Global analysis of $b \to s\mu^+\mu^-$ *anomalies*

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in collaboration with
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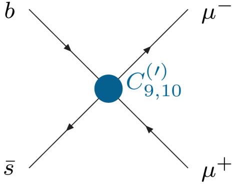
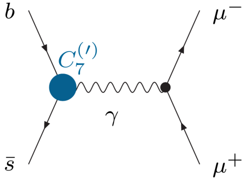
New physics in $b \to s\ell\ell$

SM and NP particles induce an effective $b\bar{s}\mu^+\mu^-$ coupling





$$\mathcal{O}_9^{(\prime)} = \frac{\alpha}{4\pi}[\bar{s}\gamma^\mu P_{L(R)} b][\bar{\mu}\gamma_\mu \mu]$$

$$\mathcal{O}_{10}^{(\prime)} = \frac{\alpha}{4\pi}[\bar{s}\gamma^\mu P_{L(R)} b][\bar{\mu}\gamma_\mu \gamma_5 \mu]$$

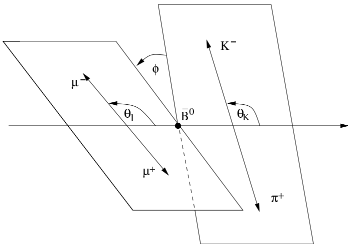
$$\mathcal{O}_7^{(\prime)} = \frac{\alpha}{4\pi} m_b [\bar{s}\sigma_{\mu\nu} P_{R(L)} b] F^{\mu\nu}$$

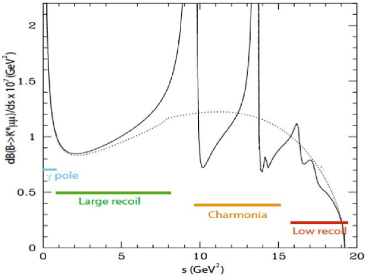
+ scalar operators
(not relevant for this talk)

processes	$\mathcal{C}_7^{(\prime)}$	$\mathcal{C}_9^{(\prime)}$	$\mathcal{C}_{10}^{(\prime)}$
$B \to X_s\gamma$, $B \to K^*\gamma$	✓		
$B \to X_s\mu^+\mu^-$	✓	✓	✓
$B_s \to \mu^+\mu^-$			✓
$B \to K^{(*)}\mu^+\mu^-$, $B_s \to \phi\mu^+\mu^-$	✓	✓	✓

$B \to K^* \mu^+ \mu^-$

4-body decay $\bar{B}_d \to \bar{K}^{*0}(\to K^- \pi^+) l^+ l^-$ with on-shell K^{*0}





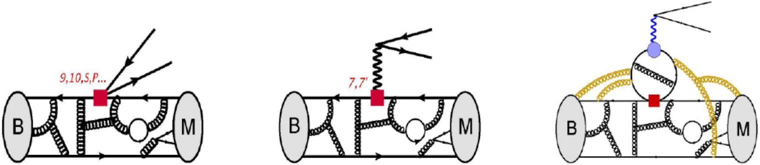
$$\frac{d^4\Gamma(\bar{B}_d)}{dq^2\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = \frac{9}{32\pi} \sum_i J_i(q^2) f_i(\theta_\ell, \theta_K, \phi)$$

invariant mass of lepton-pair q^2

angles $\theta_\ell, \theta_K, \phi$

- observables $S_i, P_i^{(\prime)}$ as ratios of J_i
- most interesting region: small $q^2 \lesssim 8\,\mathrm{GeV}$

Non-perturbative QCD



[1] Form factors: $V, A_0, A_1, A_2, T_1, T_2, T_3$

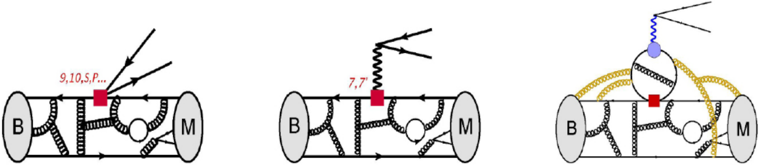
- large-recoil relations at LO, e.g.

$$\frac{m_B(m_B+m_{K^*})A_1 - 2E(m_B-m_{K^*})A_2}{m_B^2 T_2 - 2E m_B T_3} = 1 + \mathcal{O}(\alpha_s, \Lambda/m_b)$$

- construct observables involving such ratios
  $\longrightarrow$ form factors cancel at LO $\Rightarrow$ clean observables $P_i^{(\prime)}$

- correlations crucial for cancellations of FF errors

Non-perturbative QCD



$\boxed{1}$ Form factors: $V, A_0, A_1, A_2, T_1, T_2, T_3$

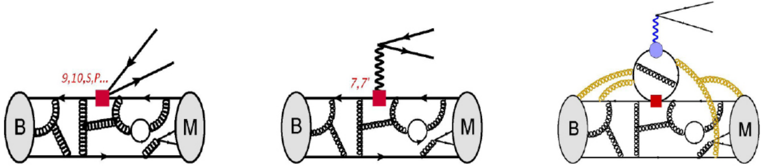
Two complementary methods to include correlations

- take correlation from particular LCSR calculation
  [Altmannshofer,Straub + Bharucha,Straub,Zwicky]

- large-recoil relations
  $+$ QCDF corrections of $\mathcal{O}(\alpha_s)$
  $+$ estimate of power corrections of $\mathcal{O}(\Lambda/m_B)$
  [Descotes-Genon,LH,Matias,Virto]

results in good agreement!

Non-perturbative QCD



[2] Long-distance charm loop effects $\mathcal{C}_9^{c\bar{c}}(q^2)$ at large recoil:

$$\mathcal{C}_9^{\rm eff}(q^2) \;=\; \mathcal{C}^{\rm eff}_{9\;{\rm SMpert.}}(q^2) \;+\; \mathcal{C}_9^{\rm NP} \;+\; \mathcal{C}_9^{c\bar{c}}(q^2)$$

- partial computation using LCSR: KMPW[Khodjamirian et al.]
  $\rightarrow$ yields $\mathcal{C}^{c\bar{c}\;i}_{9\;{\rm KMPW}} > 0$ (enhances anomalies)

- we take

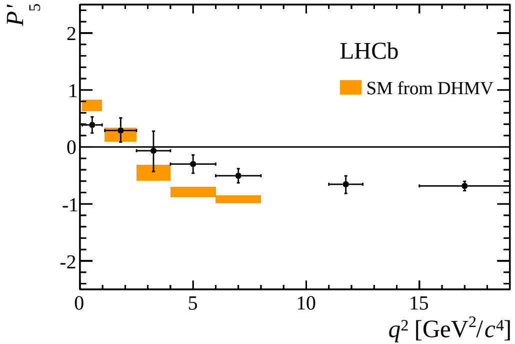
$$\mathcal{C}_9^{c\bar{c}\;i}(q^2) \;=\; s_i\;\mathcal{C}^{c\bar{c}\;i}_{9\;{\rm KMPW}}(q^2), \qquad s_i = 0 \pm 1, \qquad \text{for} \quad i = 0, \|, \perp$$

The $B \to K^* \mu^+ \mu^-$ anomaly

2013: evaluation of $1\,\text{fb}^{-1}$ data

$3.7\,\sigma$ tension in $[4, 8.3]$ GeV^2 bin of observable P_5'

2015: evaluation of $3\,\text{fb}^{-1}$ data:



$2.8\,\sigma$ in $[4, 6]$ GeV^2

$3.0\,\sigma$ in $[6, 8]$ GeV^2

tension in P_5' confirmed

$B \to K\mu^+\mu^-$ and R_K

$B^+ \to K^+\mu^+\mu^-$

$10^7 \times BR$	Theory (SM)	Experiment	Pull
$[0.1, 0.98]$	0.31 ± 0.09	0.29 ± 0.02	$+0.2$
$[1.1, 2]$	0.32 ± 0.10	0.21 ± 0.02	$+1.1$
$[2, 3]$	0.35 ± 0.11	0.28 ± 0.02	$+0.6$
$[3, 4]$	0.35 ± 0.11	0.25 ± 0.02	$+0.8$
$[4, 5]$	0.35 ± 0.11	0.22 ± 0.02	$+1.1$
$[5, 6]$	0.34 ± 0.12	0.23 ± 0.02	$+0.9$
$[6, 7]$	0.34 ± 0.12	0.25 ± 0.02	$+0.8$
$[7, 8]$	0.34 ± 0.13	0.23 ± 0.02	$+0.8$

$B^0 \to K^0\mu^+\mu^-$

$10^7 \times BR$	Theory (SM)	Experiment	Pull
$[0.1, 2]$	0.62 ± 0.19	0.23 ± 0.11	$+1.8$
$[2, 4]$	0.65 ± 0.21	0.37 ± 0.11	$+1.2$
$[4, 6]$	0.64 ± 0.22	0.35 ± 0.10	$+1.2$
$[6, 8]$	0.63 ± 0.23	0.54 ± 0.12	$+0.4$

- Agreement between theory and experiment at $\sim 1\,\sigma$
- but: experiment systematically lower than theory prediction for all available FF parametrizations:
  - LCSR FFs from KMPW[Khodjamirian et al.] and BZ[Ball,Zwicky]
  - lattice QCD[Bouchard et al.]

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- $R(K) = \mathrm{Br}(B \to K\mu^+\mu-)/\mathrm{Br}(B \to Ke^+e^-) \overset{\text{exp.}}{=} 0.75^{+0.09}_{-0.07} \pm 0.04$
  2.6 sigma deviation from clean SM prediction $R(K) = 1$

$B_s \to \phi\mu^+\mu^-$

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$[0.1, 2.]$	1.81 ± 0.36	1.11 ± 0.16	$+1.8$
$[2., 5.]$	1.88 ± 0.32	0.77 ± 0.14	$+3.2$
$[5., 8.]$	2.25 ± 0.41	0.96 ± 0.15	$+2.9$
$[15, 18.8]$	2.20 ± 0.17	1.62 ± 0.20	$+2.2$

- Tension between theory and experiment at $\sim 3\,\sigma$
- but: strong dependence on hadronic form factors (LCSR FFs from BSZ[Bharucha,Straub,Zwicky])
- better: study clean observables
  $\rightarrow$ not enough statistics yet ...
- $BR(B_s \to \phi\mu^+\mu^-)$ not conclusive as single observable, but as ingredient of global analysis

Possible explanations

- statistical fluctuation of data
  $\longrightarrow$ perform consistence checks [Matias,Serra]

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- effect from charm resonances [Lyon,Zwicky]
  + could affect the anomalous bins of P_5'
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- new physics (Z'-models, lepto-quarks)
  + can explain tension in R_K if coupled only to muons

Global Fit: Framework

Form factor input:

- large recoil: LCSR form factors mainly from KMPW
- low recoil: lattice form factors from [Horgan et al.; Bouchard et al.]

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- $B_{(s)} \to (K^*, \phi)\mu^+\mu^-$: BRs + angular observables
- $B \to K\mu^+\mu^-$: BRs charged + neutral mode
- $B \to X_s\gamma$, $B \to K^*\gamma$ (A_I and $S_{K^*\gamma}$), $B \to X_s\mu^+\mu^-$, $B_s \to \mu^+\mu^-$

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Frequentist $\Delta\chi^2$-fit:

- model hypothesis for $\{C_i\}$ with n degrees of freedom
- Experimental and theoretical correlation matrix included (theory uncertainties treated as Gaussian)
- SM-pull ($=$ by how many σ is $\{C_i^{\text{SM}}\}$ disfavoured compared to $\{C_i^{\text{fit}}\}$ under the model hypothesis)

1D scenarios

Coefficient	Best fit	1σ	3σ	Pull_{SM}
$\mathcal{C}_7^{\text{NP}}$	-0.02	$[-0.04, -0.00]$	$[-0.07, 0.03]$	1.2
$\boldsymbol{\mathcal{C}_9^{\text{NP}}}$	$\boldsymbol{-1.09}$	$[-1.29, -0.87]$	$[-1.67, -0.39]$	**4.5**
$\mathcal{C}_{10}^{\text{NP}}$	0.56	$[0.32, 0.81]$	$[-0.12, 1.36]$	2.5
$\mathcal{C}_{7'}^{\text{NP}}$	0.02	$[-0.01, 0.04]$	$[-0.06, 0.09]$	0.6
$\mathcal{C}_{9'}^{\text{NP}}$	0.46	$[0.18, 0.74]$	$[-0.36, 1.31]$	1.7
$\mathcal{C}_{10'}^{\text{NP}}$	-0.25	$[-0.44, -0.06]$	$[-0.82, 0.31]$	1.3
$\mathcal{C}_9^{\text{NP}} = \mathcal{C}_{10}^{\text{NP}}$	-0.22	$[-0.40, -0.02]$	$[-0.74, 0.50]$	1.1
$\boldsymbol{\mathcal{C}_9^{\text{NP}} = -\mathcal{C}_{10}^{\text{NP}}}$	$\boldsymbol{-0.68}$	$[-0.85, -0.50]$	$[-1.22, -0.18]$	**4.2**
$\boldsymbol{\mathcal{C}_9^{\text{NP}} = -\mathcal{C}_{9'}^{\text{NP}}}$	$\boldsymbol{-1.06}$	$[-1.25, -0.85]$	$[-1.60, -0.40]$	**4.8**

Large negative NP-contribution to $\mathcal{C}_9$ needed!

Channel decomposition

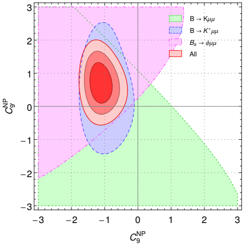
Fit	$\mathcal{C}_{9\ \mathrm{Bestfit}}^{\mathrm{NP}}$	1σ	$\mathrm{Pull}_{\mathrm{SM}}$
All $b \to s\mu\mu$	-1.09	$[-1.29, -0.87]$	4.5
All $b \to s\mu\mu$ excluding [6,8] region	-0.99	$[-1.23, -0.75]$	3.8
Only $B \to K\mu\mu$	-0.85	$[-1.67, -0.20]$	1.4
Only $B \to K^*\mu\mu$	-1.05	$[-1.27, -0.80]$	3.7
Only $B_s \to \phi\mu\mu$	-1.98	$[-2.84, -1.29]$	3.5
Only $b \to s\mu\mu$ at large recoil	-1.30	$[-1.57, -1.02]$	4.0
Only $b \to s\mu\mu$ at low recoil	-0.93	$[-1.23, -0.61]$	2.8

- different decay channels and q^2-regions point to the same NP solution
- overlap of 1σ fit regions at $\mathcal{C}_9^{\mathrm{NP}} \sim -1.1$

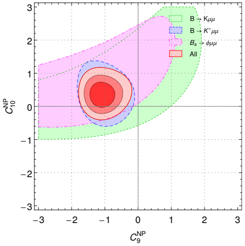
What about other Wilson coefficients?

		$\mathcal{C}_7^{\rm NP}$	$\mathcal{C}_9^{\rm NP}$	$\mathcal{C}_{10}^{\rm NP}$	$\mathcal{C}_{7'}^{\rm NP}$	$\mathcal{C}_{9'}^{\rm NP}$	$\mathcal{C}_{10'}^{\rm NP}$
$\mathcal{C}_9^{\rm NP}$	**4.47**	0.07	*	**1.54**	0.92	**2.00**	**1.89**

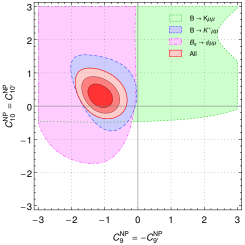
$(\mathcal{C}_9^{\rm NP}, \mathcal{C}_{9'}^{\rm NP})$
$(-1.12, 0.77)$
4.5



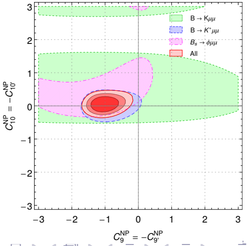
$(\mathcal{C}_9^{\rm NP}, \mathcal{C}_{10}^{\rm NP})$
$(-1.08, 0.33)$
4.3



$(\mathcal{C}_9^{\rm NP} = -\mathcal{C}_{9'}^{\rm NP}, \mathcal{C}_{10}^{\rm NP} = \mathcal{C}_{10'}^{\rm NP})$
$(-1.15, 0.34)$
4.7



$(\mathcal{C}_9^{\rm NP} = -\mathcal{C}_{9'}^{\rm NP}, \mathcal{C}_{10}^{\rm NP} = -\mathcal{C}_{10'}^{\rm NP})$
$(-1.06, 0.06)$
4.4



Complete 6D fit

Coefficient	1σ	2σ	3σ
$\mathcal{C}_7^{\text{NP}}$	$[-0.02, 0.03]$	$[-0.04, 0.04]$	$[-0.05, 0.08]$
$\mathcal{C}_9^{\text{NP}}$	$[-1.4, -1.0]$	$[-1.7, -0.7]$	$[-2.2, -0.4]$
$\mathcal{C}_{10}^{\text{NP}}$	$[-0.0, 0.9]$	$[-0.3, 1.3]$	$[-0.5, 2.0]$
$\mathcal{C}_{7'}^{\text{NP}}$	$[-0.02, 0.03]$	$[-0.04, 0.06]$	$[-0.06, 0.07]$
$\mathcal{C}_{9'}^{\text{NP}}$	$[0.3, 1.8]$	$[-0.5, 2.7]$	$[-1.3, 3.7]$
$\mathcal{C}_{10'}^{\text{NP}}$	$[-0.3, 0.9]$	$[-0.7, 1.3]$	$[-1.0, 1.6]$

- $\mathcal{C}_9$ consistent with SM only above 3σ
- All other Wilson coefficients consistent with SM ($\mathcal{C}_9'$ at 2σ)
- total SM-pull of 6D-fit: 3.6σ

New Physics vs. Charm

$$\boxed{\mathcal{C}_9^{\text{eff}}(q^2) = \mathcal{C}_{9\ \text{SMpert.}}^{\text{eff}}(q^2) + \mathcal{C}_9^{\text{NP}} + \mathcal{C}_9^{c\bar{c}}(q^2)}$$

- NP contribution $\mathcal{C}_9^{\text{NP}}$ enters always together with non-perturbative charm-contribution $\mathcal{C}_9^{c\bar{c}}(q^2)$
- $\mathcal{C}_9^{\text{NP}}$: q^2-independent
  $\mathcal{C}_9^{c\bar{c}}(q^2)$: pronounced q^2-dependence expected

New Physics vs. Charm

$$\mathcal{C}_9^{\text{eff}}(q^2) = \mathcal{C}_{9\,\text{SMpert.}}^{\text{eff}}(q^2) + \mathcal{C}_9^{\text{NP}} + \mathcal{C}_9^{c\bar{c}}(q^2)$$

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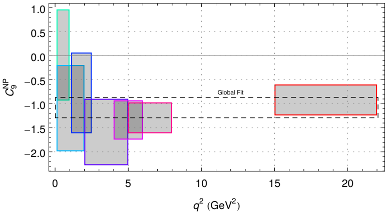
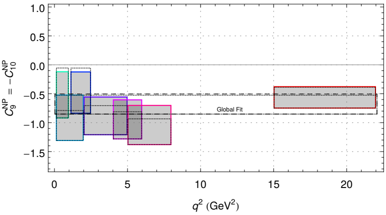
- $\mathcal{C}_9^{\text{NP}}$: q^2-independent
  $\mathcal{C}_9^{c\bar{c}}(q^2)$: pronounced q^2-dependence expected
- perform individual fits in different q^2-regions

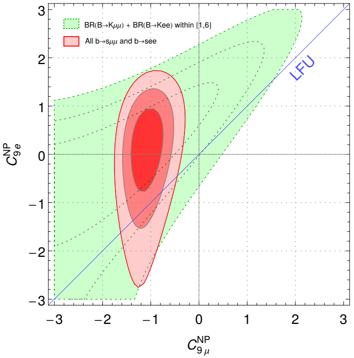


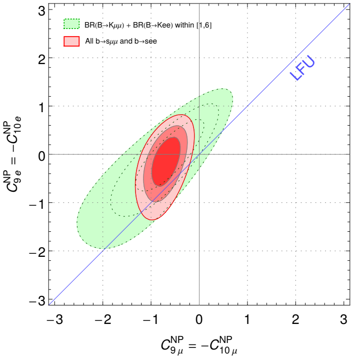


results compatible with q^2-independent shift!

Lepton-flavour non-universality

- measurement of R_K suggests violation of LFU
- allow for independent contributions $\mathcal{C}_{i\,\mu}^{\text{NP}}$ and $\mathcal{C}_{i\,e}^{\text{NP}}$ to operators
- add electron-channels $B \to K^{(*)} e^+ e^-$ to the global fit





fit prefers NP coupling to $\mu^+\mu^-$ but not to e^+e^-

(SM-pulls typically increase by $\sim 0.5\sigma$ under this hypothesis)

Conclusions

- several $\sim 3\sigma$ anomalies in $b \to s\ell^+\ell^-$ data:
  $P_5'(B \to K^*\mu^+\mu^-)$, $Br(B_s \to \phi\mu^+\mu^-)$, R_K
- global fit gives $4 - 5\sigma$ preferences for scenarios with negative $\mathcal{C}_9^{\rm NP} \sim -1.1$
- form factor uncertainties (factorizable power corrections) are under control
- alternative explanation via large charm-loop effects:
  - fit compatible with q^2-independent effect
  - cannot explain R_K
- R_K favours LFU violation with NP coupling only to $\mu^+\mu^-$, not to e^+e^- $\Rightarrow$ search for $R_{K^*}, R_\phi < 1$!

Backup

Lepton-flavour non-universality

- assume NP in $\mathcal{C}_{i\,\mu}^{\rm NP}$, but no NP in $\mathcal{C}_{i\,e}^{\rm NP}$
- Predictions for R_K, R_{K^*}, R_ϕ for best-fit points:

	$R_K[1,6]$	$R_{K^*}[1.1,6]$	$R_\phi[1.1,6]$
SM	1.00 ± 0.01	1.00 ± 0.01	1.00 ± 0.01
$\mathcal{C}_9^{\rm NP} = -1.11$	0.79 ± 0.01	0.87 ± 0.08	0.84 ± 0.02
$\mathcal{C}_9^{\rm NP} = -\mathcal{C}_{10}^{\rm NP} = -0.69$	0.67 ± 0.01	0.71 ± 0.03	0.69 ± 0.01
$\mathcal{C}_9^{\rm NP} = -1.16, \mathcal{C}_{10}^{\rm NP} = 0.35$	0.71 ± 0.01	0.78 ± 0.07	0.76 ± 0.01

$\Rightarrow$ search for $R_{K^*}, R_\phi < 1$!

Comparison with Altmannshofer/Straub

	our analysis (DHMV)	Altmannsh./Straub (AS)
FF input	mainly KMPW	BSZ
FF correlations	from large-recoil symmetries + power corrections	from BSZ calculation
$B \to K^* \mu^+ \mu^-$ observables	$P_i^{(\prime)}$ all bins	S_i bins within $[1, 6]$

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AS: + exact assessment of correlations for BSZ form factors

– depends on model-assumptions of and is limited to this particular set of form factors

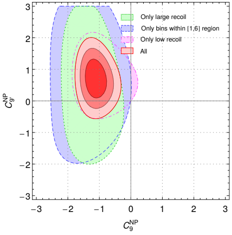
DHMV: + model-independent determination of dominant FF correlations

– correlations only up to symmetry breaking corrections of order $\mathcal{O}(\Lambda/m_b)$ which can only be estimated

$\Rightarrow$ Analyses complement each other

Comparison with Altmannshofer/Straub

DHMV



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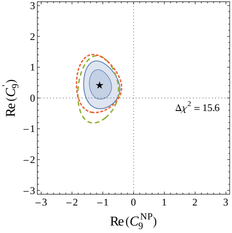

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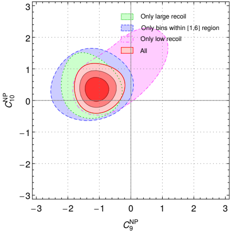

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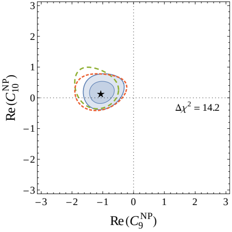

Results in reasonably good agreement!

Implementation of hadronic uncertainties

Form factors: Symmetry-breaking corrections:

$$F(q^2) = F^{\rm soft}(q^2) + \Delta F^{\alpha_s}(q^2) + a_F + b_F \frac{q^2}{m_B^2}$$

- central values for a_F, b_F from fit to the full form factor F (taken from LCSR)
- conservative error estimate:
  assign $\sim 100\%$ errors to $a_F, b_F = \mathcal{O}(\Lambda/m_B) \times F$

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Long-distance charm effects $\mathcal{C}_9^{c\bar{c}}(q^2)$ at large recoil:

- partial computation using LCSR: KMPW[Khodjamirian et al.]
  $\rightarrow$ yields $\mathcal{C}_{9\,\text{KMPW}}^{c\bar{c}\;i} > 0$ (enhances anomalies)
- we take

$$\mathcal{C}_9^{c\bar{c}\;i}(q^2) \;=\; s_i\, \mathcal{C}_{9\,\text{KMPW}}^{c\bar{c}\;i}(q^2), \qquad s_i = 0 \pm 1, \qquad \text{for} \quad i = 0, \|, \perp$$

Fit: Statistical Framework

$$\chi^2(\{C_i\}) = (\vec{O}_{\text{exp}} - \vec{O}_{\text{th}}(\{C_i\}))^T (Cov_{\text{exp}} + Cov_{\text{th}})^{-1} (\vec{O}_{\text{exp}} - \vec{O}_{\text{th}}(\{C_i\}))$$

Frequentist $\Delta\chi^2$-fit:

- model hypothesis for $\{C_i\}$ with n degrees of freedom
- Experimental correlation matrix Cov_{exp}
- Theoretical correlation matrix Cov_{th}:
  - assume $Cov_{\text{th}}(C_i) = Cov_{\text{th}}(C_i^{\text{SM}})$
    $\rightarrow$ check: repeat fit for $Cov_{\text{th}}(C_i) = Cov_{\text{th}}(C_i^{\text{fit}})$
  - treat all systematic uncertainties as Gaussian
- determine
  - best-fit point $\{C_i^{\text{fit}}\}$
  - confidence level regions
  - SM-pull (= by how many σ $\{C_i^{\text{SM}}\}$ is disfavoured compared to $\{C_i^{\text{fit}}\}$ under the model hypothesis)