Analytical calculation of massive Feynman diagrs and the NLO corrections to $H \to \gamma\gamma$ and $gg \to H$

Roberto BONCIANI

Departament de Física Teòrica, IFIC
CSIC-Universitat de València
E-46071 València, Spain

Plan of the Talk

Plan of the Talk

- Motivations

Plan of the Talk

- Motivations
- Laporta Algorithm and Differential Equations Method

Plan of the Talk

- Motivations
- Laporta Algorithm and Differential Equations Method
- Application:

Plan of the Talk

- Motivations
- Laporta Algorithm and Differential Equations Method
- Application:
  - 3-Point functions: QCD and EW two-loop corrections to $H \to \gamma\gamma$ and $gg \to H$

U. Aglietti, R. B., G. Degrassi, and A. Vicini

Plan of the Talk

- Motivations
- Laporta Algorithm and Differential Equations Method
- Application:
  - 3-Point functions: QCD and EW two-loop corrections to $H \to \gamma\gamma$ and $gg \to H$

    U. Aglietti, R. B., G. Degrassi, and A. Vicini
- Summary

Motivations

Motivations

- LHC: at the end of 2008 it will be possible to study physics at the TeV scale. Huge statistics $\Rightarrow$ observables measured with high precision in hadronic collisions. Heavy-quark factory: a pair of $t\bar{t}$ will be produced approximately every second, with $5 \cdot 10^5$ pairs of $b\bar{b}$ and $8 \cdot 10^6$ pairs of $c\bar{c}$. QCD.

Motivations

- LHC: at the end of 2008 it will be possible to study physics at the TeV scale. Huge statistics $\Rightarrow$ observables measured with high precision in hadronic collisions. Heavy-quark factory: a pair of $t\bar{t}$ will be produced approximately every second, with $5 \cdot 10^5$ pairs of $b\bar{b}$ and $8 \cdot 10^6$ pairs of $c\bar{c}$. QCD.
- ILC: we do not know when (2015?) ... very clear and precise physics at ~ 500 GeV of c.m. energy in leptonic collisions. Again, $t\bar{t}$ pairs will be produced and it will be possible to study new observables with respect to LEP (i.e. asymmetries for the top quark).

Motivations

- LHC: at the end of 2008 it will be possible to study physics at the TeV scale. Huge statistics $\Rightarrow$ observables measured with high precision in hadronic collisions. Heavy-quark factory: a pair of $t\bar{t}$ will be produced approximately every second, with $5 \cdot 10^5$ pairs of $b\bar{b}$ and $8 \cdot 10^6$ pairs of $c\bar{c}$. QCD.

- ILC: we do not know when (2015?) ... very clear and precise physics at ~ 500 GeV of c.m. energy in leptonic collisions. Again, $t\bar{t}$ pairs will be produced and it will be possible to study new observables with respect to LEP (i.e. asymmetries for the top quark).

- Heavy Quarks (top) will play a crucial role in the physics of next 10 years: as active discovery channel for new physics and as an omnipresent background for other discovery channels.

Motivations

- LHC: at the end of 2008 it will be possible to study physics at the TeV scale. Huge statistics $\Rightarrow$ observables measured with high precision in hadronic collisions. Heavy-quark factory: a pair of $t\bar{t}$ will be produced approximately every second, with $5 \cdot 10^5$ pairs of $b\bar{b}$ and $8 \cdot 10^6$ pairs of $c\bar{c}$. QCD.
- ILC: we do not know when (2015?) ... very clear and precise physics at ~ 500 GeV of c.m. energy in leptonic collisions. Again, $t\bar{t}$ pairs will be produced and it will be possible to study new observables with respect to LEP (i.e. asymmetries for the top quark).
- Heavy Quarks (top) will play a crucial role in the physics of next 10 years: as active discovery channel for new physics and as an omnipresent background for other discovery channels.
- Control on the NLO and NNLO perturbative corrections (keeping the fermion or boson masses) $\Rightarrow$ calculation of multi-loops multi-legs (massive) Feynman diagrams.

Laporta Algorithm and Diff. Equations

In the last years, for these reasons, there was a large activity in the development of methods for the calculation of Feynman diagrams.

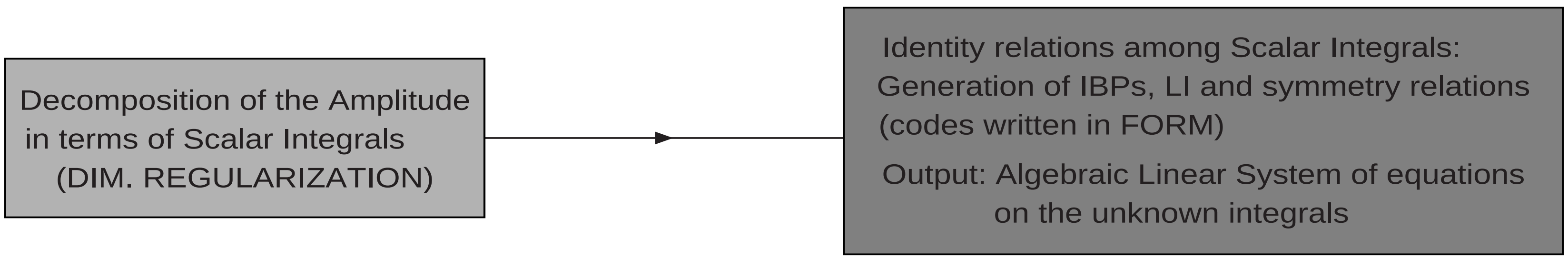
The Method based on the “Laporta algorithm” and the “Differential Equations Method” is one of the most powerful known at the moment.

It consists on the following steps:

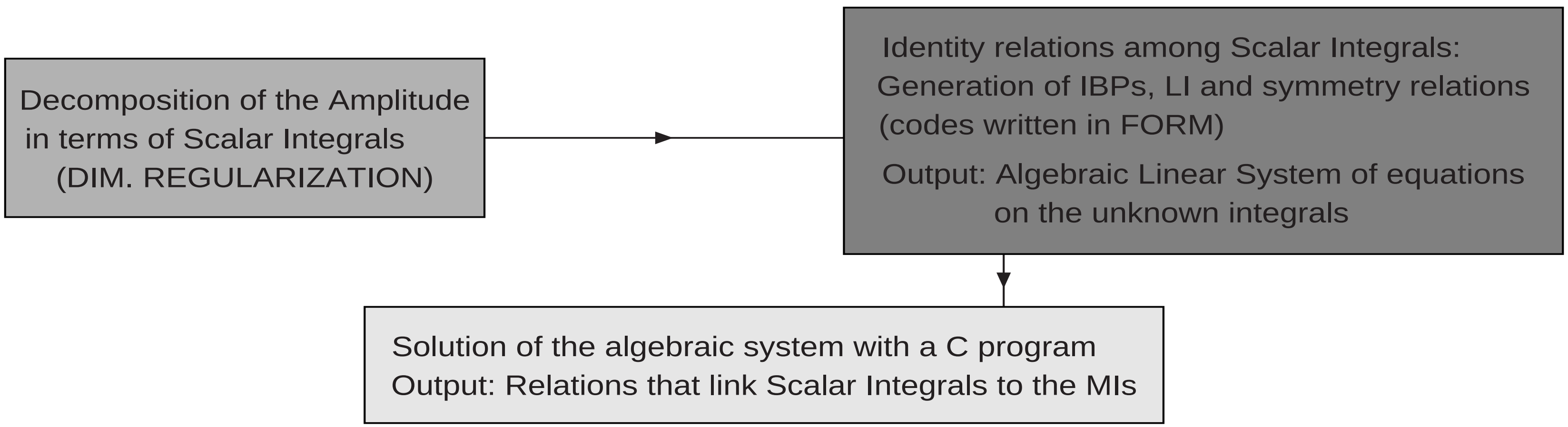
Laporta Algorithm and Diff. Equations

Decomposition of the Amplitude
in terms of Scalar Integrals
(DIM. REGULARIZATION)

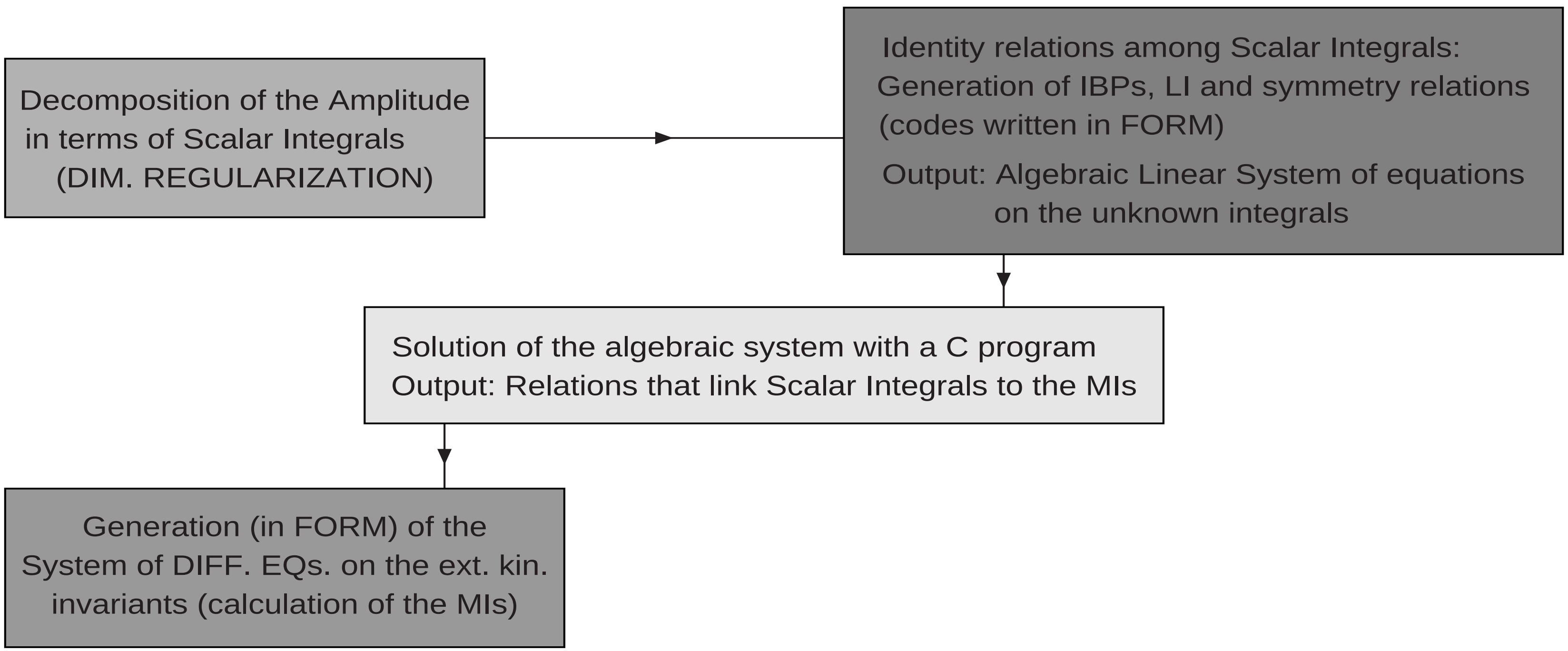
Laporta Algorithm and Diff. Equations



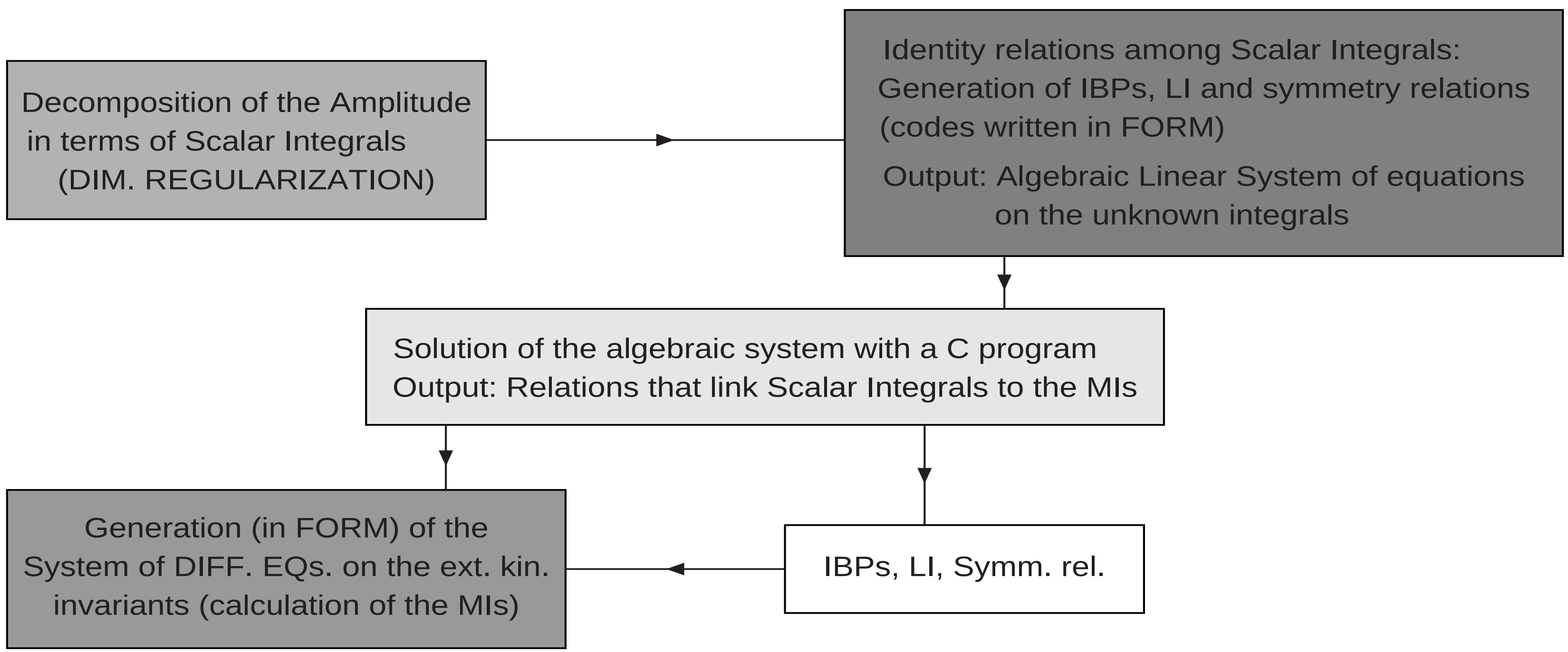
Laporta Algorithm and Diff. Equations



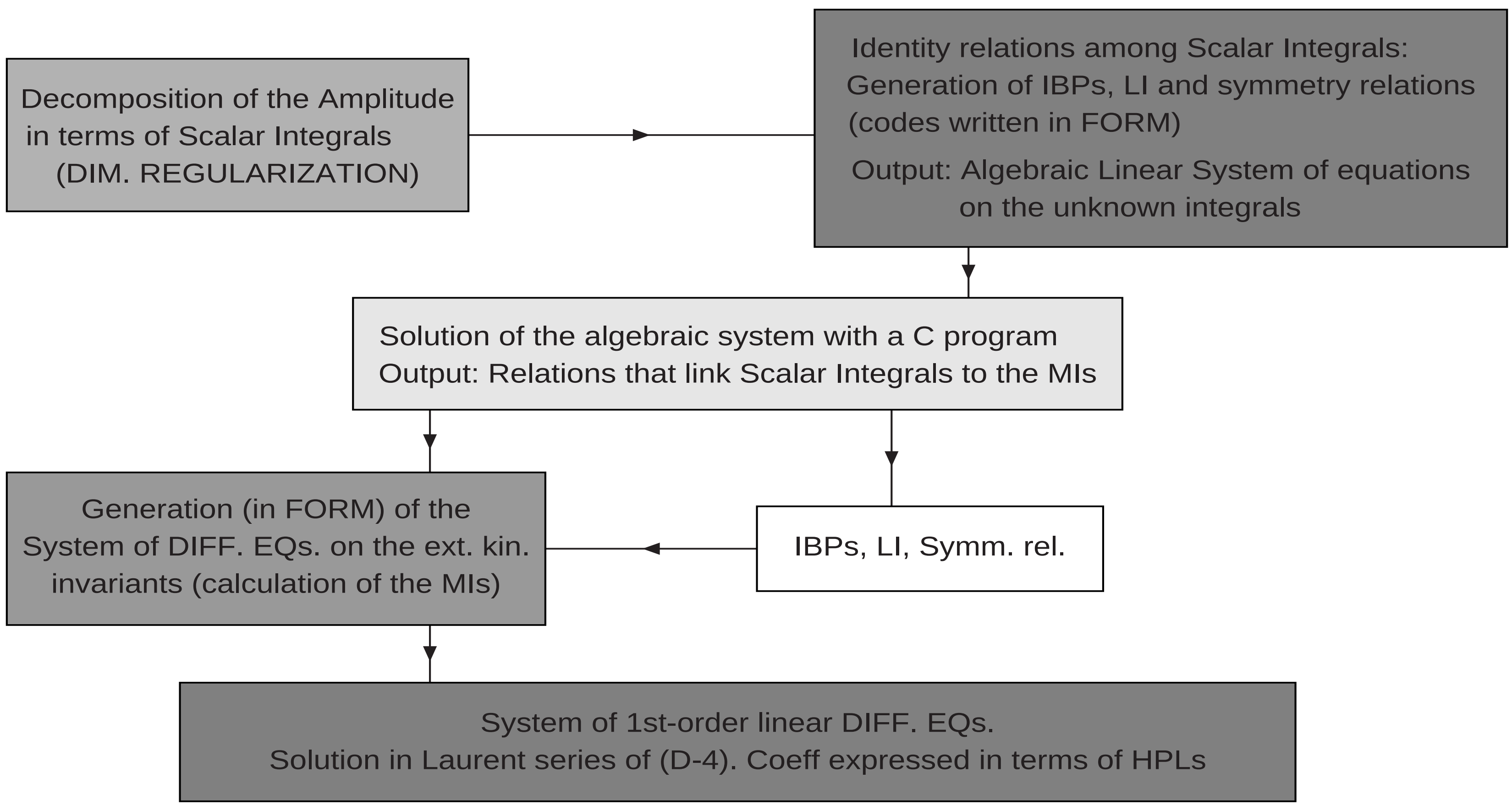
Laporta Algorithm and Diff. Equations



Laporta Algorithm and Diff. Equations



Laporta Algorithm and Diff. Equations



Pro and Contra

Pro and Contra

- ADVANTAGES: the method is completely implemented on computer codes: FORM or Maple or Mathematica. When an analytic solution is found, one has the advantages of the analytic solution (complete control on the correction). Even when an analytic solution cannot be found, the diff. eqs. give a very powerful tool for exact and fast numerical evaluation.

Pro and Contra

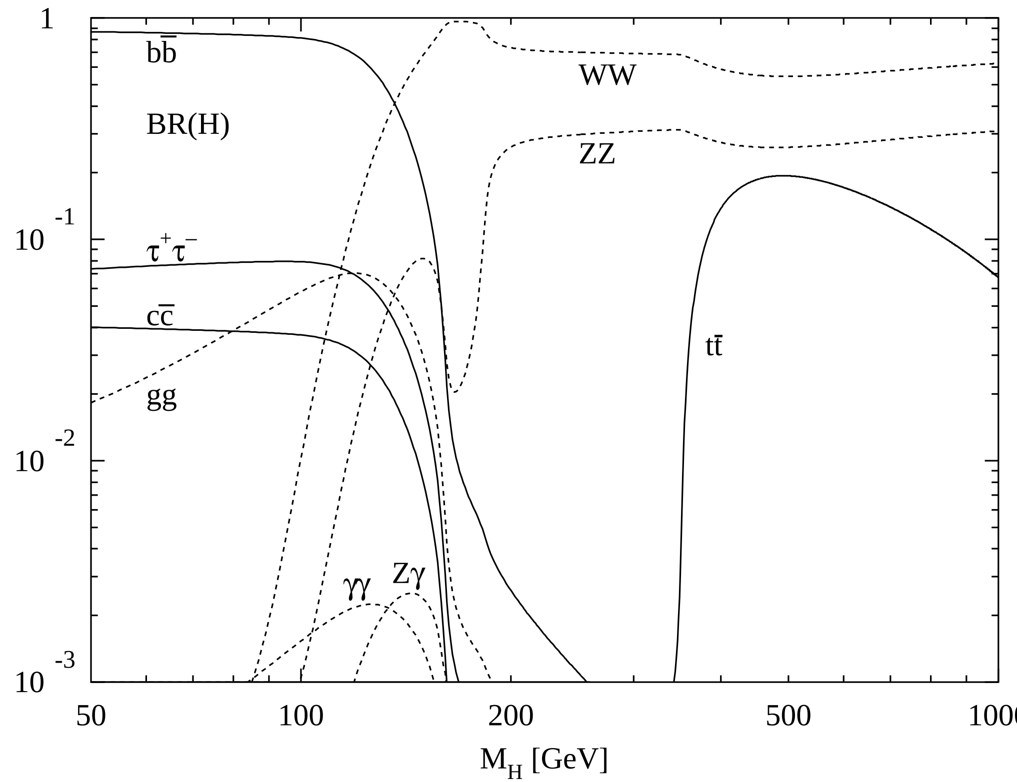
- ADVANTAGES: the method is completely implemented on computer codes: FORM or Maple or Mathematica. When an analytic solution is found, one has the advantages of the analytic solution (complete control on the correction). Even when an analytic solution cannot be found, the diff. eqs. give a very powerful tool for exact and fast numerical evaluation.

- DISADVANTAGES: altought there is no conceptual limit to the use of the method, from a practical point of view it works really good when the number of scales is limited: up to 2-3 scales in the case of 3-4 external legs. In practice, it is perfect for QCD calculations, in which only the mass of the top quark is important, with up to 4 ext legs; or EW calculations in which we consider a single bosonic mass (expanding, for instance, in the difference of gauge boson masses).

SM Higgs decays (BR)

SM Higgs decays (BR)

- In the intermediate-mass region ($m_H < 120$ GeV) $H \to b\bar{b}$ is dominant by more that one order of magnitude ($\sim 99\%$).
  The decay $H \to \gamma\gamma$ is a rare process ($BR \sim 10^{-3}$).
- For $m_H > 140$ GeV the dominant decay channels are $H \to WW$ (the main decay channel for $m_H \sim 170$ GeV) and $H \to ZZ$
  Above the kinematic threshold $H \to t\bar{t}$ can reach up to 20%



(A. Djouadi, M. Spira and P. M. Zerwas '96)

SM predictions for $\Gamma(H \to \gamma\gamma)$

SM predictions for $\Gamma(H \to \gamma\gamma)$

- One Loop (LO)
  - Ellis-Gaillard-Nanopoulos ’76, Shifman-Vainshtein-Voloshin-Zakharov ’79,
- Two-Loop QCD corrections (NLO)
  - Zheng-Wu ’90, Djouadi-Spira-van der Bij-Zerwas ’91, Dawson-Kauffman ’93, Djouadi-Spira-Zerwas ’93, Melnikov-Yakovlev ’93, Inoue-Najima-Oka-Saito ’94, Steinhauser ’96
  - Fleischer-Tarasov-Tarasov ’04, Harlander-Kant ’05, Anastasiou-Beerli-Bucherer-Daleo-Kunst ’06, Aglietti-B.-Degrassi-Vicini ’06
- Two-Loop EW corrections (NLO)
  - corrections at $\mathcal{O}(G_\mu m_t^2)$ (Liao-Li ’97)
  - corrections at $\mathcal{O}(G_\mu m_H^2)$ (Korner-Melnikov-Yakovlev ’96)
  - exact light-fermion contribution (Aglietti-B.-Degrassi-Vicini ’04)
  - top and bosonic contributions below W threshold (Degrassi-Maltoni ’05)

Decay Width

The amplitude has the following structure:

$$T^{\mu\nu} = \left[q_1^\nu \, q_2^\mu \; - (q_1 \cdot q_2) \, g^{\mu\nu} \right] \mathcal{F}$$

Once the form factor $\mathcal{F}$ is known, the Decay width can be expressed as follows:

$$\Gamma(H \to \gamma\gamma) = \frac{G_\mu \alpha^2 m_H^3}{128\sqrt{2}\pi^3} \, |\mathcal{F}|^2$$

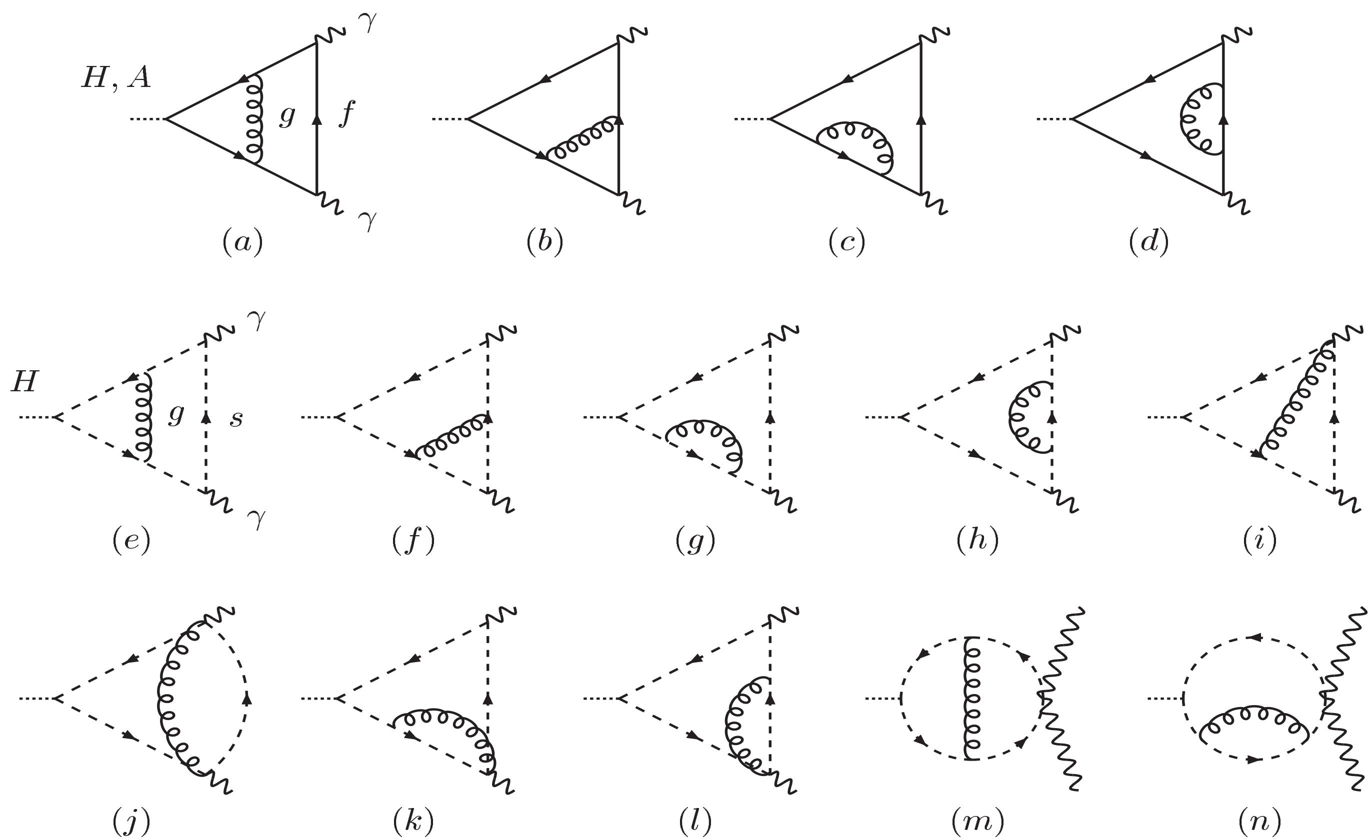
- G_μ, α and m_H are respectively the Fermi constant, fine-structure constant and mass of the Higgs boson

The form factor $\mathcal{F}$ can be calculated in perturbation theory:

$$\mathcal{F} = \mathcal{F}_{QCD}^{(1l)} + \mathcal{F}_{EW}^{(1l)} + \frac{\alpha_S}{\pi} \mathcal{F}_{QCD}^{(2l)} + \frac{\alpha m_W^2}{2\pi m_H^2 s_W^2} \mathcal{F}_{EW}^{(2l)}$$

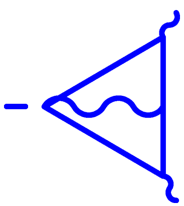
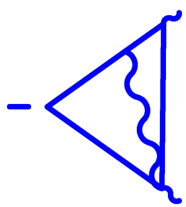
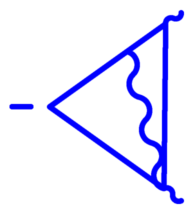
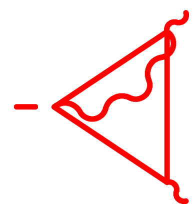
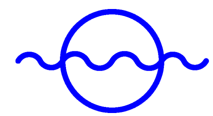
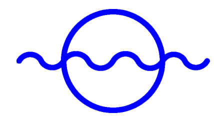
Two-Loop QCD Contributions

$$\mathcal{F} = \mathcal{F}^{(1l)}_{QCD} + \mathcal{F}^{(1l)}_{EW} + \frac{\alpha_S}{\pi}\mathcal{F}^{(2l)}_{QCD} + \frac{\alpha m_W^2}{2\pi m_H^2 s_W}\mathcal{F}^{(2l)}_{EW}$$

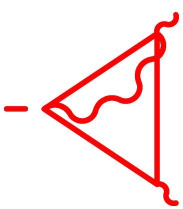
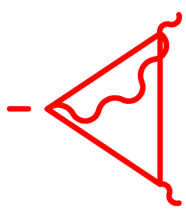
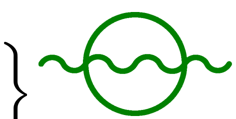


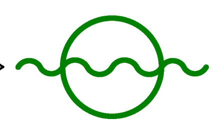
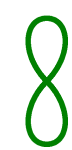
The Master Integrals

The calculation of the contributions due to the two-loop QCD Feynman diagrams can be reduced to the calculation of the following six two-loop scalar integrals (evaluated in D dimensions):

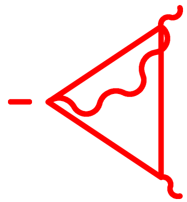
   $(k_1 \cdot k_2)$    $(p_2 \cdot k_2)$

For the 4-denominator MI we have the following Differential Equation:

$\frac{d}{ds}$  $= -\frac{1}{s}$  $-\frac{1}{4a}\left\{\frac{(D-3)}{s} + \frac{(3D-5)}{(s-4a)}\right\}$ 

$+\frac{3(D-2)}{2a^2}\left\{\frac{1}{s} - \frac{1}{(s-4a)}\right\}$  $(p_2 \cdot k_2)$ $+\frac{(D-4)}{8a^2}\left\{\frac{1}{s} - \frac{1}{(s-4a)}\right\}$ 

The Master Integrals



$$= \left(\frac{\mu^2}{a}\right)^{2\epsilon} \sum_{i=-2}^{1} \epsilon^i F_i + \mathcal{O}\left(\epsilon^2\right) , \qquad x = \frac{\sqrt{p^2+4m_t^2}-\sqrt{p^2}}{\sqrt{p^2+4m_t^2}+\sqrt{p^2}}$$

$$F_{-2} = \frac{1}{2} \qquad F_{-1} = \frac{1}{2}$$

$$F_0 = -\frac{5}{2} - \frac{4\zeta(3)}{(1-x)^2} + \frac{4\zeta(3)}{(1-x)} + \left[2 - \frac{4}{(1-x)}\right] H(0;x) - H(0,0;x)$$

$$+\left[\frac{2}{(1-x)^2} - \frac{2}{(1-x)}\right] H(0,0,0;x) + \left[\frac{4}{(1-x)^2} - \frac{4}{(1-x)}\right] H(1,0,0;x)$$

$$F_1 = -\frac{35}{2} + \frac{8\zeta^2(2)}{5(1-x)^2} - \frac{4\zeta(3)}{(1-x)^2} + \frac{4\zeta(2)}{(1-x)} - \frac{8\zeta^2(2)}{5(1-x)} + \frac{4\zeta(3)}{(1-x)} - 2\zeta(2) + 3\zeta(3) - \left[12 + \frac{24}{(1-x)}\right] H(-1,0;x)$$

$$+\left[12 - \frac{6\zeta(3)}{(1-x)^2} + \frac{6\zeta(3)}{(1-x)} - \frac{24}{(1-x)} + \zeta(2)\right] H(0;x) + 6H(0,-1,0;x) + \left[9 - \frac{2\zeta(2)}{(1-x)^2} + \frac{4}{(1-x)^2} + \frac{2\zeta(2)}{(1-x)}\right.$$

$$\left. - \frac{20}{(1-x)}\right] H(0,0;x) - \left[\frac{12}{(1-x)^2} - \frac{12}{(1-x)}\right] H(0,0,-1,0;x) - \left[3 - \frac{2}{(1-x)^2} + \frac{2}{(1-x)}\right] H(0,0,0;x) + \left[\frac{6}{(1-x)^2}\right.$$

$$\left. - \frac{6}{(1-x)}\right] H(0,0,0,0;x) + \left[\frac{4}{(1-x)^2} - \frac{4}{(1-x)}\right] H(0,0,1,0;x) - 2H(0,1,0;x) - \left[\frac{4}{(1-x)^2} - \frac{4}{(1-x)}\right] H(0,1,0,0;x)$$

$$-\left[\frac{12\zeta(3)}{(1-x)^2} - \frac{12\zeta(3)}{(1-x)}\right] H(1;x) + \left[4 - \frac{4\zeta(2)}{(1-x)^2} + \frac{4\zeta(2)}{(1-x)} - \frac{8}{(1-x)}\right] H(1,0;x) - \left[\frac{24}{(1-x)^2} - \frac{24}{(1-x)}\right] H(1,0,-1,0;x)$$

$$+\left[2 + \frac{4}{(1-x)^2} - \frac{4}{(1-x)}\right] H(1,0,0;x) + \left[\frac{12}{(1-x)^2} - \frac{12}{(1-x)}\right] H(1,0,0,0;x) + \left[\frac{8}{(1-x)^2} - \frac{8}{(1-x)}\right] H(1,0,1,0;x)$$

$$-\left[\frac{8}{(1-x)^2} - \frac{8}{(1-x)}\right] H(1,1,0,0;x)$$

Two-Loop QCD Contributions

$$\mathcal{F}^{(2l)}_{QCD} = \frac{\alpha_S}{\pi} \sum_{i=(0,1/2)} C(R_i)\, \mathcal{F}^{(2l)}_i$$

For instance in the case of on-shell quark masses the fermion contribution is:

$$\mathcal{F}^{(2l)}_{1/2} = \mathcal{F}^{(2l,a)}_{1/2}(x_{1/2}) + \frac{4}{3}\mathcal{F}^{(2l,b)}_{1/2}(x_{1/2})$$

$$\begin{aligned}
\mathcal{F}^{(2l,a)}_{1/2}(x) = \; & \frac{36x}{(x-1)^2} - \frac{4x\left(1-14x+x^2\right)}{(x-1)^4}\zeta_3 - \frac{4x(1+x)}{(x-1)^3}H(0,x) - \frac{8x\left(1+9x+x^2\right)}{(x-1)^4}H(0,0,x) \\
& + \frac{2x\left(3+25x-7x^2+3x^3\right)}{(x-1)^5}H(0,0,0,x) + \frac{4x\left(1+2x+x^2\right)}{(x-1)^4}\left[\zeta_2 H(0,x) + 4H(0,-1,0,x)\right. \\
& \left. - H(0,1,0,x)\right] + \frac{4x\left(5-6x+5x^2\right)}{(x-1)^4}H(1,0,0,x) - \frac{8x\left(1+x+x^2+x^3\right)}{(x-1)^5}\left[\frac{9}{10}{\zeta_2}^2 + 2\zeta_3 H(0,x)\right. \\
& + \zeta_2 H(0,0,x) + \frac{1}{4}H(0,0,0,0,x) + \frac{7}{2}H(0,1,0,0,x) - 2H(0,-1,0,0,x) + 4H(0,0,-1,0,x) \\
& \left. - H(0,0,1,0,x)\right]
\end{aligned}$$

$$\mathcal{F}^{(2l,b)}_{1/2}(x) = -\frac{12x}{(x-1)^2} - \frac{6x(1+x)}{(x-1)^3}H(0,x) + \frac{6x\left(1+6x+x^2\right)}{(x-1)^4}H(0,0,x)$$

Two-Loop QCD Contributions

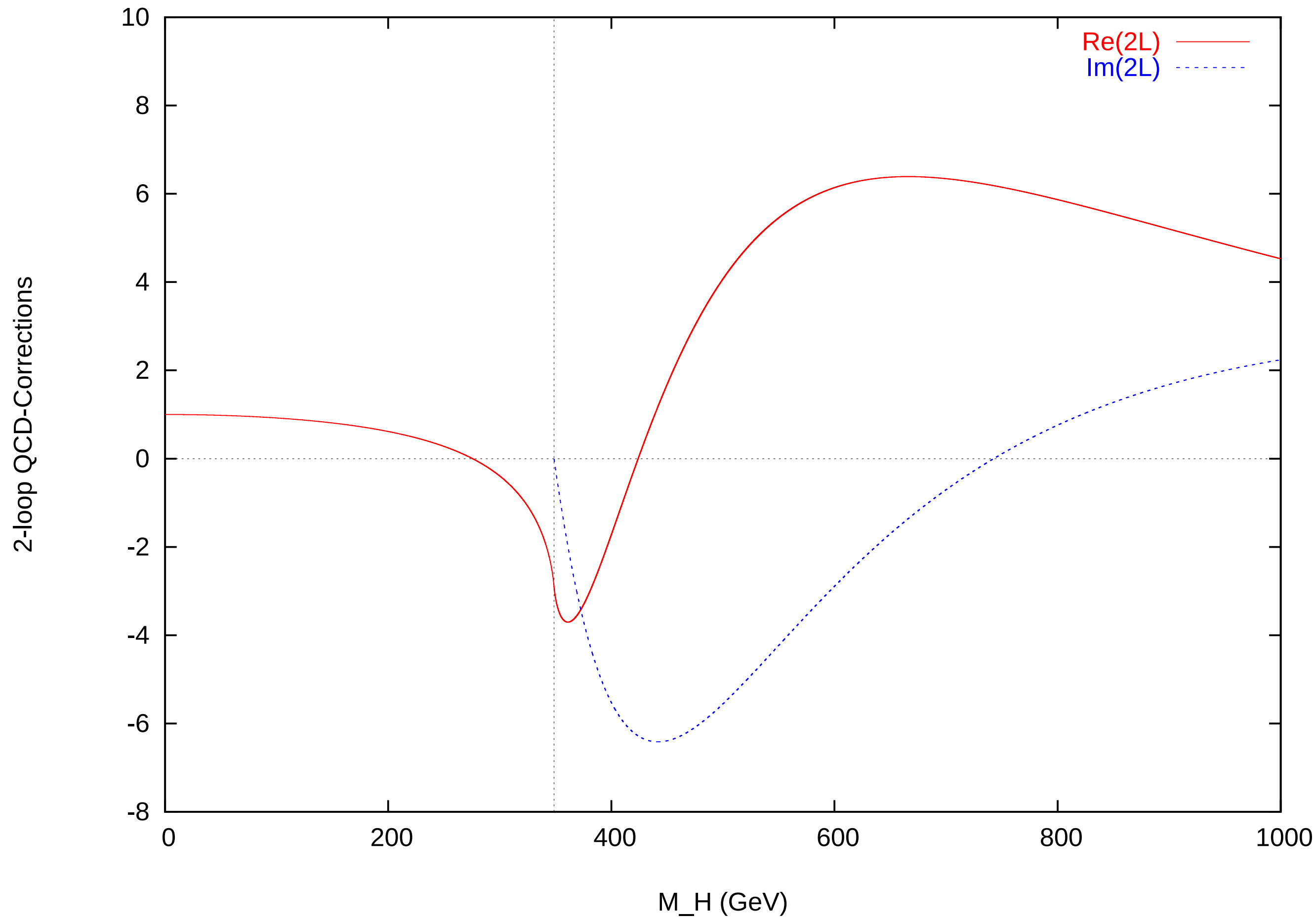
$$\mathcal{F}^{(2l)}_{QCD} = \frac{\alpha_S}{\pi} \sum_{i=(0,1/2)} C(R_i)\, \mathcal{F}^{(2l)}_i$$

For instance in the case of on-shell quark masses the fermion contribution is:

$$\mathcal{F}^{(2l)}_0 = \mathcal{F}^{(2l,a)}_0(x_0) + \frac{7}{3}\,\mathcal{F}^{(2l,b)}_0(x_0) + \mathcal{F}^{(2l,c)}_0(x_0)\ln\left(\frac{m_0^2}{\mu^2}\right)$$

$$\begin{aligned}
\mathcal{F}^{(2l,a)}_0(x) &= -\frac{14x}{(x-1)^2} - \frac{24x^2}{(x-1)^4}\zeta_3 + \frac{x\left(3-8x+3x^2\right)}{(x-1)^3(x+1)}H(0,x) + \frac{34x^2}{(x-1)^4}H(0,0,x) \\
&\quad -\frac{8x^2}{(x-1)^4}\left[\zeta_2 H(0,x) + 4H(0,-1,0,x) - H(0,1,0,x) + H(1,0,0,x)\right] \\
&\quad -\frac{2x^2(5-11x)}{(x-1)^5}H(0,0,0,x) + \frac{16x^2\left(1+x^2\right)}{(x-1)^5(x+1)}\left[\frac{9}{10}{\zeta_2}^2 + 2\zeta_3 H(0,x) + \zeta_2 H(0,0,x)\right. \\
&\quad \left. +\frac{1}{4}H(0,0,0,0,x) + \frac{7}{2}H(0,1,0,0,x) - 2H(0,-1,0,0,x) + 4H(0,0,-1,0,x) - H(0,0,1,0,x)\right] \\
\mathcal{F}^{(2l,b)}_0(x) &= \frac{6x^2}{(x-1)^3(x+1)}H(0,x) - \frac{6x^2}{(x-1)^4}H(0,0,x) \\
\mathcal{F}^{(2l,c)}_0(x) &= -\frac{3}{4}\,\mathcal{F}^{(1l)}_0
\end{aligned}$$

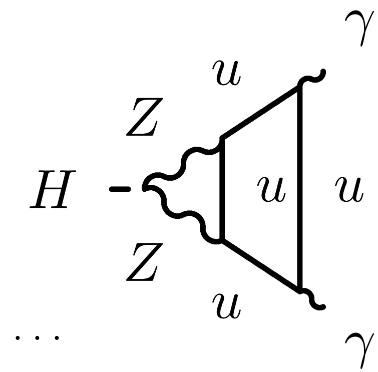
Real and Imaginary parts of $\mathcal{F}_{1/2}^{(2l)}$

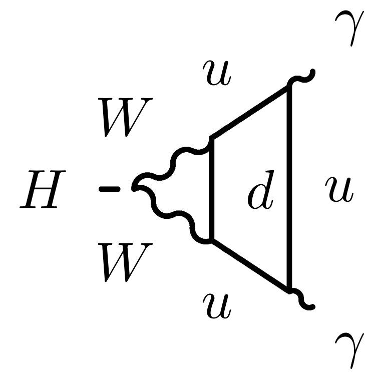


Two-Loop EW Contributions

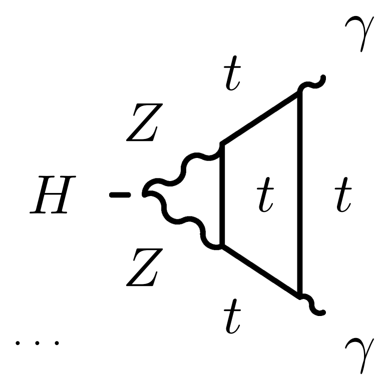
$$\mathcal{F} = \mathcal{F}^{(1l)}_{QCD} + \mathcal{F}^{(1l)}_{EW} + \frac{\alpha_S}{\pi}\mathcal{F}^{(2l)}_{QCD} + \frac{\alpha m_W^2}{2\pi m_H^2 s_W}\mathcal{F}^{(2l)}_{EW}$$

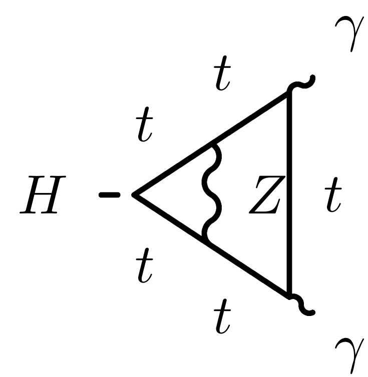
- **Light-Fermion contributions (Aglietti-B.-Degrassi-Vicini, '04)**



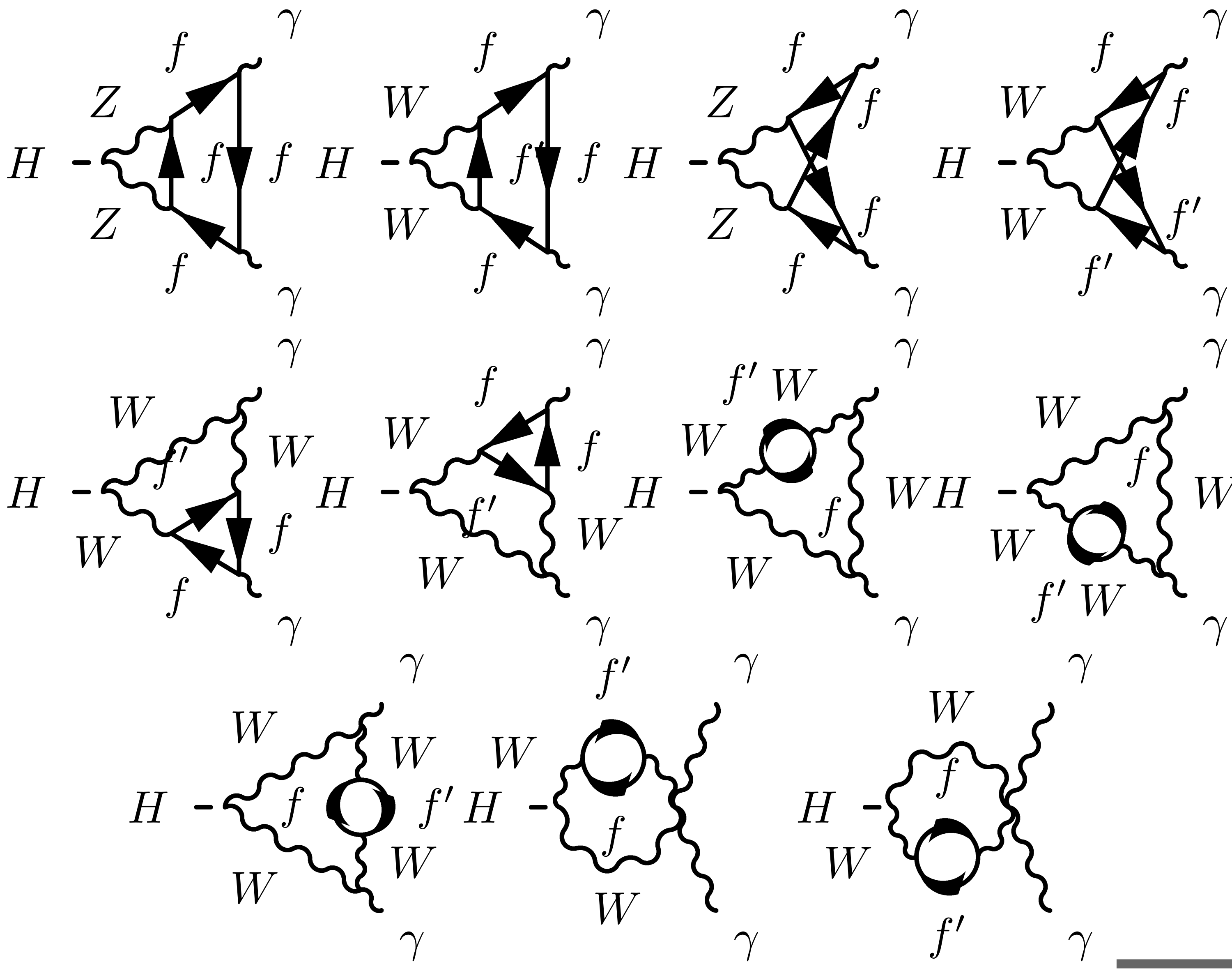


- **Top-quark and bosonic contributions (Degrassi-Maltoni, '05)**



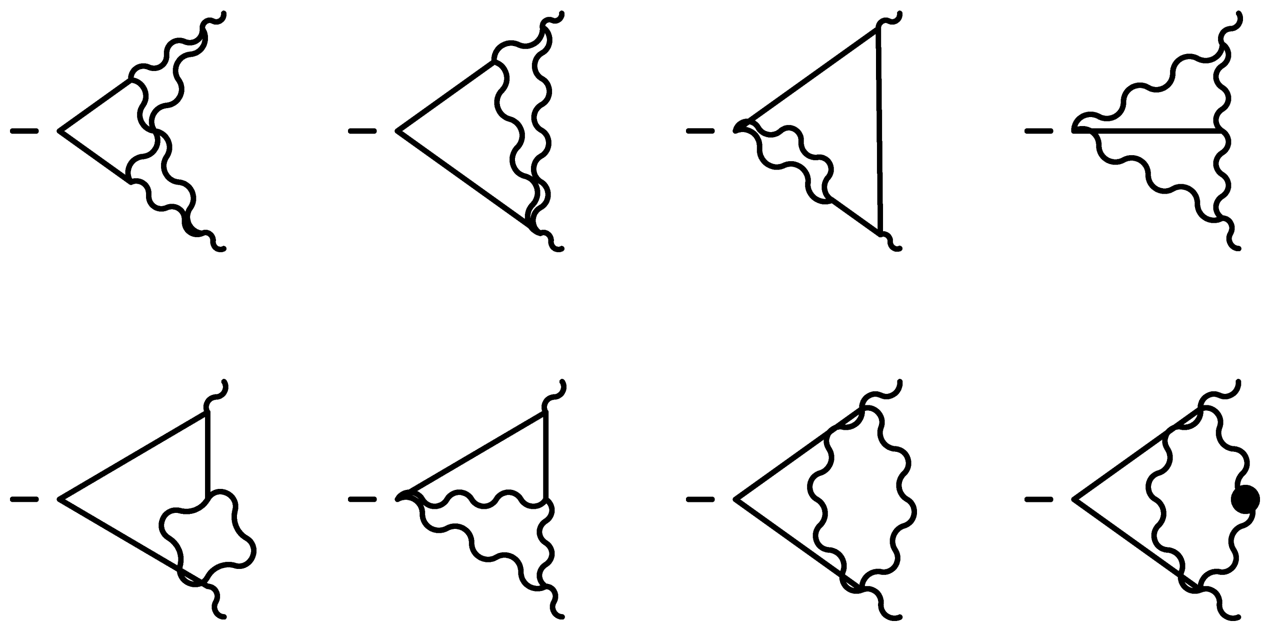


Feynman Diag. for the Light-Ferm. Contr.



The 8 Master Integrals

The reduction process leads to the evaluation of 8 new Master integrals



U. Aglietti and R. B., *Nucl. Phys.* B668 (2003) 3.
U. Aglietti and R. B., *Nucl. Phys.* B698 (2004) 277.
U. Aglietti, R. B., G. Degrassi, and A. Vicini, *Phys. Lett.* B600 (2004) 57.

EW Light Fermions Contribution

$$\mathcal{F}^{2l}_{EW} = 2N_c A_2\left[-2/9, w_H\right] + 3A_2\left[0, w_H\right] + \frac{2N_c}{c^4}\left(\frac{11}{36} - \frac{19}{27}s^2 + \frac{70}{81}s^4\right) A_1\left[z_H\right]$$
$$+\frac{3}{c^4}\left(\frac{1}{2} - 2s^2 + 4s^4\right) A_1\left[z_H\right]$$

where: $w_H = m_W^2/m_H^2$, $t_H = m_t^2/m_H^2$

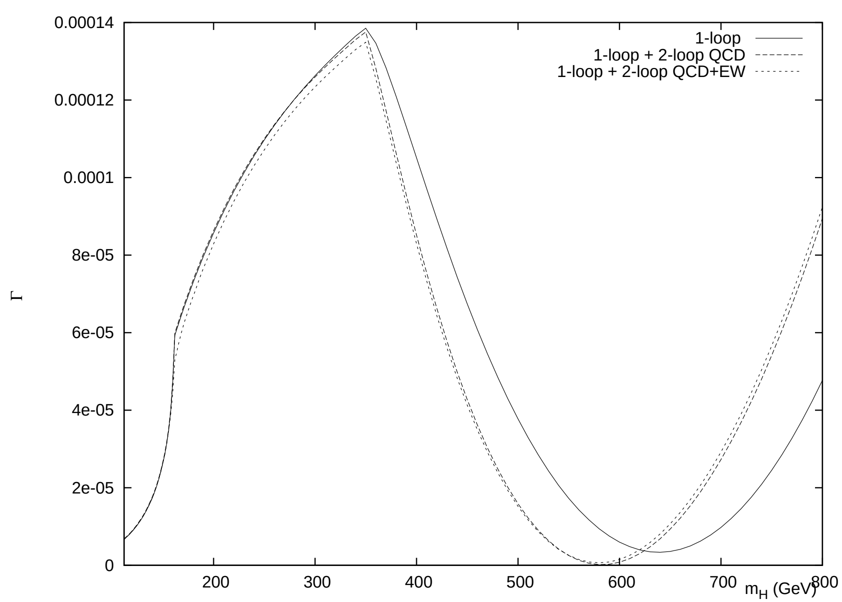
$$A_2[q,x] = -8(1+q)+4(1+q)(1-x)H\left(-1;-\frac{1}{x}\right) - 2(1+2qx)H\left(0,-1;-\frac{1}{x}\right) - \frac{2}{3}(5-12x)H\left(-r,-r;-\frac{1}{x}\right)$$
$$-6(1+q-3x-2qx)H\left(-r,-r,-1;-\frac{1}{x}\right)+2(1+2q)\left[(1-2x)H\left(0,-r,-r;-\frac{1}{x}\right)\right.$$
$$\left.+(1-3x)H\left(0,0,-1;-\frac{1}{x}\right)\right] - \sqrt{1-4x}\left\{2(1+2q)H\left(-r;-\frac{1}{x}\right) - 6q(1-2x)H\left(-4,-r,-1;-\frac{1}{x}\right)\right.$$
$$\left.+4q(1-2x)\left[H\left(-r,0,-1;-\frac{1}{x}\right) + H\left(-r,-r,-r;-\frac{1}{x}\right)\right]\right\} + \frac{6(1-2x)^2}{\sqrt{1-4x}}H\left(-r,-1;-\frac{1}{x}\right)$$

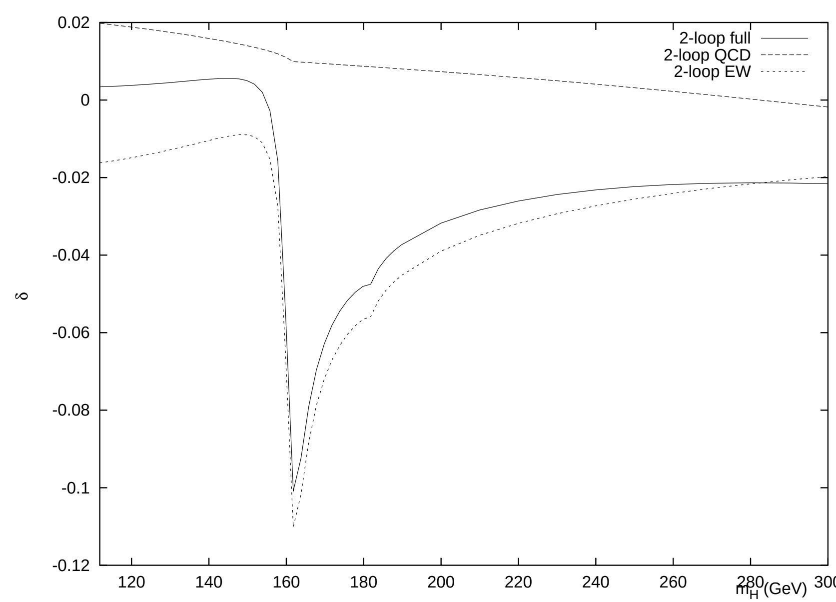
$$A_1[x] = -4+2(1-x)H\left(-1;-\frac{1}{x}\right)-2xH\left(0,-1;-\frac{1}{x}\right)+2(1-3x)H\left(0,0,-1;-\frac{1}{x}\right)$$
$$+2(1-2x)H\left(0,-r,-r;-\frac{1}{x}\right)-3(1-2x)H\left(-r,-r,-1;-\frac{1}{x}\right) - \sqrt{1-4x}\left[2H\left(-r;-\frac{1}{x}\right)\right.$$
$$\left.-3(1-2x)H\left(-4,-r,-1;-\frac{1}{x}\right)+2(1-2x)H\left(-r,0,-1;-\frac{1}{x}\right)+2(1-2x)H\left(-r,-r,-r;-\frac{1}{x}\right)\right]$$

Γ with NLO QCD+EW-Light-fermions

$$\Gamma(H\to\gamma\gamma) = \frac{G_\mu \alpha^2 m_H^3}{128\sqrt{2}\pi^3}\left|\mathcal{F}_W^{(1l)} + \frac{4}{9}N_c\mathcal{F}_t^{(1l)} + \frac{16}{27}N_c\frac{\alpha_S}{\pi}\mathcal{F}_t^{(2l)} + \frac{\alpha m_W^2}{2\pi m_H^2 s_W}\mathcal{F}_{EW}^{(2l)}\right|^2$$

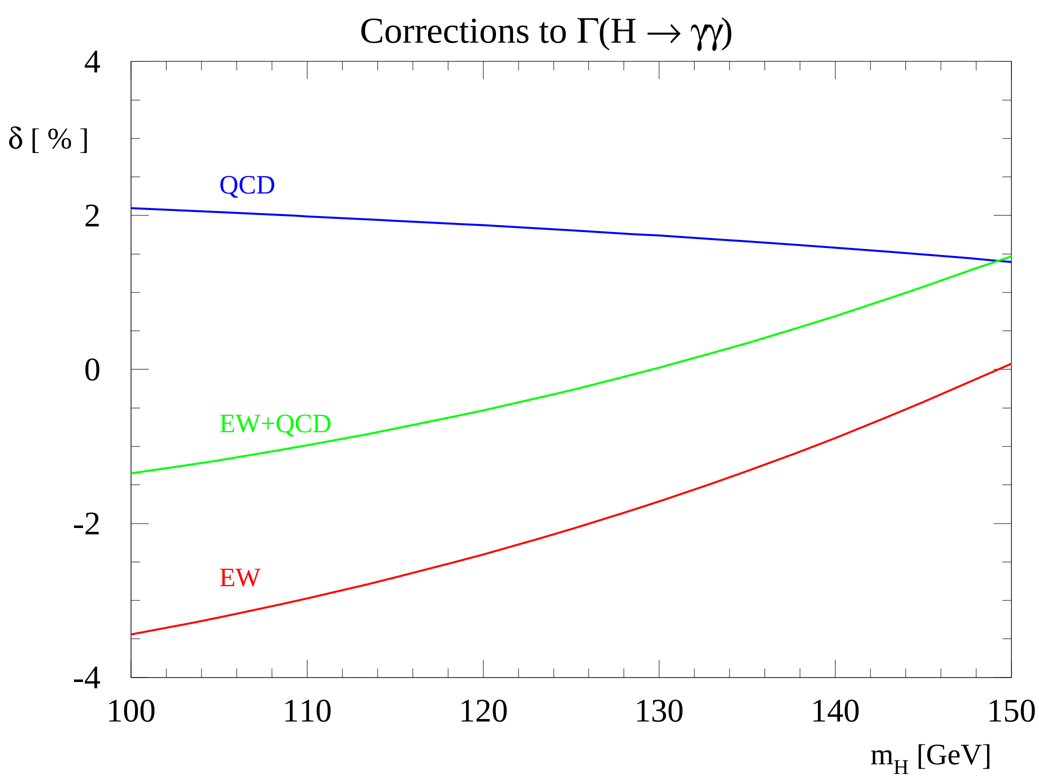
$$\delta = \frac{\Gamma(2l)-\Gamma(1l)}{\Gamma(1l)}$$





Complete EW corrections

Recently the remaining two-loop electroweak corrections (massive top and bosonic contributions) were calculated in the intermediate-mass region:

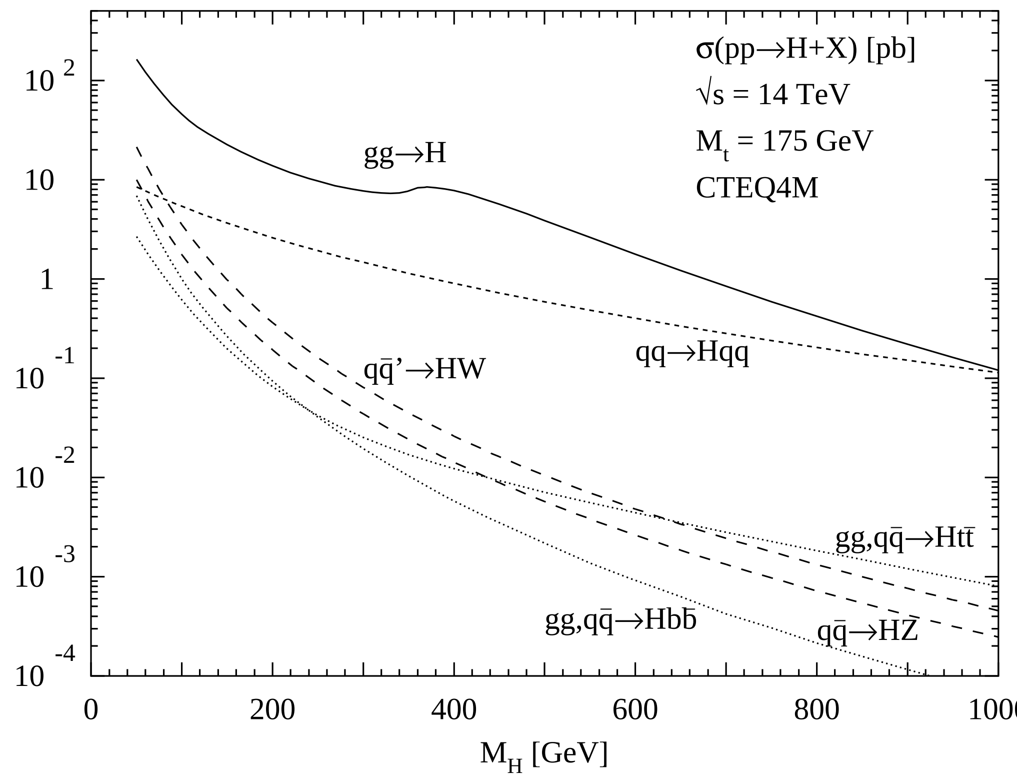


(Degrassi and Maltoni ’05)

SM Higgs production at LHC

SM Higgs production at LHC

- At LHC the gluon fusion will have the largest rate among the possible mechanisms of Higgs production. In particular in the so-called intermediate mass range it is one order of magnitude greater than the other possible channels
- NNLO QCD corrections under control. Residual theoretical uncertainty below 10%
- Important the control of NLO EW corrections



(A. Djouadi, M. Spira and P. M. Zerwas '96)

SM predictions for Higgs production

SM predictions for Higgs production

- LO corrections
  - Georgi-Glashow-Machacek-Nanopoulos '78
- NLO QCD corrections (they enhance the lowest order cross-section by 60-70%)
  - Dawson '91, Spira-Djouadi-Graudenz-Zerwas '95
- NNLO QCD corrections (they enhance the NLO by 15-25%)
  - Harlander '00, Catani-De Florian-Grazzini '01, Harlander-Kilgore '01 '02, Anastasiou-Melnikov '02, Ravindran-Smith-Van Neerven '03
- NNLO QCD corrections with soft-gluon NNLL resummation (enhancement of 6-15% and stabilization with respect to the μ)
  - Catani-De Florian-Grazzini-Nason '03
- EW corrections
  - Heavy-m_t expansion: $<$ 1% (Djouadi-Gambino '94, Djouadi-Gambino-Kniehl '98)
  - Light-F: several % (Aglietti-B.-Degrassi-Vicini '04)
  - Top-quark contribution $< 1\%$ (Degrassi-Maltoni '04)

Form Factors and Partonic CS

The amplitude has the following structure:

$$T^{\mu\nu} = \left[q_1^{\nu}\, q_2^{\mu} - (q_1 \cdot q_2)\, g^{\mu\nu}\right] T_5$$

The cross-section is given by:

$$\sigma(gg \to H) = \frac{G_\mu {\alpha_S}^2}{512\sqrt{2}\pi} \left| \mathcal{G} \right|^2$$

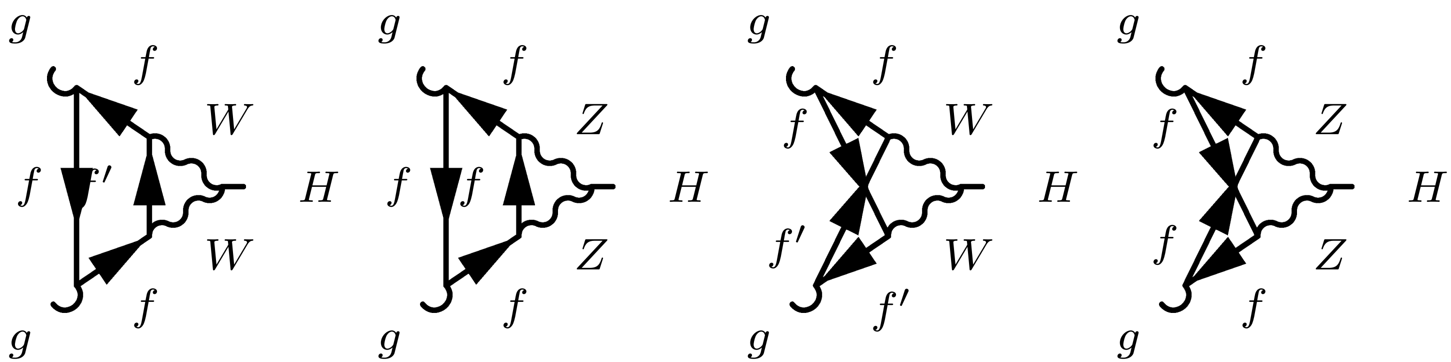
- G_μ, α_S are respectively the Fermi constant and the strong coupling constant
- $\mathcal{G} = T_5$

The form factor $\mathcal{G}$ can be calculated in perturbation theory:

$$\mathcal{G} = \mathcal{G}_{QCD}^{(1l)} + \frac{\alpha_S}{\pi}\mathcal{G}_{QCD}^{(2l)} + \frac{\alpha m_W^2}{2\pi m_H^2 s_W^2}\mathcal{G}_{EW}^{(2l)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{G}_{QCD}^{(3l)}$$

Feynman Diags for EW Light-F Contrib

$$\mathcal{G} = \mathcal{G}^{(1l)}_{QCD} + \frac{\alpha_S}{\pi}\mathcal{G}^{(2l)}_{QCD} + \frac{\alpha m_W^2}{2\pi m_H^2 s_W^2}\mathcal{G}^{(2l)}_{EW} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{G}^{(3l)}_{QCD}$$



$$\mathcal{G}^{2l}_{EW} = \frac{2}{c^4}\left(\frac{5}{4} - \frac{7}{3}\,s^2 + \frac{22}{9}s^4\right) A_1\,[z_H] + 4\,A_1\,[w_H]$$

where

$$w_H \equiv m_W^2/m_H^2\,, \quad z_H \equiv m_Z^2/m_H^2\,, \quad s^2 \equiv \sin^2\theta_W\,, \quad c^2 = 1 - s^2$$

and

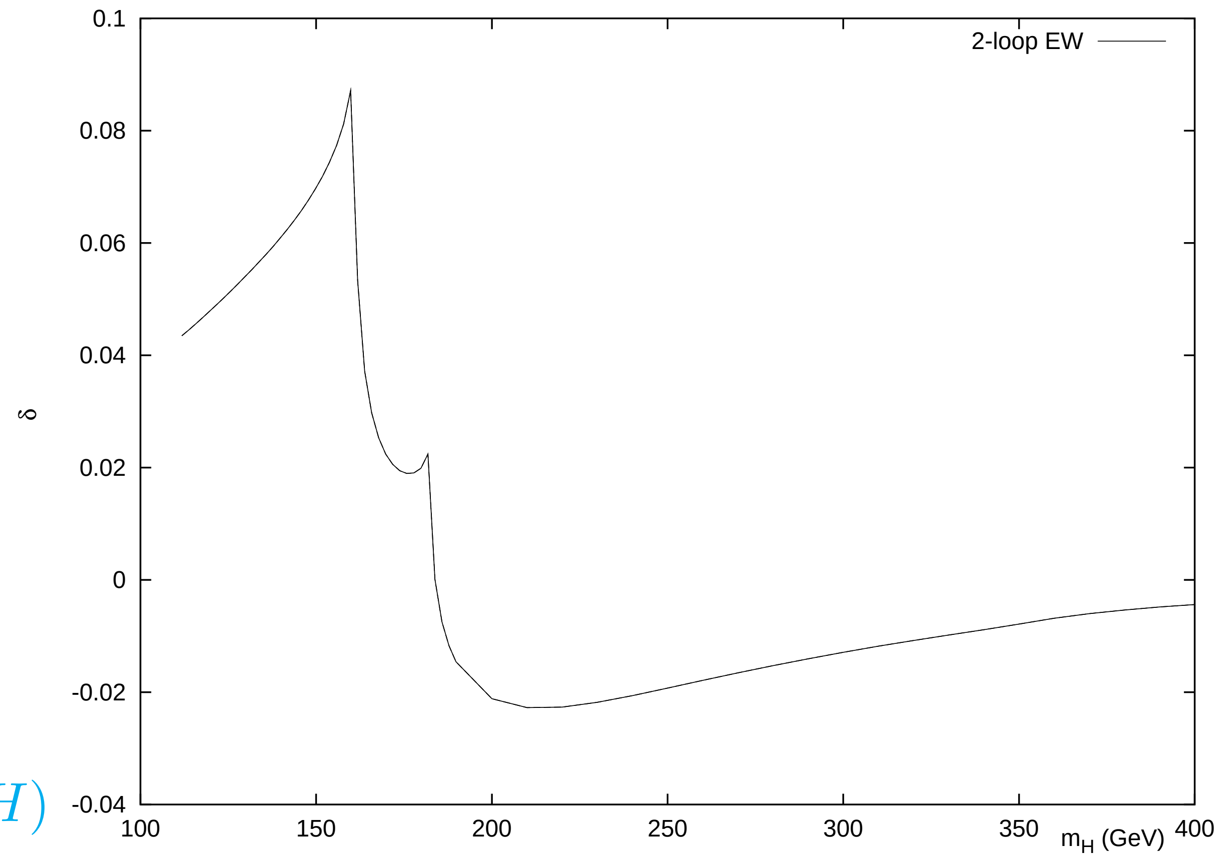
$$\begin{aligned}
A_1[x] \;=\; & -4 + 2(1-x)H\left(-1;-\frac{1}{x}\right) - 2\,x\,H\left(0,-1;-\frac{1}{x}\right) + 2(1-3x)H\left(0,0,-1;-\frac{1}{x}\right) \\
& + 2\,(1-2\,x)\,H\left(0,-r,-r;-\frac{1}{x}\right) - 3\,(1-2\,x)\,H\left(-r,-r,-1;-\frac{1}{x}\right) \\
& - \sqrt{1-4\,x}\,\left[2\,H\left(-r;-\frac{1}{x}\right) - 3\,(1-2\,x)\,H\left(-4,-r,-1;-\frac{1}{x}\right)\right. \\
& \left. + 2\,(1-2\,x)\,H\left(-r,0,-1;-\frac{1}{x}\right) + 2\,(1-2\,x)\,H\left(-r,-r,-r;-\frac{1}{x}\right)\right]
\end{aligned}$$

$\sigma(gg \to H)$ Partonic Level

$$\delta = \frac{\sigma(2l) - \sigma(1l)}{\sigma(1l)}$$

Relative corrections to the production cross-section $\sigma(gg \to H)$ and to the decay width

$$\Gamma(H \to gg) = (8m_H^3/\pi^2)\sigma(gg \to H)$$



EW Corrections

The EW corrections in the “intermediate mass range” (114 GeV $\leq m_H \leq 155$ GeV) can be parametrized by the following simple formulas:

$$\hat{\sigma}_{EW} = (1 + \delta_{EW}(m_H))\ \hat{\sigma}_0$$

$$\delta_{EW}(m_H) = 0.00961 + 6.9904 \cdot 10^{-5}\, m_H + 2.31508 \cdot 10^{-6}\, m_H^2$$

m_H (GeV)	δ_{EW}	m_H (GeV)	δ_{EW}	m_H (GeV)	δ_{EW}	m_H (GeV)	δ_{EW}
114	0.048	136	0.062	158	0.077	180	0.020
116	0.049	138	0.063	160	0.069	182	0.010
118	0.050	140	0.065	162	0.063	184	0.010
120	0.051	142	0.066	164	0.049	186	0.002
122	0.053	144	0.068	166	0.041	188	0.997
124	0.054	146	0.069	168	0.035	190	0.994
126	0.055	148	0.071	170	0.031	192	0.991
128	0.056	150	0.073	172	0.028	194	0.989
130	0.058	152	0.074	174	0.026	196	0.987
132	0.059	154	0.076	176	0.024	198	0.986
134	0.060	156	0.077	178	0.022	200	0.985

QCD Contribution

$$\hat{\sigma}_{ab}(z) = \sigma^{(0)}\, z\, G_{ab}(z)$$

$$\sigma^{(0)} = \frac{G_\mu \alpha_S^2(\mu_R^2)}{128\sqrt{2}\,\pi} \left| \sum_{i=0,1/2} \lambda_i \left(\frac{A^2}{m_0^2}\right)^{1-2i} T(R_i)\, \mathcal{G}_i^{(1l)} \right|^2$$

is the Born-level contribution with $\mathcal{G}_i^{(1l)} = \mathcal{F}_i^{(1l)}$

$$G_{ab}(z) = G_{ab}^{(0)}(z) + \frac{\alpha_s(\mu_R^2)}{\pi} G_{a,b}^{(1)}(z)$$

with

$$\begin{aligned} G_{ab}^{(0)}(z) &= \delta(1-z)\,\delta_{ag}\,\delta_{bg} \\ G_{gg}^{(1)}(z) &= \delta(1-z)\left[C_A \frac{\pi^2}{3} + \beta_0 \ln\left(\frac{\mu_R^2}{\mu_F^2}\right) + \sum_{i=0,1/2} \mathcal{G}_i^{(2l)}\right] + \ldots \end{aligned}$$

QCD Contribution

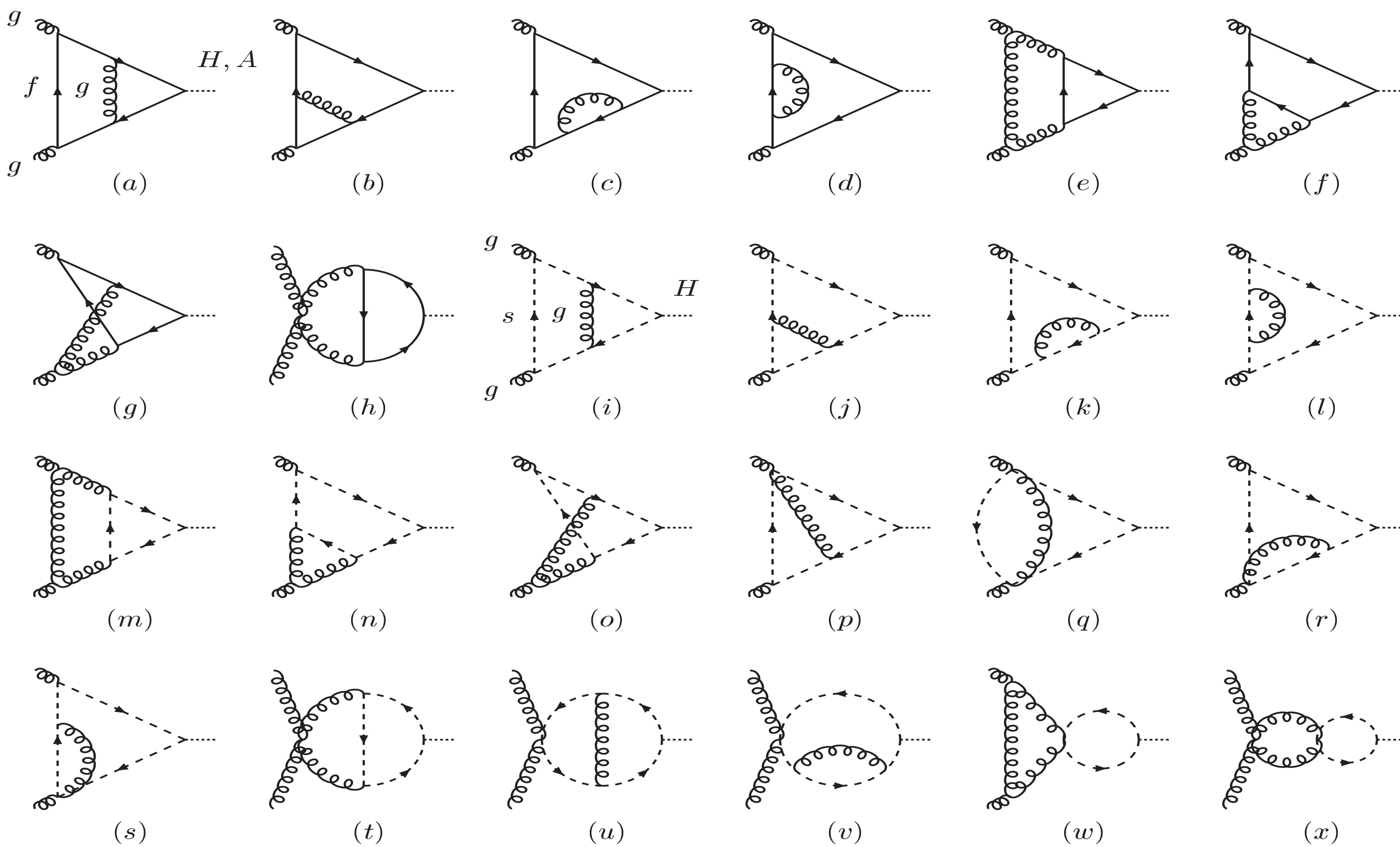
The function $\mathcal{G}_i^{(2l)}$ can be cast in the following form:

$$\mathcal{G}_i^{(2l)} = \lambda_i \left(\frac{A^2}{m_0^2}\right)^{1-2i} T(R_i) \left(C(R_i)\, \mathcal{G}_i^{(2l,C_R)}(x_i) + C_A\, \mathcal{G}_i^{(2l,C_A)}(x_i) \right) \times \left(\sum_{j=0,1/2} \lambda_j \left(\frac{A^2}{m_0^2}\right)^{1-2j} T(R_j)\, \mathcal{G}_j^{(1l)} \right)^{-1} + h.c.$$

with $\mathcal{G}_i^{(2l,C_R)} = \mathcal{F}_i^{(2l)}$.

Feynman Diags for the QCD Contribution

$$\mathcal{G} = \mathcal{G}_{QCD}^{(1l)} + \frac{\alpha_S}{\pi}\mathcal{G}_{QCD}^{(2l)} + \frac{\alpha m_W^2}{2\pi m_H^2 s_W^2}\mathcal{G}_{EW}^{(2l)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{G}_{QCD}^{(3l)}$$



QCD Contribution

After subtraction of the infrared poles:

$$\mathcal{G}_{1/2}^{(2l,C_A)}(x) = \frac{4x}{(x-1)^2}\left[3 + \frac{x(1+8x+3x^2)}{(x-1)^3}\,H(0,0,0,x) - \frac{2(1+x)^2}{(x-1)^2}\,\mathcal{H}_2(x) + \zeta_3 - H(1,0,0,x)\right]$$

$$\mathcal{G}_{0}^{(2l,C_A)}(x) = \frac{4x}{(x-1)^2}\left[-\frac{3}{2} + \frac{x(1-7x)}{(x-1)^3}H(0,0,0,x) + \frac{4x}{(x-1)^2}\mathcal{H}_2(x)\right]$$

with

$$\mathcal{H}_2(x) = \frac{4}{5}{\zeta_2}^2 + 2\zeta_3 + \frac{3\zeta_3}{2}\,H(0,x) + 3\zeta_3 H(1,x) + \zeta_2 H(1,0,x) + \frac{1}{4}\,(1+2\zeta_2)\,H(0,0,x) - 2\,H(1,0,0,x) + H(0,0,-1,0,x) + \frac{1}{4}H(0,0,0,0,x) + 2\,H(1,0,-1,0,x) - H(1,0,0,0,x)$$

Summary

- We have discussed the method for the calculation of Feynman diagrams based on the Laporta algorithm for the reduction to the MIs and on the differential equations method for their evaluation.
- The method introduced allowed to calculate the NLO EW and QCD corrections for the decay $H \to \gamma\gamma$ and for the production $gg \to H$ in a closed analytic form. The result is expressed in terms of HPLs and GHPLs
- In tha case of QCD NLO corrections we were able to give an independent check of the known results of Spira-Djouadi-Graudenz-Zerwas and Harlander-Kant
- We provided also analytic formulas for the case in which a scalar colored particle runs in the massive loops. These results have to be checked (numerically) against the results of Mühlleitner-Spira (work in progress)