BINARY BLACK HOLE MERGER: THE THEORY

INTERFACING NUMERICAL AND ANALYTICAL RELATIVITY

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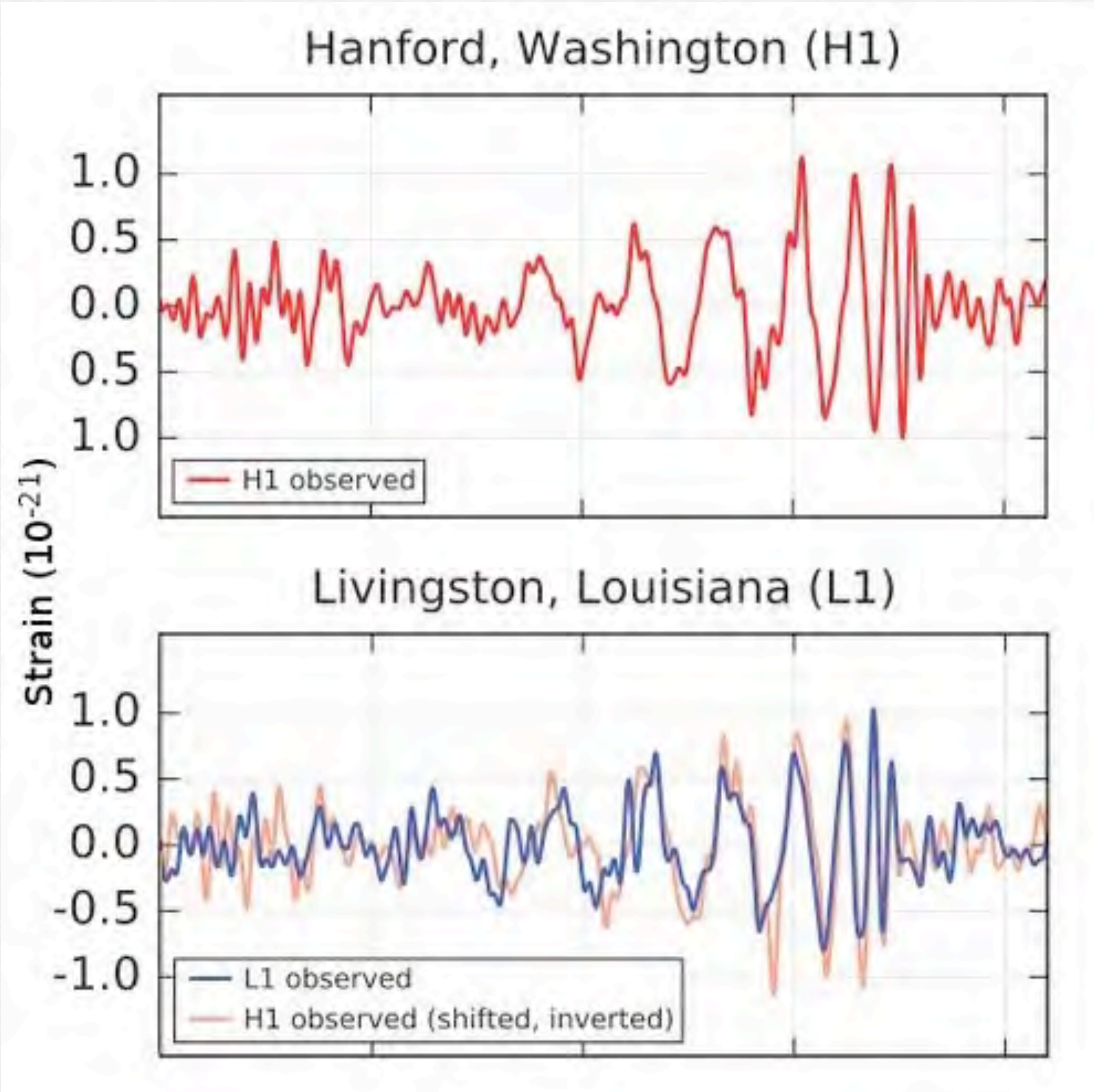
The IHES effective-one-body (EOB) code: eob.ihes.fr
T. Damour, AN, S. Bernuzzi, D. Bini...

GW150914



$$\text{strain} = \frac{\delta L}{L}$$

GW150914 parameters:

$$m_1 = 35.7 M_\odot$$
$$m_2 = 29.1 M_\odot$$
$$M_f = 61.8 M_\odot$$

$$a_1 \equiv S_1/(m_1^2) = 0.31^{+0.48}_{-0.28}$$
$$a_2 \equiv S_2/(m_2^2) = 0.46^{+0.48}_{-0.42}$$
$$a_f \equiv \frac{J_f}{M_f^2} = 0.67$$
$$q \equiv \frac{m_1}{m_2} = 1.27$$

Symmetric mass ratio

$$\nu \equiv \frac{m_1 m_2}{(m_1 + m_2)^2} = 0.2466$$

HOW TO MEASURE: MATCHED FILTERING!

To extract/do parameter estimation of the GW signal from detector's output (lost in broadband noise $S_n(f)$)

$$\langle output | h_{\rm template} \rangle = \int \frac{df}{S_n(f)} o(f) h^*_{\rm template}(f)$$

Detector's output

Template of expected GW signal

Need waveform templates!

THE THEORY...

Is needed to compute waveform templates for characterizing the source (GWs were detected...but WHAT was detected?)

Theory is needed to study the 2-body problem in General Relativity (dynamics & gravitational wave emission)

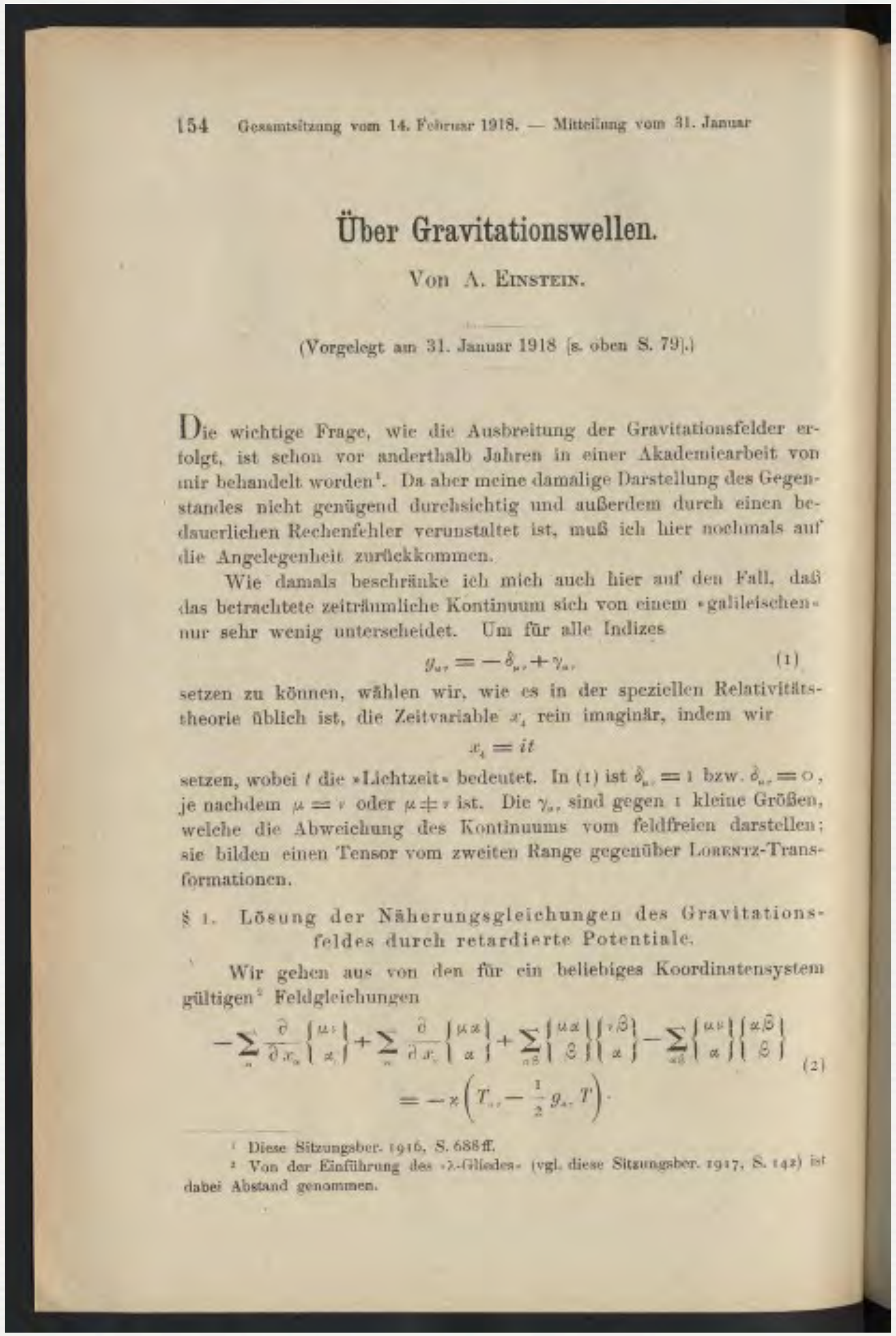
Theory: SINERGY between

Analytical and Numerical General Relativity

(AR/NR)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

UBER GRAVITATIONSWELLEN (EINSTEIN, 1918)



154 Gesamtsitzung vom 14. Februar 1918. — Mitteilung vom 31. Januar

Über Gravitationswellen.

Von A. Einstein.

(Vorgelegt am 31. Januar 1918 [s. oben S. 79].)

Die wichtige Frage, wie die Ausbreitung der Gravitationsfelder erfolgt, ist schon vor anderthalb Jahren in einer Akademiearbeit von mir behandelt worden[1]. Da aber meine damalige Darstellung des Gegenstandes nicht genügend durchsichtig und außerdem durch einen bedauerlichen Rechenfehler verunstaltet ist, muß ich hier nochmals auf die Angelegenheit zurückkommen.

Wie damals beschränke ich mich auch hier auf den Fall, daß das betrachtete zeiträumliche Kontinuum sich von einem »galileischen« nur sehr wenig unterscheidet. Um für alle Indizes

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \qquad (1)$$

setzen zu können, wählen wir, wie es in der speziellen Relativitätstheorie üblich ist, die Zeitvariable x_4 rein imaginär, indem wir

$$x_4 = it$$

setzen, wobei t die »Lichtzeit« bedeutet. In (1) ist $\delta_{\mu\nu} = 1$ bzw. $\delta_{\mu\nu} = 0$, je nachdem $\mu = \nu$ oder $\mu \neq \nu$ ist. Die $\gamma_{\mu\nu}$ sind gegen 1 kleine Größen, welche die Abweichung des Kontinuums vom feldfreien darstellen; sie bilden einen Tensor vom zweiten Range gegenüber Lorentz-Transformationen.

§ 1. Lösung der Näherungsgleichungen des Gravitationsfeldes durch retardierte Potentiale.

Wir gehen aus von den für ein beliebiges Koordinatensystem gültigen[2] Feldgleichungen

$$-\sum_{\alpha} \frac{\partial}{\partial x_\alpha} \begin{Bmatrix} \mu\nu \\ \alpha \end{Bmatrix} + \sum_{\alpha} \frac{\partial}{\partial x_\nu} \begin{Bmatrix} \mu\alpha \\ \alpha \end{Bmatrix} + \sum_{\alpha\beta} \begin{Bmatrix} \mu\alpha \\ \beta \end{Bmatrix} \begin{Bmatrix} \nu\beta \\ \alpha \end{Bmatrix} - \sum_{\alpha\beta} \begin{Bmatrix} \mu\nu \\ \alpha \end{Bmatrix} \begin{Bmatrix} \alpha\beta \\ \beta \end{Bmatrix} = -\varkappa \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right). \qquad (2)$$

[1] Diese Sitzungsber. 1916, S. 688 ff.

[2] Von der Einführung des »λ-Gliedes« (vgl. diese Sitzungsber. 1917, S. 142) ist dabei Abstand genommen.

$$g_{ij} = \delta_{ij} + h_{ij}$$

h_{ij} is transverse and traceless and propagates at the speed of light

BINARY SYSTEM: NEWTONIAN PRELIMINARIES

Newtonian binary systems in circular orbits: Kepler's law

$$GM = \Omega^2 R^3$$

$$\frac{v^2}{c^2} = \frac{GM}{c^2 R} = \left(\frac{GM\Omega}{c^3}\right)^{2/3}$$

$$M = m_1 + m_2$$

Einstein (1918) quadrupole formula: GW luminosity (energy flux)

$$P_{\rm gw} = \frac{dE_{\rm gw}}{dt} = \frac{32}{5}\frac{c^5}{G}\nu^2 x^5$$

$$x = \left(\frac{v}{c}\right)^2$$

$$\nu = \frac{\mu}{M} = \frac{m_1 m_2}{M^2}$$

GWS FROM COMPACT BINARIES: BASICS

$$E^{\rm orbital} = E^{\rm kinetic} + E^{\rm potential} = -\frac{1}{2}\frac{m_1 m_2}{R} = -\frac{1}{2}\mu x$$

Balance argument

$$\frac{dE^{\rm orbital}}{dt} = P_{GW} = \frac{dE_{\rm GW}}{dt}$$

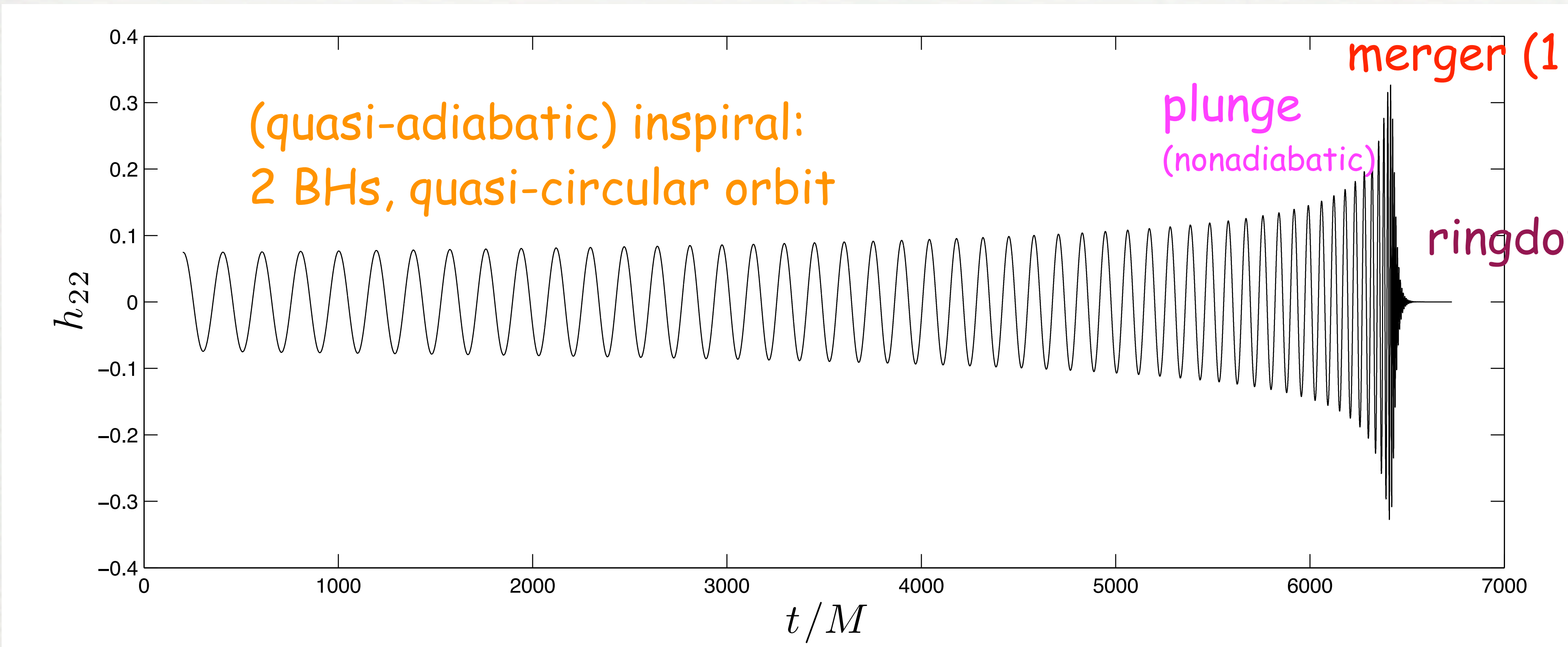
$$\omega_{22}^{\rm GW} = 2\pi f_{22}^{\rm GW} = 2\Omega^{\rm orbital}$$

$$f_{GW}^{22} = \frac{1}{\pi}\left(\frac{5}{256\nu}\right)^{3/8}\left(\frac{1}{t - t_{\rm coalescence}}\right)^{3/8}$$

MONOTONICALLY GROWING FREQUENCY: cHIRP!

BBHS: WAVEFORM OVERWIEV

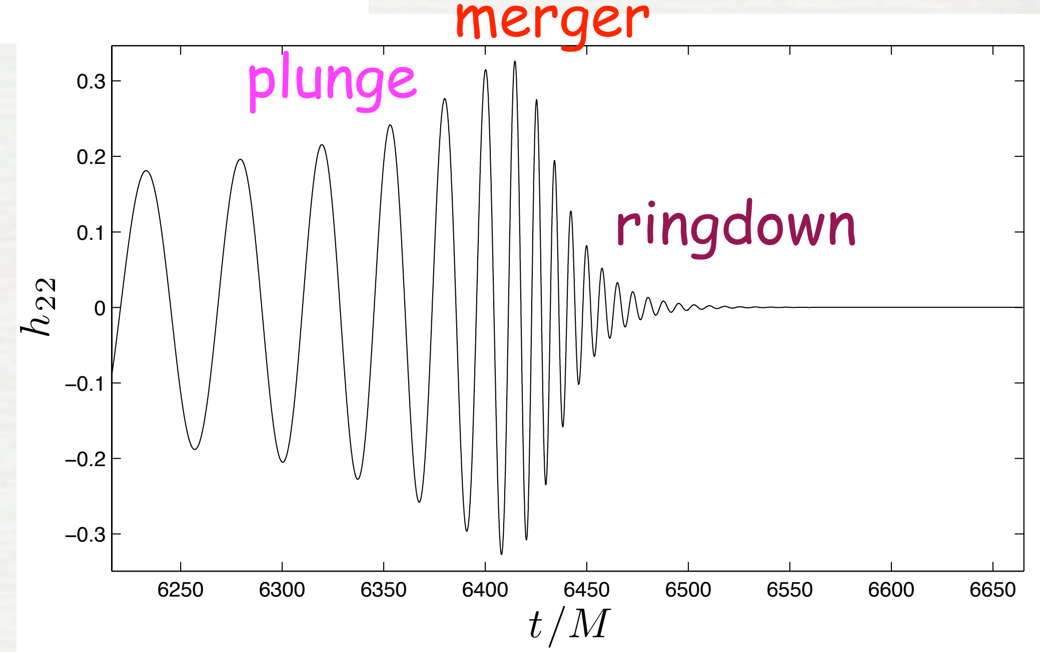
$$h_+ - ih_\times = \frac{1}{r}\sum_{\ell m} h_{\ell m}\;{}_{-2}Y_{\ell m}(\theta,\phi) \qquad h\left(m_1, m_2, \vec{S}_1, \vec{S}_2\right)$$



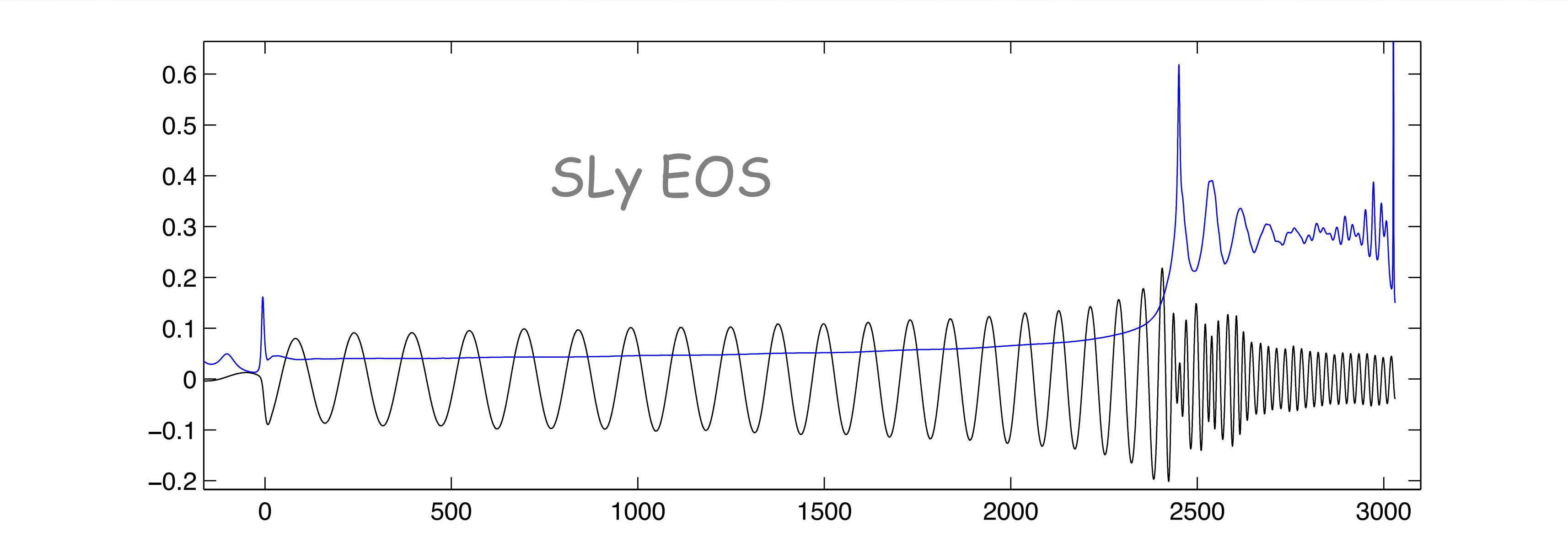
e.g: equal-mass BBH, aligned-spins

$$\chi_1 = \chi_2 = +0.98$$

- SXS (Simulating eXtreme Spacetimes) collaboration
- www.blackholes.org
- Free catalog of waveforms (downloadable)

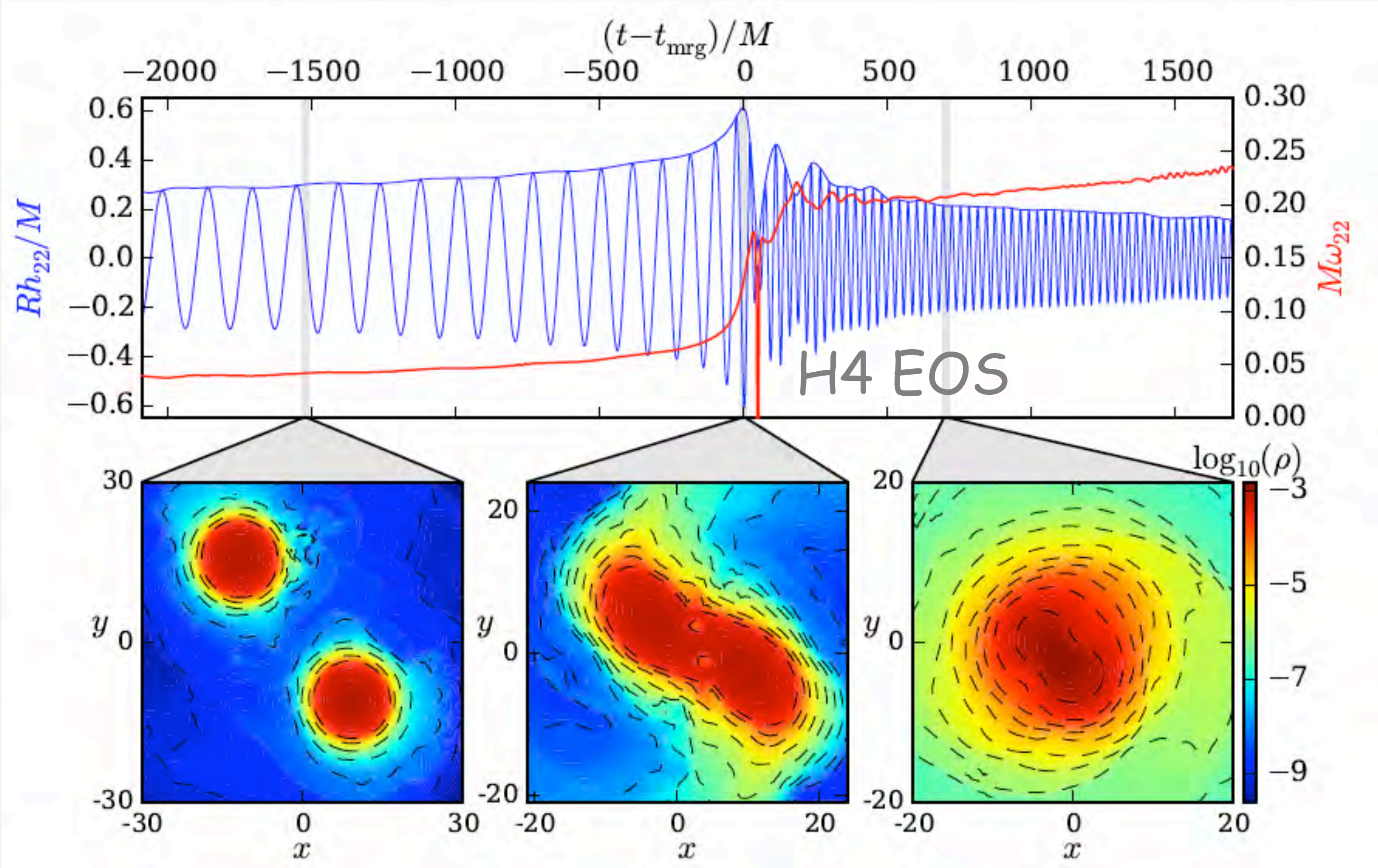


BINARY NEUTRON STARS (BNS)?



All BNS need is Love!

$$q = 1 \qquad M = 2.7M_{\odot}$$



- Tidal effects
- Love numbers (tidal "polarization" constants)
- EOS dependence & "universality"

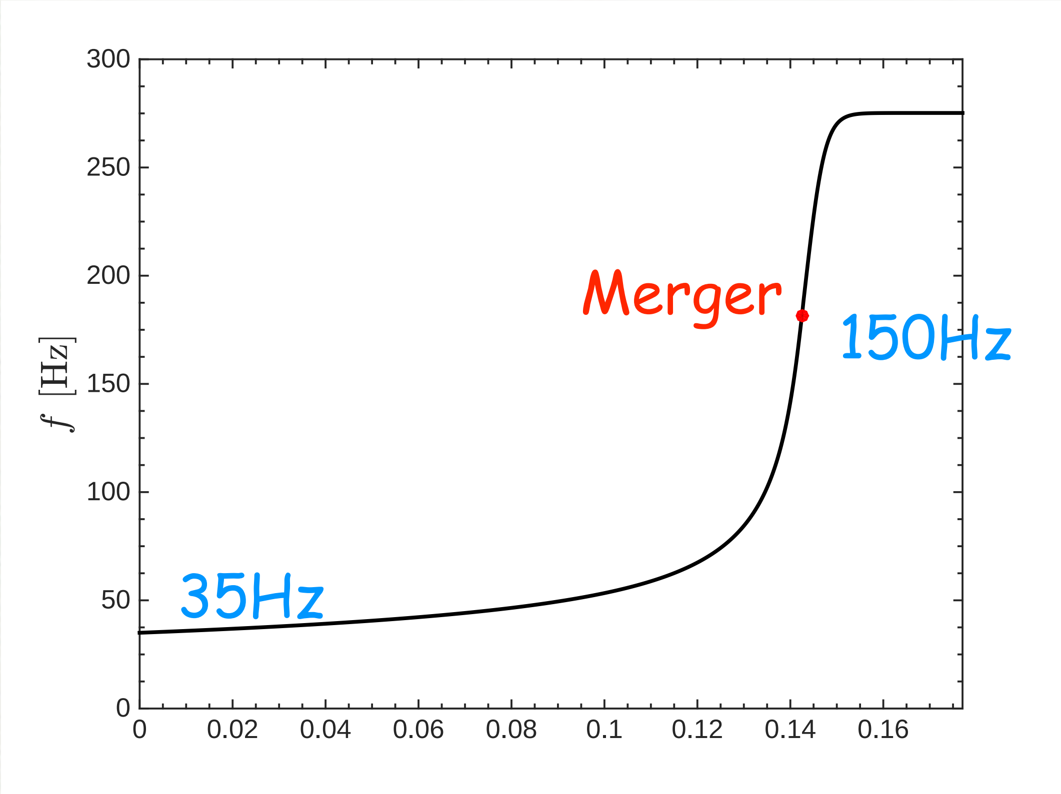
See:

Damour&Nagar, PRD 2009
Damour&Nagar, PRD 2010
Damour,Nagar et al., PRL 2011
Bini,Damour&Faye, PRD2012
Bini&Damour, PRD 2014
Bernuzzi, Nagar, et al, PRL 2014
Bernuzzi, Nagar, Dietrich, PRL 2015
Bernuzzi, Nagar, Dietrich & Damour,PRL, 2015

FAST CHIRP: COULD GW150914 BE A BNS?

The merger occurs at frequencies too low to be a "standard" BNS

GW frequency grows fro 35Hz to 150Hz around peak (factor 4) over the observed 8GWs cycles



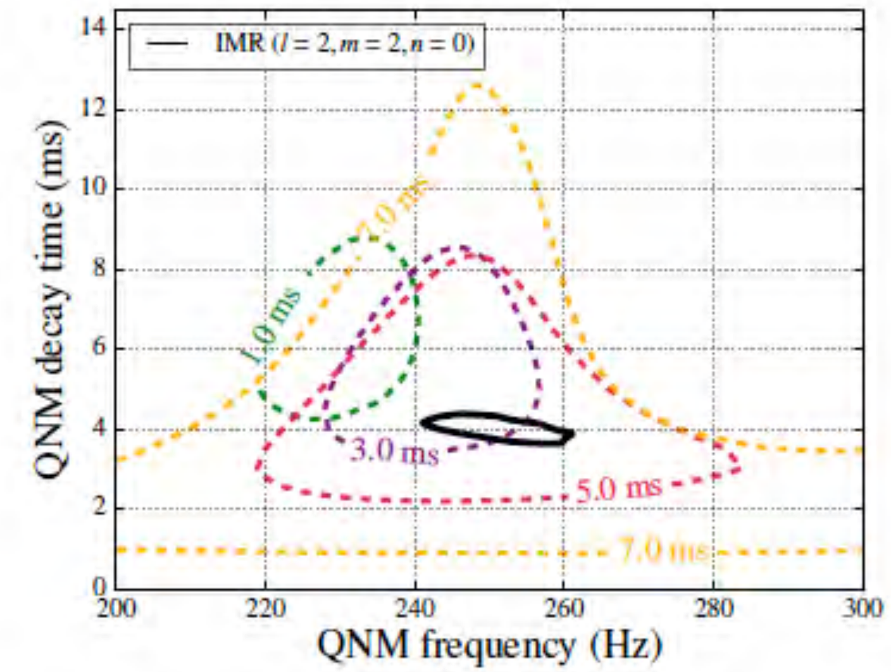


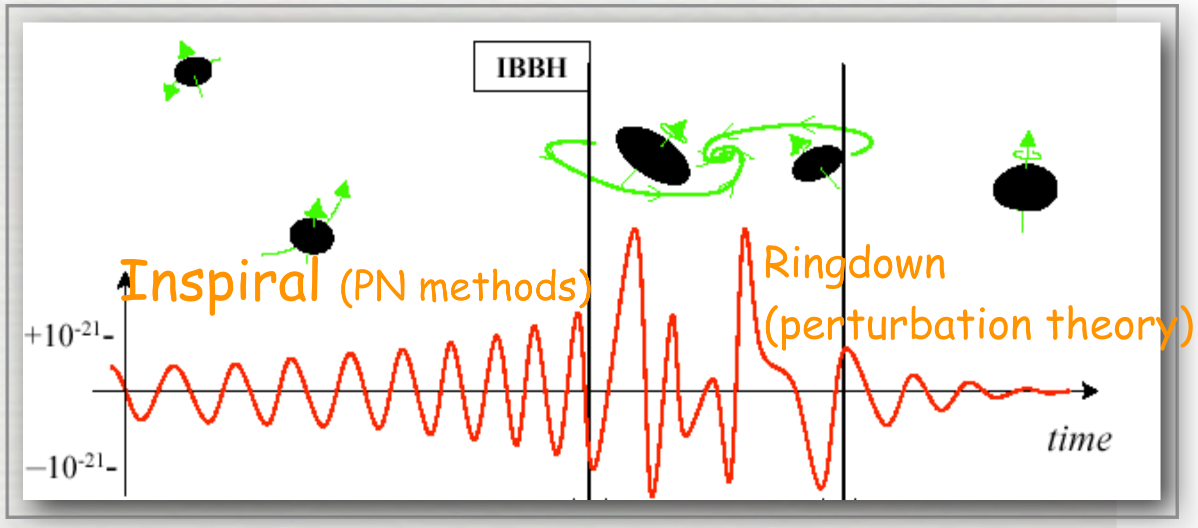
FIG. 4. We show the posterior 90% confidence regions from Bayesian parameter estimation for a damped-sinusoid model, assuming different start-times $t_0 = t_M + 1, 3, 5, 7$ ms, labeled by offset from the merger time t_M of the most-probable waveform from GW150914. The black solid line shows contours of 90% confidence region for the frequency f_0 and decay time τ of the $\ell = 2$, $m = 2$ and $n = 0$ (i.e., the least damped) QNM obtained from the inspiral-merger-ringdown waveform for the entire detector's bandwidth.

But the final answer is that consistency was found between inspiral and ringdown!

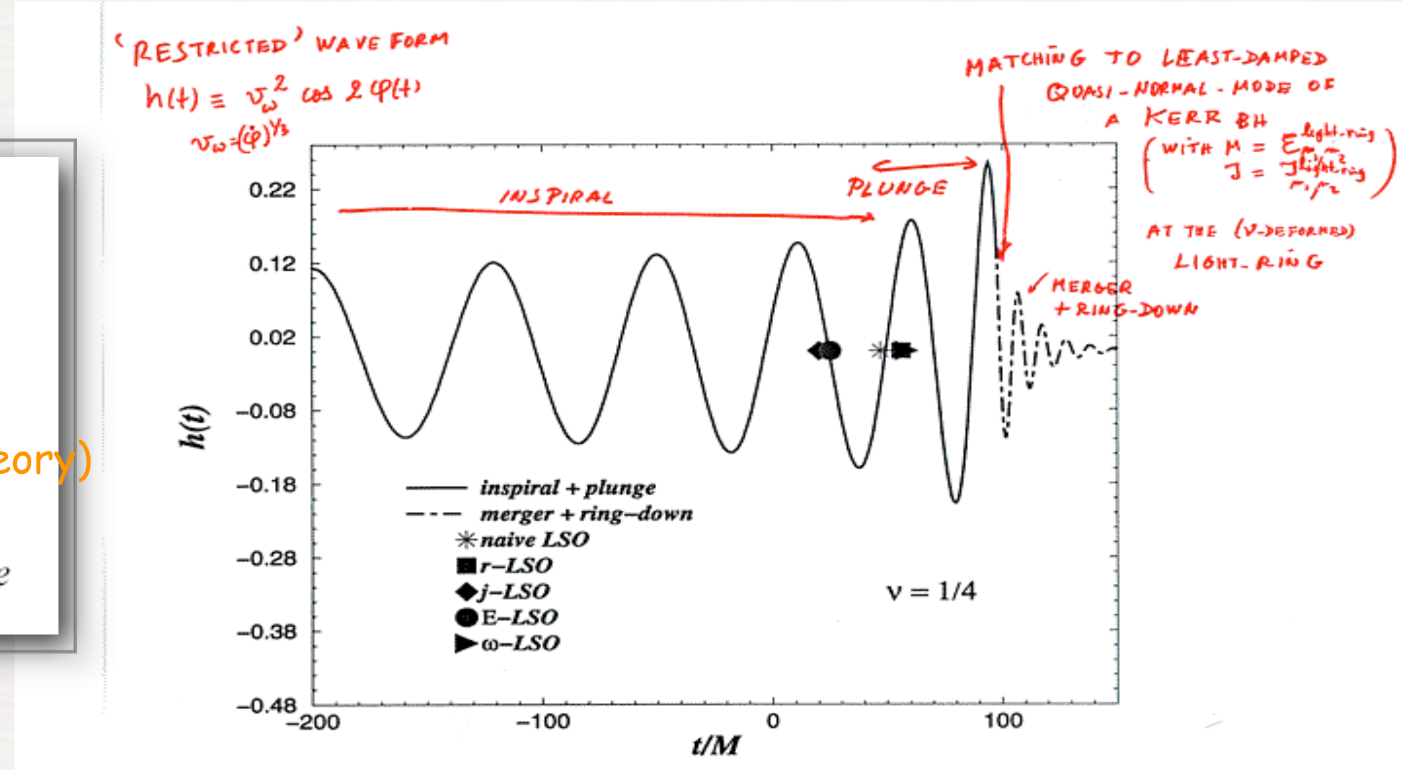
Still, one would like to put experimental limits on Love numbers (NOT DONE!)

TEMPLATES FOR GWS FROM BBH COALESCENCE

Brady, Craighton & Thorne, 1998



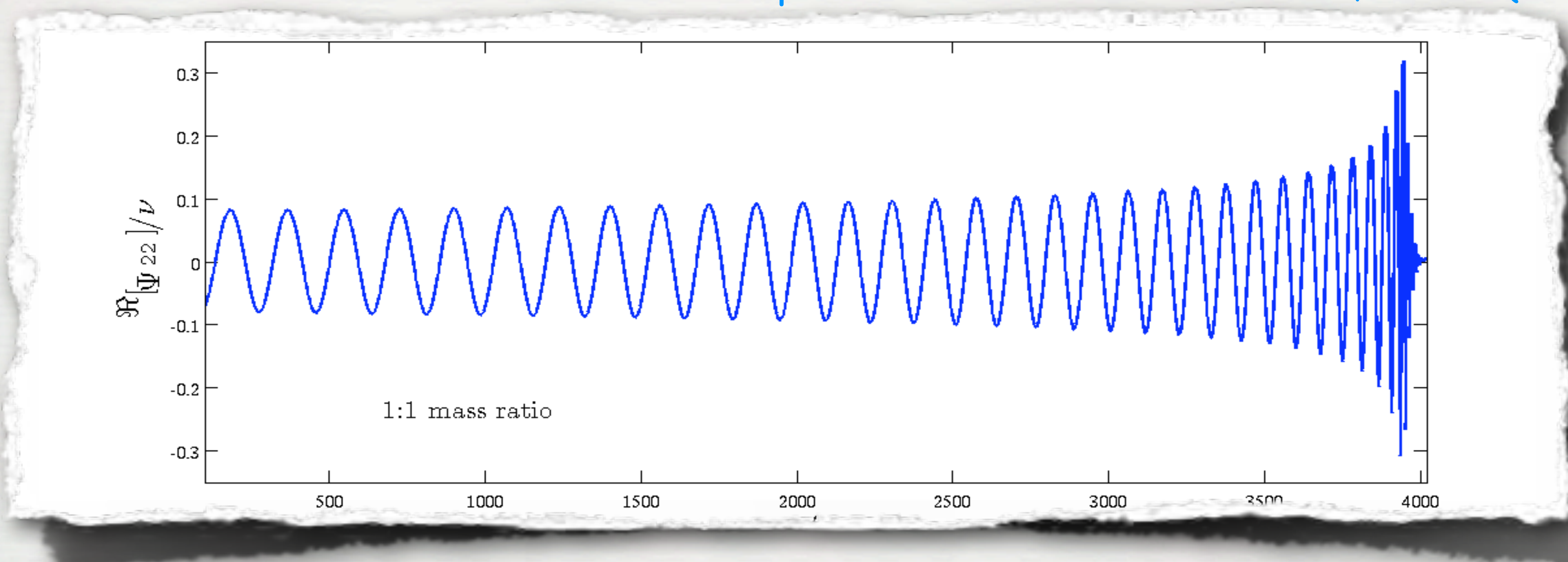
Merger:
Numerical Relativity



Effective-One-Body (Buonanno & Damour (2000))

Numerical Relativity: >= 2005 (F. Pretorius, Campanelli et al., Baker et al.)

Most accurate data: Caltech-Cornell spectral code: M. Scheel et al., 2008 (SXS collaboration)



Spectral code

Extrapolation (radius & resolution)

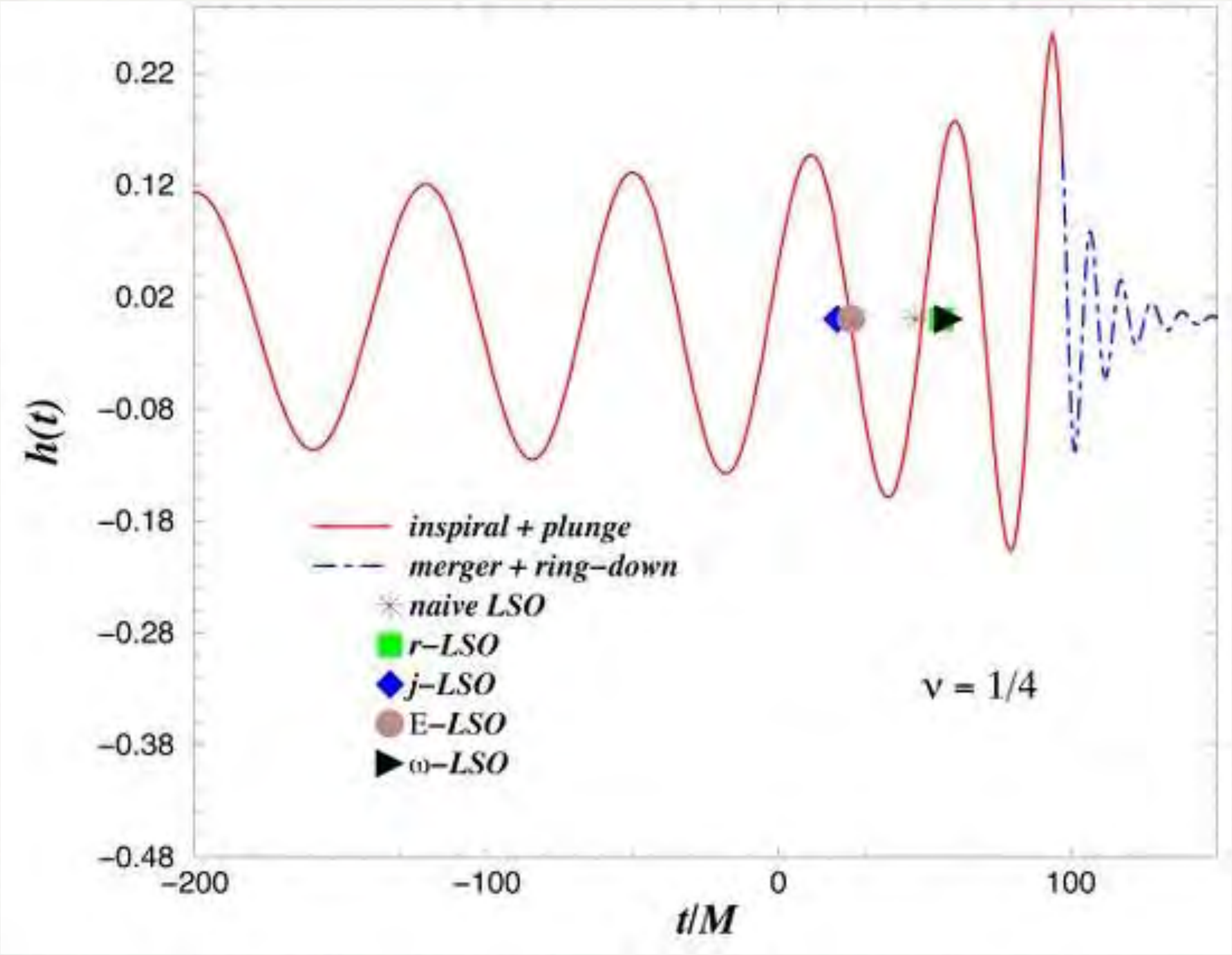
Phase error:

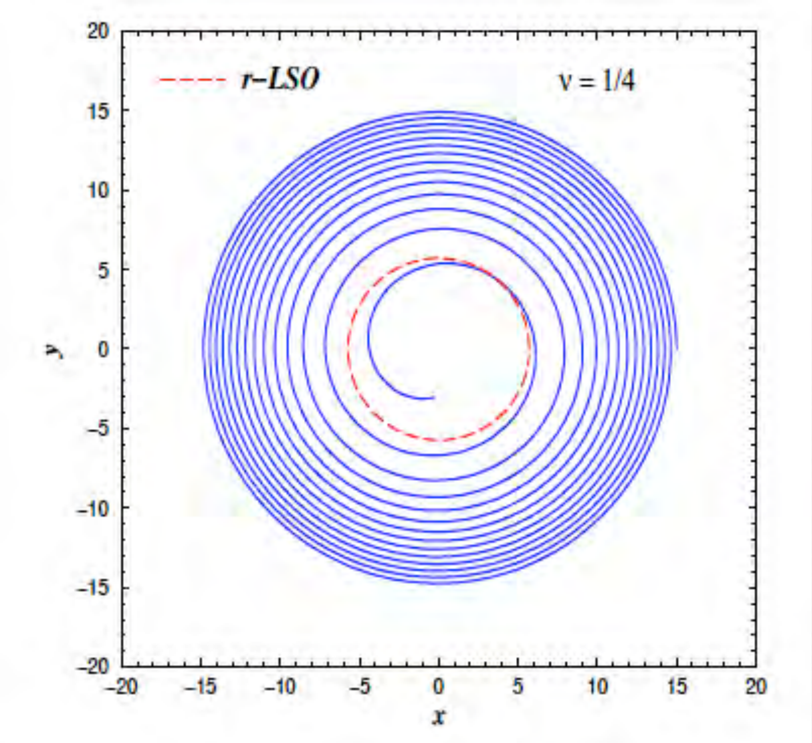
< 0.02 rad (inspiral)

<0.1 rad (ringdown)

EFFECTIVE ONE BODY (EOB): 2000

Numerical Relativity was not working (yet...)
EOB formalism was predictive, qualitatively and semi-quantitatively correct (10%)





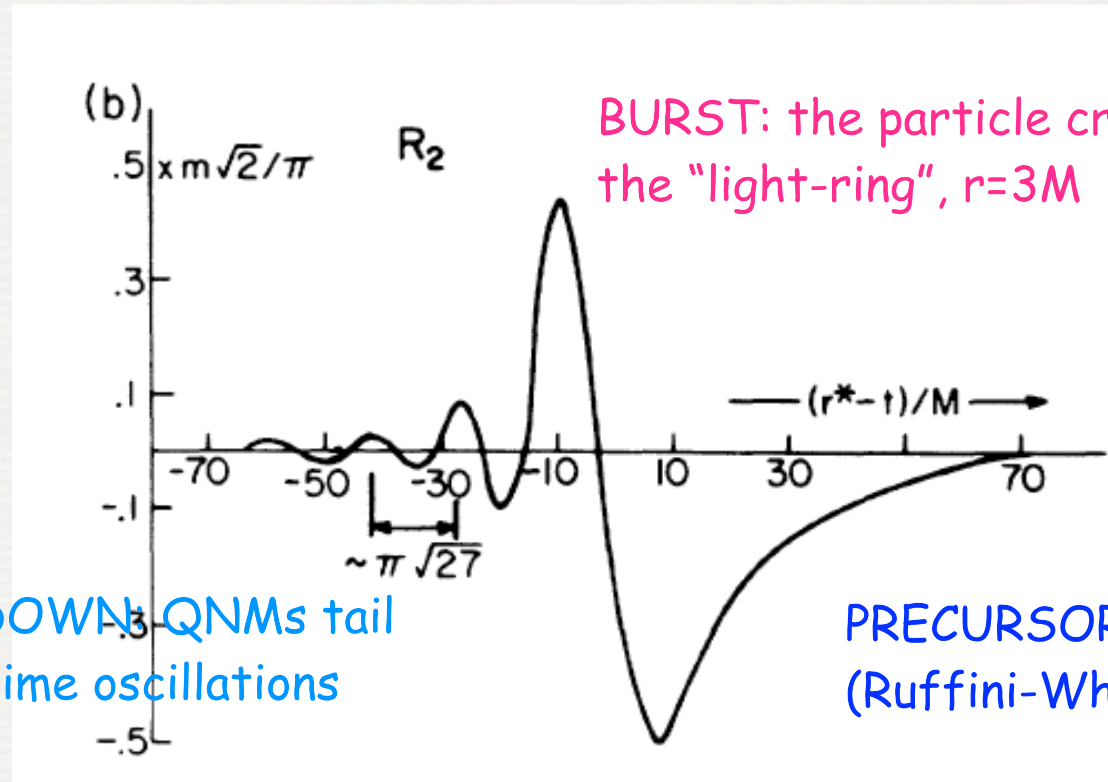
- Blurred transition from inspiral to plunge
- Final black-hole mass
- Final black hole spin
- Complete waveform

A. Buonanno & T. Damour, PRD 59 (1999) 084006
A. Buonanno & T. Damour, PRD 62 (2000) 064015

$$\nu = \frac{m_1 m_2}{(m_1 + m_2)^2} = \frac{\mu}{M}$$

PRECURSOR-BURST-RINGDOWN STRUCTURE :1972

Davis, Ruffini & Tiomno: radial plunge of a test-particle onto a Schwarzschild black hole (Regge-Wheeler-Zerilli BH perturbation theory)



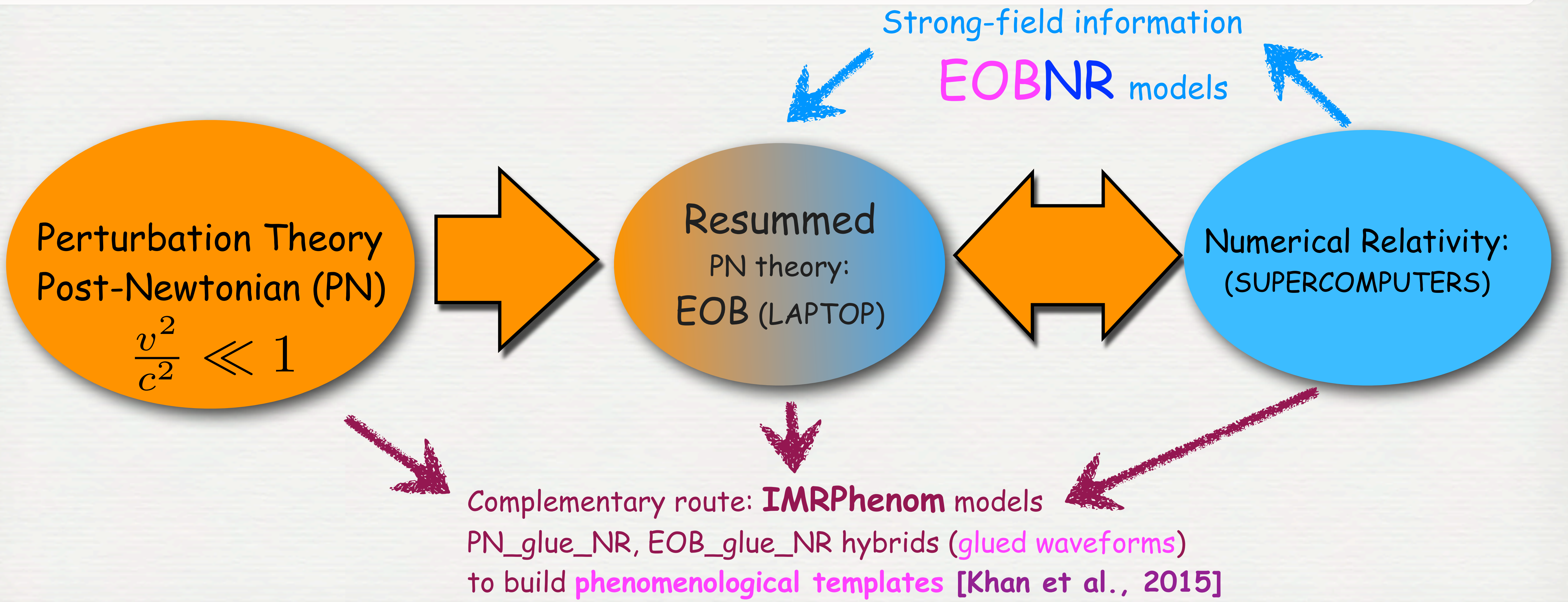
BURST: the particle crosses the “light-ring”, r=3M

RINGDOWN: QNMs tail
Spacetime oscillations

PRECURSOR: Quadrupole formula
(Ruffini-Wheeler approximation)

IMPORTANCE OF AN ANALYTICAL FORMALISM

- **Theoretical**: physical understanding of the coalescence process, especially in complicated situations (e.g., precessing spins).
- **Practical**: need many thousands of accurate GWs templates for detection and data analysis. Need analytical templates: $h\left(m_1, m_2, \vec{S}_1, \vec{S}_2\right)$
- **Solution**: synergy between analytical & numerical relativity



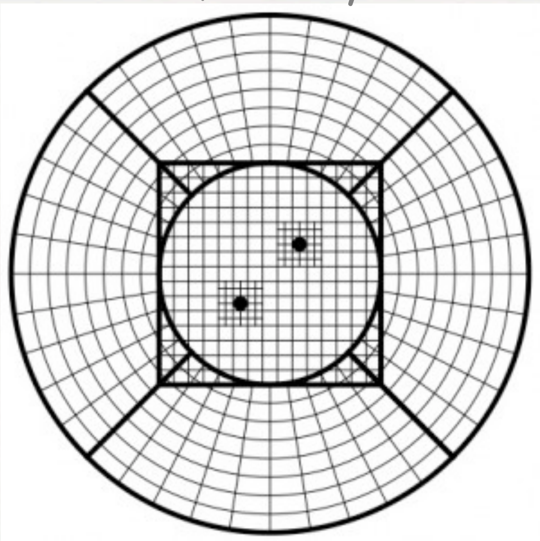
BBH & BNS COALESCENCE: NUMERICAL RELATIVITY

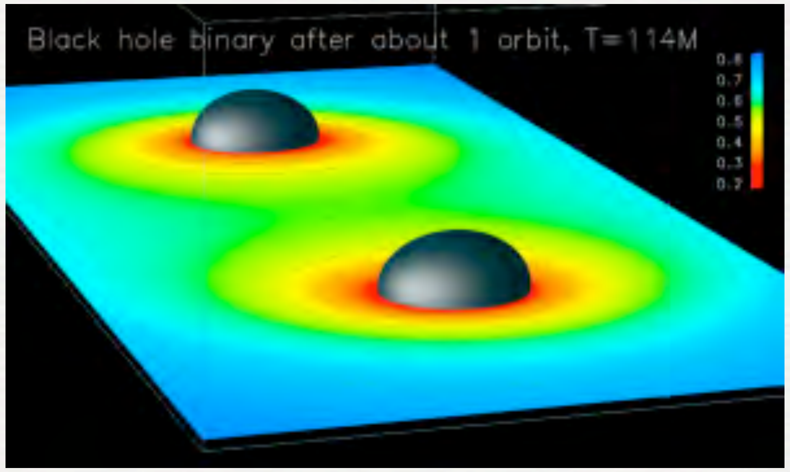
Numerical relativity is complicated & computationally expensive:

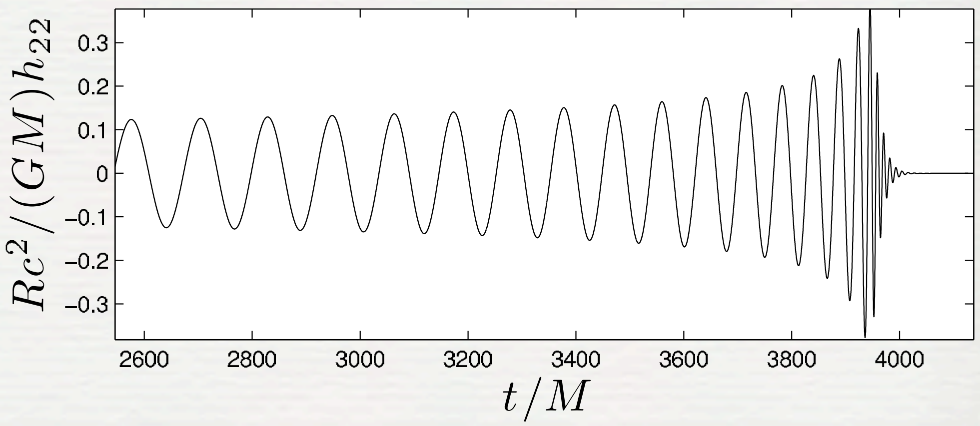
- Formulation of Einstein equations (BSSN, harmonic, Z4c,...)
- Setting up initial data (solution of the constraints)
- Gauge choice
- Numerical approach (finite-differencing (FD, e.g. Llama) vs spectral (SpEC,SXS))
- High-order FD operators
- Treatment of BH singularity (excision vs punctures)
- Wave extraction problem on finite-size grids (Cauchy-Characteristic vs extrapolation)
- Huge computational resources (mass-ratios 1:10; spin)
- Adaptive-mesh-refinement
- Error budget (convergence rates are far from clean...)
- For BNS: further complications due to GR-Hydrodynamics for matter
- Months of running/analysis to get one accurate waveform....

Multi-patch grid structure

(Llama FD code, Pollney & Reisswig)







A catalog of 171 high-quality binary black-hole simulations for gravitational-wave astronomy [PRL 111 (2013) 241104]

Abdul H. Mroué,[1] Mark A. Scheel,[2] Béla Szilágyi,[2] Harald P. Pfeiffer,[1] Michael Boyle,[3] Daniel A. Hemberger,[3] Lawrence E. Kidder,[3] Geoffrey Lovelace,[4,2] Sergei Ossokine,[1,5] Nicholas W. Taylor,[2] Anıl Zenginoğlu,[2] Luisa T. Buchman,[2] Tony Chu,[1] Evan Foley,[4] Matthew Giesler,[4] Robert Owen,[6] and Saul A. Teukolsky[3]

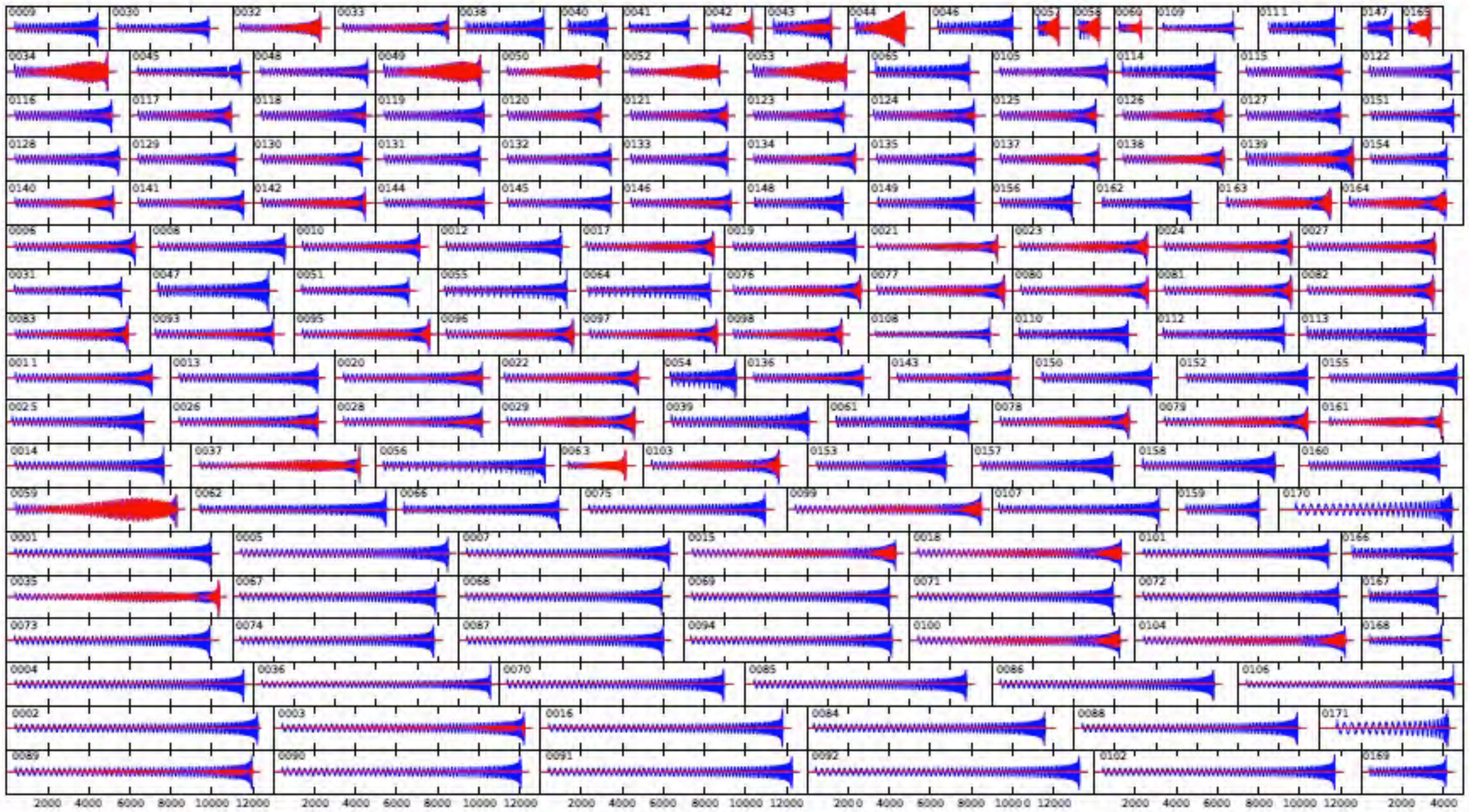


FIG. 3: Waveforms from all simulations in the catalog. Shown here are h_+ (blue) and $h_\times$ (red) in a sky direction parallel to the initial orbital plane of each simulation. All plots have the same horizontal scale, with each tick representing a time interval of $2000M$, where M is the total mass.

- www.blackholes.org

But (at least) 250.000 templates were used...

ANALYTICALLY: MOTION AND GW IN GR

Hamiltonian: conservative part of the dynamics

Radiation reaction: mechanical energy/angular momentum goes away in GWs and backreacts on the system.

The (closed) orbit CIRCULARIZES and SHRiNks with time

Waveform

General Relativity is NONLINEAR!

Post-Newtonian (PN) approximation: expansion in $\frac{v^2}{c^2}$

PROBLEM OF MOTION IN GENERAL RELATIVITY

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x) \ , \ h_{\mu\nu} \ll 1$$

$$h_{00} \sim h_{ij} \sim \frac{v^2}{c^2} \ , \ h_{0i} \sim \frac{v^3}{c^3} \ , \ \partial_0 h \sim \frac{v}{c}\partial_i h$$

Approximation methods

- post-Minkowskian (Einstein 1916)
- post-Newtonian (Droste 1916)
- Matching of asymptotic expansions: body zone/near zone/wave zone
- Numerical Relativity

One-chart versus Multi-chart approaches

Coupling between Einstein field equations and equations of motion

Strongly self-gravitating bodies: neutron stars or black holes

$$h_{\mu\nu}(x) \sim 1$$

Skeletonized: $T_{\mu\nu}$ point-masses ? delta-functions in GR

Multipolar Expansion

Need to go to very high-orders of approximation

QFT-like calculations

Use a "cocktail": PM, PN, MPM, MAE, EFT,an. reg., dim. reg.,...

POST-NEWTONIAN HAMILTONIAN (C.O.M)

$$\hat{H}^{\rm NR}_{\rm real}(\mathbf{q},\mathbf{p}) = \hat{H}_{\rm N}(\mathbf{q},\mathbf{p}) + \hat{H}_{\rm 1PN}(\mathbf{q},\mathbf{p}) + \hat{H}_{\rm 2PN}(\mathbf{q},\mathbf{p}) + \hat{H}_{\rm 3PN}(\mathbf{q},\mathbf{p}), \tag{4.27}$$

where

$$\hat{H}_{\rm N}(\mathbf{q},\mathbf{p}) = \frac{\mathbf{p}^2}{2} - \frac{1}{q}, \tag{4.28a}$$

Newton (0PN)

$$\hat{H}_{\rm 1PN}(\mathbf{q},\mathbf{p}) = \frac{1}{8}(3\nu-1)(\mathbf{p}^2)^2 - \frac{1}{2}\left[(3+\nu)\mathbf{p}^2 + \nu(\mathbf{n}\cdot\mathbf{p})^2\right]\frac{1}{q} + \frac{1}{2q^2}, \tag{4.28b}$$

(1PN, 1938) - [Einstein-Infeld-Hoffman]

$$\hat{H}_{\rm 2PN}(\mathbf{q},\mathbf{p}) = \frac{1}{16}\left(1-5\nu+5\nu^2\right)(\mathbf{p}^2)^3 + \frac{1}{8}\left[\left(5-20\nu-3\nu^2\right)(\mathbf{p}^2)^2 - 2\nu^2(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 - 3\nu^2(\mathbf{n}\cdot\mathbf{p})^4\right]\frac{1}{q} + \frac{1}{2}\left[(5+8\nu)\mathbf{p}^2 + 3\nu(\mathbf{n}\cdot\mathbf{p})^2\right]\frac{1}{q^2} - \frac{1}{4}(1+3\nu)\frac{1}{q^3}, \tag{4.28c}$$

(2PN, 1982/83) - [Damour-Deruelle]

$$\begin{aligned}\hat{H}_{\rm 3PN}(\mathbf{q},\mathbf{p}) &= \frac{1}{128}\left(-5+35\nu-70\nu^2+35\nu^3\right)(\mathbf{p}^2)^4 \\ &+ \frac{1}{16}\left[\left(-7+42\nu-53\nu^2-5\nu^3\right)(\mathbf{p}^2)^3 + (2-3\nu)\nu^2(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2 + 3(1-\nu)\nu^2(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2 - 5\nu^3(\mathbf{n}\cdot\mathbf{p})^6\right]\frac{1}{q} \\ &+ \left[\frac{1}{16}\left(-27+136\nu+109\nu^2\right)(\mathbf{p}^2)^2 + \frac{1}{16}(17+30\nu)\nu(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 + \frac{1}{12}(5+43\nu)\nu(\mathbf{n}\cdot\mathbf{p})^4\right]\frac{1}{q^2} \\ &+ \left\{\left[-\frac{25}{8} + \left(\frac{1}{64}\pi^2 - \frac{335}{48}\right)\nu - \frac{23}{8}\nu^2\right]\mathbf{p}^2 + \left(-\frac{85}{16} - \frac{3}{64}\pi^2 - \frac{7}{4}\nu\right)\nu(\mathbf{n}\cdot\mathbf{p})^2\right\}\frac{1}{q^3} \\ &+ \left[\frac{1}{8} + \left(\frac{109}{12} - \frac{21}{32}\pi^2 + \omega_{\rm static}\right)\nu\right]\frac{1}{q^4}.\end{aligned} \tag{4.28d}$$

(3PN, 2000) - [Damour, Jaranowski,Schaefer]

...and 4PN too, [Damour, Jaranowski&Schaefer 2014/2015] - 4 loop calculation

$$\mathbf{q} = \mathbf{q}_1 - \mathbf{q}_2$$

$$\mathbf{p} = \mathbf{p}_1 = -\mathbf{p}_2$$

PN-EXPANDED (CIRCULAR) ENERGY FLUX (3.5PN)

$$\frac{dE}{dt} = -\mathcal{L}$$

balance equation

Mechanical loss GW luminosity

$$\mathcal{L} = \frac{32c^5}{5G}\nu^2 x^5 \Big\{ 1 + \left(-\frac{1247}{336} - \frac{35}{12}\nu\right) x + 4\pi x^{3/2} + \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right) x^2$$
$$+ \left(-\frac{8191}{672} - \frac{583}{24}\nu\right)\pi x^{5/2}$$
$$+ \left[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}C - \frac{856}{105}\ln(16\,x)\right.$$
$$\left. + \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2\right)\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3\right] x^3$$
$$+ \left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2\right)\pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right)\Big\}.$$

Newtonian quadrupole formula

$$C = \gamma_E = 0.5772156649...$$

TAYLOR-EXPANDED (CIRCULAR) 3PN WAVEFORM

Blanchet, Iyer&Joguet, 02; Blanchet, Damour, Iyer&Esposito-Farese, 04; Kidder07; Blanchet et al.,08

$$
\begin{aligned}
h^{22} = {} & -8\sqrt{\frac{\pi}{5}}\frac{G\nu m}{c^2 R}e^{-2i\phi}x\Bigg\{1 - x\left(\frac{107}{42}-\frac{55}{42}\nu\right) + x^{3/2}\left[2\pi + 6i\ln\left(\frac{x}{x_0}\right)\right] - x^2\left(\frac{2173}{1512}+\frac{1069}{216}\nu-\frac{2047}{1512}\nu^2\right) \\
& - x^{5/2}\left[\left(\frac{107}{21}-\frac{34}{21}\nu\right)\pi + 24i\nu + \left(\frac{107i}{7}-\frac{34i}{7}\nu\right)\ln\left(\frac{x}{x_0}\right)\right] \\
& + x^3\Bigg[\frac{27\,027\,409}{646\,800} - \frac{856}{105}\gamma_E + \frac{2}{3}\pi^2 - \frac{1712}{105}\ln 2 - \frac{428}{105}\ln x \\
& - 18\left[\ln\left(\frac{x}{x_0}\right)\right]^2 - \left(\frac{278\,185}{33\,264}-\frac{41}{96}\pi^2\right)\nu - \frac{20\,261}{2772}\nu^2 + \frac{114\,635}{99\,792}\nu^3 + \frac{428i}{105}\pi + 12i\pi\ln\left(\frac{x}{x_0}\right)\Bigg] + O(\epsilon^{7/2})\Bigg\},
\end{aligned}
$$

$$x = (M\Omega)^{2/3} \sim v^2/c^2$$

$$M = m_1 + m_2$$

$$\nu = \frac{m_1 m_2}{M^2}$$

EFFECTIVE-ONE-BODY (EOB)

approach to the general relativistic two-body problem

(Buonanno-Damour 99, 00, Damour-Jaranowski-Schäfer 00, Damour 01, Damour-Nagar 07, Damour-Iyer-Nagar 08)

key ideas:

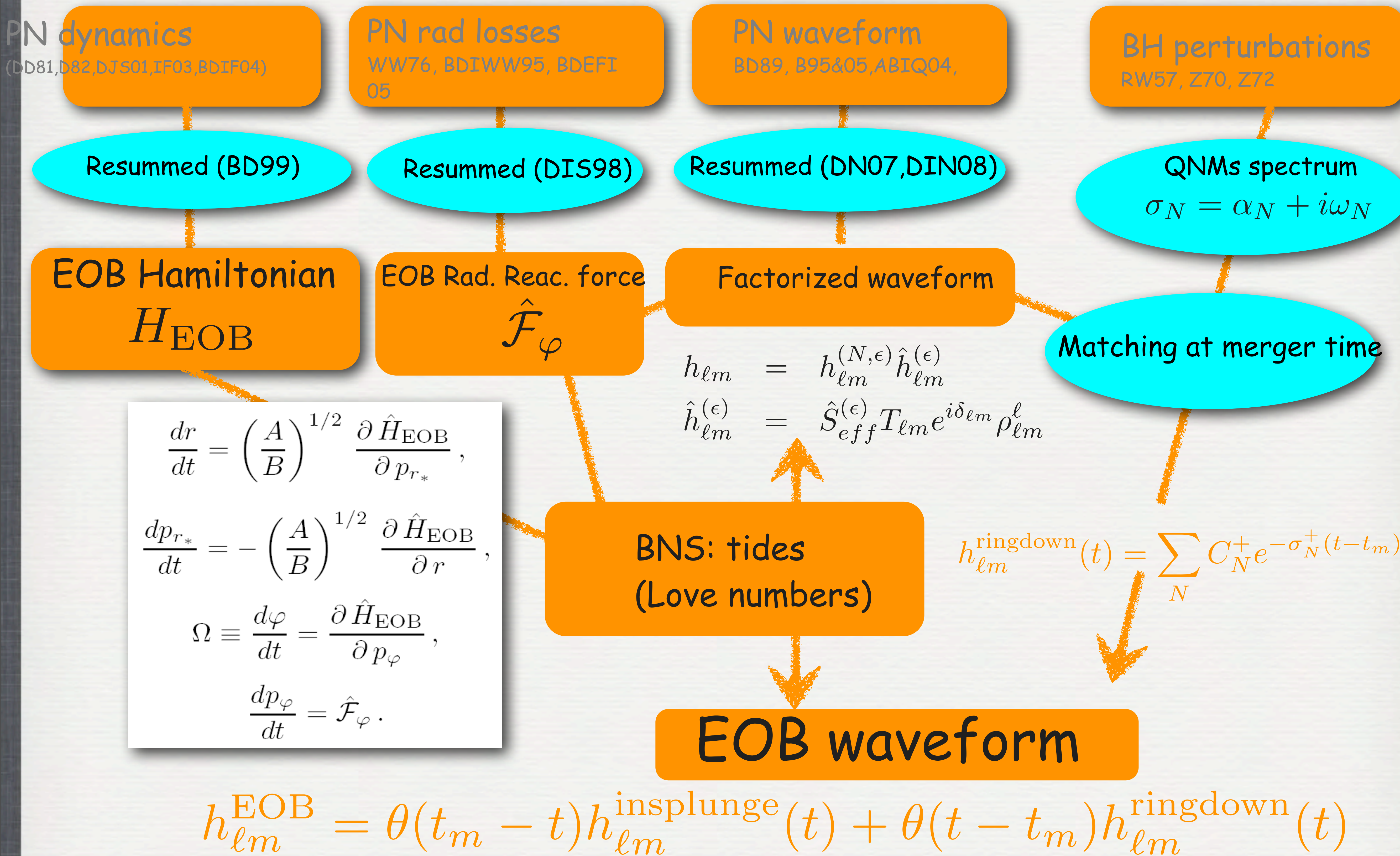
(1) Replace two-body dynamics (m_1, m_2) by dynamics of a particle $(\mu \equiv m_1\, m_2/(m_1 + m_2))$ in an effective metric $g^{\rm eff}_{\mu\nu}(u)$, with

$$u \equiv GM/c^2 R\,, \quad M \equiv m_1 + m_2$$

(2) Systematically use RESUMMATION of PN expressions (both $g^{\rm eff}_{\mu\nu}$ and $\mathcal{F}_{RR}$) based on various physical requirements

(3) Require continuous deformation w.r.t. $\nu \equiv \mu/M \equiv m_1\, m_2/(m_1 + m_2)^2$ in the interval $0 \leq \nu \leq \frac{1}{4}$

STRUCTURE OF THE EOB FORMALISM



EXPLICIT FORM OF THE EOB HAMILTONIAN

EOB Hamiltonian

$$H_{\rm EOB} = M\sqrt{1+2\nu\left(\hat{H}_{\rm eff}-1\right)}$$

All Functions are a ν-dependent deformation of the Schwarzschild ones

$$A(r) = 1 - 2u + 2\nu u^3 + a_4 \nu u^4$$

$$a_4 = \frac{94}{3} - \frac{41}{32}\pi^2 \simeq 18.6879027$$

$$A(r)B(r) = 1 - 6\nu u^2 + 2(3\nu - 26)\nu u^3$$

$$u = GM/(c^2 R)$$

Simple effective Hamiltonian:

$$\hat{H}_{\rm eff} \equiv \sqrt{p_{r_*}^2 + A(r)\left(1 + \frac{p_\varphi^2}{r^2} + z_3 \frac{p_{r_*}^4}{r^2}\right)}$$

$$p_{r_*} = \left(\frac{A}{B}\right)^{1/2} p_r$$

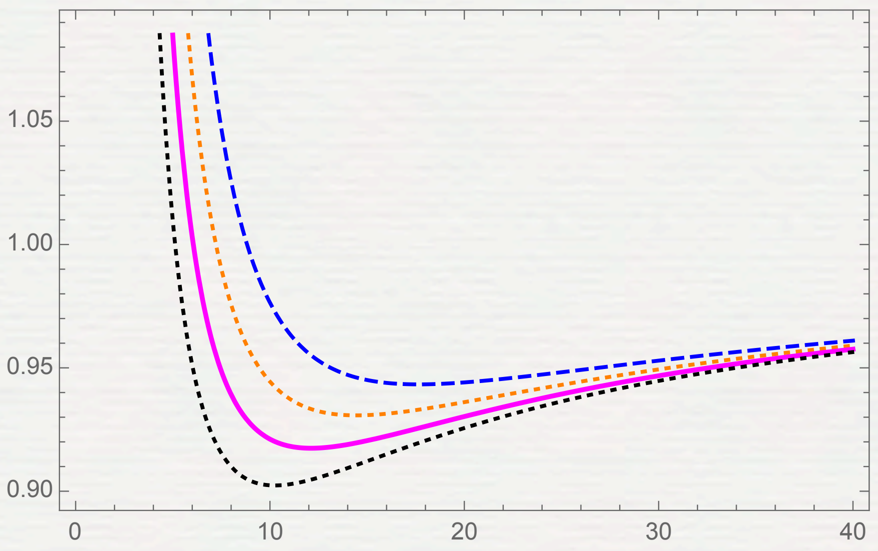
Crucial EOB radial potential

Contribution at 3PN

EFFECTIVE POTENTIALS

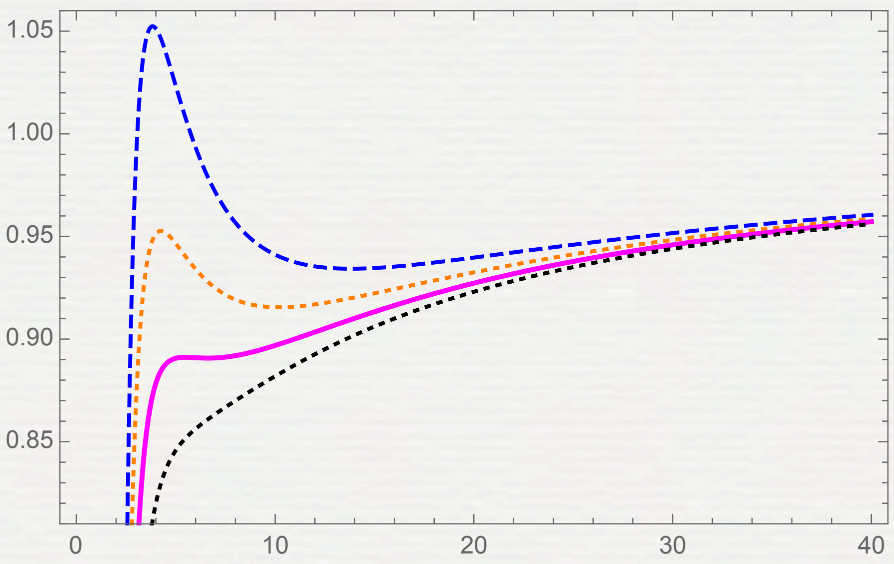
Newtonian gravity (any mass ratio):
circular orbits are always stable. No plunge.

$$W^{\rm eff}_{\rm Newt} = 1 - \frac{2}{r} + \frac{p_\varphi^2}{r^2}$$



Test-body on Schwarzschild black hole:
last stable orbit (LSO) at r=6M; plunge

$$W^{\rm eff}_{\rm Schwarzschild} = \left(1 - \frac{2}{r}\right)\left(1 + \frac{p_\varphi^2}{r^2}\right)$$

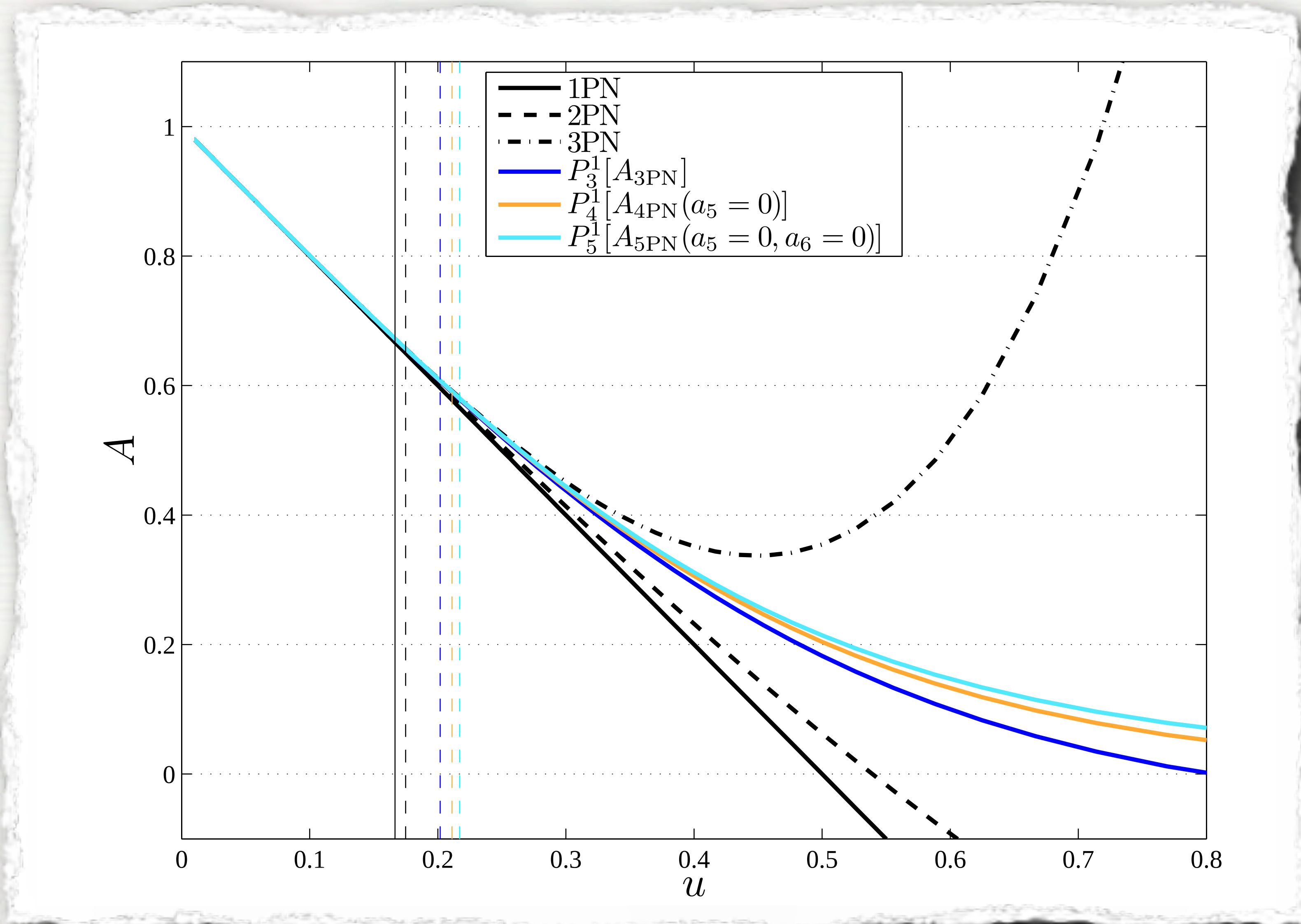


EOB, Black-hole binary, any mass ratio:
last stable orbit (LSO) at r<6M plunge

$$W^{\rm eff}_{\rm EOB} = A(r;\nu)\left(1 + \frac{p_\varphi^2}{r^2}\right)$$

ν -deformation of the Schwarzschild case!

USE OF PADE APPROXIMANTS



- Continuity with Schwarzschild metric: A(r) needs to have a zero
- Simple (possible) prescription: use a Padé representation of the potential

$$A(r) = P^1_3[A^{3\mathrm{PN}}(r)] = \frac{1 + n_1 u}{1 + d_1 u + d_2 u^2 + d_3 u^3}$$

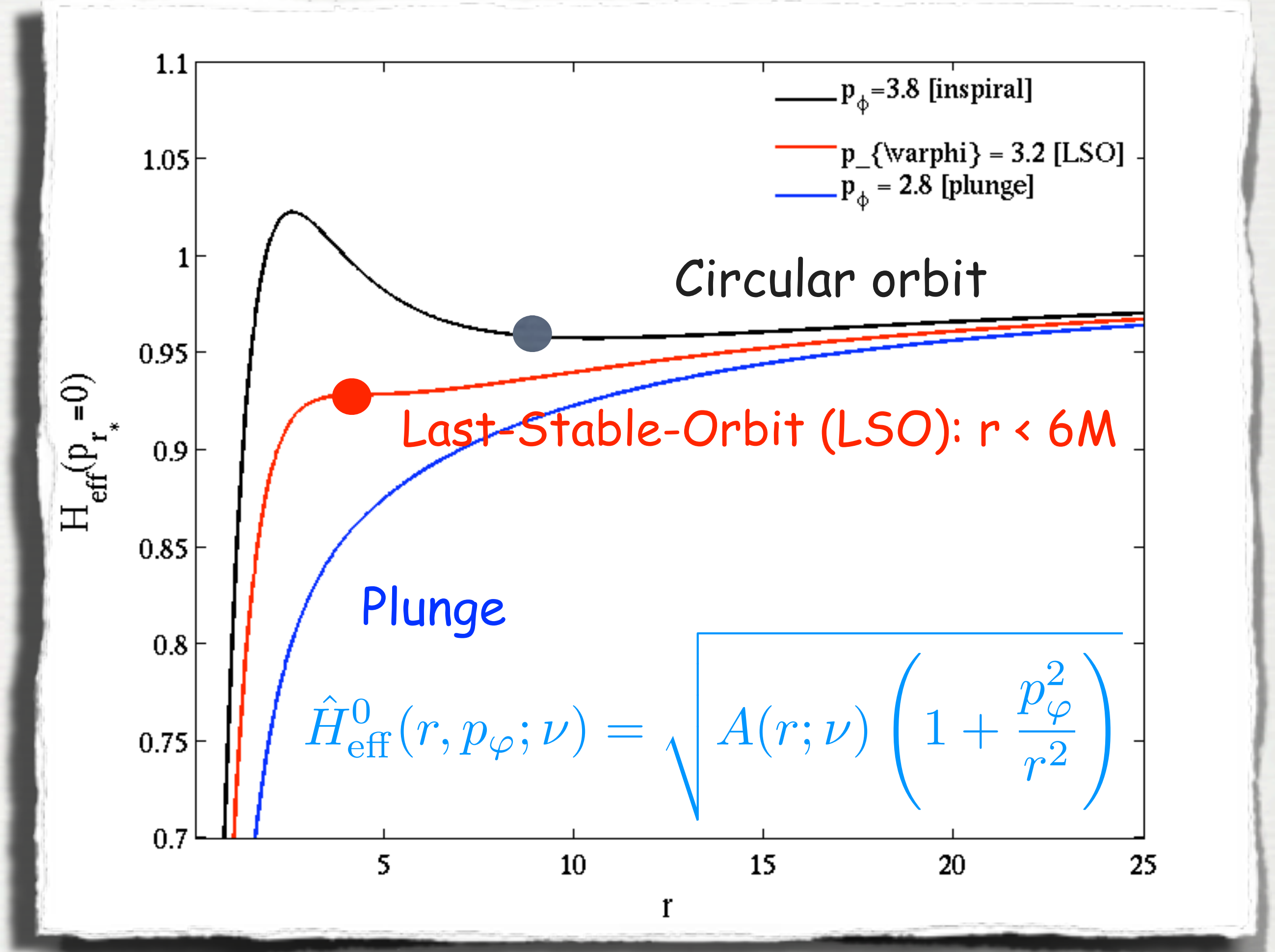
HAMILTON'S EQUATIONS & RADIATION REACTION

$$\dot{r} = \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}}$$

$$\dot{\varphi} = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{\varphi}} \equiv \Omega$$

$$\dot{p}_{r_*} = -\left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial r} + \hat{\mathcal{F}}_{r_*}$$

$$\dot{p}_{\varphi} = \hat{\mathcal{F}}_{\varphi}$$



- The system must radiate angular momentum
- How?Use PN-based (Taylor-expanded) radiation reaction force (ang-mom flux)
- Need flux resummation

$$\hat{\mathcal{F}}_{\varphi}^{\rm Taylor} = -\frac{32}{5}\nu\Omega^5 r_\Omega^4 \hat{F}^{\rm Taylor}(v_\varphi)$$

Resummation multipole by multipole (Damour&Nagar 2007, Damour, Iyer & Nagar 2008, Damour & Nagar, 2009)

Plus horizon contribution [AN&Akcay2012]

4PN analytically complete + 5PN logarithmic term in the A(u) function:

[Damour 2009, Blanchet et al. 2010, Barack, Damour & Sago 2010, Le Tiec et al. 2011, Barausse et al. 2011,Akcay et al. 2012, Bini& Damour2013, DamourJaranowski&Schaefer 2014].

$$A_{5\mathrm{PN}}^{\mathrm{Taylor}} = 1 - 2u + 2\nu u^3 + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\nu u^4 + \nu[a_5^c(\nu) + a_5^{\ln}\ln u]u^5 + \nu[a_6^c(\nu) + a_6^{\ln}\ln u]u^6$$

1PN 2PN 3PN 4PN 5PN

$$a_5^{\log} = \frac{64}{5}$$

$$a_5^c = a_{5_0}^c + \nu a_{5_1}^c$$

$$a_{5_0}^c = -\frac{4237}{60} + \frac{2275}{512}\pi^2 + \frac{256}{5}\log(2) + \frac{128}{5}\gamma$$

$$a_{5_1}^c = -\frac{221}{6} + \frac{41}{32}\pi^2$$

4PN fully known ANALYTICALLY!

$$a_6^{\log} = -\frac{7004}{105} - \frac{144}{5}\nu$$

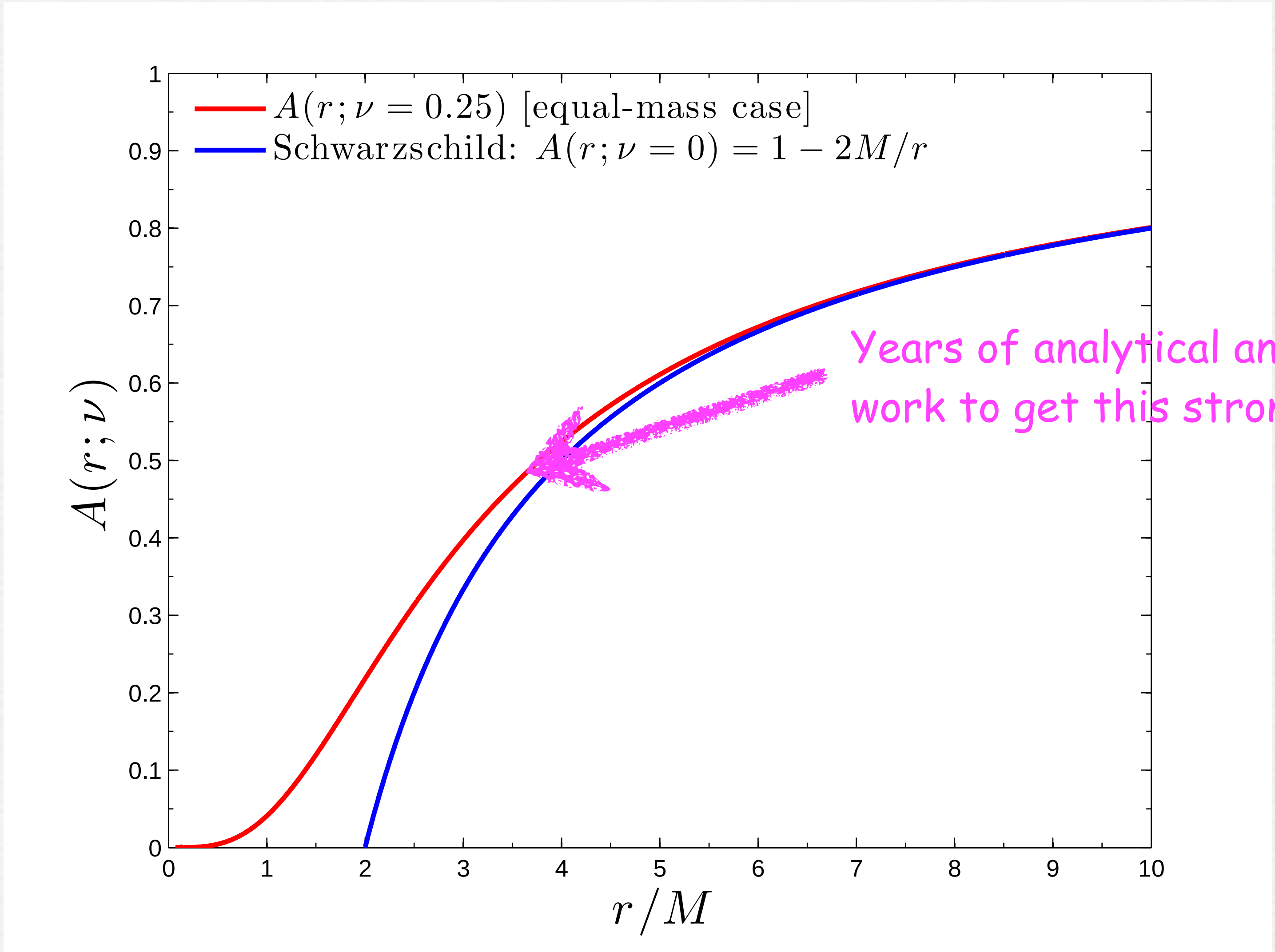
5PN logarithmic term (analytically known)

NEED ONE "effective" 5PN parameter from NR waveform data: $a_6^c(\nu)$

State-of-the-art EOB potential (5PN-resummed):

$$A(u;\nu,a_6^c) = P_5^1[A_{5\mathrm{PN}}^{\mathrm{Taylor}}(u;\nu,a_6^c)]$$

THE EOB[NR] POTENTIAL



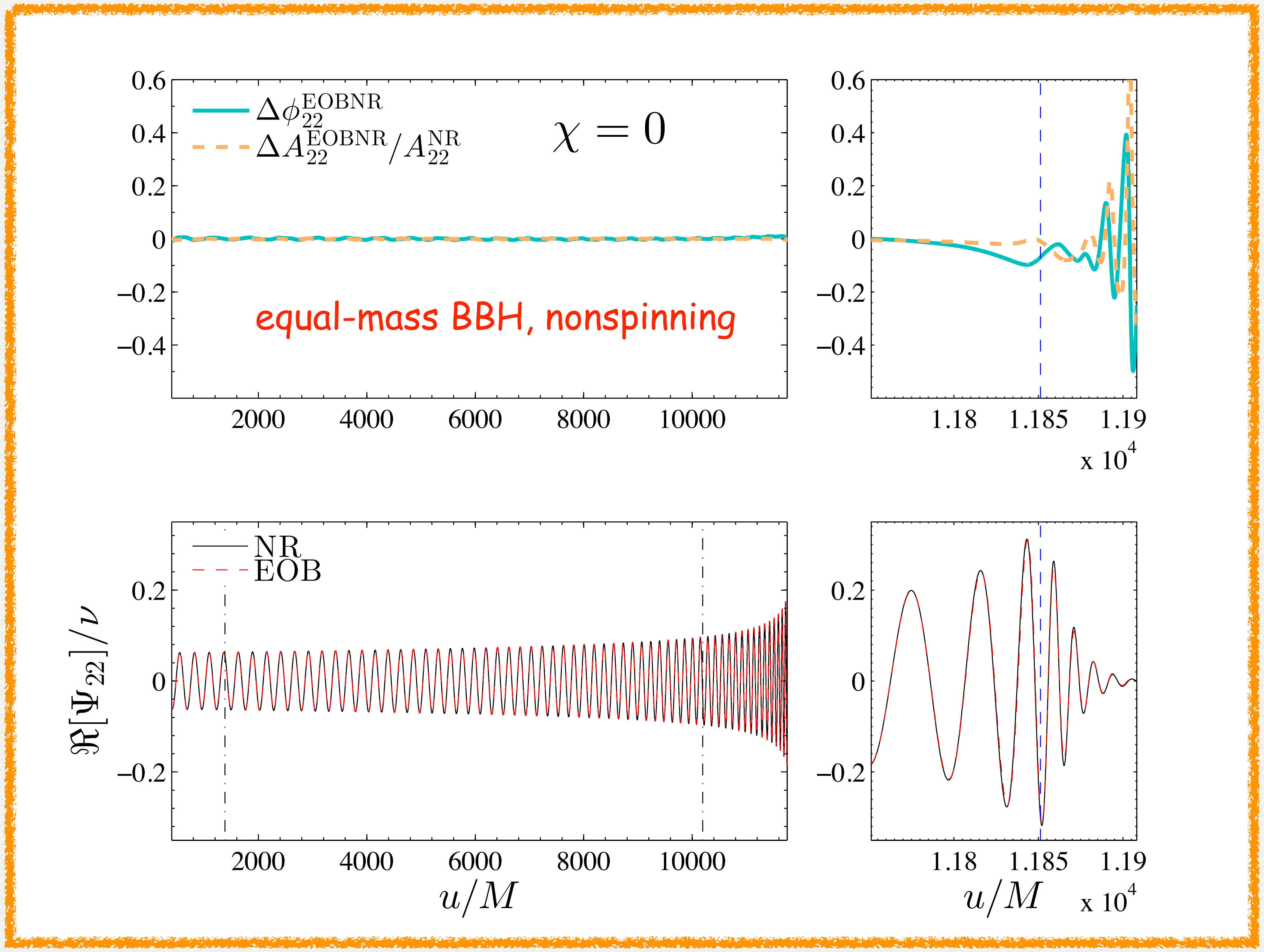
From EOB/NR-fitting: $a_6^c(\nu) = 3097.3\nu^2 - 1330.6\nu + 81.3804$

TAKE AWAY:

system is more bound, smaller "separation" and higher frequencies!

NDRP, arXiv:1506.08457

RESULTS: EOBNR/NR WAVEFORMS (NO SPIN)



equal-mass case

Nagar, Damour, Reisswig & Pollney, arXiv:1506.08457

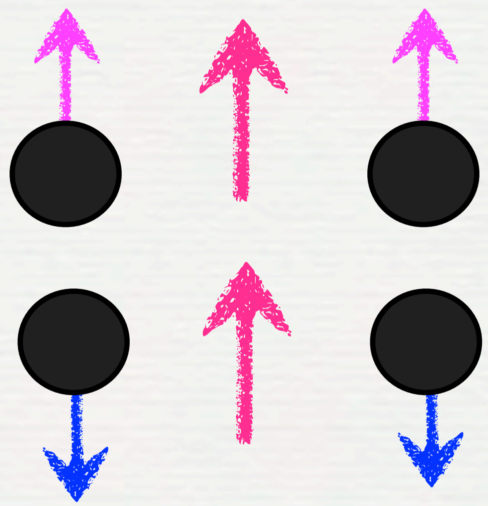
SPINNING BBHS

Spin-orbit & spin-spin couplings

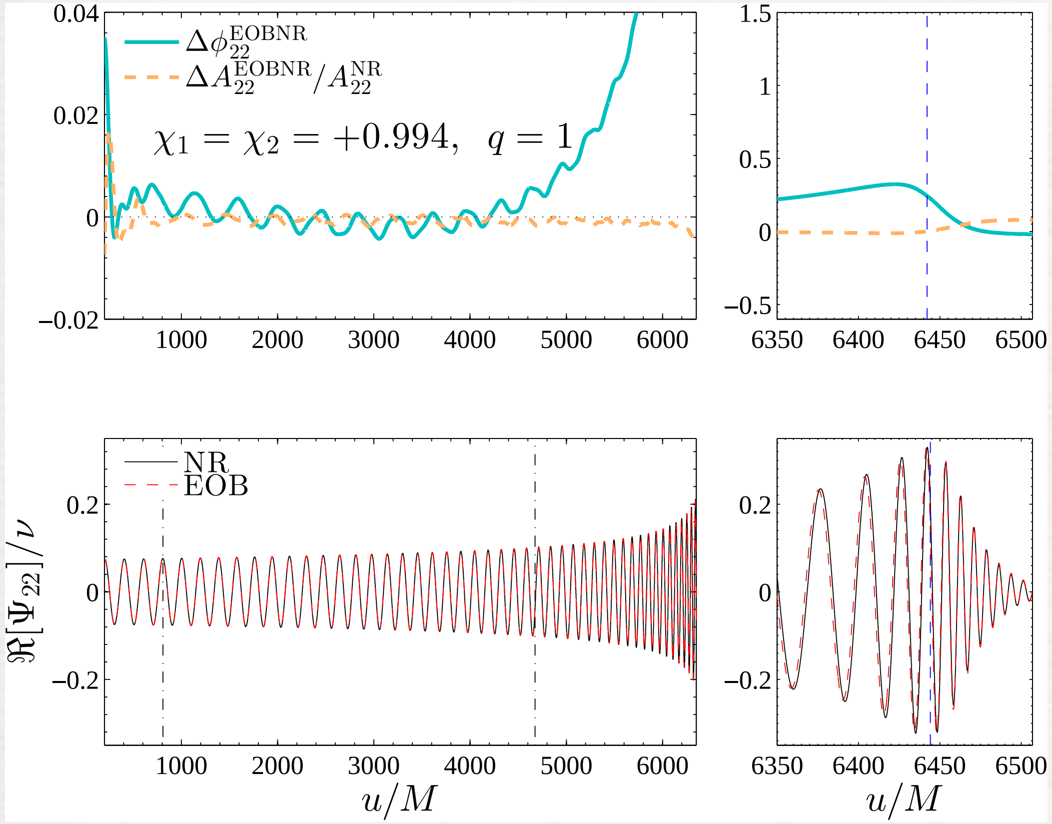
(i) Spins aligned with L: repulsive (slower) L-o-n-g-e-r INSPIRAL

(ii) Spins anti-aligned with L: attractive (faster) Shorter INSPIRAL

(iii) Misaligned spins: precession of the orbital plane (waveform modulation)



$$\chi_{1,2} = \frac{c\,\mathbf{S}_{1,2}}{G m_{1,2}^2}$$



EOB/NR agreement: sophisticated (though rather simple) model for spin-aligned binaries

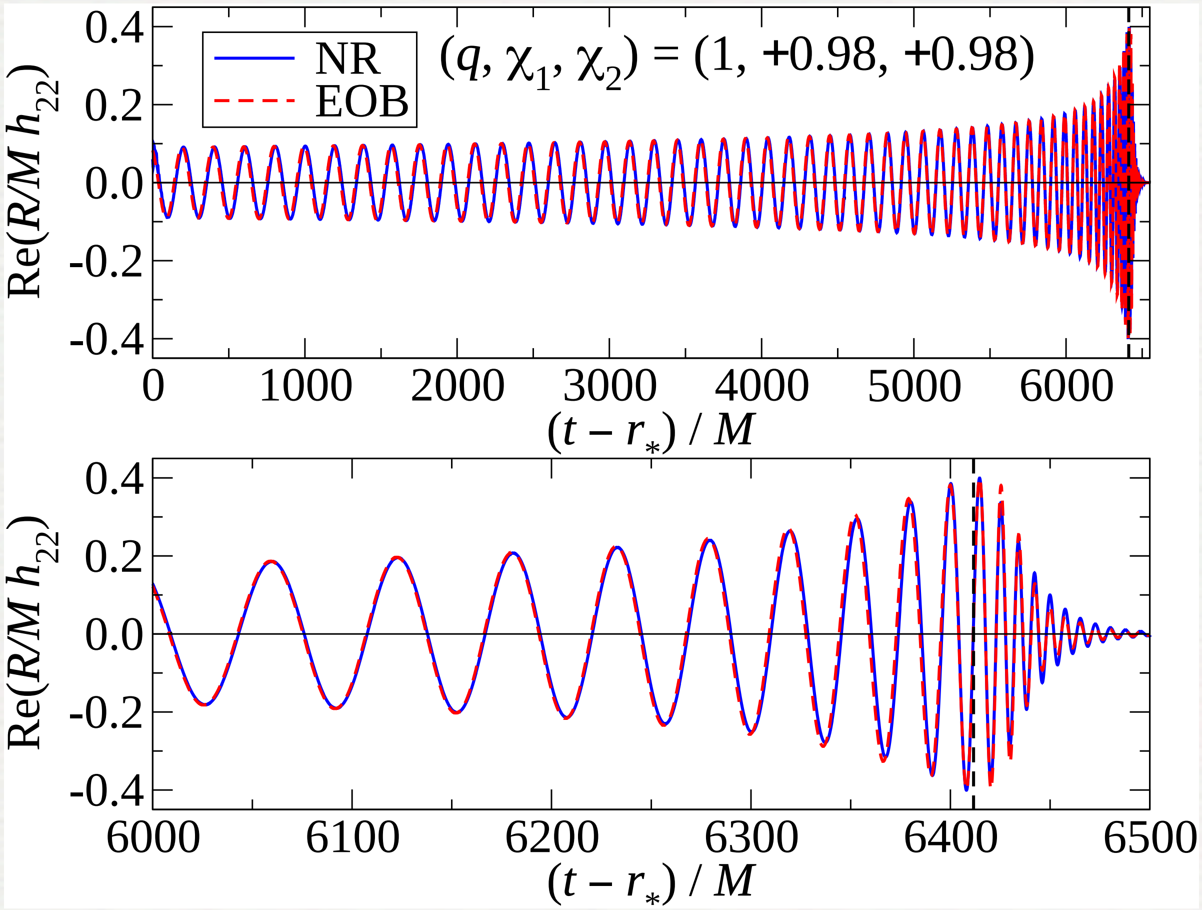
Damour&Nagar, PRD90 (2014), 024054
Damour&Nagar, PRD90 (2014), 044018
Nagar,Damour, Reisswig & Pollney, PRD 93 (2016), 044046

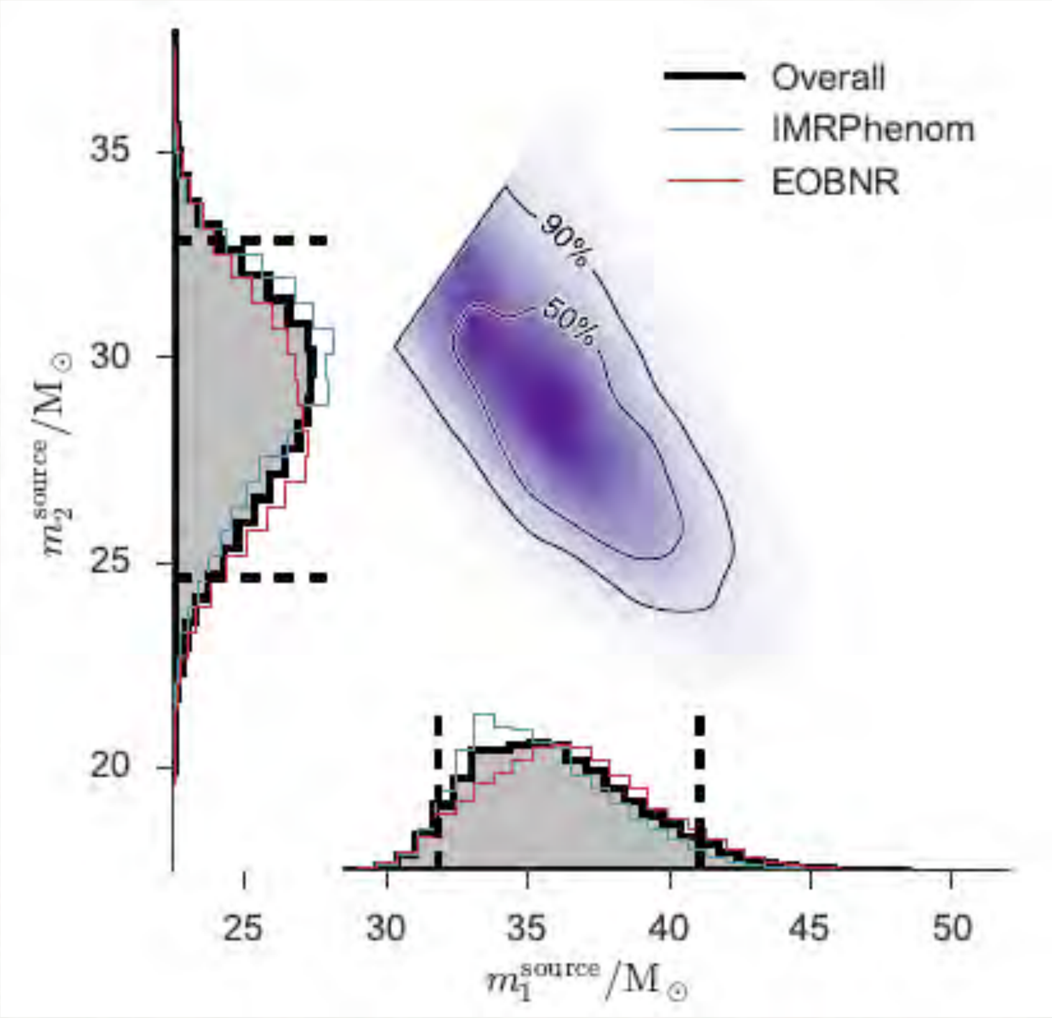
EOBNR MODEL USED FOR GW150914

Different EOB Hamiltonian [Barausse & Buonanno11, Taracchini et al.12]

SEOBNRv2: Taracchini, Buonanno et al., PRD 89, 061502 (R), 2014

SEOBNRv2_ROM_DoubleSpin: M. Puerrer, CQG 31, 195010 (2014)





Effectively used to get the masses:

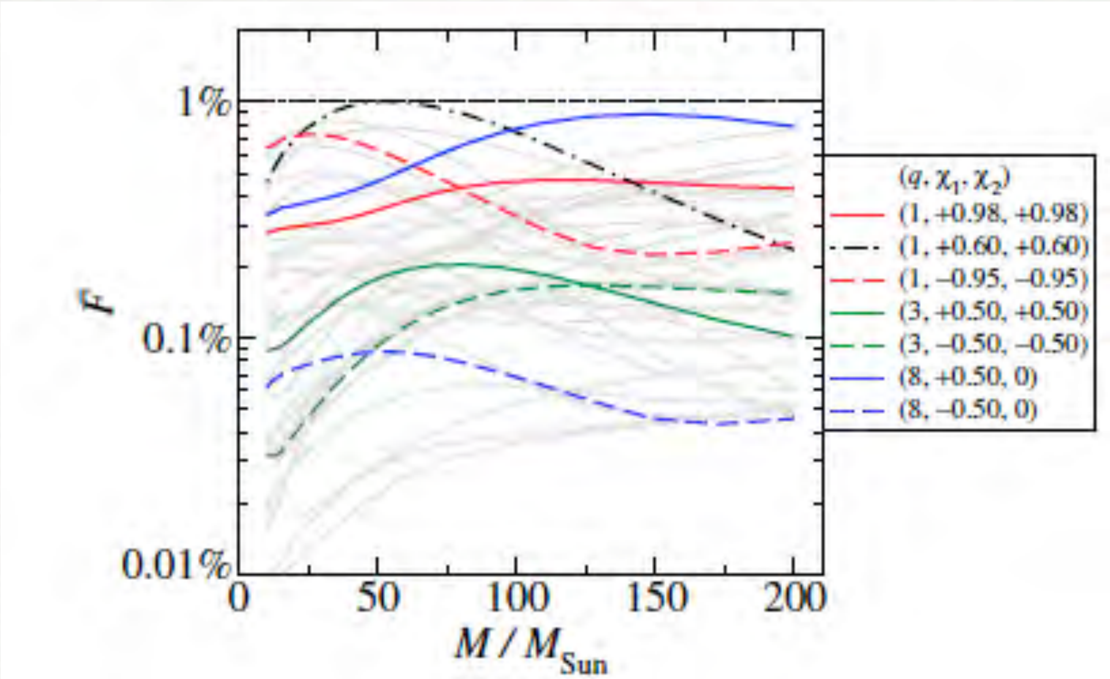
SEOBNRv2_ROM_DoubleSpin

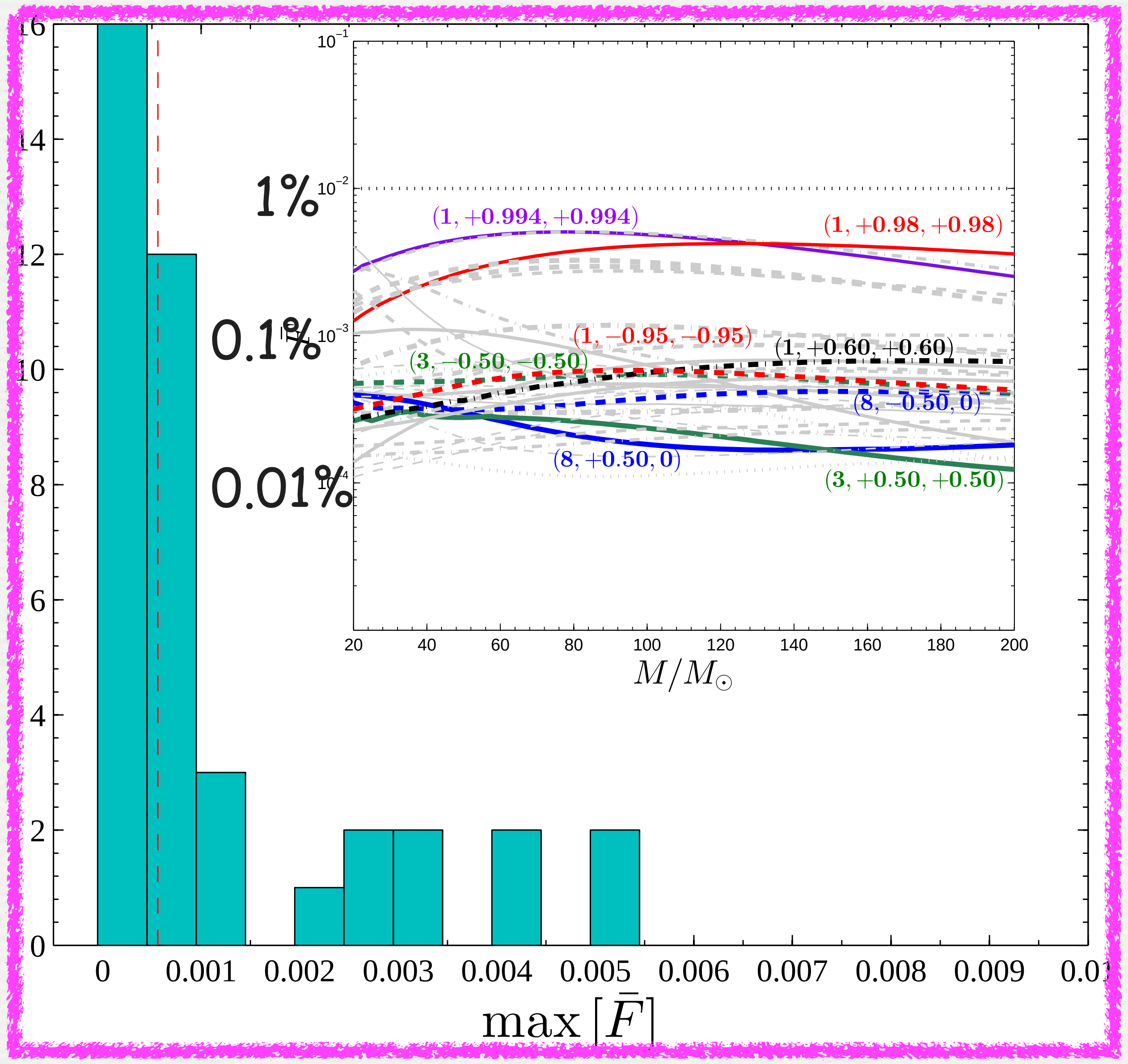
IMRPhenom (Khan et al., 2015)

just AFTER, the best choices were cross checked with NR simulations!

IHES EOBNR MODEL

Best existing EOBNR model WAS NOT used for parameter estimation:
EOB/EOBNR UNFAITHFULNESS





$$\bar{F} \equiv 1 - \max_{t_0,\phi_0} \frac{\langle h_{22}^{\rm EOB}, h_{22}^{\rm NR}\rangle}{||h_{22}^{\rm EOB}||\,||h_{22}^{\rm NR}||}$$

$$\langle h_1, h_2\rangle \equiv 4\Re \int_{f_{\rm min}}^{\infty} \tilde{h}_1(f)\tilde{h}_2^*(f)/S_n(f)\,df$$

Nagar,Damour, Reisswig & Pollney, PRD 93 (2016), 044046

PRECESSION

Different EOB Hamiltonian [Barausse & Buonanno11, Taracchini et al.12]

SEOBNRv3: Taracchini, Buonanno et al., PRD 89, 061502 (R), 2014

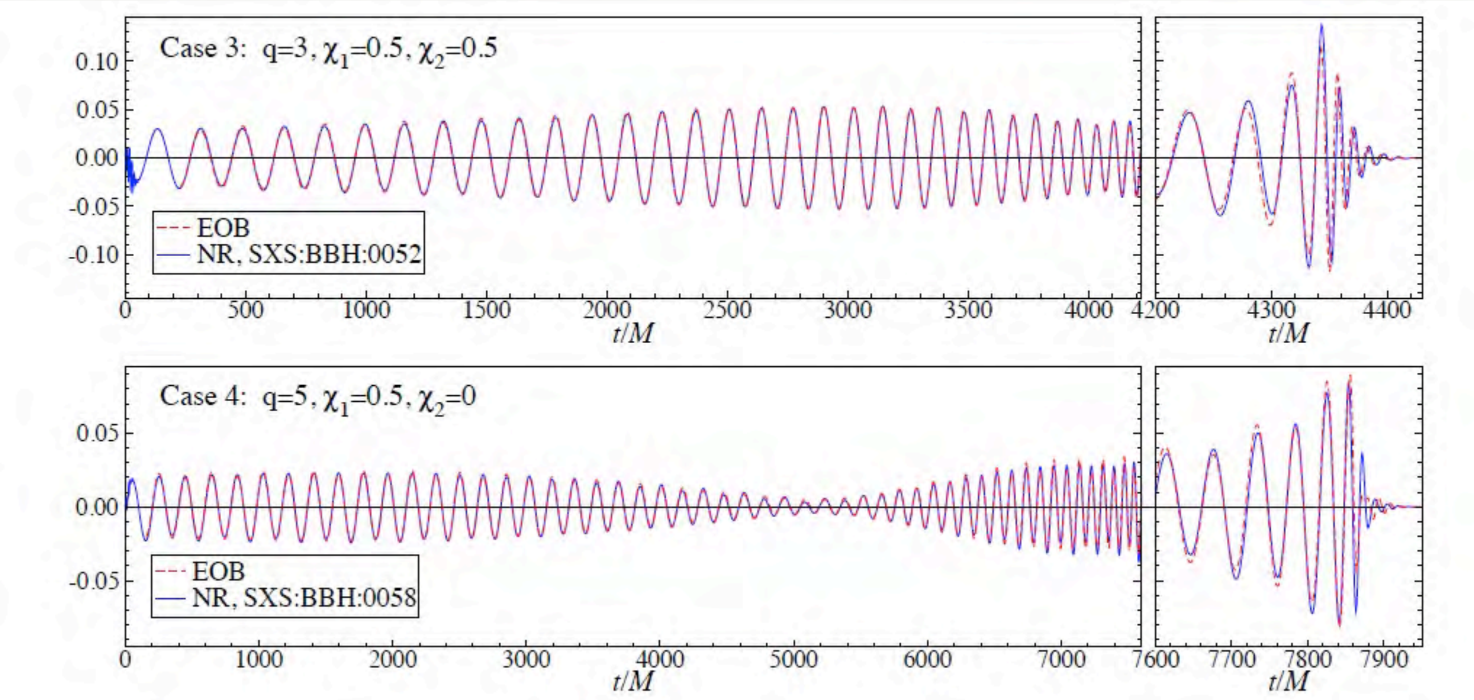


FIG. 9: We show for cases 3 and 4 of Table I the GW polarization h_+, containing contributions from $\ell = 2$ modes, that propagates along a direction $\hat{N}$ specified by spherical coordinates $\theta = \pi/3$ and $\phi = \pi/2$ associated with the inertial source frame $\{e_1^S, e_2^S, e_3^S\}$. The EOB waveforms start at the after-junk-radiation times of $t = 230M$ and $t = 160M$, respectively.

Good EOBNR/NR agreement.
The method works

Slow: not used for analyses (yet)

Improvements in the implementation are needed

PhenomP: P. Schmidt et al. 2012/2014

Phenomenological Precessing model that takes into account precession effects at leading order by "twisting" nonprecessing waveforms. **THIS IS USED FOR GW150914**. to conclude that no precession could be seen.

CONCLUSION

The wave has passed....

...and we were prepared!

Though more work to improve modelization further is needed!

Matlab EOB code (working for BNS too...), free download: https://eob.ihes.fr.
More infos: https://gravitational_waves.ihes.fr/