Weak Matrix Elements from Lattice QCD

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Outline

- Lattice QCD - Intro and Motivations
- CP violation in $K \to \pi\pi$ decays, mixing and the $\Delta I = 1/2$ rule
- Numerical results for A_2 and A_0, ε'/ε
- Tentative explanation of the $\Delta I = 1/2$ rule
- Neutral kaon mixing, SM and BSM

Lattice QCD and flavour physics

In principle, numerical simulation of Lattice QCD allows for Non-perturbative, ab-initio, computations in the strong dynamics regime of weak matrix elements

Various applications

- Central role in flavour physics, precision phenomenology

  $\longrightarrow$ see FLAG initiative : Flavour Lattice Average Group
- Tests of the Standard Model
- Constraints on New Physics Theories
- Quantitative understanding of QCD

A few words about Lattice QCD

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ab initio or model independent MonteCarlo simulation of Euclidean QCD

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ab initio or model independent MonteCarlo simulation of Euclidean QCD

- Assume we can “rotate back” to Minkowski space-time
- Take the infinite volume limit, continuum limit, large statistics limit

Lattice QCD $\Leftrightarrow$ *Euclidiean QCD*

LQCD is not a model

Therefore, in principle, for a given number of flavour N_f

Input: bare parameters of QCD Lagrangian (g, $m_i = m_u, m_d, m_s, \ldots$)

$\downarrow$

$\downarrow$

Output: Hadronic quantities, Masses, Decay constants, ...

However Lattice QCD is numerically very expensive

- Large number of d.o.f.
- Very different scales
- Inversion of large matrices ...

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The symmetries are recovered in the continuum limit $a \to 0$

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Our collaboration (RBC-UKQCD) employs a Domain-Wall formulation of QCD

- More expensive than traditional discretisations
- More realistic: "Exact" Chiral-Flavour symmetry
- Can tackle different problems

$K \to \pi\pi$ and CP violation

Background: Kaon decays and CP violation

- First discovery of CP violation was made in kaon system in 1964 (Christenson, Cronin, Fitch and Turlay)

- Noble prize in 1980 (Cronin and Fitch)

- Direct CP violation discovered in kaon decays [NA31, KTeV, NA48, '90-99]

- Very nice measurements of both direct and indirect CP violation (numbers from [PDG 2011])

$$\begin{cases} \textit{Indirect} \quad |\varepsilon| = (2.228 \pm 0.011) \times 10^{-3} \\ \textit{Direct} \quad Re\left(\frac{\varepsilon'}{\varepsilon}\right) = (1.65 \pm 0.26) \times 10^{-3} \end{cases}$$

- Theoretically:

  Relate indirect CP violation parameter (ε) to neutral kaon mixing (B_K)

  B_K is now computed on the lattice with a few-percent precision

  But the first realistic theoretical computstion of ε' has only been achieved last year

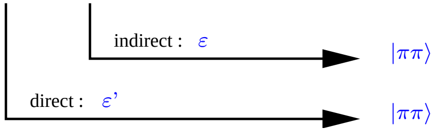
- Sensitivity to new physics expected

Flavour eigenstates $\begin{pmatrix} K^0 = \bar{s}\gamma_5 d \\ \overline{K}^0 = \bar{d}\gamma_5 s \end{pmatrix}$ $\neq$ CP eigenstates $|K^0_\pm\rangle = \frac{1}{\sqrt{2}}\{|K^0\rangle \mp |\overline{K}^0\rangle\}$

They are mixed in the physical eigenstates $\begin{cases} |K_L\rangle & \sim & |K^0_-\rangle + \bar{\varepsilon}|K^0_+\rangle \\ |K_S\rangle & \sim & |K^0_+\rangle + \bar{\varepsilon}|K^0_-\rangle \end{cases}$

Direct and indirect CP violation in $K \to \pi\pi$

$$|K_L\rangle \propto |K_-\rangle + \varepsilon|K_+\rangle$$



$$\varepsilon = \frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})} = |\varepsilon| e^{i\phi_\varepsilon} \sim \bar{\varepsilon}$$

$K \to \pi\pi$ amplitudes

Two isospin channels: $\Delta I = 1/2$ and $\Delta I = 3/2$

$$K \to (\pi\pi)_{\mathrm{I}=0,2}$$

Corresponding amplitudes defined as

$$A[K \to (\pi\pi)_{\mathrm{I}}] = A_{\mathrm{I}} \exp(i\delta_{\mathrm{I}}) \qquad \text{/w } \mathrm{I} = 0, 2 \qquad \delta = \text{strong phases}$$

$\Delta I = 1/2$ rule

$$\omega = \frac{\mathrm{Re}A_2}{\mathrm{Re}A_o} \sim 1/22 \qquad \text{(experimental number)}$$

Amplitudes are related to the parameters of CP violation $\varepsilon, \varepsilon'$ via

$$\varepsilon' = \frac{i\omega \exp(i\delta_2 - \delta_0)}{\sqrt{2}} \left[\frac{\mathrm{Im}(A_2)}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right]$$

$$\varepsilon = e^{i\phi_\varepsilon} \left[\frac{\mathrm{Im}\langle \bar{K}^0 | H_{\mathrm{eff}}^{\Delta S=2} | K^0 \rangle}{\Delta m_K} + \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right]$$

$\Rightarrow$ Related to $K^0 - \bar{K}^0$ mixing

The $\Delta I = 1/2$ rule

- In $K \to \pi\pi$ decays, the final state can have isospin 0 or 2
- Experimentally we observe that

$$\mathbb{P}[K \to (\pi\pi)_{\mathrm{I=0}}] \sim 450 \times \mathbb{P}[K \to (\pi\pi)_{\mathrm{I=2}}]$$

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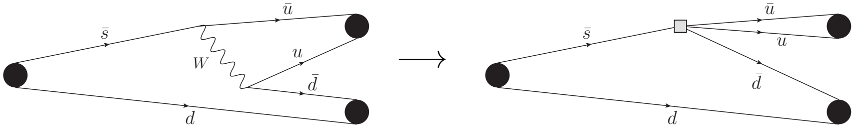
Is the remaining contribution coming from non-perturbative QCD ?

Why so different from the Perturbative estimate ?

- Operator Product expansion



- Describe $K \to (\pi\pi)_{\mathrm{I}=0,2}$ with an effective Hamiltonian [Buchalla, Buras, Lautenbacher '96]

$$H^{\Delta s=1} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{i=1}^{10} \left(V_{ud} V_{us}^* z_i(\mu) - V_{td} V_{ts}^* y_i(\mu) \right) Q_i(\mu) \right\}$$

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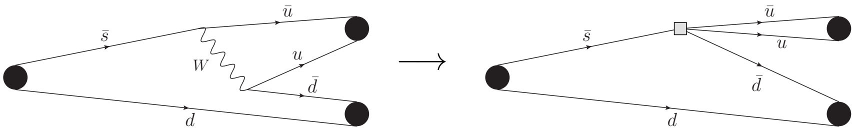

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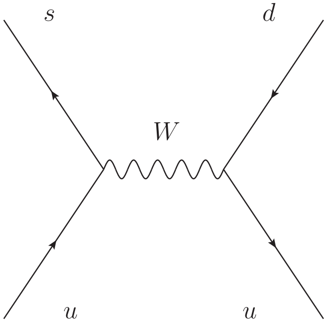
- Amplitude given by $A \propto \langle \pi\pi | H^{\Delta s=1} | K \rangle$
- Short distance effects factorized in the Wilson coefficients y_i, z_i , computed at NLO in [BBL '96]
- Long distance effects factorized in the matrix elements

$$\langle \pi\pi | Q_i(\mu) | K \rangle \longrightarrow \text{task for the Lattice}$$

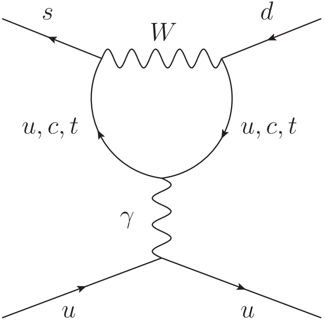
See reviews by [Christ @ Kaon'09, Lellouch @ Les Houches'09, Sachrajda @ Lattice '10], ...

Current diagrams



$Q_1 = (\bar{s}d)_{\mathrm{V-A}}(\bar{u}u)_{\mathrm{V-A}}$ $Q_2 =$ color mixed

Electroweak penguins

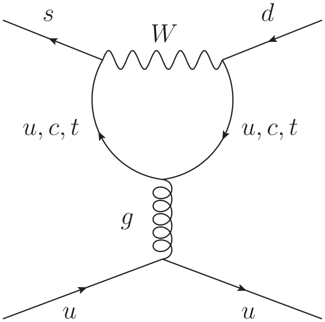


$$Q_7 = \frac{3}{2}(\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} e_q(\bar{q}q)_{\mathrm{V+A}} \qquad Q_8 = \text{color mixed}$$

$$Q_9 = \frac{3}{2}(\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} e_q(\bar{q}q)_{\mathrm{V-A}} \qquad Q_{10} = \text{color mixed}$$

4-quark operators

QCD penguins



$$Q_3 = (\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} (\bar{q}q)_{\mathrm{V-A}} \qquad Q_4 = \text{color mixed}$$

$$Q_5 = (\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} (\bar{q}q)_{\mathrm{V+A}} \qquad Q_6 = \text{color mixed}$$

$SU(3)_L \otimes SU(3)_R$ and isospin decomposition

Irrep of $SU(3)_L \otimes SU(3)_R$

$$\begin{aligned} \overline{3} \otimes 3 &= 8 + 1 \\ \overline{8} \otimes 8 &= 27 + \overline{10} + 10 + 8 + 8 + 1 \end{aligned}$$

Relevant operators transform under $(27,1), (8,8)$ and $(8,1)$ of $SU(3)_L \otimes SU(3)_R$

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Decomposition of the 4-quark operators gives

$$\begin{aligned}Q_{1,2} &= Q_{1,2}^{(27,1),\Delta I=3/2} + Q_{1,2}^{(27,1),\Delta I=1/2} + Q_{1,2}^{(8,8),\Delta I=1/2} \\ Q_{3,4} &= Q_{3,4}^{(8,1),\Delta I=1/2} \\ Q_{5,6} &= Q_{5,6}^{(8,1),\Delta I=1/2} \\ Q_{7,8} &= Q_{7,8}^{(8,8),\Delta I=3/2} + Q_{7,8}^{(8,8),\Delta I=1/2} \\ Q_{9,10} &= Q_{9,10}^{(27,1),\Delta I=3/2} + Q_{9,10}^{(27,1),\Delta I=1/2} + Q_{9,10}^{(8,8),\Delta I=1/2}\end{aligned}$$

see eg [Claude Bernard @ TASI'89] and [RBC'01]

$SU(3)_L \otimes SU(3)_R$ and isospin decomposition

Only 7 are independent: one $(27, 1)$ four $(8, 1)$, and two $(8, 8)$, $\Rightarrow$ we call them Q'

$$
\begin{array}{llcl}
(27,1) & Q'_1 & = & Q'^{(27,1),\Delta I=3/2}_1 + Q'^{(27,1),\Delta I=1/2}_1 \\
(8,1) & Q'_2 & = & Q'^{(8,1),\Delta I=1/2}_2 \\
 & Q'_3 & = & Q'^{(8,1),\Delta I=1/2}_3 \\
 & Q'_5 & = & Q'^{(8,1),\Delta I=1/2}_5 \\
 & Q'_6 & = & Q'^{(8,1),\Delta I=1/2}_6 \\
(8,8) & Q'_7 & = & Q'^{(8,8),\Delta I=3/2}_7 + Q'^{(8,8),\Delta I=1/2}_7 \\
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\end{array}
$$

Only 3 operators contribute to the $\Delta I = 3/2$ chanel

A challenge !

Many obstacles:

- Final state with two pions
- Many operators that mix under renormalisation
- Require the evaluation of disconnected graphs

Need to preserve chiral-flavour symetry at finite lattice spacing

Plus the usual difficulties: light dynamical quarks, large volume, . . .

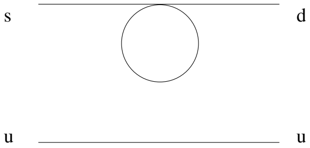
A challenge !

Many strategies have been proposed over the year

- Using SU(4) Chiral PT [Giusti et al ' 04 Hernandez et al '08, Endress and Pena '14]
- Relate $K \to \pi\pi$ to $K \to \pi$ matrix elements [RBC'01] and others
- ...

Isospin channels

- Only 3 of these operators contribute to the $\Delta I = 3/2$ channel
  - A tree-level operator
  - 2 electroweak penguins
- No disconnect graphs contribute to the $\Delta I = 3/2$ channel



$\Rightarrow$ A_2 is much simpler than A_0

Still highly non-trivial, but perfect challenge for lattice QCD with chiral fermions

$K \to (\pi\pi)_{I=2}$ Results

- First computation (2012): Physical kinematic, Near physical vaule of the pion mass

  But only one coarse lattice spacing

  IDSDR $32^3 \times 64$, with $a^{-1} \sim 1.37\ \mathrm{GeV} \Rightarrow a \sim 0.14\ \mathrm{fm}$, $L \sim 4.6\ \mathrm{fm}$

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- New computation:

  two lattice spacing, $n_f = 2+1$, large volume at the physical point

  New discretisation of the Domain-Wall fermion forumlation: Möbius Brower, Neff, Orginos '12

  - $48^3 \times 96$, with $a^{-1} \sim 1.729$ GeV $\Rightarrow a \sim 0.11$ fm, $L \sim 5.5$ fm
  - $64^3 \times 128$ with $a^{-1} \sim 2.358$ GeV $\Rightarrow a \sim 0.084$ fm, $L \sim 5.4$ fm
  - $am_{res} \sim 10^{-4}$

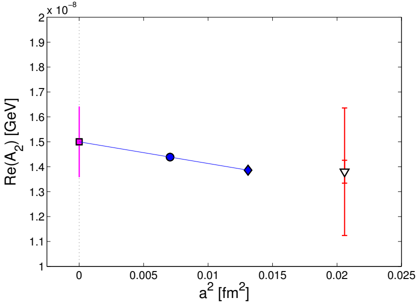
$K \to (\pi\pi)_{I=2}$ 2015 Results

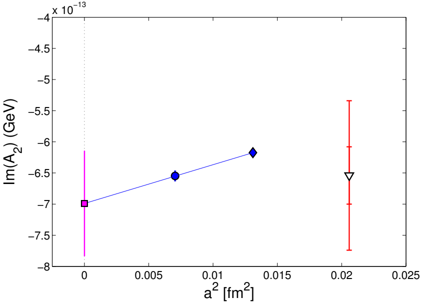
2012 Blum, Boyle, Christ, N.G.,Goode, Izubuchi, Jung, Kelly, Lehner, Lightman, Liu, Lytle, Mawhinney, Sachrajda, Soni, Sturm, *PRL'12, PRD'12*

$\mathrm{Re}\, A_2 = 1.381(46)_{\mathrm{stat}}(258)_{\mathrm{syst}}\ 10^{-8}\ \mathrm{GeV}$ $\qquad$ $\mathrm{Im}\, A_2 = -6.54(46)_{\mathrm{stat}}(120)_{\mathrm{syst}} 10^{-13}\ \mathrm{GeV}$

2015 Blum, Boyle, Christ, Frison, N.G., Janowski, Jung, Kelly, Lehner, Lytle, Mawhinney, Sachrajda, Soni, Hin, Zhang, *PRD'15*

$\mathrm{Re}\, A_2 = 1.50(4)_{\mathrm{stat}}(14)_{\mathrm{syst}}\ 10^{-8}\ \mathrm{GeV}$ $\qquad$ $\mathrm{Im}\, A_2 = -6.99(20)_{\mathrm{stat}}(84)_{\mathrm{syst}} 10^{-13}\ \mathrm{GeV}$





see also talk by T.Janowski @ lat'13

- First complete computation of $K \to \pi\pi$ (both isospin channel) with physical kinematics

  Bai, Blum, Boyle, Christ, Frison, N.G., Izubuchi, Jung, Kelly, Lehner, Mawhinney, Sachrajda, Soni, Zhang

- Pion mass $m_\pi = 143.1(2.0)\ \mathrm{MeV}$, single lattice spacing $a \sim 0.14\ \mathrm{fm}$
- Physical kinematics achieved with G-Parity boundary conditions

  Kim, Christ, '03 and '09

- Requires algorithmic development, dedicated generation of gauge configurations, . . .
- See talk by C.Kelly and proceeding from Lattice'14

A_0, 2015

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Another computation, [Ishizuka, Ishikawa, Ukawa, Yoshié '15] with Wilson fermions at threshold (unphysical kinematics)

A_0, 2015 update

After renormalisation at $\mu \sim 1.5\ \mathrm{GeV}$, we combine with the Wilson coefficients and find

i	Re(A_0)(GeV)	Im(A_0)(GeV)
1	$1.02(0.20)(0.07) \times 10^{-7}$	0
2	$3.63(0.91)(0.28) \times 10^{-7}$	0
3	$-1.19(1.58)(1.12) \times 10^{-10}$	$1.54(2.04)(1.45) \times 10^{-12}$
4	$-1.86(0.63)(0.33) \times 10^{-9}$	$1.82(0.62)(0.32) \times 10^{-11}$
5	$-8.72(2.17)(1.80) \times 10^{-10}$	$1.57(0.39)(0.32) \times 10^{-12}$
6	$3.33(0.85)(0.22) \times 10^{-9}$	$-3.57(0.91)(0.24) \times 10^{-11}$
7	$2.40(0.41)(0.00) \times 10^{-11}$	$8.55(1.45)(0.00) \times 10^{-14}$
8	$-1.33(0.04)(0.00) \times 10^{-10}$	$-1.71(0.05)(0.00) \times 10^{-12}$
9	$-7.12(1.90)(0.46) \times 10^{-12}$	$-2.43(0.65)(0.16) \times 10^{-12}$
10	$7.57(2.72)(0.71) \times 10^{-12}$	$-4.74(1.70)(0.44) \times 10^{-13}$
Tot	$4.66(0.96)(0.27) \times 10^{-7}$	$-1.90(1.19)(0.32) \times 10^{-11}$
Exp	$3.3201(18) \times 10^{-7}$	-

Standard model prediction for ε'/ε

ε'/ε can be computed from

$$Re(\varepsilon'/\varepsilon) = Re\left\{ \frac{i\omega \exp(i\delta_2 - \delta_0)}{\sqrt{2}\varepsilon} \left[\frac{\mathrm{Im}(A_2)}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right] \right\}$$

Combining our new value of $\mathrm{Im}A_0$ and δ_0 with

- our continuum value for $\mathrm{Im}A_2$
- the experimental value for $\mathrm{ReA_0}$, $\mathrm{ReA_2}$ and their ratio ω

we find

$$Re(\varepsilon'/\varepsilon) = 1.38(5.15)(4.43) \times 10^{-4}$$

whereas the experimental value is

$$Re(\varepsilon'/\varepsilon) = 16.6(2.3) \times 10^{-4}$$

$\sim 2.1\sigma$

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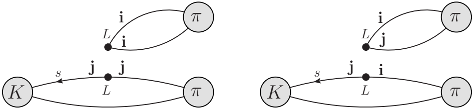
Our errors are large, but are expected to dectrease rapidly

Emerging understanding of the $\Delta I = 1/2$ rule

Two kinds of contraction for each $\Delta I = 3/2$ operator



Contraction ① Contraction ②

Toward an quantitative understanding of the $\Delta I = 1/2$ rule

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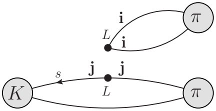
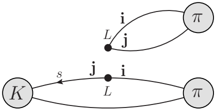


Contraction ①



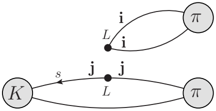
Contraction ②

- Re A_2 is dominated by the tree level operator (EWP $\sim 1\%$)

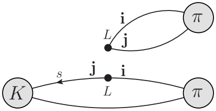
$$ReA_2 \sim ① + ②$$

Toward an quantitative understanding of the $\Delta I = 1/2$ rule

Two kinds of contraction for each $\Delta I = 3/2$ operator



Contraction ①



Contraction ②

- Re A_2 is dominated by the tree level operator (EWP $\sim 1\%$)

$$ReA_2 \sim ① + ②$$

- Naive factorisation approach: ② $\sim 1/3$①

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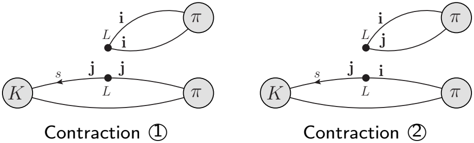

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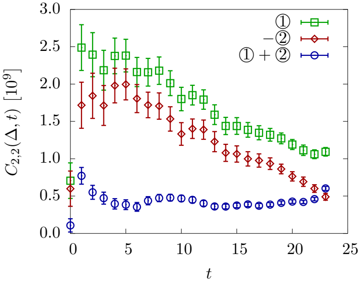
$$ReA_2 \sim ① + ②$$

- Naive factorisation approach: ② $\sim 1/3$①
- Our computation: ② $\sim -0.7$①

$\Rightarrow$ large cancellation in ReA_2



ReA_0 is also dominated by the tree level operators

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Dominant contribution to $Q_2^{\rm lat}$ is $\propto$ (2② − ①) ⇒ Enhancement in ReA_0

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Dominant contribution to $Q_2^{\rm lat}$ is $\propto$ (2② − ①) $\Rightarrow$ Enhancement in ReA_0

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} \sim \frac{2② - ①}{① + ②}$$

With this unphysical computation (kinematics, masses) we find

$$\begin{aligned}\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} &= 9.1(2.1) \text{ for } m_K = 878 \ \mathrm{MeV}\ m_\pi = 422 \ \mathrm{MeV} \\ &= 12.0(1.7) \text{ for } m_K = 662 \ \mathrm{MeV}\ m_\pi = 329 \ \mathrm{MeV}\end{aligned}$$

Emerging understanding of the $\Delta I = 1/2$ rule

- Relative sign between ① and ② implies both a cancellation in $\mathrm{Re}A_2$ and an enhancement in $\mathrm{Re}A_0$
- Analytic work in that direction, e.g. Pich, de Rafael '96, Bardeen, Buras, Gerard '87
- See also discussion in Lellouch @ Les Houches '09

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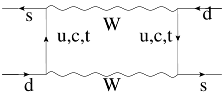
- See also discussion in Lellouch @ Les Houches '09
- Similar observation done by the other lattice computation Ishizuka, Ishikawa, Ukawa, Yoshié '15

  $K \to \pi\pi$ amplitudes with unphysical kinematics (and Wilson fermions)

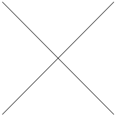
Neutral kaon mixing

Indirect CP violation related to neutral kaon oscillations

in the SM this occurs though box diagrams with W exchange



$\mu \ll M_W$ $\longrightarrow$
OPE

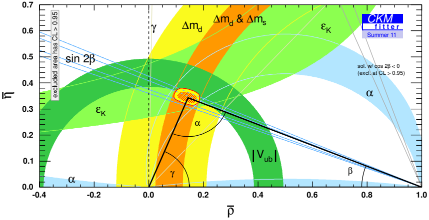


Factorise the non-perturbative contribution into

$$\langle \overline{K}^0 | \mathcal{O}_{LL}^{\Delta S=2}(\mu) | K^0 \rangle = \frac{8}{3} F_K^2 M_K^2 B_K(\mu) \qquad \text{w/ } \mathcal{O}_{LL}^{\Delta S=2} = (\bar{s}\gamma_\mu(1-\gamma_5)d)(\bar{s}\gamma^\mu(1-\gamma_5)d)$$

Related to ε via CKM parameters, schematically

$$\varepsilon \sim \text{ known factors} \times V_{\mathrm{CKM}} \times C(\mu) \times B_K(\mu)$$



[CKMfitter'11]

and beyond

In the SM, neutral kaon mixing occurs through W-exchanges $\to$ $(V-A)$

$$O_1^{\Delta s=2} = (\bar{s}\,(V-A)\,d)\,(\bar{s}\,(V-A)\,d)$$

Beyond the SM, other Dirac structure appear in the generic Hamiltonian $H^{\Delta s=2} = \sum_{i=1}^{5} C_i(\mu)\, O_i^{\Delta s=2}(\mu)\,.$

We express them in terms of Lorentz matrices Vector, Axial, Scalar, Pseudo-scalar, Tensor

$$\begin{array}{l} (V-A)\times(V+A) \\ (S-P)\times(S+P) \\ (S-P)\times(S-P) \\ TT\times TT \end{array}$$

On the lattice, we compute $\langle \bar{K}^0 | O_i^{\Delta s=2} | K^0 \rangle$

FLAG 2013 quotes an error of 1.3% dominated by the perturbative matching

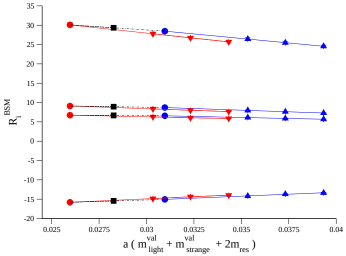
Most recent determinations, in $\overline{\mathrm{MS}}$ at 3 GeV

Collaboration	N_f	Discretisation	Result
RBC-UKQCD	$2+1$	Domain-Wall	$0.5293(17)_{stat+syst}(106)_{PT}$
SWME	$2+1$	Staggered	$0.518(3)_{stat}(26)_{syst}$
ETM	$2+1+1$	Twisted Mass	$0.506(17)_{stat+syst}(3)_{PT}$

Results

We show the ratios

$$R_i^{\mathrm{BSM}} \sim \frac{\langle \bar{K}^0 | O_i | K^0 \rangle}{\langle \bar{K}^0 | O_1 | K^0 \rangle} \sim \mathrm{BSM/SM}$$



plot from [RBC-UKQCD, Boyle, N.G., Hudspith'12]

$$R_i^{\mathrm{BSM}} \sim O(10)$$

$\Rightarrow$ combined with experimental values of Δm_K and ε, provides important constraints on BSM theories.

3 lattice groups are working on this computation: ETMc, RBC-UKQCD, SWME

BSM kaon mixing - Results

	ETM 12	ETM 15	RBC − UKQCD 12	SWME 15
	RI − MOM	*RI − MOM*	*RI − MOM*	1 − *loop*
B_2	0.47(2)	0.46(3)	0.43(5)	0.525(1)(23)
B_3	0.78(4)	0.79(5)	0.75(9)	0.772(5)(35)
B_4	0.75(3)	0.78(5)	0.69(7)	0.981(3)(61)
B_5	0.60(3)	0.49(4)	0.47(6)	0.751(8)(68)

Difference seems to come from the renormalisation procedure

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New RBC-UKQCD results [N.G., R.Hudspith, A.T. Lytle] (in preparation)

	RBC − UKQCD 16(prelim.) $RI-SMOM$	$RI-MOM$
B_2	0.488(7)(17)(2)	0.417(6)(2)(2)
B_3	0.743(14)(64)(3)	0.655(12)(44)(2)
B_4	0.920(12)(12)(4)	0.745(9)(28)(3)
B_5	0.707(8)(34)(3)	0.555(6)(53)(2)

Difference seems to come from the renormalisation procedure

Conclusions, outlook

- Tremendous progress achieved by the lattice community in the light quark sector

  Physical quark masses, electromagnetic effects,. . .

- B_K determined at the sub-percent level

- Discrepancy on the BSM $\Delta S = 2$ matrix element has be resolved

- Computations of hadronic decay amplitudes

  $A \to (\pi\pi)_{I=2}$ becoming a "mature" quantity

  First realistic computation of $A \to (\pi\pi)_{I=0}$ and ε'/ε

  Error on ε'/ε should decrease rapidly

  Crucial Test of the SM (very nice experimental measurement)

- Proposals for rare kaon decays

$\Rightarrow$Lattice QCD continues to provide Non-trivial tests of the SM and constraints on BSM theories

Backup

Standard model prediction for ε'/ε

	$1/a$ [GeV]	m_π [MeV]	m_K [MeV]	ReA_2 [10^{-8}GeV]	ReA_0 [10^{-8} GeV]	$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2}$	kinematics
16^3 **IW**	1.73(3)	422(7)	878(15)	4.911(31)	45(10)	9.1(2.1)	threshold
24^3 **IW**	1.73(3)	329(6)	662(11)	2.668(14)	32.1(4.6)	12.0(1.7)	threshold
32^3 **ID**	1.36(1)	142.9(1.1)	511.3(3.9)	1.38(5)(26)	-	-	physical
cont.	-	139	497-507	1.50(4)(14)	-	-	physical
32^3 **ID**	1.36(1)	143.1(2.0)	490.6(2.4)	-	46.6(10.0)(12.1)	-	physical
Exp	–	135 - 140	494 - 498	1.479(4)	33.2(2)	22.45(6)	

RBC-UKQCD setup - History- Present

$2 + 1$ Domain-Wall fermions

Chiral-Flavour symmetry (almost) exact at finite lattice spacing

Finite fith dimension L_s $\rightarrow$ small additive quark mass renormalisation m_{res}

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- 2008: IW $a^{-1} = 1.729(18)$ GeV $\leftrightarrow$ $a \sim 0.1145$ fm, on $24^3 \times 64 \times 16$, ie $L \sim 2.74$ fm

  Unitary pion masses $m_\pi = 331, 419, (557)$ MeV

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  Unitary pion masses $m_\pi = 290, 345, 394$ MeV

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- 2012: IDSDR $a^{-1} = 1.372(10)$ GeV $\leftrightarrow$ $a \sim 0.144$ fm, on $32^3 \times 64 \times 32$, ie $L \sim 4.62$ fm

  Unitary pion mass $m_\pi = 171$ MeV

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  Unitary pion mass $m_\pi = 171$ MeV

- 2014: Möbius, Unitary pion mass 139 MeV
  - $a^{-1} = 1.730(4)$ GeV $\leftrightarrow a \sim 0.1145$ fm, on $48^3 \times 96 \times 24$ ie $L \sim 4.62$ fm
  - $a^{-1} = 2.359(7)$ GeV $\leftrightarrow a \sim 0.0839$ fm, on $64^3 \times 128 \times 12$ ie $L \sim 5.475$ fm

$R^{\mathrm{SUSY}}(3\ \mathrm{GeV})$	$\mathrm{SMOM} - \gamma_\mu$	$\overline{\mathrm{MS}}$
R_2	−19.11(43)(19)(25)	−19.48(44)(20)(25)
R_3	5.76(14)(15)(07)	6.08(15)(16)(08)
R_4	40.1(08)(17)(09)	43.1(09)(18)(10)
R_5	11.13(21)(79)(25)	10.99(20)(78)(25)

Table: RBC-UKQCD preliminary results for the BSM $\Delta S = 2$ ratio R in the SUSY basis, in the γ_μ-SMOM scheme and in $\overline{\mathrm{MS}}$ at 3 GeV. The quantities R are the ratios of the BSM matrix elements over the SM contributions. Errors are statistics, discretisation, chiral, respectively.

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- $K \to \pi\pi$ decays
- Neutral kaon mixing within and beyond the SM
- Kaon semi-leptonic decays
- $K_L - K_S$ mass difference
- Rare kaon decays