
Precise Measurement of the Top Mass at Proton and Electron Colliders

Top Quark Mass Calibration for Monte-Carlo Event Generators

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Outline

- Why precise top quark mass?
- Methods for top mass measurements
- Monte Carlo and the top quark mass
- Calibration of the Monte Carlo top mass parameter
- Preliminary results of first serious analysis

In collaboration with:

M. Butenschön

B. Dehnadi,

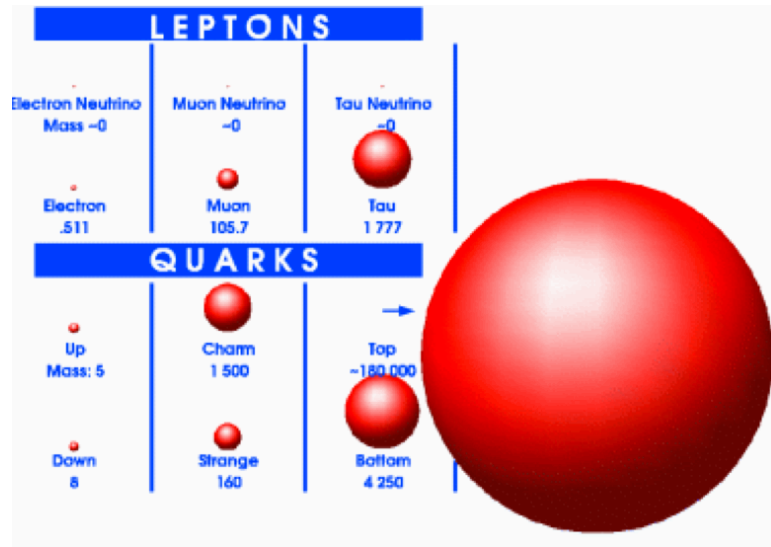
V. Mateu,

M. Preisser

I. Stewart

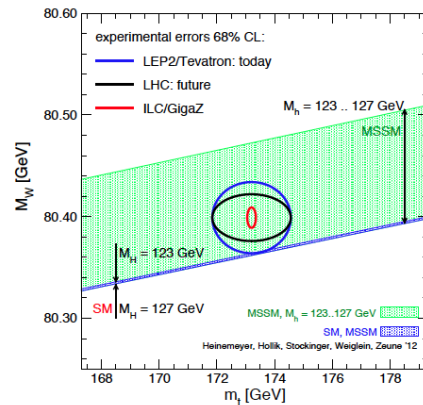


Why the top quark is not just heavy

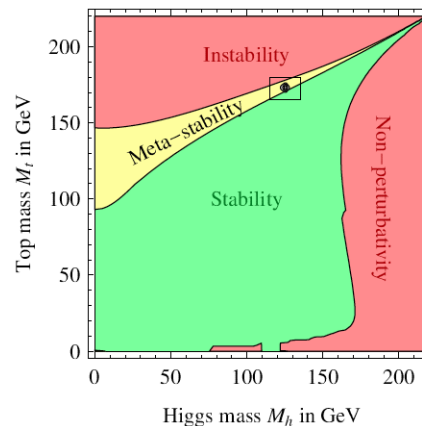


- Top quark: heaviest known particle
- Most sensitive to the mechanism of mass generation
- Peculiar role in the generation of flavor.
- Top might not be the SM-Top, but have a non-SM component.
- Top as calibration tool for new physics particles (SUSY and other exotics)
- Top production major background in new physics searches
- One of crucial motivations for SUSY

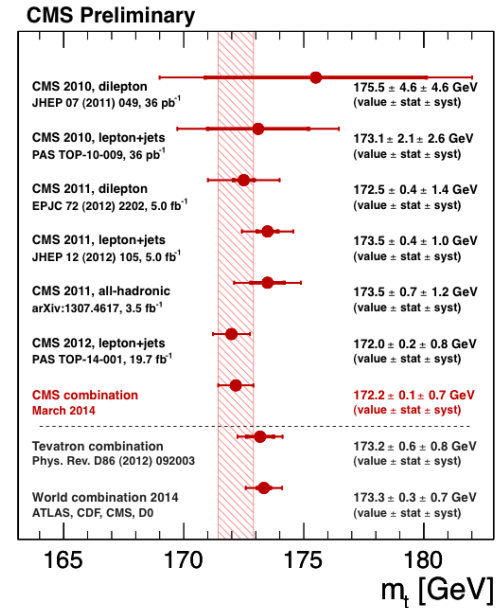
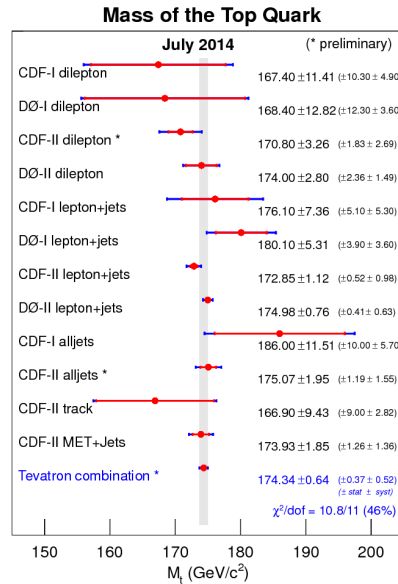
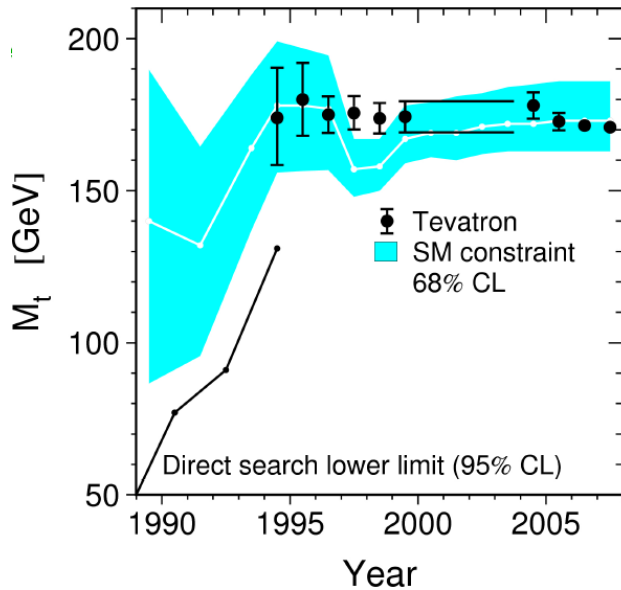
Electroweak precision observables



SM vacuum stability

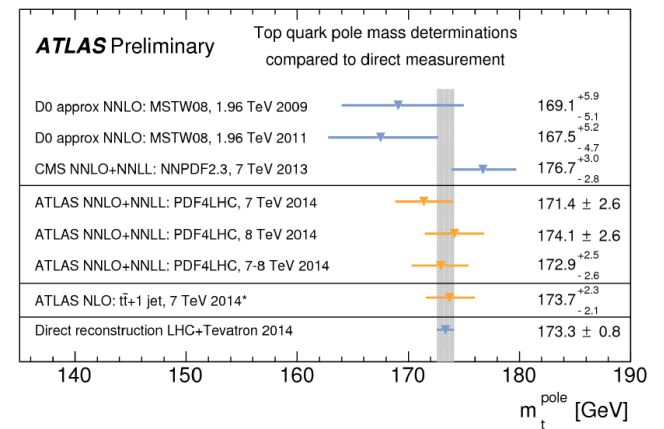


A small history on top mass reconstruction



- Many individual measurements with uncertainty below 1 GeV.
- World combination 2014:

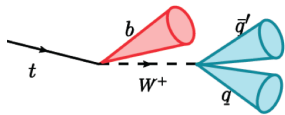
$$m_t^{\text{MC}} = 173.34 \pm 0.76 \text{ GeV}$$



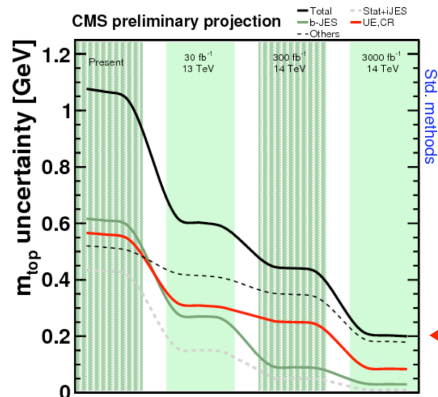
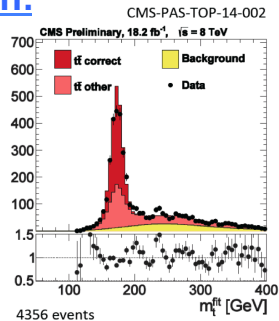
Top Mass Measurements Methods

LHC+Tevatron

Direct Reconstruction:



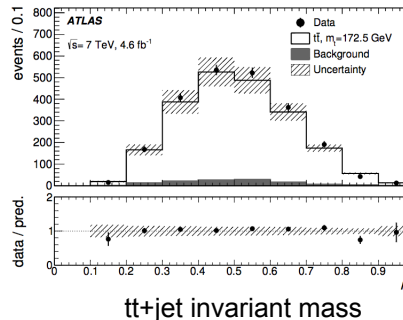
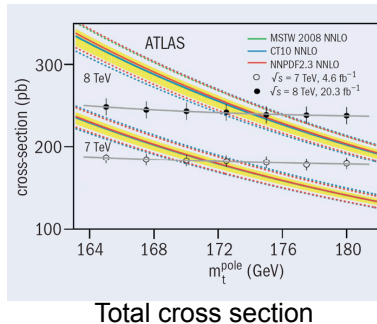
kinematic mass determination



- ⊕ High top mass sensitivity
- ⊖ Precision of MC ?
- ⊖ Meaning of m_t^{MC} ?
- $\Delta m_t \sim 0.5 \text{ GeV}$
- $\Delta m_t \sim 200 \text{ MeV (projection)}$

Indirect Mass Fit:

global mass dependence

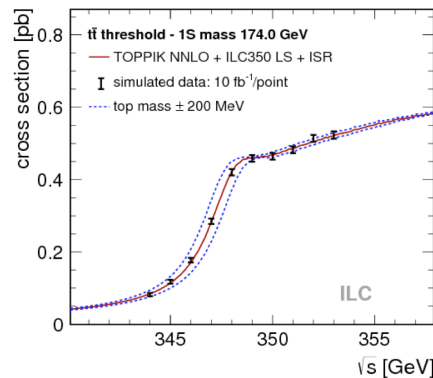


- ⊕ pQCD calculations dominate
- ⊕ Control of mass scheme
- ⊖ Lower top mass sensitivity
- ⊖ High sensitivity to norm errors
- $\Delta m_t \sim 1\text{-}2 \text{ GeV}$

Future Linear Collider:

Top Pair Threshold:

kinematic mass determination
perturbative toponium

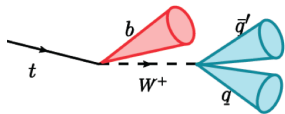


- ⊕ High top mass sensitivity
- ⊕ pQCD calculations dominate
- ⊕ Control of mass scheme
- $\Delta m_t \sim 100 \text{ MeV}$

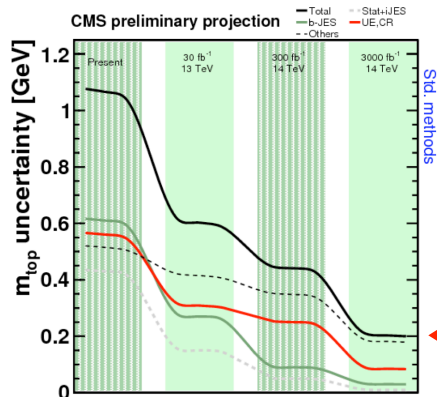
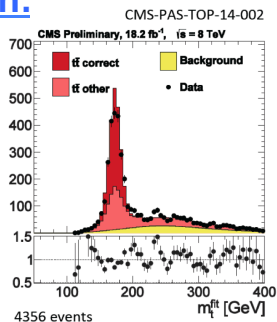
Top Mass Measurements Methods

LHC+Tevatron

Direct Reconstruction:



kinematic mass determination



⊕ High top mass sensitivity

⊖ Precision of MC ?

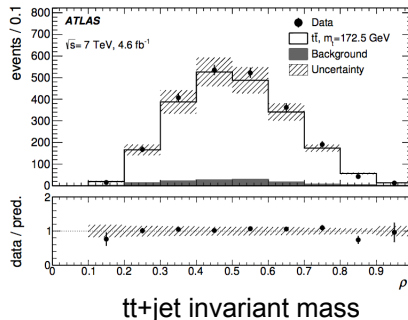
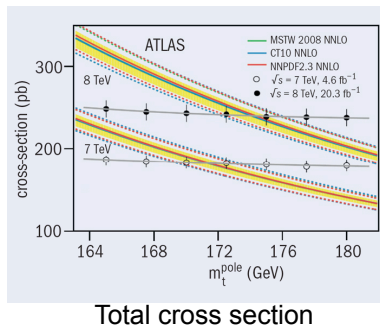
⊖ Meaning of m_t^{MC} ?

$$\Delta m_t \sim 0.5 \text{ GeV}$$

$$\Delta m_t \sim 200 \text{ MeV (projection)}$$

Indirect Mass Fit:

global mass dependence



⊕ pQCD calculations dominate

⊕ Control of mass scheme

⊖ Lower top mass sensitivity

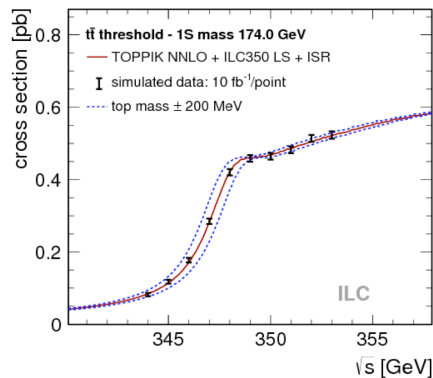
⊖ High sensitivity to norm errors

$$\Delta m_t \sim 1\text{-}2 \text{ GeV}$$

Future Linear Collider:

Top Pair Threshold:

kinematic mass determination
perturbative toponium



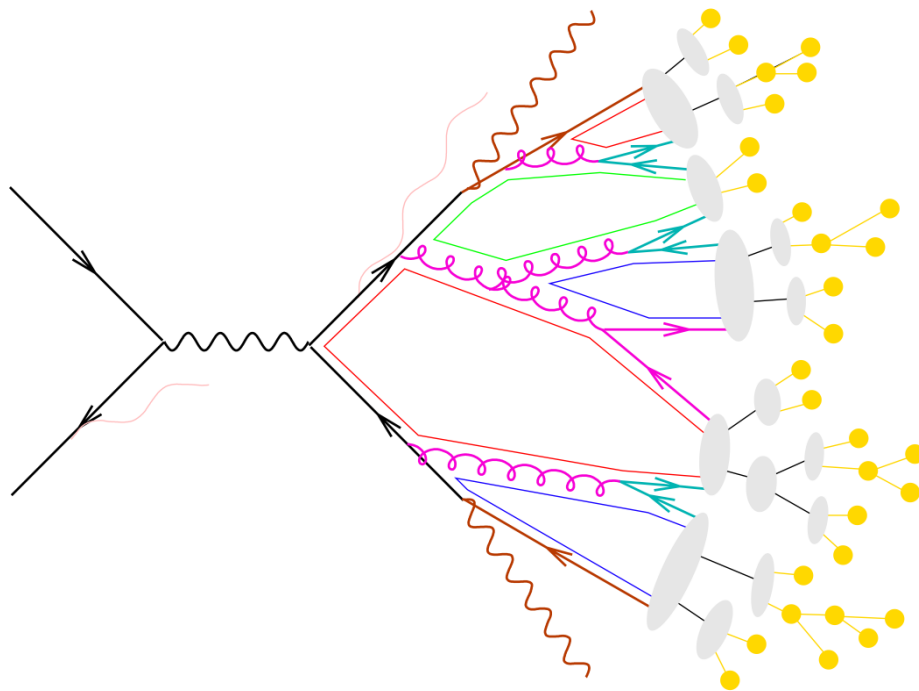
⊕ High top mass sensitivity

⊕ pQCD calculations dominate

⊕ Control of mass scheme

$$\Delta m_t \sim 100 \text{ MeV}$$

Monte-Carlo Event Generators



- hard scattering
- (QED) initial/final state radiation
- partonic decays, e.g. $t \rightarrow bW$
- parton shower evolution
- nonperturbative gluon splitting
- colour singlets
- colourless clusters
- cluster fission
- cluster \rightarrow hadrons
- hadronic decays

- Full simulation of all processes (all experimental aspects accessible)
- QCD-inspired: partly first principles QCD \Leftrightarrow partly model
- Description power of data better than intrinsic accuracy. (But how precise?)
- Top quark: treated like a real particle ($m_t^{\text{MC}} \approx m_t^{\text{pole}}$).

But pole mass ambiguous by $O(1 \text{ GeV})$ due to confinement.

MC Top Quark Mass

$$m_t^{\text{MC}} = m_t^{\text{MSR}}(R = 1 \text{ GeV}) + \Delta_{t,\text{MC}}(R = 1 \text{ GeV})$$

$$\Delta_{t,\text{MC}}(1 \text{ GeV}) \sim \mathcal{O}(1 \text{ GeV})$$

AHH, Stewart 2008
AHH, 2014

- small size
- Renormalon-free
- little parametric dependence on other parameters

MSR Mass Definition

MS Scheme: $(\mu > \bar{m}(\bar{m}))$

$$\bar{m}(\bar{m}) - m^{\text{pole}} = -\bar{m}(\bar{m}) [0.42441 \alpha_s(\bar{m}) + 0.8345 \alpha_s^2(\bar{m}) + 2.368 \alpha_s^3(\bar{m}) + \dots]$$

MSR Scheme: $(R < \bar{m}(\bar{m}))$



$$m_{\text{MSR}}(R) - m^{\text{pole}} = -R [0.42441 \alpha_s(R) + 0.8345 \alpha_s^2(R) + 2.368 \alpha_s^3(R) + \dots]$$

$$m_{\text{MSR}}(m_{\text{MSR}}) = \bar{m}(\bar{m})$$

\Rightarrow $m_{\text{MSR}}(R)$ Short-distance mass that smoothly interpolates all R scales

Calibration of the MC Top Mass

Method:

- 1) **Strongly mass-sensitive observable** (closely related to reconstructed invariant mass distribution !)
- 2) Accurate **analytic hadron level QCD predictions at \geq NLL/NLO** with **full control over the quark mass scheme dependence**.
- 3) QCD masses as function of m_t^{MC} from **fits** of observable.
- 4) Cross check observable independence

$$m_t^{\text{MC}} = m_t^{\text{MSR}}(R = 1 \text{ GeV}) + \Delta_{t,\text{MC}}(R = 1 \text{ GeV})$$

$$\Delta_{t,\text{MC}}(1 \text{ GeV}) = \bar{\Delta} + \delta\Delta_{\text{MC}} + \delta\Delta_{\text{pQCD}} + \delta\Delta_{\text{param}}$$

- different tunings
- parton showers
- color reconnection
- ...

- perturbative error
- scale uncertainties
- electroweak effects

- strong coupling α_s
- Non-perturbative parameters

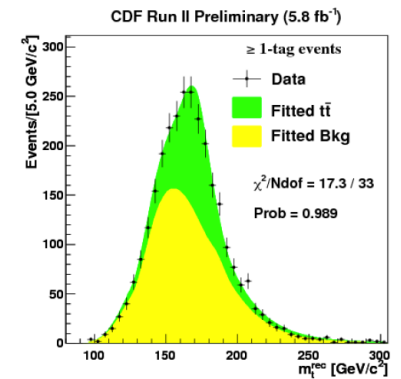
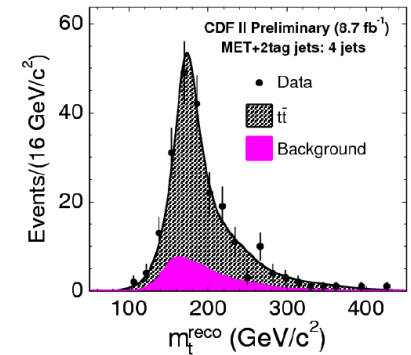
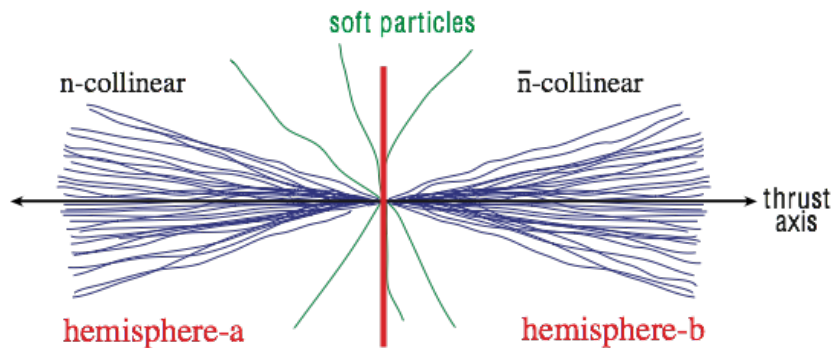
Thrust Distribution

Observable: 2-jettiness in e+e- (event shape distributions)

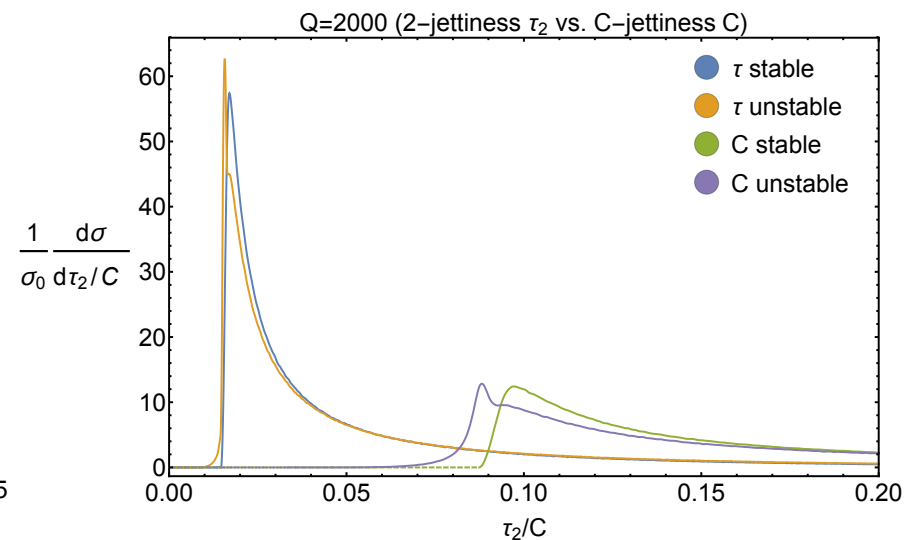
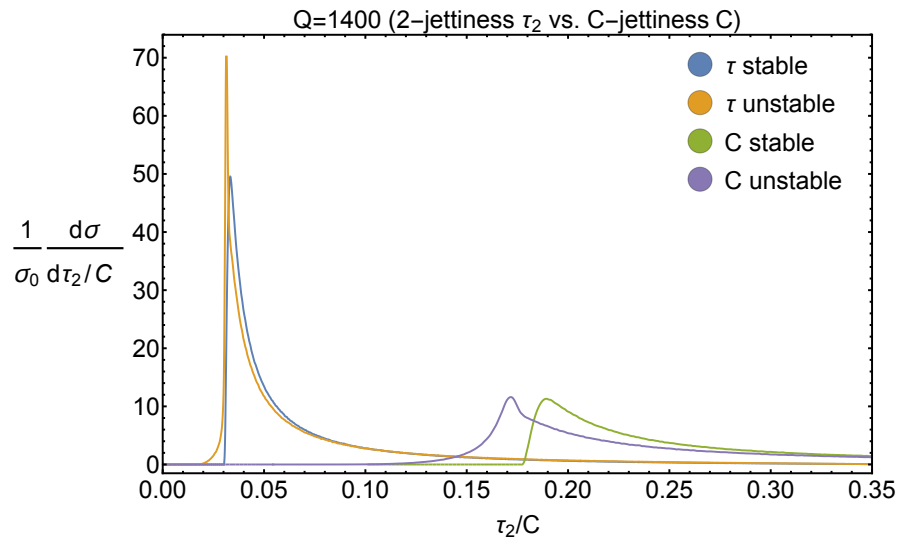
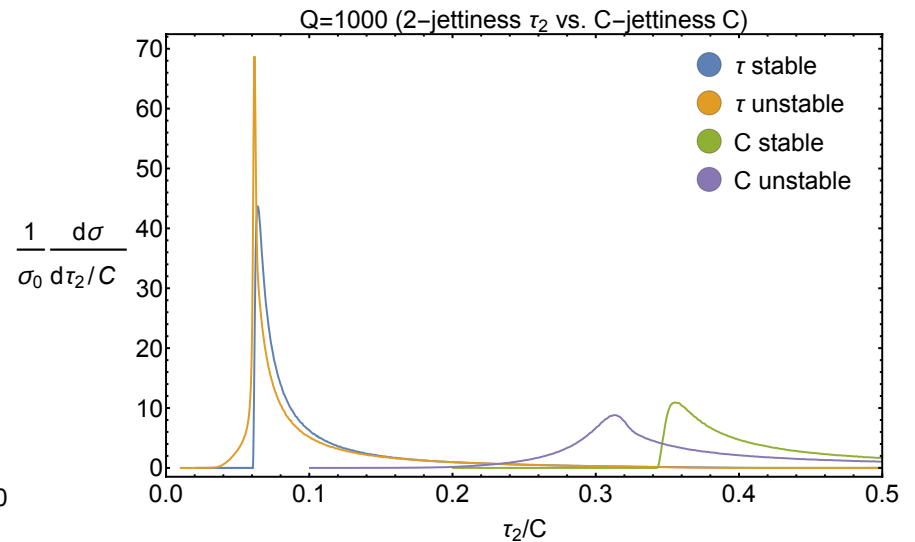
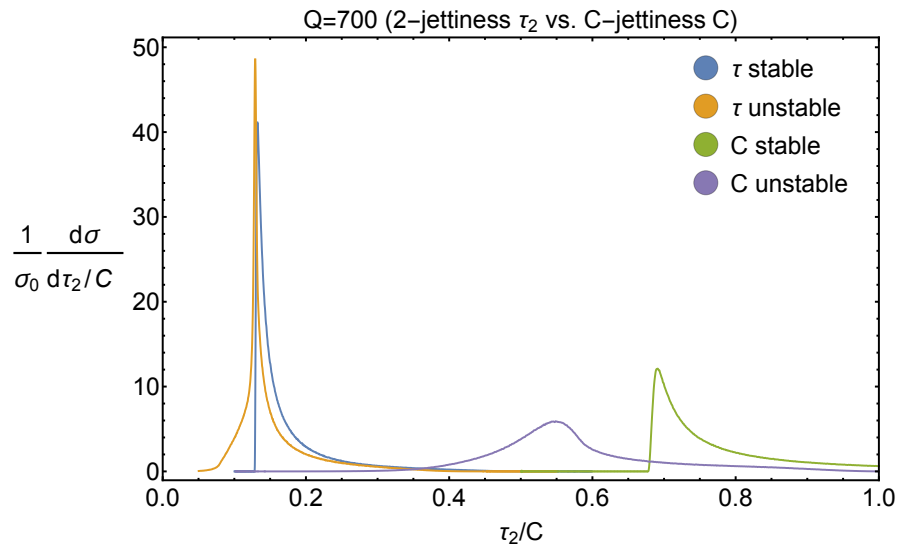
$$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{Q}$$

$$\tau \xrightarrow{\rightarrow 0} \frac{M_1^2 + M_2^2}{Q^2}$$

Invariant mass distribution in the resonance region !

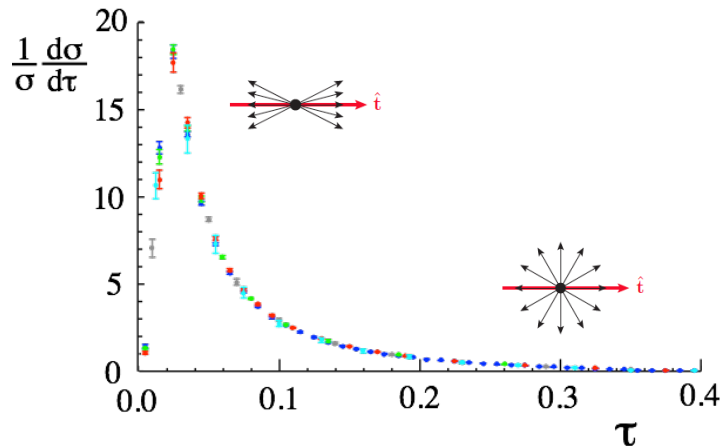


Event Shape Distributions (Pythia 8.2)



Factorization for Event Shapes

$$\frac{d\sigma}{d\tau} = Q^2 \sigma_0 H_0(Q, \mu) \int dl J_0(Ql, \mu) S_0(Q\tau - l, \mu)$$



Massless quarks:

Korshemski, Sterman 1995-2000

Bauer, Fleming, Lee, Sterman (2008)

Becher, Schwartz (2008)

Abbate, AHH, Fickinger, Mateu, Stewart 2010

Extension to massive quarks:

- VFNS for final state jets (with massive quarks): log summation incl. mass
- Boosted fat top jets

Fleming, AHH, Mantry, Stewart 2007

Gritschacher, AHH, Jemos, Mateu Pietrulewicz 2013-2014

Butenschön, Dehnadi, AHH, Mateu 2016 (to appear)

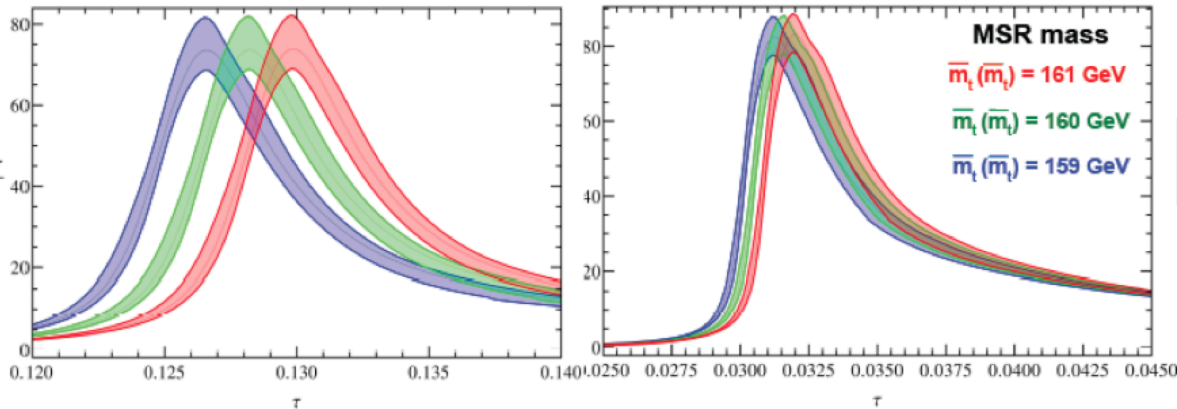
➔ NNLL + NLO + non-singular + hadronization + renormalon-subtraction

2-Jettiness for Top Production (QCD)

$$\frac{d\sigma}{d\tau_2} = f(\underbrace{m_t^{\text{MSR}}(R)}_{\text{any scheme possible}}, \underbrace{\alpha_s(M_Z), \Omega_1, \Omega_2, \dots}_{\text{Non-perturbative}}, \underbrace{\mu_h, \mu_j, \mu_s, \mu_m, R, \Gamma_t}_{\text{renorm. scales finite lifetime}})$$

Q=700 GeV

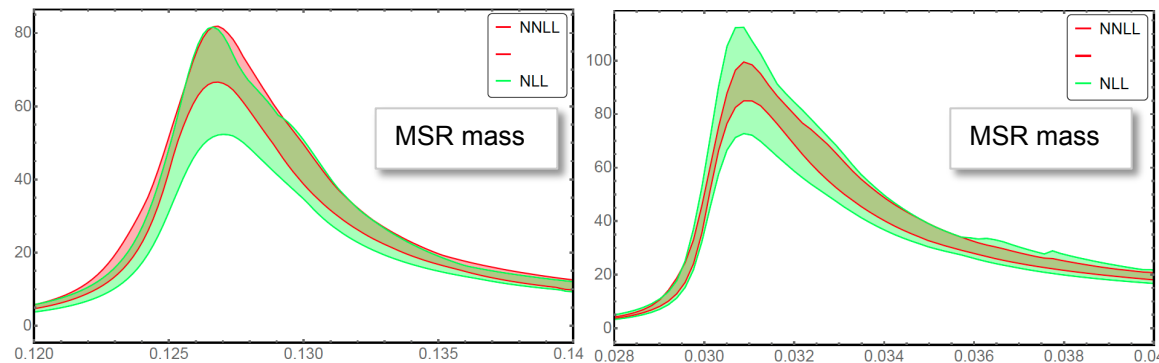
Q=1400 GeV



- Higher mass sensitivity for lower Q (p_T)
- Finite lifetime effects included
- Dependence on non-perturbative parameters
- Convergence: $\Omega_{1,2,\dots}$

Q=700 GeV

Q=1400 GeV



- Good convergence
- Reduction of scale uncertainty (NLL to NNLL)
- Control over whole distribution

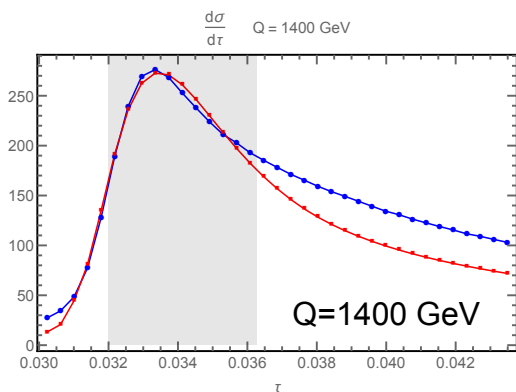
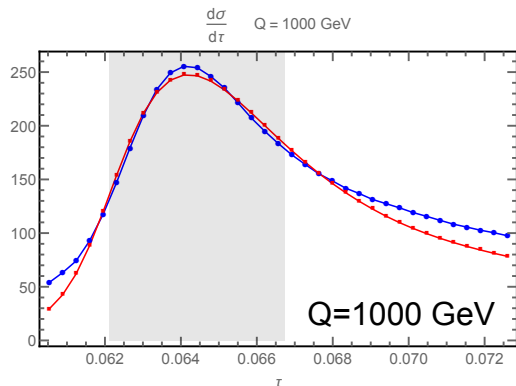
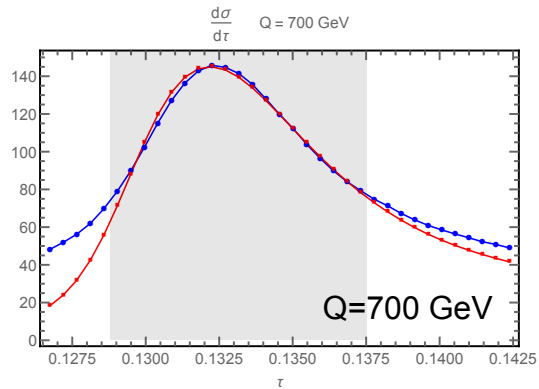
Fit Procedure Details

$$\frac{d\sigma}{d\tau_2} = f(\underbrace{m_t^{\text{MSR}}(R)}_{\text{any scheme possible}}, \underbrace{\alpha_s(M_Z), \Omega_1, \Omega_2, \dots}_{\text{Non-perturbative}}, \underbrace{\mu_h, \mu_j, \mu_s, \mu_m}_{\text{renorm. scales}}, R, \Gamma_t)_{\text{finite lifetime}}$$

QCD parameters measured from Pythia

- Fit parameters: $m_t^{\text{MSR}}(R), \alpha_s(M_Z), \Omega_1, \Omega_2, \dots,$
- Perturbative error: fits for 500 randomly picked sets of renorm. scales
- Tunings: 1, 3, 7 (default)
- Top quark width: $\Gamma_t =$ dynamical (default), 0.7, 1.4, 2.0 GeV
- External smearing (Detector effects): $\Omega_{1,\text{smear}} = 0, 0.5, \dots, 3.0, 3.5, \text{ GeV}$
- Pythia masses: $m_t^{\text{Pythia}} = 170, \dots, 175 \text{ GeV}$
- Fit possible for any mass scheme

Preliminary Peak Fits



Pythia
QCD

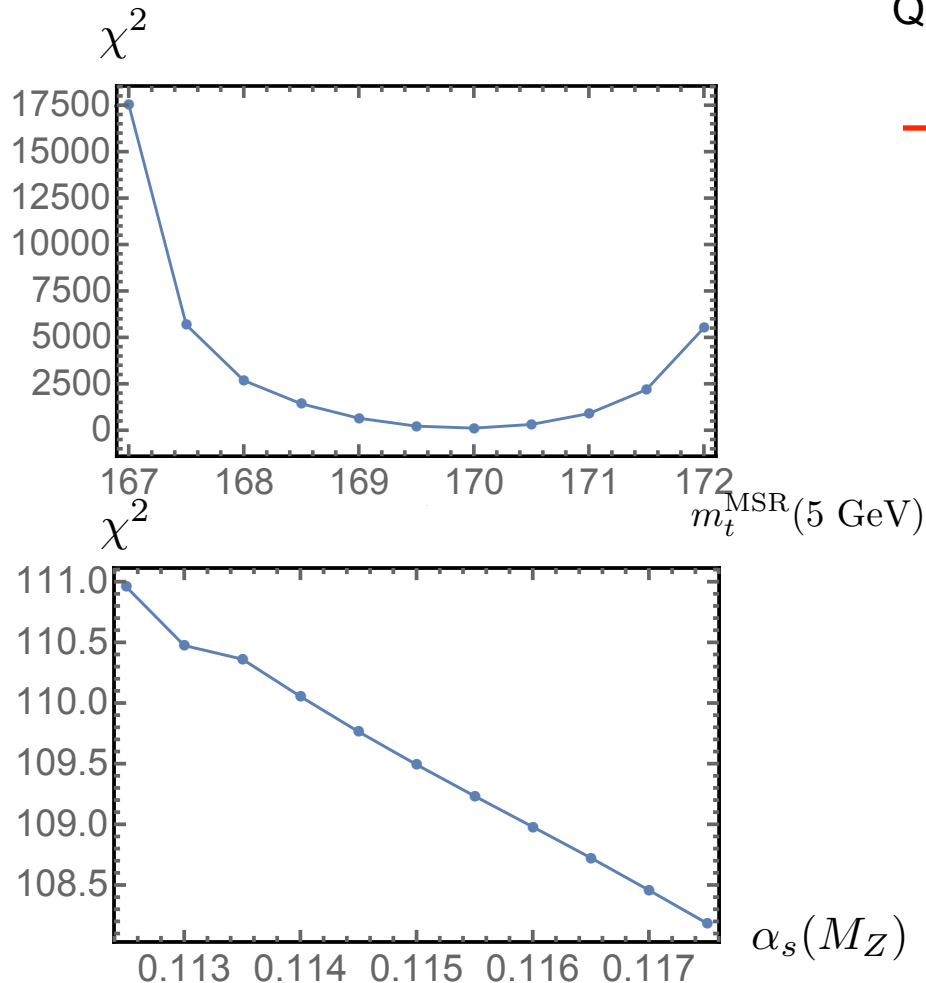
Default renormalization scales; $\Gamma_t = 1.4$ GeV, tune 7, $\Omega_{1,\text{smear}} = 2.5$ GeV, $m_t^{\text{Pythia}} = 171$ GeV, $Q = \{700, 1000, 1400\}$ GeV, peak fit (60/80)%

- Good agreement of Pythia 8.2 with NNLL+NLO QCD description
- Pythia statistics: 10^6 events
- Discrepancies in distribution tail and for higher energies (Pythia is less reliable where fixed-order results valid, well reliable in soft-collinear limit)
- Excellent sensitivity to the top quark mass.

Preliminary

Peak Fits

Default renormalization scales; $\Gamma_t=1.4$ GeV, tune 7, $\Omega_{1,\text{smear}}=2.5$ GeV, $m_t^{\text{Pythia}}=171$ GeV, $Q=\{700, 1000, 1400\}$ GeV, peak fit (60/80)%



→ $\chi^2_{\text{min}} \sim O(100)$

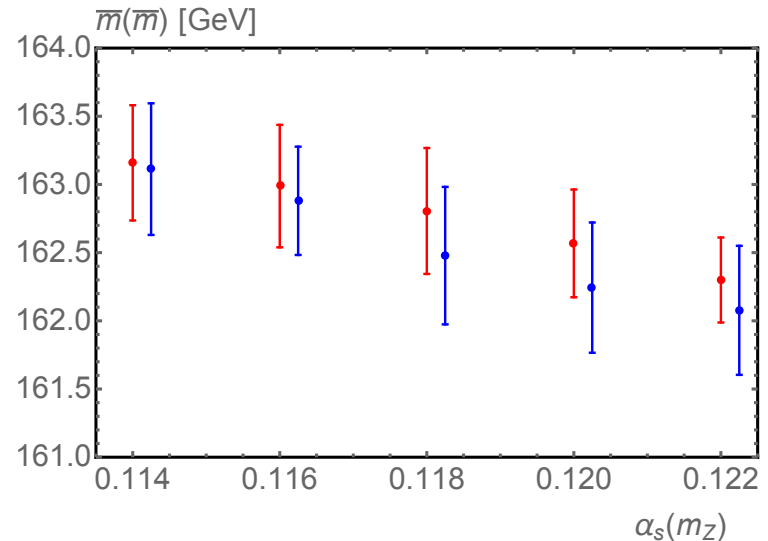
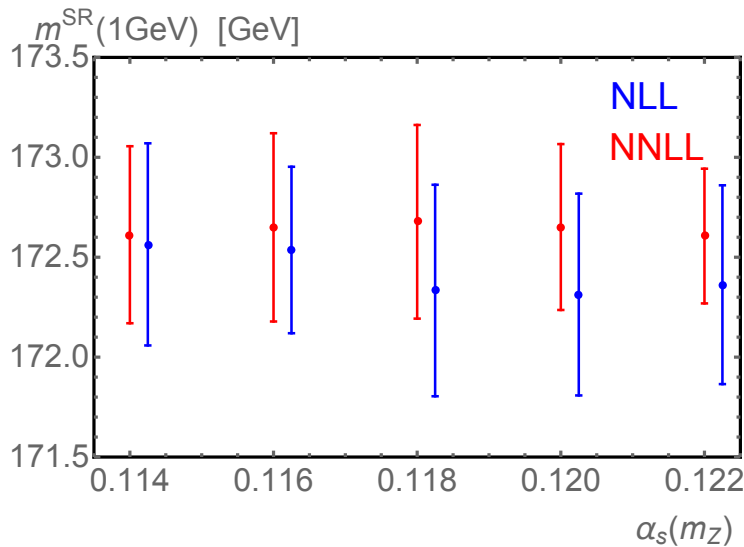
- Very strong sensitivity to m_t
- Low sensitivity to strong coupling
- Take strong coupling as input
- χ^2_{min} and δm_t^{stat} do not have any physical meaning
- We use rescaled χ^2/dof (PDG prescription) to define “intrinsic MC compatibility uncertainty”

Preliminary

Peak Fits

First serious run: $\Gamma_t=1.4$ GeV, tunes 1, 3, 7,
 $\Omega_{1,\text{smear}}=1.5, 2.0, 2.5, 3.0, 3.5$ GeV,
 $Q=\{700, 1000, 1400\}$ GeV, peak fit (60/80)%
 $m_t^{\text{Pythia}}=173$ GeV,
NLL: 177 scan survivors, NNLL: 254 scan survivors

Preliminary



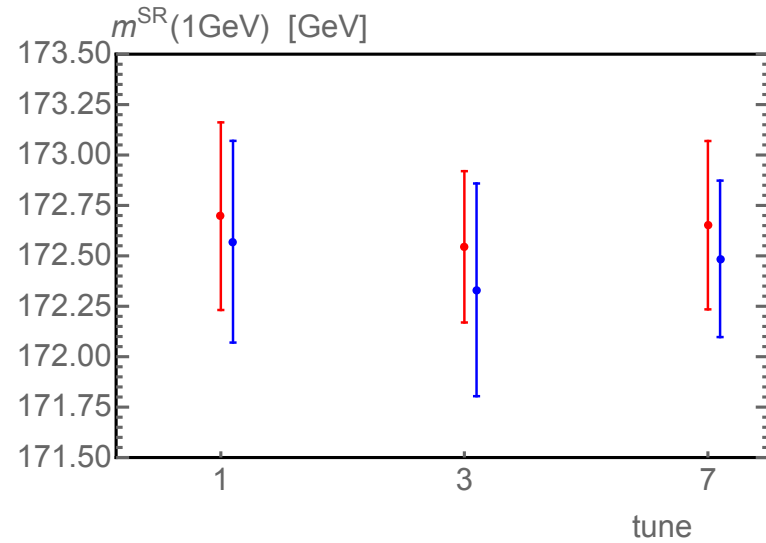
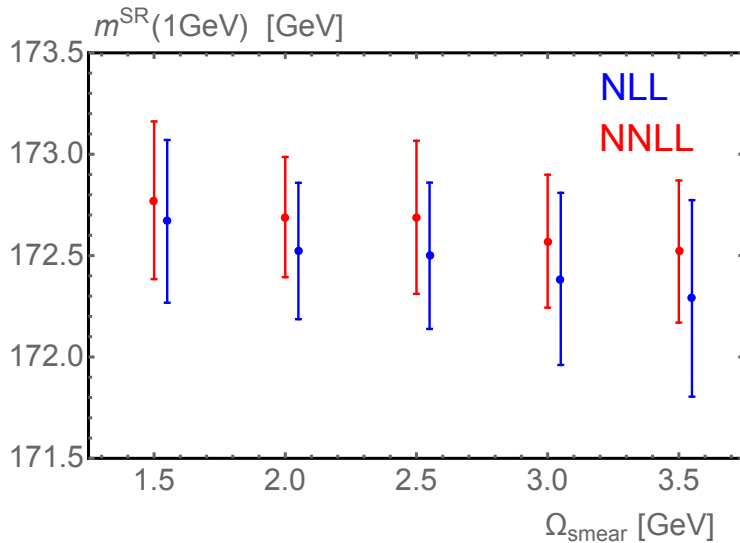
- Very low sensitivity of $m_t^{\text{MSR}}(5\text{GeV})$ on $\alpha_s(M_Z)$. ✓
- Large sensitivity of MSbar mass on $\alpha_s(M_Z)$. ✓

MC top mass indeed closely related to $m_t^{\text{MSR}}(R\sim 1 \text{ GeV})$!!

Peak Fits

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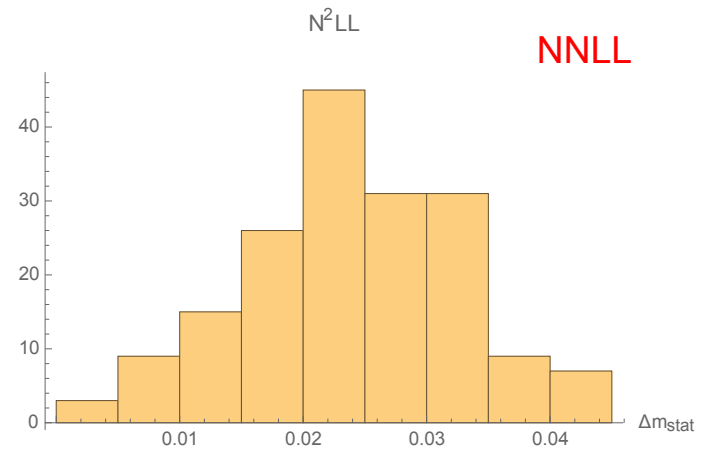
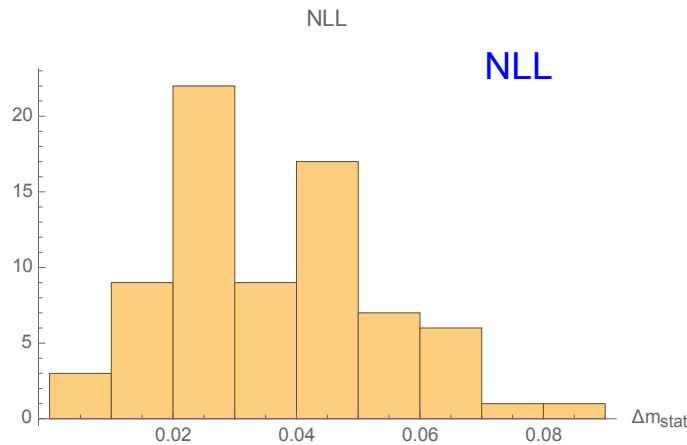
- “Detector effects” \ll perturbative uncertainty. ✓
- MC tune dependence \ll perturbative uncertainty. ✓

MC top mass indeed closely related to $m_t^{\text{MSR}}(R \sim 1 \text{ GeV})$!!

Peak Fits

First serious run: $\Gamma_t = 1.4$ GeV, tunes 1, 3, 7,
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NLL: 177 scan survivors, NNLL: 254 scan survivors

Preliminary



- “MC compatibility error” ~ tuning error ~ detector effect error ✓
- Effects < 100 MeV. (Maybe estimate for ultimate precision)

Peak Fits

First serious run:

$\Gamma_t = 1.4$ GeV,

tunes 1, 3, 7,

$\Omega_{1,\text{smear}} = 1.5, 2.0, 2.5, 3.0, 3.5$ GeV,

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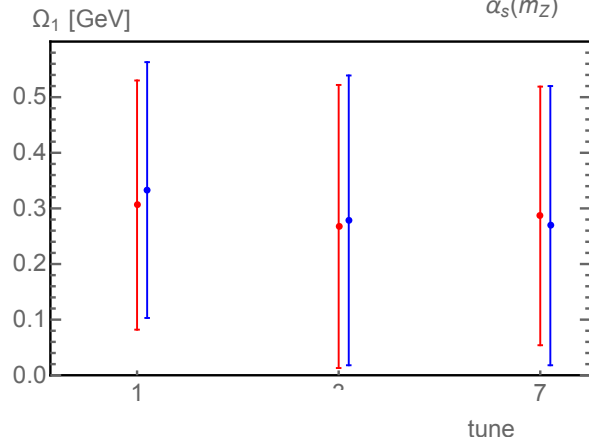
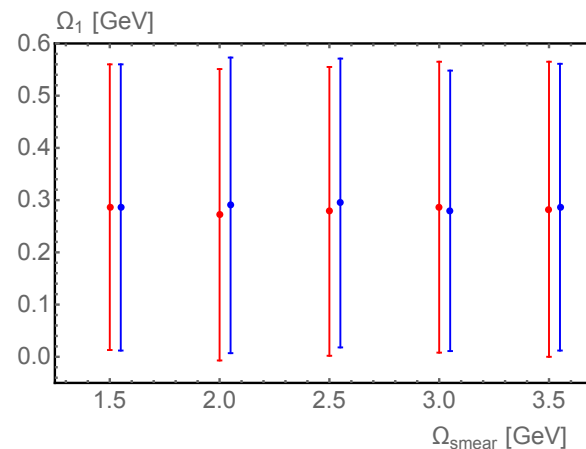
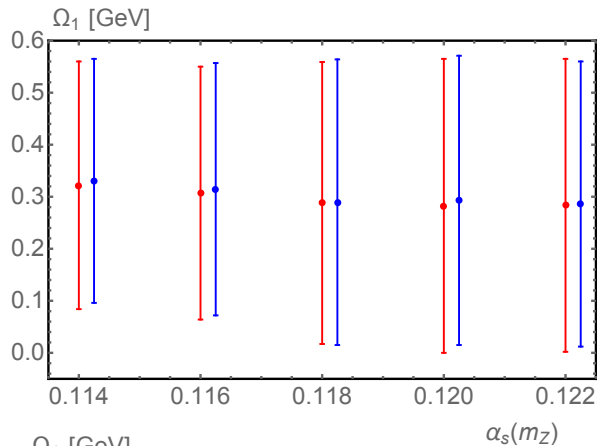
NLL

NNLL

$m_t^{\text{Pythia}} = 173$ GeV,

NLL: 177 scan survivors, NNLL: 254 scan survivors

Preliminary



- Reliable determination of non-perturbative matrix element Ω_1 (hadronization effects)
- Expected: $\delta m_t \sim \delta \Omega_1$ ✓
- Compatible with α_s -fits to e^+e^- data tail fits (Abbate et al, AHH et al.), larger err.

Peak Fits

First serious run:

$$\Gamma_t = 1.4 \text{ GeV},$$

tunes 1, 3, 7,

$$\Omega_{1,\text{smear}} = 1.5, 2.0, 2.5, 3.0, 3.5 \text{ GeV},$$

$$Q = \{700, 1000, 1400\} \text{ GeV}, \quad \text{peak fit (60/80)\%}$$

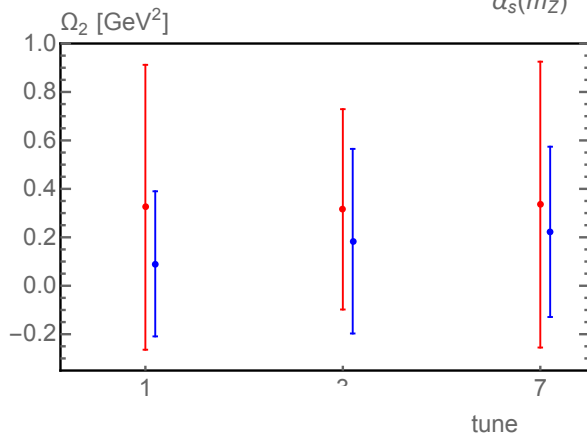
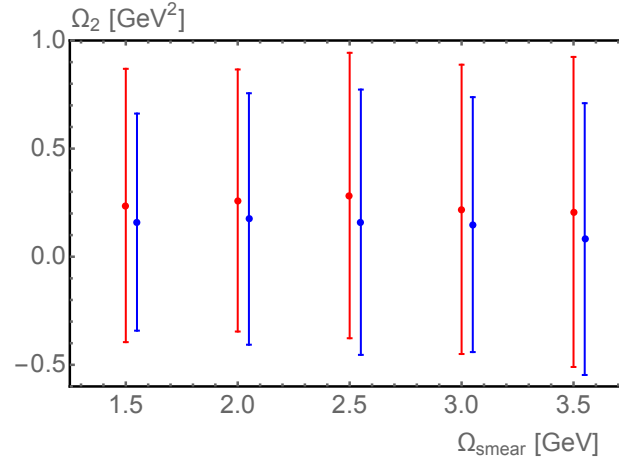
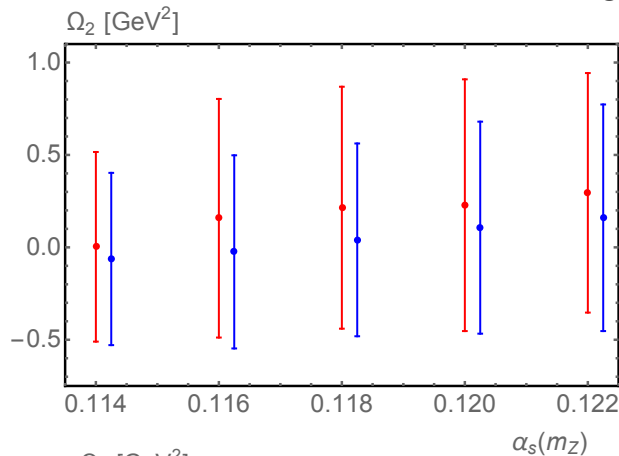
NLL

NNLL

$$m_t^{\text{Pythia}} = 173 \text{ GeV},$$

NLL: 177 scan survivors, NNLL: 254 scan survivors

Preliminary



- Reliable determination of non-perturbative matrix element Ω_2 (hadronization effects)
- Found to be have huge error as expected due to little sensitivity ✓
-

Peak Fits

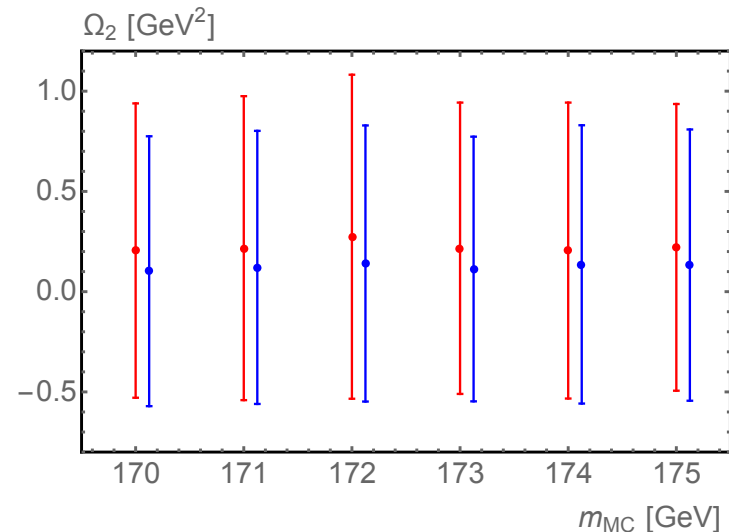
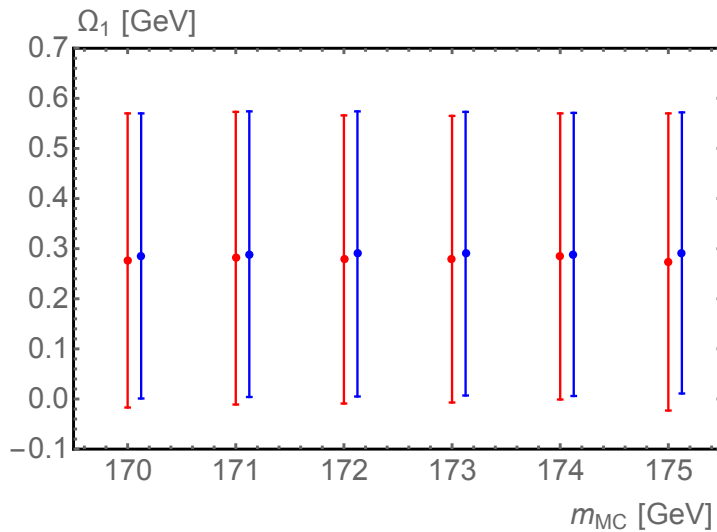
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$m_t^{\text{Pythia}}=170, 171, 172, 173, 174, 175$ GeV

NLL: 177 scan survivors, NNLL: 254 scan survivors

NLL
NNLL

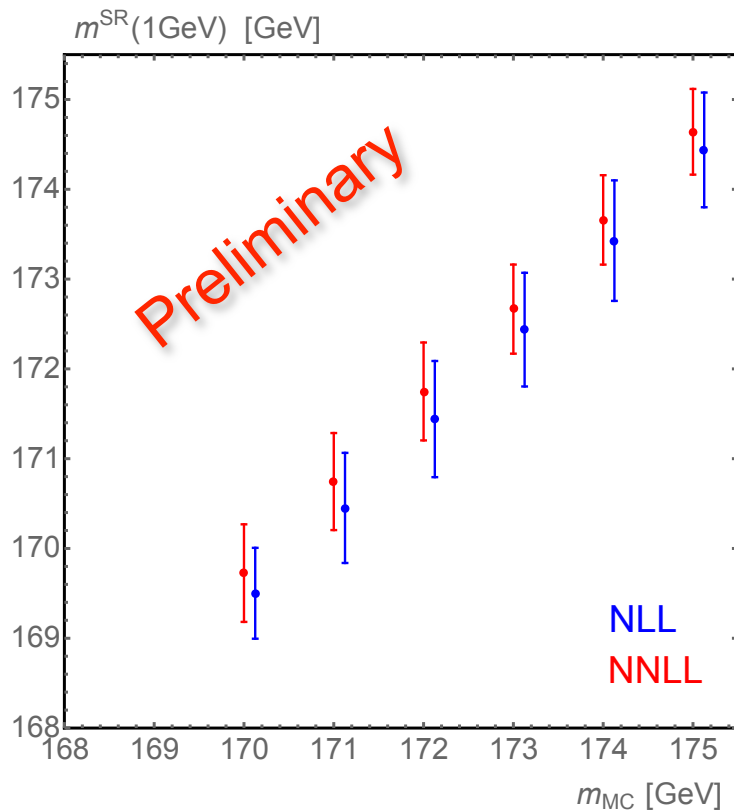
Preliminary



- Non-pert. matrix elements $\Omega_{1,2}$ independent of top mass. ✓

Peak Fits

First serious run: $\Gamma_t=1.4$ GeV, tunes 1, 3, 7,
 $\Omega_{1,\text{smear}}=1.5, 2.0, 2.5, 3.0, 3.5$ GeV,
 $Q=\{700, 1000, 1400\}$ GeV, peak fit (60/80)%
 $m_t^{\text{Pythia}}=170, 171, 172, 173, 174, 175$ GeV
NLL: 177 scan survivors, NNLL: 254 scan survivors



- Many more cross checks to be done.
- Calibration error: 0.5 GeV seems feasible at NNLL !

Conclusions & Outlook

- First serious precise MC top quark mass calibration based on e^+e^- 2-jettiness: preliminary results.
- NNLL+NLO QCD calculations based on an extension of the SCET approach concerning massive quark effects (all large logs incl. $\ln(m)$'s summed systematically).
- The Monte Carlo top mass calibration in terms of MSR mass with perturbative error $O(500 \text{ MeV})$ appears feasible at NNLL+NLO
- Intrinsic MC error seems $O(100 \text{ MeV})$.

Outlook:

- Full verified error analysis @ NNLL+NLO on the way
- Calibration for other MC generators
- Heavy jet mass, C-parameter (NNLL), pp-2 jettiness analysis (NLL) w.i.p.
- NNNLL+NNLO (2jettiness) w.i.p
- Mass (+ Yukawa coupling) conversions w. QCD + electroweak