Ultra-relativistic Heavy Ion Collisions

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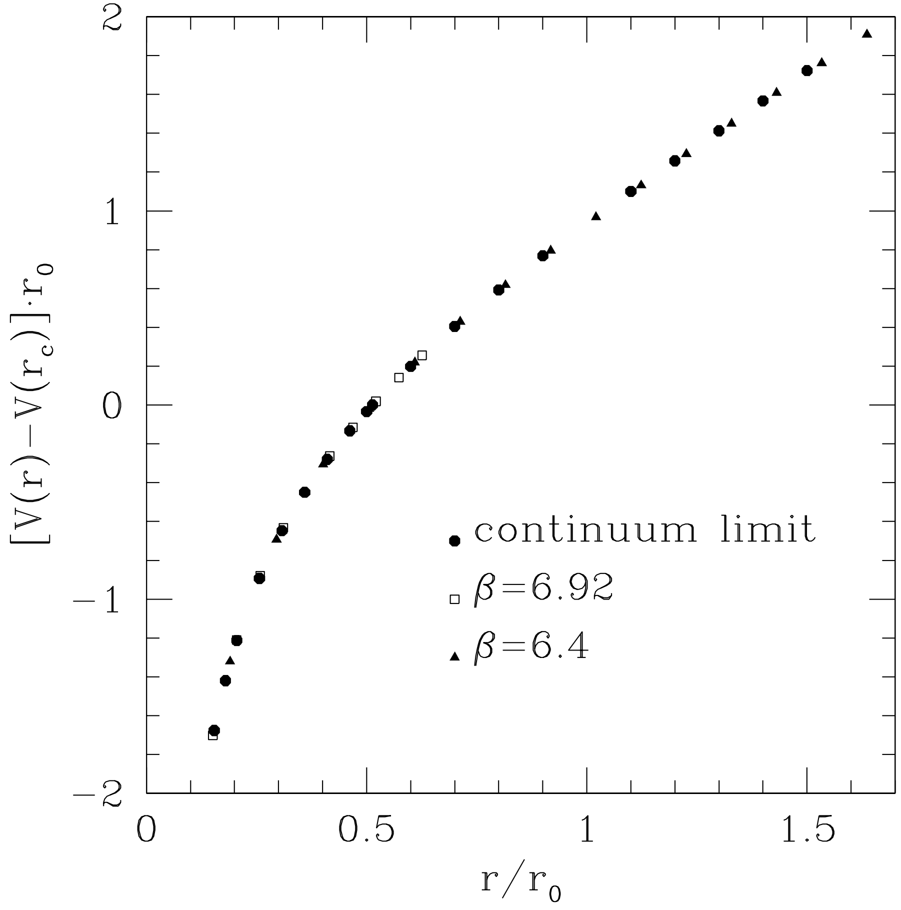
Outline

- Deconfinement transition
- Heavy ion collisions
- Physics of the QGP
- Experimental signatures
- ADS/CFT duality and the QGP
- Is the QGP a perfect fluid?
- Color Glass Condensate, and formation of the QGP

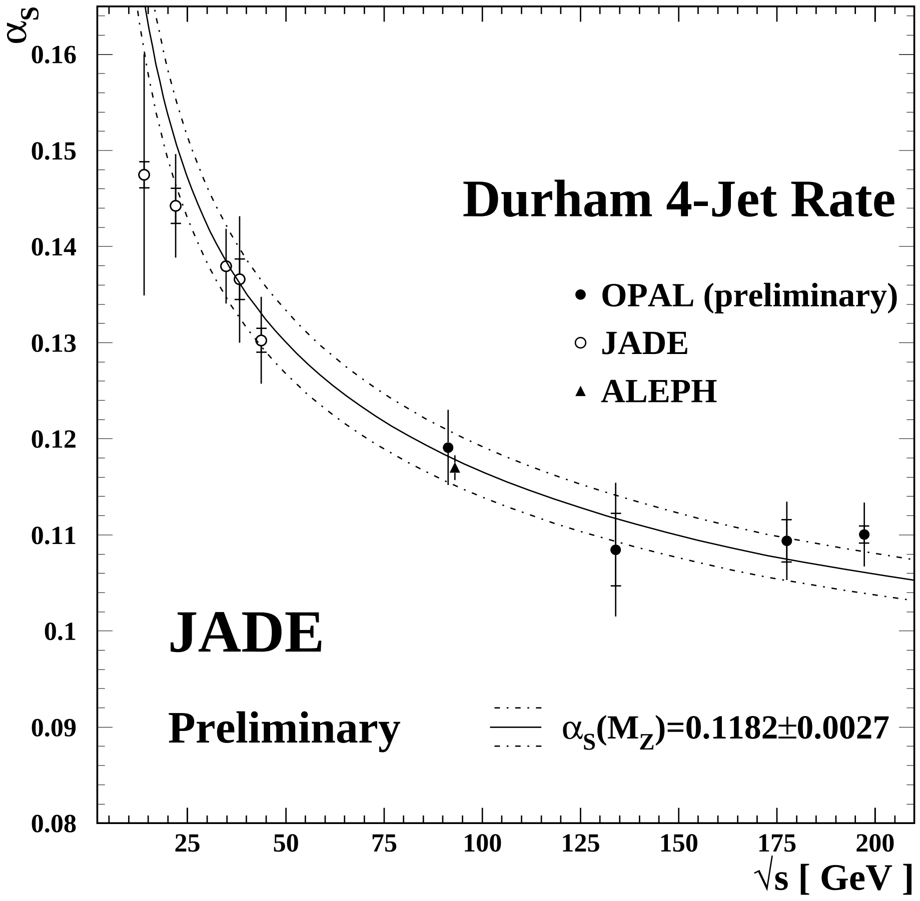
Deconfinement transition

Quark confinement



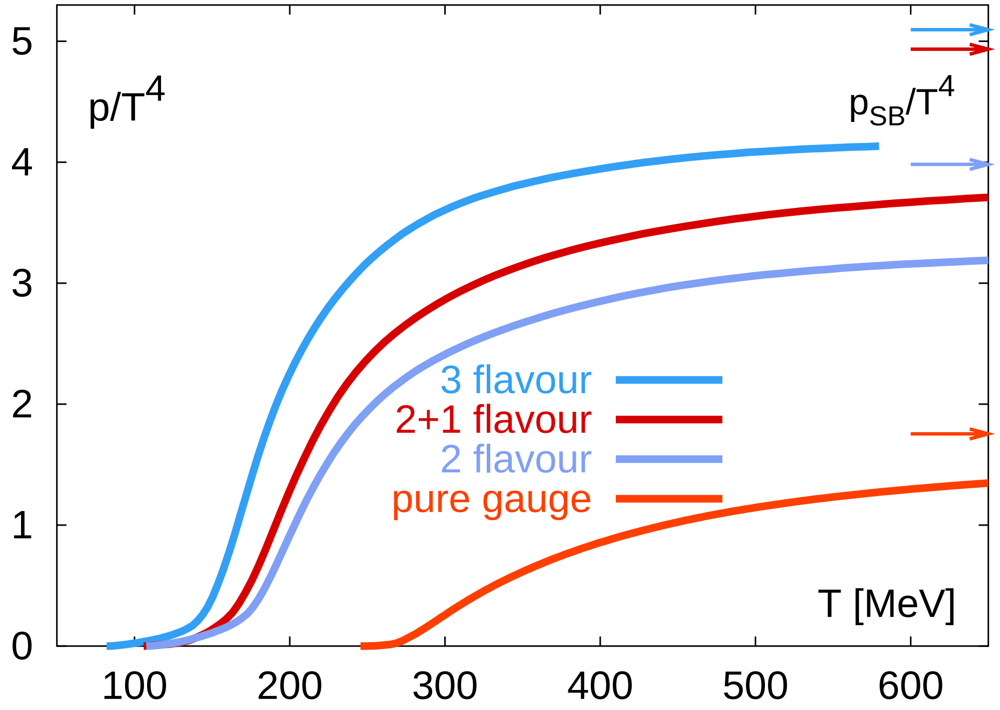
- The quark potential increases linearly with distance
- Quarks are confined into color singlet hadrons

Asymptotic freedom



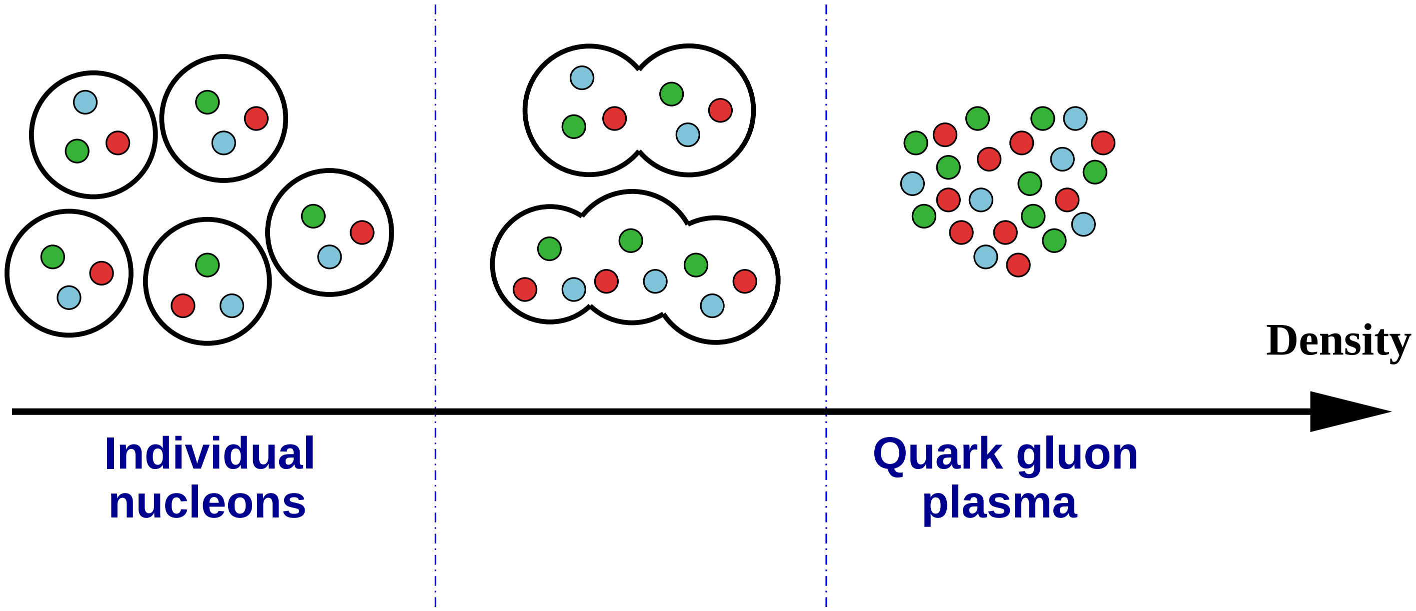
- The coupling constant is small at short distances
- At high density, a hadron gas may undergo deconfinement
  ▷ quark gluon plasma

Deconfinement



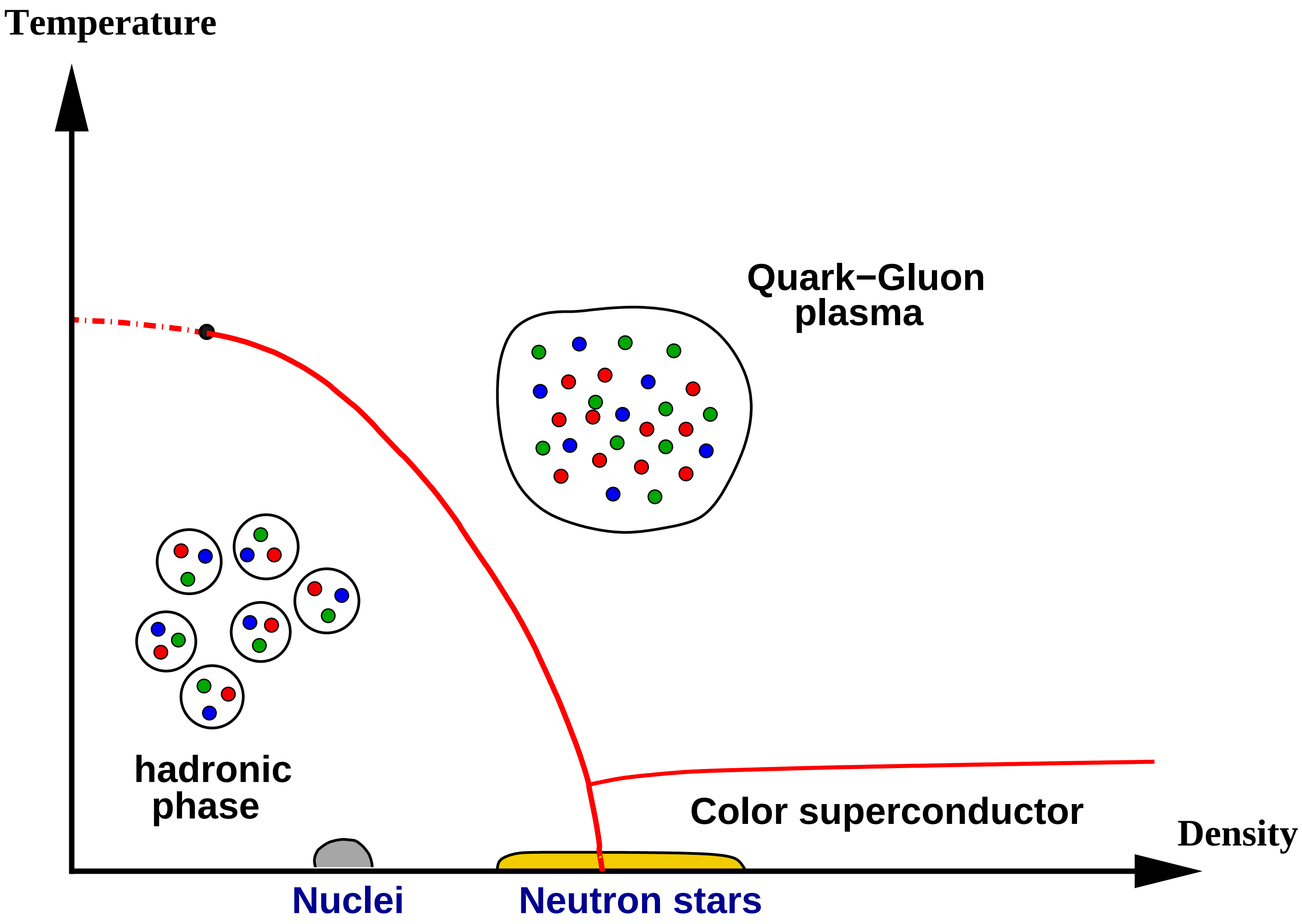
- Fast increase of the pressure :
  - at $T \sim 270$ MeV, if there are only gluons
  - at $T \sim 150$–180 MeV, depending on the number of light quarks

Deconfinement

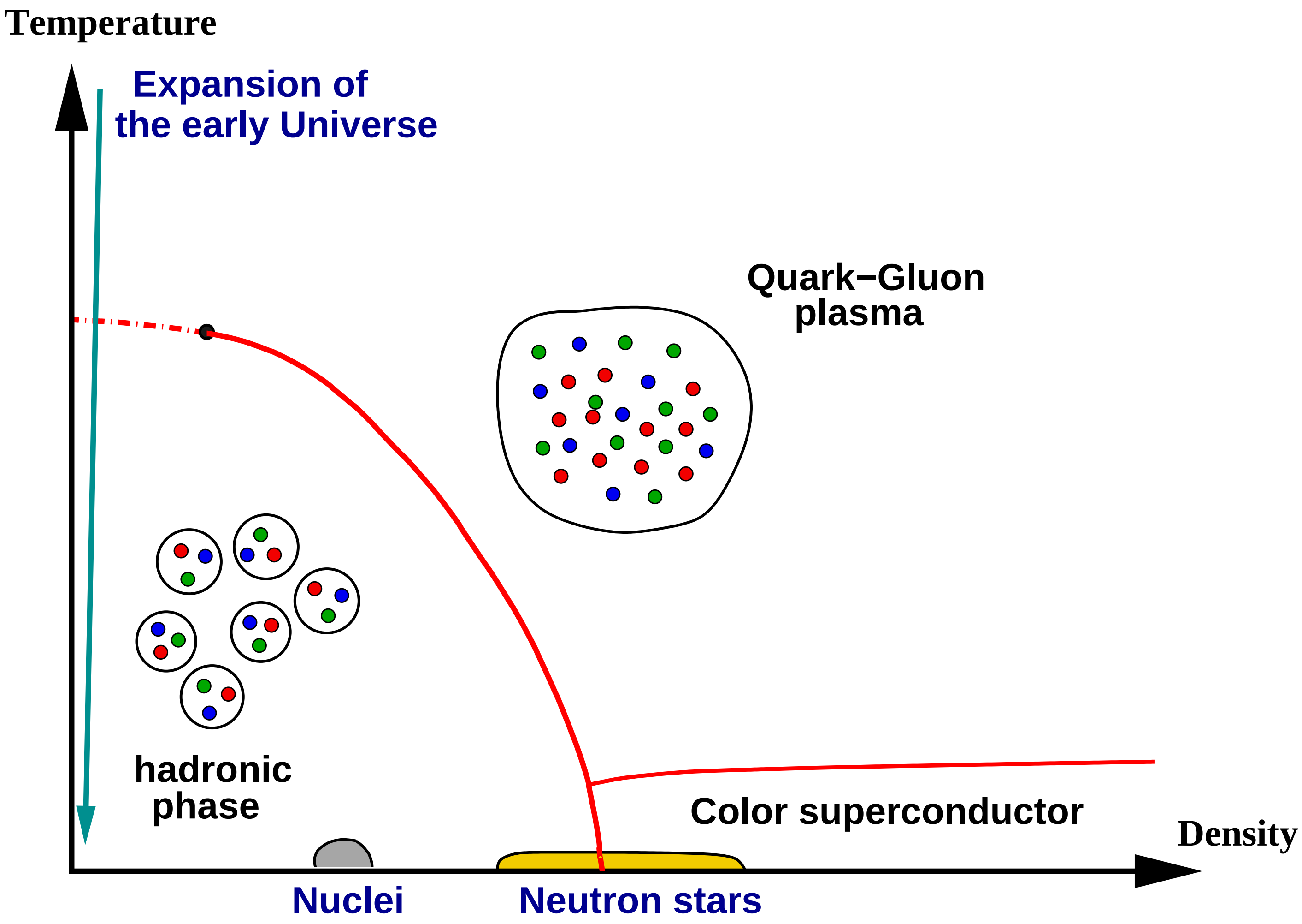


- When the nucleon density increases, they merge, enabling quarks and gluons to hop freely from a nucleon to its neighbors
- This phenomenon extends to the whole volume when the phase transition ends
- Note: if the transition is first order, it goes through a mixed phase containing a mixture of nucleons and plasma

QCD phase diagram



The QGP in the early universe



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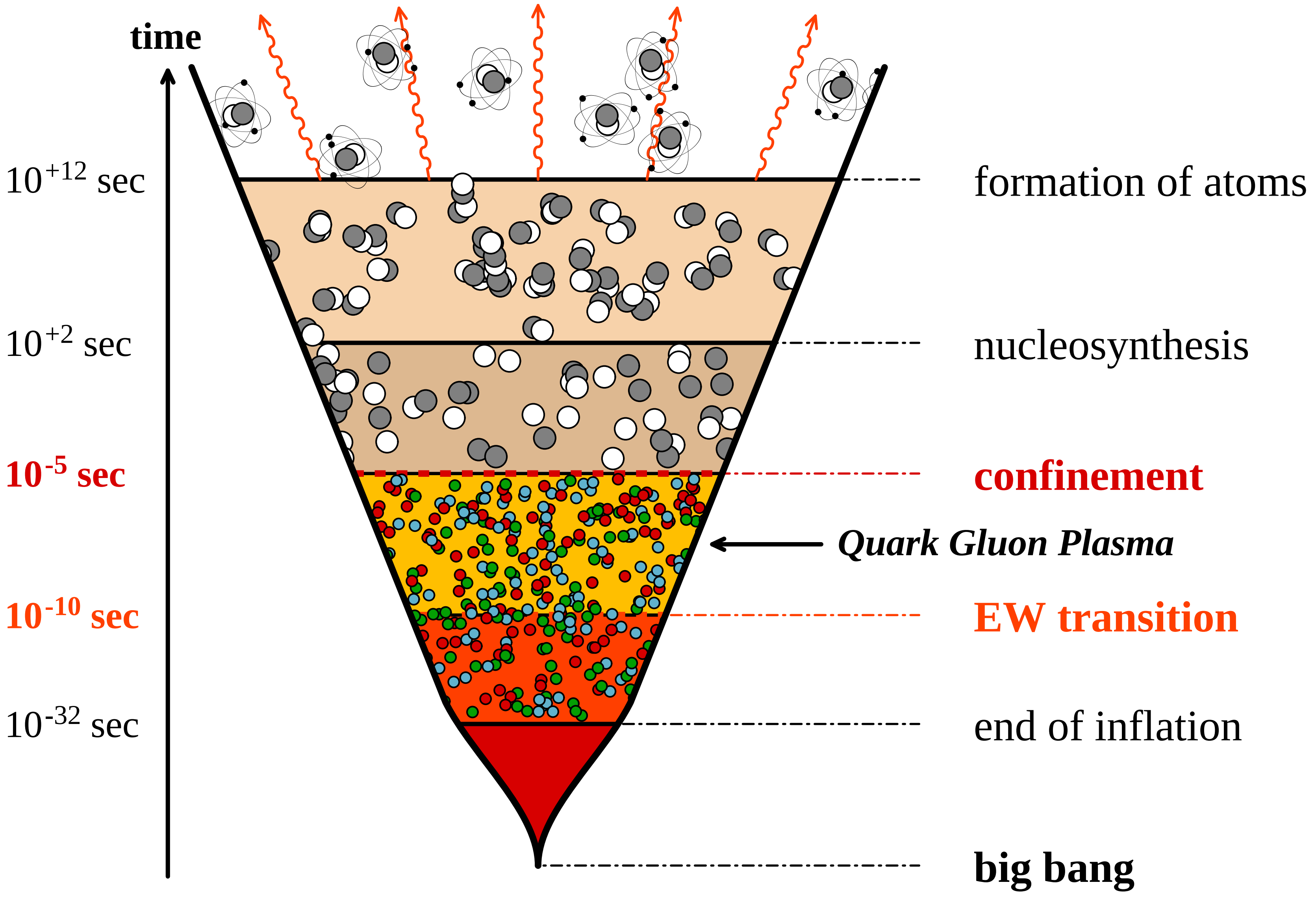
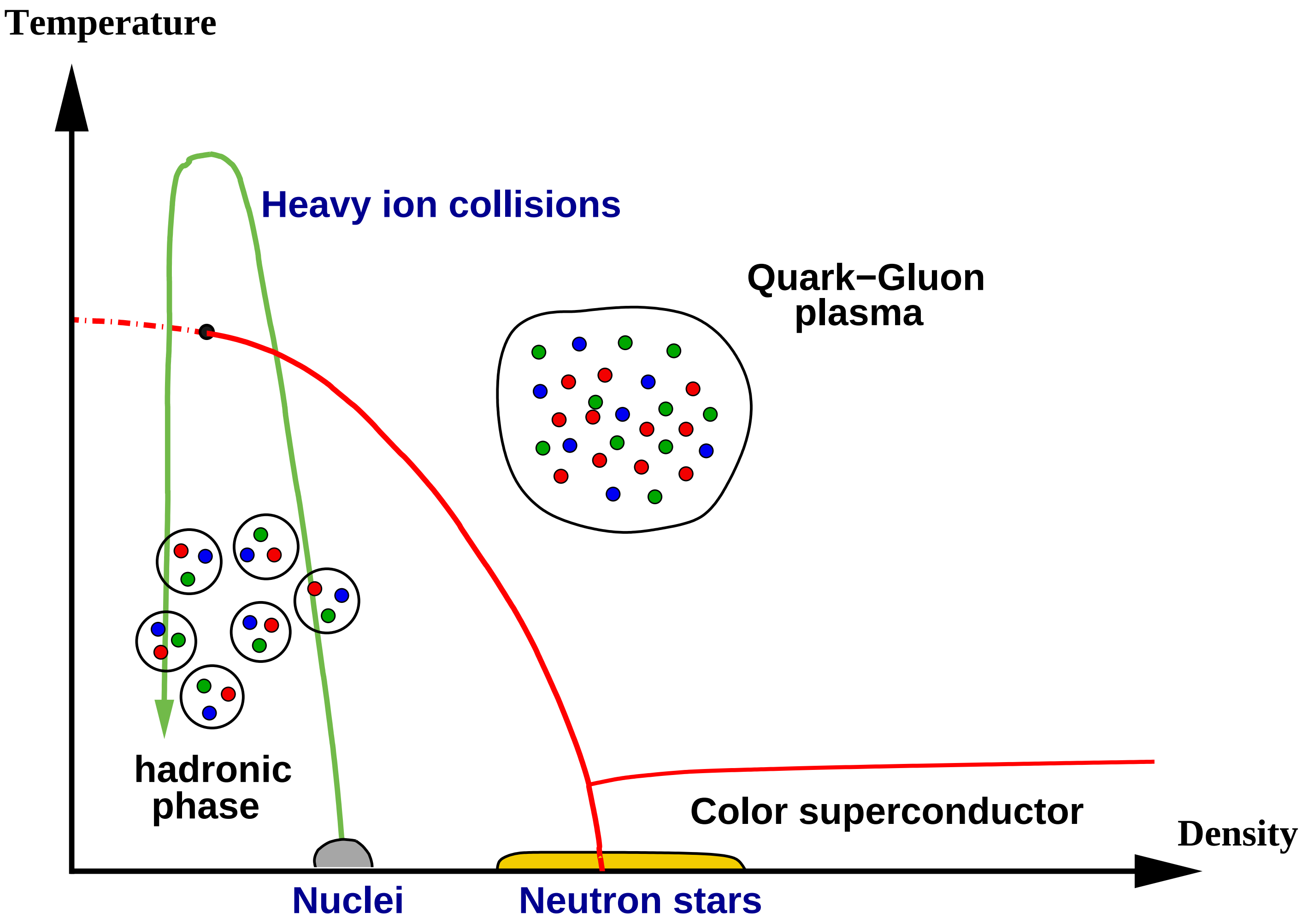
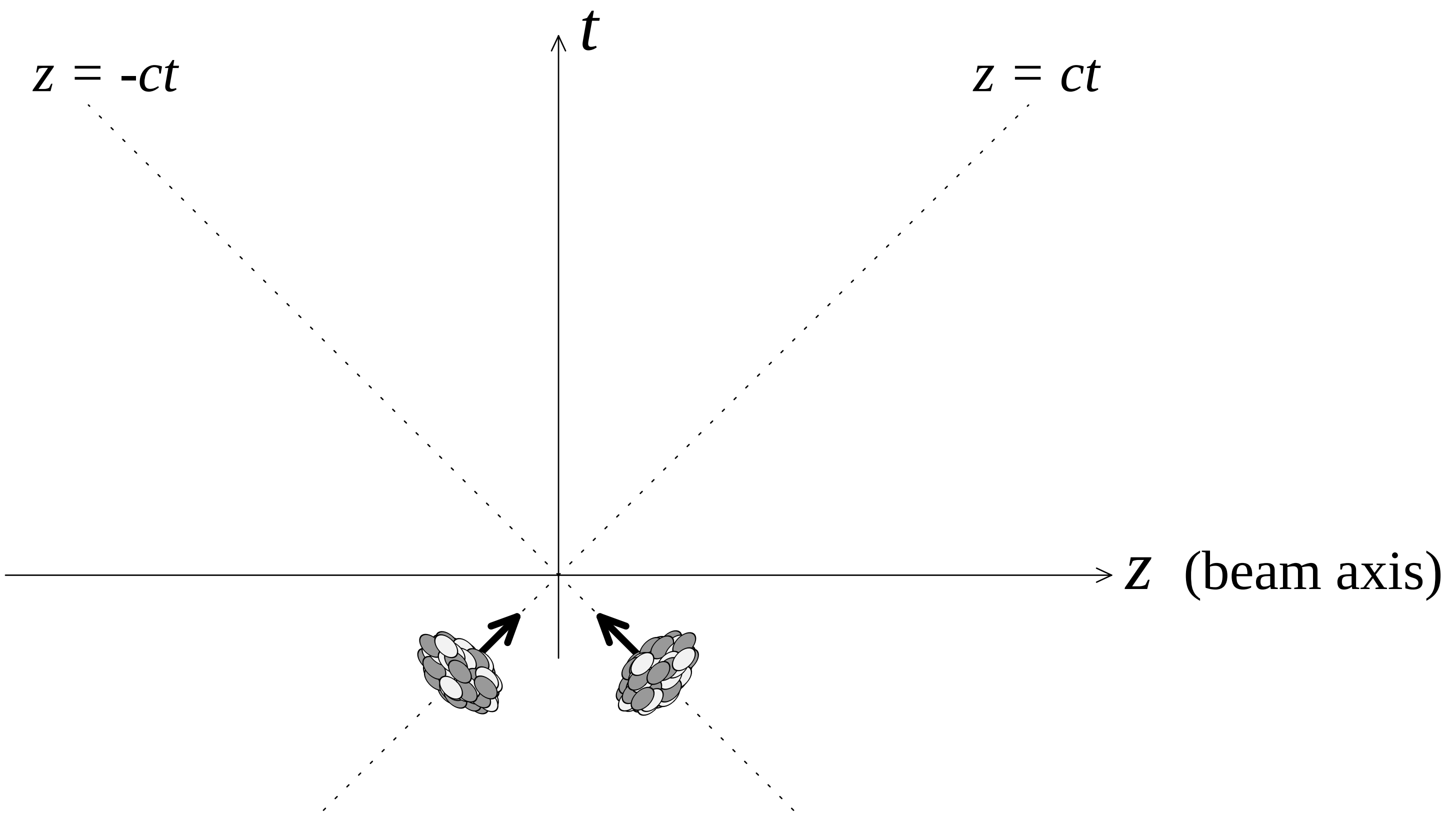


Heavy ion collisions

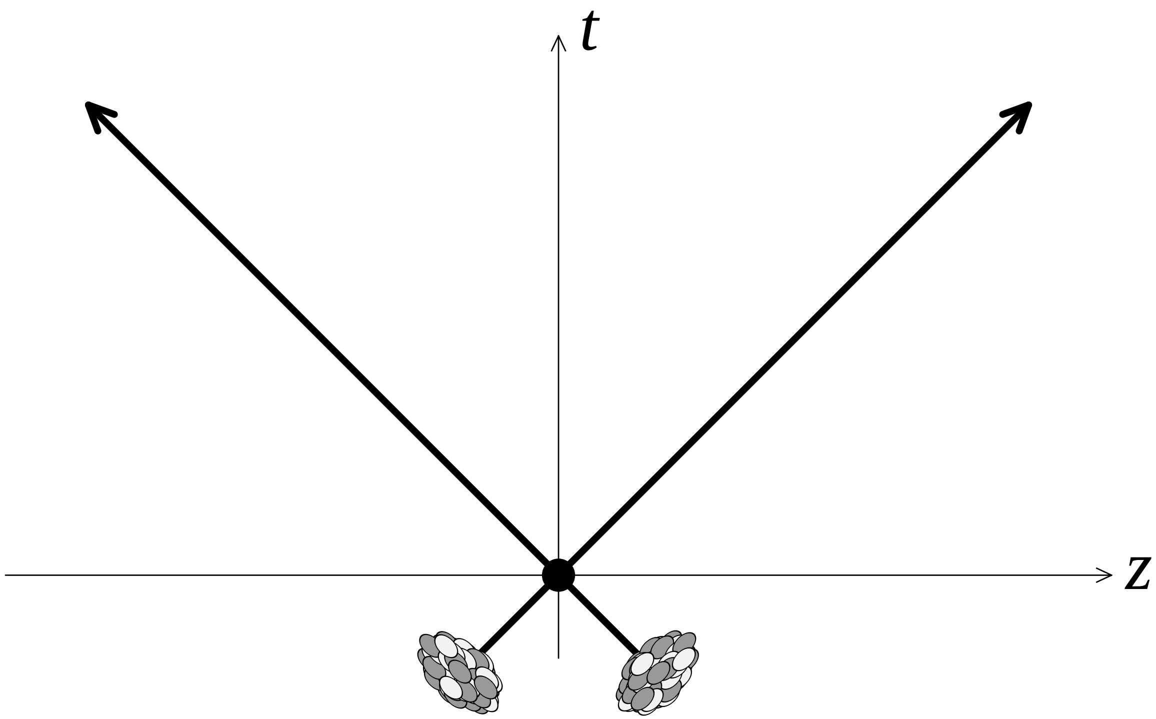


Heavy Ion Collisions

Stages of a nucleus-nucleus collision

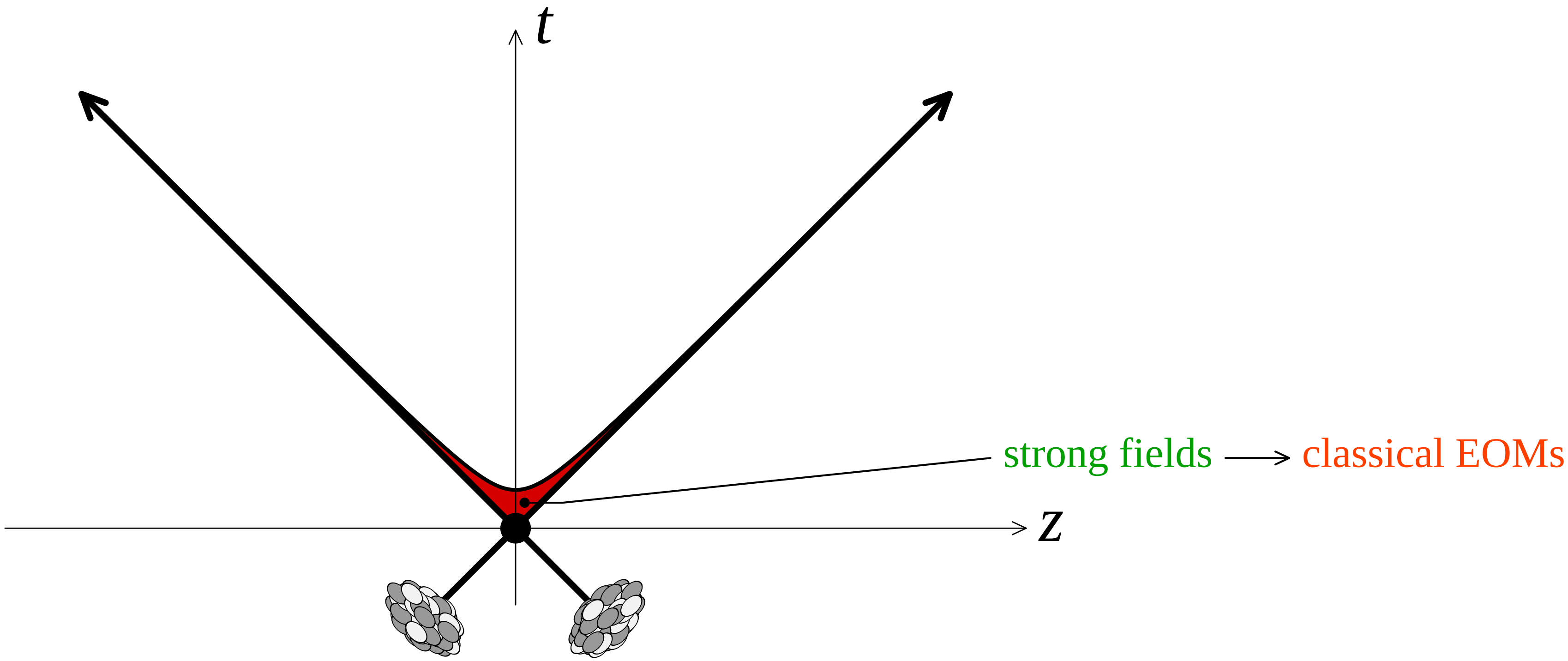


Stages of a nucleus-nucleus collision



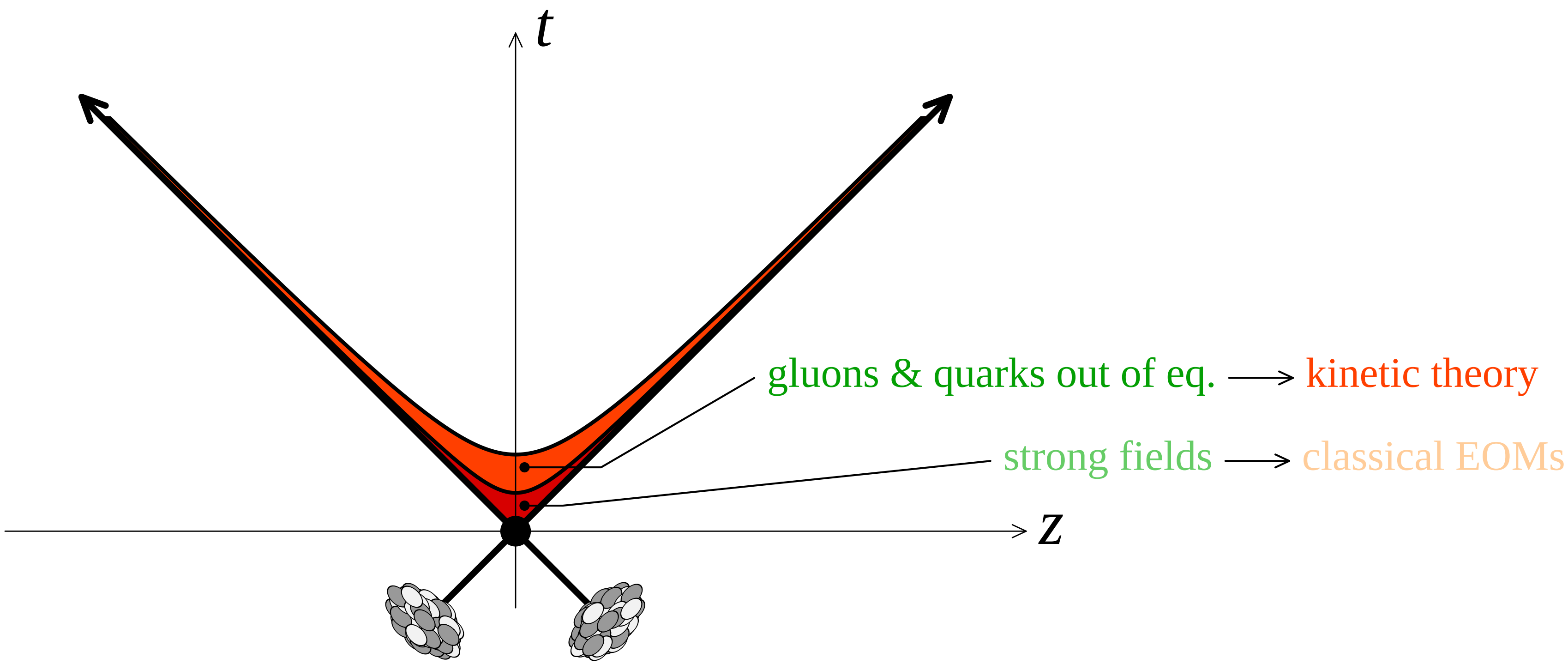
- $\tau \sim 0$ fm/c
- Production of hard particles :
  - jets, direct photons
  - heavy quarks
- calculable with perturbative QCD (leading twist)

Stages of a nucleus-nucleus collision



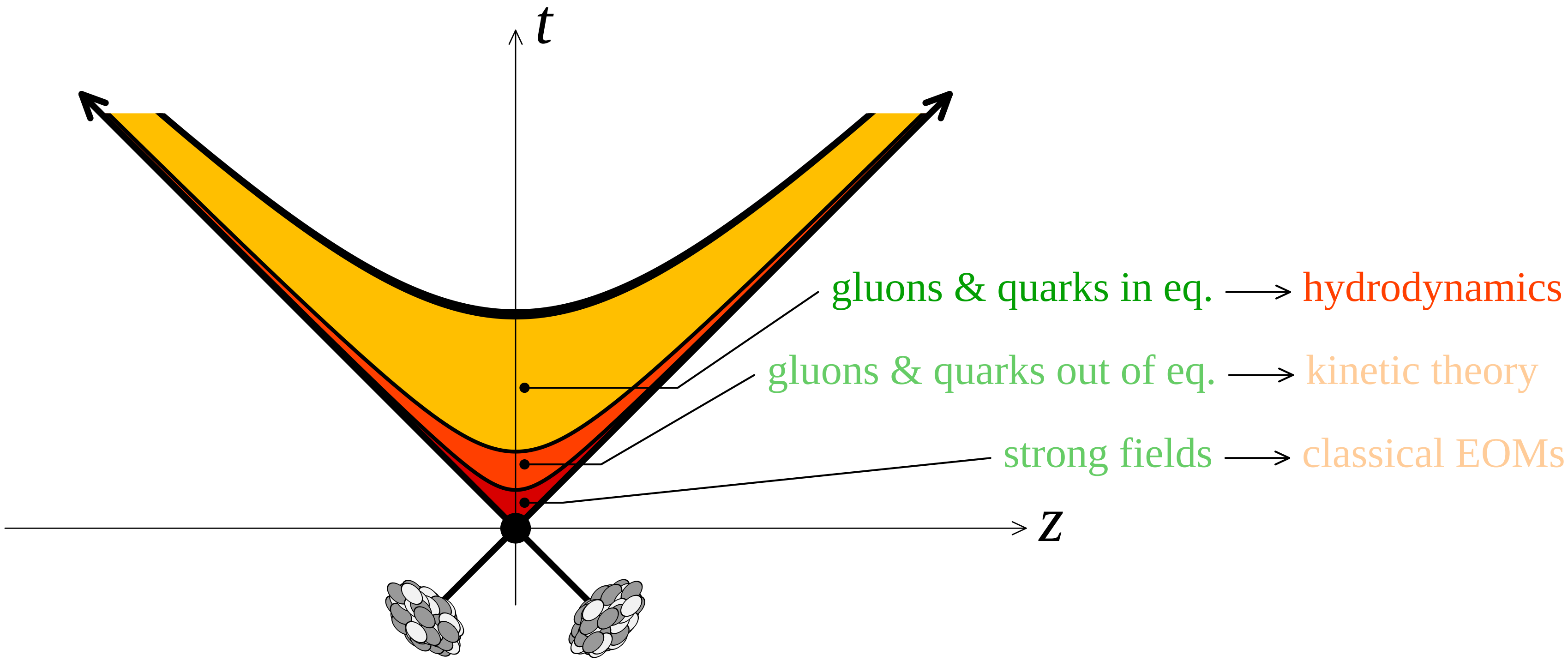
- $\tau \sim 0.2$ fm/c
- Production of semi-hard particles : gluons, light quarks
- relatively small momentum : $p_\perp \lesssim 2$–3 GeV
- make up for most of the multiplicity
- sensitive to the physics of saturation (higher twist)

Stages of a nucleus-nucleus collision



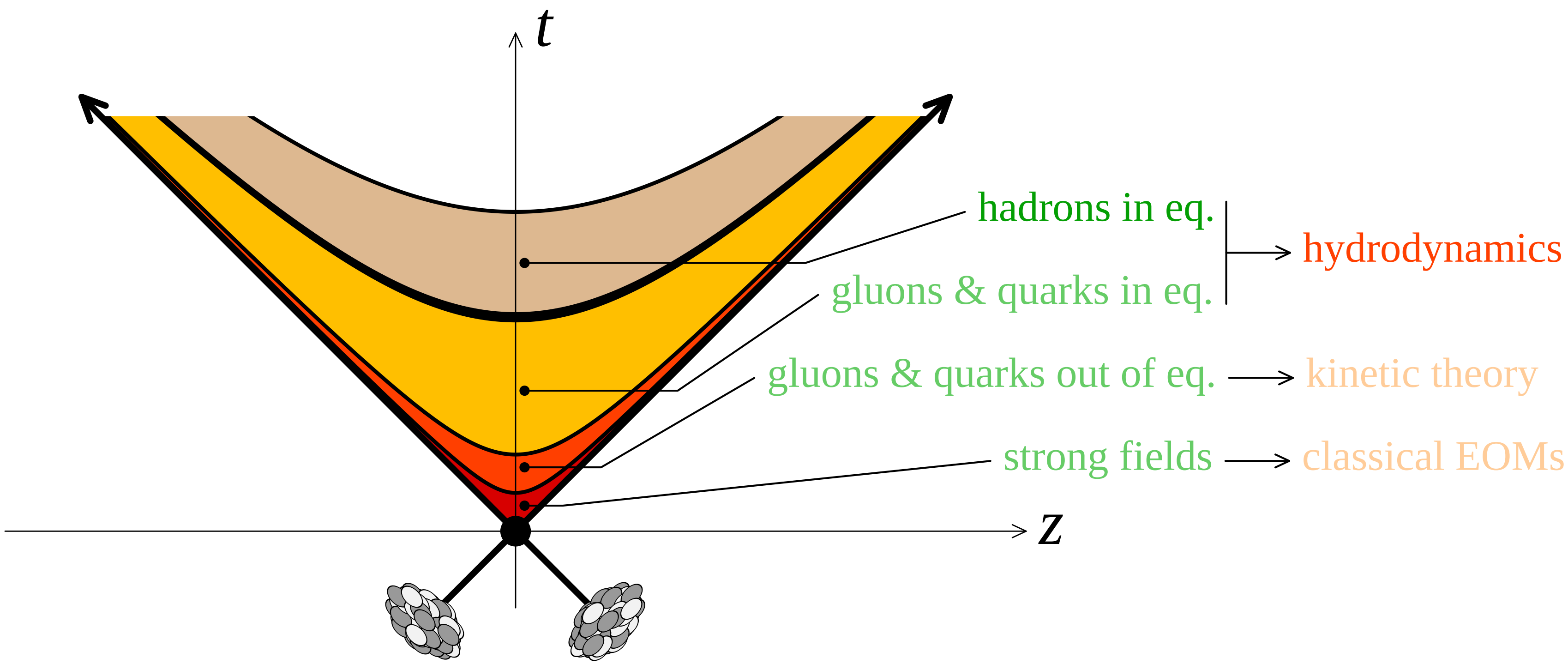
- $\tau \sim 1\text{–}2$ fm/c
- Thermalization
  - experiments suggest a fast thermalization
  - but this is still not understood from QCD

Stages of a nucleus-nucleus collision



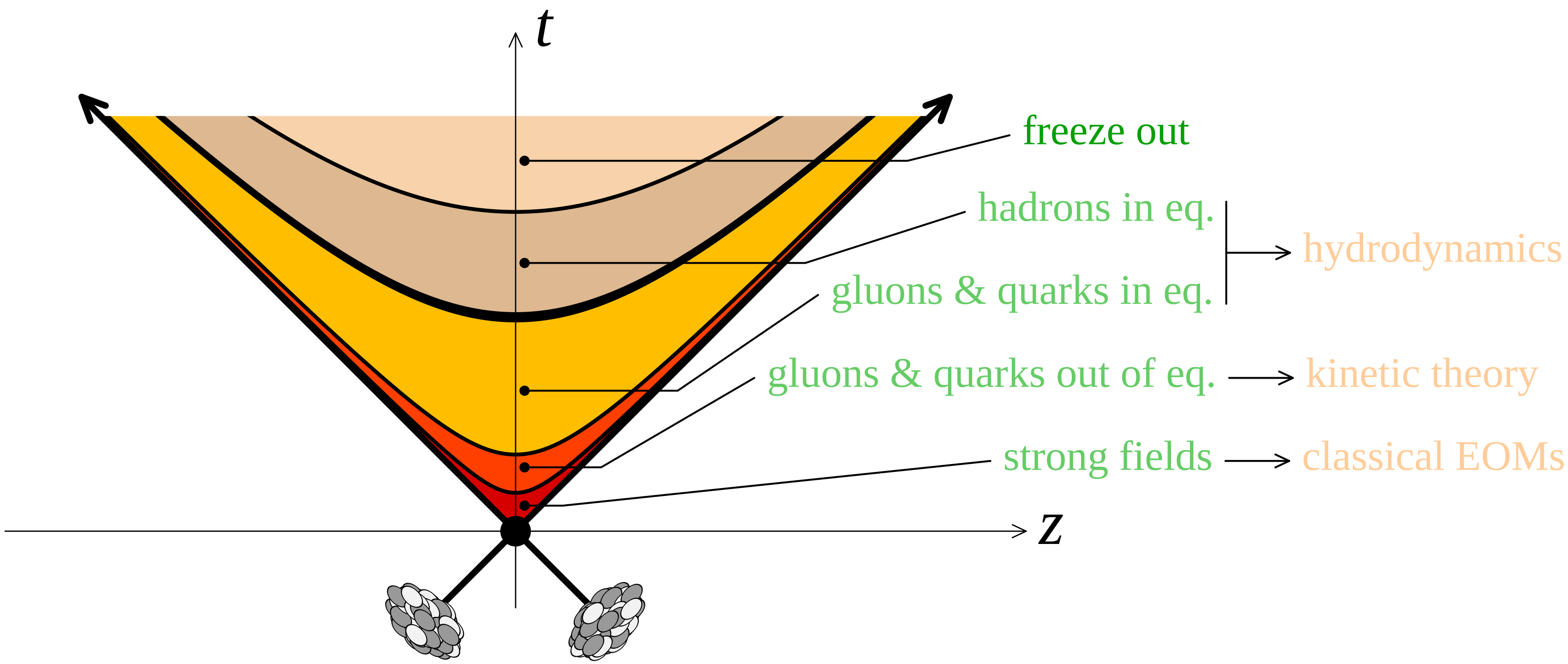
- $2 \leq \tau \lesssim 10$ fm/c
- Quark gluon plasma

Stages of a nucleus-nucleus collision



- $10 \lesssim \tau \lesssim 20$ fm/c
- Hot hadron gas

Stages of a nucleus-nucleus collision



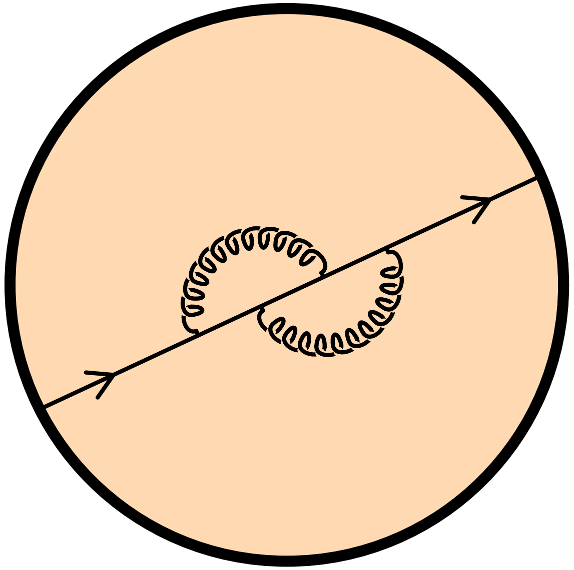
- $\tau \rightarrow +\infty$
- Chemical freeze-out :
  density too small to have inelastic interactions
- Kinetic freeze-out :
  no more elastic interactions

Physics of the QGP

Length scales

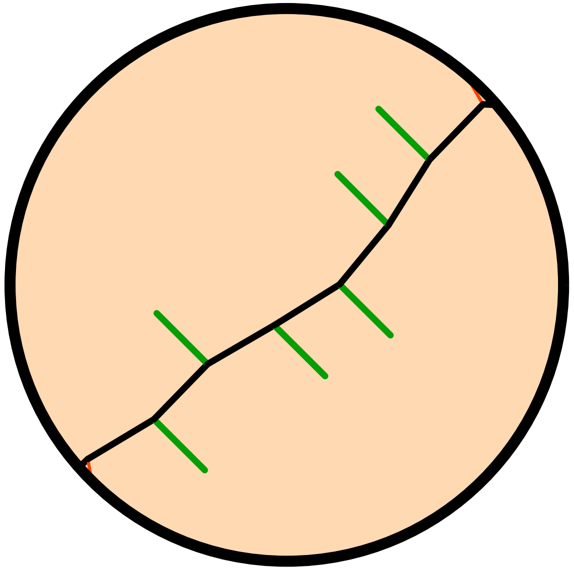
- $1/T$: average distance between particles
- $1/gT$: typical distance for collective phenomena
  - Thermal masses of quasi-particles
  - Screening phenomena
  - Damping of waves
- $1/g^2T$: distance between two small angle scatterings
  - Color transport
  - Photon emission
- $1/g^4T$: distance between two large angle scatterings
  - Momentum, electric charge transport
    - ▷ characteristic scale of hydrodynamic modes
- In the weak coupling limit ($g \ll 1$), there is a clear hierarchy between these scales
- Distinct effective theories according to the characteristic scale of the problem under study

Vacuum fluctuations



- At distances scales $\ell \lesssim 1/T$, medium effects are irrelevant
- At such scales the dynamics is simply described by the usual QCD in the vacuum

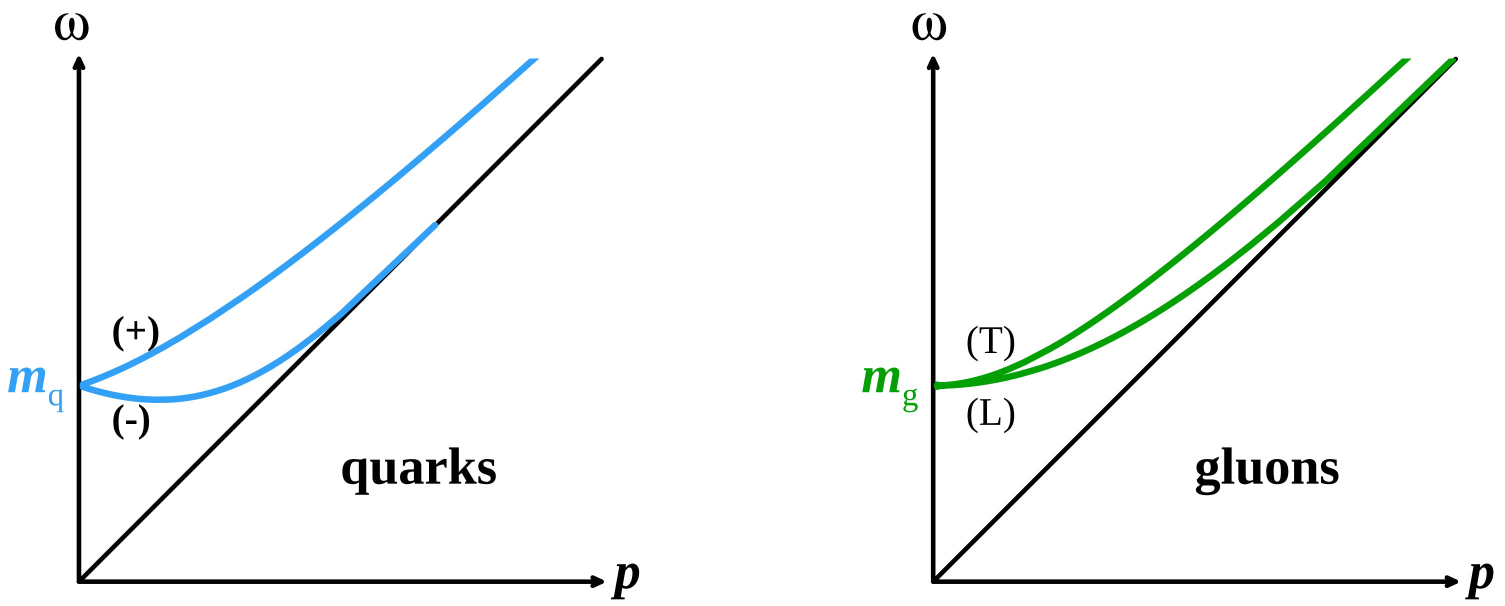
Thermal fluctuations



- Distance scales $1/T \lesssim \ell \lesssim 1/gT$ control the bulk thermodynamic properties. The system can be studied by QCD at finite temperature
- The leading thermal effects can be treated by an effective theory that encompasses the main collective effects, and that has the form of a collision-less Vlasov equation

Quasi-particles

- Dispersion curves of particles in the plasma :

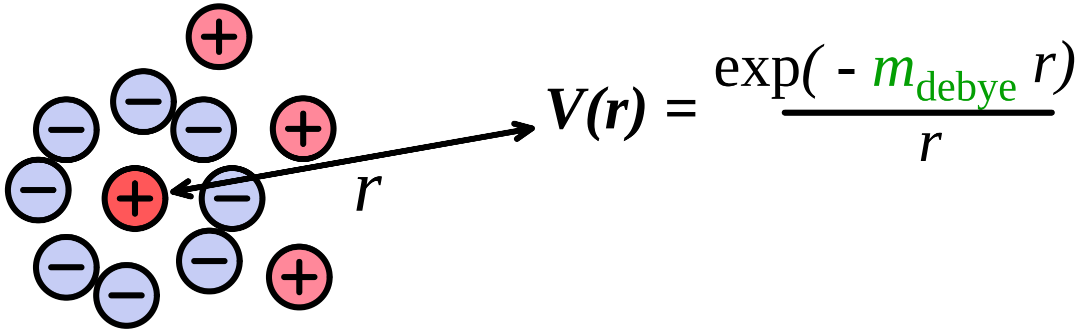


- Thermal masses due to interactions with the other particles in the plasma :

$$m_q \sim m_g \sim gT$$

Debye screening

- A test charge polarizes the particles of the plasma in its vicinity, in order to screen its charge :

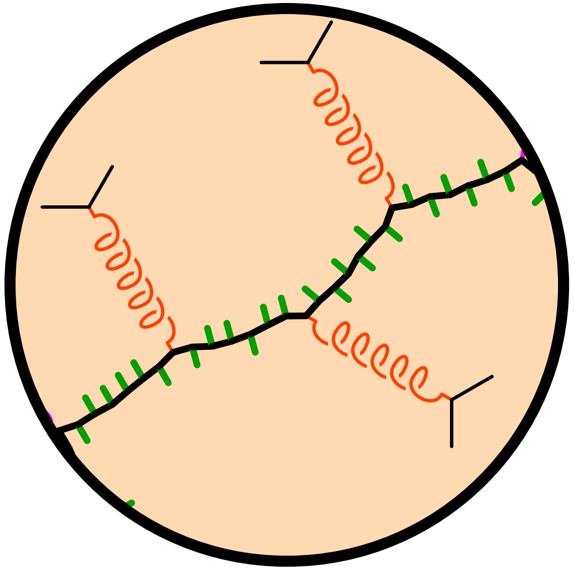


$$V(r) = \frac{\exp(-m_{\text{debye}}\, r)}{r}$$

- The Coulomb potential of the test charge decreases exponentially at large distance. The effective interaction range is :

$$\ell \sim 1/m_{\text{debye}} \sim 1/gT$$

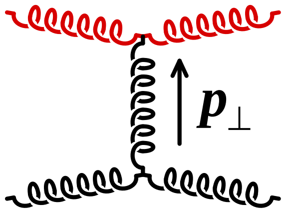
Small angle scatterings



- When we follow a plasma particle over distances $1/g^2T \lesssim \ell$, it is necessary to account for soft (small angle) collisions with other particles of the plasma
- This can be done simply by adding a collision term to the previous Vlasov equation

Collisional width

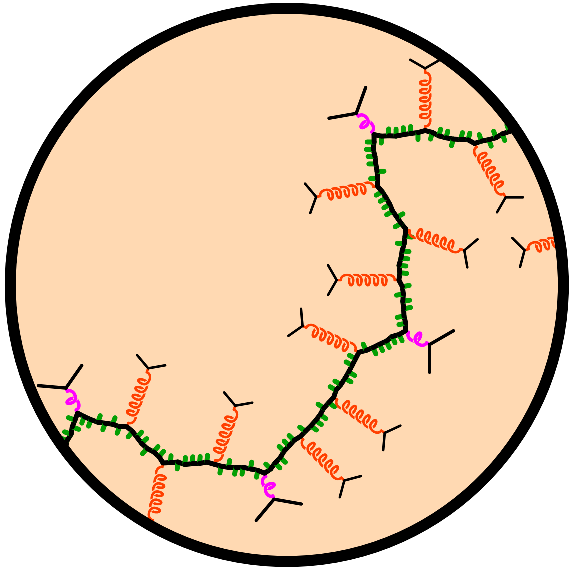
- Collisional width :



$$\Gamma_{\rm coll} = \left| \quad \right|^2 \sim g^4\, T^3 \int_{m_{\rm debye}} \frac{d^2\vec{\boldsymbol{p}}_\perp}{p_\perp^4} \sim g^2 T$$

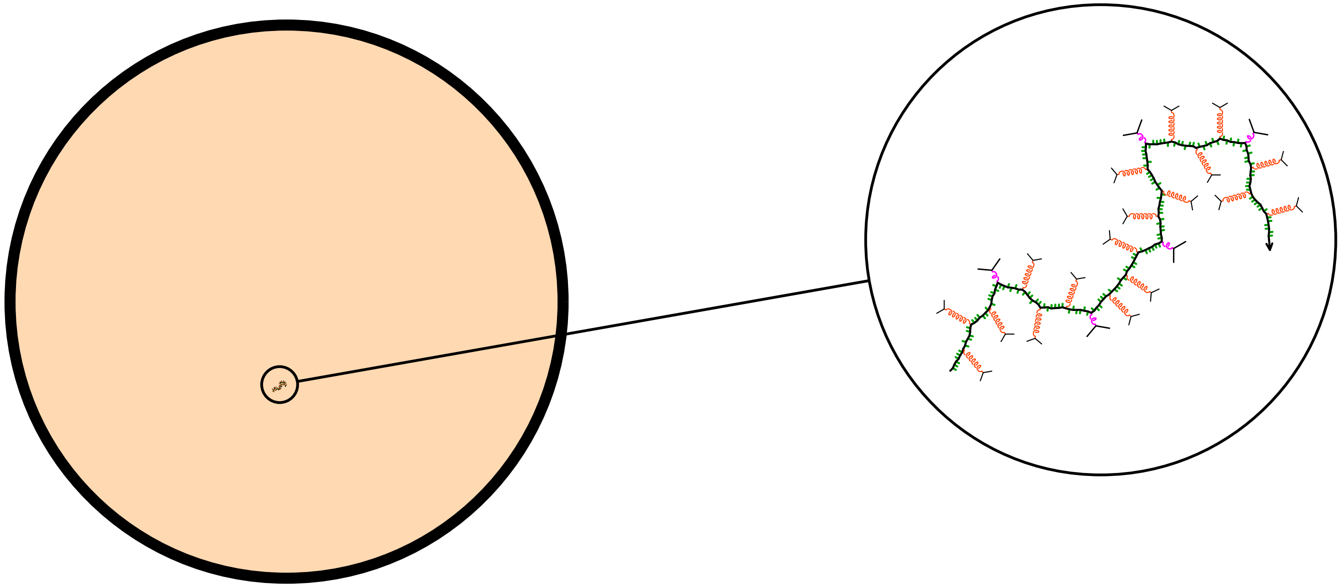
- $\lambda \equiv 1/\Gamma_{\rm coll}$ is the mean free path between two small angle scatterings ($\theta \sim g$)
- Note : the mean free path between two large angle scatterings ($\theta \sim 1$) is $\sim 1/g^4 T$

Large angle scatterings



- Over distance scales $\ell \sim 1/g^4T$, one must take into account the large angle collisions, that change significantly the direction of motion of the particle (this is necessary e.g. for calculating transport coefficients)
- The most efficient way to describe the system over these scales is via a Boltzmann equation for color/spin averaged particle distributions

Hydrodynamical regime



- The hydrodynamical regime is reached for length scales that are much larger than the mean free path : $1/g^4T \ll \ell$
- In order to describe the system at such scales, one needs :
  - Hydrodynamical equations (Euler, Navier-Stokes)
  - Conservation equations for the various currents
  - Equation of state, viscosity

Experimental signatures

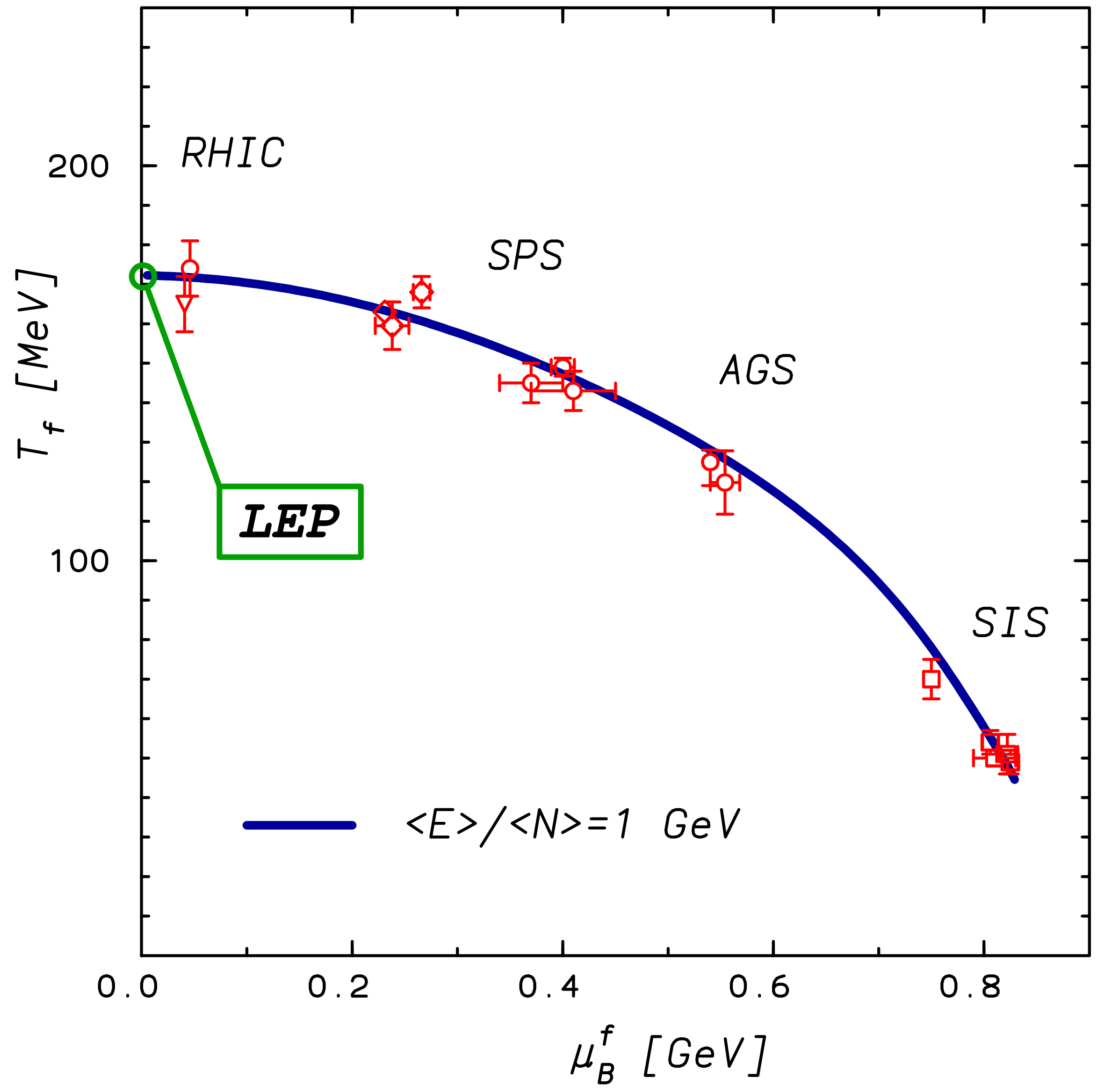
Statistical models

- One assumes that particles are produced by a thermalized system with temperature T and baryon chemical potential μ_B
- The number of particles of mass m per unit volume is :

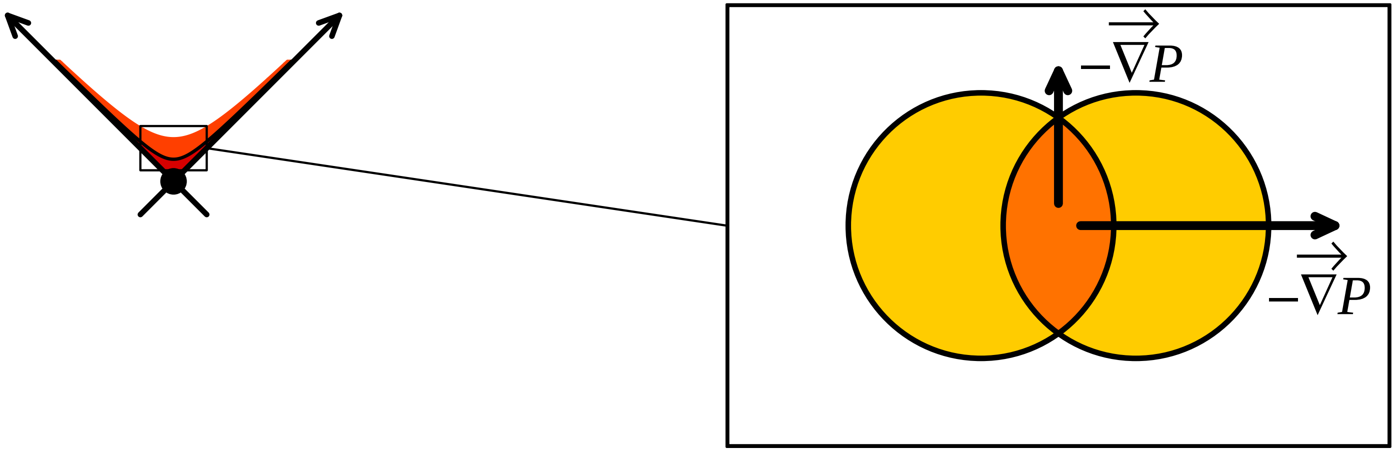
$$\frac{dN}{d^3\vec{\boldsymbol{x}}} = \int \frac{d^3\vec{\boldsymbol{p}}}{(2\pi)^3} \frac{1}{e^{(\sqrt{p^2+m^2}-\mu_B Q)/T} \pm 1}$$

- These models reproduce the ratios of particle yields with only two parameters
- The same models also work for e^+e^- collisions
  - Standard explanation: randomly filling a phase space leads to exponential distributions
  - However, this argument alone does not explain why the value of T that comes out is the same as in nucleus-nucleus collisions ▷ dynamical arguments (about the properties of the vacuum?) may be involved here...

Freeze-out parameters



Collective flow

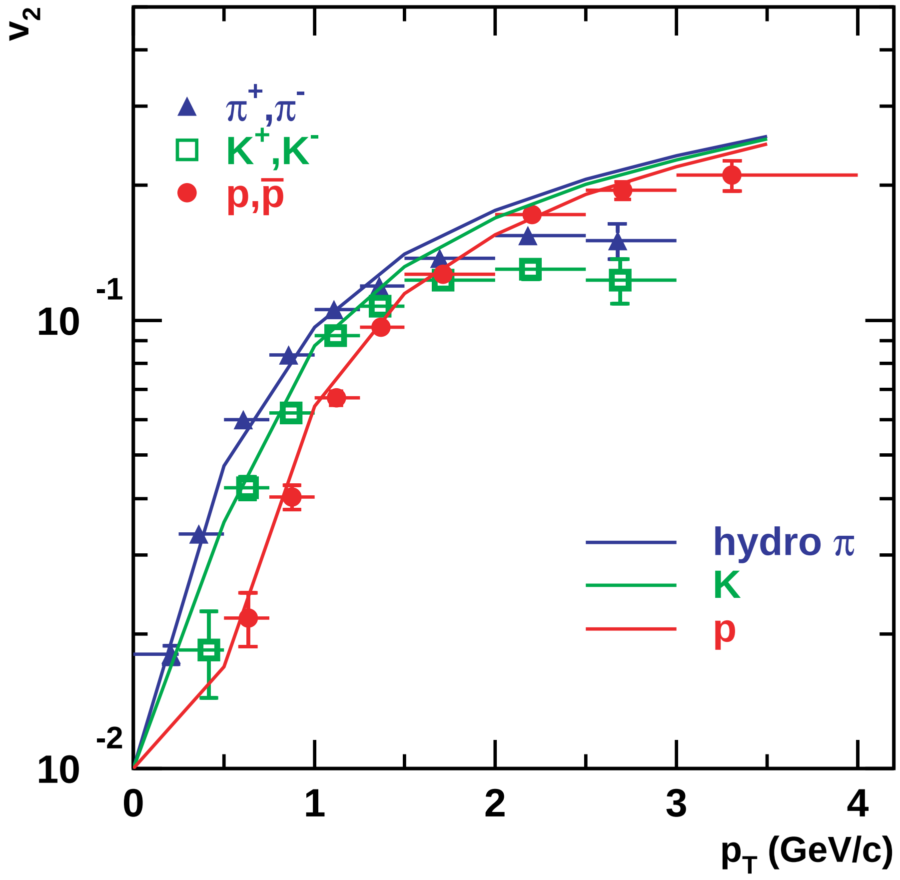


- In non-central collisions, pressure turns a spatial anisotropy into an anisotropy of the momenta (Ollitrault (1992))
- Observable: $2^{\rm nd}$ harmonic of the azimuthal distribution

$$dN/d\varphi \sim 1 + 2v_1\cos(\varphi) + 2v_2\cos(2\varphi) + \cdots$$

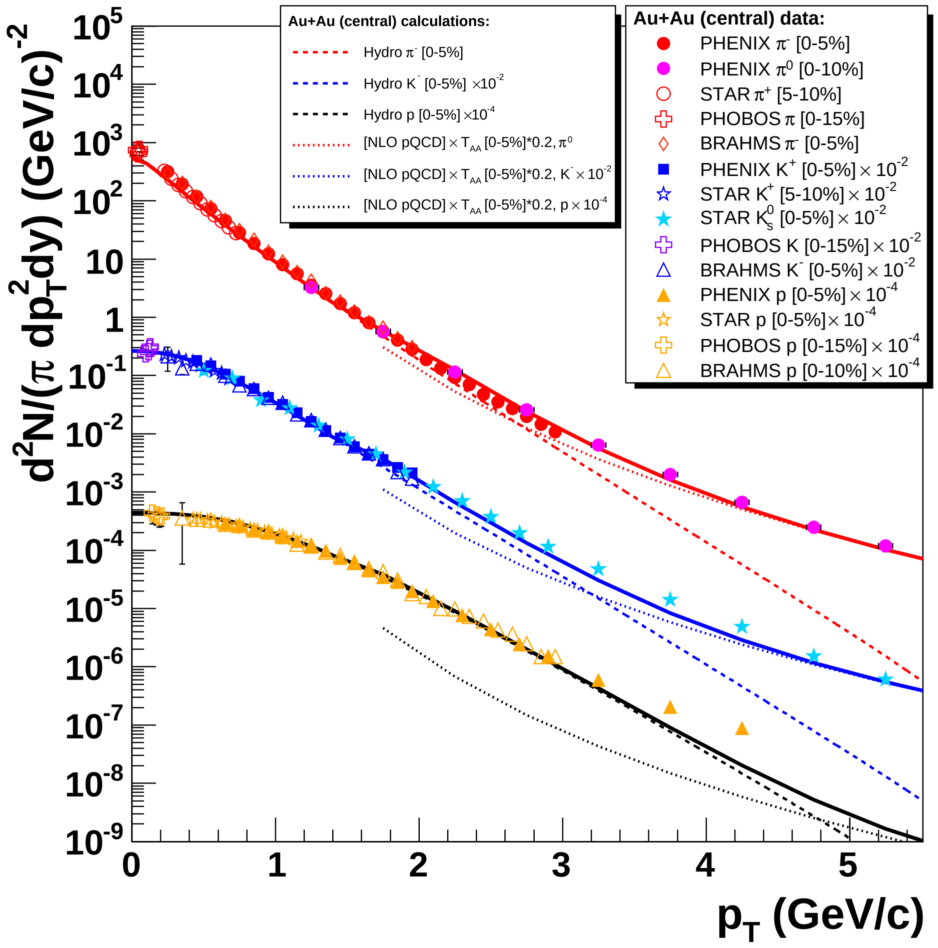
- **Note:** a large v_2 implies a strong transverse pressure, but says very little on the longitudinal degrees of freedom
  ▷ does not imply a tri-dimensional thermalization...

Collective flow

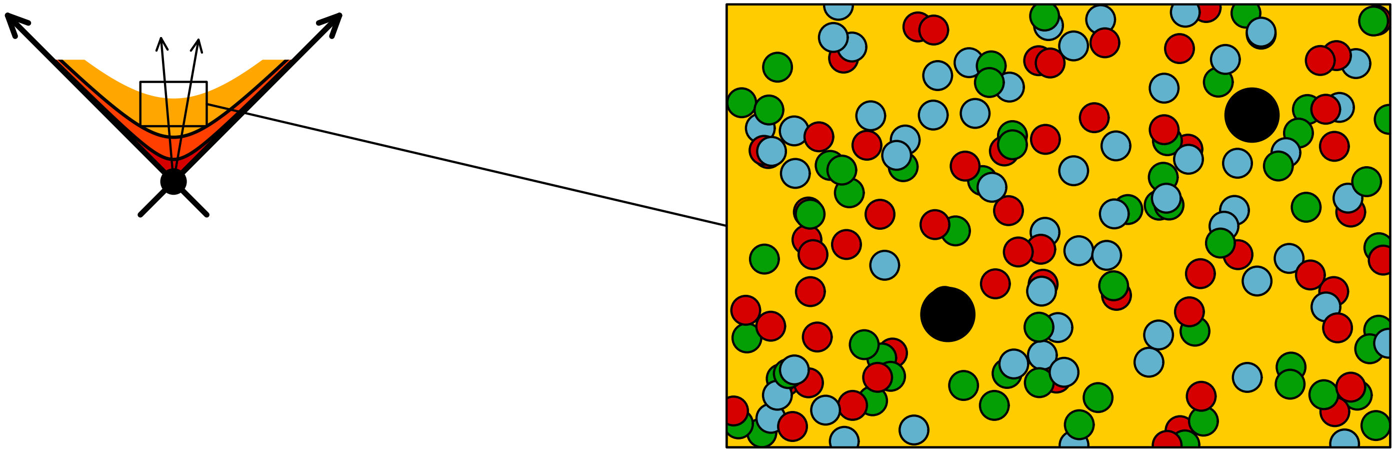


Another success of hydrodynamics

Deconfinement transition
Heavy Ion Collisions
Physics of the QGP
Experimental signatures
● Chemical freeze-out
○ Transverse pressure
● Deconfinement
● Energy density
● Temperature
AdS/CFT duality and the QGP
Is the QGP a perfect fluid?
Color Glass Condensate
Summary

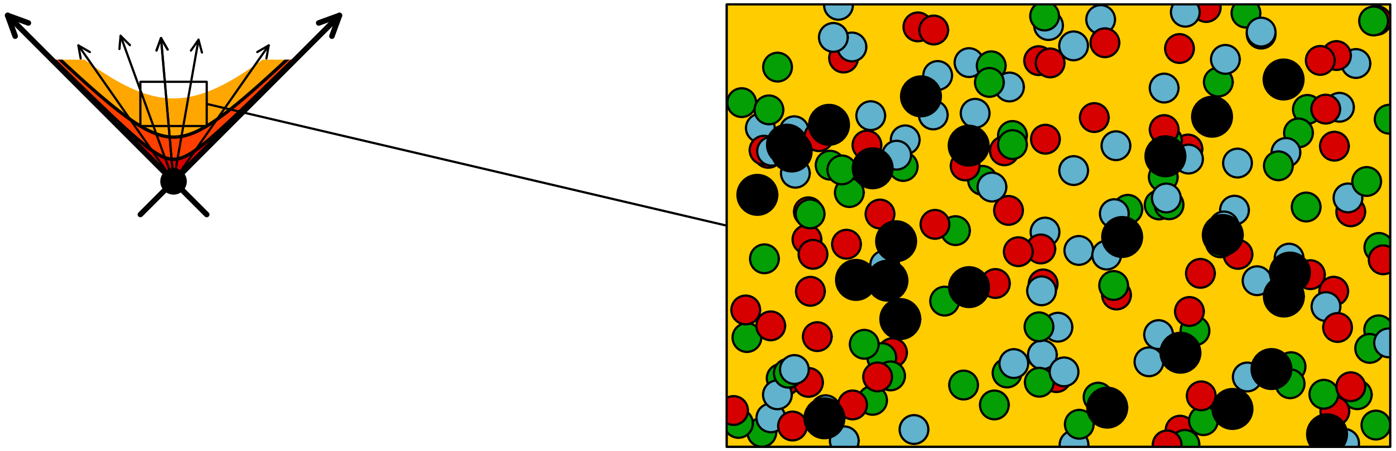


J/Psi suppression



- Debye screening prevents the $Q\overline{Q}$ pair from forming a bound state Matsui, Satz (1986)
  - each heavy quark pairs with a light quark in order to form a D meson
- The inter-quark potential can be calculated using lattice QCD
- Possible observable : $[J/\psi]$ / [Open charm]
  ▷ complication : there is also a suppression in proton-nucleus collisions, due to multiple scattering

... or enhancement ?

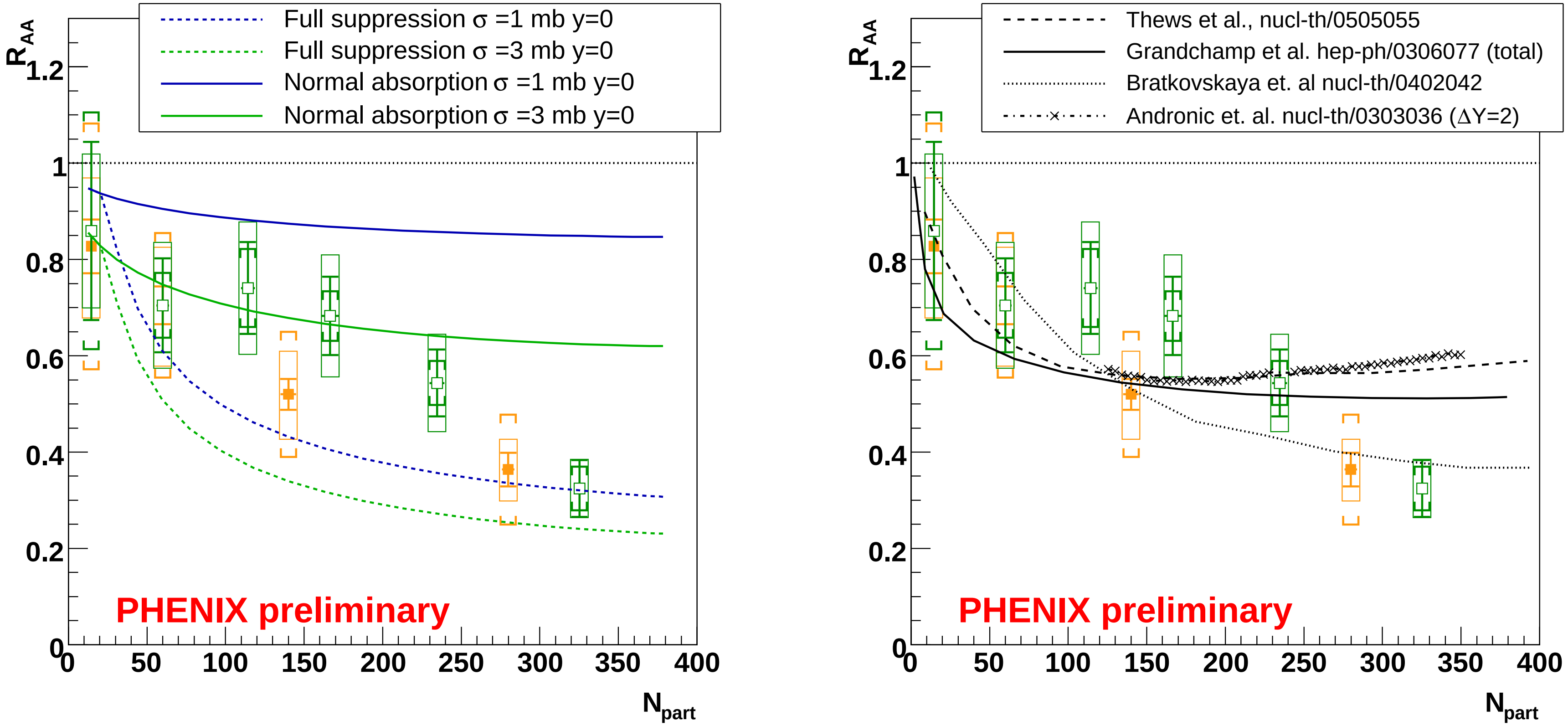


- Many $Q\overline{Q}$ pairs may be produced in each AA collision

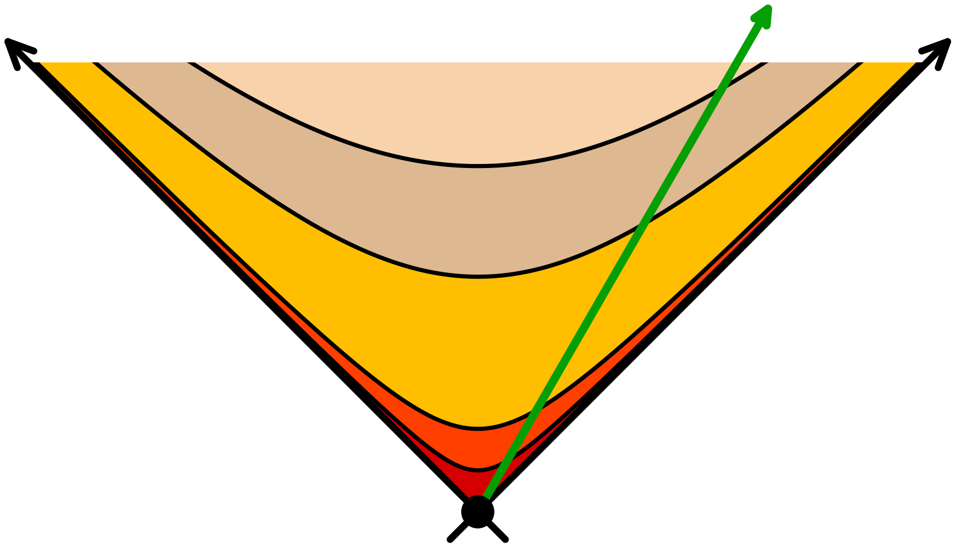
  Braun-Munzinger, Stachel (2000)

  Thews, Schroedter, Rafelski (2001)
  - A Q from one pair may recombine with a $\overline{Q}$ from another pair
- Avoids the conclusion of Matsui and Satz's scenario, provided that the average distance between heavy quarks is smaller than the Debye screening length
- May lead to an enhancement of J/ψ production

J/Psi measurements at RHIC

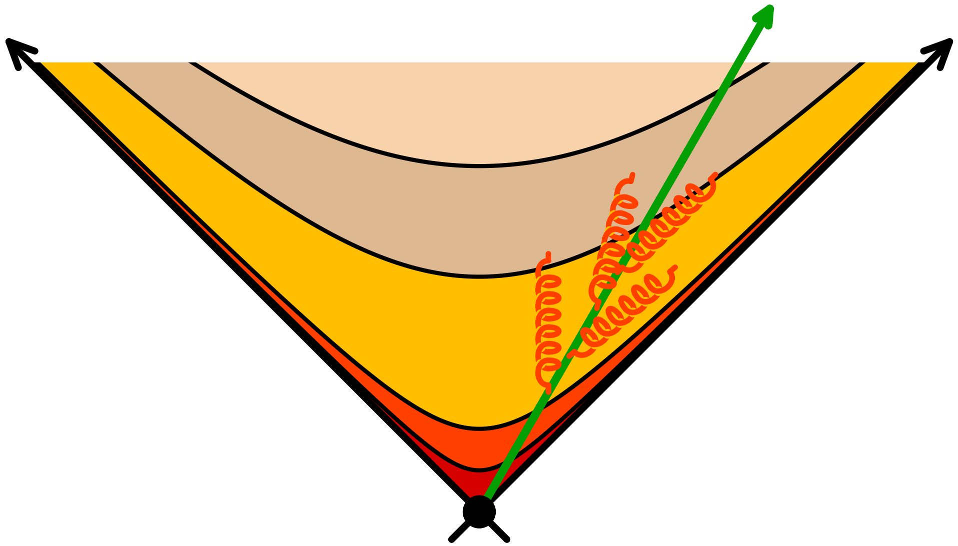


Jet quenching



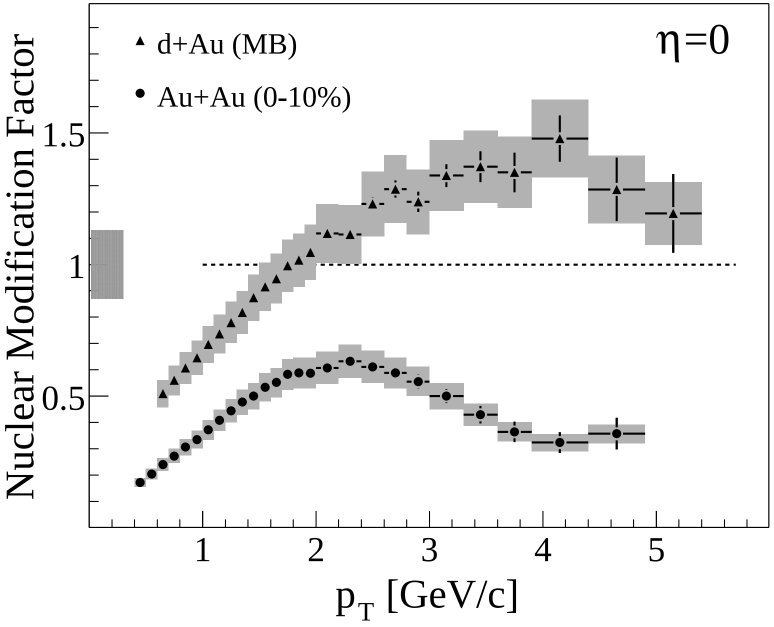
- Jets are produced at the initial impact
  - Not very interesting by themselves...

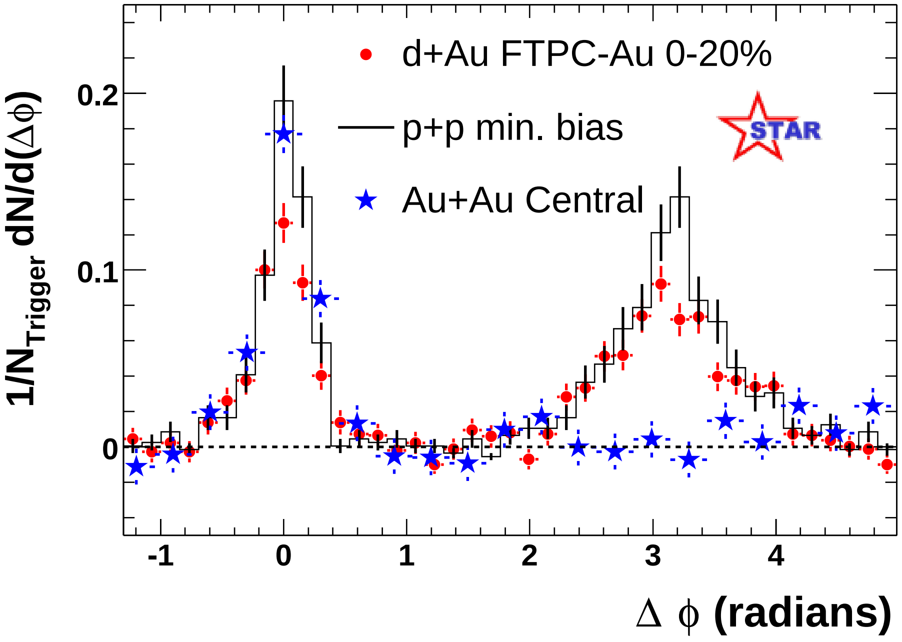
Jet quenching



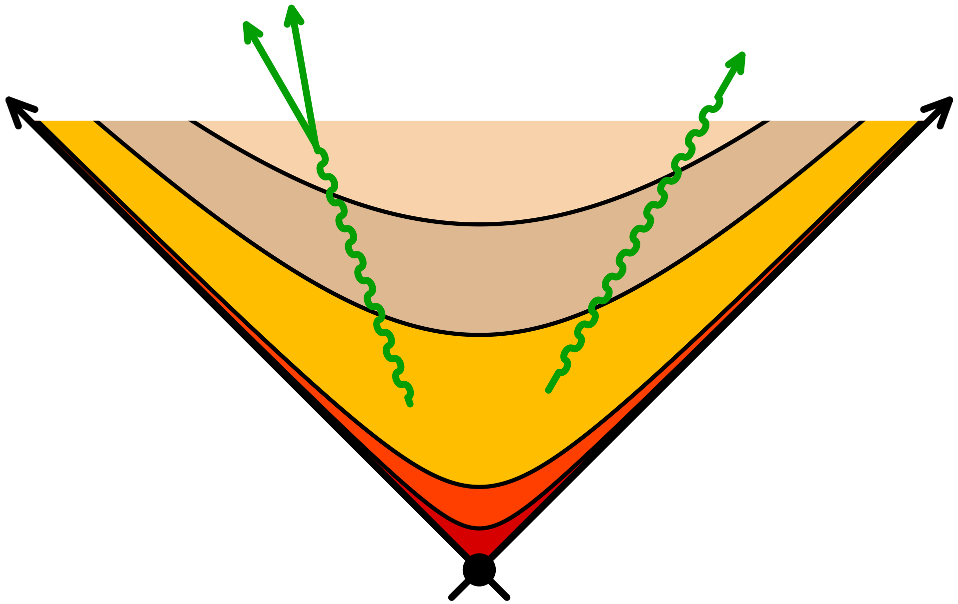
- Jets are produced at the initial impact
  - Not very interesting by themselves...
- Radiative energy loss when they travel through the QGP
  - Sensitive to the energy density of the medium
  - Depends on the path length as L^2
  - Important modification of the azimuthal correlations (at RHIC, complete absorption of the opposite jet)

Jet quenching



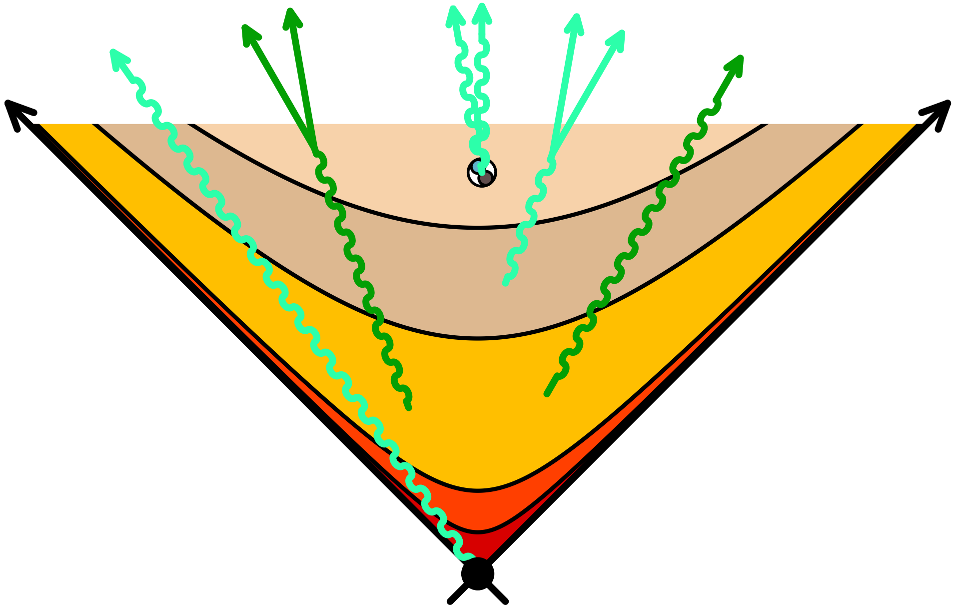


Thermal photons



- Photons produced by the QGP :
  - Rate determined by physics at the scale g^2T
  - Very sensitive to the temperature : $dN_\gamma/dtd^3\vec{\boldsymbol{x}} \sim T^4$

Thermal photons



- Photons produced by the QGP :
  - Rate determined by physics at the scale g^2T
  - Very sensitive to the temperature : $dN_\gamma/dtd^3\vec{\boldsymbol{x}} \sim T^4$
- But very important background...
  - initial photons
  - photons produced by in-medium jet fragmentation
  - photons produced by the hadron gas
  - meson decays

Direct photons at RHIC

Deconfinement transition

Heavy Ion Collisions

Physics of the QGP

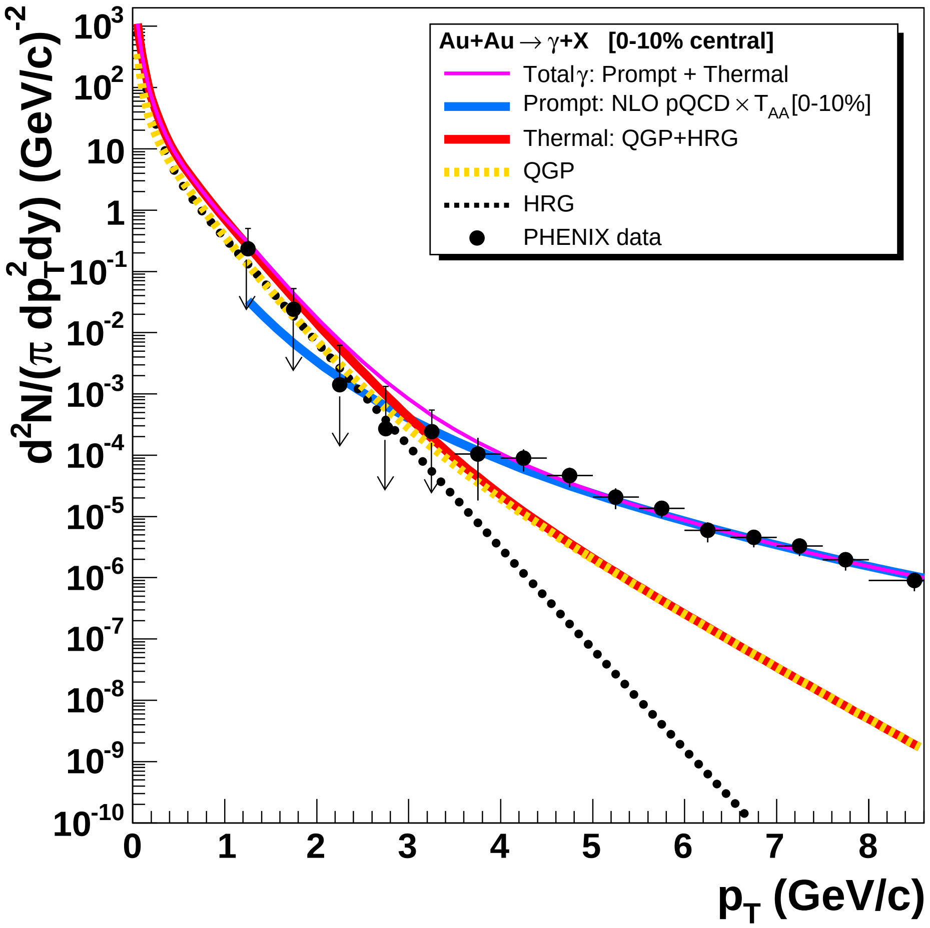
Experimental signatures
- Chemical freeze-out
- Transverse pressure
- Deconfinement
- Energy density
- Temperature

AdS/CFT duality and the QGP

Is the QGP a perfect fluid?

Color Glass Condensate

Summary



AdS/CFT duality and the QGP

AdS/CFT conjecture

- Maximally super-symmetric $SU(N)$ Yang-Mills theories in the limit $g^2N \to +\infty$ are dual to classical super-gravity on an $\mathrm{AdS}_5 \times S_5$ manifold with metric

$$ds^2 = \frac{R^2}{z^2}\big(\underbrace{-dt^2 + d\vec{\boldsymbol{x}}^2}_{\text{we live here... (at z=0)}} + dz^2\big) + R^2 d\Omega_5^2$$

Note: this metric has Poincaré invariance in $(t, \vec{\boldsymbol{x}})$

- If a field ϕ on $\mathrm{AdS}_5 \times S_5$ is coupled to an operator $\boldsymbol{\mathcal{O}}$ on the boundary, the correspondence states that :

$$e^{-S_{\mathrm{cl}}[\phi]} = \left\langle e^{\int_{\mathrm{boundary}} \boldsymbol{\mathcal{O}}\ \phi(z=0)} \right\rangle$$

  - The right hand side is a generating functional for the correlators of operators $\boldsymbol{\mathcal{O}}$ in the D=4 super Yang-Mills theory
  - The left hand side is calculable in the gravity dual (solve the classical EOM for ϕ with the boundary condition $\phi(z=0)$)

AdS/CFT conjecture

- At finite temperature T

$$-dt^2+dz^2 \to -f(z)dt^2+dz^2/f(z) \qquad \text{with} \quad f(z)=1-(\pi z T)^4$$

- $f(z)=0$ at $z=1/\pi T \quad \Rightarrow \quad$ horizon
  - ▷ translationally invariant in $\vec{\boldsymbol{x}}$
  - ▷ “black brane”

- A stronger form of the conjecture relates an N=4 $SU(N)$ Yang-Mills theory at finite coupling to a type IIB string theory
  - Parameters : g, N for Yang-Mills, $\; g_s, l_s=\sqrt{\alpha'}, R$ for strings
  - Dictionary : $g^2=4\pi g_s, \;\; g^2N=R^4/l_s^4$
  - The string theory simplifies into classical super-gravity when
    $g_s \ll 1, \;\; R \gg l_s$
    i.e. $g \ll 1, \;\; g^2N \gg 1$

Shear viscosity

- In Quantum Field Theory, the shear viscosity can be obtained via Kubo's formula :

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt d^3\vec{\boldsymbol{x}}\ e^{i\omega t} \ \Big\langle \Big[T_{xy}(t, \vec{\boldsymbol{x}}), T_{xy}(0, \vec{\boldsymbol{0}}) \Big] \Big\rangle$$

(linear response theory)

- In the dual theory, T_{xy} couples to metric perturbations. One can relate the above correlation function to the absorption cross-section of a graviton of energy ω :

$$\sigma_{\rm abs}(\omega) = \frac{8\pi G}{\omega} \int dt d^3\vec{\boldsymbol{x}}\ e^{i\omega t} \ \Big\langle \Big[T_{xy}(t, \vec{\boldsymbol{x}}), T_{xy}(0, \vec{\boldsymbol{0}}) \Big] \Big\rangle$$

Hence $\eta = \sigma_{\rm abs}(0)/16\pi G$

- In the classical limit, this absorption cross-section at $\omega = 0$ is the area of the horizon. Moreover, the area of a black-hole is related to its entropy via $s = \mathcal{A}/4G$
- Combining everything, one obtains $\eta/s = 1/4\pi$

Minimal eta/s

- The first subleading correction in the large N_c expansion has also been calculated :

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 + \frac{135\zeta(3)}{8(2g^2N_c)^{3/2}} + \cdots\right]$$

$\triangleright$ the correction is positive

- Reminder : in the weak coupling regime, $\eta/s = \mathrm{const}/(g^4\ln(1/g)) \to +\infty$
- Conjecture : $1/4\pi$ is the lowest possible value for this ratio
- Handwaving argument for the existence of a minimum :
  - $\eta \sim nm\overline{v}\lambda \sim n\overline{p}\lambda$ (for a dilute gas)
  - $s \sim n$

  Hence $\eta/s \sim \overline{p}\lambda \geq 1$
- All the known substances have a viscosity to entropy ratio (much) larger than the bound

  $\triangleright$ led to the idea that the QGP may be the “perfect fluid”

Limitations

- The correspondence is only a conjecture (so far)
- It only applies to maximally super-symmetric Yang-Mills theories. Such theories are conformally invariant, have no running coupling, and no confinement
- Whether what we learn about these theories is relevant for QCD (that has broken scale invariance, running coupling, confinement...) is at best a wishful thinking
- There are some dissenting views about whether the physics of the QGP at $T \sim 2-3T_c$ is really strongly coupled. For quantities such as the entropy or the pressure, perturbative techniques (+resummations) lead to accurate results in this region

Is the QGP a perfect fluid?

Elliptic flow

- A perfect fluid is a fluid with a very small viscosity, that can be described with Euler equations (ideal hydrodynamics) (Note: a perfect fluid has a very small mean free path, i.e. large microscopic cross-section)
- The elliptic flow coefficient v_2 measured at RHIC is reasonably well predicted by ideal hydrodynamics, that has no viscosity at all
  - It has been concluded from there that the QGP has a ratio η/s which is close to the conjectured lower bound $1/4\pi$
  - Note: the predicted v_2 is proportional to the spatial eccentricity of the initial overlap region of the two nuclei $\triangleright$ one needs to correctly calculate this eccentricity

What if this is indeed true?

- For a relativistic plasma, $\eta/s \sim \lambda T$
  ▷ hence, at $T = 200$ MeV, $\lambda \approx 0.1$ fm if the AdS/CFT lower bound is reached

- For a system with transverse size ~ 5 fm, this would mean an average of 50 collisions per particle
  ▷ very good local thermalization

- Since $\lambda = 1/n\sigma$ and $n \sim T^3$, one would get a cross-section $\sigma \approx 30$ mb among constituents
  ▷ 10 times larger than the typical perturbative partonic cross-section

What about v4 ?

- It can be shown that if ideal hydrodynamics applies, then the fourth harmonic coefficient should be $v_4 = v_2^2/2$ at large $p_\perp$ (Borghini, Ollitrault (2005))

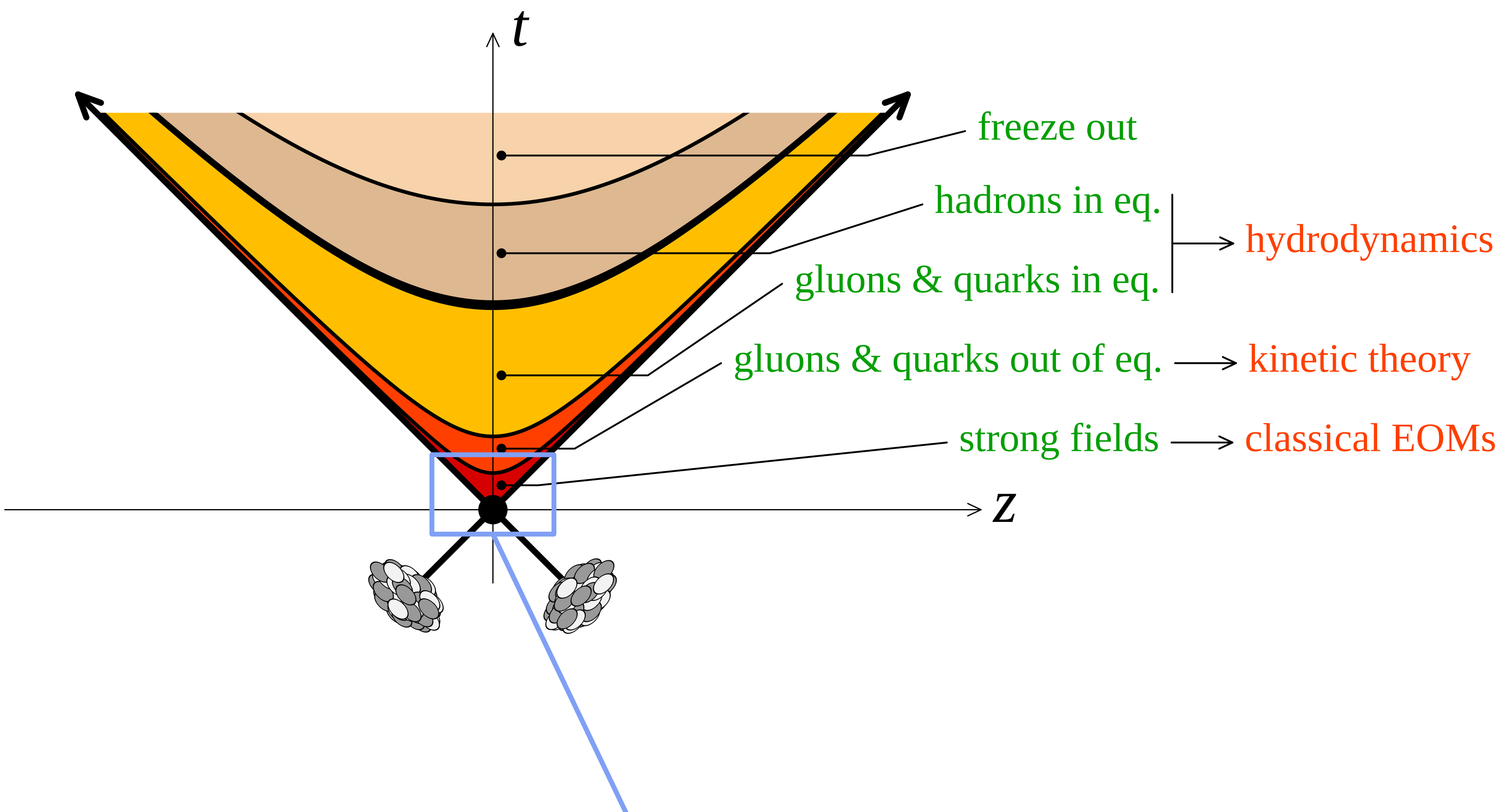
- However, at RHIC, the measured v_4 is about twice larger than this prediction, which seems to suggest that there are only a few collisions per particle. This would imply
  - larger mean free path
  - smaller cross-sections
  - higher viscosity and less perfect thermalization

- How to accommodate the fact that ideal hydrodynamics predicts the measured v_2 ?
  - Non-ideal hydrodynamics would lead to a similar v_2 provided the initial eccentricity is higher
    Problem: there is no hydrodynamical code including viscous effects in order to test this possibility quantitatively...

Color Glass Condensate,

Formation of the QGP

Before the QGP



freeze out

hadrons in eq.
gluons & quarks in eq. → hydrodynamics

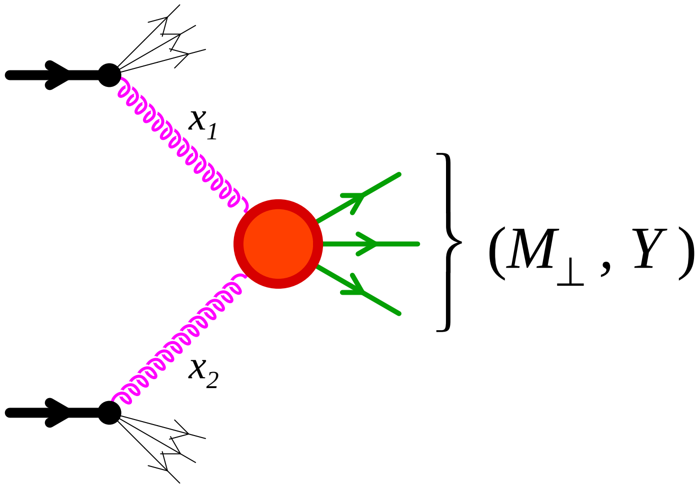
gluons & quarks out of eq. → kinetic theory

strong fields → classical EOMs

- calculate the initial production of semi-hard particles
- prepare the stage for kinetic theory or hydrodynamics

Infrared and collinear divergences

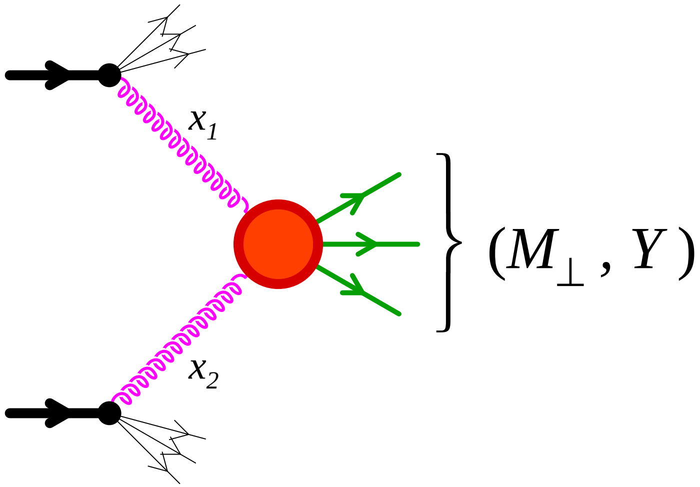
- Calculation of some process at LO :



$$\begin{cases} x_1 = M_\perp\, e^{+Y}/\sqrt{s} \\ x_2 = M_\perp\, e^{-Y}/\sqrt{s} \end{cases}$$

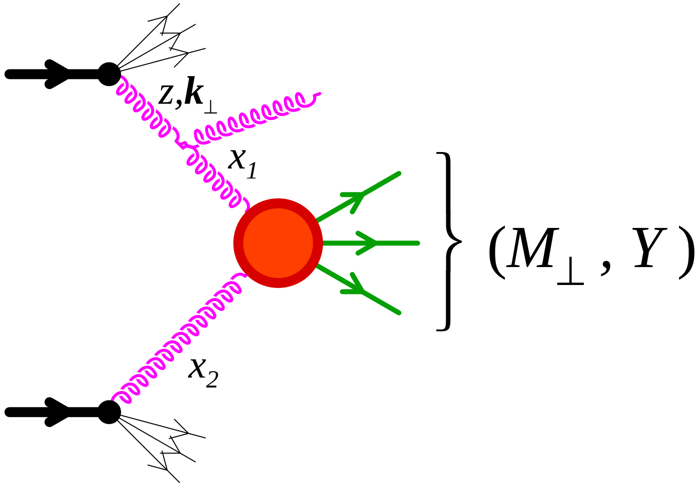
Infrared and collinear divergences

- Calculation of some process at LO :



$$\begin{cases} x_1 = M_\perp\, e^{+Y}/\sqrt{s} \\ x_2 = M_\perp\, e^{-Y}/\sqrt{s} \end{cases}$$

- Radiation of an extra gluon :



$$\Longrightarrow \quad \alpha_s \int_{x_1} \frac{dz}{z} \int^{M_\perp} \frac{d^2\vec{\boldsymbol{k}}_\perp}{\boldsymbol{k}_\perp^2}$$

Infrared and collinear divergences

- Large $\log(M_\perp)$ when $M_\perp$ is large
- Large $\log(1/x_1)$ when $x_1 \ll 1$

  ▷ these logs can compensate the additional α_s, and void the naive application of perturbation theory

  ▷ resummations are necessary

- Logs of $M_\perp \Longrightarrow$ DGLAP. Important when :
  - $M_\perp \gg \Lambda_{QCD}$
  - x_1, x_2 are rather large
- Logs of $1/x \Longrightarrow$ BFKL. Important when :
  - $M_\perp$ remains moderate
  - x_1 or x_2 (or both) are small
- Physical interpretation :
  - The physical process can resolve the gluon splitting if $M_\perp \gg k_\perp$
  - If $x_1 \ll 1$, the gluon that initiates the process is likely to result from bremsstrahlung from another parent gluon

Factorization

- Logs of $M_\perp$ can be resummed by :
  - promoting $f(x_1)$ to $f(x_1, M_\perp^2)$
  - letting $f(x_1, M_\perp^2)$ evolve with $M_\perp$ according to the DGLAP equation

$$\frac{\partial f(x, M^2)}{\partial \ln(M^2)} = \alpha_s(M^2) \int_x^1 \frac{dz}{z} P(x/z) \otimes f(z, M^2)$$

▷ collinear factorization

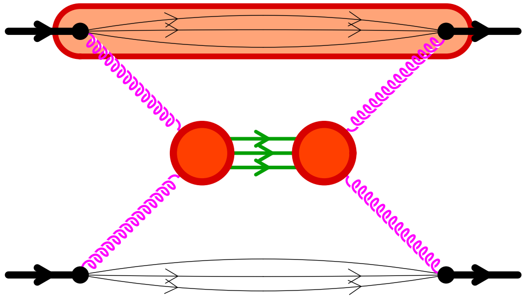
- Logs of x_1 can be resummed by :
  - promoting $f(x_1)$ to a non integrated distribution $\varphi(x_1, \vec{\boldsymbol{k}}_\perp)$
  - letting $\varphi(x_1, \vec{\boldsymbol{k}}_\perp)$ evolve with x_1 according to the BFKL equation

$$\frac{\partial \varphi(x, k_\perp)}{\partial \ln(1/x)} = \alpha_s \int \frac{d^2\vec{\boldsymbol{p}}_\perp}{(2\pi)^2} K(\vec{\boldsymbol{k}}_\perp, \vec{\boldsymbol{p}}_\perp) \otimes \varphi(x, \vec{\boldsymbol{p}}_\perp)$$

▷ $k_\perp$-factorization

Higher twist corrections

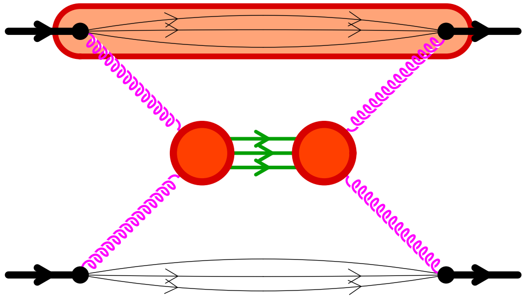
- Leading twist :



▷ 2-point function in the projectile ▷ gluon number

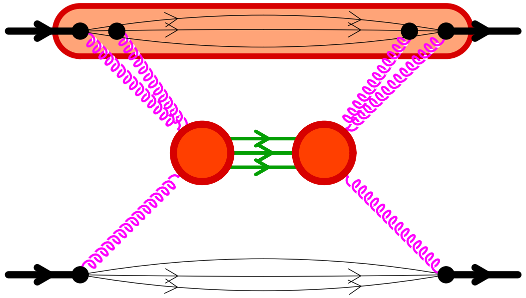
Higher twist corrections

- Leading twist :



▷ 2-point function in the projectile ▷ gluon number

- Higher twist contributions :



▷ 4-point function in the projectile ▷ higher correlation
▷ multiple scattering in the projectile

Higher twist corrections

- Power counting : rescattering corrections are suppressed by inverse powers of the typical mass scale in the process :

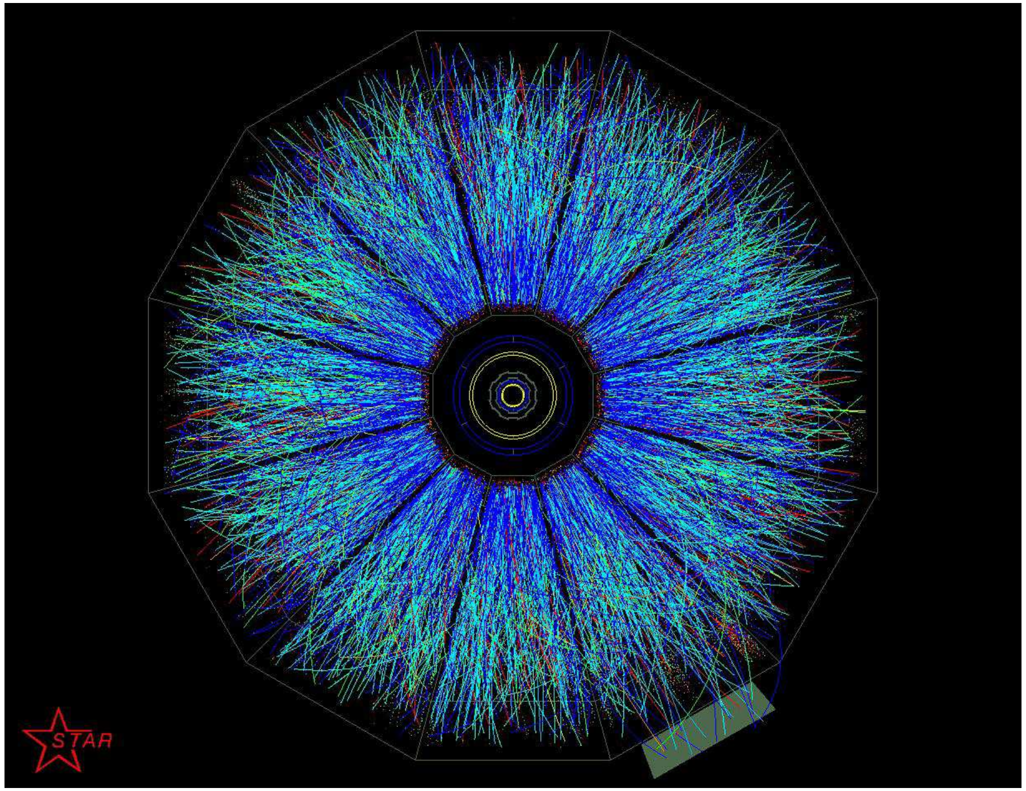
$$\left[\frac{\mu^2}{M_\perp^2}\right]^n$$

- The parameter μ^2 has a factor of α_s, and a factor proportional to the gluon density $\triangleright$ rescatterings are important at high density
- Relative order of magnitude :

$$\frac{\text{twist 4}}{\text{twist 2}} \sim \frac{Q_s^2}{M_\perp^2} \quad \text{with} \quad Q_s^2 \sim \alpha_s \frac{xG(x,Q_s^2)}{\pi R^2}$$

- When this ratio becomes ~ 1, all the rescattering corrections become important
- These effects are not accounted for in DGLAP or BFKL

Higher twist corrections

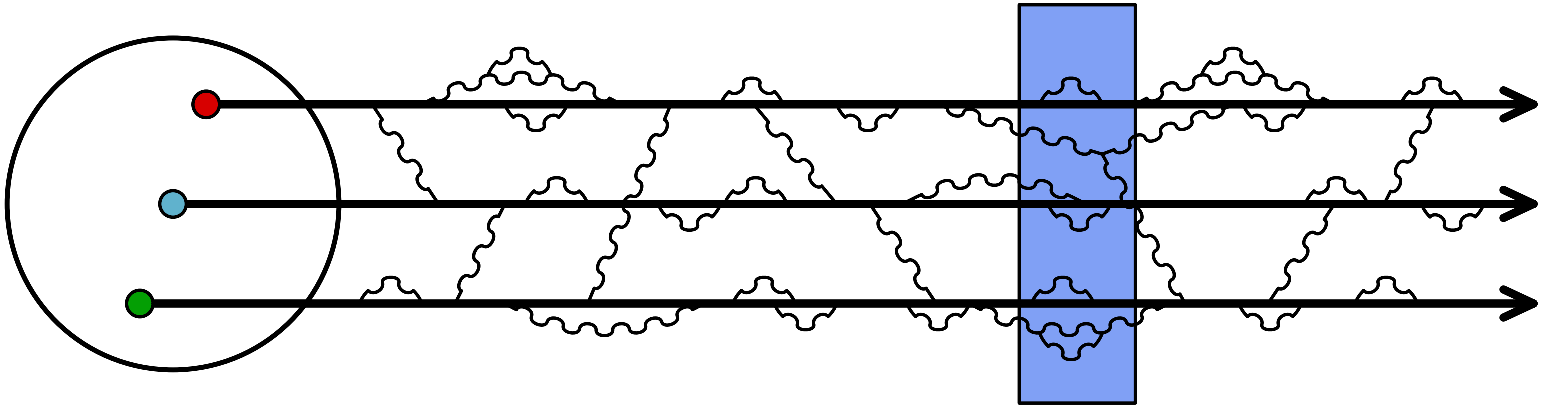


- 99% of the multiplicity below $p_\perp \sim 2$ GeV
- Q_s^2 might be as large as 5 GeV^2 at the LHC ($\sqrt{s} = 5.5$ TeV)
  $\triangleright$ rescatterings are important, and one should also resum logs of $1/x$

Goals

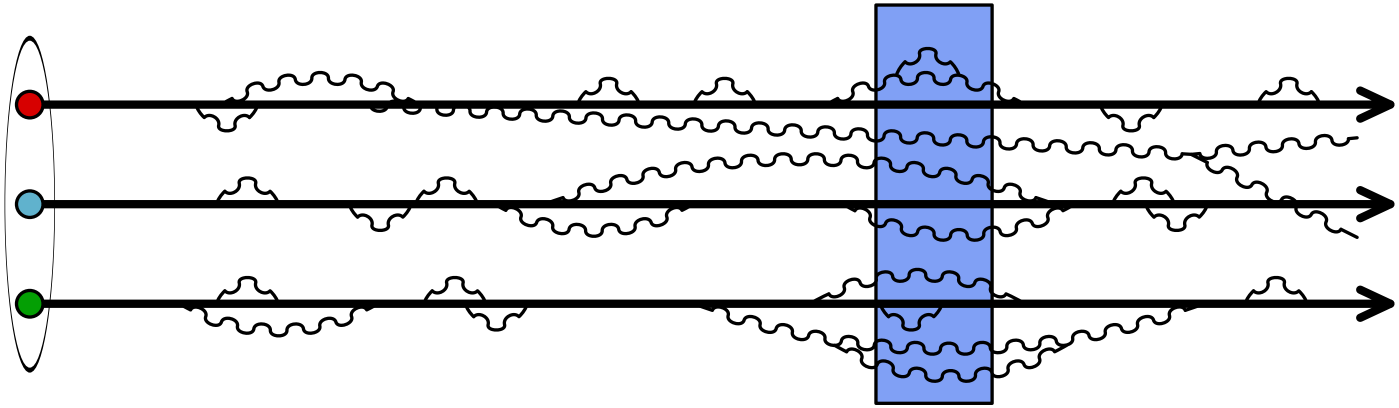
- The Color Glass Condensate framework aims at resumming all the $[\alpha_s \ln(1/x)]^m \; [Q_s/\boldsymbol{p}_\perp]^n$ corrections
- Generalize the concept of “parton distribution”
  - Due to the high density of partons, observables depend on higher correlations (beyond the usual parton distributions, which are 2-point correlation functions)
- If logs of $1/x$ show up in loop corrections, one should be able to factor them out into the evolution of these distributions
- These distributions should be universal, with non-perturbative information relegated into the initial condition for the evolution
- There may possibly be extra divergences associated with the evolution of the final state

Nucleon at rest



- Very complicated non-perturbative object, that contains fluctuations at all space-time scales smaller than its own size
- Only the fluctuations that are longer lived than the external probe participate in the interaction process
- Interactions are very complicated if the constituents of the nucleon have a non trivial dynamics over time-scales comparable to those of the probe

Nucleon at high energy



- Dilation of all internal time-scales of the nucleon
- The constituents behave as if they were free over time-scales comparable to the interaction time
- Many fluctuations live long enough to be seen by the probe.
  The nucleon appears denser at high energy.
  Pre-existing fluctuations act as static sources of new partons
- In a nucleus, soft gluons (long wavelength) belonging to different nucleons overlap in the longitudinal direction
  ▷ coherent effects ▷ saturation

Degrees of freedom and their interplay

Deconfinement transition

Heavy Ion Collisions

Physics of the QGP

Experimental signatures

AdS/CFT duality and the QGP

Is the QGP a perfect fluid?

Color Glass Condensate
- IR & Coll. divergences
- Higher twist corrections
- Degrees of freedom
- Particle production
- Gluon spectrum at LO
- 1-loop corrections

Summary

McLerran, Venugopalan (1994), Iancu, Leonidov, McLerran (2001)

- Soft modes have a large occupation number
  ▷ they are described by a classical color field A^μ that obeys Yang-Mills's equation:

$$[D_\nu, F^{\nu\mu}]_a = J_a^\mu$$

- The source term J_a^μ comes from the faster partons. The hard modes, slowed down by time dilation, are described as frozen color sources ρ_a. Hence :

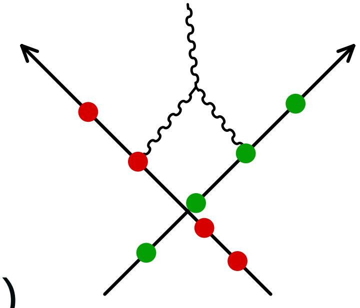
$$J_a^\mu = \delta^{\mu+}\delta(x^-)\rho_a(\vec{\boldsymbol{x}}_\perp) \qquad (x^- \equiv (t-z)/\sqrt{2})$$

- The color sources ρ_a are random, and described by a distribution functional $W_Y[\rho]$, with Y the rapidity that separates "soft" and "hard". Evolution equation (JIMWLK) :

$$\frac{\partial W_Y[\rho]}{\partial Y} = \mathcal{H}[\rho] \;\; W_Y[\rho]$$

Description of hadronic collisions

- Compute the observable $\mathcal{O}$ of interest for a configuration of the sources ρ_1, ρ_2. Note : the sources are $\sim 1/g$ $\triangleright$ weak coupling but strong interactions
- At LO, this requires to solve the classical Yang-Mills equations in the presence of the following current :



$$J^\mu \equiv \delta^{\mu+}\delta(x^-)\,\rho_1(\vec{\boldsymbol{x}}_\perp) + \delta^{\mu-}\delta(x^+)\,\rho_2(\vec{\boldsymbol{x}}_\perp)$$

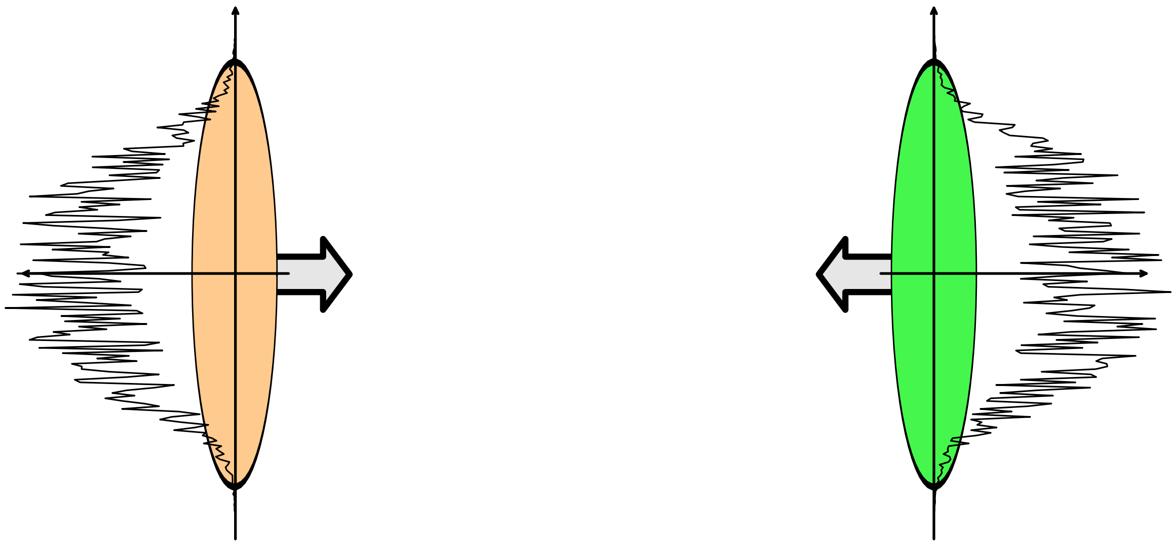
(Note: the boundary condition depend on the observable)

- Average over the sources ρ_1, ρ_2

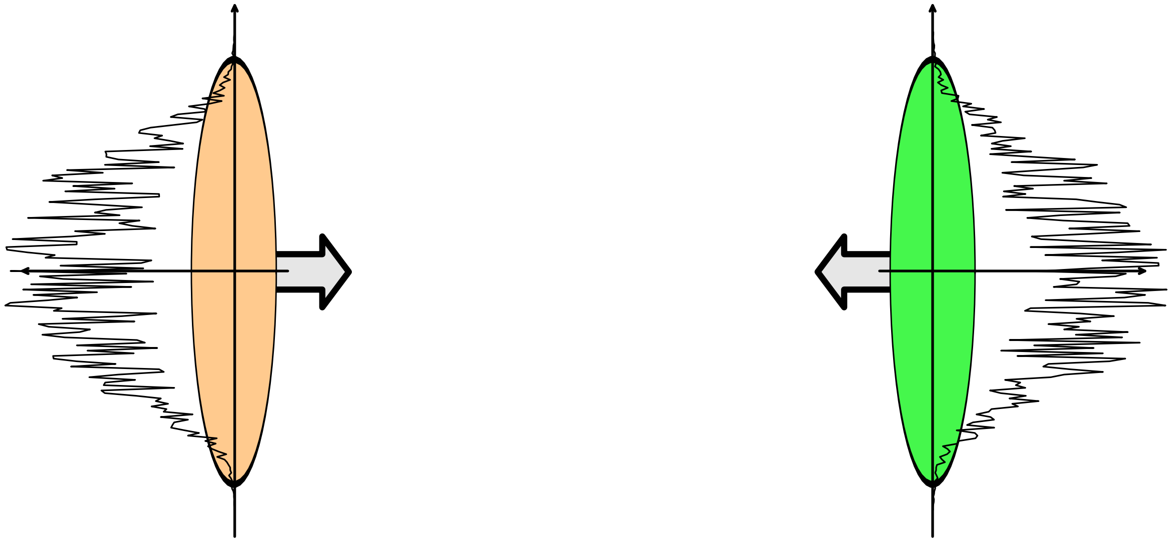
$$\langle \mathcal{O}_Y \rangle = \int [D\rho_1]\,[D\rho_2]\;W_{Y_{\rm beam}-Y}[\rho_1]\;W_{Y+Y_{\rm beam}}[\rho_2]\;\mathcal{O}[\rho_1,\rho_2]$$

- Can this procedure – and in particular the above factorization formula – be justified ?

Description of hadronic collisions



Description of hadronic collisions



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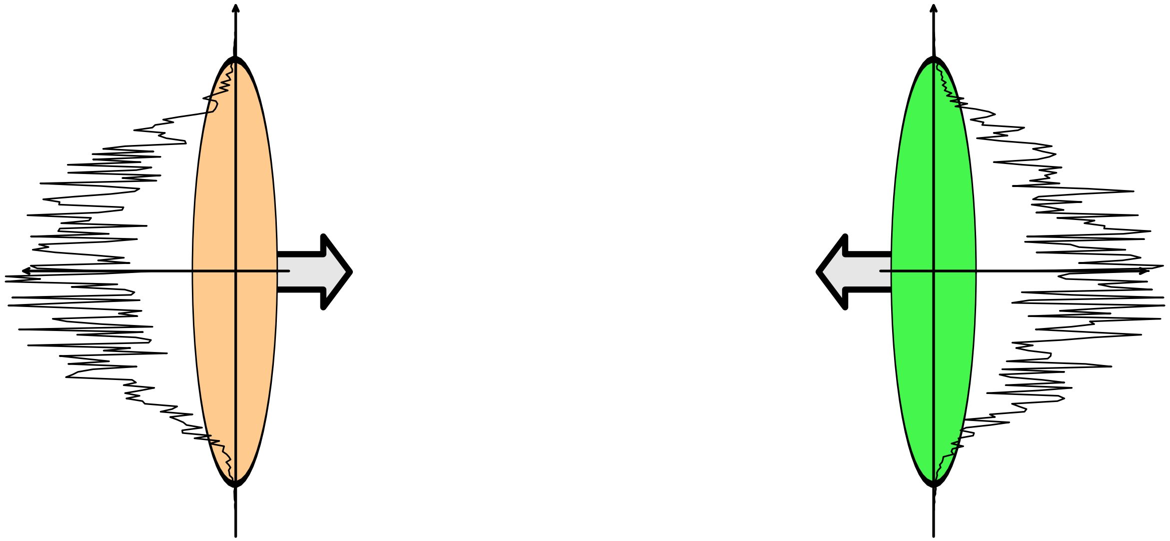

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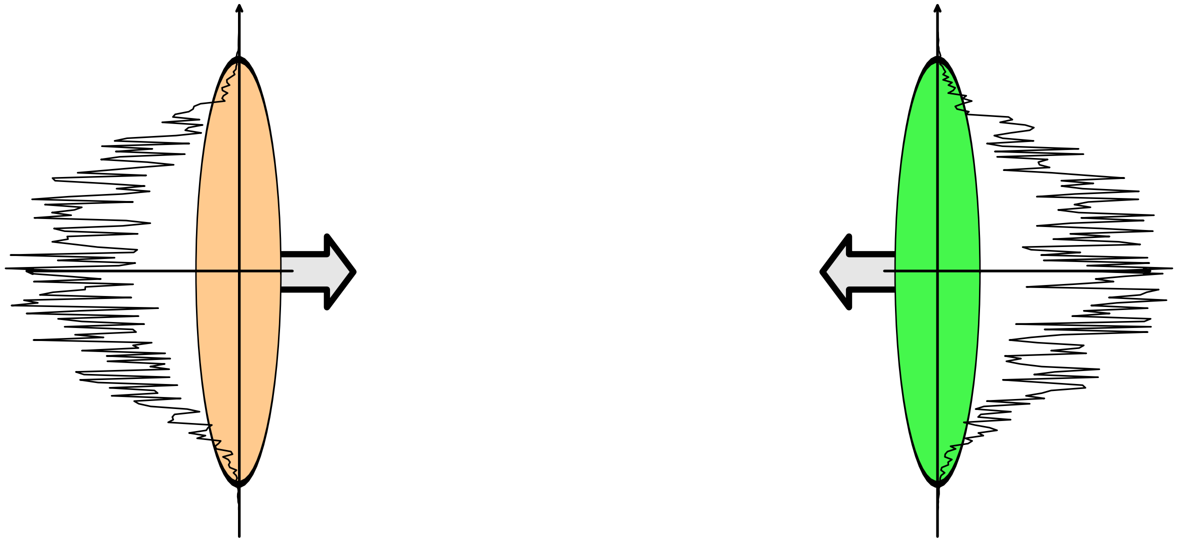

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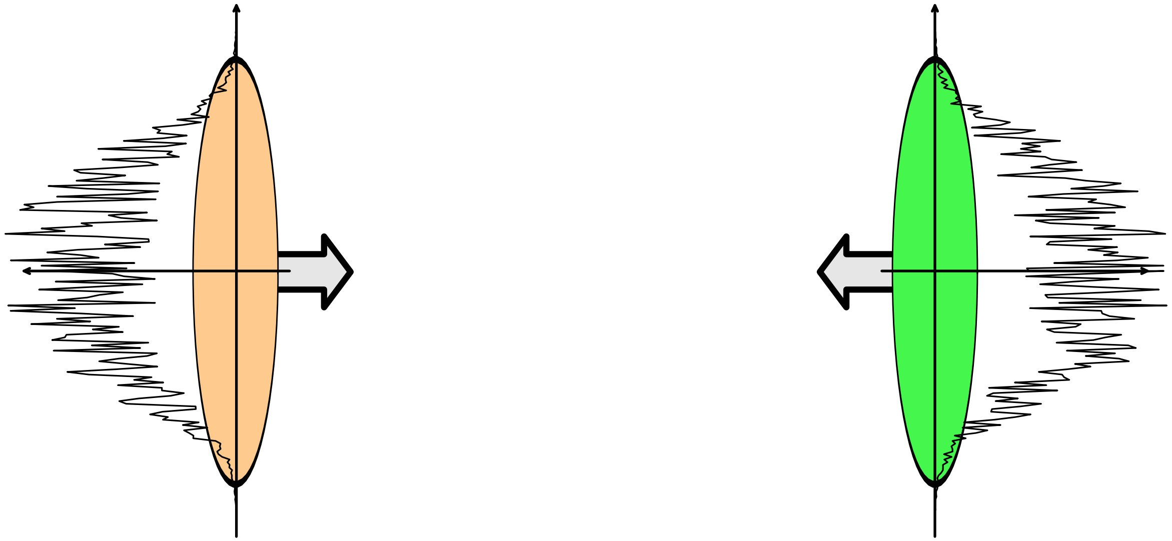

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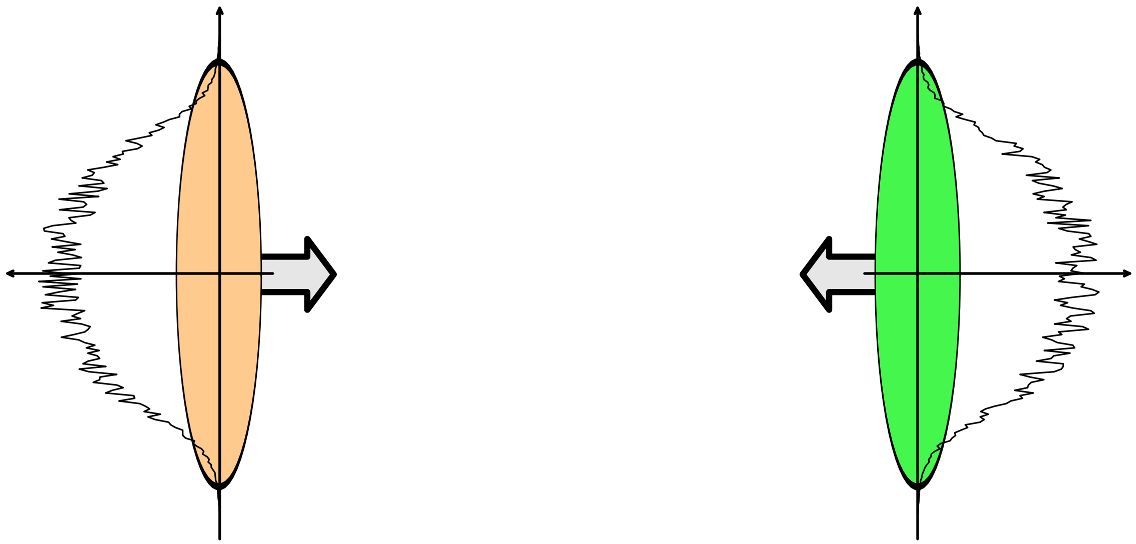
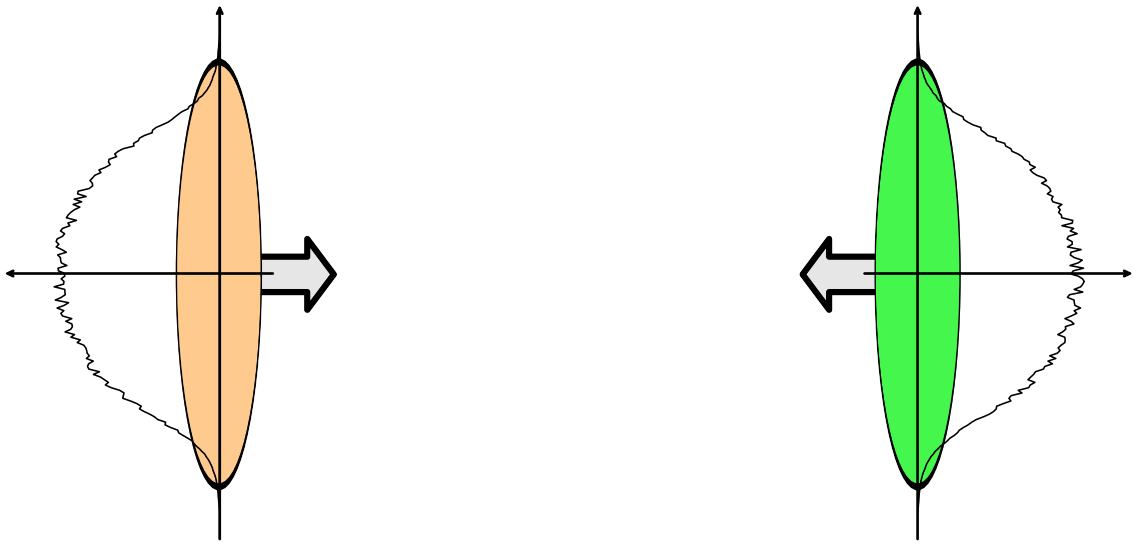


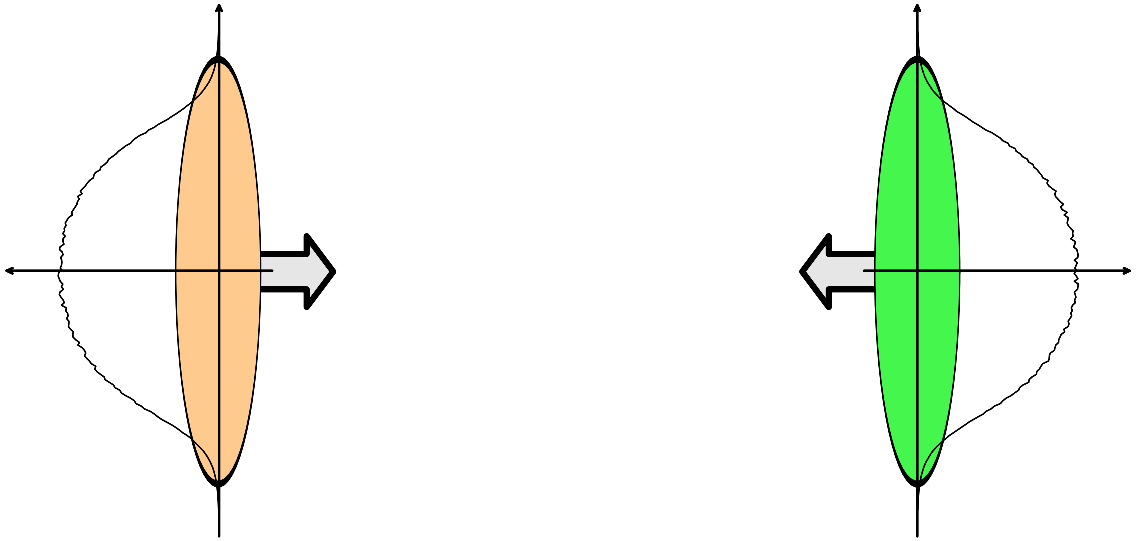
10 configurations

Description of hadronic collisions



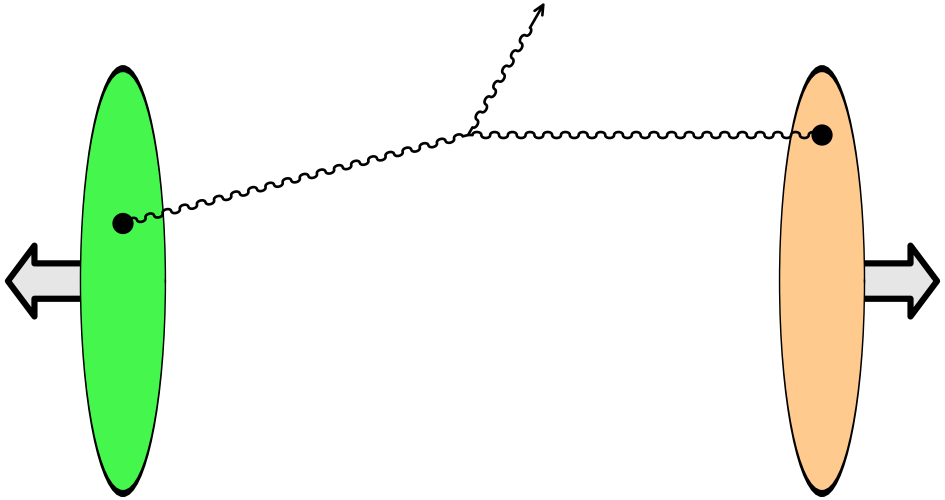
100 configurations

Description of hadronic collisions



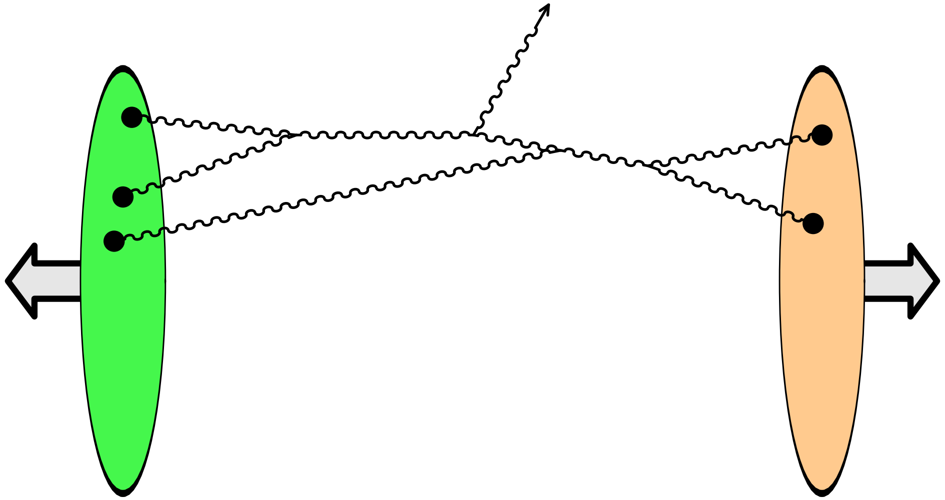
1000 configurations

Particle production at high energy



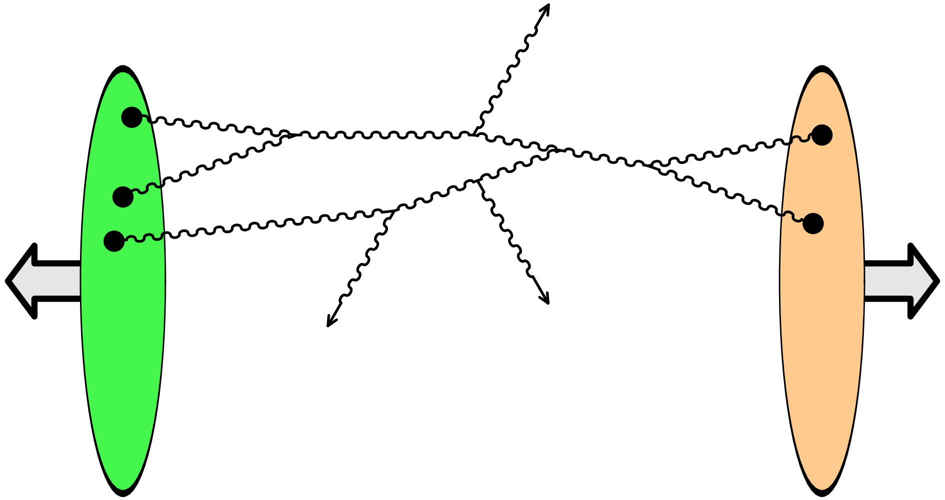
- Dilute regime : one source in each projectile interact

Particle production at high energy



- Dilute regime : one source in each projectile interact
- Dense regime : non linearities are important

Particle production at high energy



- Dilute regime : one source in each projectile interact
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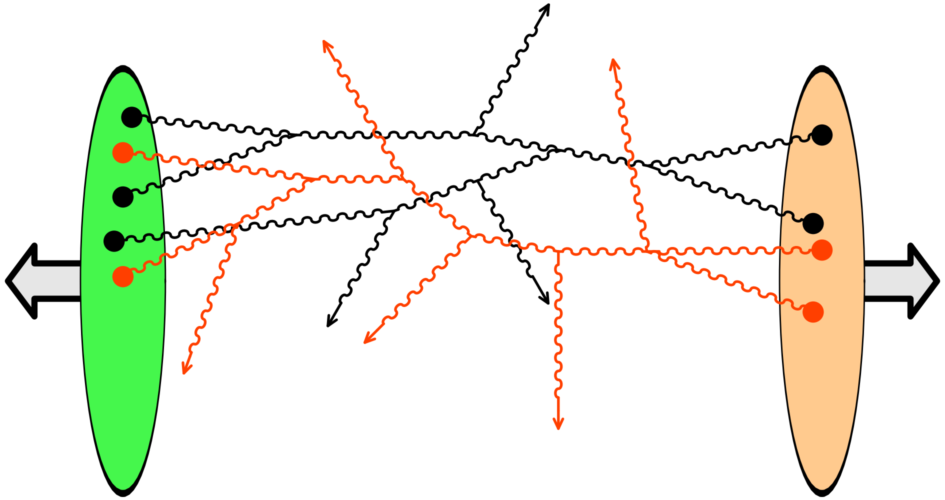
Particle production at high energy



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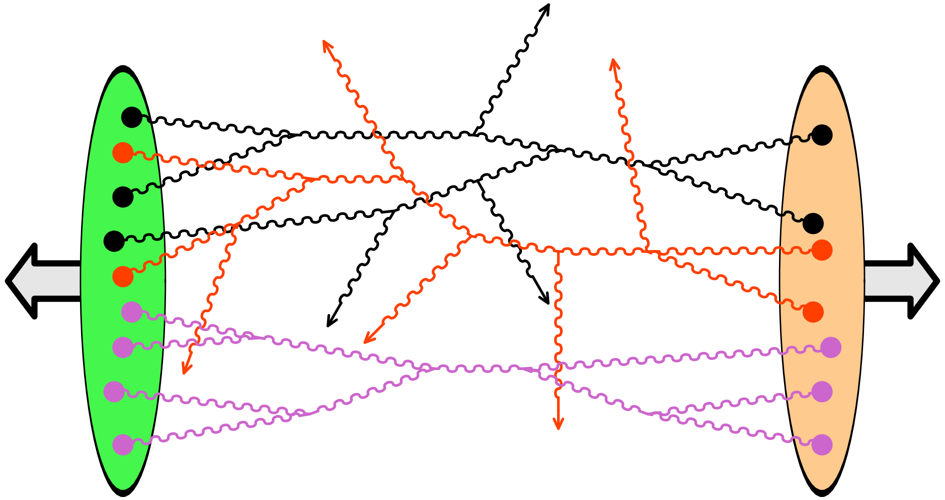
Particle production at high energy



- Dilute regime : one source in each projectile interact
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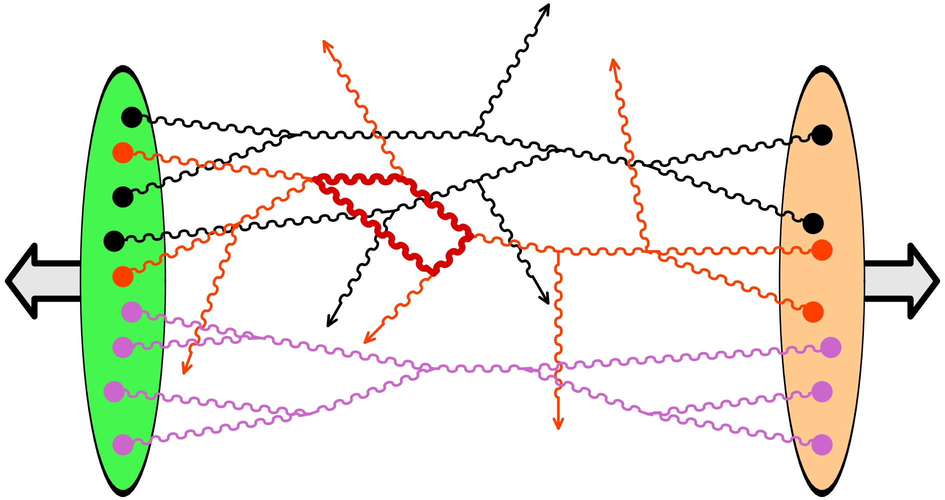
- Some of them may not produce anything (vacuum diagrams)

Particle production at high energy



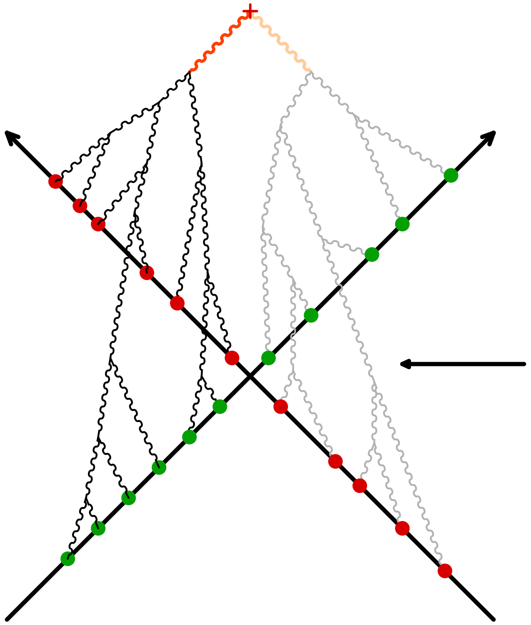
- Dilute regime : one source in each projectile interact
- Dense regime : non linearities are important
- Many gluons can be produced from the same diagram
- There can be many simultaneous disconnected diagrams
- Some of them may not produce anything (vacuum diagrams)
- All these diagrams can have loops (not at LO though)

Gluon multiplicity at LO

Krasnitz, Nara, Venugopalan (1999 – 2001), Lappi (2003)

$$\frac{d\overline{N}_{LO}}{d^3\vec{\boldsymbol{p}}} \propto \int_{x,y} e^{ip\cdot(x-y)} \cdots \mathcal{A}_\mu(x)\mathcal{A}_\nu(y)$$

- $\mathcal{A}^\mu(x) = $ retarded solution of Yang-Mills equations



only tree diagrams at LO

Gluon multiplicity at LO

Krasnitz, Nara, Venugopalan (1999 – 2001), Lappi (2003)

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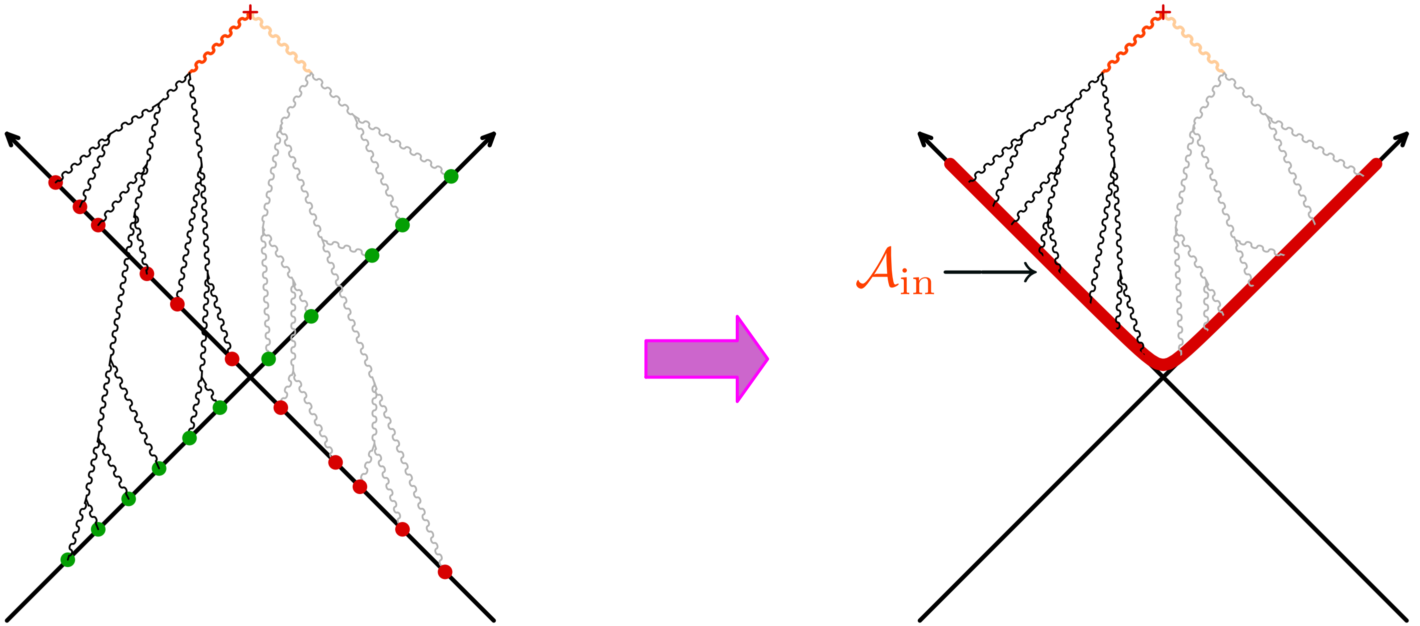
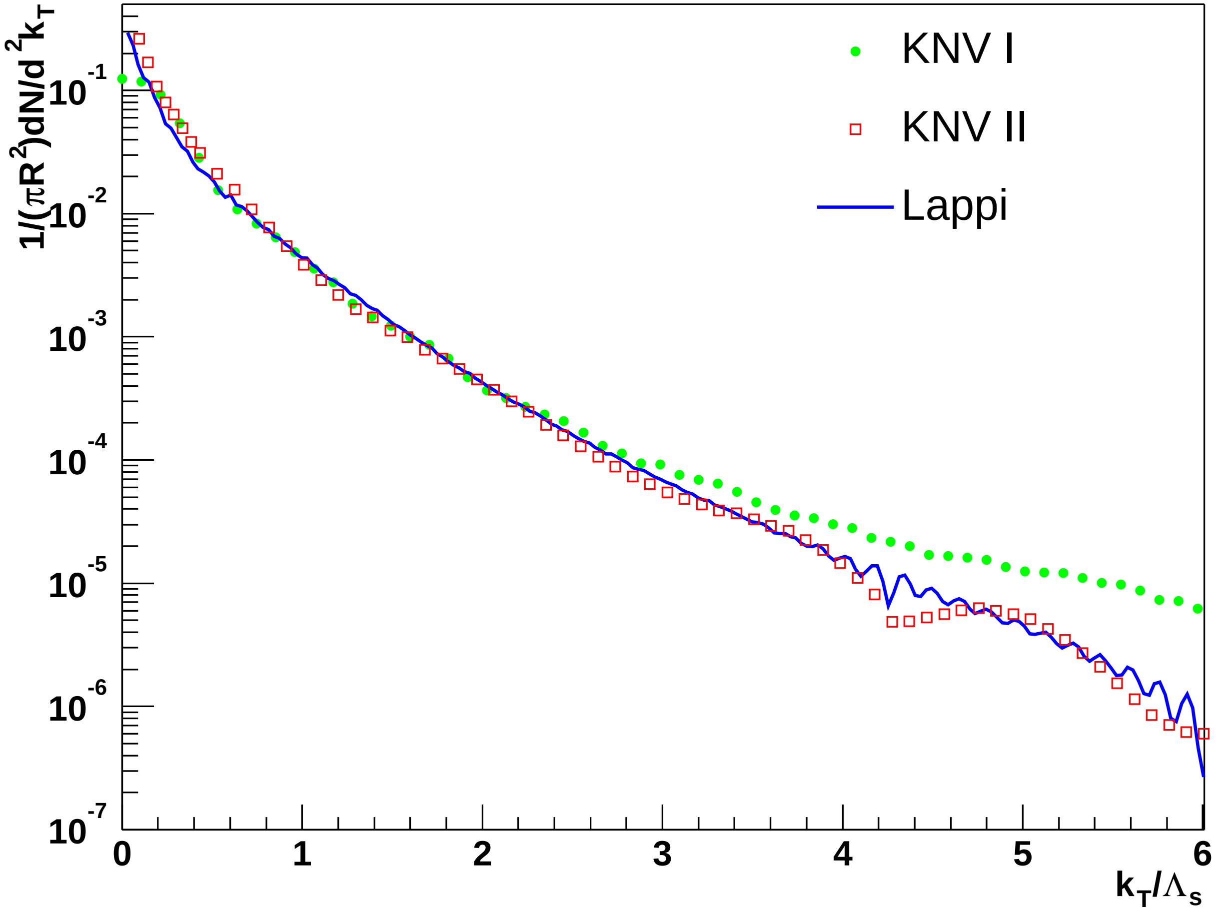
- $\mathcal{A}^\mu(x) =$ retarded solution of Yang-Mills equations
  - ▷ can be cast into an initial value problem on the light-cone

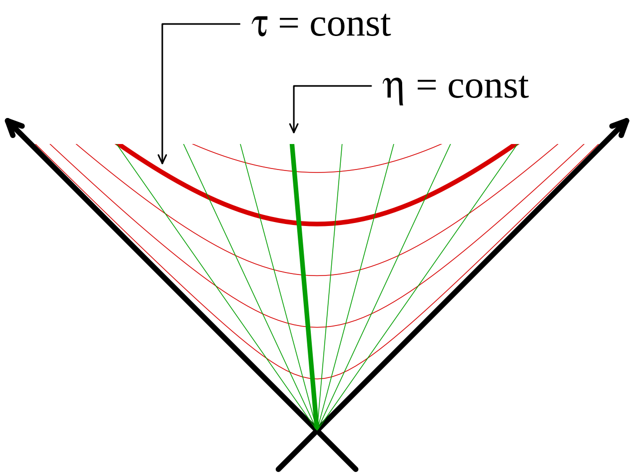


Gluon multiplicity at LO



- Important softening at small $k_\perp$ compared to pQCD (saturation)
- Quark production has also been computed (FG, Kajantie, Lappi (2005))

Initial conditions and boost invariance



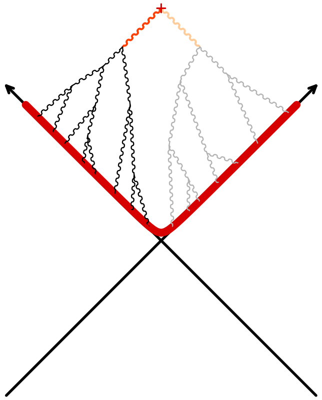
- The color field at $\tau = 0$ does not depend on the rapidity η

  ▷ it remains independent of η at all times (invariance under boosts in the z direction)

  ▷ numerical resolution performed in $2 + 1$ dimensions

1-loop corrections to N

- The 1-loop correction to $\overline{N}$ can be written as a perturbation of the initial value problem encountered at LO :



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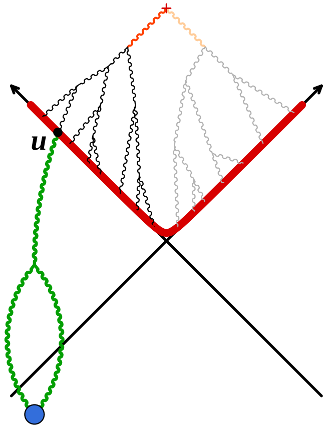

$$\delta\overline{N} = \Bigg[\int_{\vec{\boldsymbol{u}}\,\in\,\text{light cone}} \delta\mathcal{A}_{\rm in}(\vec{\boldsymbol{u}})\;\boldsymbol{T}_{\vec{\boldsymbol{u}}} \Bigg]\;\overline{N}_{LO}$$

- ◆ $\overline{N}_{LO}$ is a functional of the initial fields $\mathcal{A}_{\rm in}(\vec{\boldsymbol{u}})$ on the light-cone
- ◆ $\boldsymbol{T}_{\vec{\boldsymbol{u}}}$ is the generator of shifts of the initial condition at the point $\vec{\boldsymbol{u}}$ on the light-cone, i.e. : $\boldsymbol{T}_{\vec{\boldsymbol{u}}} \sim \delta/\delta\mathcal{A}_{\rm in}(\vec{\boldsymbol{u}})$

1-loop corrections to N

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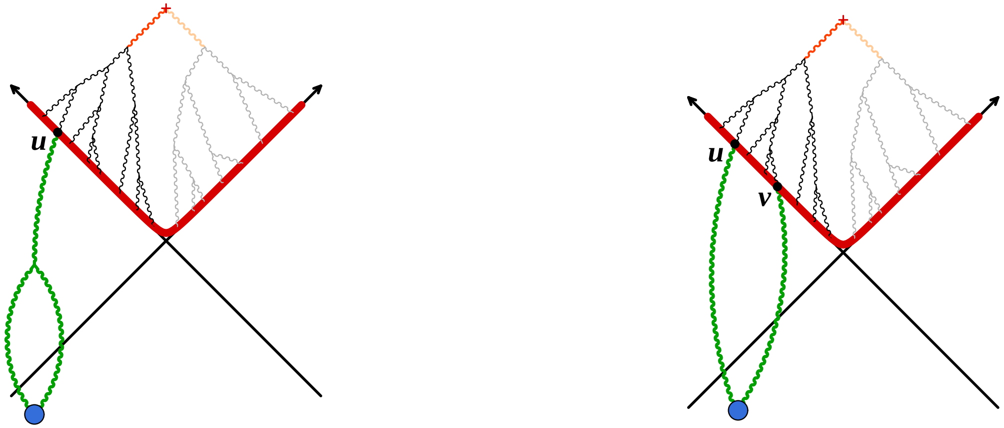


$$\delta\overline{N} = \left[\int\limits_{\vec{\boldsymbol{u}}\,\in\,\text{light cone}} \delta\mathcal{A}_{\rm in}(\vec{\boldsymbol{u}})\;\boldsymbol{T}_{\vec{\boldsymbol{u}}} + \int\limits_{\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}\,\in\,\text{light cone}} \frac{1}{2}\;\boldsymbol{\Sigma}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}})\;\boldsymbol{T}_{\vec{\boldsymbol{u}}}\,\boldsymbol{T}_{\vec{\boldsymbol{v}}}\right]\overline{N}_{LO}$$

- $\overline{N}_{LO}$ is a functional of the initial fields $\mathcal{A}_{\rm in}(\vec{\boldsymbol{u}})$ on the light-cone
- $\boldsymbol{T}_{\vec{\boldsymbol{u}}}$ is the generator of shifts of the initial condition at the point $\vec{\boldsymbol{u}}$ on the light-cone, i.e. : $\boldsymbol{T}_{\vec{\boldsymbol{u}}} \sim \delta/\delta\mathcal{A}_{\rm in}(\vec{\boldsymbol{u}})$
- $\delta\mathcal{A}_{\rm in}(\vec{\boldsymbol{u}})$ and $\boldsymbol{\Sigma}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}})$ are in principle calculable analytically

Divergences

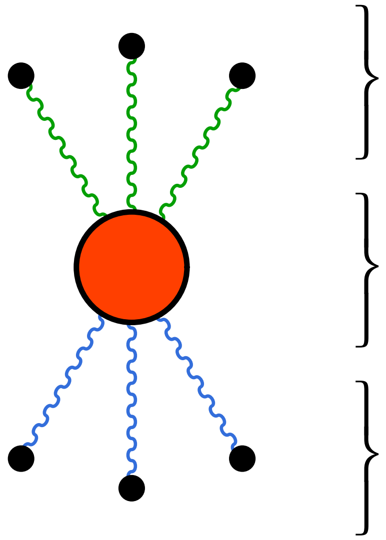
- If taken at face value, this 1-loop correction is plagued by several divergences :
  - The two coefficients $\delta\mathcal{A}_{\rm in}(\vec{\boldsymbol{x}})$ and $\boldsymbol{\Sigma}(\vec{\boldsymbol{x}},\vec{\boldsymbol{y}})$ are infinite, because of an unbounded integration over a rapidity variable
  - At late times, $\boldsymbol{T}_{\vec{\boldsymbol{x}}}\mathcal{A}(\tau,\vec{\boldsymbol{y}})$ diverges exponentially,

$$\boldsymbol{T}_{\vec{\boldsymbol{x}}}\mathcal{A}(\tau,\vec{\boldsymbol{y}}) \underset{\tau\to+\infty}{\sim} e^{\sqrt{\mu\tau}}$$

because of an instability of the classical solution of Yang-Mills equations under rapidity dependent perturbations (Romatschke, Venugopalan (2005))

Initial state factorization

- Anatomy of the full calculation :



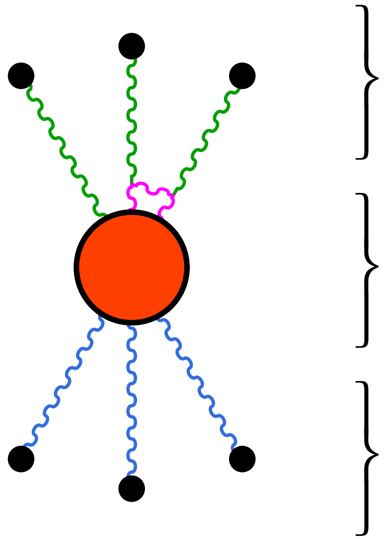
$\mathbf{W}_{\mathbf{Y}_{\text{beam}}\mathbf{-Y}}[\rho_1]$

$\mathbf{N}[\ \mathbf{A}_{\text{in}}(\rho_1\ \mathbf{,}\ \rho_2)\ \mathbf{]}$

$\mathbf{W}_{\mathbf{Y}_{\text{beam}}\mathbf{+Y}}[\rho_2]$

Initial state factorization

- Anatomy of the full calculation :



$$\mathbf{W}_{\mathbf{Y}_{\text{beam}}\mathbf{-Y}}[\rho_1]$$

$$\mathbf{N}[\ \mathbf{A}_{\text{in}}(\rho_1\ ,\ \rho_2)\ ] + \delta\,\mathbf{N}$$

$$\mathbf{W}_{\mathbf{Y}_{\text{beam}}\mathbf{+Y}}[\rho_2]$$

- When the observable $\overline{N}[\mathcal{A}_{\text{in}}(\rho_1,\rho_2)]$ is corrected by an extra gluon, one gets divergences of the form $\alpha_s \int dY$ in $\delta\overline{N}$
  ▷ one would like to be able to absorb these divergences into the Y dependence of the source densities $W_Y[\rho_{1,2}]$

Initial state factorization

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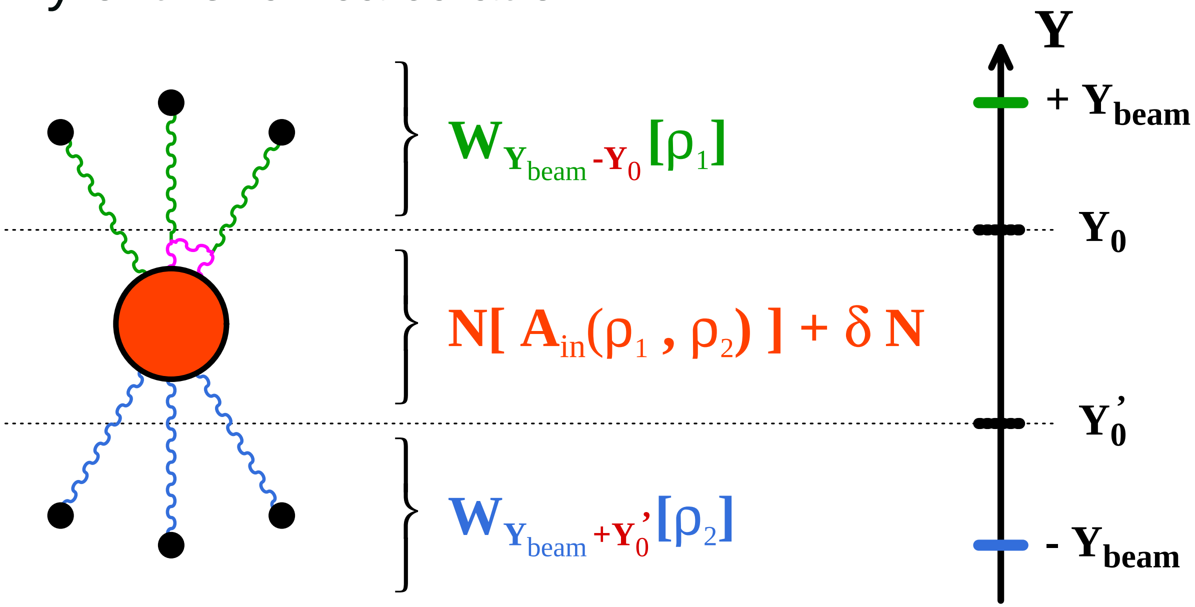


$W_{Y_{\rm beam}-Y_0}[\rho_1]$

$N[\,A_{\rm in}(\rho_1\,,\,\rho_2)\,]+\delta N$

$W_{Y_{\rm beam}+Y_0'}[\rho_2]$

- When the observable $\overline{N}[\mathcal{A}_{\rm in}(\rho_1,\rho_2)]$ is corrected by an extra gluon, one gets divergences of the form $\alpha_s\int dY$ in $\delta\overline{N}$
  $\triangleright$ one would like to be able to absorb these divergences into the Y dependence of the source densities $W_Y[\rho_{1,2}]$
- Equivalently, if one puts some arbitrary frontier Y_0 between the "observable" and the "source distributions", the dependence on Y_0 should cancel between the various factors

Initial state factorization

- The two kind of divergences don't mix, because the divergent part of the coefficients is boost invariant.

  Given their structure, the divergent coefficients seem related to the evolution of the sources in the initial state

- In order to prove the factorization of these divergences in the initial state distributions of sources, one needs to establish :

$$\Big[\delta\overline{N}\Big]_{\substack{\text{divergent}\\ \text{coefficients}}} = \Big[(Y_0 - Y)\,\mathcal{H}^\dagger[\rho_1] + (Y - Y_0')\,\mathcal{H}^\dagger[\rho_2]\Big]\;\overline{N}_{LO}$$

where $\mathcal{H}[\rho]$ is the Hamiltonian that governs the rapidity dependence of the source distribution $W_{_Y}[\rho]$:

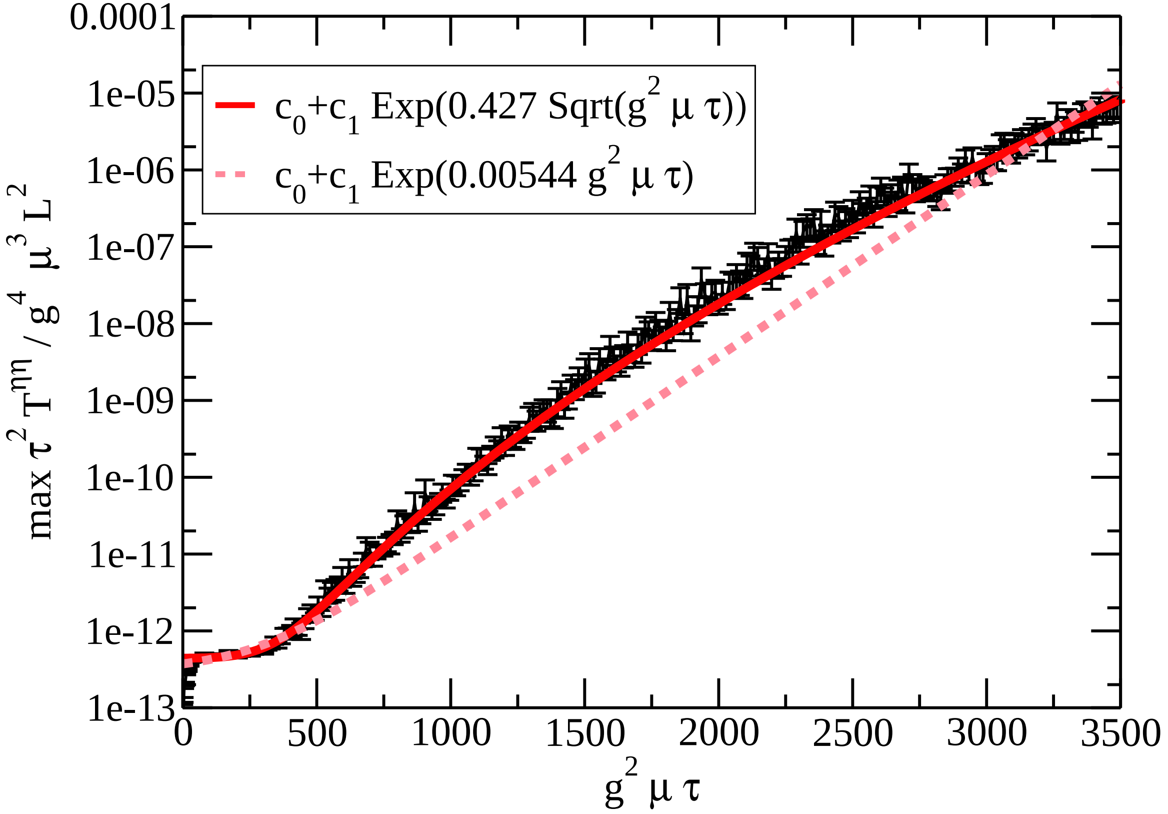
$$\frac{\partial W_{_Y}[\rho]}{\partial Y} = \mathcal{H}[\rho]\;W_{_Y}[\rho]$$

FG, Lappi, Venugopalan (work in progress)

Unstable modes

Romatschke, Venugopalan (2005)

- Rapidity dependent perturbations to the classical fields grow like $\exp(\#\sqrt{\tau})$ until the non-linearities become important :



Unstable modes

- The coefficient $\delta\mathcal{A}_{\rm in}(\vec{\boldsymbol{x}})$ is boost invariant.
  Hence, the divergences due to the unstable modes all come from the quadratic term in $\delta\overline{N}$:

$$\left[\delta\overline{N}\right]_{\substack{\text{unstable}\\ \text{modes}}} = \left\{\frac{1}{2}\int_{\vec{\boldsymbol{x}},\vec{\boldsymbol{y}}} \boldsymbol{\Sigma}(\vec{\boldsymbol{x}},\vec{\boldsymbol{y}})\; \boldsymbol{T}_{\vec{\boldsymbol{x}}}\boldsymbol{T}_{\vec{\boldsymbol{y}}}\right\} \overline{N}_{LO}[\mathcal{A}_{\rm in}(\rho_1,\rho_2)]$$

- When summed to all orders, this becomes a certain functional $Z[\boldsymbol{T}_{\vec{\boldsymbol{x}}}]$:

$$\left[\delta\overline{N}\right]_{\substack{\text{unstable}\\ \text{modes}}} = Z[\boldsymbol{T}_{\vec{\boldsymbol{x}}}]\;\; \overline{N}_{LO}[\mathcal{A}_{\rm in}(\rho_1,\rho_2)]$$

Unstable modes

- This can be arranged in a more intuitive way :

$$\left[\delta \overline{N}\right]_{\substack{\text{unstable}\\ \text{modes}}} = \int [Da]\ \widetilde{Z}[a(\vec{\boldsymbol{x}})]\ e^{i\int_{\vec{\boldsymbol{x}}} a(\vec{\boldsymbol{x}})\, \boldsymbol{T}_{\vec{\boldsymbol{x}}}}\ \overline{N}_{LO}[\mathcal{A}_{\rm in}(\rho_1,\rho_2)]$$

$$= \int [Da]\ \widetilde{Z}[a(\vec{\boldsymbol{x}})]\ \overline{N}_{LO}[\mathcal{A}_{\rm in}(\rho_1,\rho_2)+a]$$

▷ summing these divergences simply requires to add fluctuations to the initial condition for the classical problem

▷ the fact that $\delta\mathcal{A}_{\rm in}(\vec{\boldsymbol{x}})$ does not contribute implies that the distribution of fluctuations is real

- Interpretation :

Despite the fact that the fields are coupled to strong sources, the classical approximation alone is not good enough, because the classical solution has unstable modes that can be triggered by the quantum fluctuations

Unstable modes

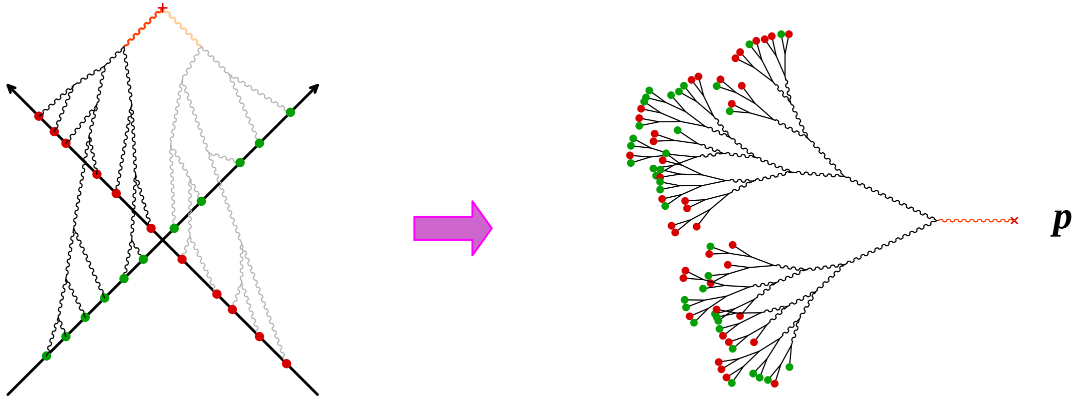
- Combining everything, one should write

$$\frac{d\overline{N}}{dY\,d^2\vec{\boldsymbol{p}}_\perp} = \int [D\rho_1]\,[D\rho_2]\;\; W_{Y_{\rm beam}-Y}[\rho_1]\; W_{Y_{\rm beam}+Y}[\rho_2] \times \int [Da]\;\; \widetilde{Z}[a]\;\; \frac{d\overline{N}[\mathcal{A}_{\rm in}(\rho_1,\rho_2)+a]}{dY\,d^2\vec{\boldsymbol{p}}_\perp}$$

▷ This formula resums (all?) the divergences that occur at one loop, both in the initial and final state

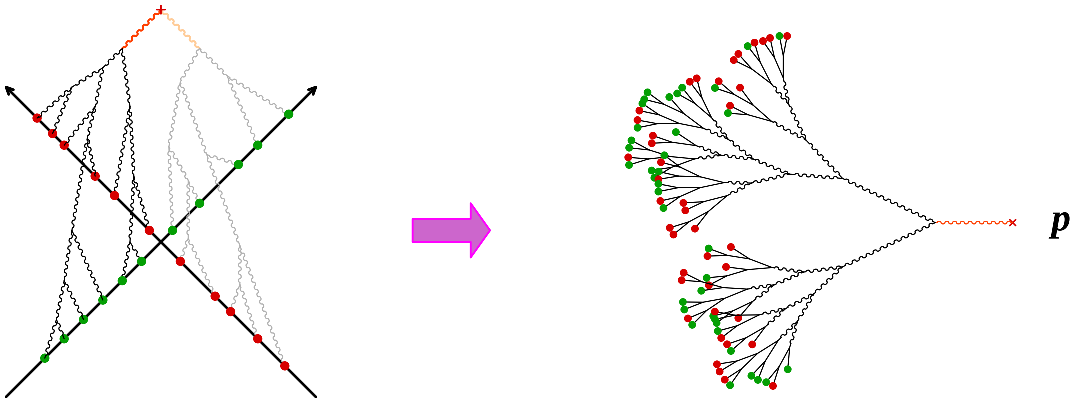
Unstable modes

- Tree level :

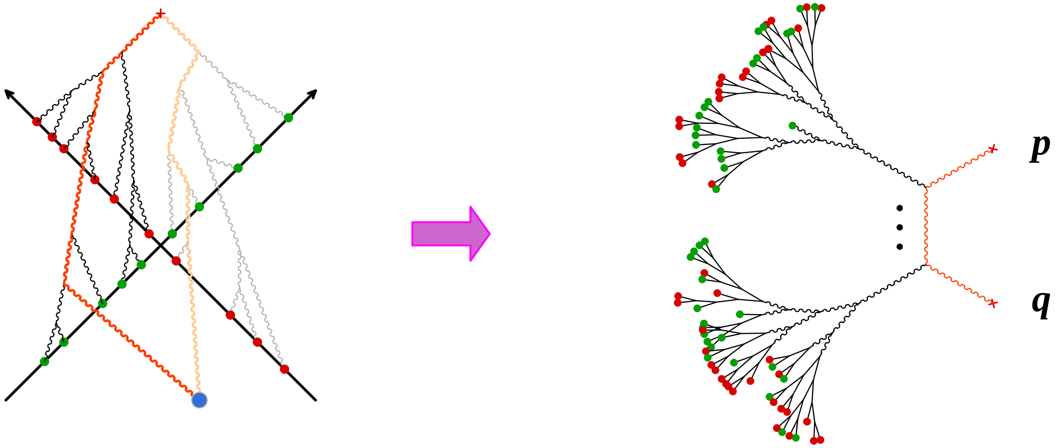


Unstable modes

- Tree level :



- One loop :



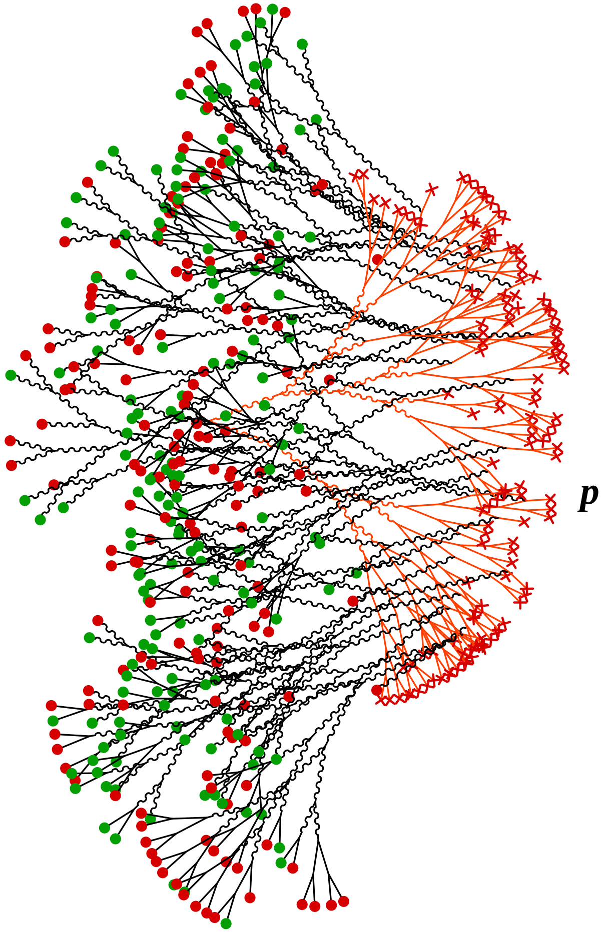
▷ The momentum $\vec{q}$ is integrated out

▷ If $\alpha_s^{-1} \lesssim \left|y_{\boldsymbol{p}} - y_{\boldsymbol{q}}\right|$, the correction is absorbed in $W[\rho_{1,2}]$

▷ If $\left|y_{\boldsymbol{p}} - y_{\boldsymbol{q}}\right| \lesssim \alpha_s^{-1}$: gluon splitting in the final state

Unstable modes

- After summing the fluctuations, things may look like this :



▷ these splittings may help to fight against the expansion ?

Note : the timescale for this process is $\tau \sim Q_s^{-1} \ln^2(1/\alpha_s)$

Summary

Summary

- Very rich (and complicated, sometimes messy...) physics
- Some aspects can be assessed by perturbative methods

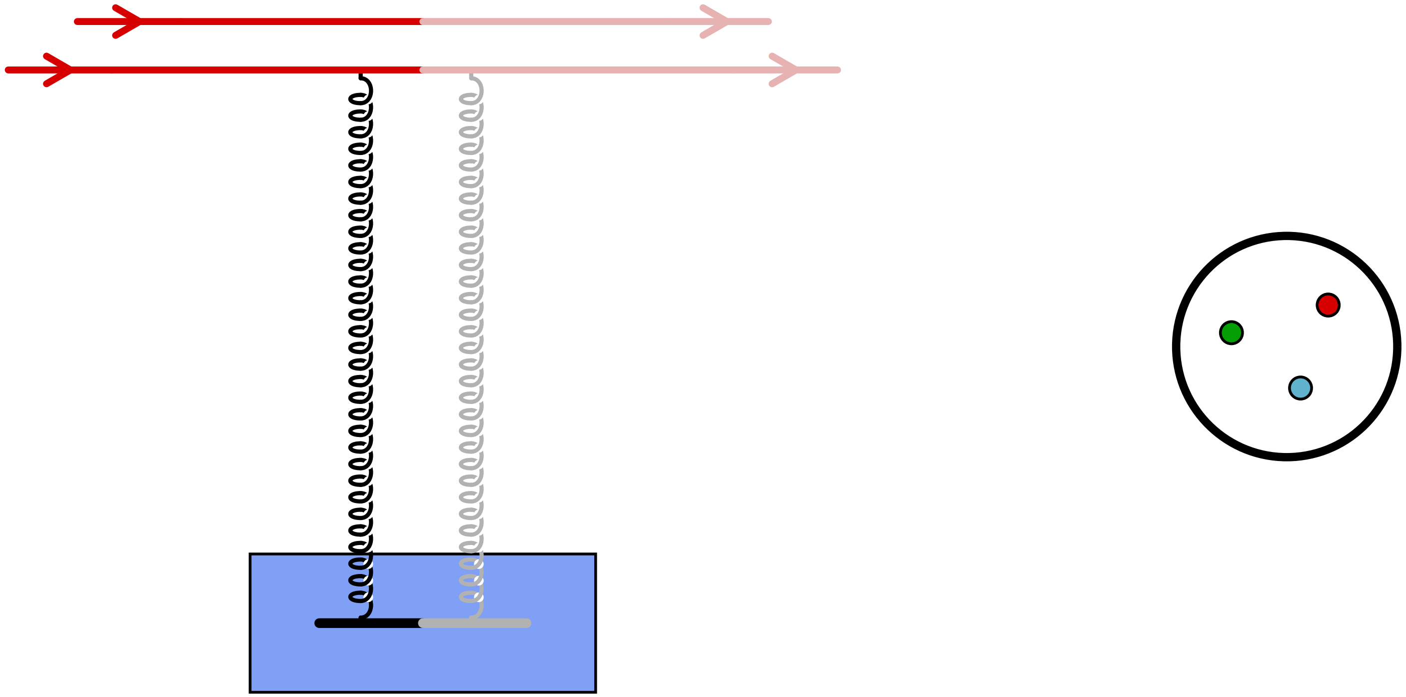
  ▷ these “perturbative” techniques usually require resummations, in order to handle the many scales involved in the problem
- Interesting applications of string-inspired calculational methods. Two things to keep in mind :

  ▷ $\neq$ “strings as the fundamental theory of the early universe”

  ▷ reliability of these results for QCD ?
- Many experimental results from RHIC, that suggest :
  - extremely large energy density
  - low viscosity, almost perfect fluid (?)
  - early thermalization (?)
- One of the biggest theoretical challenges is to understand whether thermalization occurs, by which mechanism and how fast (CGC, plasma instabilities)

Extra bits

Parton evolution

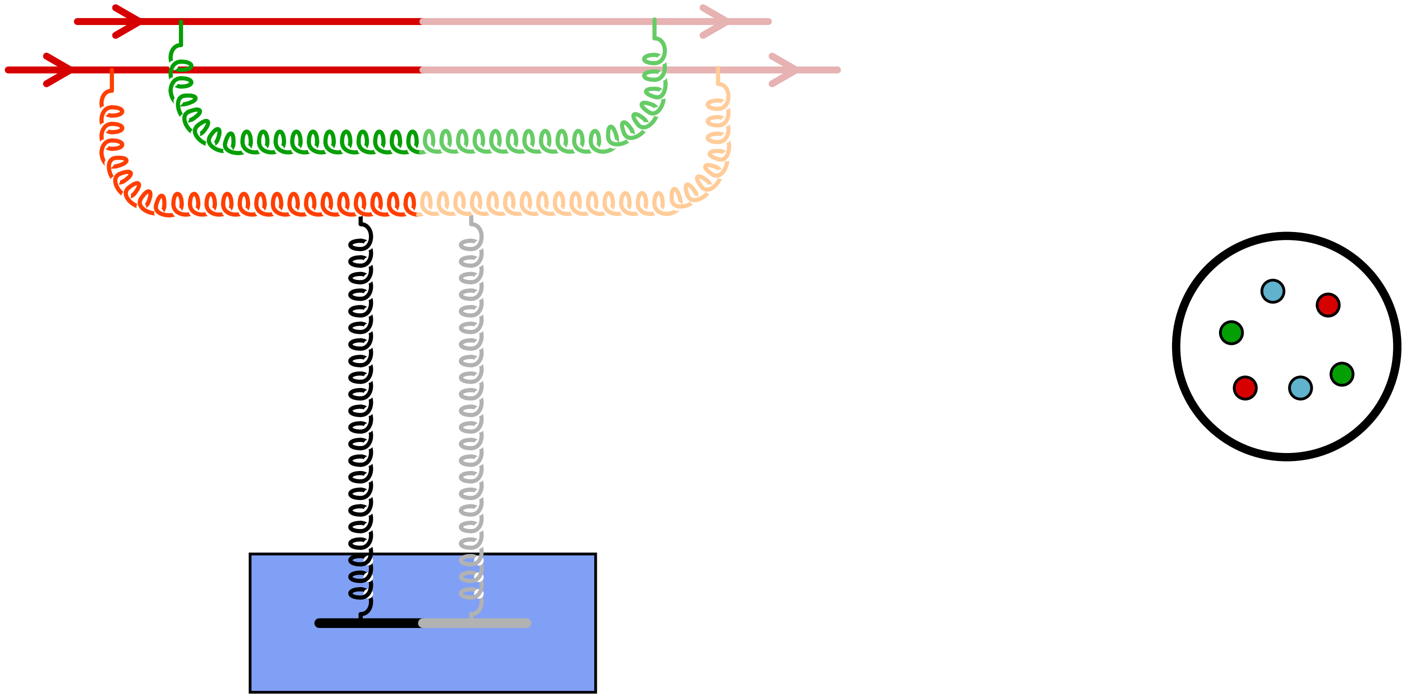


▷ assume that the projectile is big, e.g. a nucleus, and has many valence quarks (only two are represented)

▷ on the contrary, consider a small probe, with few partons

▷ at low energy, only valence quarks are present in the hadron wave function

Parton evolution

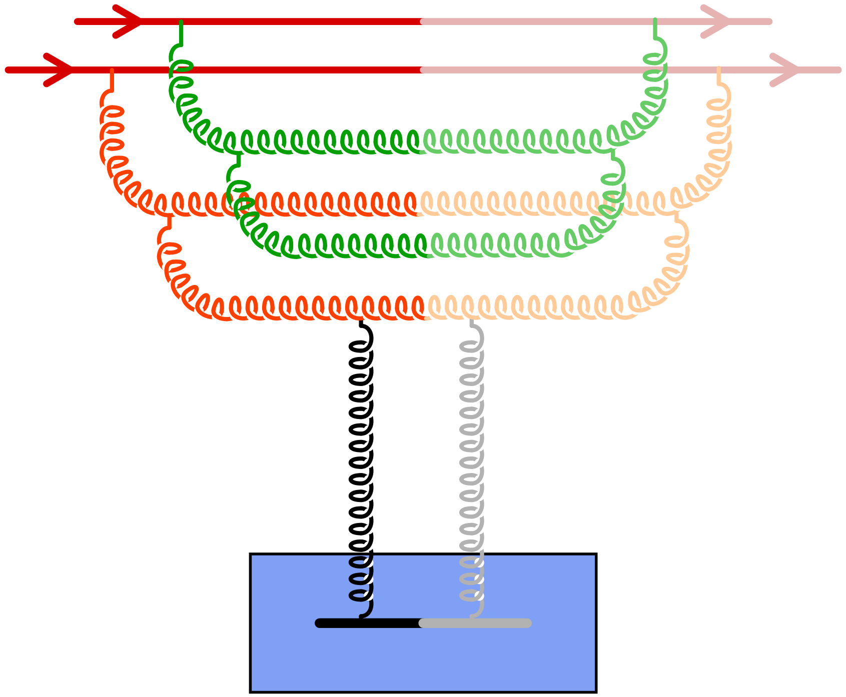


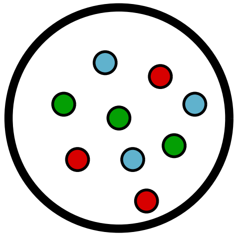
▷ when energy increases, new partons are emitted

▷ the emission probability is $\alpha_s \int \frac{dx}{x} \sim \alpha_s \ln(\frac{1}{x})$, with x the longitudinal momentum fraction of the gluon

▷ at small-x (i.e. high energy), these logs need to be resummed

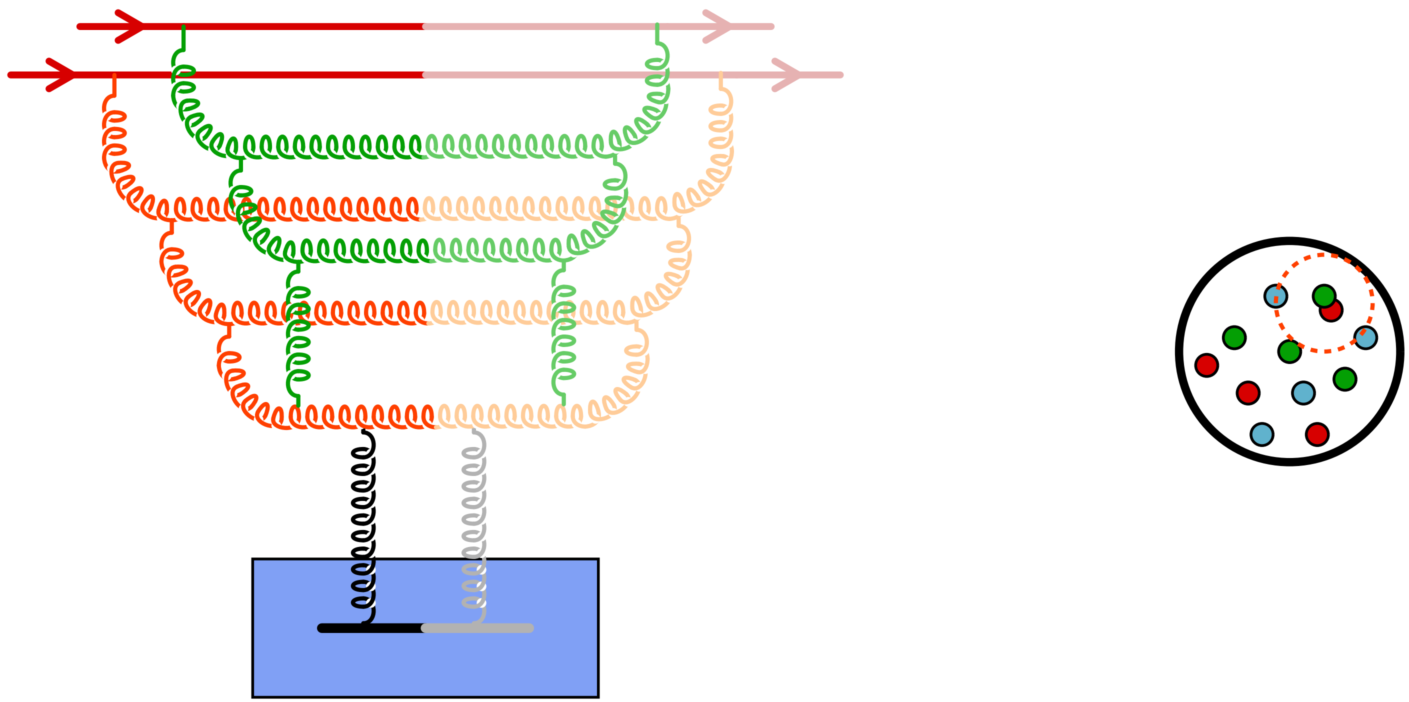
Parton evolution





▷ as long as the density of constituents remains small, the evolution is linear: the number of partons produced at a given step is proportional to the number of partons at the previous step (BFKL)

Parton evolution



- ▷ eventually, the partons start overlapping in phase-space
- ▷ parton recombination becomes favorable
- ▷ after this point, the evolution is non-linear:

the number of partons created at a given step depends non-linearly on the number of partons present previously

Saturation criterion

Gribov, Levin, Ryskin (1983)

- Number of gluons per unit area:

$$\rho \sim \frac{xG_A(x,Q^2)}{\pi R_A^2}$$

- Recombination cross-section:

$$\sigma_{gg\to g} \sim \frac{\alpha_s}{Q^2}$$

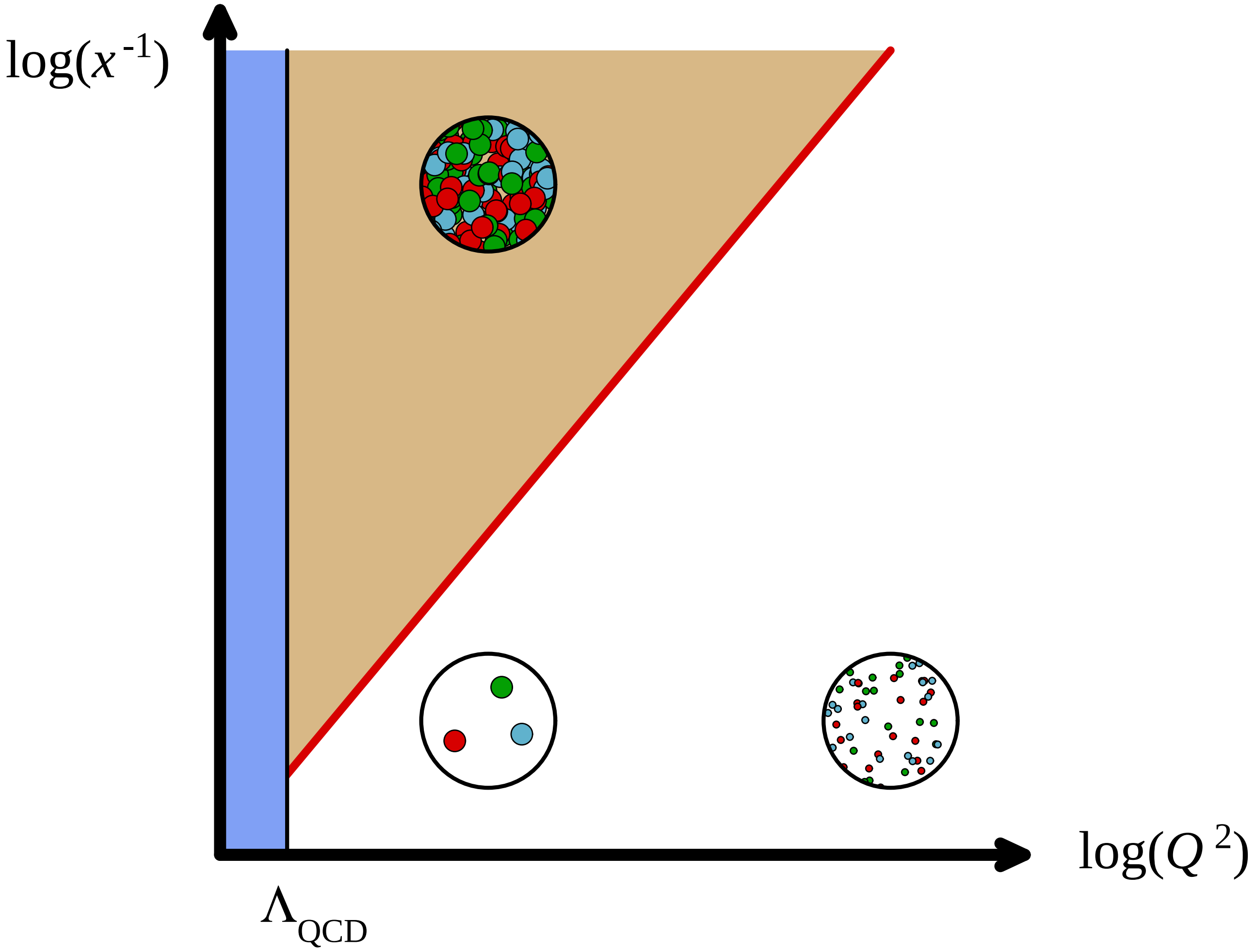
- Recombination happens if $\rho\sigma_{gg\to g} \gtrsim 1$, i.e. $Q^2 \lesssim Q_s^2$, with:

$$Q_s^2 \sim \frac{\alpha_s xG_A(x,Q_s^2)}{\pi R_A^2} \sim A^{1/3}\frac{1}{x^{0.3}}$$

- At saturation, the phase-space density is:

$$\frac{dN_g}{d^2\vec{\boldsymbol{x}}_\perp d^2\vec{\boldsymbol{p}}_\perp} \sim \frac{\rho}{Q^2} \sim \frac{1}{\alpha_s}$$

Saturation domain



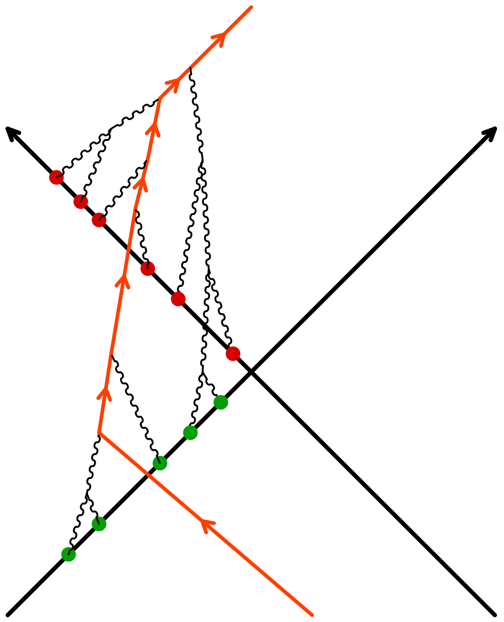
Quark production

Deconfinement transition
Heavy Ion Collisions
Physics of the QGP
Experimental signatures
AdS/CFT duality and the QGP
Is the QGP a perfect fluid?
Color Glass Condensate
Summary
Extra bits
● Parton saturation
○ Quark production
● Longitudinal expansion

FG, Kajantie, Lappi (2004, 2005)

$$E_{\boldsymbol{p}} \frac{d\langle n_{\text{quarks}}\rangle}{d^3\vec{\boldsymbol{p}}} = \frac{1}{16\pi^3} \int_{x,y} e^{ip\cdot(x-y)}\, \partial\!\!\!/_x \partial\!\!\!/_y \,\langle \overline{\psi}(x)\psi(y)\rangle$$

- Dirac equation in the classical color field :



Quark production

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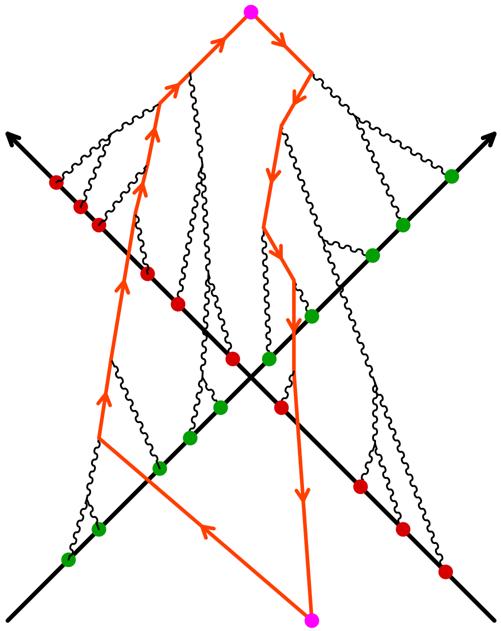
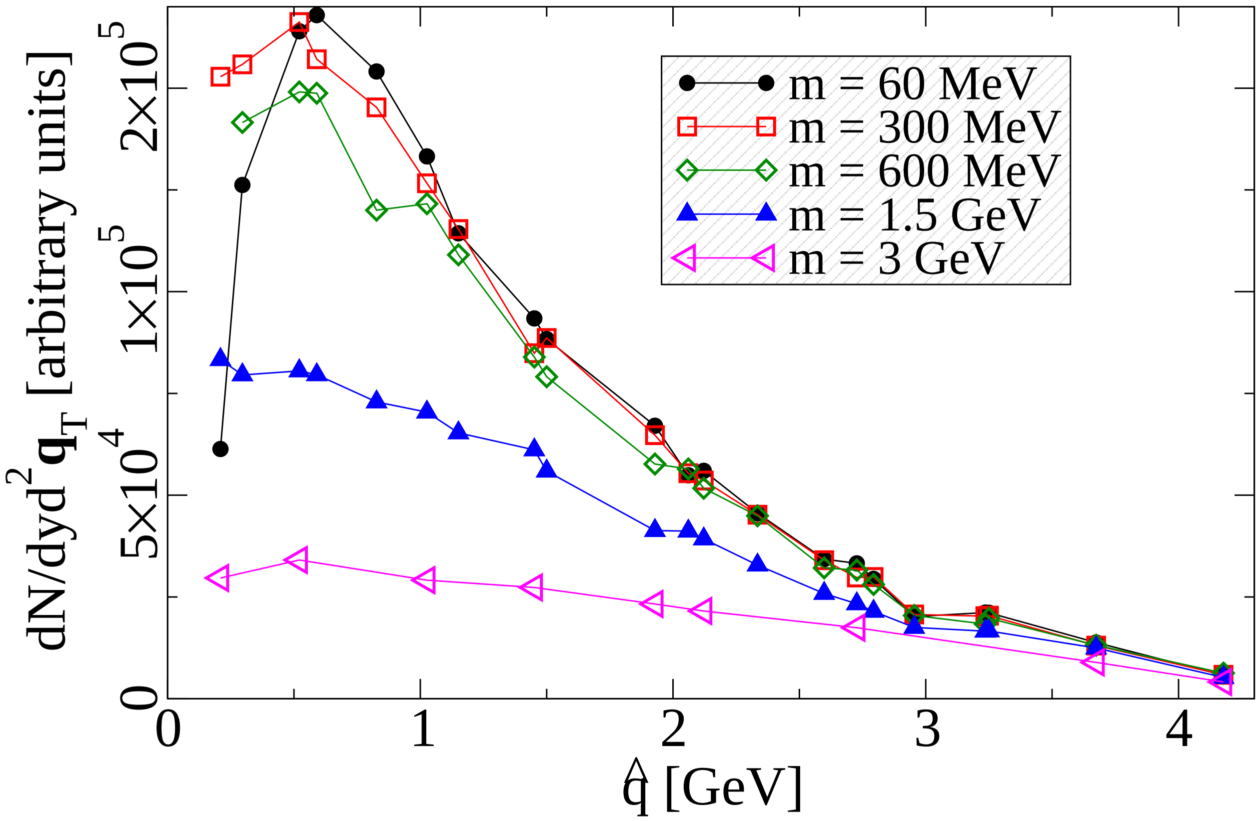


Spectra for various quark masses



Longitudinal expansion

- For a system finite in the η direction, the gluons will have a longitudinal velocity tied to their space-time rapidity

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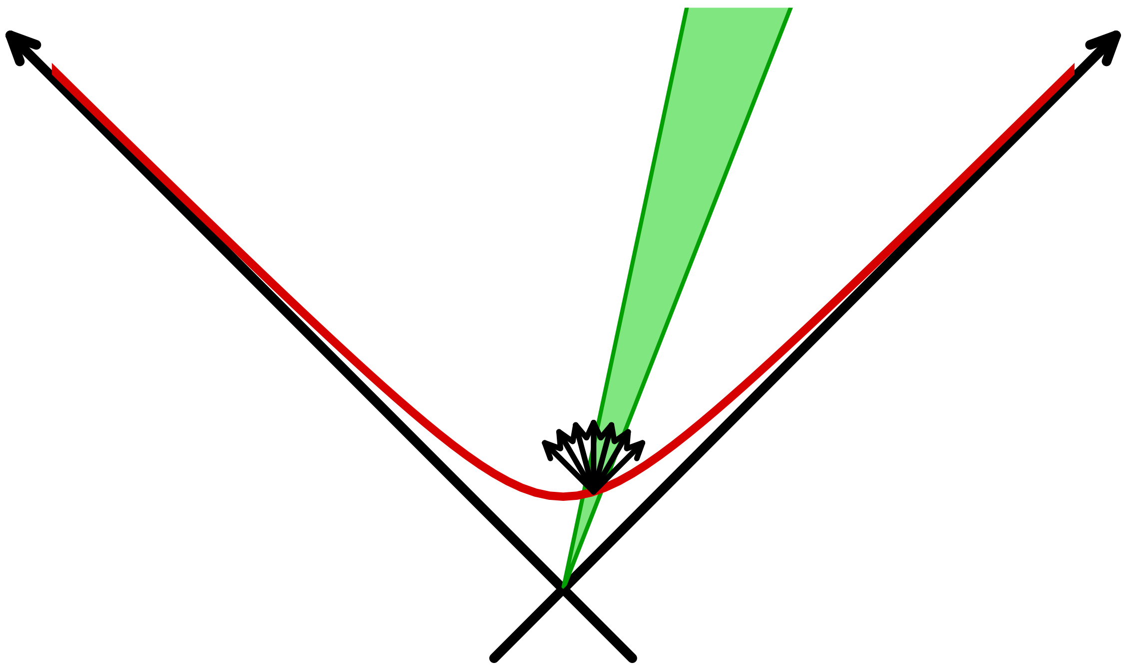

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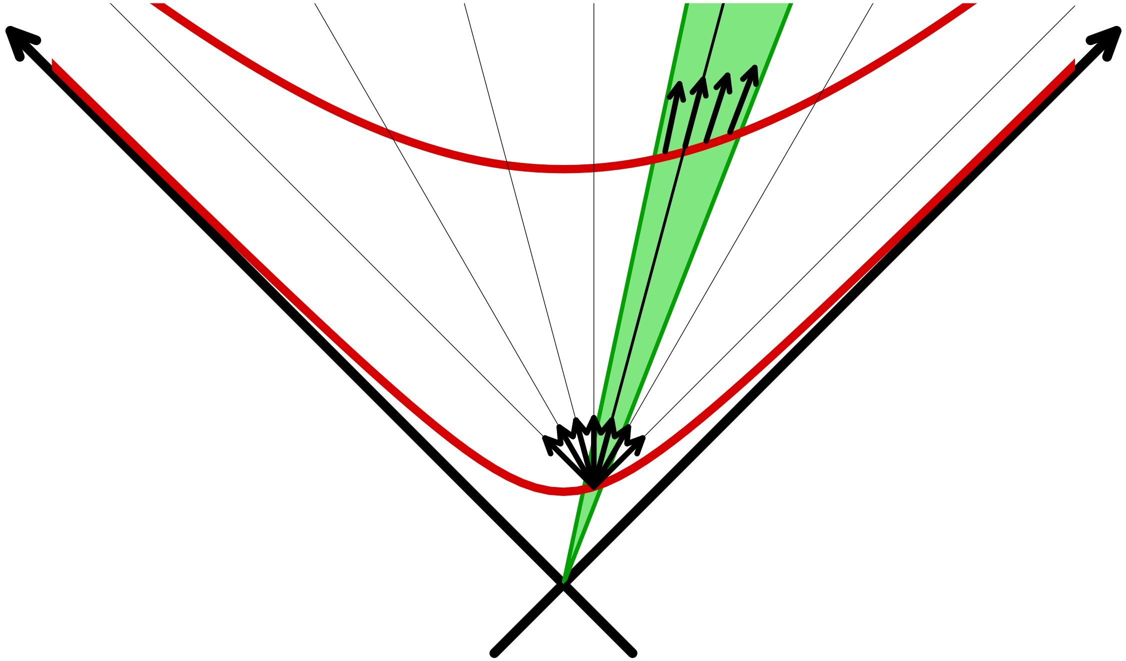

▷ at late times : if particles fly freely, only one longitudinal velocity can exist at a given η : $v_z = \tanh(\eta)$

▷ the expansion of the system is the main obstacle to local isotropy