Mixing angles for non-degenerate coupled systems in QFT

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- Q. Duret & B. Machet, Phys. Lett. B 643 (2006) 303-310, hep-ph/0606303.
- Q. Duret & B. Machet, Phys. Lett. B 642 (2006) 469-471, hep-ph/0610148.

Introduction

Non-degenerate coupled systems :

- Bosons, such as binary systems of neutral kaons $K_L - K_S$
- Fermions in the standard model (coupled through non-diagonal Yukawa couplings in flavour space)

A fundamental feature :

In QFT, due to the mass differences between particles, mixing matrices of such systems should *a priori* never be considered as unitary

$\Longrightarrow$ In the following approach, considering massive fermions in the SM :

- We parametrize mixing matrices as non-unitary
- We derive, through basic physical requirements, some significant results on the value of the mixing angles.

Flavour and mass states in QFT

Two types of states :

Flavour eigenstates		Mass eigenstates
$(e^-_f, \mu^-_f, \nu_{e,f}, \nu_{\mu,f} \ldots)$	**VS**	$(e^-_m, \mu^-_m, \nu_{e,m}, \nu_{\mu,m} \ldots)$
=		=
gauge interaction eigenstates		propagating eigenstates

In QFT the physical masses are the poles of the full renormalized propagator, i.e. the values of $z = q^2$ which satisfy

$$\det \Delta^{-1}(z) = 0, \text{ for } z = z_i, \tag{1}$$

The mass eigenstates are the corresponding eigenvectors :

$$\Delta^{-1}(z = z_i)\varphi^i_m = 0. \tag{2}$$

In terms of the renormalized quadratic lagrangian $L^{(2)}(z) = \Delta^{-1}(z)$:

$$\det L^{(2)}(z) = 0 \qquad\qquad L^{(2)}(z = z_i)\varphi^i_m = 0. \tag{3}$$

Why mixing matrices have no reasons to be unitary

The mixing matrices connect flavour eigenstates (Ψ_f) to mass eigenstates (Ψ_m) :

$$\Psi_f = K\Psi_m. \tag{4}$$

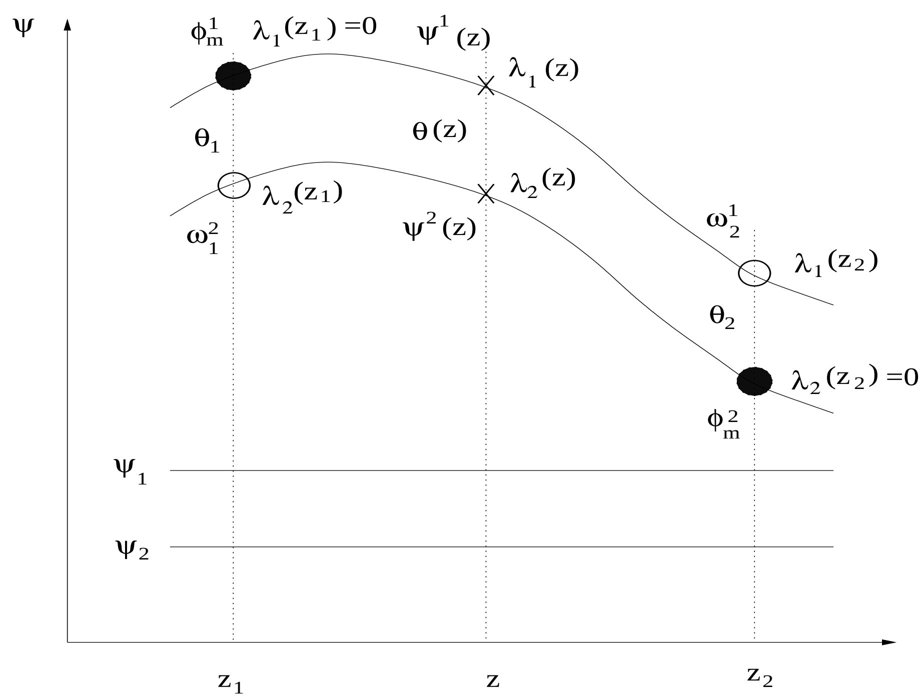
$L^{(2)}(z = q^2)$ hermitian

$\Longrightarrow$ at each z, the eigenstates of $L^{(2)}(z)$ form an orthonormal basis $\Psi(z)$.

The mass eigenstates respectively belong to different orthonormal bases

$\Longrightarrow$ they do no form themselves an orthonormal basis

If the flavour states form an orthonormal basis,
the mixing matrix K cannot be unitary.



Leptonic weak neutral currents I

$$\Psi_f = J\Psi_m, \quad \Psi_f = \begin{pmatrix} \nu_{e,f} \\ \nu_{\mu,f} \\ e^-_f \\ \mu^-_f \end{pmatrix}, \quad \Psi_m = \begin{pmatrix} \nu_{e,m} \\ \nu_{\mu,m} \\ e^-_m \\ \mu^-_m \end{pmatrix}, \quad J = \left(\begin{array}{c|c} K_\nu & \\ \hline & K_\ell \end{array}\right), \tag{5}$$

- Parametrize the mixing matrices as non-unitary with two angles instead of one (preserving a unit norm for all states) :

$$K_\nu = \begin{pmatrix} e^{i\alpha}c_1 & e^{i\delta}s_1 \\ -e^{i\beta}s_2 & e^{i\gamma}c_2 \end{pmatrix}, \qquad K_\ell = \begin{pmatrix} e^{i\theta}c_3 & e^{i\zeta}s_3 \\ -e^{i\chi}s_4 & e^{i\omega}c_4 \end{pmatrix}. \tag{6}$$

- Neutral currents $(W^3_\mu) \quad \Longrightarrow \quad K^\dagger_\nu K_\nu$, $K^\dagger_l K_l$

- Two characteristics in flavour space : universality and absence of FCNC.

– If K unitary $\quad \Longrightarrow \quad$ automatically achieved in mass space too.
– If K non-unitary $\quad \Longrightarrow \quad$ no longer automatic.

Hence we impose **I** : universality of neutral currents, and **II** : absence of ”FCNC” in the space of mass eigenstates (experimentally observed).

Leptonic weak neutral currents II

Given

$$K_\nu^\dagger K_\nu = \begin{pmatrix} c_1^2 + s_2^2 & c_1 s_1 e^{i(\delta-\alpha)} - c_2 s_2 e^{i(\gamma-\beta)} \\ c_1 s_1 e^{i(\alpha-\delta)} - c_2 s_2 e^{i(\beta-\gamma)} & s_1^2 + c_2^2 \end{pmatrix} \tag{7}$$

these constraints translate into :

– **I** : identity of diagonal elements : $c_1^2 + s_2^2 = c_2^2 + s_1^2$

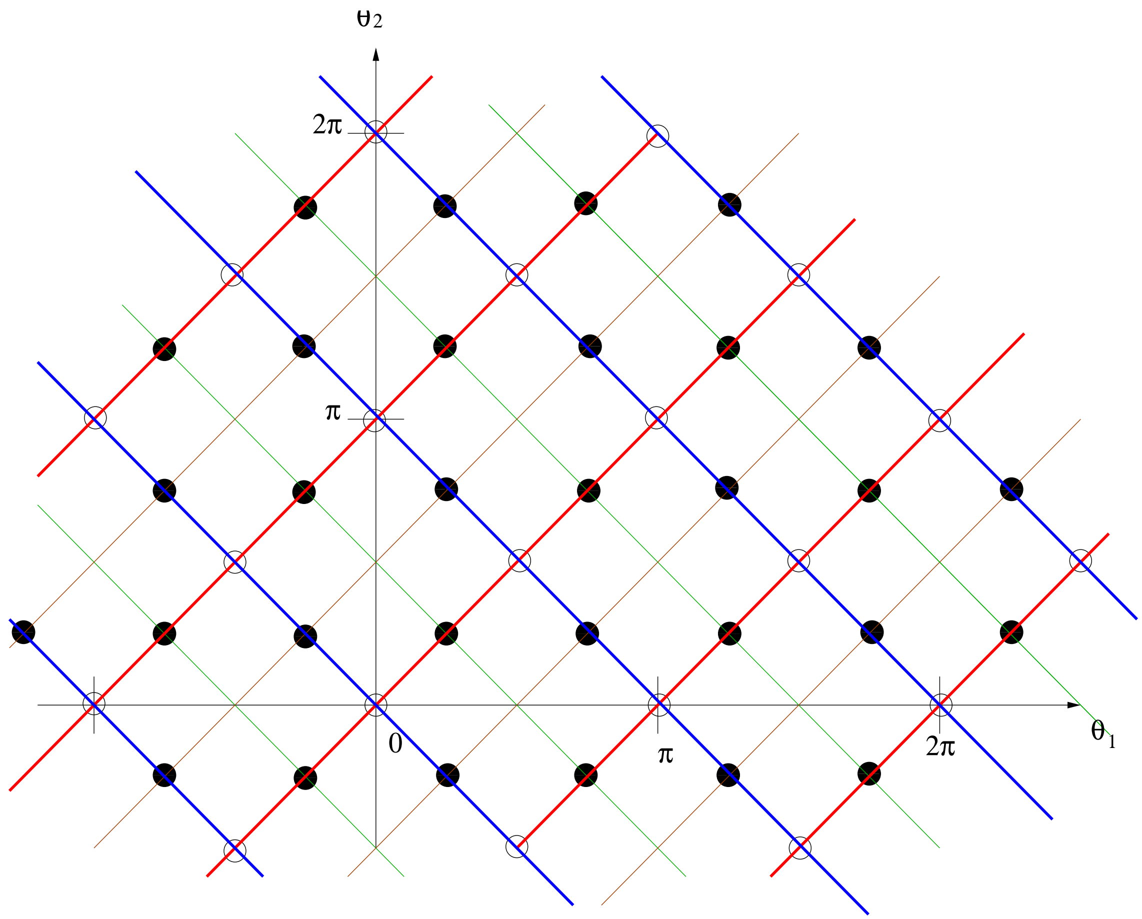
– **II** : vanishing of non-diagonal elements : $c_1 s_1 = c_2 s_2$ or $c_1 s_1 = -c_2 s_2$.

Two sets of solutions arise :

- One-parameter ("Cabibbo-like") solutions : $\theta_2 = \pm\theta_1 + k\pi$ for which **I** and **II** coincide.

- Two-parameter solutions, for which **I** and **II** are independent.
  They are of the form $\theta_1 = \pm\frac{\pi}{4}$; $\theta_2 = \pm\theta_1 + k\pi$, i.e. give rise to maximal mixing.

Mixing angles



Constraints given by the two conditions of universality and absence of FCNC's.

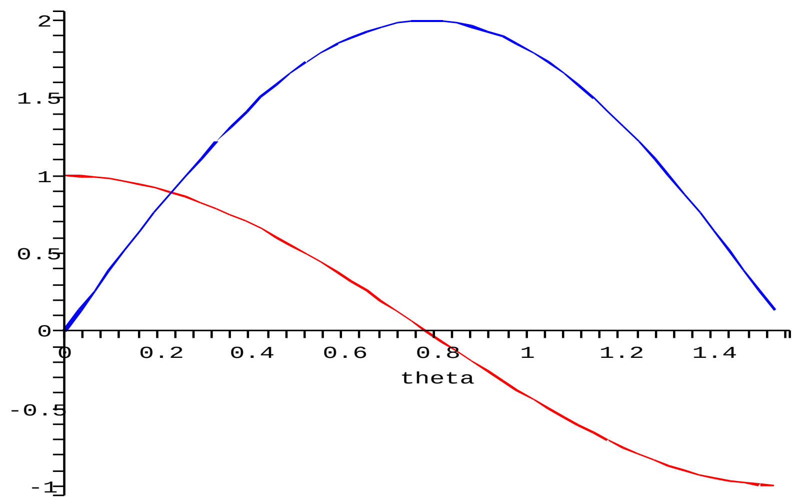
Getting the Cabibbo angle

Neighborhood of the Cabibbo case : $\theta_2 = \pm\theta_1 + \epsilon$.

$$\Longrightarrow \quad K \text{ deviates from unitarity by } K^\dagger K = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \epsilon \begin{pmatrix} \sin(2\theta_c) & -a\cos(2\theta_c) \\ -a^*\cos(2\theta_c) & -\sin(2\theta_c) \end{pmatrix}.$$

- Conditions **I** and **II** cannot any more be simultaneously fulfilled

- But **I** and **II** reduce to a single condition for a value of θ_c which turns out to be that of the Cabibbo angle experimentally measured.



$$\boxed{\tan(2\theta_c) = \frac{1}{2} \qquad \Longrightarrow \qquad \cos\theta_c = 0.9732.}$$

Conclusion

For three generations, the same type of conditions leads to

- a configuration $(31.7, 45, 0)$ which matches quite well the mixing matrix measured at present for neutrinos
- an exact realization of the Quark-Lepton Complementarity

$\ldots$?