H Effective Theory and the Geometry of Scalar Field Space

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Outline

- Effective Field Theory: Motivation
- Effective Field Theory Generalizations of SM Higgs Sector: SMEFT and HEFT
- Geometric Invariants of Scalar Manifold: Curvature of Scalar Field Space, Connection
- 1-Loop Renormalization for a Curved Scalar Manifold
- Two Concrete Examples of HEFT: Positive and Negative Curvature
- Conclusions

R. Alonso, EJ, A. Manohar, 1511.00724, 1602.00706

Effective Field Theory: Motivation

- SM renormalizable theory gives a good description of particle physics, including the Higgs boson h, up to energy scale $\Lambda \sim 1$ TeV. Why introduce EFT?
- Current LHC at 13 TeV run can
  (1) produce any accessible new particles (direct discovery).
  (2) probe new interactions of SM particles resulting from unobserved particles at inaccessible higher energy scale Λ (indirect discovery).
- EFT classifies all possible interactions for a given set of fields. (Classification enables one to pursue objective (2) for a given set of particles with known symmetry properties.)
- EFT tells you what modifications to SM couplings are possible.
  (EFT only valid to energy scale Λ, where it is replaced by another EFT.)

Effective Field Theory: Motivation

- Generalize SM by constructing EFT generalizations. Add all possible non-renormalizable higher dimension operators $d > 4$ with arbitrary coefficients and power suppressions $1/\Lambda^{d-4}$. Use experiment to bound these new interactions. Effectively probes low-energy $E < \Lambda$ consequences of fundamental high energy theory with new particles of mass $\sim \Lambda$.

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^{d-4}} \sum_i C_i O_i^{(d)}$$

- Any fundamental high energy theory with new particles of mass Λ yields EFT description at $E < \Lambda$. Analysis is model independent.

SM Higgs Sector

SM written in terms of scalar Higgs doublet

$$H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi^+(x) \\ v + h(x) + i\varphi^0(x) \end{pmatrix}.$$

4 scalar d.o.f in H: 3 Goldstone bosons $\varphi^\pm, \varphi^0$ and h.

3 Goldstone boson directions are in $W^\pm, Z$ gauge directions $\Rightarrow$
3 Goldstone bosons are third (longitudinal) polarization states of massive $W^\pm, Z$

$$\varphi^\pm, \varphi^0 \sim W_L^\pm, Z_L^0$$

$$\mathcal{L}_{\rm SM} = D_\mu H^\dagger D^\mu H - \lambda \left(H^\dagger H - \frac{v^2}{2} \right)^2$$

SM Higgs Sector

SM written in terms of scalar four-dimensional irrep ϕ of $O(4)$.

$$\mathcal{L}_{\rm SM} = \frac{1}{2}\partial_\mu \boldsymbol{\phi} \cdot \partial^\mu \boldsymbol{\phi} - \frac{1}{4}\lambda \left(\boldsymbol{\phi}\cdot\boldsymbol{\phi} - v^2\right)^2, \qquad \boldsymbol{\phi} = \begin{pmatrix} \phi^1 \\ \phi^2 \\ \phi^3 \\ \phi^4 \end{pmatrix}$$

Cartesian Coordinates

$$\boldsymbol{\phi} = \begin{pmatrix} \varphi^1 \\ \varphi^2 \\ \varphi^3 \\ v + h \end{pmatrix}$$

Polar Coordinates

$$\boldsymbol{\phi} = (v + h)\ \boldsymbol{n}(\varphi), \qquad \boldsymbol{n}(\varphi) \in S^3 \Rightarrow \boldsymbol{n}\cdot\boldsymbol{n} = 1.$$

SM Higgs Sector

Cartesian Coordinates

$$\phi = \begin{pmatrix} \varphi^1 \\ \varphi^2 \\ \varphi^3 \\ v + h \end{pmatrix}$$

$$\mathcal{L}_{\rm SM} = \frac{1}{2}\partial_\mu h\, \partial^\mu h + \frac{1}{2}\partial_\mu \boldsymbol{\varphi} \cdot \partial^\mu \boldsymbol{\varphi} - \frac{1}{4}\lambda \left(2vh + h^2 + \boldsymbol{\varphi} \cdot \boldsymbol{\varphi}\right)^2$$

- ϕ transforms linearly as **4** under $\mathcal{G} = O(4)$.
- h invariant (singlet) under $\mathcal{H} = O(3)$.
- $\varphi = (\varphi^1, \varphi^2, \varphi^3)$ transforms linearly as triplet under $\mathcal{H} = O(3)$.

⋆ $V(\phi)$ depends on $\boldsymbol{\varphi} \cdot \boldsymbol{\varphi}$ and h. There are non-derivative couplings of φ.

⋆ $m_h^2 = 2\lambda v^2$, $m_\varphi^2 = 0$.

Polar Coordinates

$$\phi = (v + h)\ \boldsymbol{n}(\varphi), \qquad \boldsymbol{n}(\varphi) \in S^3 \Rightarrow \boldsymbol{n} \cdot \boldsymbol{n} = 1.$$

$$\mathcal{L}_{\rm SM} = \frac{1}{2}(\partial_\mu h)^2 + (v + h)^2 \partial_\mu \boldsymbol{n} \cdot \partial^\mu \boldsymbol{n} - \frac{1}{4}\lambda \left(2vh + h^2\right)^2$$

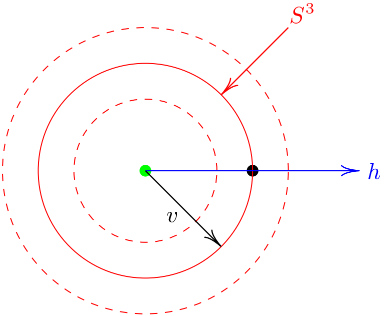
$$h \to h, \qquad \boldsymbol{n}(\varphi) \to O\,\boldsymbol{n}(\varphi), \quad O^T O = 1.$$

- h invariant (singlet) under $\mathcal{G} = O(4)$ and $\mathcal{H} = O(3)$.
- $\boldsymbol{n}(\varphi)$ transforms non-linearly under $\mathcal{G} = O(4)$, but $\varphi = (\varphi^1, \varphi^2, \varphi^3)$ transforms linearly as triplet under $\mathcal{H} = O(3)$.

⋆ $V(\phi)$ depends only on h. Goldstone bosons are derivatively coupled.

⋆ $m_h^2 = 2\lambda v^2$, $m_\varphi^2 = 0$.

SM Higgs Sector



$$\mathcal{G}/\mathcal{H} = S^3 \subset \mathcal{M} = \mathbb{R}^4$$

$$(v + h) = v\left(1 + \frac{h}{v}\right) \equiv v\, F_{\rm SM}(h)$$

Custodial Symmetry

SM has enhanced global symmetry

$$\mathcal{G} \to \mathcal{H}$$

$$O(4) \to O(3)$$

$$SU(2)_L \times SU(2)_R \to SU(2)_V$$

$$6 \to 3$$

$$\mathcal{G}_{\text{gauge}} \to \mathcal{H}_{\text{gauge}}$$

$$SU(2)_L \times U(1)_Y \to U(1)_Q$$

$$4 \to 1$$

2 additional global symmetry generators

$$M_W = M_Z \cos\theta_W$$

SM Higgs Sector

- Scalar fields are coordinates ϕ^i of scalar manifold $\mathcal{M}$. Scalar field redefinitions are change of coordinates. Physical observables independent of coordinates ($\equiv$ scalar fields) used.
- Tangent vectors to $\mathcal{M}$ define scalar metric $g_{ij}(\phi)$. Parallel transport on $\mathcal{M}$ depends on geometric invariants (curvature) of $\mathcal{M}$ which are independent of the choice of coordinates ($\equiv$ scalar fields).
- Scalar metric determined from scalar KE term. SM scalar manifold is flat $\Rightarrow$ curvature vanishes. Same result of zero curvature for both Cartesian and polar coordinates. No indication for any new physics scale Λ at which theory breaks down. Hallmark of renormalizable theory.

HEFT

HEFT written in terms of singlet h and triplet $\boldsymbol{\varphi} = (\varphi^1, \varphi^2, \varphi^3)$.

- Non-trivial scalar metric since $\exists$ higher dimensional 2-derivative scalar operators $\Rightarrow$ scalar manifold $\mathcal{M}$ is curved.
- Coordinate-independent Riemann curvature tensor determines physical observables.

$$\mathcal{L}_{\rm KE} = g_{ij}(\phi)\,\partial_\mu \phi^i\,\partial^\mu \phi^j = v^2 F(h)^2\,\partial_\mu \boldsymbol{n}(\varphi)\cdot \partial^\mu \boldsymbol{n}(\varphi) + \frac{1}{2}(\partial_\mu h)^2$$

$$F(h) = 1 + c_1 \frac{h}{v} + \frac{1}{2} c_2 \frac{h^2}{v^2} + \cdots \qquad \text{vs.} \qquad F_{\rm SM}(h) = 1 + \frac{h}{v}$$

$$g_{ij}(\phi) = \begin{bmatrix} F(h)^2 g_{ab}(\varphi) & 0 \\ 0 & 1 \end{bmatrix}$$

where $g_{ab}(\varphi)$ is metric on $\mathcal{G}/\mathcal{H} = S^3 \subset \mathcal{M}$.
3 angular coordinates of S^3 are $\boldsymbol{\varphi}/v$.

Curved Spaces: Analogy with General Relativity

General Relativity

- Spacetime metric $g_{\mu\nu}(x)$
- Curvature of spacetime: $R_{\mu\nu\sigma\tau}(x)$, $\Gamma^{\kappa}_{\mu\nu}(x)$
- Point particles travel on geodesics of curved spacetime.
- Spacetime curvature sourced by $T_{\mu\nu}$.

HEFT

- Scalar manifold metric $g_{ij}(\phi)$
- Curvature of scalar manifold $\mathcal{M}$: $R_{ijkl}(\phi)$, $\Gamma^{i}_{jk}(\phi)$
- Scalar fields travel on geodesics of curved scalar manifold $\mathcal{M}$.
- Scalar manifold curvature sourced by BSM Physics at Energy Scale $\Lambda \equiv 4\pi f > v$. This BSM Physics appears in HEFT as higher dimension (non-renormalizable) contributions to the two-derivative scalar KE term.

1-Loop Renormalization For Curved Scalar Manifold

- Introduce covariant formalism for curved scalar manifold $\mathcal{M}$.
- EOM of scalar field is geodesic of $\mathcal{M}$.

$$g_{ij}(\phi)\left(\partial^2\phi^i + \Gamma^i_{jk}\left(\partial_\mu\phi^j\right)\left(\partial^\mu\phi^k\right)\right) - \mathcal{I}_{,j} = 0$$

- Variation of action using geodesic fluctuation of scalar field to maintain covariance. For example, in sigma model with S^3 submanifold

$$\frac{d^2\varphi^a}{d\lambda^2} + \Gamma^a_{bc}(\phi)\frac{d\varphi^b}{d\lambda}\frac{d\varphi^c}{d\lambda} = 0$$

$$\varphi^a(\lambda) = \tilde{\varphi}^a + \lambda\zeta^a - \frac{1}{2}\lambda^2\Gamma^a_{bc}\zeta^b\zeta^c + O(\zeta^3)$$

$$\delta S = S[\varphi] + \frac{\delta S}{\delta\varphi^a}\left(\zeta^a - \frac{1}{2}\Gamma^a_{bc}\zeta^b\zeta^c\right) + \frac{\delta^2 S}{\delta\varphi^b\delta\varphi^c}\zeta^b\zeta^c + O(\zeta^3)$$

1-Loop Renormalization For Curved Scalar Manifold

$$\delta^2 S = \frac{1}{2}\int d^4x\left[g_{ab}\,(\mathscr{D}_\mu\zeta)^a\,(\mathscr{D}^\mu\zeta)^b - R_{abcd}\,D_\mu\tilde{\varphi}^a D^\mu\tilde{\varphi}^c\zeta^b\zeta^d\right]$$

$$\mathscr{D}_\mu\zeta^a = (D_\mu\zeta)^a + \Gamma^a_{bc}(\tilde{\varphi})\,(D_\mu\tilde{\varphi})^b\,\zeta^c,\quad (D_\mu\zeta)^a = \partial_\mu\zeta^a + A^B_\mu t^a_B$$

No non-covariant EOM terms proportional to $\delta S/\delta\varphi^a$ which vanish on-shell and do not contribute to S-matrix.

- 1-loop correction

$$\Delta\Gamma = \frac{1}{16\pi^2\epsilon}\int d^4x\,\mathrm{Tr}\left\{\frac{\Gamma^2_{\mu\nu}}{24} + \frac{X^2}{4}\right\}$$

$$[\Gamma_{\mu\nu}]^i{}_j = [\mathscr{D}_\mu, \mathscr{D}_\nu]^i{}_j = R^i{}_{jkl}\,(D_\mu\phi\,)^k\,(D_\nu\phi)^l + (\mathscr{A}_{\mu\nu})^i{}_j$$

$$(\mathscr{A}_{\mu\nu})^i{}_j = \left(\partial_{[\mu}A^B_{\nu]} + f^B_{CD}A^C_\mu A^D_\nu\right)\,t^i_{B;j}$$

$$X^i{}_j = \nabla^i\nabla_j\mathcal{I} - R^i{}_{kjl}\,(D^\mu\phi)^k\,(D_\mu\phi)^l$$

1-Loop Renormalization For Curved Scalar Manifold

1-loop correction of HEFT encapsulates many interesting special cases in a unified way.

$$L = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}F(h)^2 g_{ab}(\varphi) D_\mu \varphi^a D^\mu \varphi^b - V(h) + K(h) w^i n^i$$

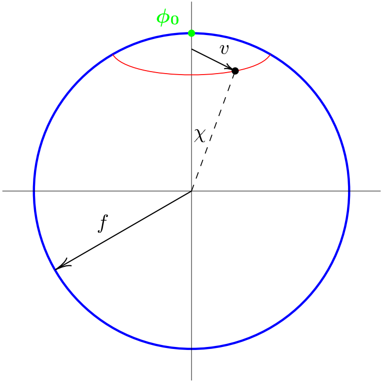
$$w^i = \bar{q}_L \sigma^i Y_q q_R + \bar{\ell}_L \sigma^i Y_\ell \ell_R$$

Need to specify $F(h)$, $K(h)$, $V(h)$.

- Chiral Perturbation Theory (theory of pions with no h): $F(h)$,$K(h)$= constant
- SM Higgs boson h: $F(h) = K(h) = v + h$, $V(h) = \lambda/4(h^2 + 2vh)^2$
- Goldstone boson h of enlarged global symmetry: $F(h) = \sin h/f$
- Dilaton h: Introduce scale invariance via $v \to ve^{\tau/v}$, $h/v = e^{\tau/v} - 1$

Example of HEFT with Positive Curvature

$$O(5)/O(4) = S^4$$



$$f^2 = \sum_{i=1}^{5} (\phi^i)^2$$

Agashe, Contino and Pomarol (2005)

$O(5) \to O(4)$ Composite Higgs Model

$$\phi = \begin{bmatrix} \phi^1 \\ \phi^2 \\ \phi^3 \\ \phi^4 \\ \phi^5 \end{bmatrix} = \begin{bmatrix} f \sin\chi\, \boldsymbol{n} \\ f \cos\chi \end{bmatrix}, \qquad \langle \phi_0 \rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ f \end{bmatrix}$$

- Vacuum manifold of Composite Higgs Model $O(5)/O(4) = S^4$ with constant positive curvature set by scale f. Point ϕ_0 is $O(4)$ invariant point $\Rightarrow$ SMEFT. There are 4 (approximate) Goldstone bosons: $\phi^{1,2,3,4}$.

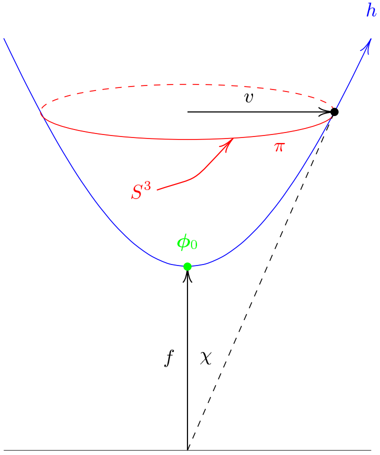
$$L_{\mathrm{KE}} = \frac{1}{2}\left[f^2 \left(\partial_\mu \chi\right)^2 + f^2 \sin^2\chi \partial_\mu \boldsymbol{n} \partial^\mu \boldsymbol{n}\right]$$

- HEFT vacuum manifold $O(4)/O(3) = S^3$ with curvature set by EW vev $v = f \sin\chi = f \sin\left(\chi_0 + \frac{h}{f}\right) \approx f\chi_0 + h$.

$$f^2 \sin^2\chi = v^2 F(h)^2 \Rightarrow F(h) = 1 + \frac{h}{v}\sqrt{1 - \frac{v^2}{f^2}} - \frac{h^2}{2f^2} + \cdots$$

Example of HEFT with Negative Curvature

$$O(4,1)/O(4) = \mathbb{H}^4$$



$$f^2 = (\phi^5)^2 - \sum_{i=1}^{4} \left(\phi^i\right)^2$$

Example of HEFT with Negative Curvature

$O(4,1) \to O(4)$ Model

$$\phi = \begin{bmatrix} \phi^1 \\ \phi^2 \\ \phi^3 \\ \phi^4 \\ \phi^5 \end{bmatrix} = \begin{bmatrix} f \sinh \chi \, \boldsymbol{n} \\ f \cosh \chi \end{bmatrix}, \qquad \langle \phi_0 \rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ f \end{bmatrix}$$

- Vacuum manifold $O(4,1)/O(4) = \mathbb{H}^4$ with constant negative curvature set by scale f, $f^2 = \left(\phi^5\right)^2 - \left(\phi^i\right)^2$. Point ϕ_0 is $O(4)$ invariant point $\Rightarrow$ SMEFT. There are 4 (approximate) Goldstone bosons: $\phi^{1,2,3,4}$.

$$L_{\text{KE}} = \frac{1}{2}\left[f^2 \left(\partial_\mu \chi\right)^2 + f^2 \sin^2 \chi \partial_\mu \boldsymbol{n} \partial^\mu \boldsymbol{n}\right]$$

- HEFT vacuum manifold $O(4)/O(3) = S^3$ with curvature set by EW vev $v = f \sinh \chi$.

$$f^2 \sinh^2 \chi = v^2 F(h)^2 \Rightarrow F(h) = 1 + \frac{h}{v}\sqrt{1 + \frac{v^2}{f^2}} + \frac{h^2}{2f^2} + \cdots$$

Summary

- $SM \subset SMEFT \subset HEFT$.
- New Physics models for Higgs boson h described by EFT.
- EFT scalar kinetic energy term defines scalar metric $g_{ij}(\phi)$.

  ⋆ SM metric is trivial $g_{ij}(\phi) = \delta_{ij}$ and $\mathcal{M}$ is flat. $R_{ijkl} = 0$.
  SM (renormalizable theory) valid to arbitrarily high energies.

  ⋆ HEFT metric is non-trivial and $\mathcal{M}$ is curved.

$$g_{ij}(\phi) = \begin{pmatrix} F(h)^2 g_{ab}(\varphi) & 0 \\ 0 & 1 \end{pmatrix}$$

  $R_{ijkl} \neq 0 \Rightarrow \exists$ new physics scale f (in addition to v) at which EFT breaks down.
  Different predictions for $W_L W_L \to W_L W_L$ and $W_L W_L \to hh$ scattering, h couplings, etc. from SM.

- Covariant description on curved manifold maintains manifest coordinate invariance and gauge invariance.
- Curvature: Gauge curvature (gauge field strength) and Scalar manifold curvature both contribute.
- Interesting possibility of negative curvature arises.