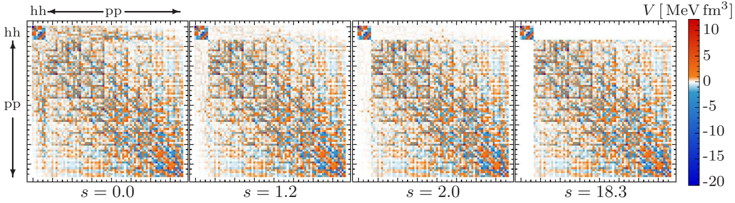
1. Calculate low-lying spectra of ^{56}Ni + 1 and 2 nucleons (and 3 nucleons in some approximation), where full calculation feasible.
2. Do Lee-Suzuki mapping of lowest eigenstates onto $f_{5/2}pg_{9/2}$ shell, determine effective Hamiltonian and decay operator.

   Lee-Suzuki maps *d* lowest eigenvectors to orthogonal vectors in shell model space in way that minimizes difference between mapped and original vectors.
3. Use these operators in shell-model calculation of matrix element for ^{76}Ge (with analogous plans for other elements).

Option 2: In-Medium Similarity Renormalization Group

Flow equation for effective Hamiltonian. Asymptotically decouples shell-model space.

$$\frac{d}{ds}H(s) = [\eta(s), H(s)]\,, \qquad \eta(s) = [H_d(s), H_{od}(s)]\,, \quad H(\infty) = H_{\text{eff}}$$

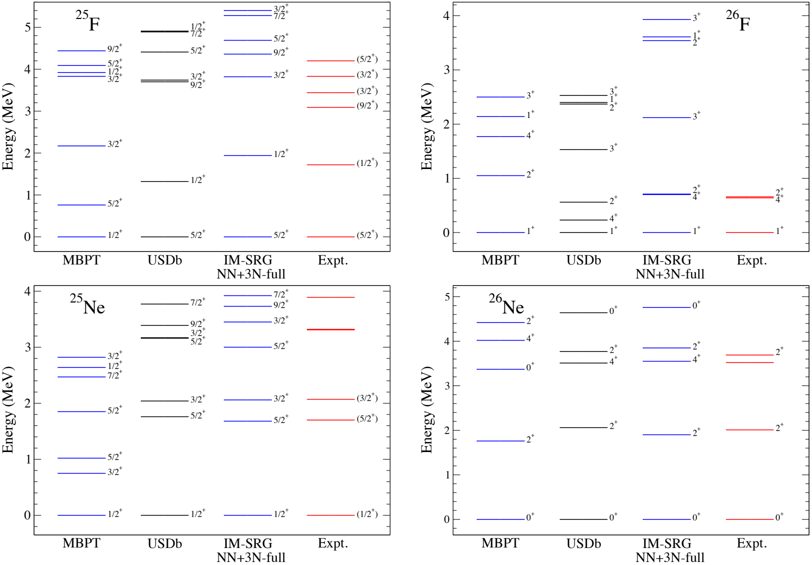


Hergert et al.

If shell-model space contains just a single state, approach yields ground-state energy. If it is a typical valence space, result is effective interaction and operators.

Development about as far along as coupled clusters.
Beginning to look at renormalization of double-beta operators.

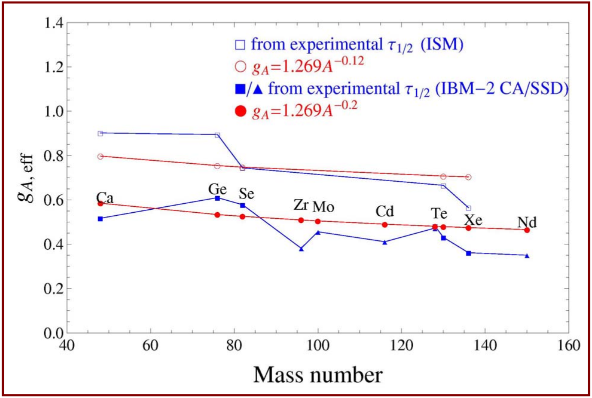
Preliminary Results in sd Shell



Bogner, Hergert, et al.

Issue Facing All Models: “g_A”

40-Year-Old Problem Particularly Important in $\beta\beta$ Decay:
Effective g_A needed for two-neutrino decay in shell model and IBM



F. Iachello, MEDEX’13 meeting

If 0ν matrix elements quenched by same amount, experiments will be less sensitive; rates go like fourth power of g_A.

We Should Resolve the Issue Soon

Problem must be due to some combination of:

1. Truncation of model space.

   Should be fixable in ab-initio shell model, which compensates effects of truncation via effective operators. Will calculate β, $2\nu\beta\beta$, and $0\nu\beta\beta$ decay, e.g., in *sd* shell and compare results with those of phenomenological shell-model with bare decay operators.

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2. Many-body weak currents.

   Size still not clear, particularly for $0\nu\beta\beta$ decay, where current is needed at finite momentum transfer q.

   Leading terms in chiral EFT for finite q only recently worked out. Careful fits and use in decay computations will happen in next year or two.

Finally. . .

Existence of topical collaboration will speed progress in next few years on this and other fronts:

- g_A problem
- Uncertainty quantification
- Other mechanisms for $\beta\beta$ decay, short-range physics

$\vdots$

Goal is accurate matrix elements with quantified uncertainty by end of collaboration (5 years from now).

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That's all; thanks for listening.