

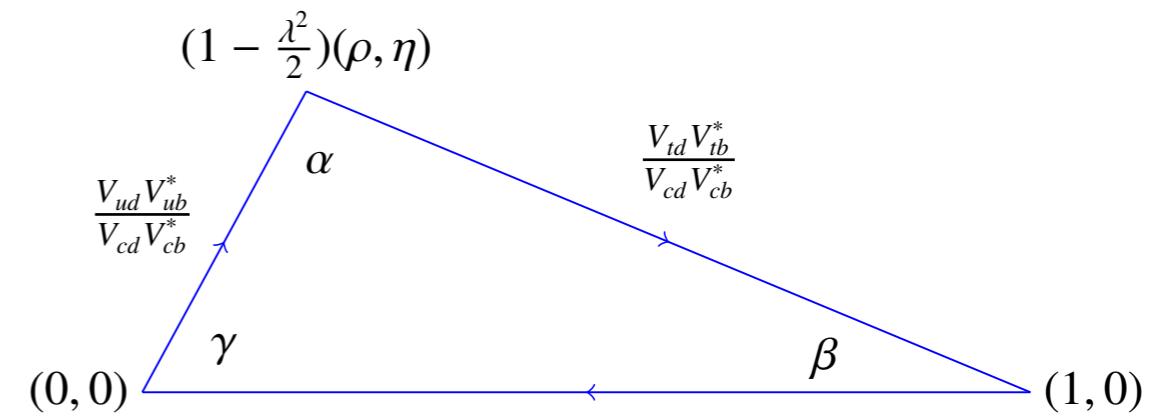
The CKM angle γ

Malcolm John, University of Oxford
on behalf of the LHCb collaboration

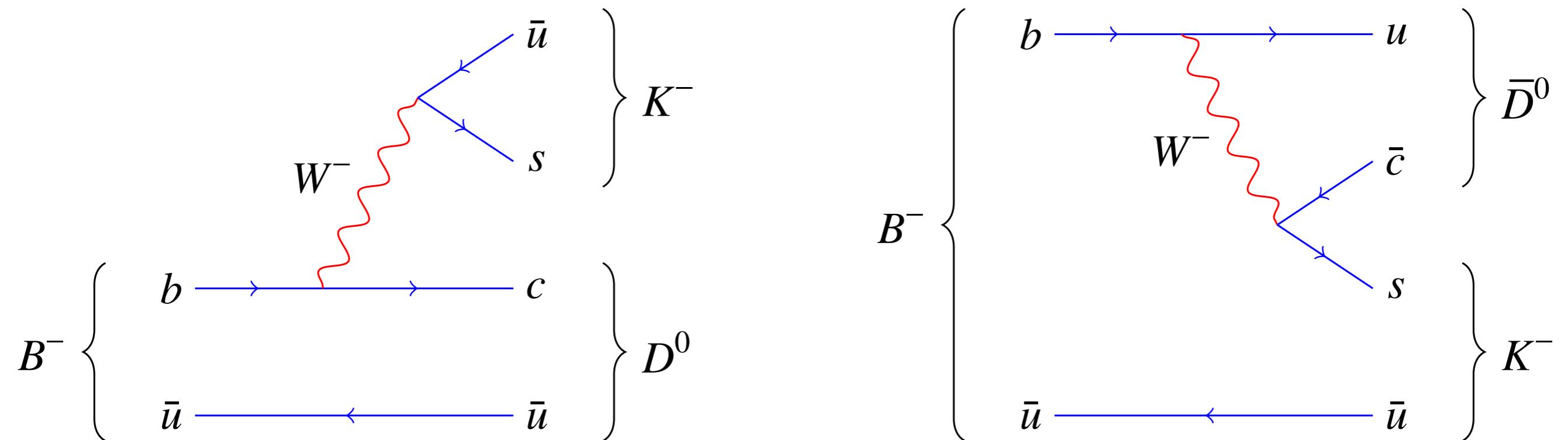
Rencontres de Moriond, 13th March 2016

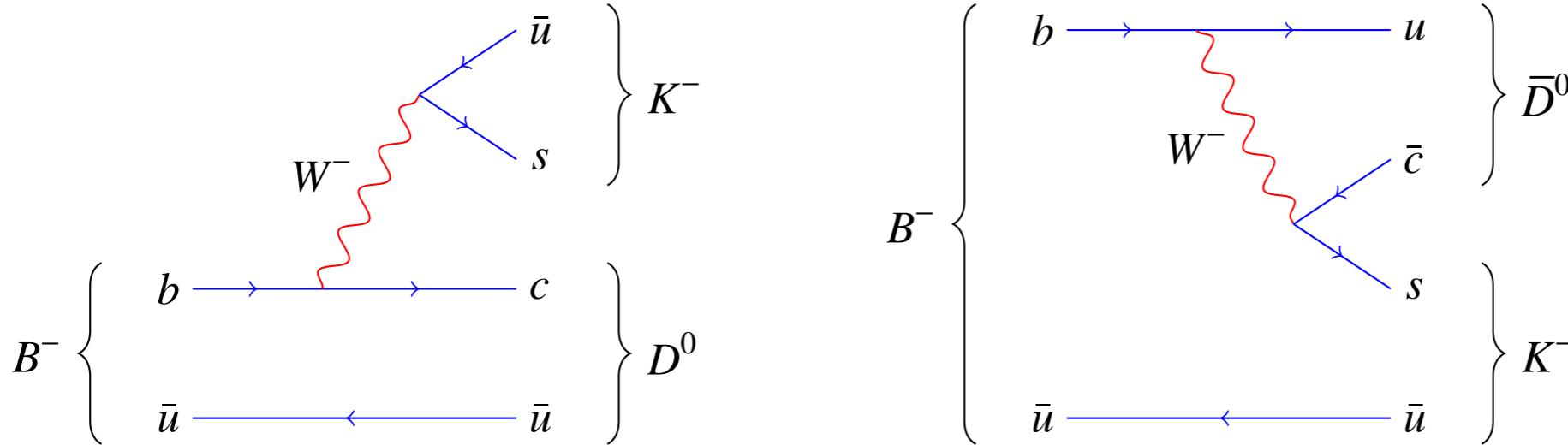


$$\gamma \equiv \arg \left(-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right) \approx \arg \left(-\frac{V_{ub}^*}{V_{cb}^*} \right)$$



- Accessible in decays where $b \rightarrow u$ and $b \rightarrow c$ transitions interfere to give CP violation
- No dependence on coupling to top so γ can be determined from direct CPV in tree decays
- $B \rightarrow DX$ decays satisfy these criteria and a few are known to exhibit large CP violation. The most studied case is $B^- \rightarrow DK^-$ decays,





- The hadronic parameters are the amplitude ratio r_B and the CP-conserving phase δ_B ,

$$\frac{A(B^- \rightarrow \bar{D}^0 K^-)}{A(B^- \rightarrow D^0 K^-)} = r_B e^{i(\delta_B - \gamma)}$$

- To unambiguously determine γ , measurements of partial width ratios and charge asymmetries are needed from many final states of the D meson. Analyses of different types of D decay require different techniques and are categorised as,

GLW: $D \rightarrow K^+ K^-$
 $\pi^+ \pi^-$
 $K_S \pi^0$

GGSZ $D \rightarrow K_S \pi^+ \pi^-$
 $K_S K^+ K^-$

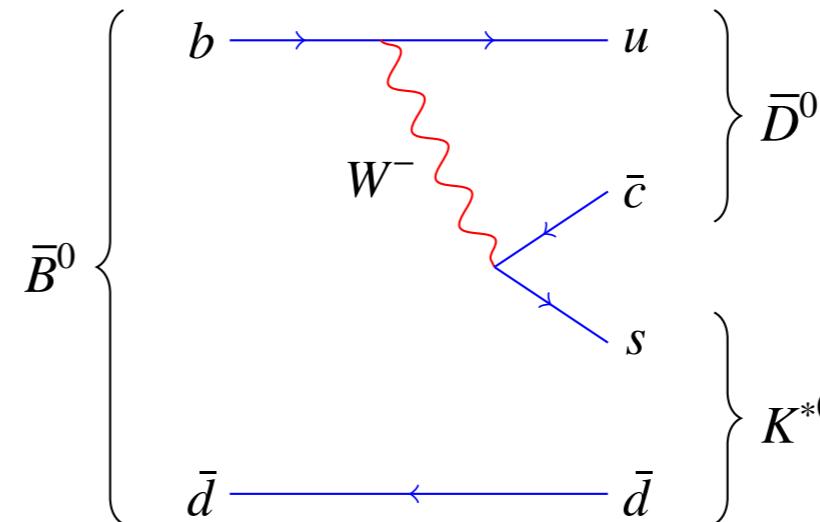
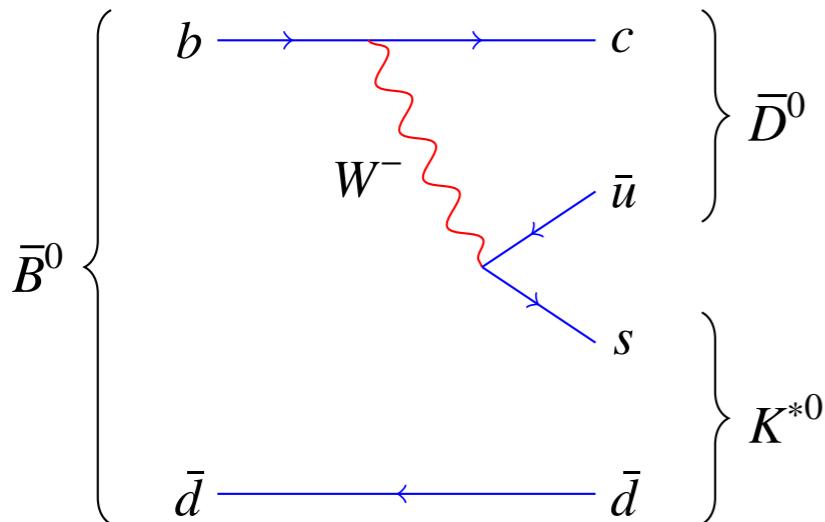
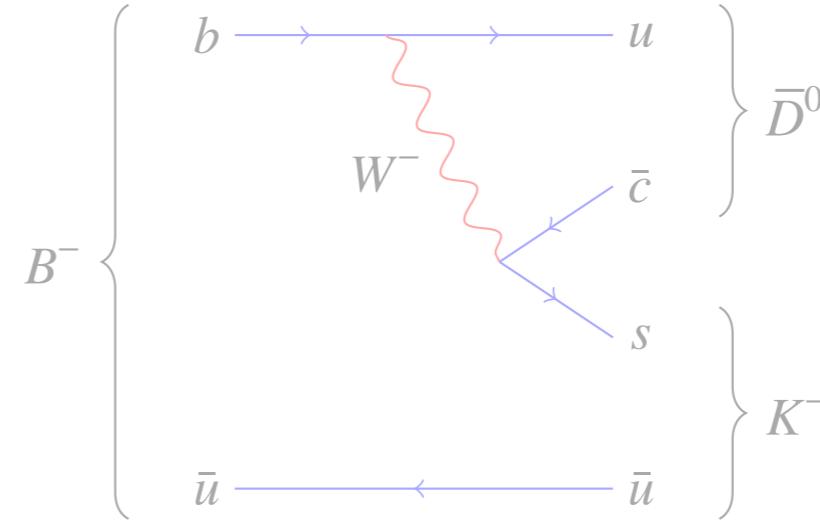
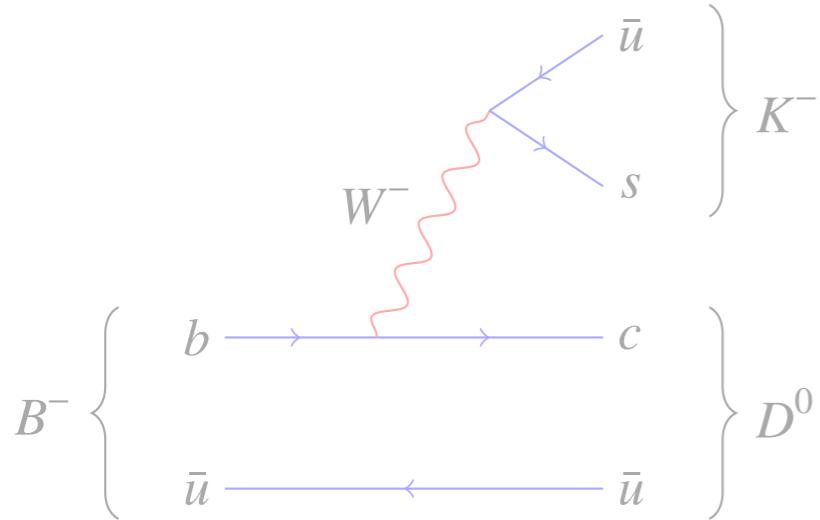
ADS: $D \rightarrow \pi^- K^+$

quasi-GLW $D \rightarrow \pi^+ \pi^- \pi^+ \pi^-$

quasi-ADS $D \rightarrow \pi^- K^+ \pi^+ \pi^-$
 $\pi^- K^+ \pi^0$

$K^+ K^- \pi^0$
 $\pi^+ \pi^- \pi^0$

GLS $D \rightarrow K_S K^- \pi^+$
 $K_S \pi^- K^+$



- The LHCb data sample size is such that some B^0 analysis becomes viable for measuring γ

Dalitz-GLW: $D \rightarrow K^+ K^- \pi^+ \pi^- K_S \pi^0$

GGSZ $D \rightarrow K_S \pi^+ \pi^- \pi^- K_S K^+ K^-$

ADS: $D \rightarrow \pi^- K^+$

quasi-GLW

K
 π

quasi-ADS

π

GLS

K

Four new papers are described today (final Run 1 analyses),

Constraints on the unitarity triangle angle γ from Dalitz plot analysis of $B^0 \rightarrow DK^+\pi^-$ decays

LHCb-PAPER-2015-059

arXiv:1602.03455

$B^0 \rightarrow [K^+K^-]K\pi$

$B^0 \rightarrow [\pi^+\pi^-]K\pi$

Measurement of CP observables in $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ with two- and four-body D decays

LHCb-PAPER-2016-003

$B^\pm \rightarrow [h^\pm h^\pm]h^\pm$

$B^\pm \rightarrow [K^+\pi^-\pi^+\pi^-]h^\pm$

$B^\pm \rightarrow [\pi^+\pi^-\pi^+\pi^-]h^\pm$

Model-independent measurement of the CKM angle γ using $B^0 \rightarrow DK^{*0}$, $D \rightarrow K_S^0\pi^+\pi^-$, $K_S^0K^+K^-$ decays

LHCb-PAPER-2016-006

$B^0 \rightarrow [K_S^0h^+h^-]K^{*0}$

Measurement of the CKM angle γ using $B^0 \rightarrow DK^{*0}$ with $D \rightarrow K_S^0\pi^+\pi^-$ decays

LHCb-PAPER-2016-007

$B^0 \rightarrow [K_S^0\pi^+\pi^-]K^{*0}$

which add to five published papers,

A study of CP violation in $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ decays with $D \rightarrow K_S^0K^\pm\pi^\mp$ final states
Phys. Lett. B733 (2014) 36 arXiv:1402.2982

$B^\pm \rightarrow [K_S^0K^+\pi^-]h^\pm$

Measurement of CP asymmetry in $B_s^0 \rightarrow D_s^\mp K^\pm$ decays
JHEP 11 (2014) 060 arXiv:1407.6127

$B_s \rightarrow D_s K$

Measurement of CP violation parameters in $B^0 \rightarrow DK^{*0}$ decays
Phys. Rev. D90 (2014) 112002 arXiv:1407.8136

$B^0 \rightarrow [K^\pm\pi^\pm]K^{*0}$

Measurement of the CKM angle γ using $B^\pm \rightarrow DK^\pm$ with $D \rightarrow K_S^0\pi^+\pi^-$, $K_S^0K^+K^-$ decays
JHEP 10 (2014) 097 arXiv:1408.2748

$B^\pm \rightarrow [K_S^0h^+h^-]K^\pm$

A study of CP violation in $B^\mp \rightarrow Dh^\mp$ ($h = K, \pi$) with the modes $D \rightarrow K^\mp\pi^\pm\pi^0$, $D \rightarrow \pi^+\pi^-\pi^0$ and $D \rightarrow K^+K^-\pi^0$
Phys. Rev. D91 (2015) 112014 arXiv:1504.05442

$B^\pm \rightarrow [h^\pm h^\pm\pi^0]h^\pm$

to be combined today in,

Measurement of the CKM angle γ from a combination of $B \rightarrow DK$ analyses

LHCb-CONF-2016-001

$B^\pm \rightarrow Dh^\pm, D \rightarrow 2\text{-body}$

Many observables that have direct sensitivity to γ

$$\frac{\Gamma(B^- \rightarrow [K^+ K^-]_D K^-) + \Gamma(B^+ \rightarrow [K^+ K^-]_D K^+)}{\Gamma(B^- \rightarrow [K^- \pi^+]_D \pi^-) + \Gamma(B^+ \rightarrow [K^+ \pi^-]_D \pi^+)} = R_{CP}$$

$$\frac{\Gamma(B^- \rightarrow [\pi^+ \pi^-]_D K^-) + \Gamma(B^+ \rightarrow [\pi^+ \pi^-]_D K^+)}{\Gamma(B^- \rightarrow [K^- \pi^+]_D \pi^-) + \Gamma(B^+ \rightarrow [K^+ \pi^-]_D \pi^+)} = R_{CP}$$

$$\frac{\Gamma(B^- \rightarrow [K^- \pi^+]_D K^-) - \Gamma(B^+ \rightarrow [K^+ \pi^-]_D K^+)}{\Gamma(B^- \rightarrow [K^- \pi^+]_D K^-) + \Gamma(B^+ \rightarrow [K^+ \pi^-]_D K^+)} = A_K^{K\pi}$$

$$\frac{\Gamma(B^- \rightarrow [K^- K^+]_D K^-) - \Gamma(B^+ \rightarrow [K^+ K^-]_D K^+)}{\Gamma(B^- \rightarrow [K^- K^+]_D K^-) + \Gamma(B^+ \rightarrow [K^+ K^-]_D K^+)} = A_K^{KK}$$

$$\frac{\Gamma(B^- \rightarrow [\pi^- \pi^+]_D K^-) - \Gamma(B^+ \rightarrow [\pi^+ \pi^-]_D K^+)}{\Gamma(B^- \rightarrow [\pi^- \pi^+]_D K^-) + \Gamma(B^+ \rightarrow [\pi^+ \pi^-]_D K^+)} = A_K^{\pi\pi}$$

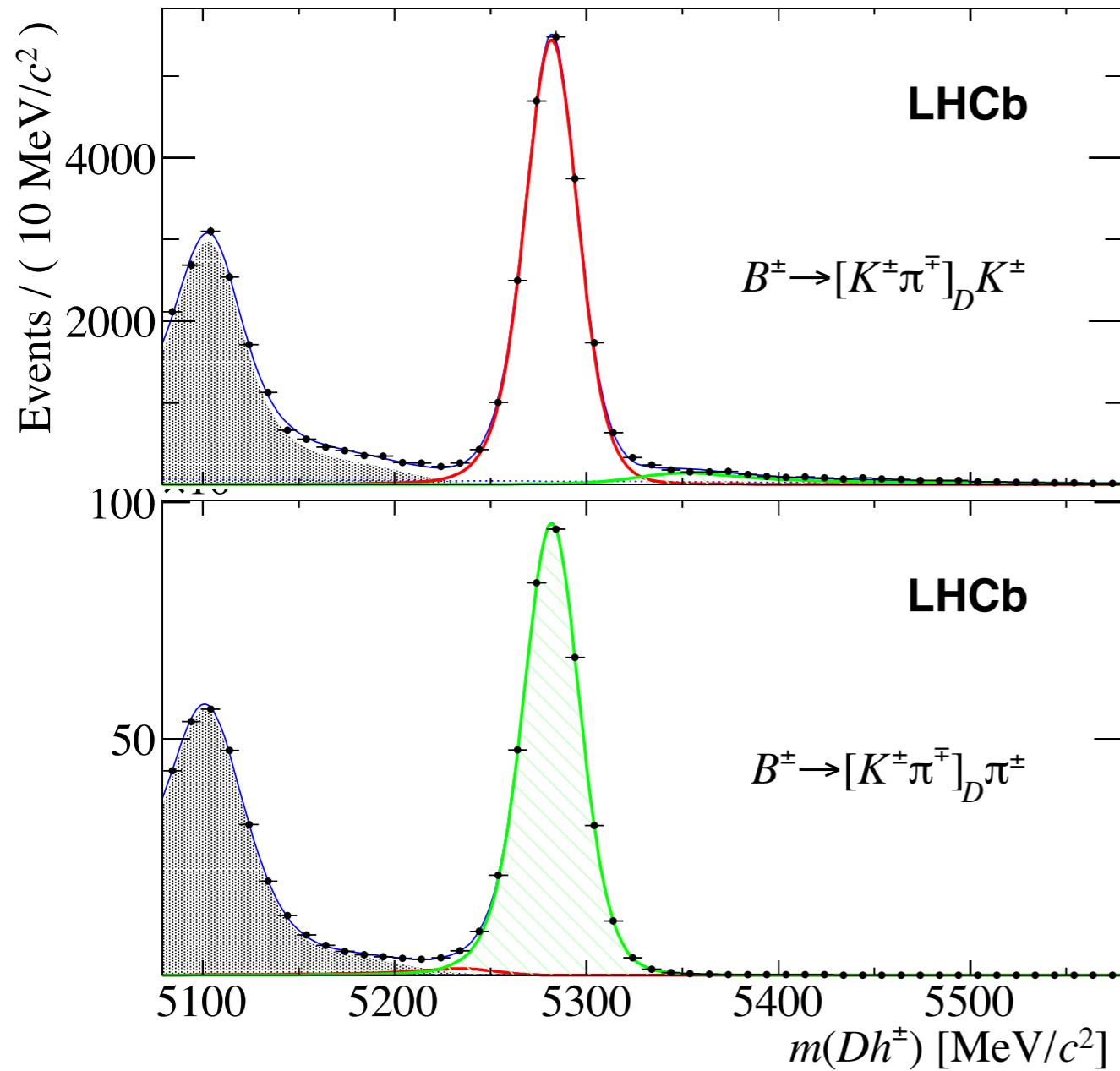
$$\frac{\Gamma(B^- \rightarrow [\pi^- K^+]_D K^-) + \Gamma(B^+ \rightarrow [\pi^+ K^-]_D K^+)}{\Gamma(B^- \rightarrow [K^- \pi^+]_D K^-) + \Gamma(B^+ \rightarrow [K^+ \pi^-]_D K^+)} = R_{ADS(K)}^{\pi K}$$

$$\frac{\Gamma(B^- \rightarrow [\pi^- K^+]_D K^-) - \Gamma(B^+ \rightarrow [\pi^+ K^-]_D K^+)}{\Gamma(B^- \rightarrow [\pi^- K^+]_D K^-) + \Gamma(B^+ \rightarrow [\pi^+ K^-]_D K^+)} = A_{ADS(K)}^{\pi K}$$

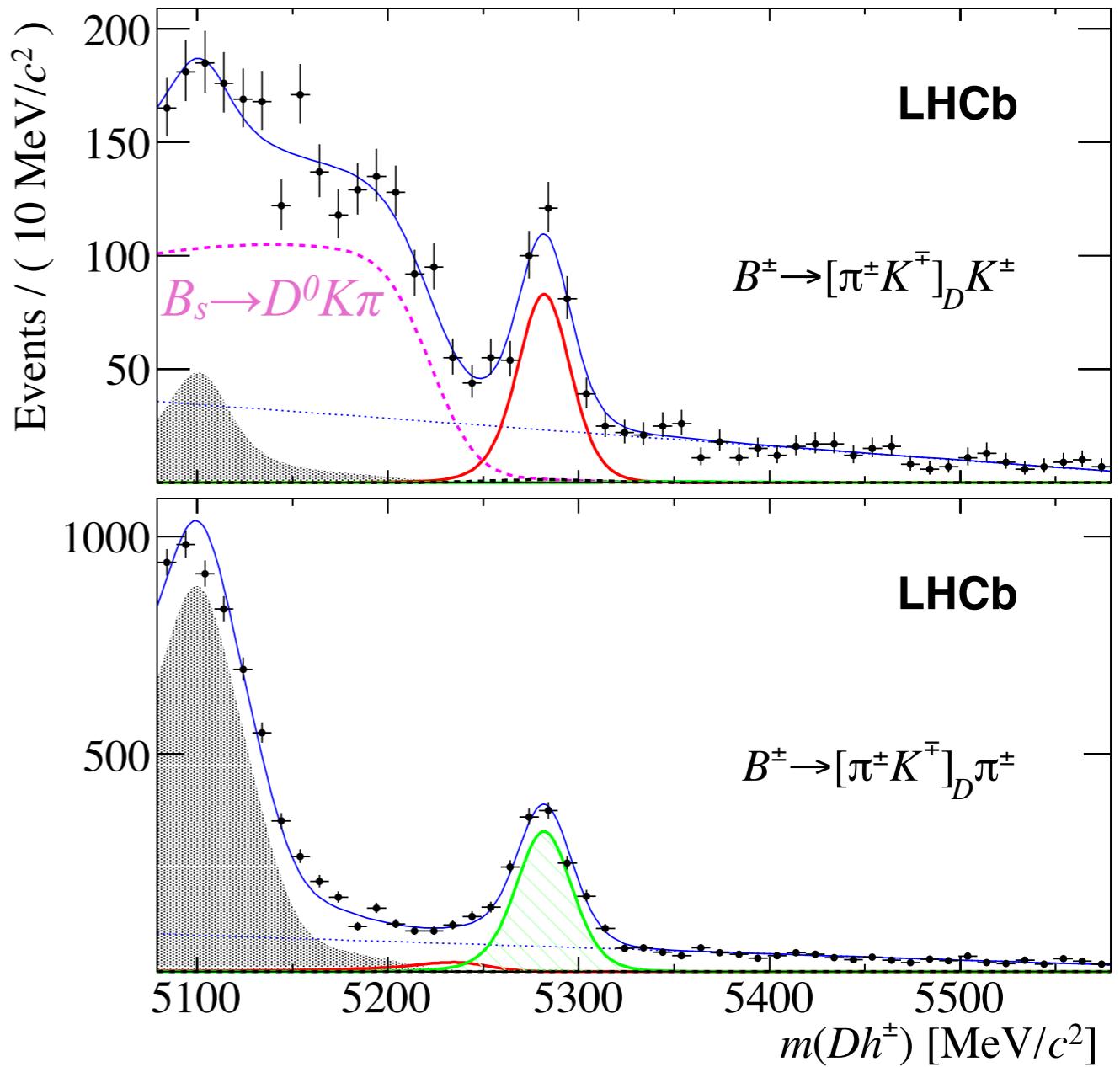
$$\Gamma\left(B^\pm \rightarrow [f]_D K^\pm\right) = r_D^2 + r_B^2 + 2\kappa_D r_D r_B \cos(\delta_B + \delta_D \pm \gamma)$$

ADS: $B^\pm \rightarrow Dh^\pm, D \rightarrow \pi^+ K^-$

29 500 $B^\pm \rightarrow DK^\pm, D \rightarrow K^+ \pi^-$



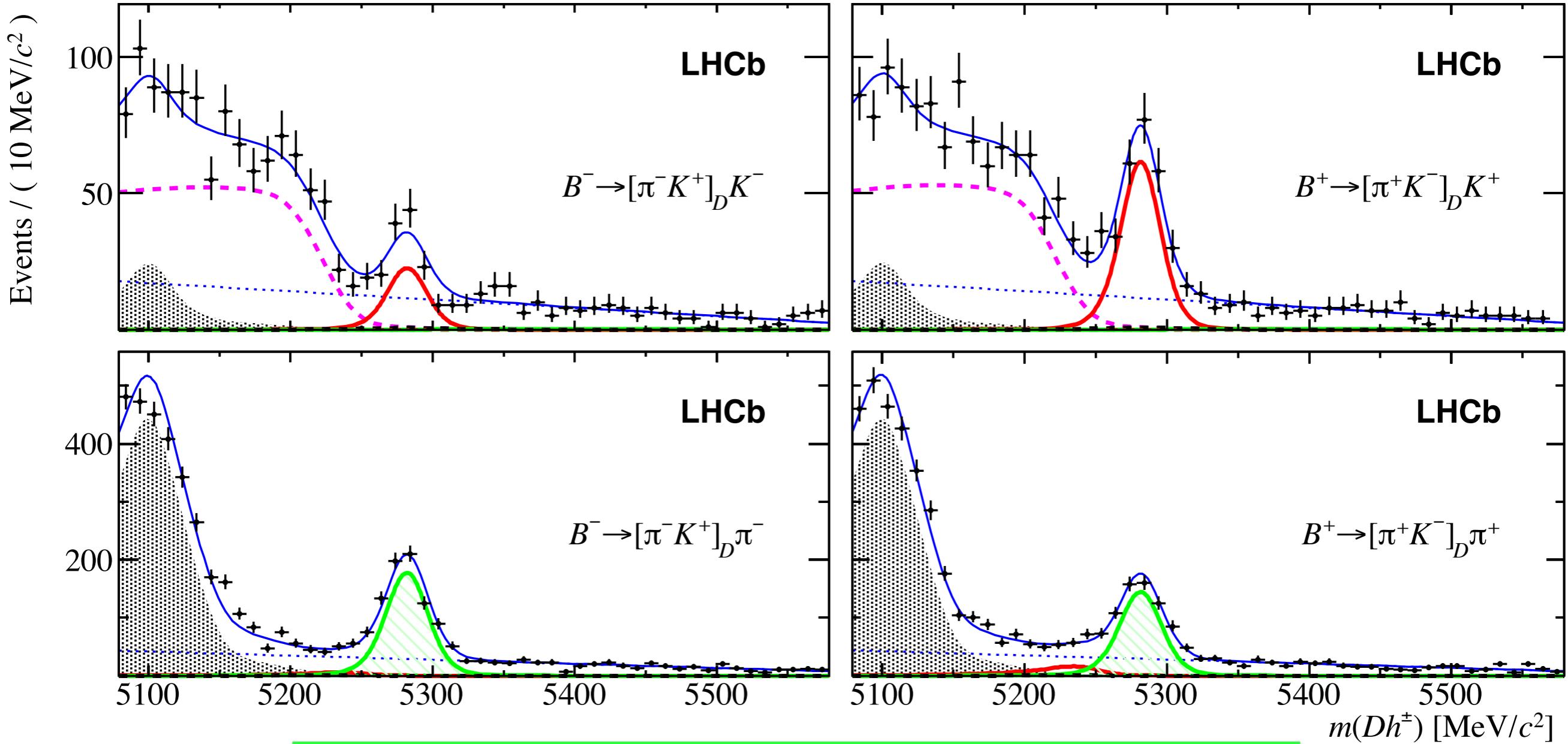
550 $B^\pm \rightarrow DK^\pm, D \rightarrow \pi^+ K^-$



ADS: $B^\pm \rightarrow Dh^\pm, D \rightarrow \pi^+ K^-$

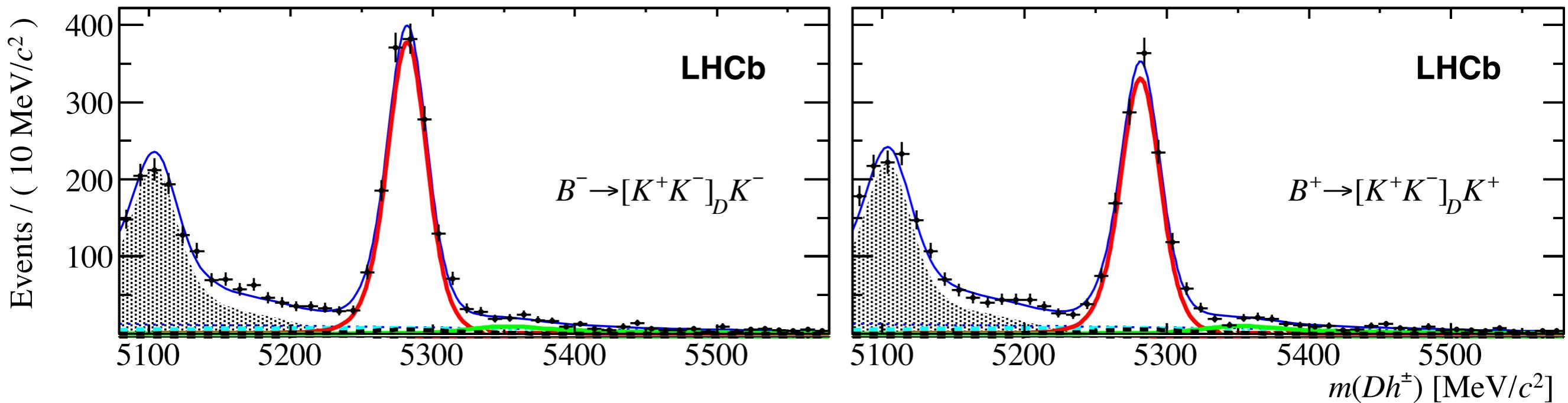


$$A_{\text{ADS}(K)}^{\pi K} = -0.403 \quad \pm 0.056 \quad \pm 0.011$$



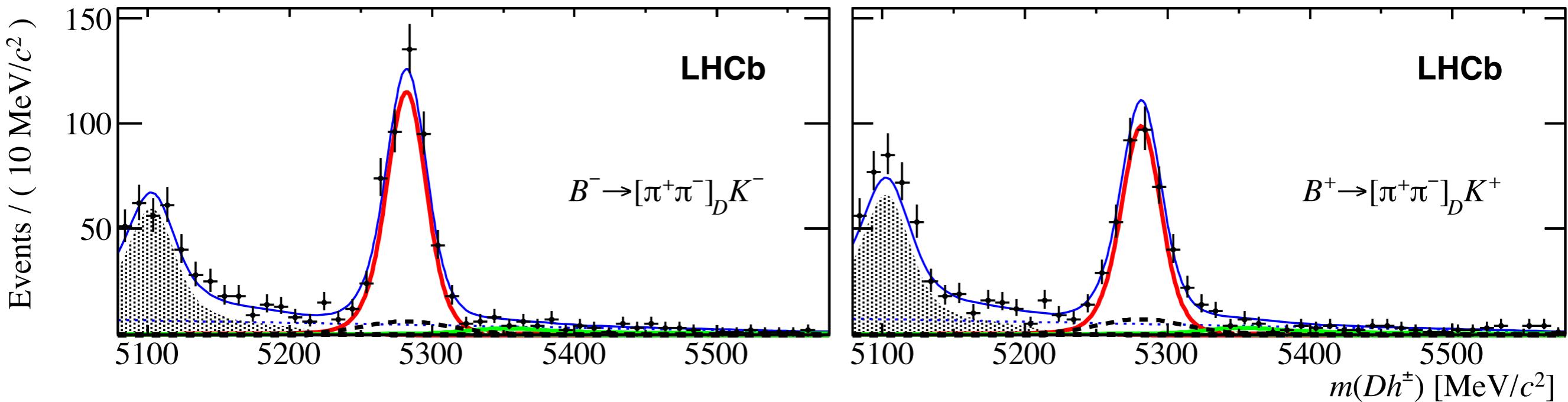
$$A_{\text{ADS}(\pi)}^{\pi K} = 0.100 \quad \pm 0.031 \quad \pm 0.009$$

GLW: $B^\pm \rightarrow D K^\pm, D \rightarrow K^+ K^-, \pi^+ \pi^-$



$$A_K^{KK} = 0.087 \pm 0.020 \pm 0.008$$

$$A_K^{\pi\pi} = 0.128 \pm 0.037 \pm 0.012$$



$B^\pm \rightarrow Dh^\pm, D \rightarrow 4\text{-body}$

Same observables as in the 2-body analysis and similarly sensitivity to γ

$$\frac{\Gamma(B^- \rightarrow [\pi^-\pi^+\pi\pi]_D K^-) + \Gamma(B^+ \rightarrow [\pi^+\pi^-\pi\pi]_D K^+)}{\Gamma(B^- \rightarrow [K^-\pi^+\pi\pi]_D \pi^-) + \Gamma(B^+ \rightarrow [K^+\pi^-\pi\pi]_D \pi^+)} = R_{CP}^{\pi\pi\pi\pi}$$

$$\frac{\Gamma(B^- \rightarrow [K^-\pi^+\pi\pi]_D K^-) - \Gamma(B^+ \rightarrow [K^+\pi^-\pi\pi]_D K^+)}{\Gamma(B^- \rightarrow [K^-\pi^+\pi\pi]_D K^-) + \Gamma(B^+ \rightarrow [K^+\pi^-\pi\pi]_D K^+)} = A_K^{K\pi\pi\pi}$$

$$\frac{\Gamma(B^- \rightarrow [\pi^-\pi^+\pi\pi]_D K^-) - \Gamma(B^+ \rightarrow [\pi^+\pi^-\pi\pi]_D K^+)}{\Gamma(B^- \rightarrow [\pi^-\pi^+\pi\pi]_D K^-) + \Gamma(B^+ \rightarrow [\pi^+\pi^-\pi\pi]_D K^+)} = A_K^{\pi\pi\pi\pi}$$

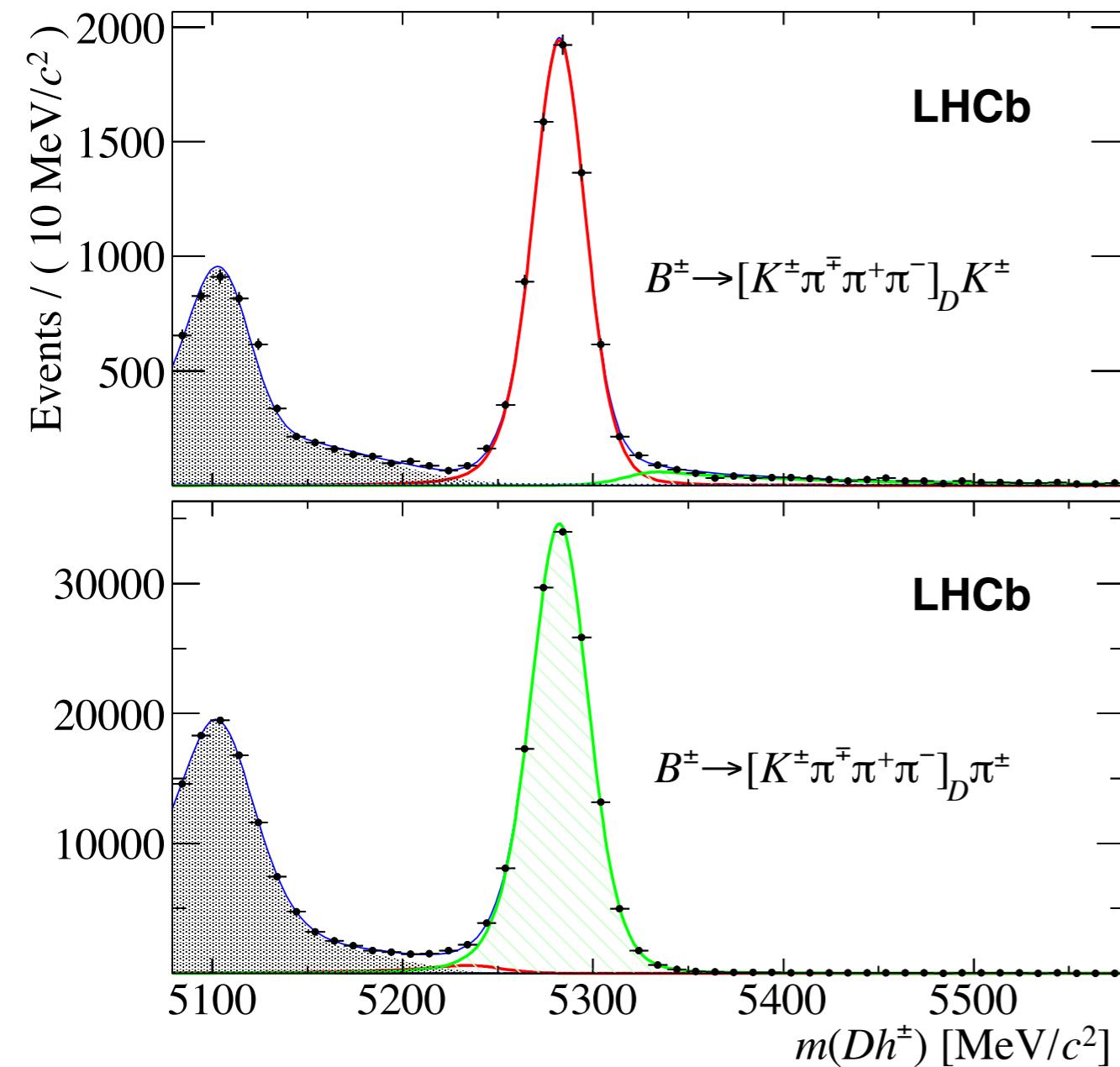
$$\frac{\Gamma(B^- \rightarrow [\pi^-K^+\pi\pi]_D K^-) + \Gamma(B^+ \rightarrow [\pi^+K^-\pi\pi]_D K^+)}{\Gamma(B^- \rightarrow [K^-\pi^+\pi\pi]_D K^-) + \Gamma(B^+ \rightarrow [K^+\pi^-\pi\pi]_D K^+)} = R_{ADS(K)}^{\pi K \pi\pi}$$

$$\frac{\Gamma(B^- \rightarrow [\pi^-K^+\pi\pi]_D K^-) - \Gamma(B^+ \rightarrow [\pi^+K^-\pi\pi]_D K^+)}{\Gamma(B^- \rightarrow [\pi^-K^+\pi\pi]_D K^-) + \Gamma(B^+ \rightarrow [\pi^+K^-\pi\pi]_D K^+)} = A_{ADS(K)}^{\pi K \pi\pi}$$

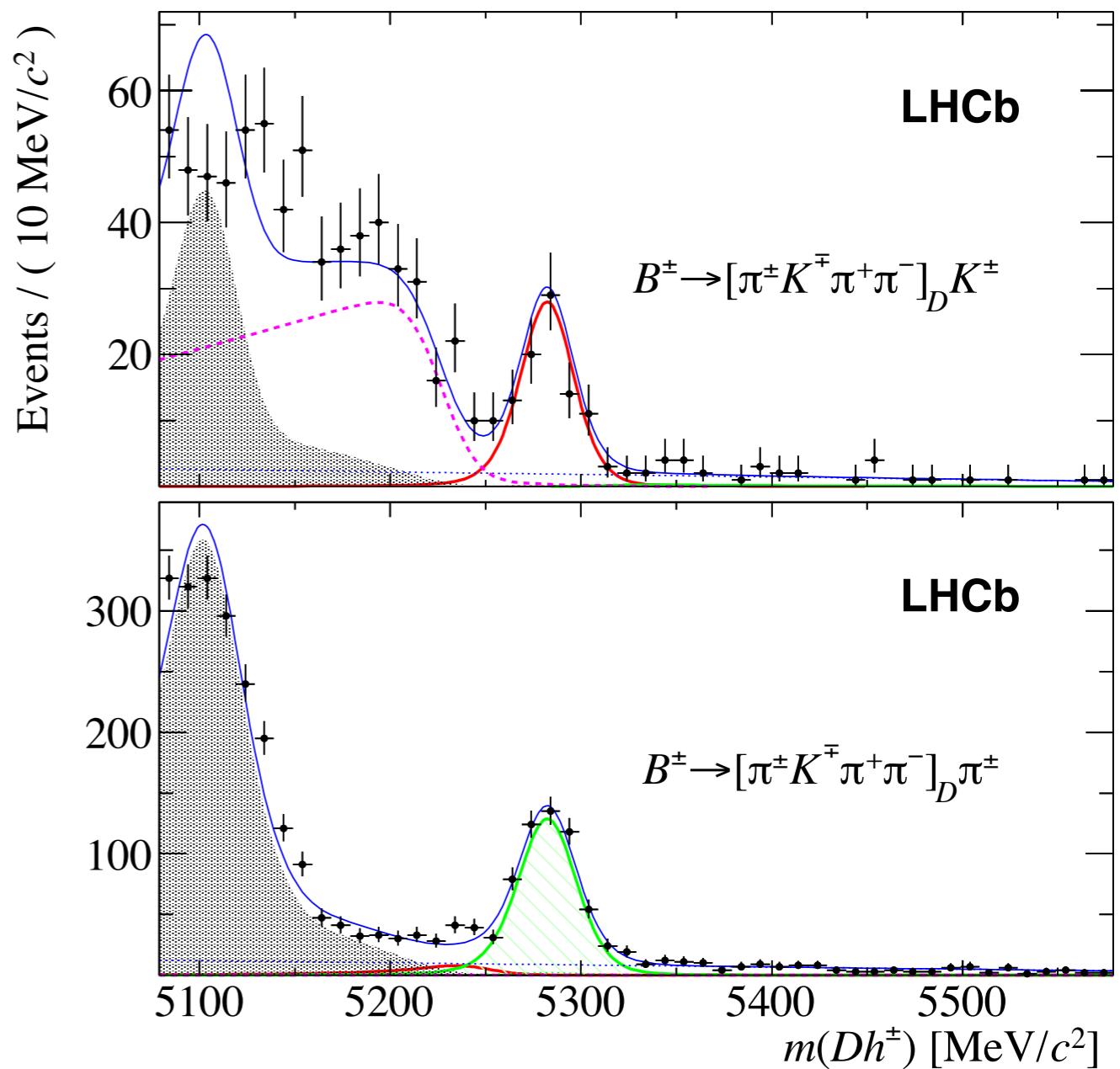
$$\Gamma\left(B^\pm \rightarrow [f]_D K^\pm\right) = r_D^2 + r_B^2 + 2\kappa_D r_D r_B \cos(\delta_B + \delta_D \pm \gamma)$$

qADS: $B^\pm \rightarrow Dh^\pm, D \rightarrow \pi^+ K^- \pi^+ \pi^-$

11 300 $B^\pm \rightarrow DK^\pm, D \rightarrow K^+ \pi^- \pi^+ \pi^-$

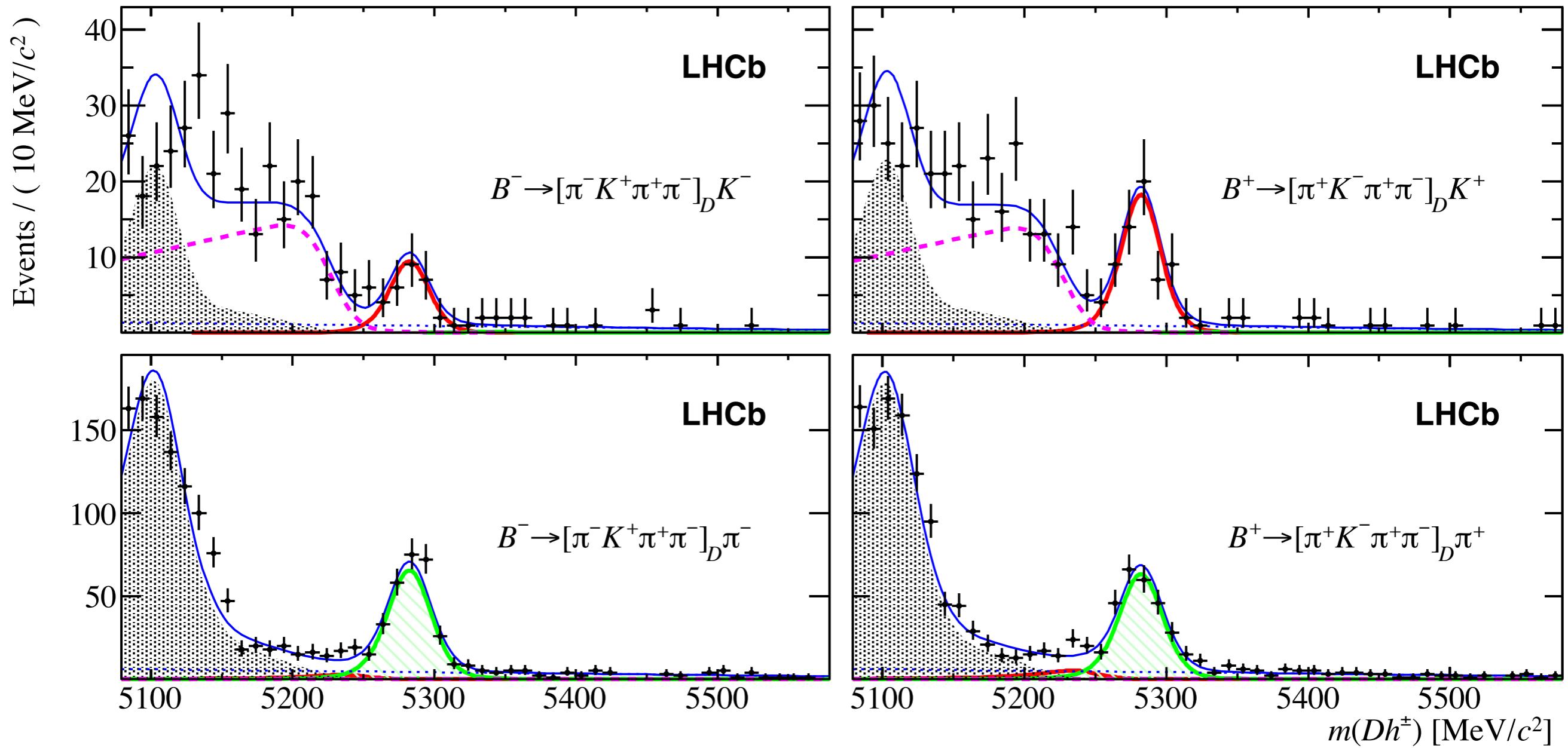


160 $B^\pm \rightarrow DK^\pm, D \rightarrow \pi^+ K^- \pi^+ \pi^-$



qADS: $B^\pm \rightarrow Dh^\pm, D \rightarrow \pi^+ K^- \pi^+ \pi^-$

$$A_{\text{ADS}(K)}^{\pi K \pi\pi} = -0.313 \pm 0.102 \pm 0.038$$

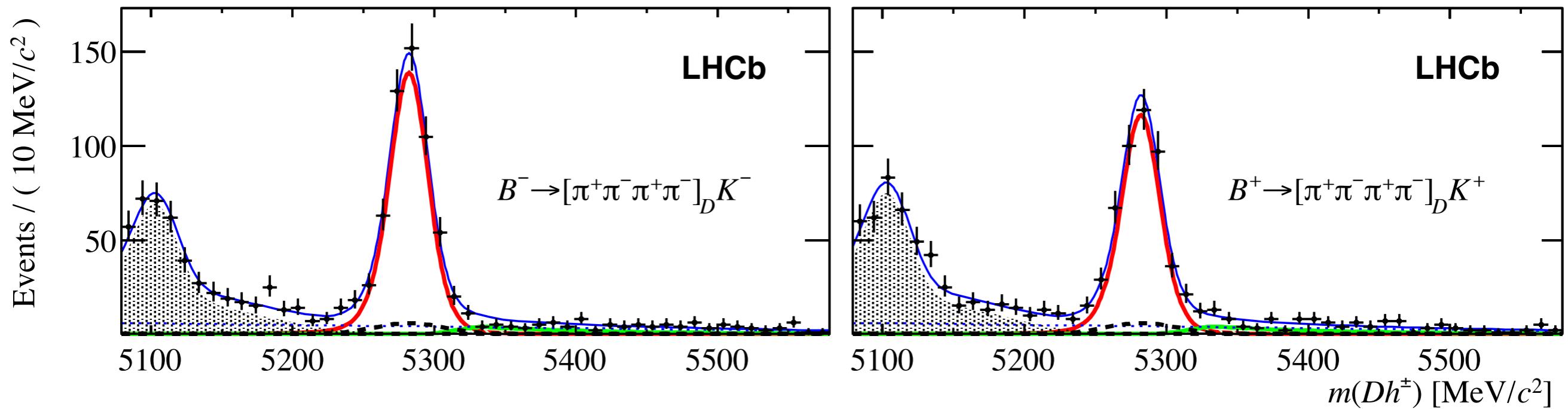


Expect a negative A_{ADS} , like the 2-body ADS mode, given knowledge of δ_D , arXiv:1602.07430

$$A_{\text{ADS}(K)}^{\pi K} = -0.403 \pm 0.056 \pm 0.011$$

qGLW: $B^\pm \rightarrow Dh^\pm$, $D \rightarrow \pi^+\pi^-\pi^+\pi^-$ First analysis of this mode

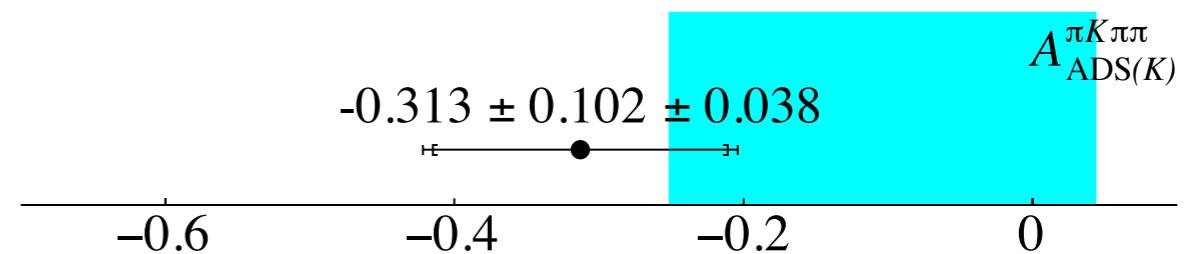
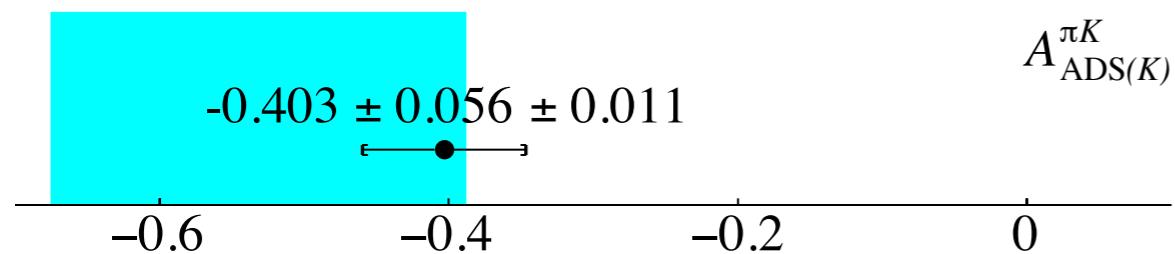
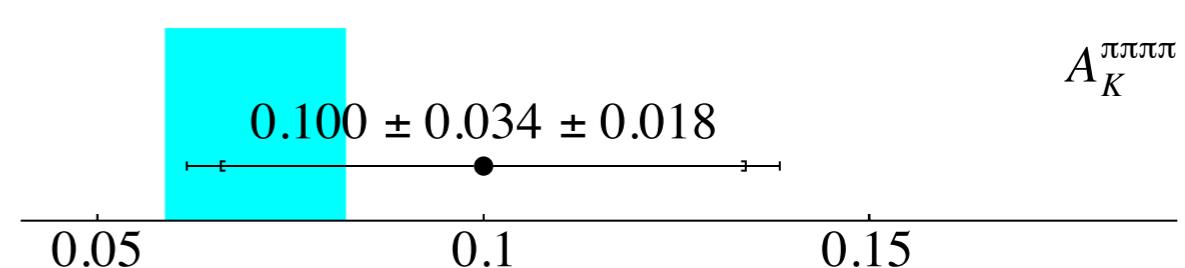
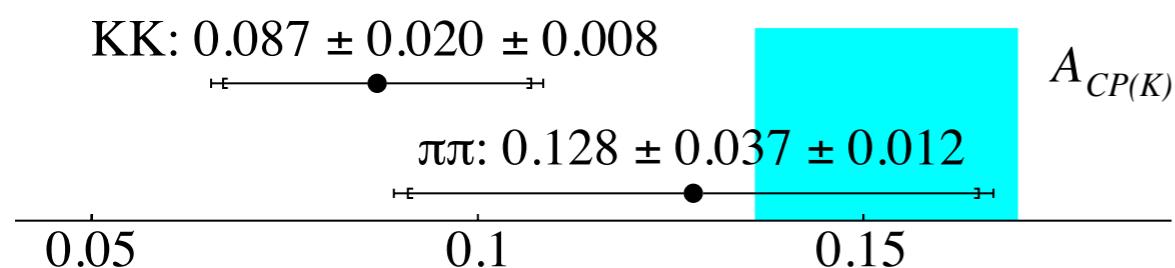
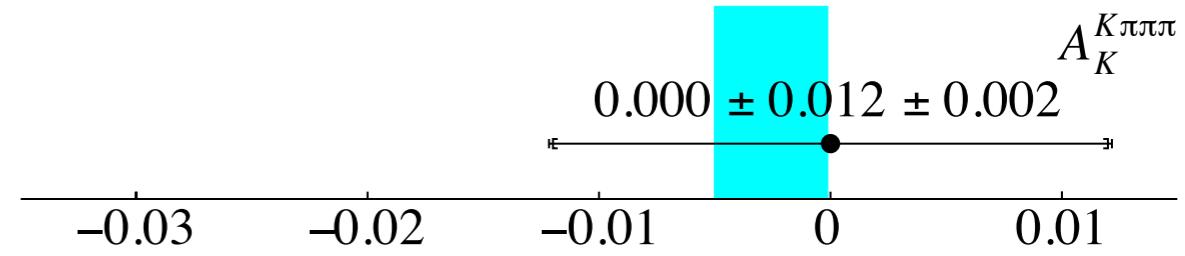
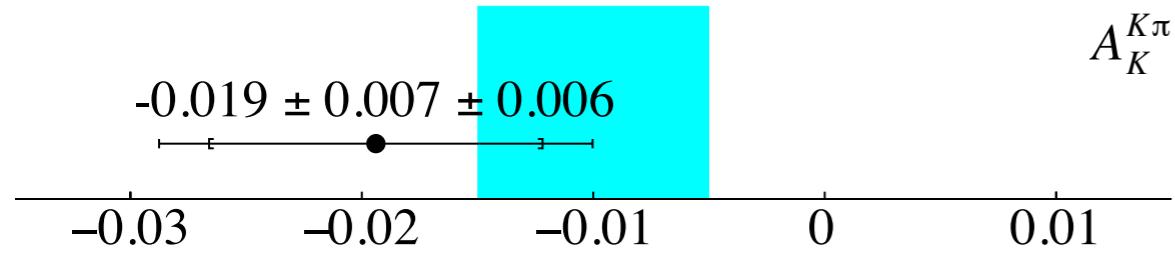
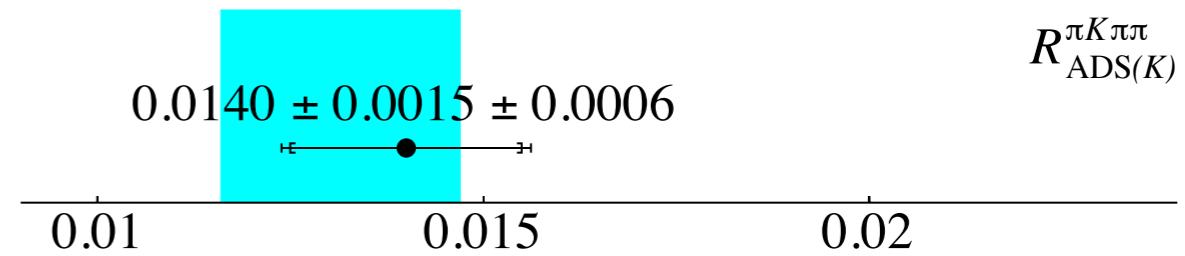
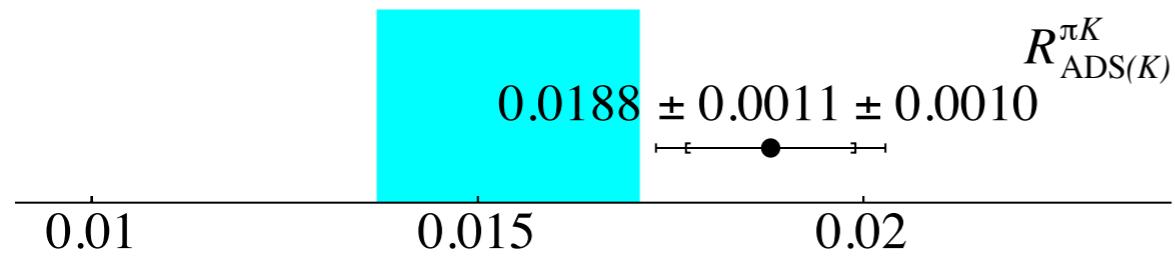
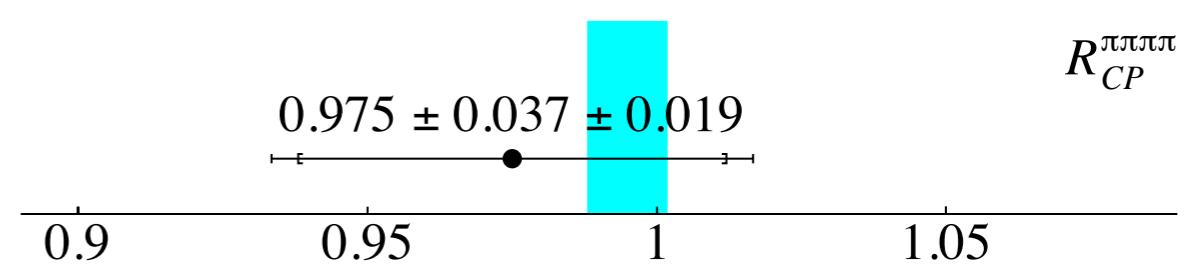
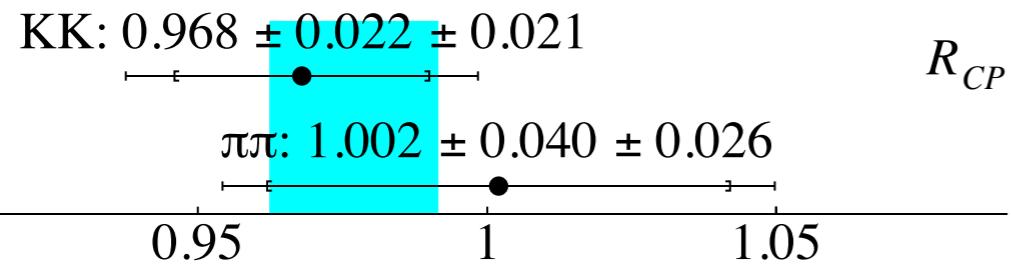
$$A_K^{\pi\pi\pi\pi} = 0.100 \pm 0.034 \pm 0.018$$

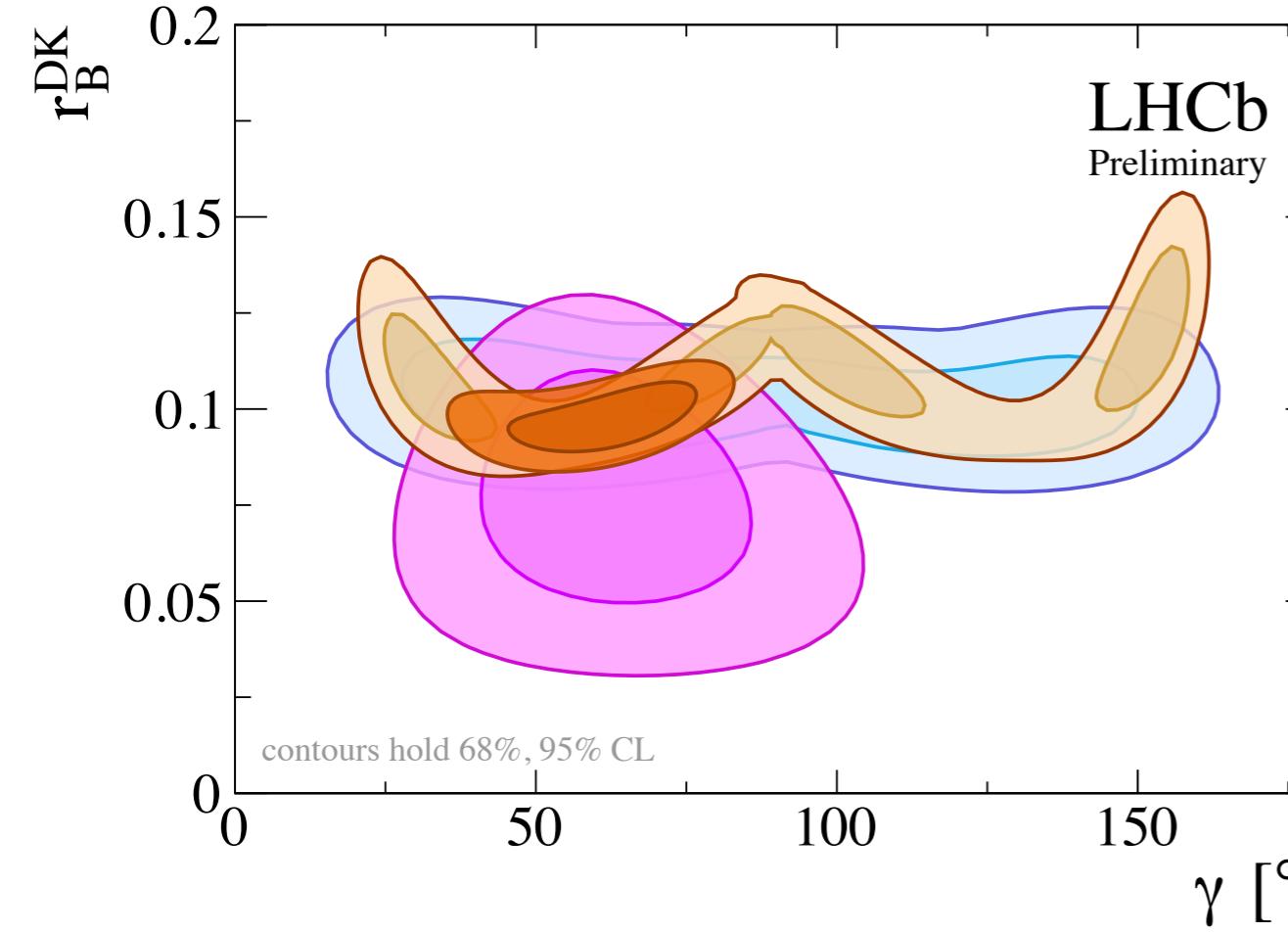


Compared to 2-body GLW modes; interference expected to be diluted by ~0.5 , arXiv:1504.05878

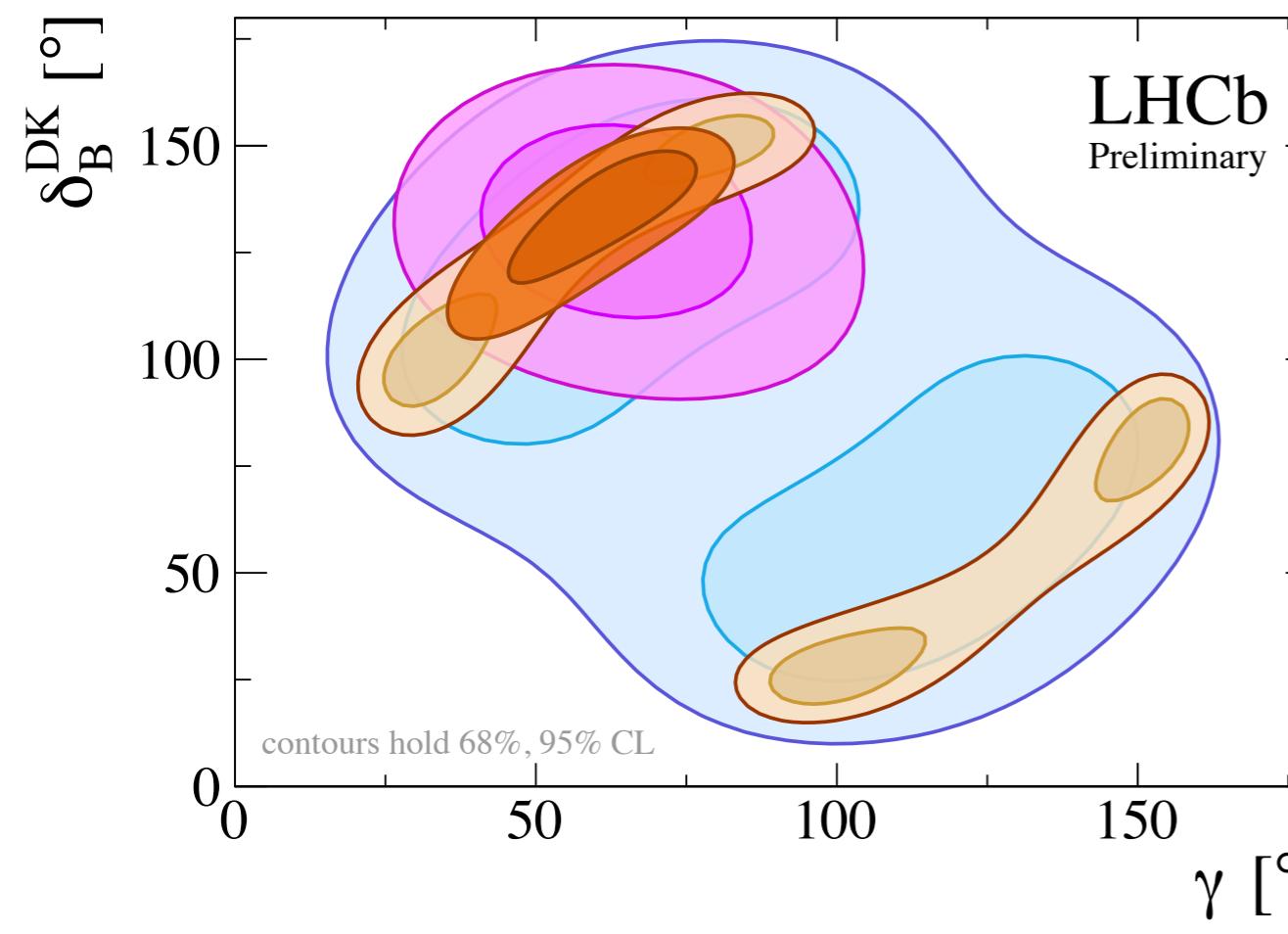
$$A_K^{KK} = 0.087 \pm 0.020 \pm 0.008$$

$$A_K^{\pi\pi} = 0.128 \pm 0.037 \pm 0.012$$





B[±] combination



$$B^0 \rightarrow D K^{*0}, K_S^0 h^+ h^-$$

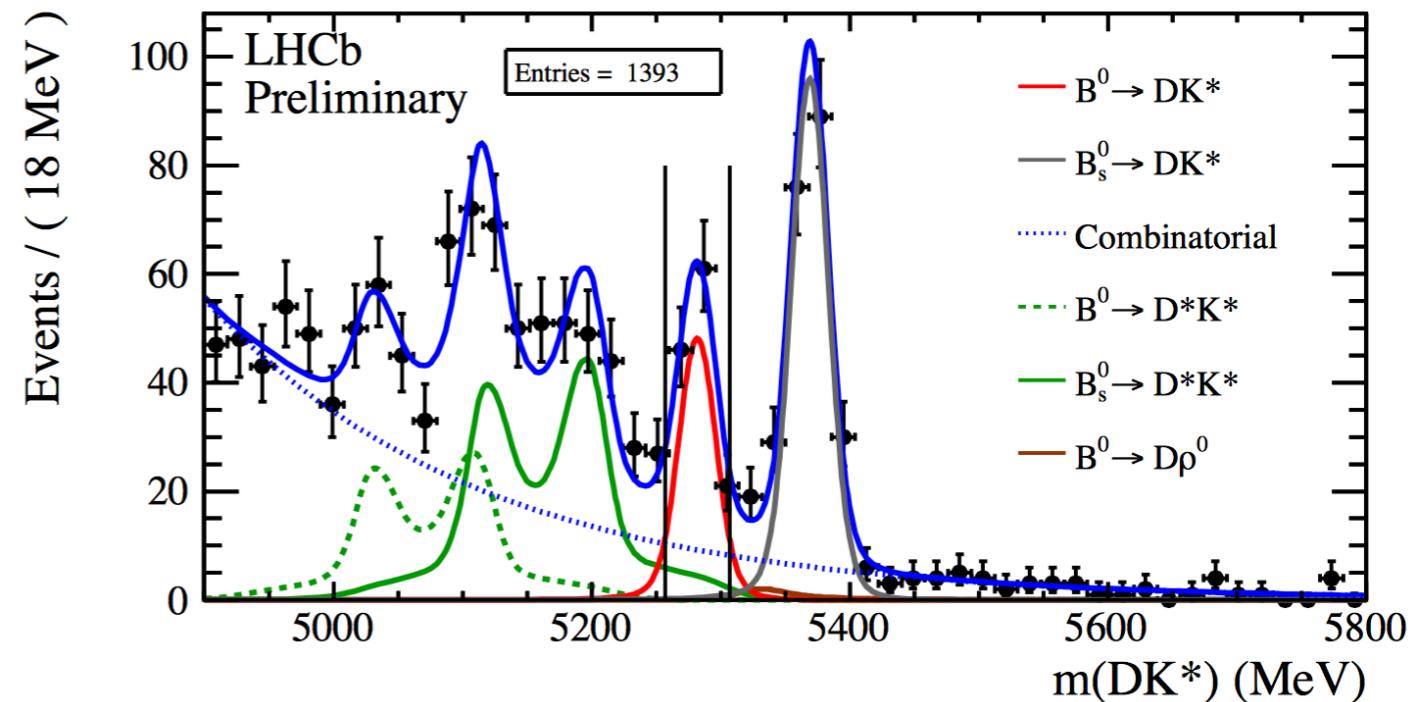
- Two methods attempted using the 3.0fb^{-1} data
 - Model-dependent (uses an amplitude model to fit the Dalitz distribution)
 - Model-independent (counts events in bin across the Dalitz plane)
- Both analyses extract a set of four “cartesian” observables

$$x_{\pm} = r_B \cos(\delta_B \pm \gamma),$$

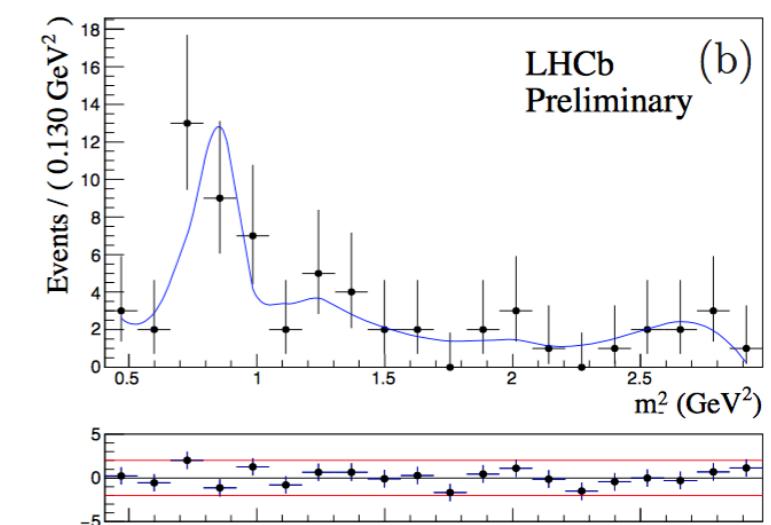
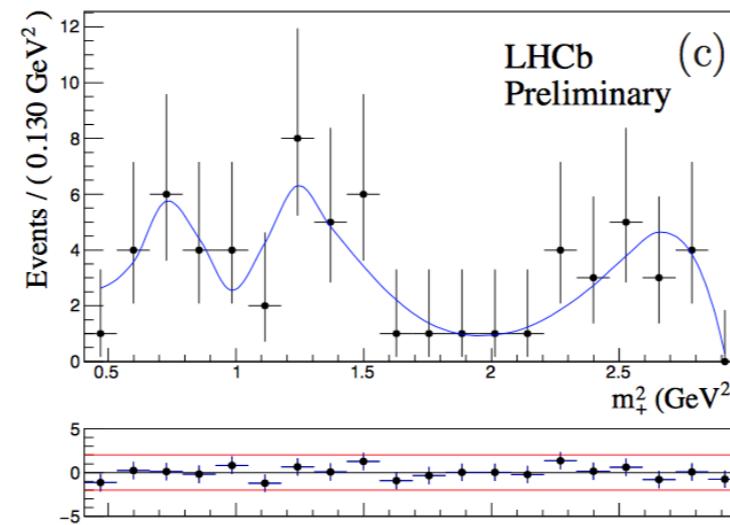
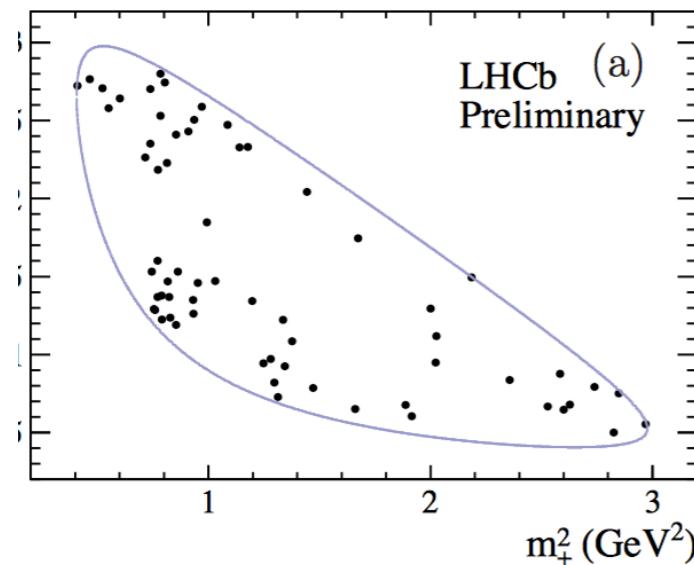
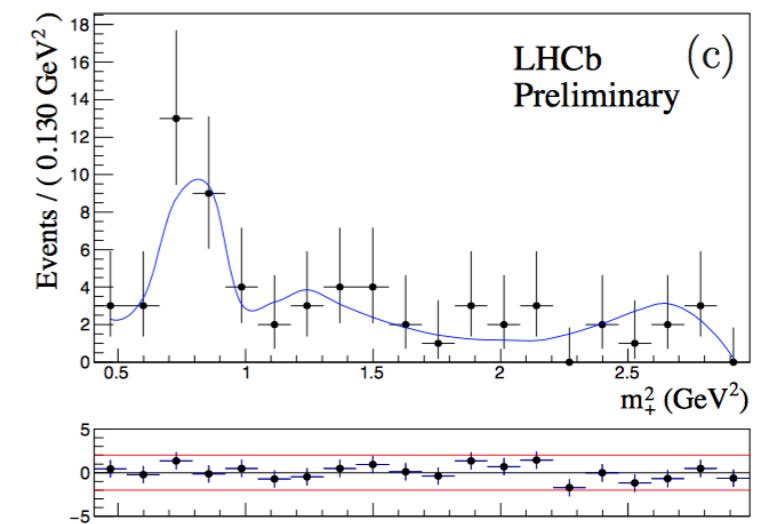
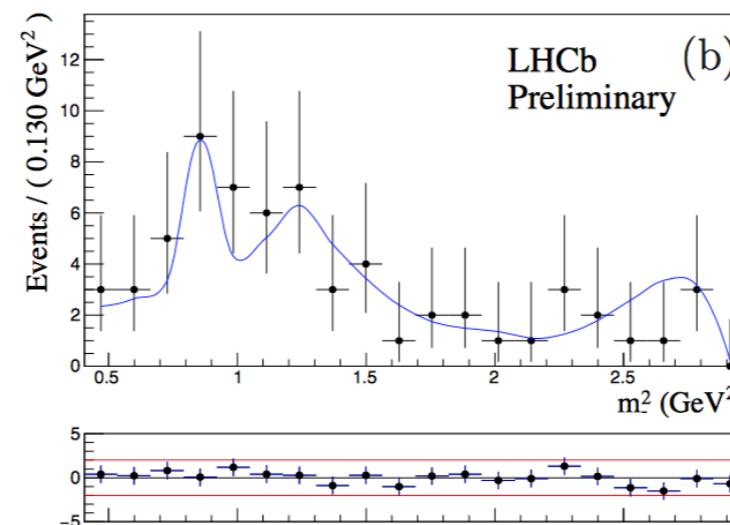
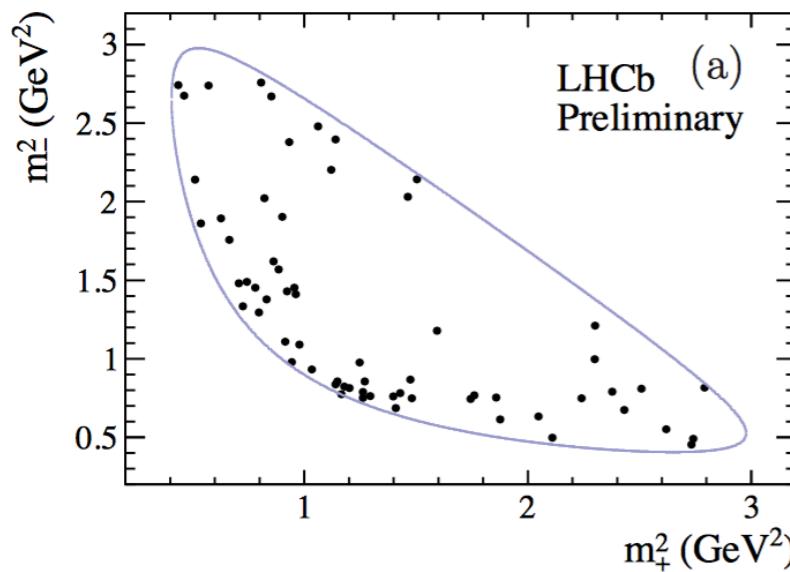
$$y_{\pm} = r_B \sin(\delta_B \pm \gamma)$$

or $z_{\pm} = r_{B^0} e^{i(\delta_{B^0} \pm \gamma)}$

- Model-dependent analysis (MD) starts by extracting the signal (red) from an invariant mass fit, then fitting an amplitude model to the Dalitz distribution



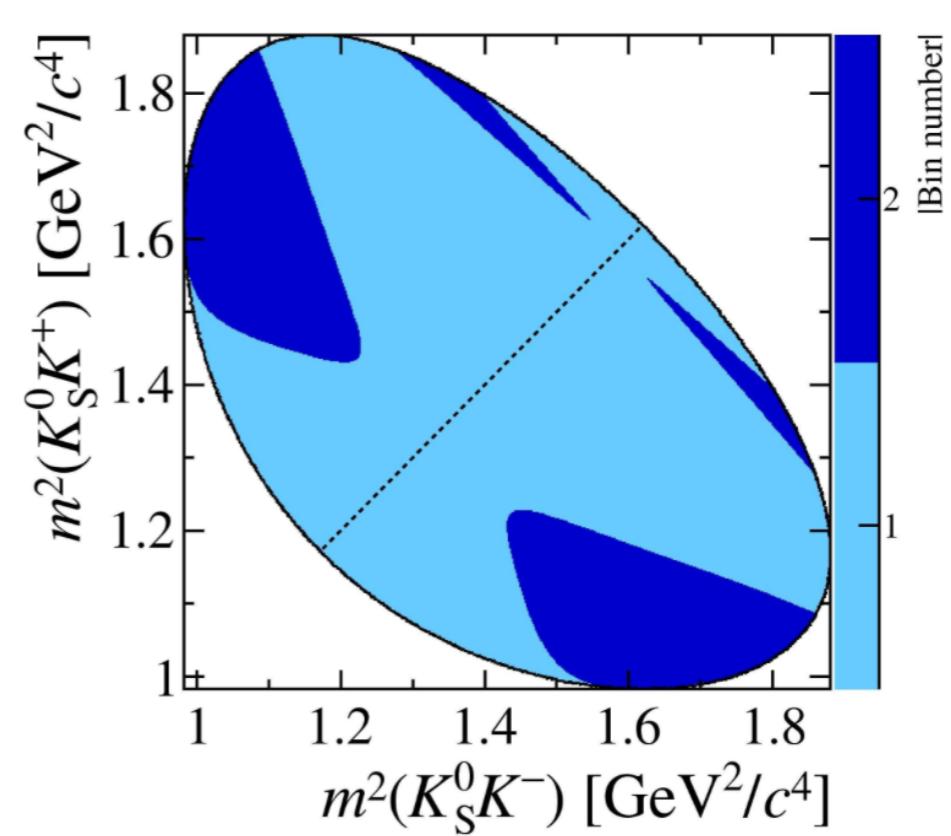
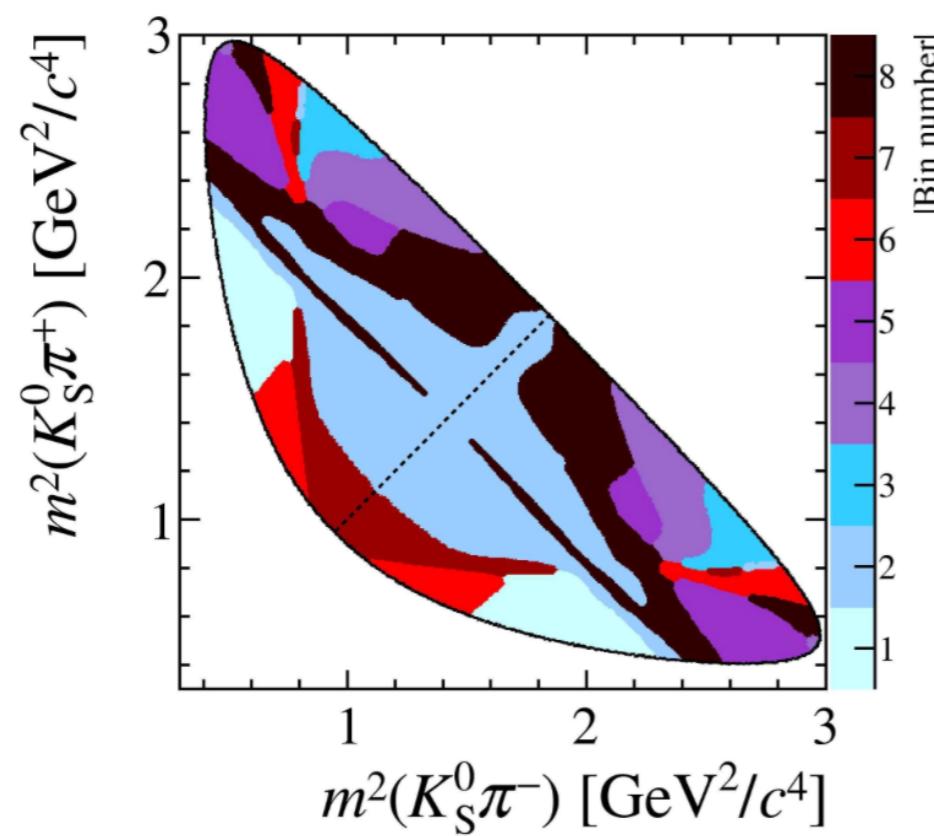
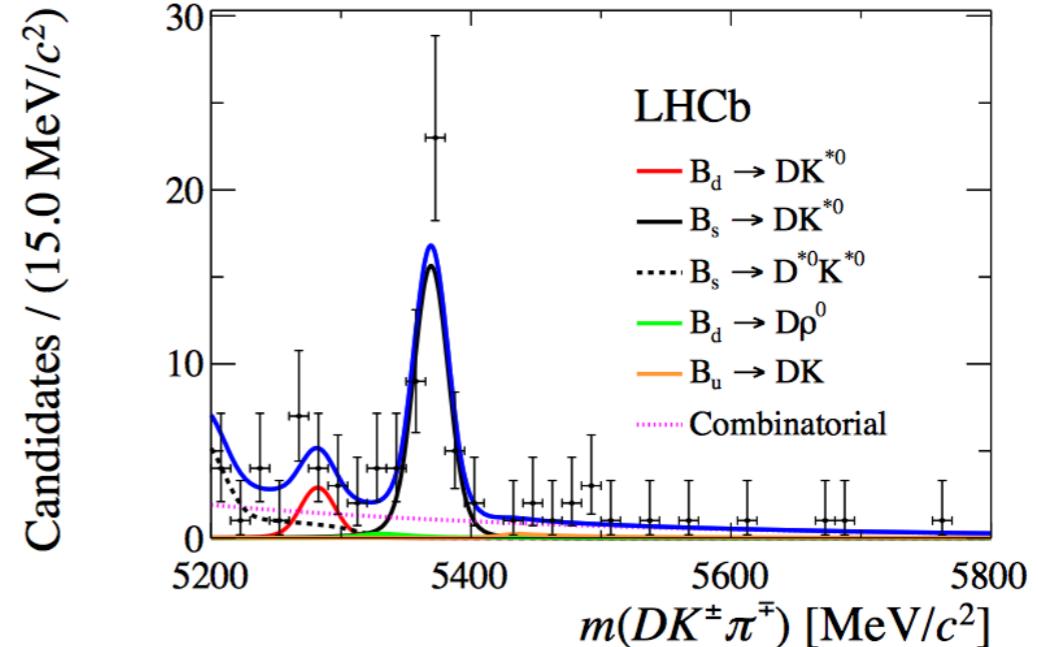
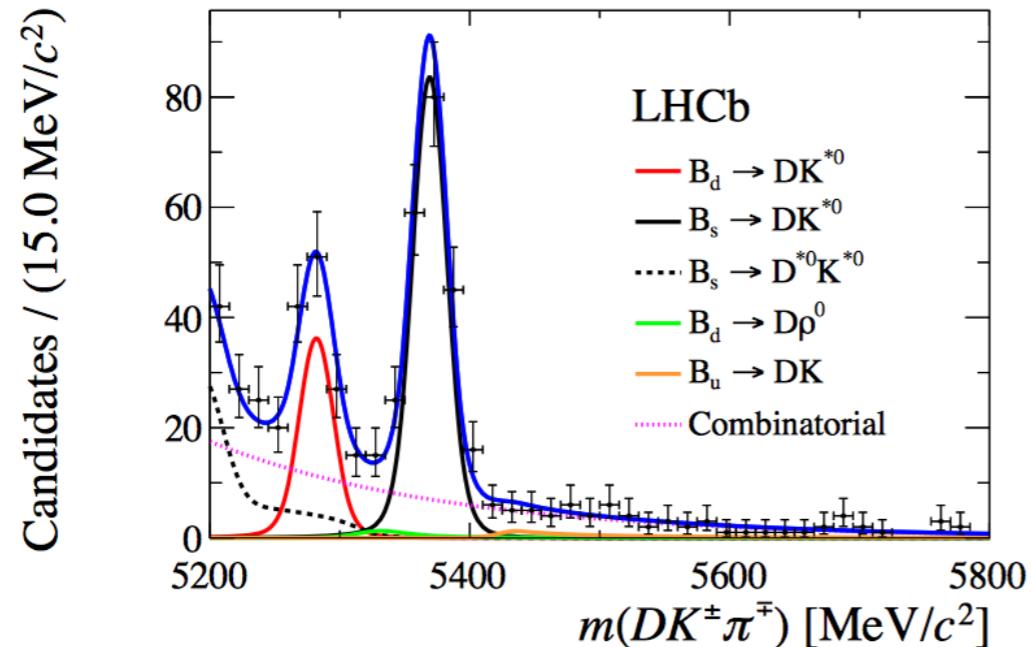
$B^0 \rightarrow DK^{*0}, K_S^0 \pi^+ \pi^-$ amplitude analysis (MD)



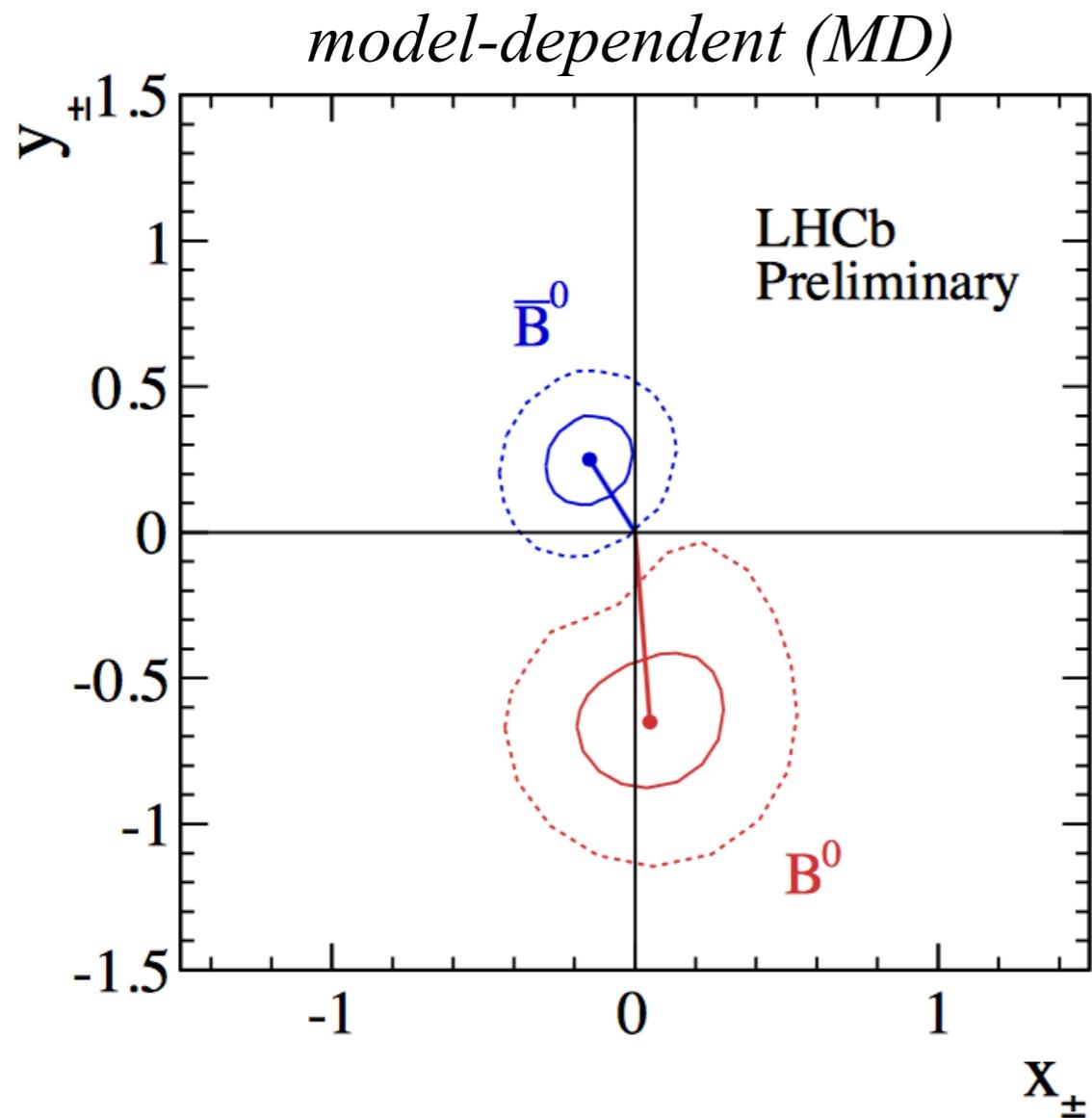
$$\begin{aligned}\mathcal{P}_{\bar{B}^0}(m_-^2, m_+^2) &\propto \mathcal{P}(A_f, z_-, \kappa) \\ \mathcal{P}_{B^0}(m_-^2, m_+^2) &\propto \mathcal{P}(\bar{A}_f, z_+, \kappa)\end{aligned}$$

$$\mathcal{P}(A, z, \kappa) = |A|^2 + |z|^2 |\bar{A}|^2 + 2\kappa \mathcal{R}e [z A^\star \bar{A}], \quad z_\pm = r_{B^0} e^{i(\delta_{B^0} \pm \gamma)}$$

$B^0 \rightarrow DK^{*0}$, $K_S^0\pi^+\pi^-$ and $K_S^0K^+K^-$ mass fits (MI)



Cartesian coordinates result, note: different scale

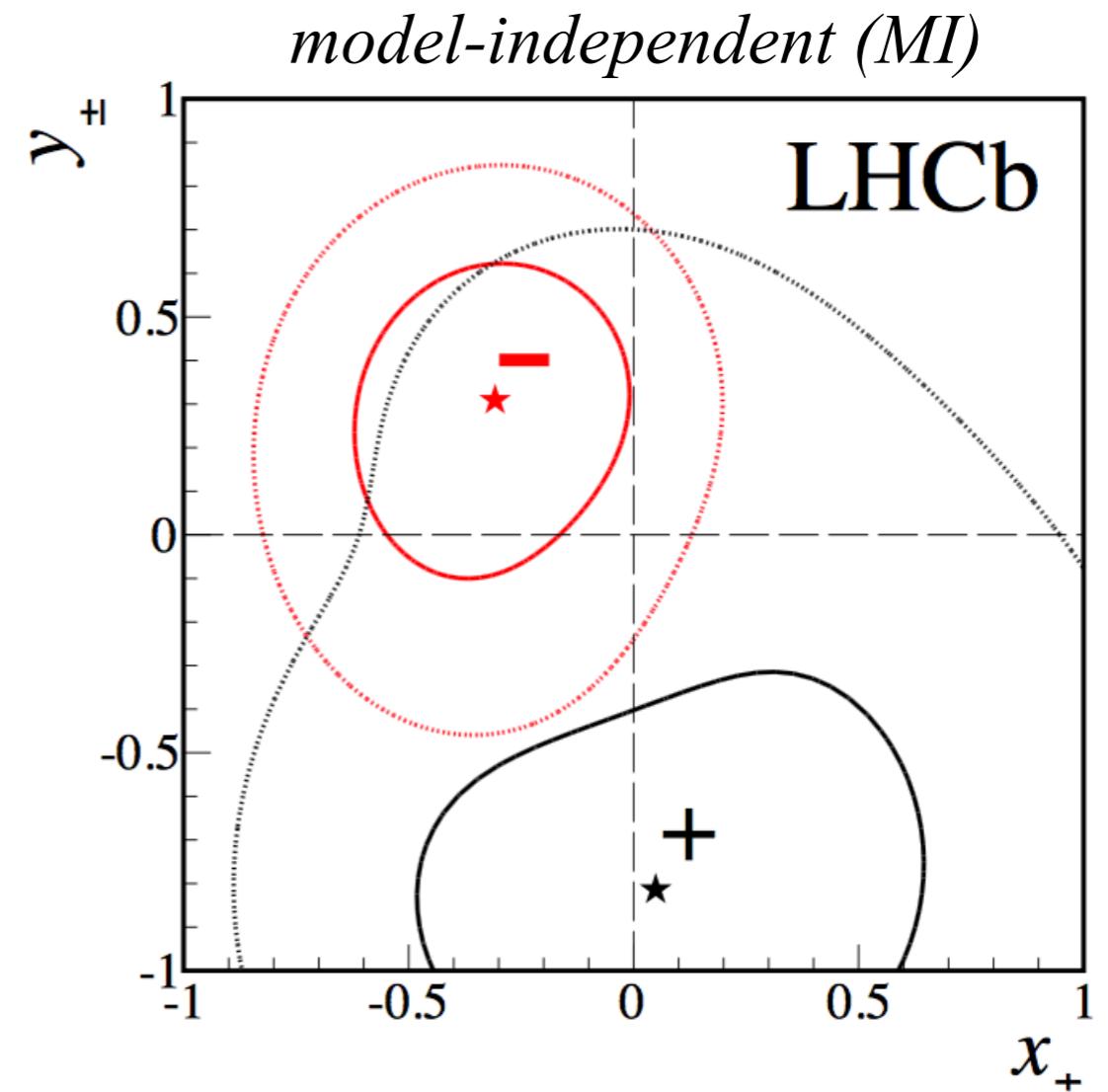


$$x_+ = 0.05 \pm 0.24 \pm 0.04 \pm 0.01,$$

$$x_- = -0.15 \pm 0.14 \pm 0.03 \pm 0.01,$$

$$y_+ = -0.65^{+0.24}_{-0.23} \pm 0.08 \pm 0.01,$$

$$y_- = 0.25 \pm 0.15 \pm 0.06 \pm 0.01,$$



$$x_+ = 0.05 \pm 0.35 \pm 0.02,$$

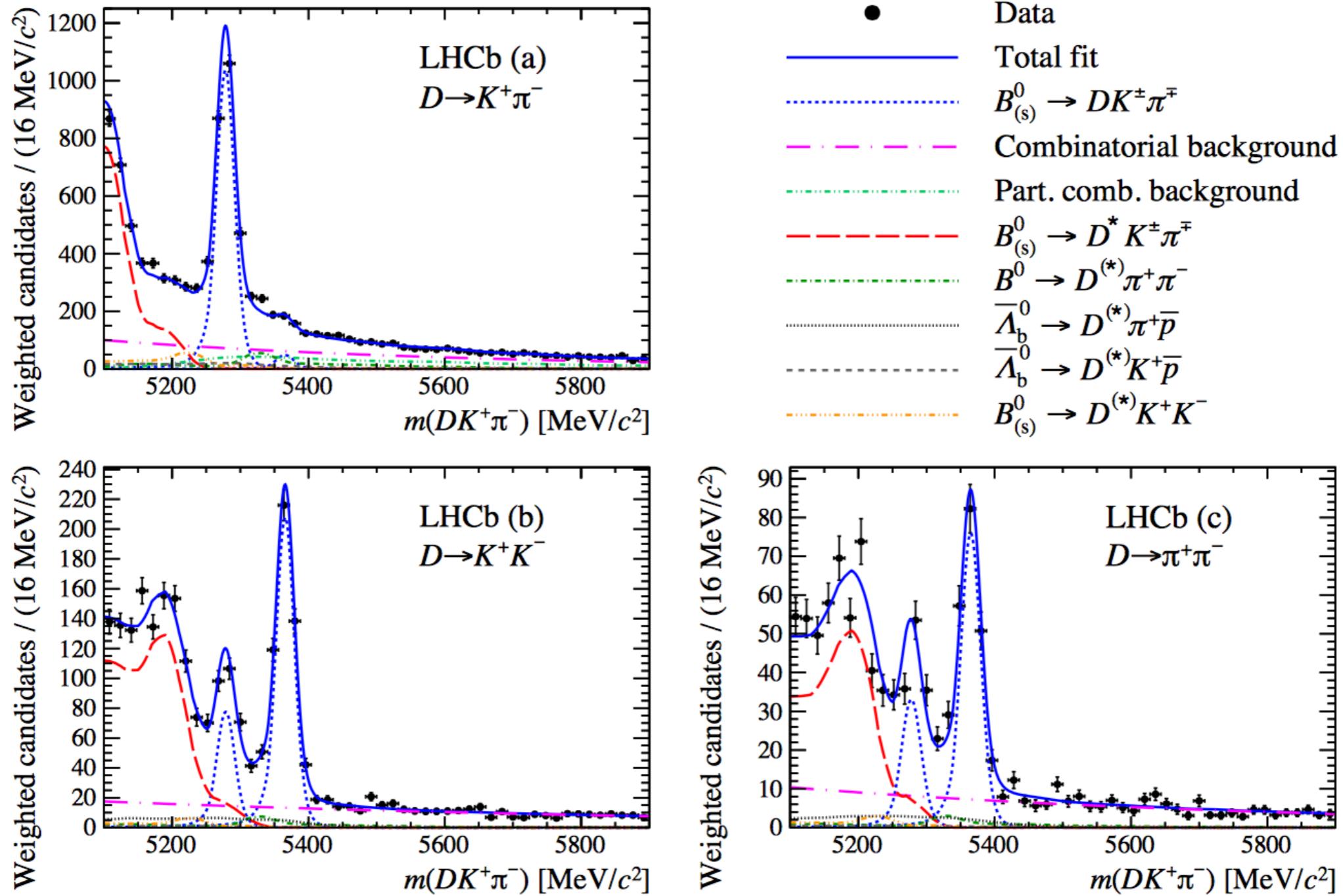
$$x_- = -0.31 \pm 0.20 \pm 0.04,$$

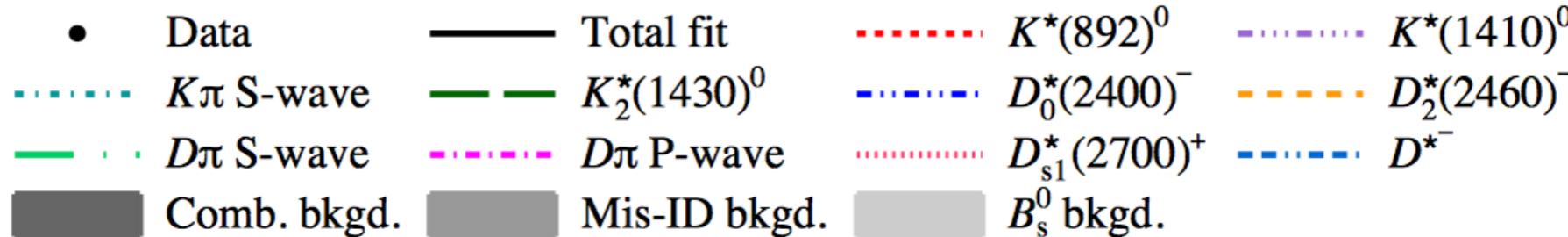
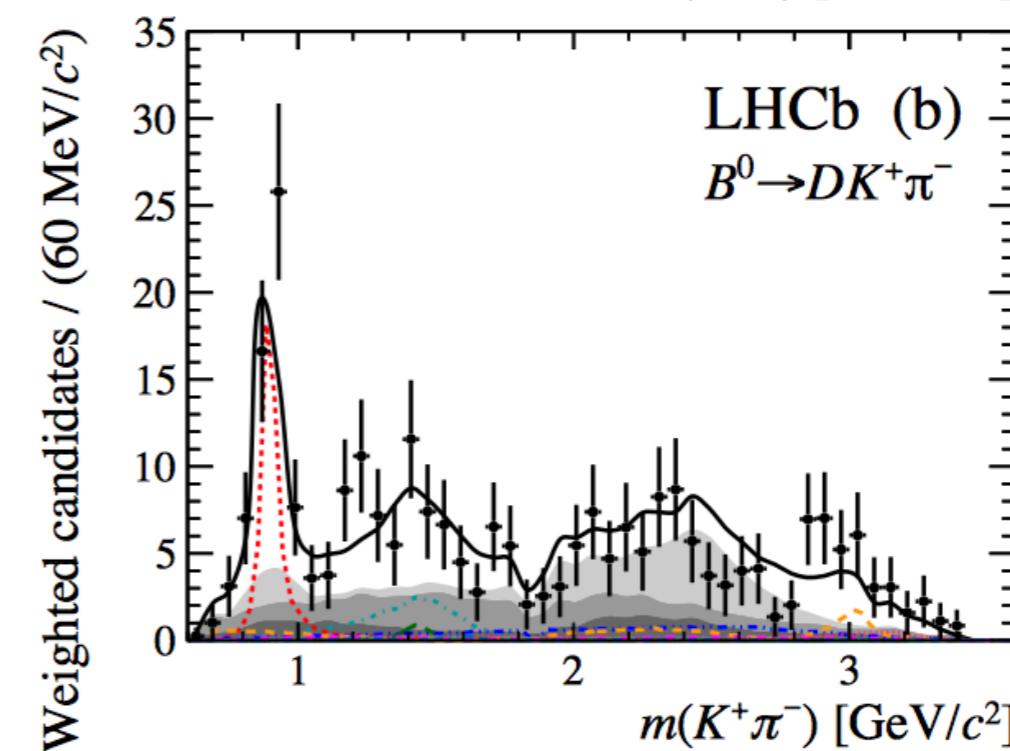
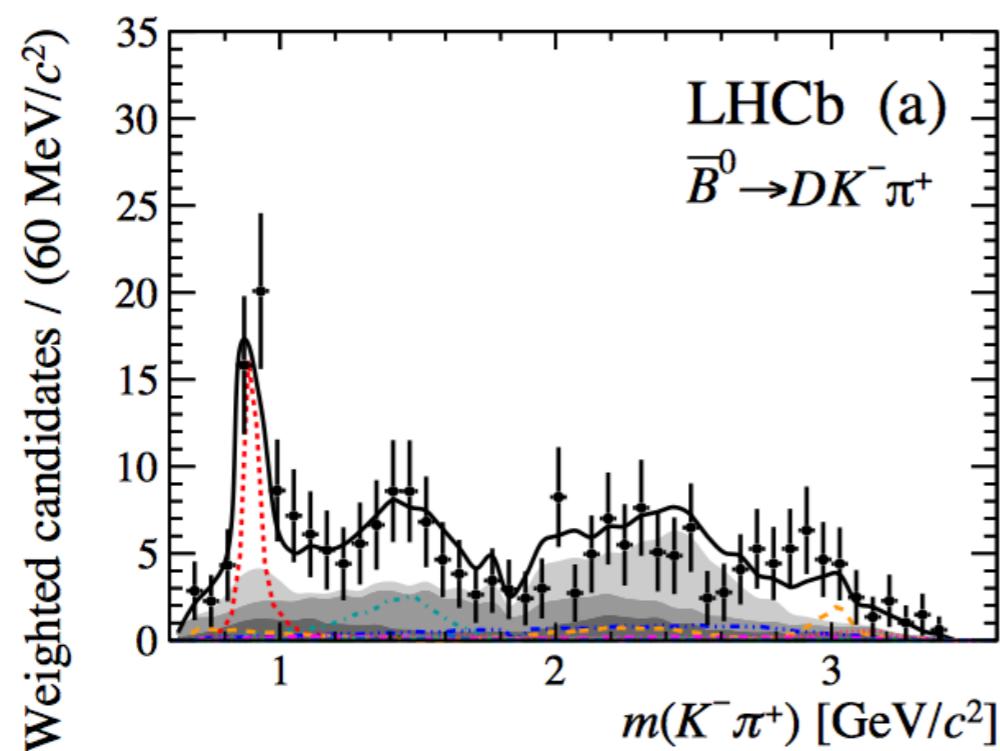
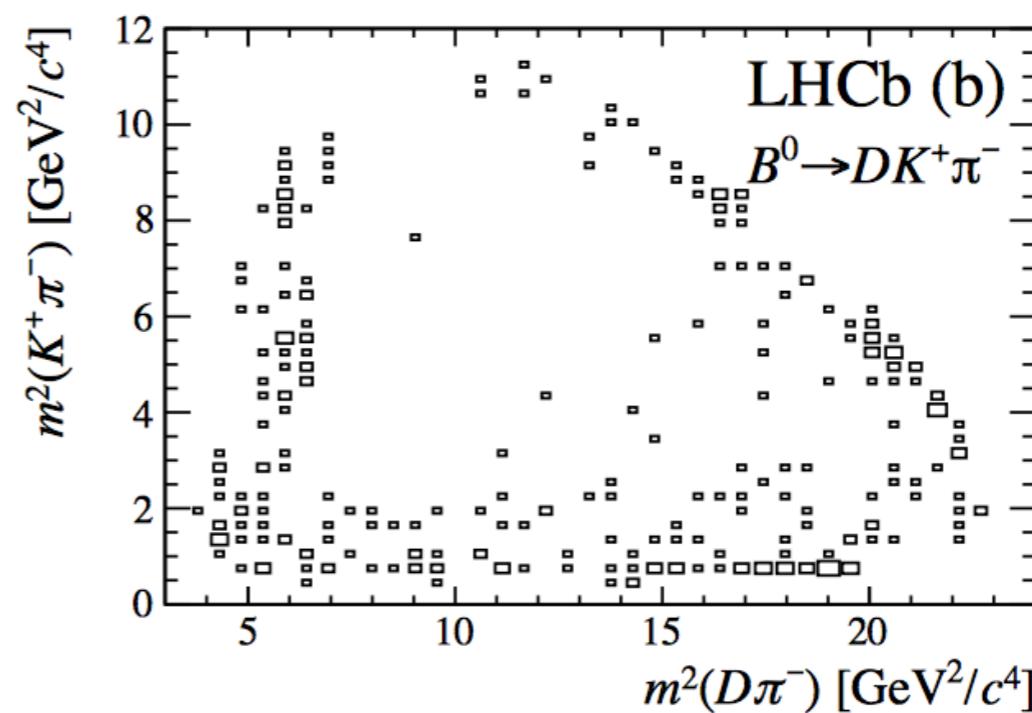
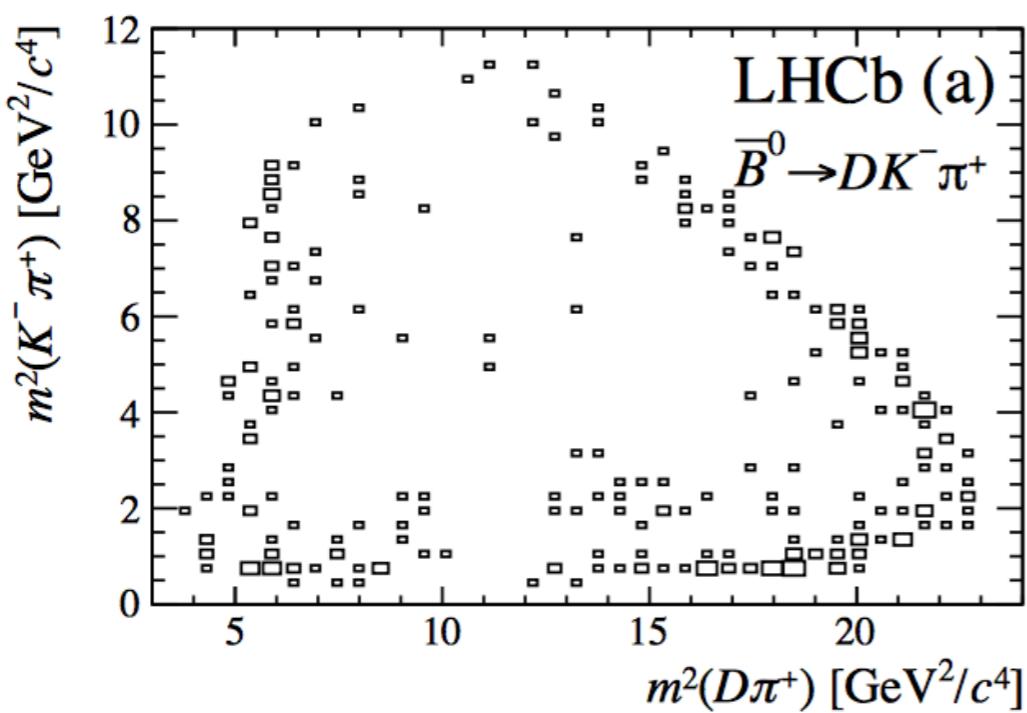
$$y_+ = -0.81 \pm 0.28 \pm 0.06,$$

$$y_- = 0.31 \pm 0.21 \pm 0.05,$$

$B^0 \rightarrow D_{CP} K^+ \pi^-$, $D_{CP} \rightarrow K^+ K^-$ and $D_{CP} \rightarrow \pi^+ \pi^-$

First use of a B^0 amplitude analysis to target γ

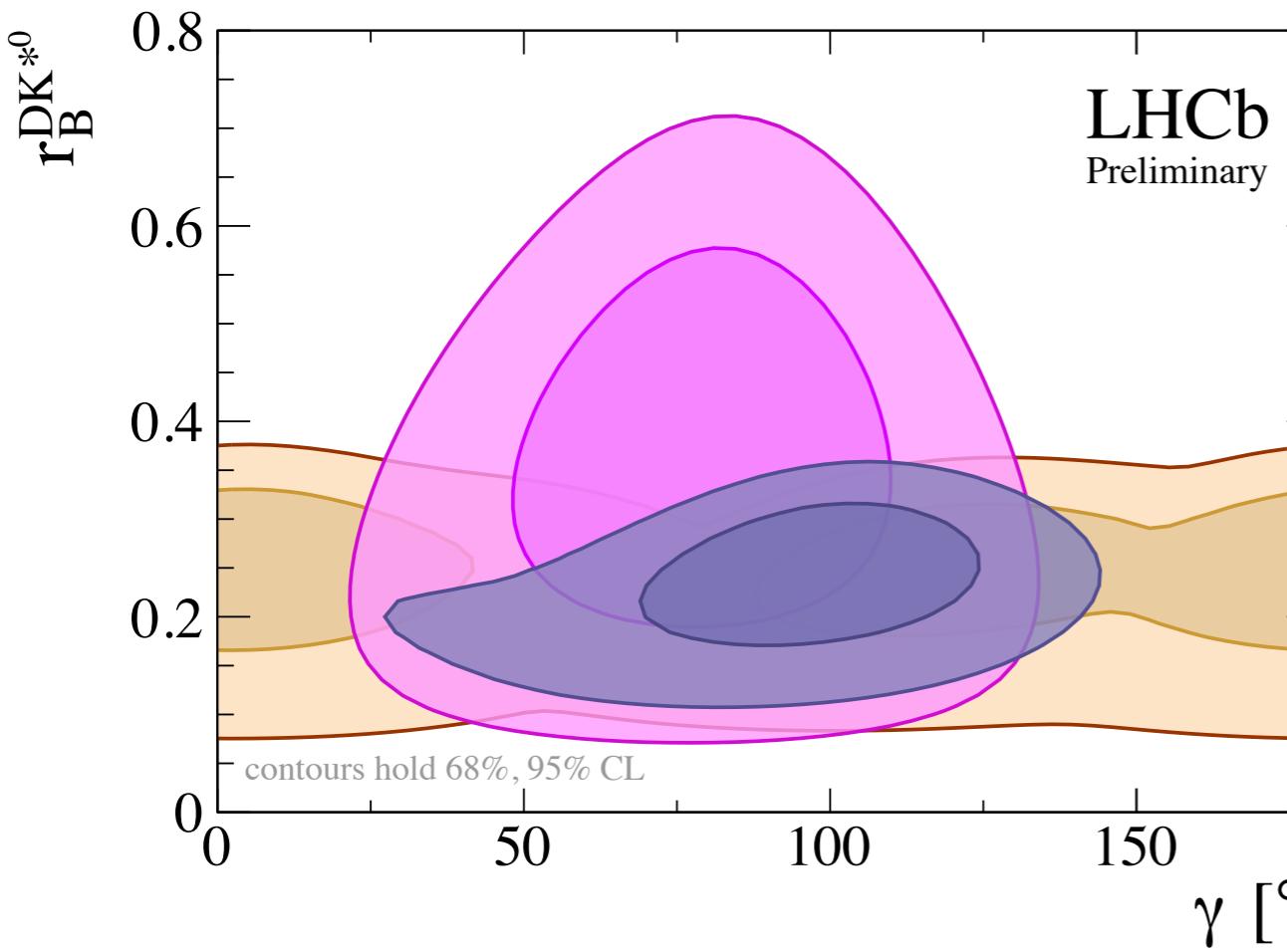




For the K^{*0} region:

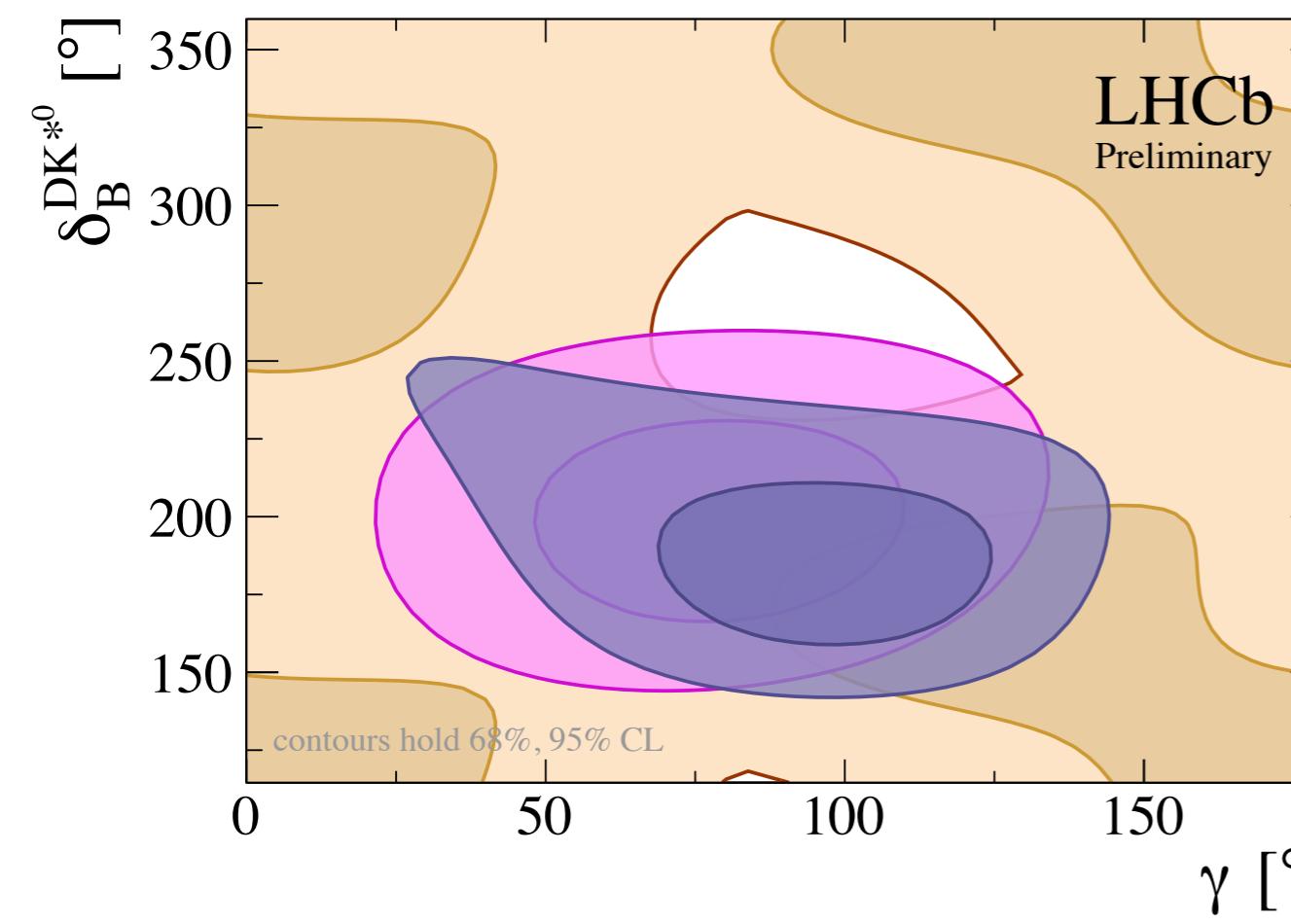
$$x_+ = 0.04 \pm 0.16 \pm 0.11, \quad y_+ = -0.47 \pm 0.28 \pm 0.22,$$

$$x_- = -0.02 \pm 0.13 \pm 0.14, \quad y_- = -0.35 \pm 0.26 \pm 0.41,$$

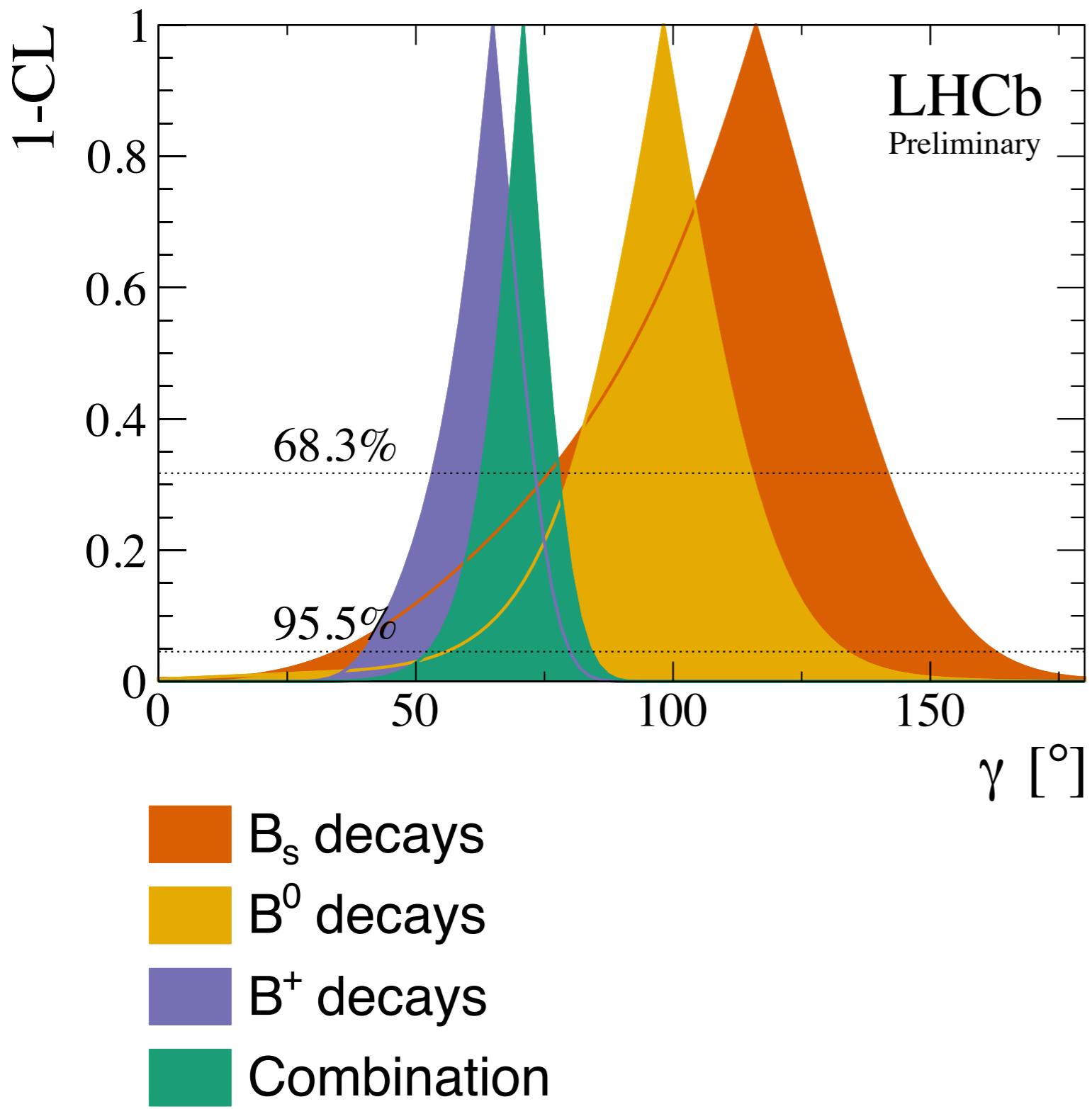


B^0 combination

- [Orange square] $B^0 \rightarrow DK^{*0}, D \rightarrow KK/K\pi/\pi\pi$
- [Pink square] $B^0 \rightarrow DK^{*0}, D \rightarrow K_S\pi\pi$ (MD)
- [Purple square] All B^0 modes



Full combination



Quantity	Value
γ (°)	70.9
68% CL (°)	[62.4, 78.0]
95% CL (°)	[51.0, 85.0]
r_B^{DK}	0.1006
68% CL	[0.0946, 0.1065]
95% CL	[0.0890, 0.1120]
δ_B^{DK} (°)	141.1
68% CL (°)	[133.4, 147.2]
95% CL (°)	[122.0, 153.0]
r_B^{DK*0}	0.217
68% CL	[0.169, 0.261]
95% CL	[0.115, 0.303]
δ_B^{DK*0} (°)	189.0
68% CL (°)	[169.0, 213.0]
95% CL (°)	[149.0, 243.0]

Conclusion

- A world-leading measurement of γ is made from a combination of LHCb analysis, concluding with

$$\gamma = 70.9^{+7.1}_{-8.5}$$

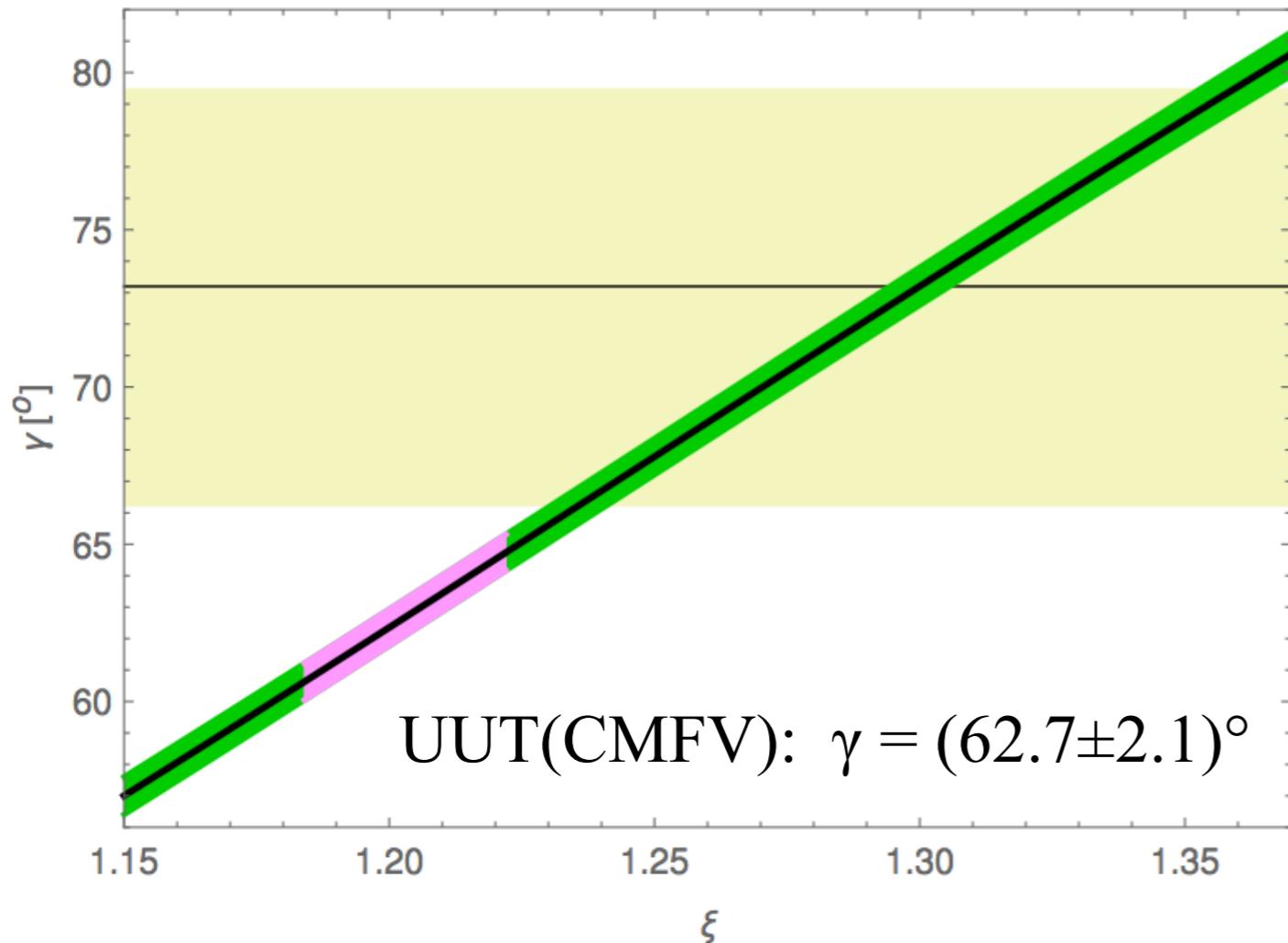
which improved the previous LHCb-only conclusion by 2°

- Inline with B-factory conclusions from $B \rightarrow D\bar{K}$,
 - BaBar: $\gamma = (70 \pm 18)^\circ$
 - Belle: $\gamma = (73^{+13}_{-15})^\circ$
- But $\sim 1\sigma$ above expectation (with Constrained Minimal Flavour Violation) for $\sin 2\beta = 0.691 \pm 0.017$ and new lattice determinations of hadronic matrix elements in B mixing ([Fermilab-MILC arXiv:1602.03560](#))
 - UUT(CMFV): $\gamma = (62.7 \pm 2.1)^\circ$ ([Blanke/Buras arXiv:1602.0402](#))
- Large collaborative effort in this area:
 - update to $B_s \rightarrow D_s K$ expected soon
 - new (for LHCb) $B \rightarrow D^{(*)} X^{(*)}$ modes are in development
 - on-going investigating of $B^\pm \rightarrow D \pi^\pm$ for γ

BACKUP

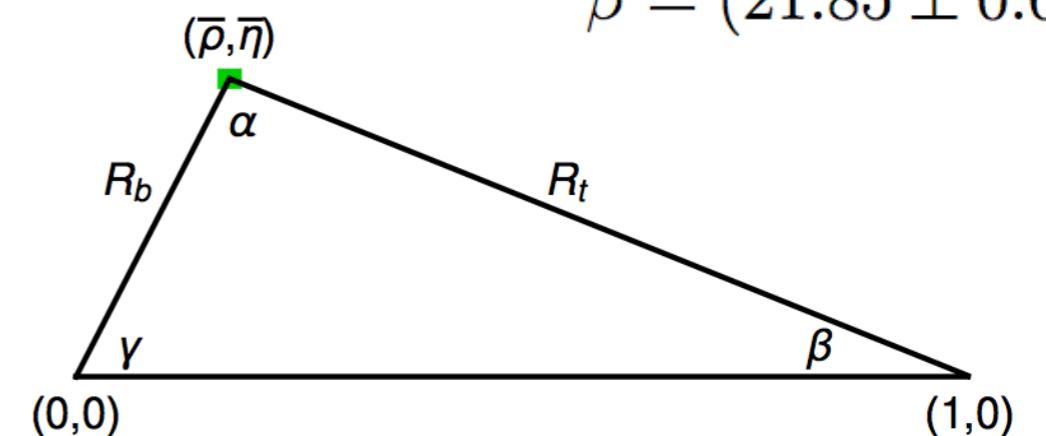
Universal Unitarity Triangle 2016 and the Tension Between $\Delta M_{s,d}$ and ε_K in CMFV Models

(Blanke/Buras arXiv:1602.0402)



$$\cot \gamma = \frac{1 - R_t \cos \beta}{R_t \sin \beta}$$

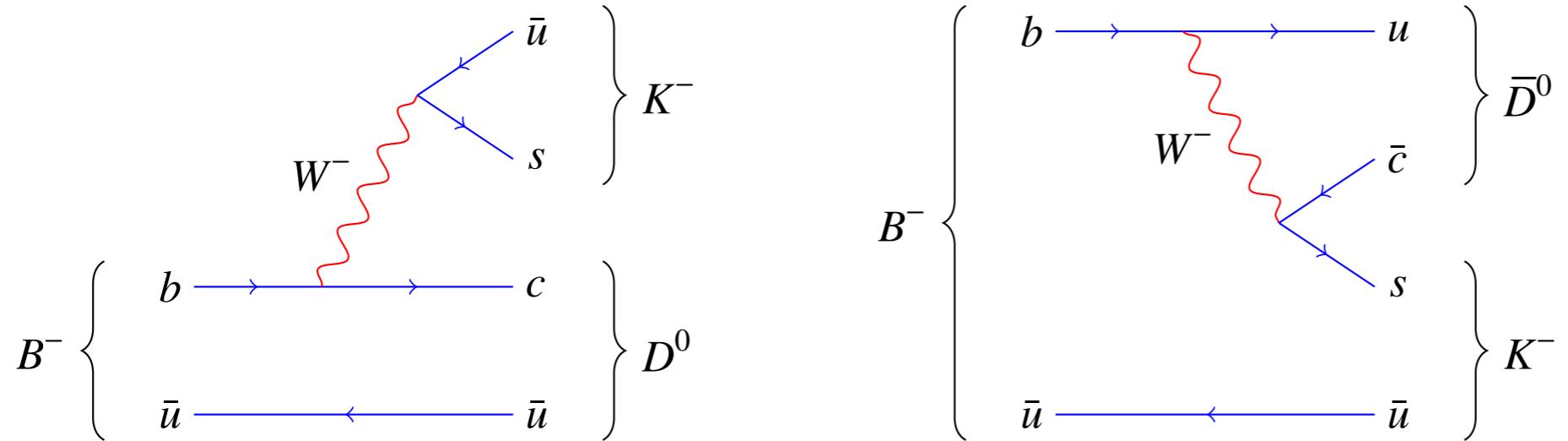
$$\beta = (21.85 \pm 0.67)^\circ$$



$$R_t = 0.741 \quad \xi = 0.893 \pm 0.013$$

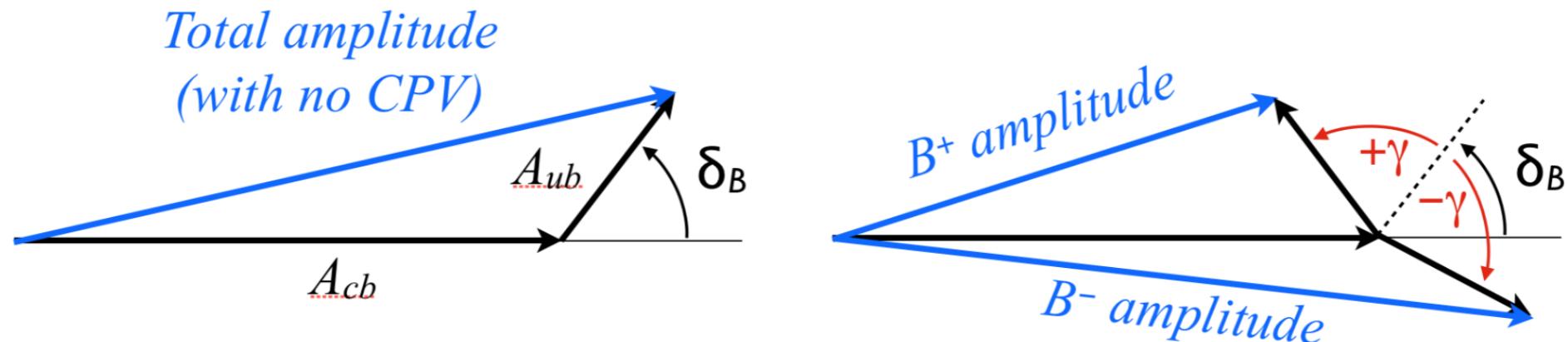
$$\xi = \frac{F_{B_s} \sqrt{\hat{B}_{B_s}}}{F_{B_d} \sqrt{\hat{B}_{B_d}}} = 1.203 \pm 0.019$$

(Fermilab-MILC arXiv:1602.03560)



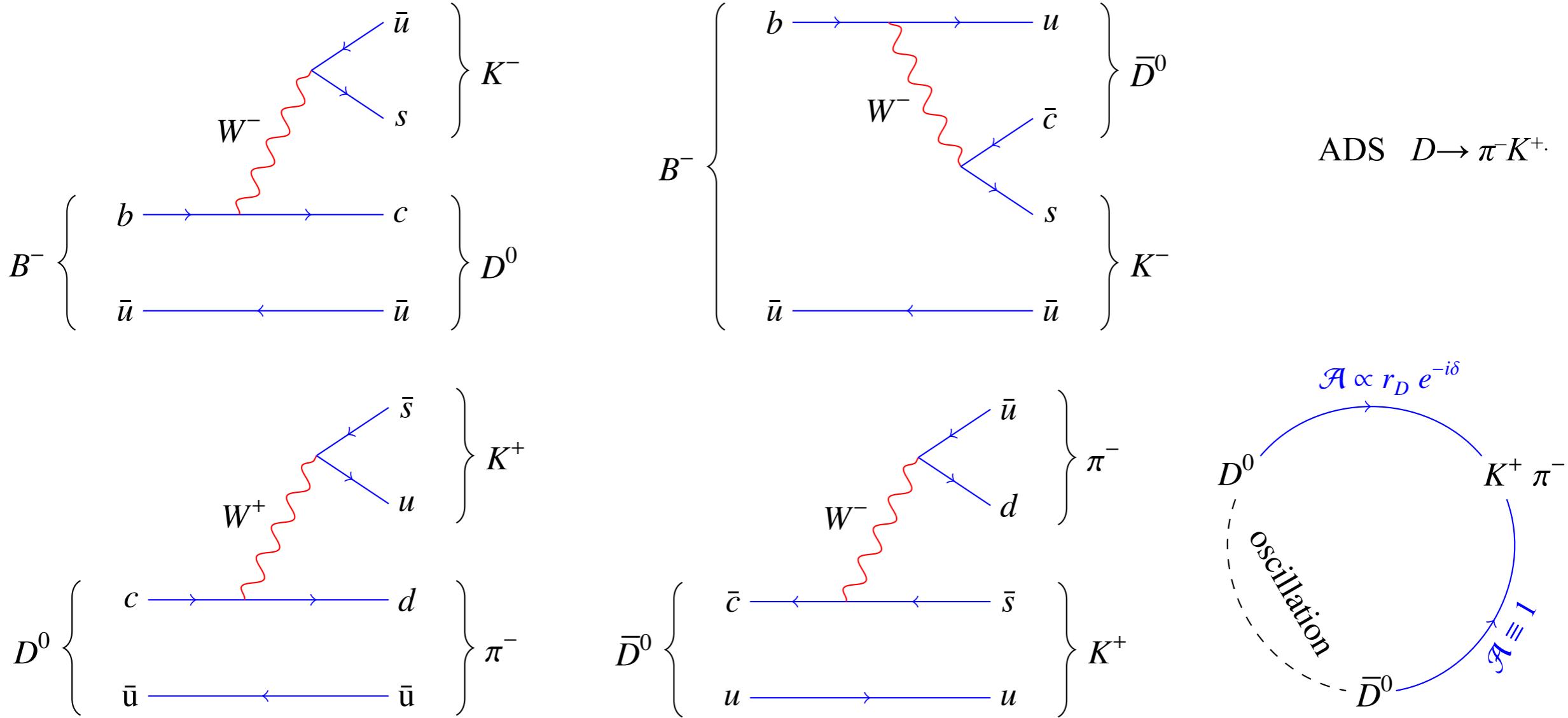
The ancillary “hadronic parameters” are the amplitude magnitude ratio and the CP-conserving part of the phase,

$$\frac{A(B^- \rightarrow \bar{D}^0 K^-)}{A(B^- \rightarrow D^0 K^-)} = r_B e^{i(\delta_B - \gamma)}$$



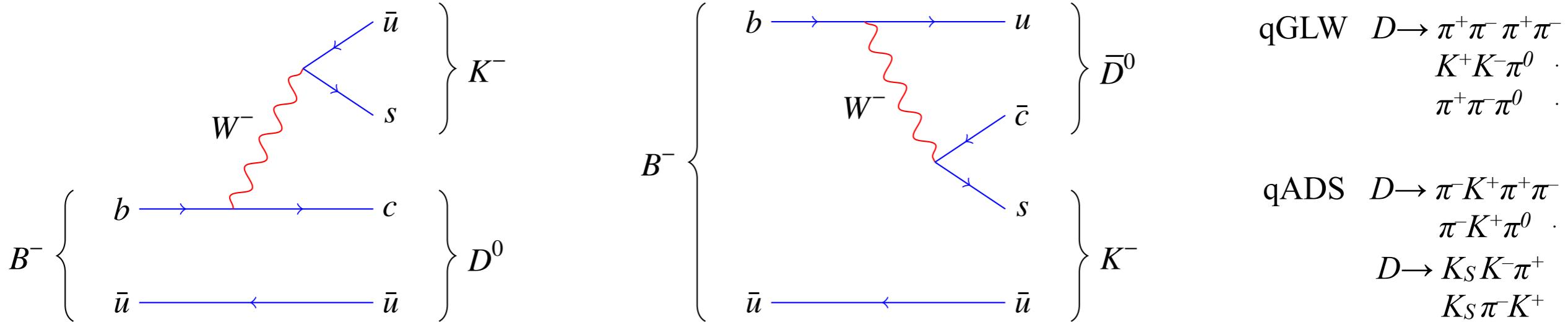
GLW analysis uses CP eigenstate D decays, equally accessible to the D^0 and \bar{D}^0

$$\Gamma(B^\pm \rightarrow [f]_D K^\pm) = 1 + r_B^2 + 2r_B \cos(\delta_B) \pm \gamma)$$



The rates equations can be generalised to non- CP eigenstate D decays. So-called ADS modes may have large asymmetry due to the interplay of suppressed and favoured B and D decays.

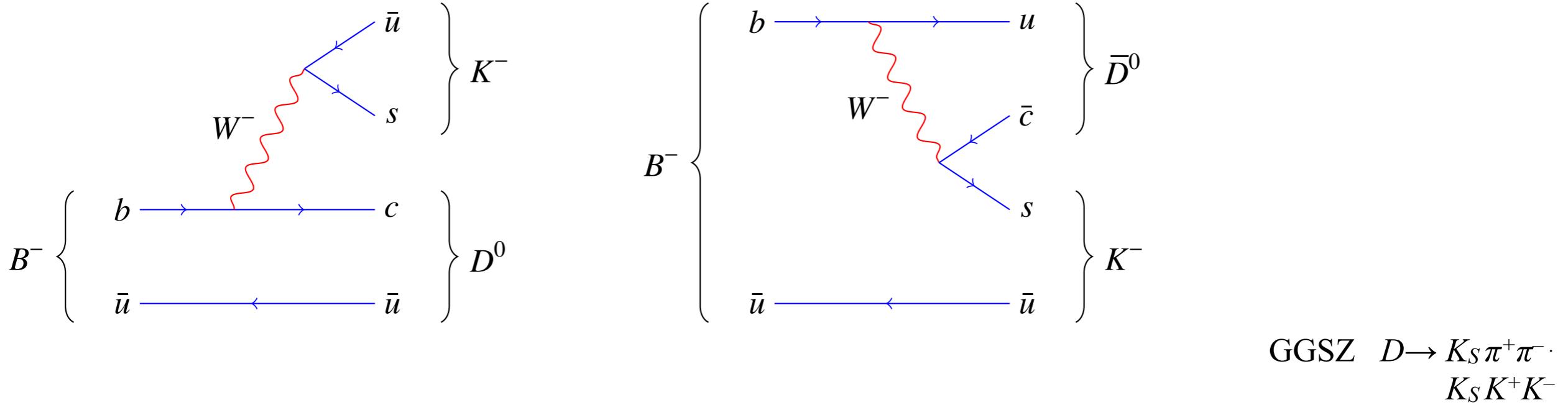
$$\Gamma(B^\pm \rightarrow [f]_D K^\pm) = r_D^2 + r_B^2 + 2 r_D r_B \cos(\delta_B + \delta_D \pm \gamma)$$



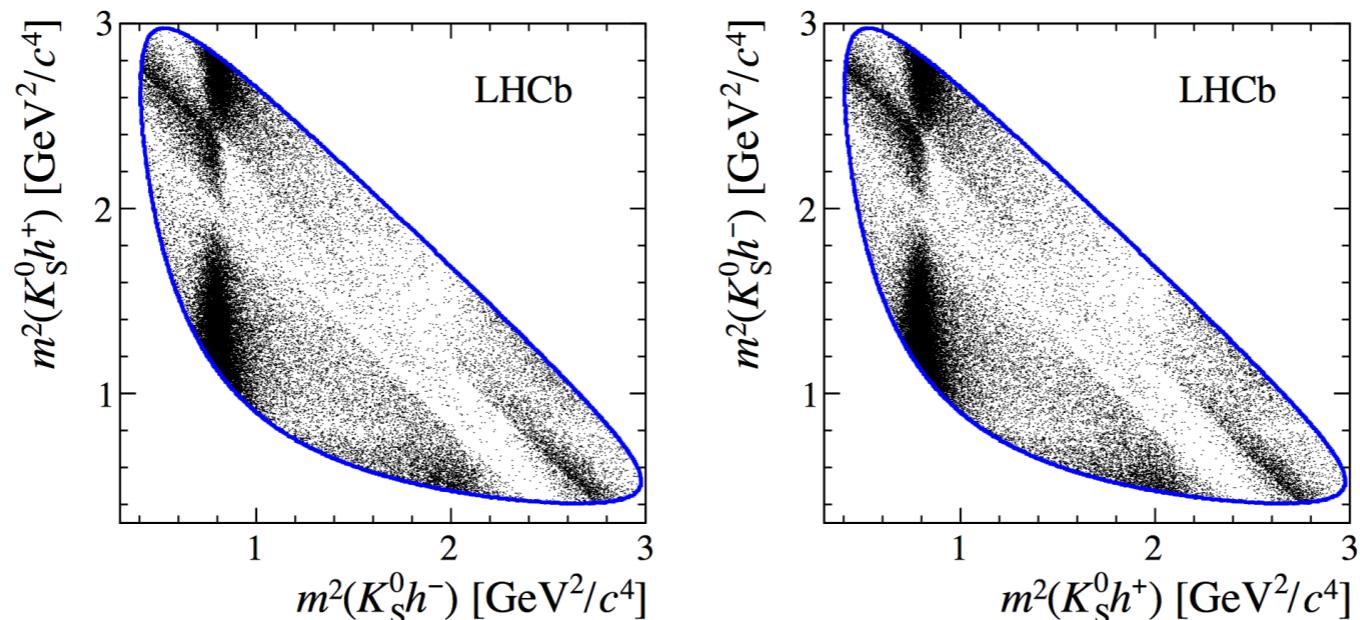
“quasi” ADS/GLW analysis are applicable for 3- or 4-body D decays and are effective if external measurements of the D-decay parameters are available

- for qADS analyses, $\kappa_D^f e^{i\delta_D^f} = \frac{\int A_f(\mathbf{x}) A_{\bar{f}}(\mathbf{x}) d\mathbf{x}}{A_f A_{\bar{f}}}$
- for qGLW analyses, $\kappa_D = (2F_+ - 1)$ where, $F_+^f \equiv \frac{\int_{\mathbf{x} \in \mathcal{D}} |\langle f(\mathbf{x}) | D_{CP+} \rangle|^2 d\mathbf{x}}{\int_{\mathbf{x} \in \mathcal{D}} |\langle f(\mathbf{x}) | D_{CP+} \rangle|^2 + |\langle f(\mathbf{x}) | D_{CP-} \rangle|^2 d\mathbf{x}}$
- for classic ADS and GLW analyses, $\kappa_D = 1$

$$\Gamma(B^\pm \rightarrow [f]_D K^\pm) = r_D^2 + r_B^2 + 2\kappa_D r_D r_B \cos(\delta_B + \delta_D \pm \gamma)$$



GGSZ analysis uses multi-body self-conjugate final states and looks for asymmetries in the D-decay Dalitz plot from b -quark decays vs. that from \bar{b} -quark decays



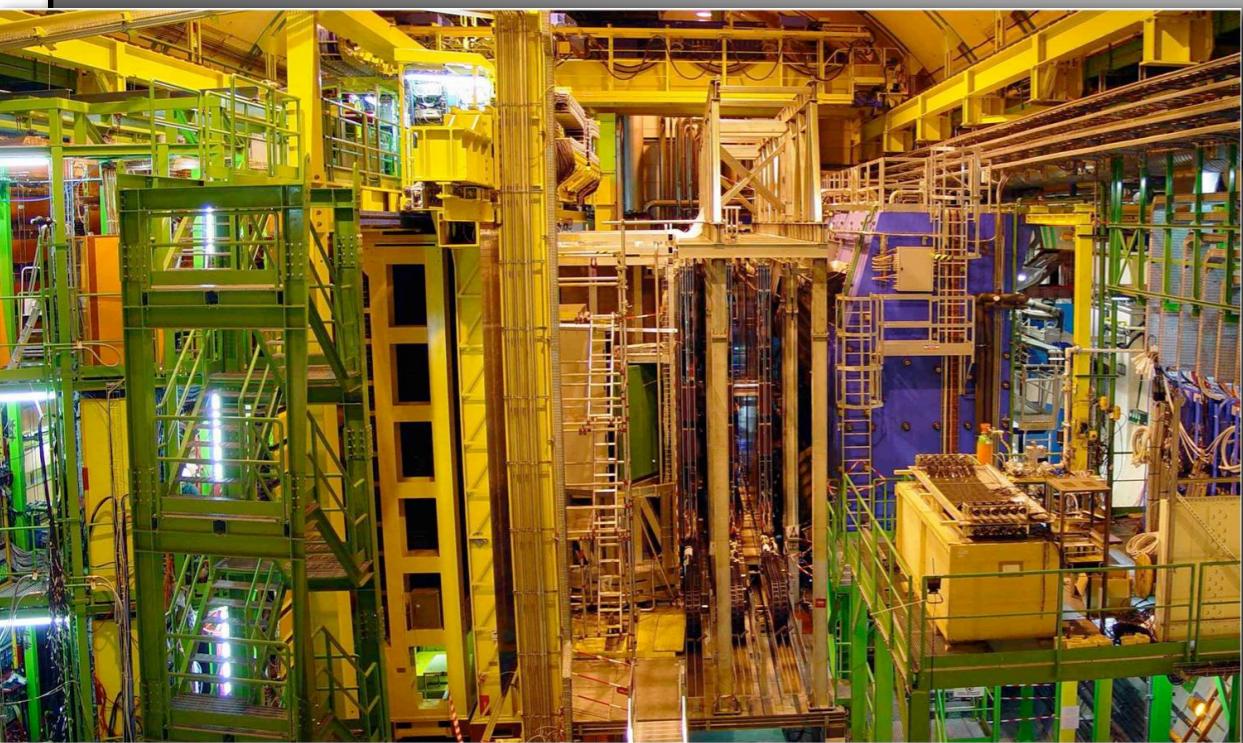
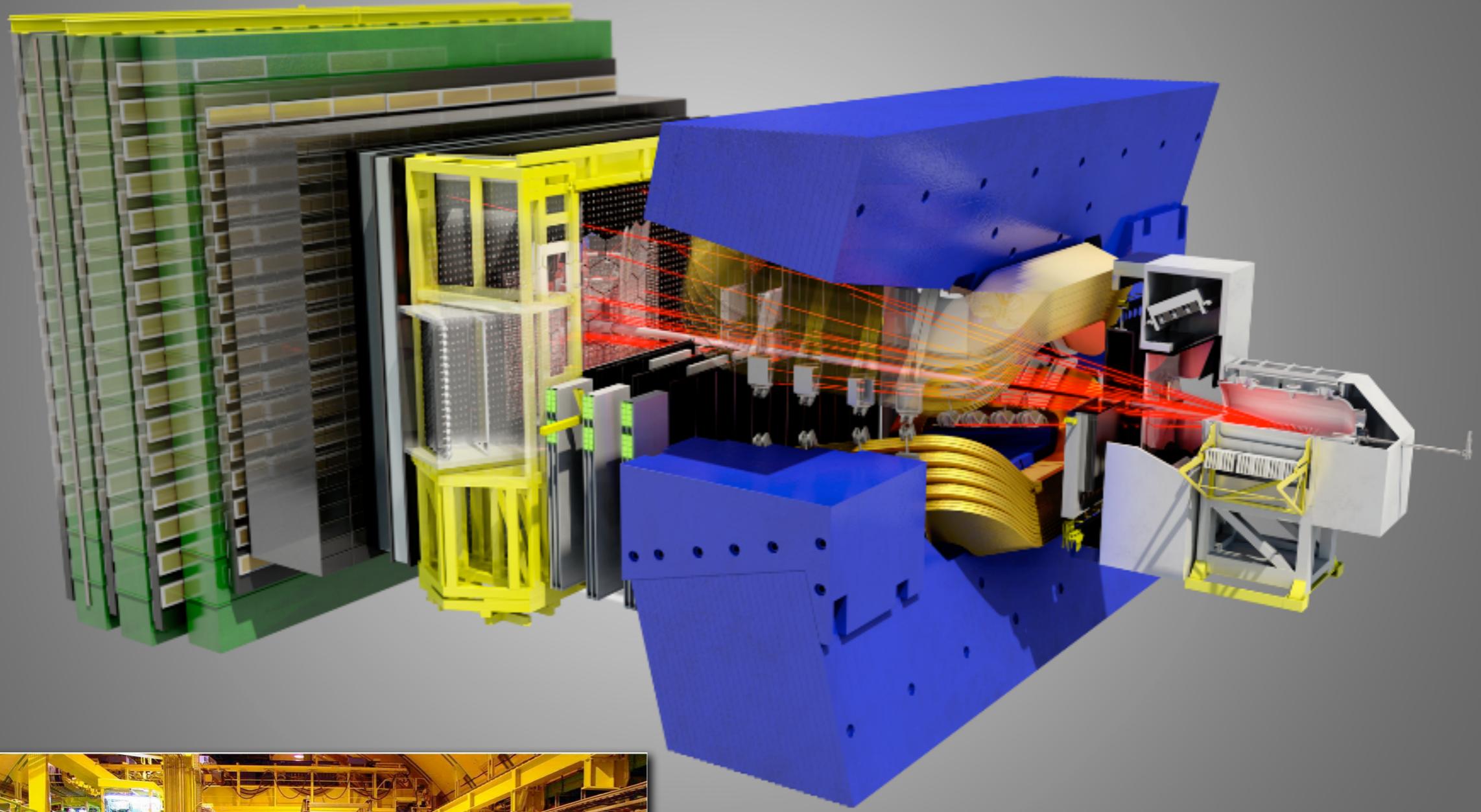
illustrative plots from flavour-tagged $D \rightarrow K_S \pi^+ \pi^-$ decays

$$x_{\pm} = r_B \cos(\delta_B \pm \gamma),$$

$$y_{\pm} = r_B \sin(\delta_B \pm \gamma)$$

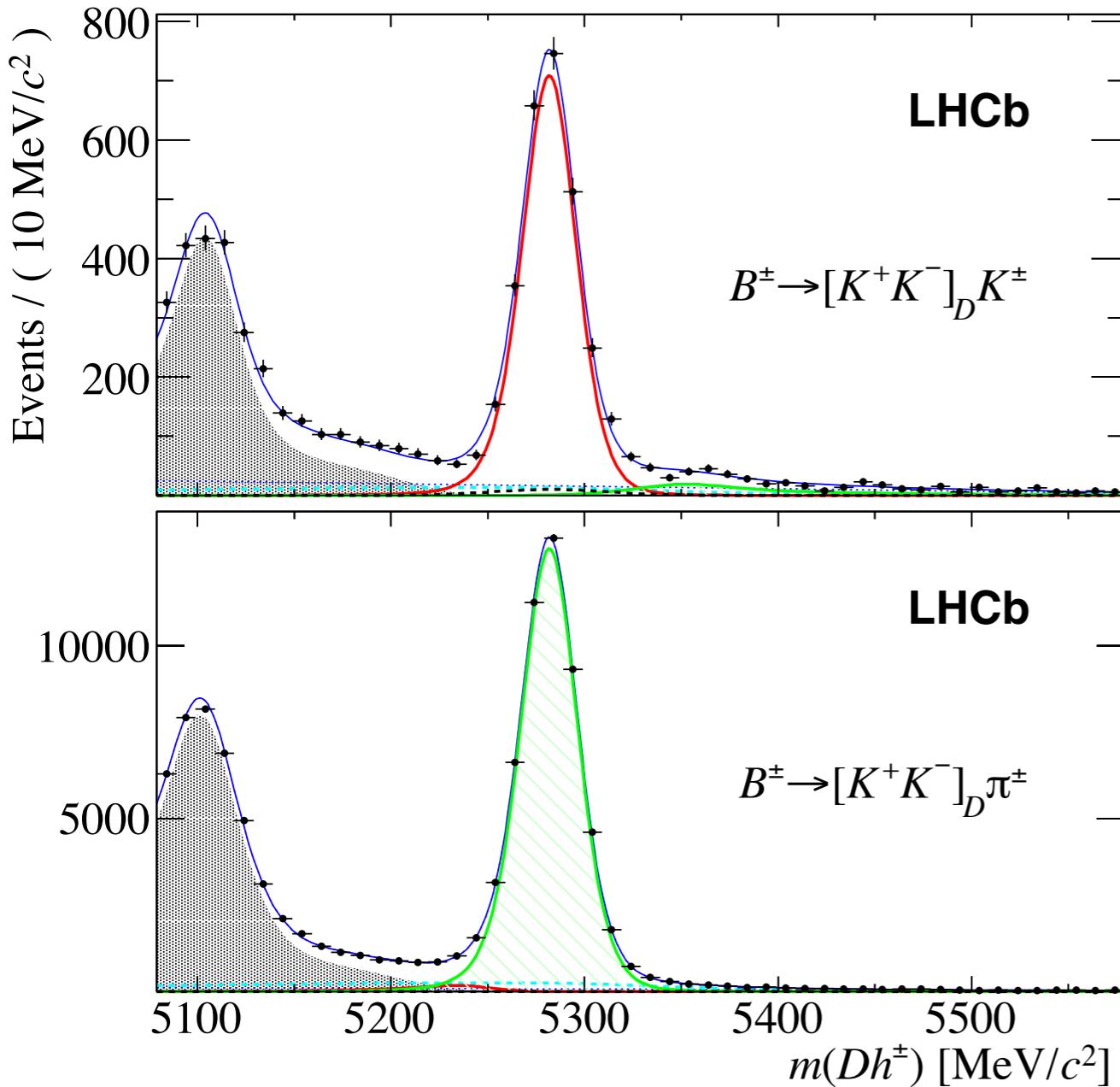
Common themes of $B \rightarrow DX$ analyses

- Use multivariate background rejection algorithms to minimise combinatorial background
- The LHCb RICH system is extensively exploited to minimise backgrounds from other D-decays and crucially, to separate the kinematically similar $B \rightarrow DK$ and $B \rightarrow D\pi$ decays
- The LHCb vertex detector is vital for topologically distinguishing the B and D decay vertex. This is vital to control charmless physics backgrounds
- K_S^0 are reconstructed either in decays inside the vertex detector or further downstream leading to a significant boost in event yield
- Signals are measured using fits to invariant mass spectra
 - The high-precision momentum measurement from the tracking system, plus the use of decay-tree refits leads to a invariant mass resolution of B candidates of around $15 \text{ MeV}/c^2$



GLW: $B^\pm \rightarrow Dh^\pm, D \rightarrow K^+K^-, \pi^+\pi^-$

3800 $B^\pm \rightarrow DK^\pm, D \rightarrow K^+K^-$



1160 $B^\pm \rightarrow DK^\pm, D \rightarrow \pi^+\pi^-$

