Lepton distribution from top-quark

A probe of new physics and top-polarization

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by

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top-quark : A looking glass

The mass of the top-quark is very large ($m_t \sim 175$ GeV)

- top-mass being close to electro-weak scale, its couplings are sensitive to EWSB. Any new physics of EWSB (or mass generation) affects top-couplings with other particles.

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We have a clean looking glass for new physics.

Anomalous t-decay

Anomalous tbW vertex :

$$\Gamma^\mu = \frac{g}{\sqrt{2}} \left[\gamma^\mu (f_{1L} P_L + f_{1R} P_R) - \frac{i\sigma^{\mu\nu}}{m_W} (p_t - p_b)_\nu \, (f_{2L} P_L + f_{2R} P_R)\right]$$

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- In the SM, $f_{1L} = 1$, $f_{1R} = 0$, $f_{2L} = 0$, $f_{2R} = 0$.
- Contribution from $f_{1R}, \; f_{2L}$ are proportional to m_b.

Lepton distribution

$$AB \longrightarrow t \; P_1 \ldots P_{n-1}$$
$$\quad\quad\quad\quad \hookrightarrow b \; W^+$$
$$\quad\quad\quad\quad\quad\quad\quad \hookrightarrow l^+ \nu$$

Lepton distribution is independent of anomalous tbW coupling if

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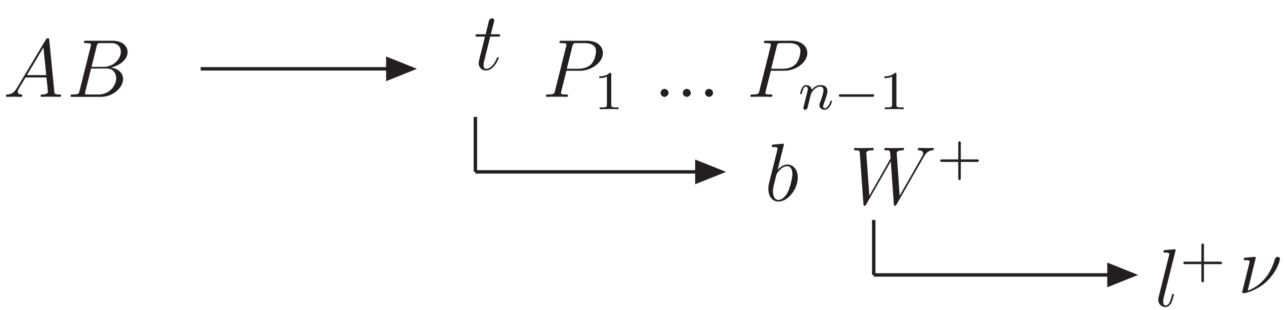

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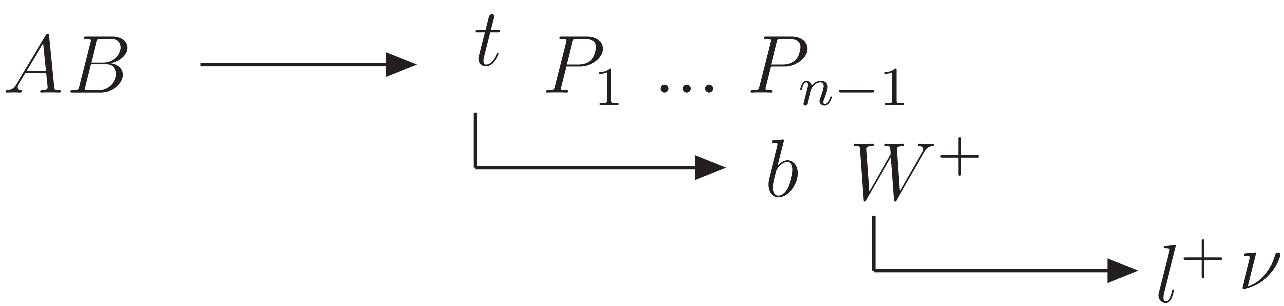

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- anomalous couplings f_{1R}, f_{2R} and f_{2L} are small,
- narrow-width approximation for W-boson,
- b-quark is mass-less,
- $t \to bW(\ell\nu_\ell)$ is the only decay channel for t-quark.

Decay distribution

Narrow-width approximation for t-quark $\Rightarrow$

$$\overline{|\mathcal{M}|^2} = \frac{\pi\delta(p_t^2 - m_t^2)}{\Gamma_t m_t} \sum_{\lambda,\lambda'} \rho(\lambda,\lambda')\Gamma(\lambda,\lambda')$$

where,

$$\rho(\lambda,\lambda') = M_\rho(\lambda)\, M_\rho^*(\lambda') \quad \text{and} \quad \Gamma(\lambda,\lambda') = M_\Gamma(\lambda)\, M_\Gamma^*(\lambda').$$

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$$\begin{aligned} d\sigma \;=\; & \sum_{\lambda,\lambda'} \left[\frac{(2\pi)^4}{2I} \rho(\lambda,\lambda') \delta^4(k_A + k_B - p_t - \sum_i^{n-1} p_i) \frac{d^3 p_t}{2E_t (2\pi)^3} \prod_i^{n-1} \frac{d^3 p_i}{2E_i (2\pi)^3} \right] \\ \times \; & \left[\frac{1}{\Gamma_t} \left(\frac{(2\pi)^4}{2m_t} \Gamma(\lambda,\lambda') \delta^4(p_t - p_b - p_\nu - p_\ell) \frac{d^3 p_b}{2E_b (2\pi)^3} \frac{d^3 p_\nu}{2E_\nu (2\pi)^3} \right) \frac{d^3 p_\ell}{2E_\ell (2\pi)^3} \right]. \end{aligned}$$

Decay distribution

Production part ($\phi_t = 0$) :

$$\int \frac{d^3 p_t}{2E_t (2\pi)^3} \prod_i^{n-1} \frac{d^3 p_i}{2E_i (2\pi)^3} \; \frac{(2\pi)^4}{2I} \rho(\lambda, \lambda') \delta^4 \left(k_A + k_B - p_t - \left(\sum_i^{n-1} p_i \right) \right)$$

$$= d\sigma_{2\to n}(\lambda, \lambda') \; dE_t \; d\cos\theta_t .$$

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Decay part (in rest rest frame of t-quark) :

$$\frac{1}{\Gamma_t}\;\frac{(2\pi)^4}{2m_t}\int \frac{d^3p_\ell}{2E_\ell(2\pi)^3}\;\frac{d^3p_b}{2E_b(2\pi)^3}\;\frac{d^3p_\nu}{2E_\nu(2\pi)^3}\;\Gamma(\lambda,\lambda')\delta^4(p_t - p_b - p_\nu - p_\ell)$$

$$= \frac{1}{32\Gamma_t m_t}\;\frac{E_\ell}{(2\pi)^4}\,\frac{\langle\Gamma(\lambda,\lambda')\rangle}{m_t E_\ell}\;dE_\ell\; d\Omega_\ell\; dp_W^2\ .$$

Angular brackets stands for averaging over $\phi = (\phi_b - \phi_\ell)$.

Decay density matrix

In the rest frame of t-quark, we have

$$\begin{aligned}
\langle\Gamma(\pm,\pm)\rangle &= g^4 m_t E_\ell^0\, |\Delta_W(p_W^2)|^2\, (1 \pm \cos\theta_l) \;\times\; F(E_\ell^0),\\
\langle\Gamma(\pm,\mp)\rangle &= g^4 m_t E_\ell^0\, |\Delta_W(p_W^2)|^2\, (\sin\theta_l e^{\pm i\phi_l}) \;\times\; F(E_\ell^0).
\end{aligned}$$

where $\Delta_W(p_W^2) = \frac{1}{p_W^2 - m_W^2 + i\Gamma_W m_W}$

$$\begin{aligned}
F(E_\ell^0) &= \Bigg[(m_t^2 - m_b^2 - 2p_t\cdot p_l)\left(|f_{1L}|^2 + \Re(f_{1L}f_{2R}^*)\frac{m_t}{m_W}\frac{p_W^2}{p_t.p_l}\right)\\
&- \quad 2\Re(f_{1L}f_{2L}^*)\,\frac{m_b}{m_W}\,p_W^2 - \Re(f_{1L}f_{1R}^*)\,\frac{m_b\,m_t}{p_t.p_l}\,p_W^2\Bigg]
\end{aligned}$$

In general,

$$\langle\Gamma(\lambda,\lambda')\rangle = (m_t E_\ell^0)\,|\Delta(p_W^2)|^2\, g^4\, A(\lambda,\lambda')\, F(E_\ell^0)$$

Angular distribution of lepton

Combining production and decay part, we have

$$
\begin{aligned}
d\sigma &= \frac{1}{32\,\Gamma_t m_t (2\pi)^4}\left[\sum_{\lambda,\lambda'} d\sigma_{2\to n}(\lambda,\lambda') \times g^4 A^{c.m.}(\lambda,\lambda')\right] \\
&\times \; dE_t \, d\cos\theta_t \, d\cos\theta_\ell \, d\phi_\ell \\
&\times \; E_\ell \, F(E_\ell) \, |\Delta(p_W^2)|^2 \, dE_\ell \, dp_W^2
\end{aligned}
$$

and

$$
\Gamma_t \propto \int E_\ell \, F(E_\ell) \, |\Delta(p_W^2)|^2 \, dE_\ell \, dp_W^2
$$

Contribution from anomalous tbW couplings cancels between numerator and denominator, if $t \to bW$ is the only decay channel.

$\Rightarrow$ Lepton angular distribution is independent of anomalous tbW interactions.

Energy distribution of lepton

The E_ℓ distribution (in the lab frame) depends both on

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The E_ℓ^0 distribution (in the top-rest-frame) depends only on the possible **new physics in t-decay.**

$$\frac{d\sigma}{dE_\ell^0} \propto \int E_l^0 F(E_l^0)\, |\Delta(p_W^2)|^2 \, dp_W^2$$

Independent of production mechanism of t-quark !!

Polarization of t-quark

Polarized cross-sections :

$$\int \frac{d^3p_t}{2E_t(2\pi)^3} \left(\prod_{i=1}^{n-1} \frac{d^3p_i}{2E_i(2\pi)^3} \right) \frac{(2\pi)^4}{2I} \rho(\lambda,\lambda') \; \delta^4 \left(k_A + k_B - p_t - \left(\sum_{i=1}^{n-1} p_i \right) \right) = \sigma(\lambda,\lambda').$$

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Total cross-section :

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Polarization density matrix :

$$P_t = \frac{1}{2} \begin{pmatrix} 1+\eta_3 & \eta_1 - i\eta_2 \\ \eta_1 + i\eta_2 & 1-\eta_3 \end{pmatrix},$$

$$\eta_3 = \left(\sigma(+,+) - \sigma(-,-)\right)/\sigma_{tot}$$

$$\eta_1 = \left(\sigma(+,-) + \sigma(-,+)\right)/\sigma_{tot}$$

$$i\,\eta_2 = \left(\sigma(+,-) - \sigma(-,+)\right)/\sigma_{tot}$$

Polarization of t-quark

Polarization through leptonic decay of t-quark :

$$\frac{\eta_3}{2} = \frac{\sigma(p_\ell.s_3 < 0) - \sigma(p_\ell.s_3 > 0)}{\sigma(p_\ell.s_3 < 0) + \sigma(p_\ell.s_3 > 0)}$$

$$\frac{\eta_2}{2} = \frac{\sigma(p_\ell.s_2 < 0) - \sigma(p_\ell.s_2 > 0)}{\sigma(p_\ell.s_2 < 0) + \sigma(p_\ell.s_2 > 0)}$$

$$\frac{\eta_1}{2} = \frac{\sigma(p_\ell.s_1 < 0) - \sigma(p_\ell.s_1 > 0)}{\sigma(p_\ell.s_1 < 0) + \sigma(p_\ell.s_1 > 0)}$$

$s_i.s_j = -\delta_{ij} \qquad p_t.s_i = 0$

For $p_t^\mu = E_t(1, \beta_t \sin\theta_t, 0, \beta_t \cos\theta_t)$, we have

$$s_1^\mu = (0, -\cos\theta_t, 0, \sin\theta_t),\ s_2^\mu = (0,0,1,0),\ s_3^\mu = E_t(\beta_t, \sin\theta_t, 0, \cos\theta_t)/m_t.$$

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requires reconstruction of t-production plane;

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Angular distribution in lab frame can be used as a qualitative measure of the t-polarization.

Polarization through angular distribution

For demonstration, we chose $\gamma\gamma \to t\bar{t}$ process with/without Higgs exchange contribution.

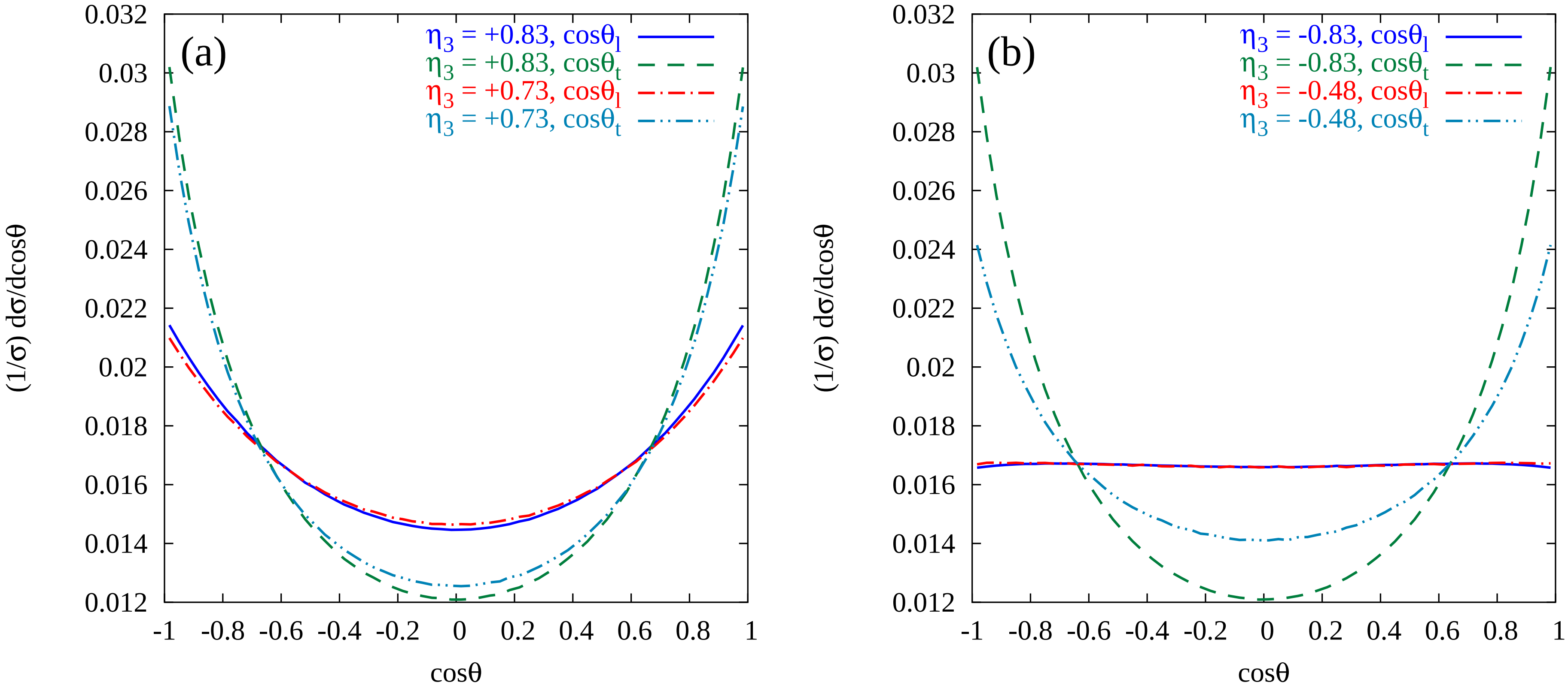
$m_\phi = 500\,\text{GeV}; \Gamma_\phi = 2.5\,\text{GeV},$
$S_t = 0.2,\ P_t = 0.4,\ S_\gamma = 4.0 + i\,0.5$ and $P_\gamma = 1.25 + i\,2.0.$

Polarized ideal photon spectrum is used.

Assumptions :

- t-quark is on-shell
- anomalous tbW couplings are small
- W-boson is on-shell
- b-quark is mass-less and
- $t \to bW$ is the only decay channel for t-decay

Polarization through angular distribution



$\eta_1 = 0$ and $\eta_2 = 0$

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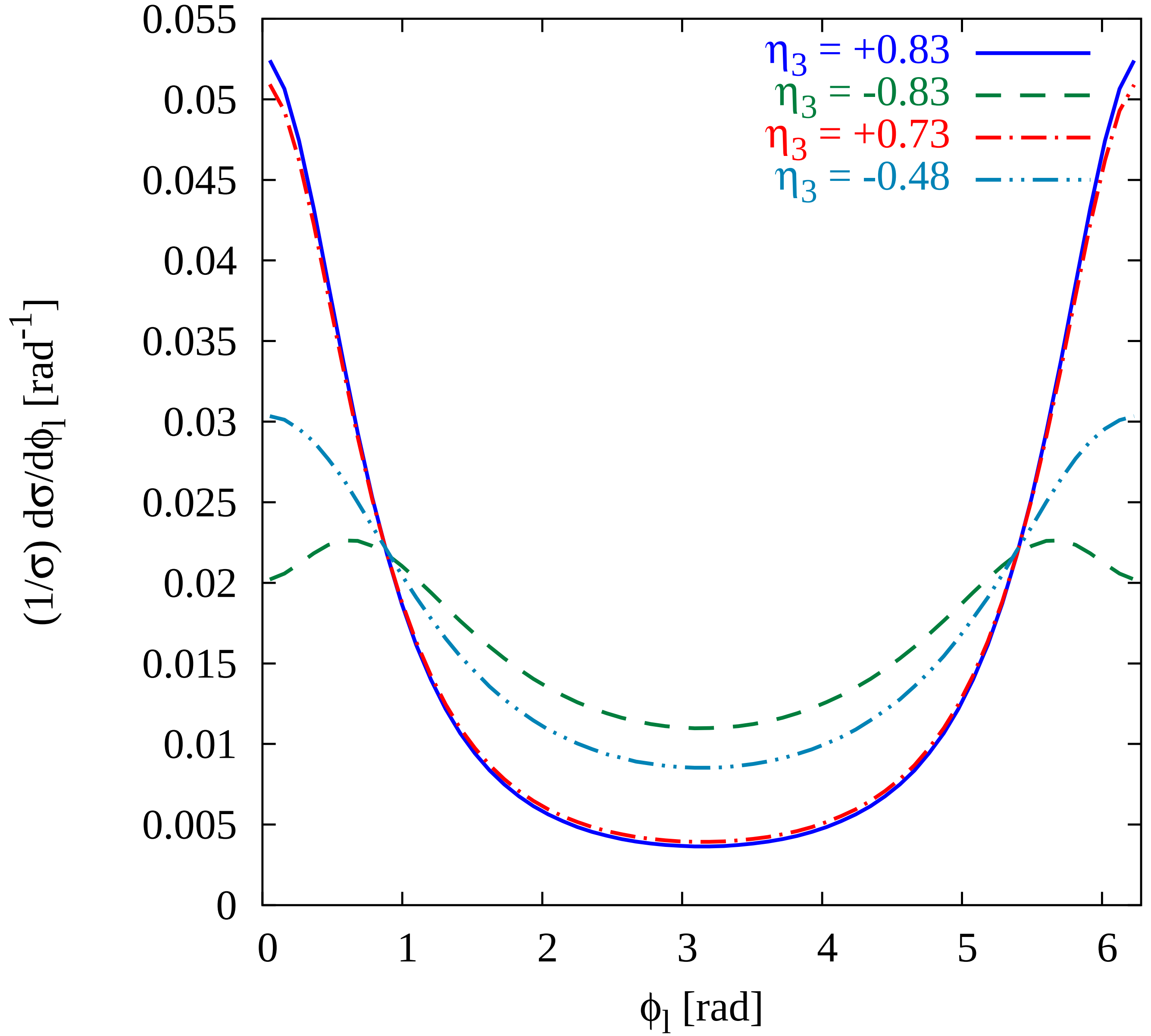

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- Polarization of t-quark can be measured (quantitatively) through angular asymmetries of decay leptons.
- Angular distribution of decay lepton in the lab-frame is a good qualitative probe of t-polarization; quantitatively better for negative polarizations.