Production of ρ-meson pairs in $\gamma^* \gamma^*$ collisions

Rencontres de Physique des Particules,
LPSC, Grenoble
1 Mars 2007

M. Segond, LPT Orsay

in a collaboration with :
B.Pire ,
L.Szymanowski and
S.Wallon

Introduction

The exclusive reaction of ρ mesons electroproduction in $\gamma^*\gamma^*$ collisions is a beautiful laboratory to study different dynamics and factorization properties in HE QCD.

It seems to be a promising probe of the BFKL effects which could be studied in the next generation of e^+e^- colliders (ILC) and at lower energy of other kind of QCD factorizations involving GDA and TDA, which could be observed at Babar or Belle.

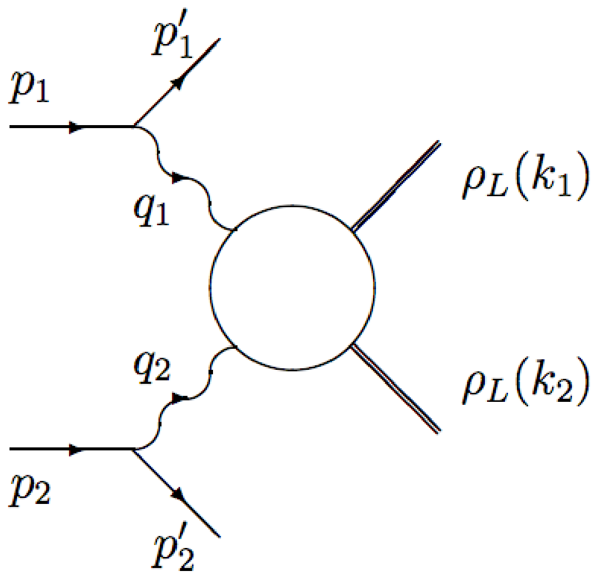
We consider the following process : $e^+e^- \to e^+e^-\rho_L\rho_L$

In the Regge limit, we expect to ‘observe’ an exchange of a BFKL Pomeron in the t-channel.

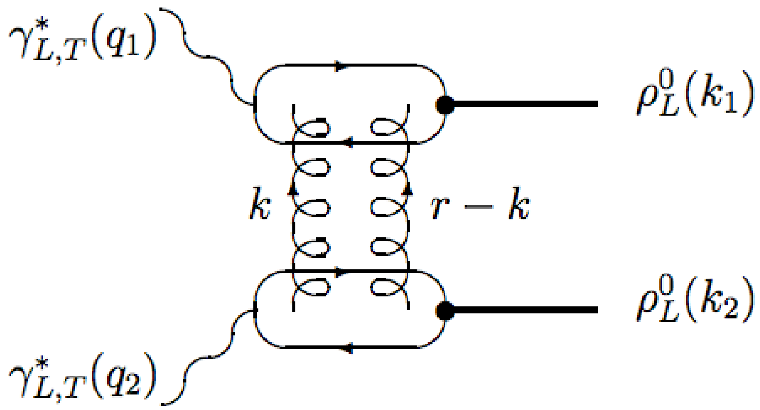
We compute the scattering amplitude in a complete analytical way at the Born order.

This process has already been studied until NLO but in the forward case.

D.Ivanov, A.Papa



Study of the process $\gamma^*_L(q_1)\,\gamma^*_L(q_2) \to \rho_L(k_1)\,\rho_L(k_2)$



Selection of events in which two vectors ρ mesons with longitudinal polarization are produced in the final state with a big gap in rapidity.

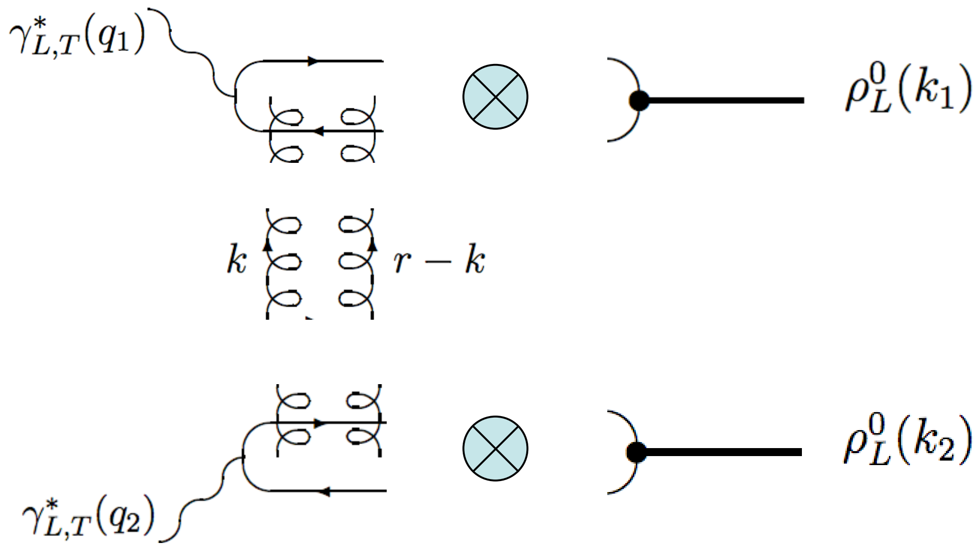
Double tagging of final leptons ➔ photons polarization

Highly virtual photons $Q_1^2\,, Q_2^2 \gg \Lambda^2_{QCD}$ ➔ perturbative computation

$Q_1^2 \sim Q_2^2$ ➔ neglect DGLAP partonic evolution

➔ In the Regge limit $s \gg -t,\ Q_1^2\,, Q_2^2$, the process is dominated by BFKL evolution.

Amplitude of the process at the Born order



Integration over the internal moments :

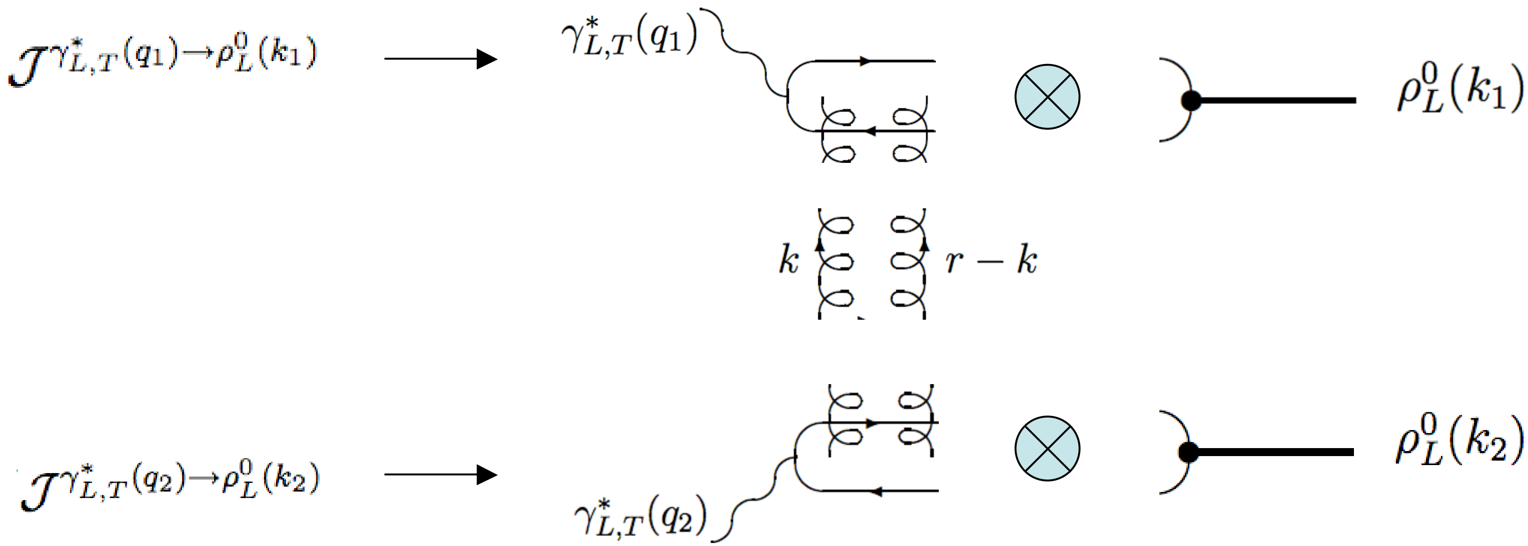
◆Sudakov basis $k = \alpha q'_1 + \beta q'_2 + k_\perp$ $\quad q'^2_1 = q'^2_2 = 0$

- In the BFKL dynamics the longitudinal momenta of the gluons are strongly ordered.

➔ kT-factorization in transverse momentum cf. $\int d^4k = \int d\alpha d\beta d\underline{k}^2$

- Collinear approximation ➔ we neglect transverse momentum of quark inside the mesons.

Factorization of the amplitude



$$\mathcal{M} = is \int \frac{d^2\,\underline{k}}{(2\pi)^4 \underline{k}^2\,(\underline{r}-\underline{k})^2} \mathcal{J}^{\gamma^*_{L,T}(q_1)\to\rho^0_L(k_1)}(\underline{k},\underline{r}-\underline{k})\;\mathcal{J}^{\gamma^*_{L,T}(q_2)\to\rho^0_L(k_2)}(-\underline{k},-\underline{r}+\underline{k})$$

Every impact factor $\mathcal{J}^{\gamma^*_{L,T}(q_1)\to\rho^0_L(k_1)}$ is written as a convolution of the DA of the meson with the more simple impact factor corresponding to the quark-antiquark opened pair production from one polarized photon with two gluons exchanged in the t channel.

We use the matrix element of the non local correlator of quarks fields on the light cone to define the DA of the meson as :

$$\langle\rho(k_2)|\bar{q}(-\frac{z}{2})\;\gamma^\mu\;q(\frac{z}{2})|0\rangle = f_\rho\,k_2^\mu \int_0^1 du e^{i(1-2u)(k_2\frac{z}{2})}\phi(u)$$

- In the case of longitudinaly polarized photons, they read :

$$\mathcal{J}^{\gamma_L^*(q_i)\to\rho_L(k_i)}(\underline{k},\underline{r}-\underline{k})$$

$$= 8\pi^2\alpha_s \frac{e}{\sqrt{2}}\frac{\delta^{ab}}{2N_c} Q_i\, f_\rho \alpha(k_i) \int_0^1 dz_i z_i\, \bar{z}_i\, \phi(z_i) \mathrm{P_P}(\mathrm{z_i}, \underline{\mathrm{k}}, \underline{\mathrm{r}}, \mu_\mathrm{i})$$

with $\mathrm{P_P}(\mathrm{z_i}, \underline{\mathrm{k}}, \underline{\mathrm{r}}, \mu_\mathrm{i}) = \frac{1}{\mathrm{z_i^2}\underline{\mathrm{r}}^2 + \mu_\mathrm{i}^2} + \frac{1}{\bar{\mathrm{z}}_\mathrm{i}^2\underline{\mathrm{r}}^2 + \mu_\mathrm{i}^2} - \frac{1}{(\mathrm{z_i}\underline{\mathrm{r}} - \underline{\mathrm{k}})^2 + \mu_\mathrm{i}^2} - \frac{1}{(\bar{\mathrm{z}}_\mathrm{i}\underline{\mathrm{r}} - \underline{\mathrm{k}})^2 + \mu_\mathrm{i}^2}$

where $\mu_i^2 = Q_i^2\, z_i\, \bar{z}_i + m^2$

- For transversely polarized photons, one obtains :

$$\mathcal{J}^{\gamma_T^*(q_i)\to\rho_L(k_i)}(\underline{k},\underline{r}-\underline{k})$$

$$= 4\pi^2\alpha_s \frac{e}{\sqrt{2}}\frac{\delta^{ab}}{2N_c}\, f_\rho \alpha(k_i) \int_0^1 dz_i\, (z_i - \bar{z}_i)\, \phi(z_i)\, \underline{\epsilon} \cdot \underline{\mathrm{Q}}(z_i, \underline{k}, \underline{r}, \mu_i)$$

with $\underline{Q}(z_i, \underline{k}, \underline{r}, \mu_i) = \frac{z_i\, \underline{r}}{z_i^2\underline{r}^2 + \mu_i^2} - \frac{\bar{z}_i\, \underline{r}}{\bar{z}_i^2\underline{r}^2 + \mu_i^2} + \frac{\underline{k} - z_i\, \underline{r}}{(z_i\underline{r} - \underline{k})^2 + \mu_i^2} - \frac{\underline{k} - \bar{z}_i\underline{r}}{(\bar{z}_i\, \underline{r} - \underline{k})^2 + \mu_i^2}$

Both Impact factor vanish when $\underline{k} \to 0$ or $\underline{r} - \underline{k} \to 0$ due to QCD gauge invariance

To compute the scattering amplitude $M_{\lambda_1\lambda_2}$ we have to perform analytically the 2D integration over the transverse momentum.

Analytical computation of the 2D integrals involved is performed after the use of conformal transformations in the transverse momentum space.

This reduces the number of propagators .

For example , we have to compute this kind of integrals :

$$J_{3m} = \int \frac{d^2\underline{k}}{\underline{k}^2(\underline{k}-\underline{r})^2}\left(\frac{1}{(\underline{k}-\underline{r}a)^2+m^2} - \frac{1}{\underline{r}^2+m^2} + (a \leftrightarrow \bar{a})\right)$$

Inversion on the integration variable and vector parameter

$$\underline{k} \to \frac{\underline{K}}{\underline{K}^2}, \quad \underline{r} \to \frac{\underline{R}}{\underline{R}^2}, \quad m \to \frac{1}{M}$$

Then we perform the shift of variable $\underline{K} = \underline{R} + \underline{k}'$

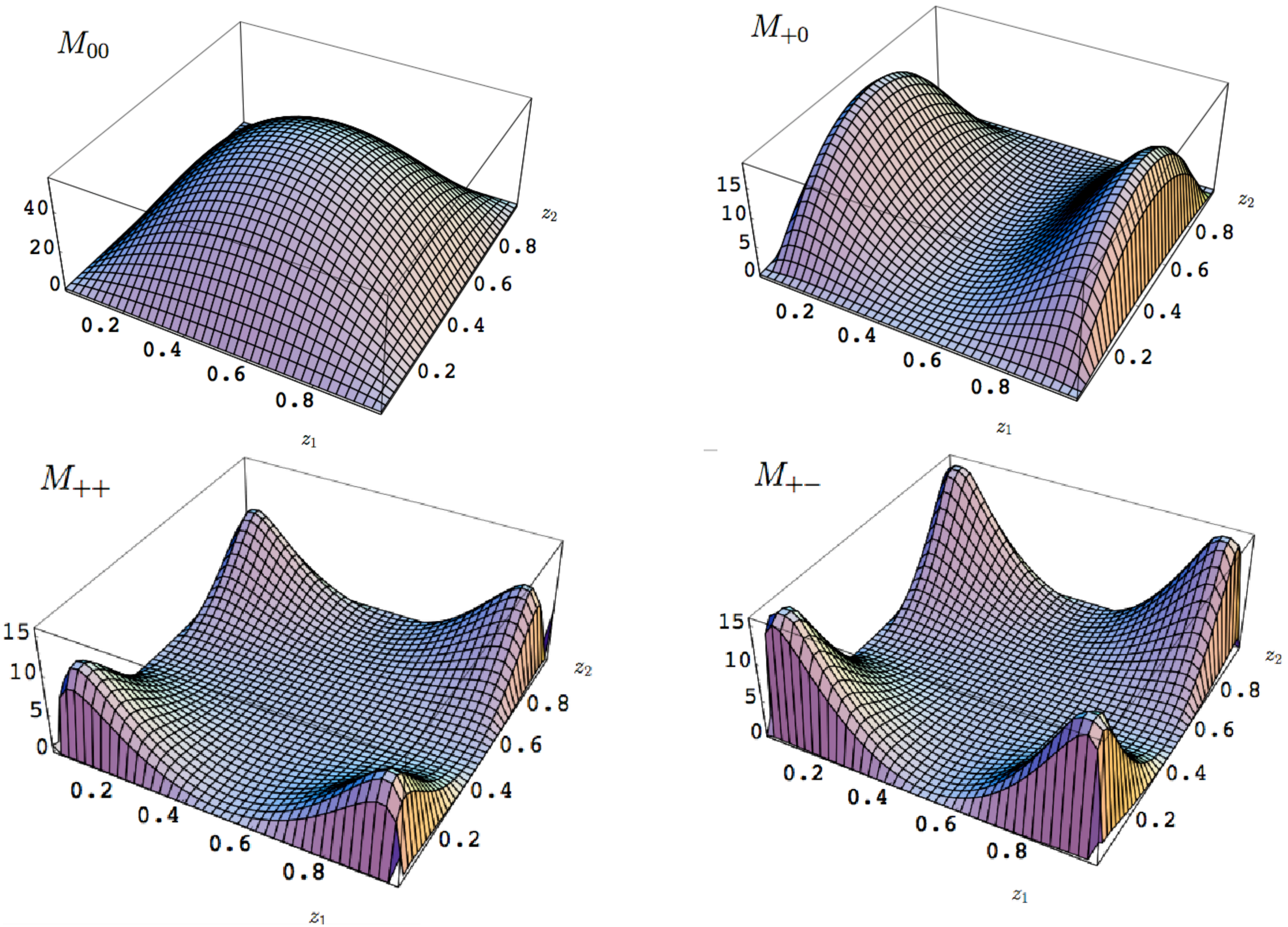
And an other inversion

Finally we obtain

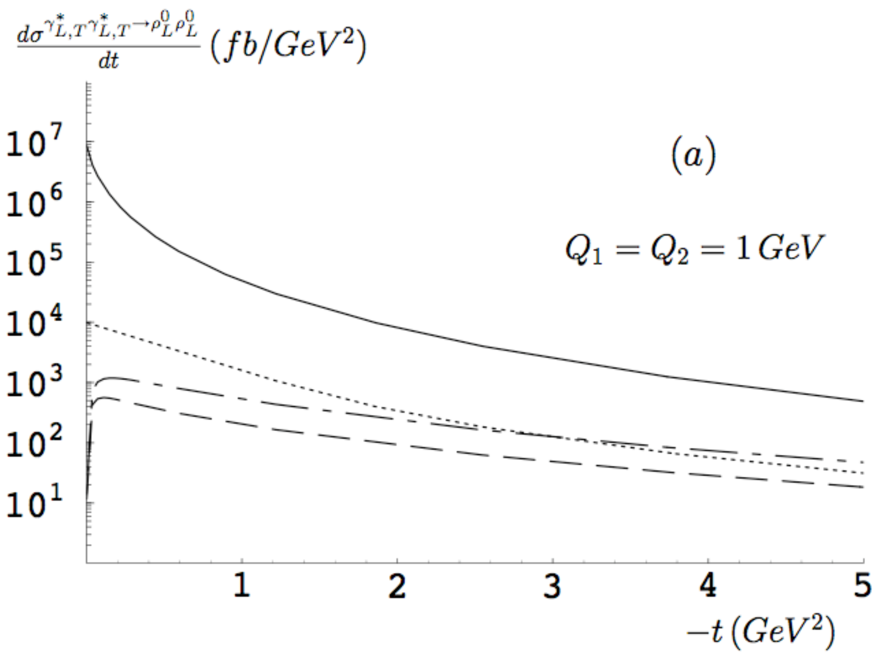
$$J_{3m} = \frac{1}{r^2}\int \frac{d^2\underline{k}}{\underline{k}^2}\left[\frac{(\underline{r}+\underline{k})^2}{(r^2a^2+m^2)\left(\left(\underline{k}-\underline{r}\frac{r^2a\,\underline{a}-m^2}{r^2\underline{a}^2+m^2}\right)^2+\frac{m^2r^4}{(r^2\underline{a}^2+m^2)^2}\right)} - \frac{1}{a^2r^2+m^2} + (a \leftrightarrow \bar{a})\right]$$

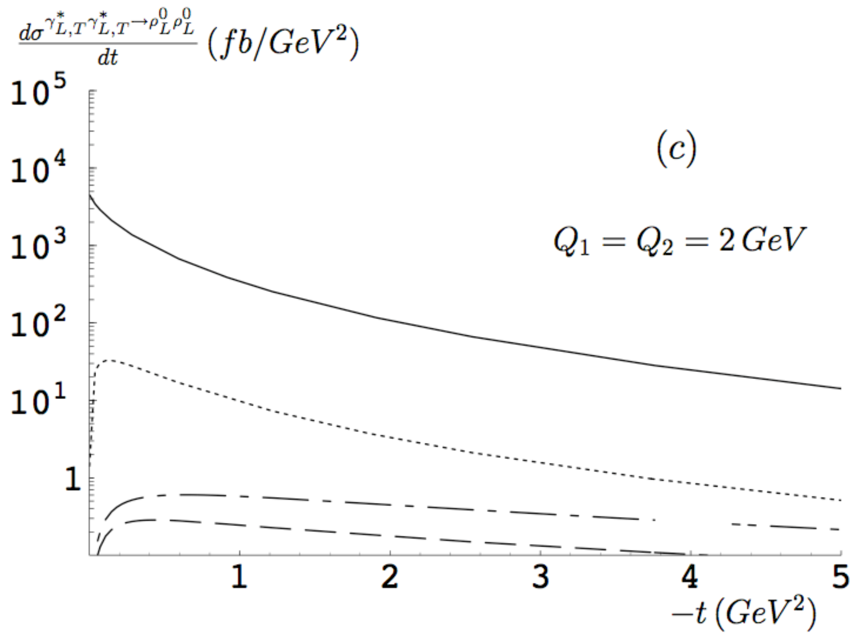
It is now possible to compute this integral by using standard technique

Shape of the k_T-integrated amplitudes



Differential cross section for the different polarizations of the virtual photons





solid curve : LL mode
dotted curve : LT mode
dashed and dashed-dotted curves :TT mode

Non-forward cross-sections at ILC for $e^+e^- \to e^+e^- \rho_L^0\ \rho_L^0$

We use the same Sudakov basis

$$\frac{d\sigma(e^+e^- \to e^+e^- \rho_L \rho_L)}{dy_1 dy_2 dQ_1^2 dQ_2^2}$$
$$= \frac{1}{y_1 y_2\, Q_1^2 Q_2^2} \left(\frac{\alpha}{\pi}\right)^2 [l_1(y_1)\, l_2(y_2) \sigma(\gamma_L^* \gamma_L^* \to \rho_L \rho_L) + t_1(y_1)\, l_2(y_2)\, \sigma(\gamma_T^* \gamma_L^* \to \rho_L \rho_L)$$
$$+ l_1(y_1)\, t_2(y_2)\, \sigma(\gamma_L^* \gamma_T^* \to \rho_L \rho_L) + t_1(y_1)\, t_2(y_2)\, \sigma(\gamma_T^* \gamma_T^* \to \rho_L \rho_L)] \ .$$

with the usual photons flux factors $\quad t_i = \frac{1+(1-y_i)^2}{2}, \quad l_i = 1 - y_i$

Weizsacker-Wiliams

Kinematical constraints coming from experimental features of the ILC collider are used to perform the phase-space integration.

Photons momentum fractions

$$y_i = \frac{E - E'_i \cos^2(\theta_i/2)}{E}$$

and virtualities

$$Q_i^2 = 4EE'_i \sin^2(\theta_i/2)$$

In the cms frame, kinematical constraints coming from the minimal detection angle around the beampipe and from the conditions on the energies of the scattered leptons and the Regge limit.

$$y_{i\,max} = 1 - \frac{E_{min}}{E}$$

$$y_{1\,min} = \max\left(f(Q_1), 1 - \frac{E_{max}}{E}\right)$$

$$y_{2\,min} = \max\left(f(Q_2), 1 - \frac{E_{max}}{E}, \frac{c\,Q_1\,Q_2}{s\,y_1}\right)$$

with

$$f(Q_i) = 1 - \frac{Q_i^2}{s\tan^2(\theta_{min}/2)}$$

$$\frac{d\sigma^{e^+e^-\to e^+e^-\rho_L\rho_L}}{dt} = \int_{Q^2_{1min}}^{Q^2_{1max}} dQ_1^2 \int_{Q^2_{2min}}^{Q^2_{2max}} dQ_2^2 \int_{\epsilon}^{y_{max}} dy_1 \int_{\frac{Q_1 Q_2}{s y_1}}^{y_{max}} dy_2 \frac{d\sigma^{e^+e^-\to e^+e^-\rho_L\rho_L}}{dt\, dy_1\, dy_2\, dQ_1^2\, dQ_2^2}$$

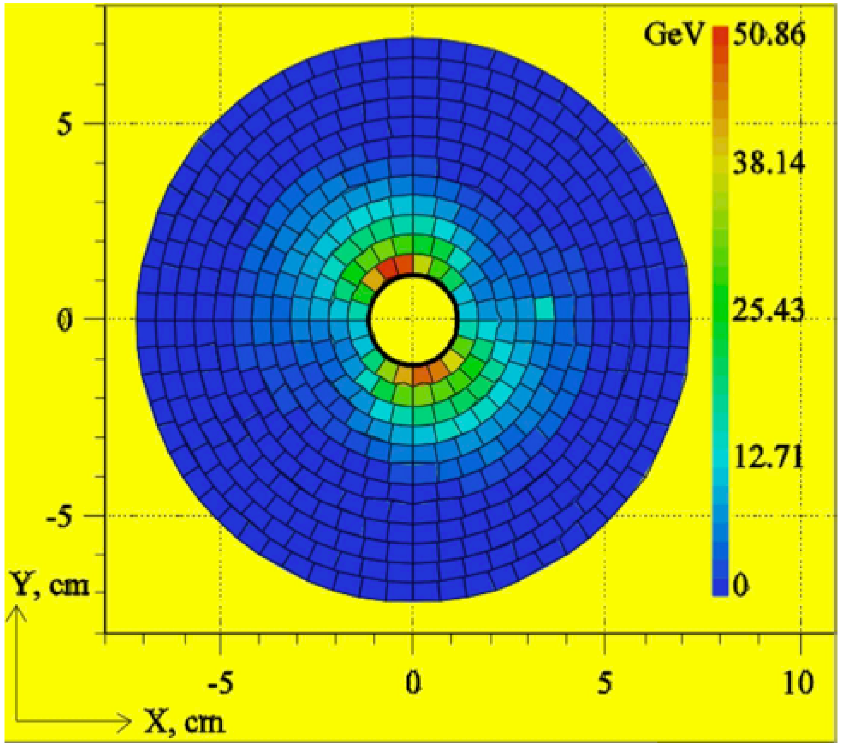
Experimental features of the ILC collider

Foreseen cms energy $\sqrt{s} = 2E = 500\,GeV$

LDC detector:
emc BeamCal around
the beampipe at 355cm from the vertex

Energy deposit in BeamCal :

F.Richard, R.Poeschl

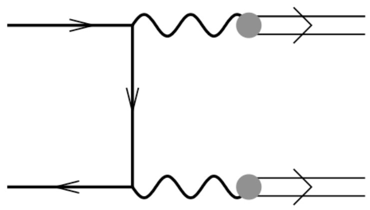


$$E_{min} = 100\,GeV$$
$$\theta_{min} = 4 \text{ mrad}$$

$$\frac{d\sigma^{e^+e^-\to e^+e^-\rho_L\rho_L}}{dt} = \int_{Q^2_{1min}}^{Q^2_{1max}} dQ_1^2 \int_{Q^2_{2min}}^{Q^2_{2max}} dQ_2^2 \int_{\epsilon}^{y_{max}} dy_1 \int_{\frac{Q_1Q_2}{sy_1}}^{y_{max}} dy_2 \frac{d\sigma^{e^+e^-\to e^+e^-\rho_L\rho_L}}{dt\,dy_1\,dy_2\,dQ_1^2\,dQ_2^2}$$

Background in the detector

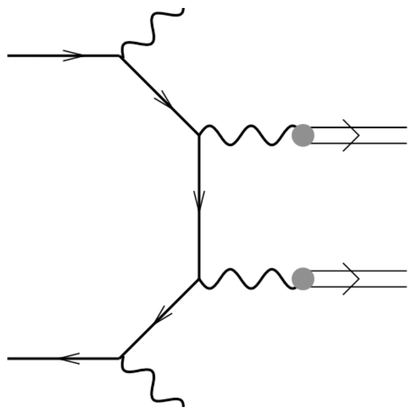
$$e^+e^- \to \rho_L^0\ \rho_L^0$$



$$\frac{d\sigma}{dt} = \frac{\alpha_{em}^4 f_\rho^4}{s^2 m_\rho^4}$$

competitor process

$$e^+e^- \to \gamma\gamma\rho_L^0\ \rho_L^0$$



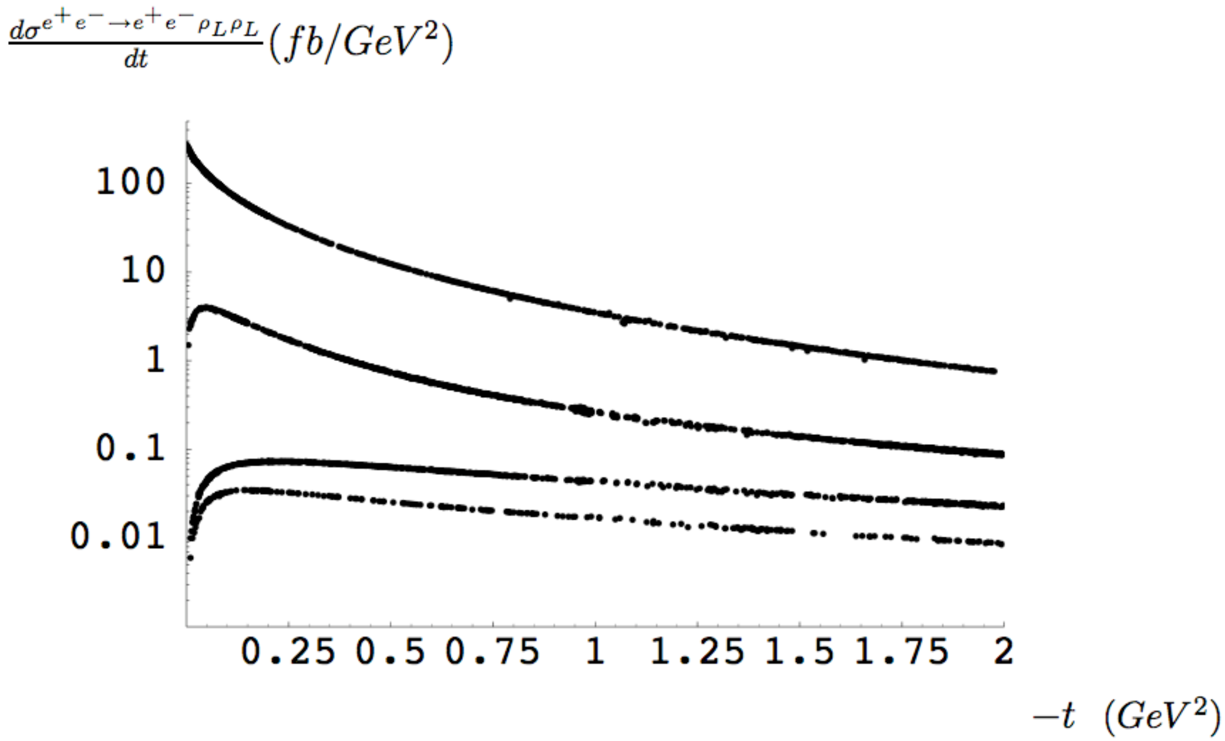
$$\frac{d\sigma^{e^+e^-\to\gamma\gamma\rho_L\rho_L}}{dt\,dy_1\,dy_2\,dQ_1^2\,dQ_2^2} \Big/ \frac{d\sigma^{e^+e^-\to e^+e^-\rho_L\rho_L}}{dt\,dy_1\,dy_2\,dQ_1^2\,dQ_2^2} \simeq \frac{\alpha_{em}^2 Q_1^4 Q_2^4}{\alpha_s^4 s^2 m_\rho^4}$$

M.Davier, M.Peskin

G.T.Bodwin, E.Braaten

Results for non-forward cross-sections at ILC for

$$e^+e^- \to e^+e^- \rho_L^0\, \rho_L^0$$



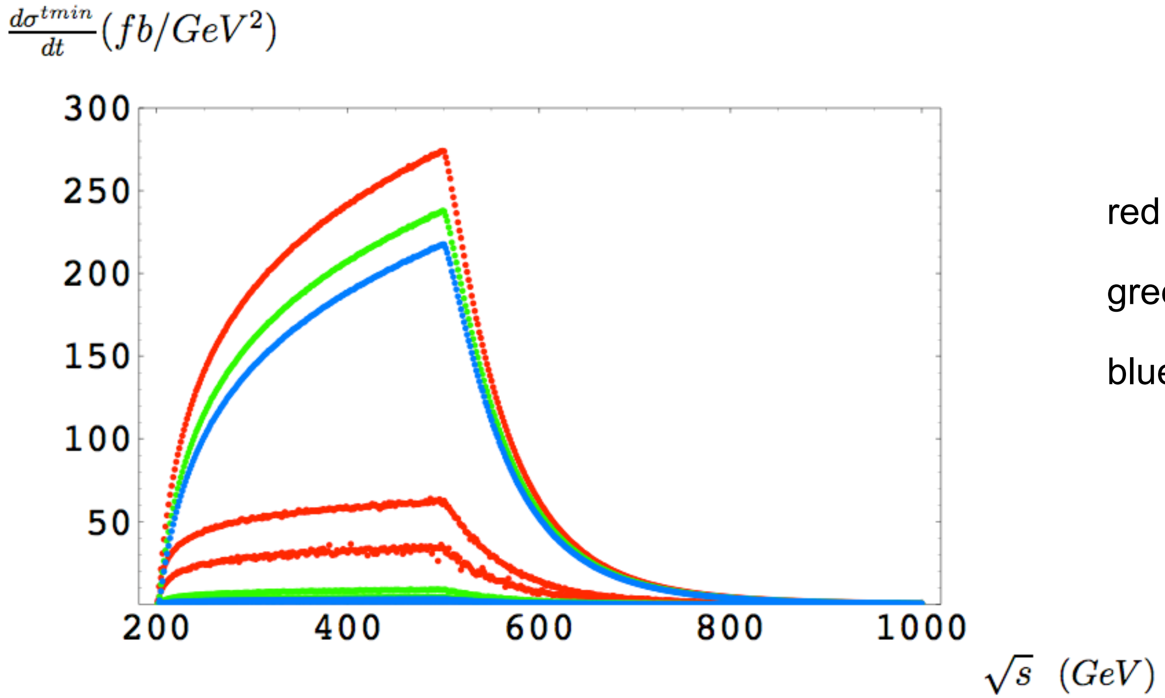
$$\sigma^{LL} = 32.39 \pm 0.12\, fb$$
$$\sigma^{LT} = 1.49 \pm 0.04\, fb$$
$$\sigma^{TT} = 0.199 \pm 0.007\, fb$$

with
- $\alpha_s(\sqrt{Q_1 Q_2})$ running at three loops
- $\sqrt{s} = 500\mathrm{GeV}$
- $c = 1$

$$\sigma^{Total} = 34.08 \pm 0.17\, fb$$

$\longrightarrow$ $4.26\, 10^3$ events per year with foreseen luminosity

Effects of parameters on the non-forward cross-sections for $e^+e^- \to e^+e^-\rho_L^0\,\rho_L^0$ at t_{min}



red curve: c = 1

green curve: c = 2

blue curve: c = 3

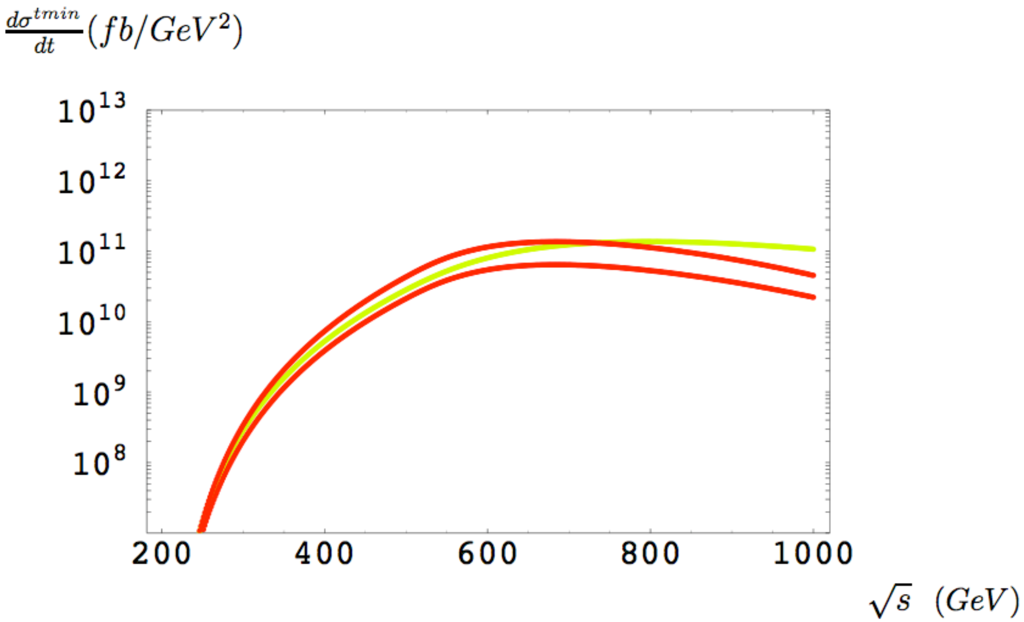
For the total cross-section:

$\alpha_s(\sqrt{Q_1Q_2})$ running at one loop $\longrightarrow$ $\sigma^{Total} = 34.08 \pm 0.17\, fb$

$\alpha_s(\sqrt{Q_1Q_2})$ running at three loops $\longrightarrow$ $\sigma^{Total} = 35.63 \pm 0.17\, fb$

$\alpha_s(\sqrt{Q_1Q_2})$ running at three loops and c = 2 $\longrightarrow$ $\sigma^{Total} = 29.55 \pm 0.13\, fb$

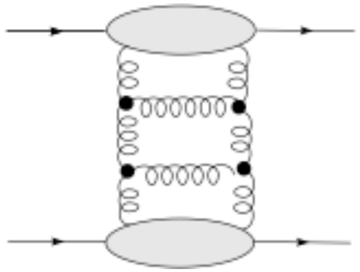
Effects of parameters on the non-forward cross-sections for $e^+e^- \rightarrow e^+e^-\rho_L^0\,\rho_L^0$ with LO BFKL evolution at t_{min}



upper red curve: $\alpha_s(\sqrt{Q_1Q_2})$ running at one loop

lower red curve: $\alpha_s(\sqrt{Q_1Q_2})$ running at three loops

green curve: fixed value of $\alpha_s = 0.46$



Forward case BFKL amplitude in the saddle-point approximation:

$$A(s, t = t_{min}, Q_1, Q_2) \sim is\, \frac{\alpha_s^2 \alpha_{em} f_\rho^2}{Q_1^2 Q_2^2} \frac{e^{4\ln 2\,\bar{\alpha}_s Y}}{\sqrt{14\bar{\alpha}_s\zeta(3)Y}} \exp\left(-\frac{\ln^2 R}{14\bar{\alpha}_s\zeta(3)Y}\right)$$

R.Enberg et al.

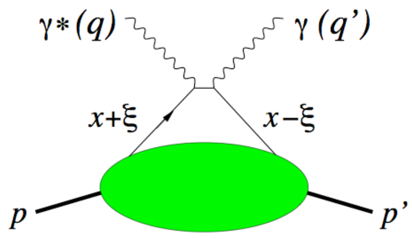
Second part: Motivation and origins

- DIS : **inclusive** process t = 0

Structure function = perturbative CF ⁎ PDF

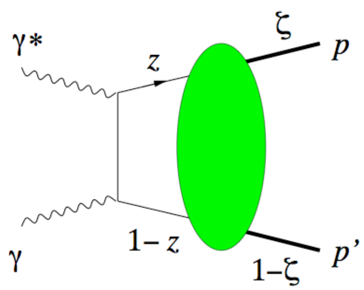
- DVCS : **exclusive** process at small t

Amplitude = perturbative CF ⁎ GPD



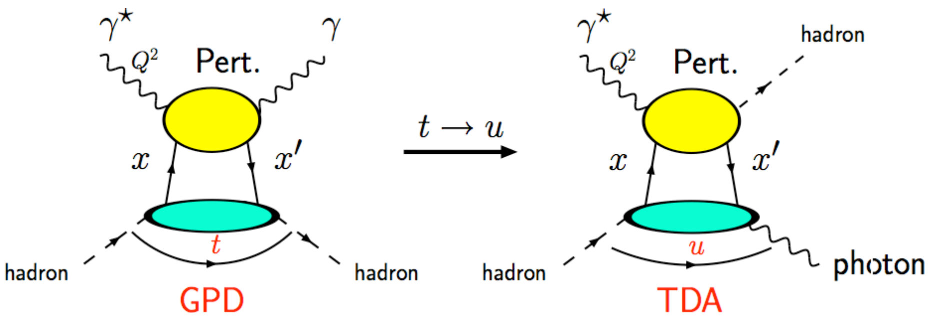
Crossing symmetry, small cm energy

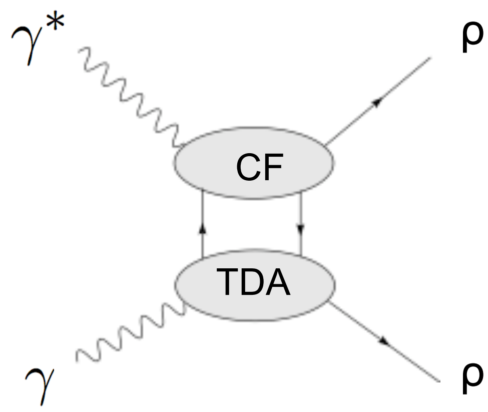
Amplitude = perturbative CF ⁎ GDA



- Generalization of DVCS, small t

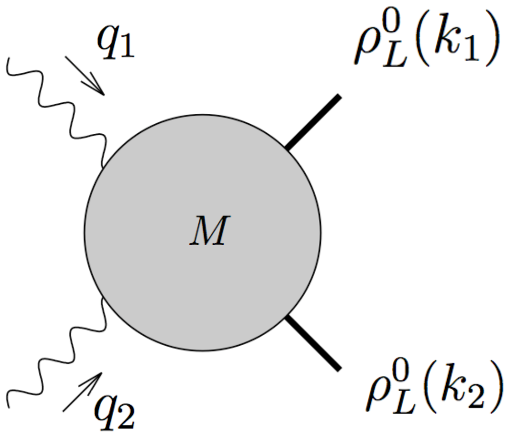
Amplitude = perturbative CF ⁎ TDA





Plan

$\gamma^*(Q_1)\gamma^*(Q_2) \to \rho_L^0(k_1)\rho_L^0(k_2)$



Exclusive reaction at Born order
quark exchange contribution

Q1,Q2 hard scales

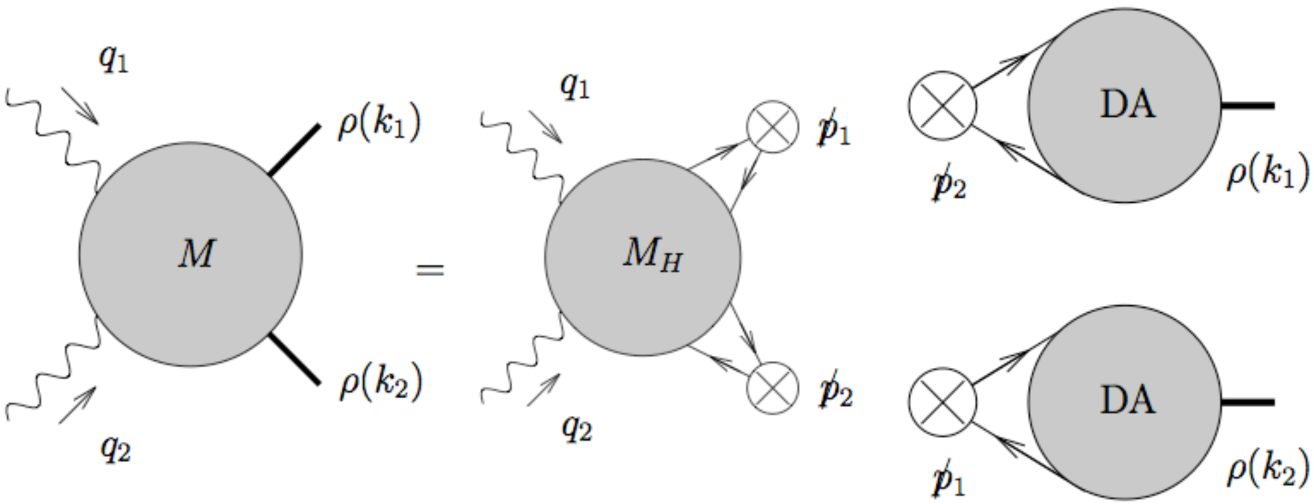
- Direct calculation and factorization with two ρ DAs
longitudinal mesons ⟶ twist 2

Brodsky-Lepage

- Factorization with ρρ GDA for transverse γ^*

- Factorization with $\gamma^* \to \rho$ TDA for longitudinal γ^*

Amplitude of the process in the collinear factorization with DA



The scattering amplitude reads

$$\mathcal{A} = T^{\mu\,\nu}\epsilon_\mu(q_1)\epsilon_\nu(q_2)$$

where

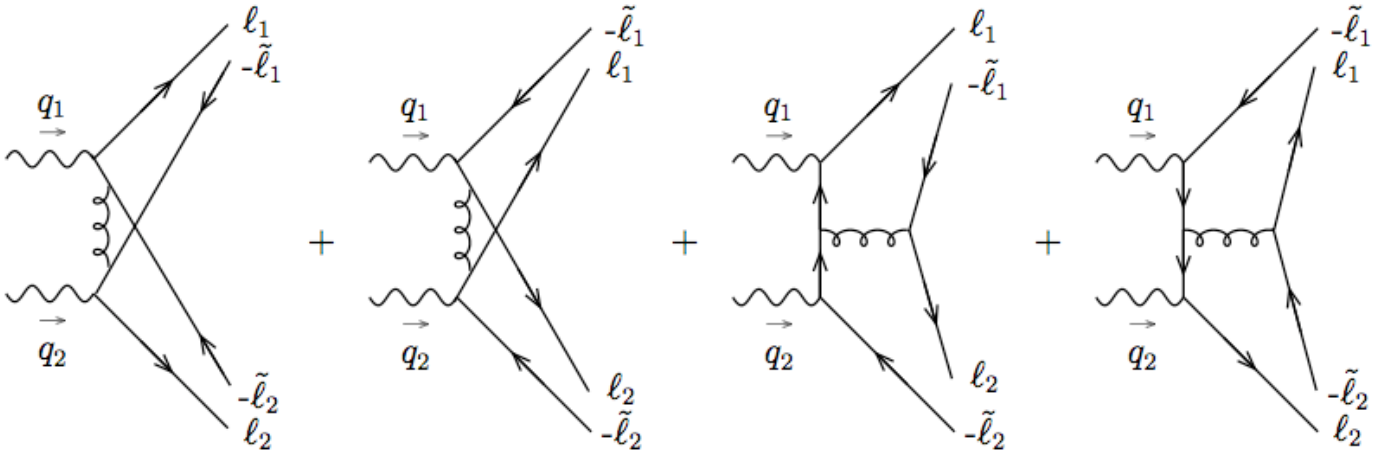
$$T^{\mu\,\nu} =$$

$$\tfrac{1}{2} g_T^{\mu\,\nu}\,(T^{\alpha\,\beta} g_{T\,\alpha\,\beta}) + (p_1^\mu + \tfrac{Q_1^2}{s} p_2^\mu)(p_2^\nu + \tfrac{Q_2^2}{s} p_1^\nu)\,\tfrac{4}{s^2}(T^{\alpha\,\beta} p_{2\,\alpha}\, p_{1\,\beta})$$

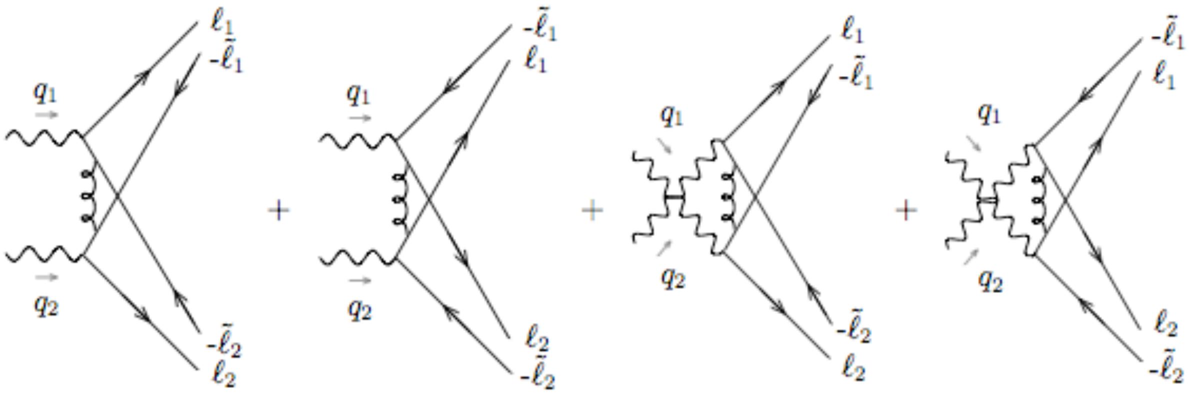
and

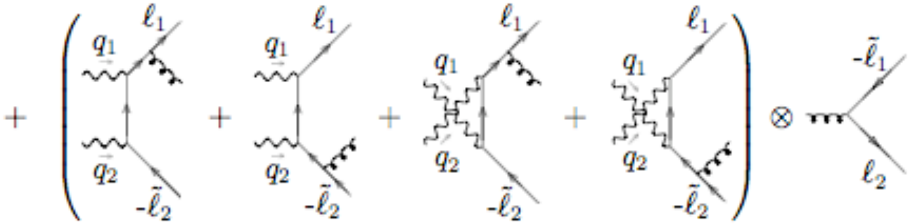
$$g_T^{\mu\,\nu} = g^{\mu\nu} - (p_1^\mu p_2^\nu + p_1^\nu p_2^\mu)/(p_1 . p_2)$$

longitudinally polarized photons

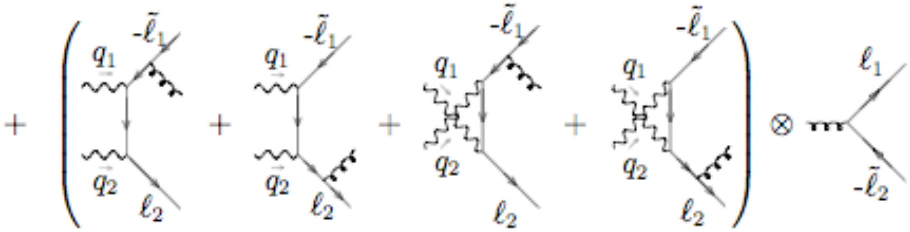


transversally polarized photons





$(s1)$ $(s3)$ $(s4)$ $(s6)$



$(s1')$ $(s3')$ $(s4')$ $(s6')$

Analytic expressions in collinear factorization

The virtualities Q_1 and Q_2 of the photons supply the hard scale

Collinear approximation $\longrightarrow$ neglect transverse relative quark momenta in the rho mesons

$$\ell_1 \sim z_1 k_1, \qquad \ell_2 \sim z_2 k_2$$
$$\tilde{\ell}_1 \sim \bar{z}_1 k_1, \qquad \tilde{\ell}_2 \sim \bar{z}_2 k_2$$

We use the matrix element of the non local correlator of quarks fields on the light cone to define the DA of the meson as

$$\langle \rho_L^0(k) | \bar{q}(x) \gamma^\mu q(0) | 0 \rangle = \frac{f_\rho}{\sqrt{2}} k^\mu \int_0^1 dz\, e^{iz(kx)} \phi(z)$$

Analytic expressions in collinear approximation

Transverse photons

$$T^{\alpha\,\beta} g_{T\,\alpha\,\beta} = -\frac{e^2 (Q_u^2+Q_d^2)\, g^2\, C_F\, f_\rho^2}{4\, N_c\, s} \int_0^1 dz_1\, dz_2\, \phi(z_1)\, \phi(z_2)$$

$$\left\{ 2\left(1-\frac{Q_2^2}{s}\right)\left(1-\frac{Q_1^2}{s}\right)\left[\frac{1}{(z_2+\bar{z}_2\frac{Q_1^2}{s})^2(z_1+\bar{z}_1\frac{Q_2^2}{s})^2}+\frac{1}{(\bar{z}_2+z_2\frac{Q_1^2}{s})^2(\bar{z}_1+z_1\frac{Q_2^2}{s})^2}\right] + \right.$$

$$\left.\left(\frac{1}{\bar{z}_2\, z_1}-\frac{1}{\bar{z}_1\, z_2}\right)\left[\frac{1}{1-\frac{Q_2^2}{s}}\left(\frac{1}{\bar{z}_2+\frac{z_2 Q_1^2}{s}}-\frac{1}{z_2+\frac{\bar{z}_2 Q_1^2}{s}}\right)-\frac{1}{1-\frac{Q_1^2}{s}}\left(\frac{1}{\bar{z}_1+\frac{z_1 Q_2^2}{s}}-\frac{1}{z_1+\frac{\bar{z}_1 Q_2^2}{s}}\right)\right]\right\}$$

Non-zero values of Q_1 and Q_2

Behaviour of the rho DAs

→ no end-point singularities

Analytic expressions in collinear approximation

Longitudinal photons

$$T^{\alpha\,\beta} p_{2\,\alpha}\, p_{1\,\beta} = -\frac{s^2 f_\rho^2 C_F e^2 g^2 (Q_u^2+Q_d^2)}{8 N_c Q_1^2 Q_2^2} \int\limits_0^1 dz_1\, dz_2\, \phi(z_1)\, \phi(z_2)$$

$$\times \left\{ \frac{(1-\frac{Q_1^2}{s})(1-\frac{Q_2^2}{s})}{(z_1+\bar{z}_1\frac{Q_2^2}{s})(z_2+\bar{z}_2\frac{Q_1^2}{s})} + \frac{(1-\frac{Q_1^2}{s})(1-\frac{Q_2^2}{s})}{(\bar{z}_1+z_1\frac{Q_2^2}{s})(\bar{z}_2+z_2\frac{Q_1^2}{s})} + \frac{1}{z_2\bar{z}_1} + \frac{1}{z_1\bar{z}_2} \right\}$$

Non-zero values of Q_1 and Q_2

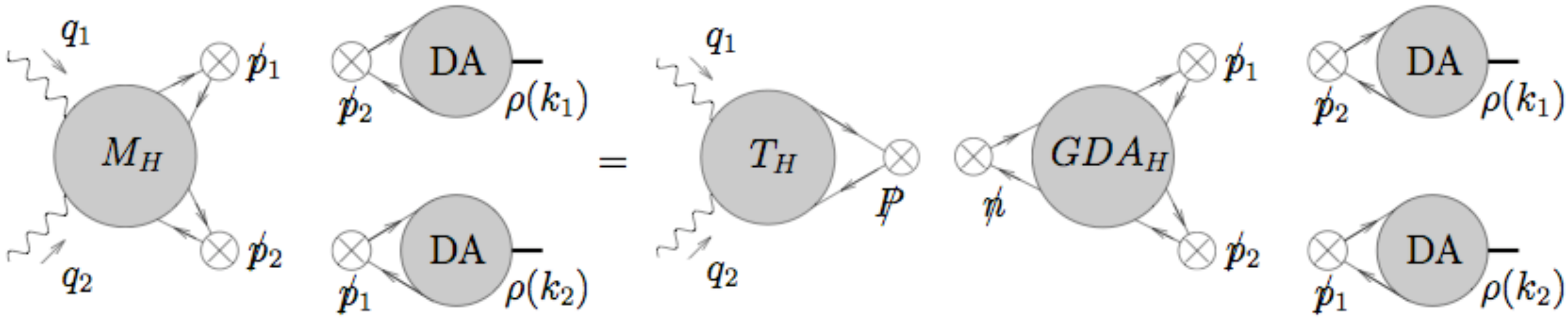
Behaviour of the rho DAs

→ no end-point singularities

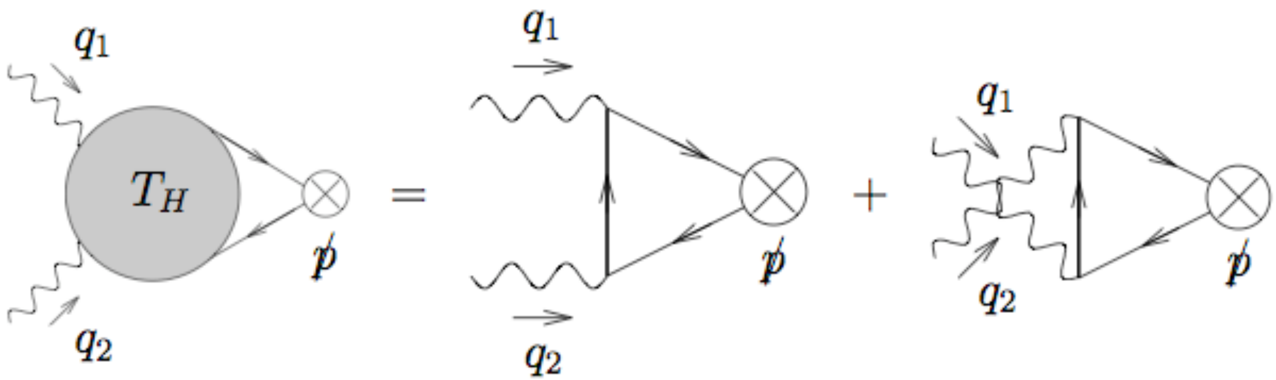
Factorization of the amplitude in terms of a GDA in the generalized Bjorken limit

In the kinematical region where the scattering energy is small

$$\frac{W^2}{Q_1^2} = \frac{s}{Q_1^2}\left(1 - \frac{Q_1^2}{s}\right)\left(1 - \frac{Q_2^2}{s}\right) \approx 1 - \frac{Q_1^2}{s} \ll 1$$



Expansion of the Hard Part at Born order



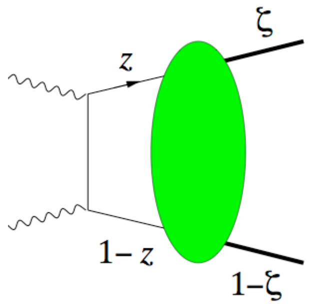
For a one flavored quarks , it equals

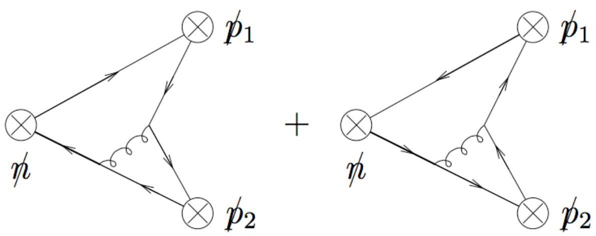
$$T_H(z) = -4\, e^2\, N_c\, Q_q^2 \left(\frac{1}{\bar{z} + z\, \frac{Q_2^2}{s}} - \frac{1}{z + \bar{z}\, \frac{Q_2^2}{s}} \right)$$

Definition of the leading twist GDA calculated in the Born order of the pertubation theory

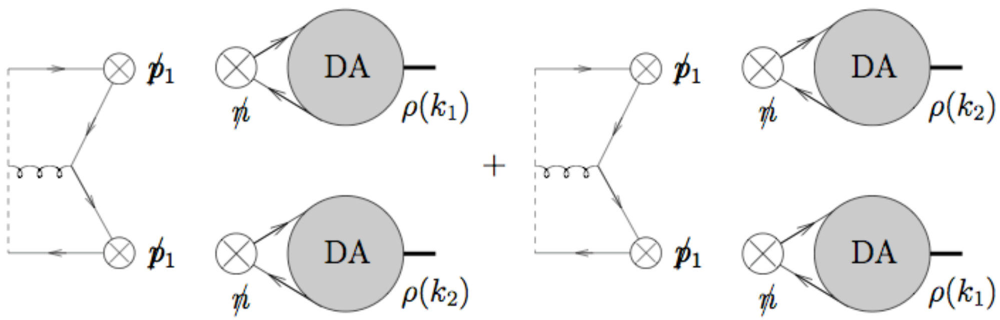
$$\langle \rho_L^0(k_1)\, \rho_L^0(k_2) | \bar{q}(-\alpha\, n/2)\, \hat{n} \exp\Big(ig \int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} dy\, n_\nu\, A^\nu(y) \Big)\, q(\alpha\, n/2) | 0 \rangle$$

$$= \int_0^1 dz\, e^{-i(2z-1)\alpha(nP)/2} \Phi^{\rho_L \rho_L}(z, \zeta, W^2)$$





→ The QCD Wilson line vanishes



= 0

Summary of the factorization with GDA for transverse virtual photons

factorization of $T^{\alpha\,\beta} g_{T\,\alpha\,\beta}$ into the Hard Part and the GDA

$$T^{\alpha\,\beta} g_{T\,\alpha\,\beta} = \frac{e^2}{2}\,(Q_u^2 + Q_d^2)\int_0^1 dz\,\left(\frac{1}{\bar{z} + z\frac{Q_2^2}{s}} - \frac{1}{z + \bar{z}\frac{Q_2^2}{s}}\right)\Phi^{\rho_L\rho_L}(z,\zeta\approx 1, W^2)$$

with $\Phi^{\rho_L\rho_L}(z,\zeta \approx 1, W^2) = -\frac{f_\rho^2\,g^2\,C_F}{2\,N_c\,W^2}\int_0^1 dz_2\,\phi(z)\,\phi(z_2)\left[\frac{1}{z\bar{z}_2} - \frac{1}{\bar{z}z_2}\right]$

Limiting case of the original equation by D.Müller et al (1994)

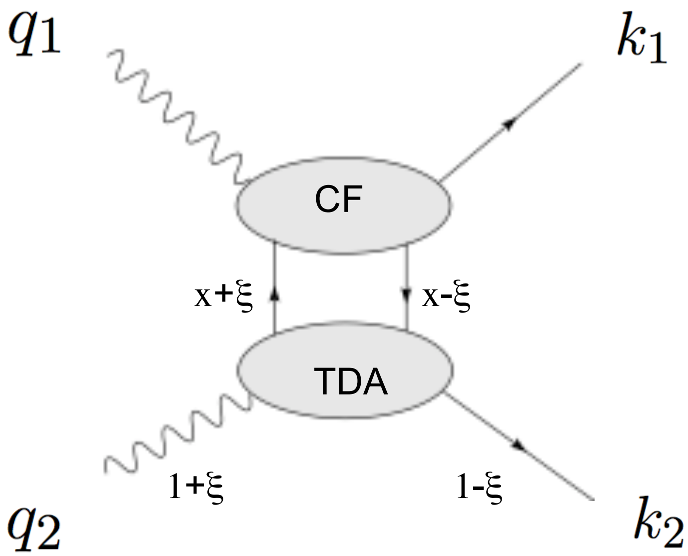
Extension of the studies of $\gamma^*\gamma \to \pi\pi$ by M.Diehl et al (2000) for virtual photons at $t = t_{min}$

Factorization of the amplitude in terms of a TDA for longitudinal virtual photons

- Kinematical regime $Q_1^2 \quad >> \quad Q_2^2 \quad , \qquad t = t_{min}$

  ⟶ second factorization

- Amplitude = convolution of Hard Part and TDA

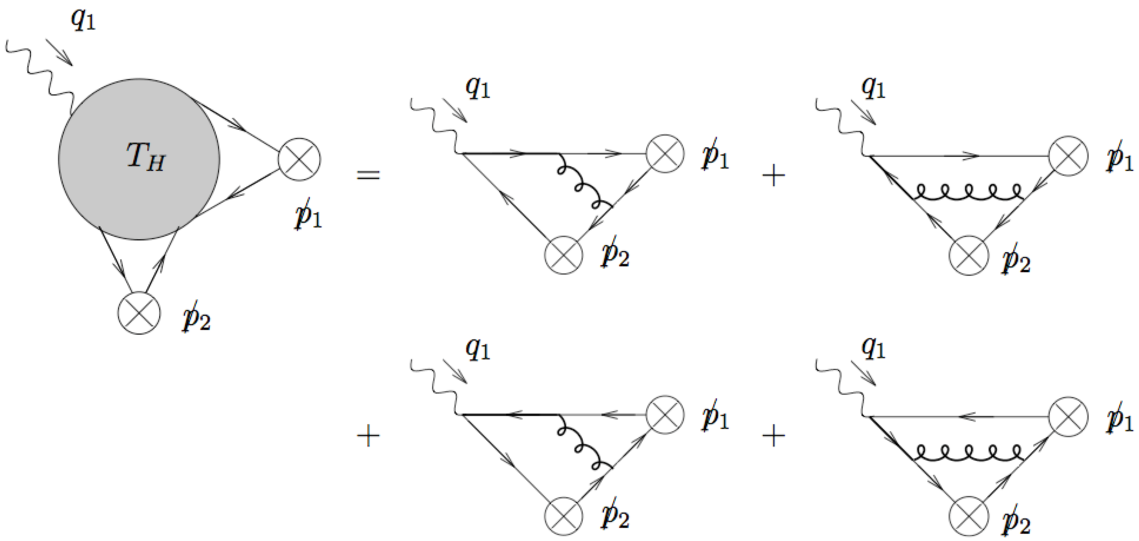


TDA kinematics = GPD kinematics

$$\int \frac{dz^-}{2\pi} e^{-ixP^+z^-} < \rho(p_2)|\bar{q}(-z^-/2)\gamma^+ q(z^-/2))|\ \gamma(q_2) >$$

Proof of the factorization in terms of a TDA

Hard Part



$$T_H(z_1,x) = -i\, f_\rho\, g^2\, e\, Q_q\, \frac{C_F\,\phi(z_1)}{2\,N_c\,Q_1^2}\epsilon^\mu(q_1)\left(2\xi\, n_{2\,\mu} + \frac{1}{1+\xi}n_{1\,\mu}\right)$$

$$\times\left[\frac{1}{z_1(x+\xi-i\epsilon)} + \frac{1}{\bar{z}_1(x-\xi+i\epsilon)}\right]$$

Proof of the factorization in terms of a TDA

definition of the TDA

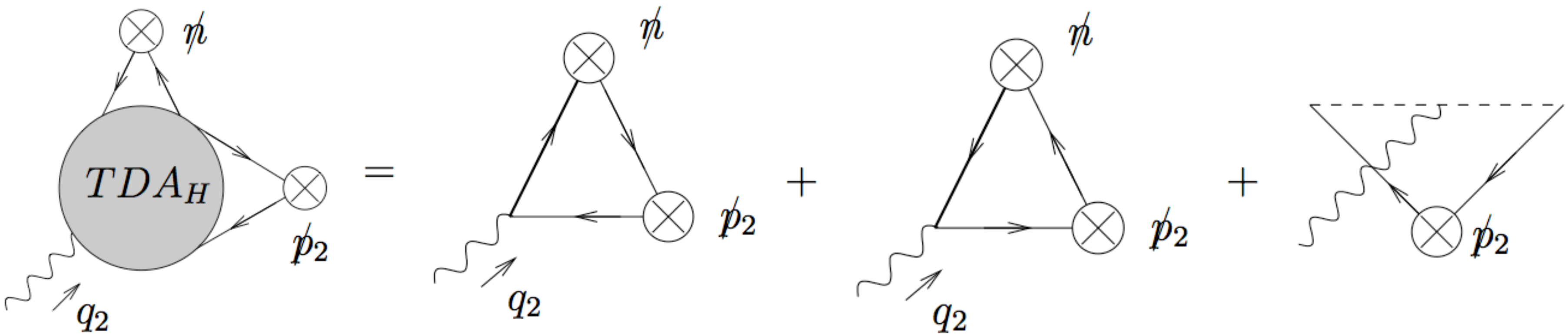
$$\int \frac{dz^-}{2\pi}\, e^{ix(P.z)}\, \langle \rho^q_L(k_2)|\bar{q}(-z/2)\hat{n}\;\; e^{-ieQ_q \int_{z/2}^{-z/2} dy_\mu\, A^\mu(y)}\, q(z/2)|\gamma^*(q_2)\rangle$$

$$= \frac{e\,Q_q\, f_\rho}{P^+}\,\frac{2}{Q_2^2}\,\epsilon_\nu(q_2)\left((1+\xi)n_2^\nu + \frac{Q_2^2}{s(1+\xi)}n_1^\nu\right)\; T(x,\xi,t_{min})$$

with $T(x,\xi,t_{min}) \equiv$

$$N_c\left[\Theta(1\geq x\geq \xi)\phi\left(\tfrac{x-\xi}{1-\xi}\right) - \Theta(-\xi\geq x\geq -1)\phi\left(\tfrac{1+x}{1-\xi}\right)\right]$$

QED Wilson line



Summary of the factorization with TDA for longitudinal virtual photons

Factorized form of the amplitude involving a TDA

$$T^{\alpha\beta} p_{2\,\alpha} p_{1\,\beta} =$$

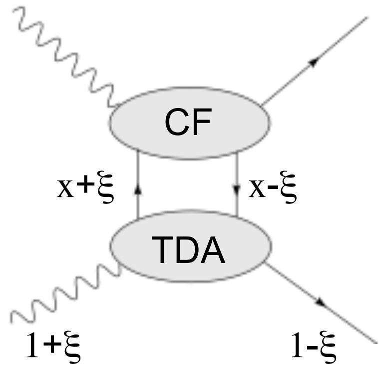
$$-if_\rho^2 e^2 (Q_u^2 + Q_d^2) g^2 \frac{C_F}{8N_c} \int\limits_{-1}^{1} dx \int\limits_{0}^{1} dz_1 \left[\frac{1}{\bar{z}_1(x-\xi)} + \frac{1}{z_1(x+\xi)} \right] T(x, \xi, t_{min})$$

with $T(x, \xi, t_{min}) \equiv$

$$N_c \left[\Theta(1 \geq x \geq \xi) \phi\left(\frac{x-\xi}{1-\xi}\right) - \Theta(-\xi \geq x \geq -1) \phi\left(\frac{1+x}{1-\xi}\right) \right]$$

Only the DGLAP part of the TDA contributes because we use the ρ-mesons DAs.

$$-\xi \geq x \geq -1$$
$$1 \geq x \geq \xi$$



We get the same kind of factorization for an opposite ordering of the photons virtualities.

Conclusions

We gave a precise estimation of the two gluon t-channel exchange in the exclusive reaction $\gamma^*\,\gamma^* \to \rho_L^0 \rho_L^0$, which dominated at HE and corresponds to the BFKL background; since the impact factor are completely known in a pertubative way, not only the behaviour with energy but the complete amplitude can be analytically computed. ⟶ Clean test of the BFKL resummation scheme at ILC.

We demonstated the measurability of this process at ILC and predicted the number of events.

Born order evaluation ⟶ NLO BFKL evolution fot any t.

At lower energy, quark exchange processes appear at lower order in α_s :

- The perturbative analysis of $\gamma^*\,\gamma^* \to \rho_L^0 \rho_L^0$ at the Born order leads to two different types of factorization.
- Not only kinematics but polarization states also dictate the type of the factorization.
- Usually they are applied for two different kinematics but the arbitrariness in choosing the photons virtualities shows that there may exist an intersection region.
- Generalization for transverse mesons, non-forward kinematics and charged mesons pair.