

Parametric representation and dimensional regularization of noncommutative QFT

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in collaboration with:

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submitted to *Comm. Math. Phys.*

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(work in progress)

Grenoble, RPP 2007

Plan

- Introduction: noncommutativity and renormalizability; conventions
- Parametric representation
- Dimensional regularization
- Conclusions and perspectives

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Introduction and motivation: noncommutativity

noncommutativity of space-time

$$[x^\mu, x^\nu] \neq 0$$

NCQFT - one of the most appealing candidates for New Physics beyond the Standard Model

possible framework for the quantization of gravity

noncommutativity of space-time at (below) the Planck scale ?

string theory context:

effective theories for strings (*Seiberg-Witten map*)

better adapted for *non-local* effective interactions
(*fractional quantum Hall effect - FQHE*)

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how to chose between all possible Lagrangians and geometries?

we propose to use renormalizability

renormalizable theories - generic building blocks of physics

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we propose to use **renormalizability**

renormalizable theories - generic building blocks of physics

we place ourselves on the 4D Moyal space:

$$[x^\mu, x^\nu] = i\Theta^{\mu\nu}, \quad \Theta = \begin{pmatrix} \Theta_2 & 0 \\ 0 & \Theta_2 \end{pmatrix}, \quad \Theta_2 = \begin{pmatrix} 0 & -\theta \\ \theta & 0 \end{pmatrix}.$$

the Φ^4 model

$$\mathcal{L} = \frac{1}{2}\partial_\mu\Phi \star \partial^\mu\Phi + \frac{1}{4!}\Phi \star \Phi \star \Phi \star \Phi \star \Phi$$

\star - the Moyal product

UV/IR mixing; non-renormalizable ! (hep-th/9912072)

additional harmonic term - the Grosse-Wulkenhaar model

(hep-th/0305066, 0401128):

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi \star \partial^\mu\phi + \frac{\Omega^2}{2}(\tilde{x}_\mu\phi) \star (\tilde{x}^\mu\phi) + \frac{\lambda}{4}\phi \star \phi \star \phi \star \phi$$

where $\tilde{x}_\mu = 2(\Theta^{-1})_{\mu\nu}x^\nu$

modification of the propagator - the model becomes renormalizable!

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the Langmann-Szabo-Zarembo (LSZ) model

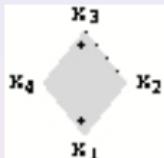
hep-th/0205287, 0308043, 0308042

scalar complex field ϕ

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\Phi + \Omega\tilde{x}_\mu\Phi)(-\partial^\mu\bar{\Phi} + \Omega\tilde{x}^\mu\bar{\Phi}) + \frac{1}{4!}\bar{\Phi} \star \Phi \star \bar{\Phi} \star \Phi$$

constant magnetic background

the contribution of a vertex V (in position space)



the Moyal product on the interaction term:

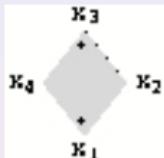
$$\delta(x_1^V - x_2^V + x_3^V - x_4^V) e^{2i \sum_{1 \leq i < j \leq 4} (-1)^{i+j+1} x_i^V \Theta^{-1} x_j^V}$$

- ↔ non-locality
- ↔ restricted invariance: only under cyclic permutation
- ribbon graphs
- clear distinction between planar and non-planar graphs

vertex $V \Leftrightarrow$ hypermomenta p_V

$$\delta(x_1^V - x_2^V + x_3^V - x_4^V) = \int \frac{dp_V}{(2\pi)^4} e^{p_V \sigma(x_1^V - x_2^V + x_3^V - x_4^V)}$$

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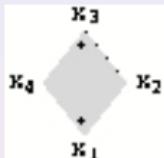
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Feynman graphs

n - number of vertices,

L - number of internal lines,

F - number of faces,

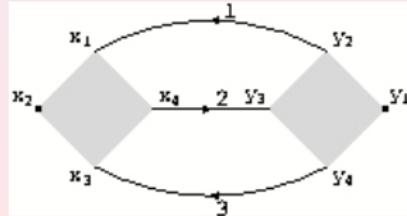
b - number of loops

$$2 - 2g = n - L + F, \quad b = F - 1 + 2g$$

$g \in \mathbb{N}$ - graph genus

$g = 0$ - planar graph

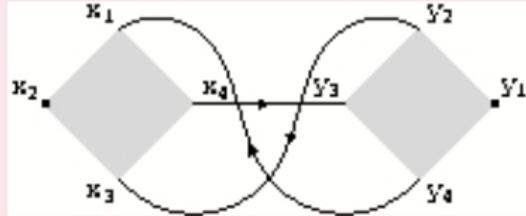
example:



$$n = 4, \quad L = 3, \quad F = 3, \quad g = 0$$

$g \geq 1$ - non-planar graph

example:



$$n = 4, \quad L = 3, \quad F = 1, \quad g = 1$$

(Spanning) tree

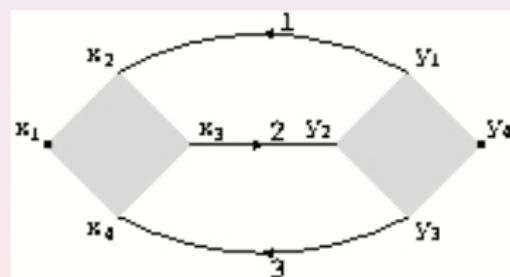
$n - 1$ internal lines, with no loop

root vertex \bar{V}

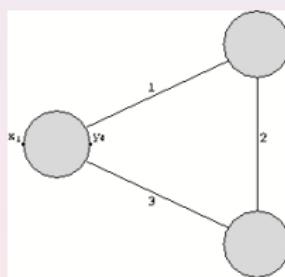
Feynman graphs - dual graph

face (in the direct graph) \Leftrightarrow vertex (in the dual graph)

example:



$$n = 2, F = 3, L = 3$$



$$n = 3, F = 2, L = 3$$

categories of NCQFT

depending on the propagator form in position space

(hep-th/0512071, 0512271, math-ph/0606069)

$$① C_\ell(x, y) = \int_0^\infty d\alpha_\ell \frac{\tilde{\Omega}(1-t_\ell)^2}{(4\pi t_\ell)^2} e^{-\frac{\tilde{\Omega}}{4} \frac{1}{t_\ell} (x-y)^2 - \frac{\tilde{\Omega}}{4} t_\ell (x+y)^2}$$

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$$\text{with } \tilde{\Omega} = \frac{2\Omega}{\theta}, \quad t_\ell = \tanh \frac{\alpha_\ell}{2}$$

2nd case: harder to investigate (the vertex and propagator contributions mix)

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Parametric representation

another way of approaching renormalization ...

→ natural frame to define
dimensional regularization and renormalization
(preserves the symmetries of gauge theories)

→ very important technical tool in QFT

Parametric representation for commutative QFT

$$\mathcal{A}(p) = \delta\left(\sum p\right) \int_0^\infty \frac{e^{-V(p,\alpha)/U(\alpha)}}{U(\alpha)^{\frac{D}{2}}} \prod_{\ell=1}^L d\alpha_\ell$$

U, V - polynomials of the α_ℓ 's

$$U = \sum_T \prod_{\ell \notin T} \alpha_\ell ,$$

\mathcal{T} - a (spanning) tree

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Parametric representation

- most condensed form of perturbation theory: positions and internal momenta are integrated out
- it leads to **trees**;
 “**democracy**” between them
- dimensionality of space-time - just a parameter

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Parametric representation for NCQFT

putting together the propagators and vertices of a graph:

$$\mathcal{A}_G = \int \prod_{\ell=1}^L \left[\frac{1-t_\ell^2}{t_\ell} \right]^{\frac{D}{2}} d\alpha \int dx dp e^{-\frac{\tilde{\Omega}}{2} X G X^t}$$

where

$$t_\ell = \tanh \frac{\alpha_\ell}{2}, \quad \ell = 1, \dots, L$$

$$X = (x_e \quad p_{\bar{V}} \quad x_i \quad p_V) \quad , \quad G = \begin{pmatrix} M & P \\ P^t & Q \end{pmatrix}$$

x_e — external positions

x_i — internal positions

p_V — hypermomenta associated to a vertex V

$p_{\bar{V}}$ — hypermomenta associated to the root vertex \bar{V}

Definition of the noncommutative polynomials

after Gaussian integration:

$$\mathcal{A} = \int_0^\infty \prod_{\ell=1}^L \left[\frac{1-t_\ell^2}{t_\ell} \right]^{\frac{D}{2}} d\alpha_\ell \frac{1}{\sqrt{\det Q}} e^{-\frac{\tilde{\Omega}}{2} \begin{pmatrix} x_e & p_{\bar{V}} \end{pmatrix} (M - PQ^{-1}P^t) \begin{pmatrix} x_e \\ p_{\bar{V}} \end{pmatrix}}$$

we put:

$$\mathcal{A}_G = K \int_0^\infty \prod_{\ell} [d\alpha_\ell (1-t_\ell^2)^{\frac{D}{2}}] \frac{1}{HU(t)^{\frac{D}{2}}} e^{-\frac{HV(t, x_e)}{HU(t)}}$$

HU and HV are polynomials in t_ℓ

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The noncommutative polynomial HU

$$HU = (\det Q)^{\frac{1}{D}} \prod_{\ell=1}^L t_\ell$$

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↪ development of the determinant

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↪ development of the determinant

identification of **leading terms** (lowest global degree in the t_ℓ variables)

(UV regime)

$J_0 = \{\ell_1, \dots, \ell_{F-1}\}$ - **admissible set**, i.e.

- it contains a tree \tilde{T} in the dual graph and
- its complement contains a tree T in the direct graph

$$HU(t_\ell) \geq \sum_{J_0 \text{ admissible}} s^{2[g+(F-1)]} \left(2^g \prod 2(\Omega \pm 1) \right)^2 \\ \prod_{\ell \in \{1, \dots, L\} \setminus J_0} \frac{1 + t_\ell^2}{2t_\ell} \prod_{\ell' \in \{1, \dots, L\}}$$

where $s = \frac{2}{\theta\Omega}$

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$$\omega = 4g + \frac{N - 4}{2}$$

- it leads to “*admissible subgraphs*”
“democracy” between them
- space-time dimension is just a parameter → dimensional regularization

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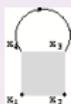
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Some examples

- one loop



$$HU = 4s^2t(\Omega + 1)^2$$



$$HU = 4s^2t(\Omega - 1)^2$$

- bubble graph



$$HU = 2s^2(t_1 + t_2 + t_1^2 t_2 + t_1 t_2^2)(\Omega - 1)^2$$

Dimensional regularization

- analytical continuation for $D \in \mathbb{C}$
- definition of an appropriate subtraction operator R
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- other models, other non-commutative geometries ...

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Je vous remercie pour votre
attention

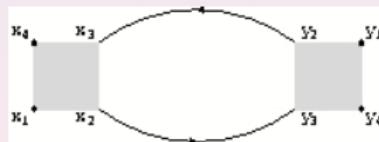
power counting:

$$\omega = 4g + \frac{N - 4}{2} + (B - 1)$$

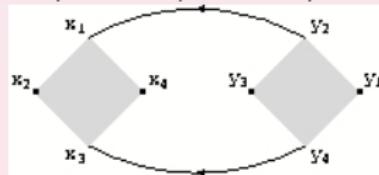
B - number of broken faces

improved factor in the broken faces

example:



$$n = 2, L = 2, F = 2, B = 1$$



$$n = 2, L = 2, F = 2, B = 2$$