

Yang-Mills propagators close to the phase transition

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Introduction

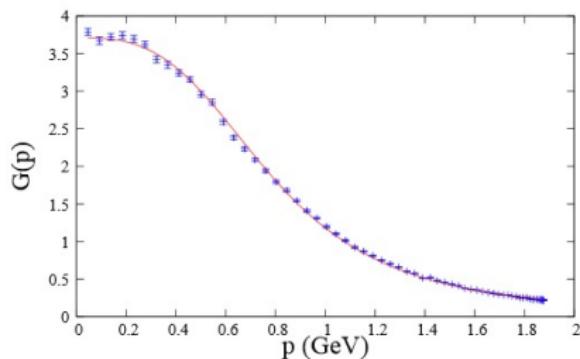
- We study Yang-Mills (YM) theories at finite temperature close to the phase transition.
- We use an analytic (perturbative) approach in order to access the Green functions.
- Our approach is motivated by previous studies at zero temperature, where the deep infrared regime was probed with simple perturbation theory.

Preliminary: vacuum study

- Gauge-fixed lattice simulations (Landau gauge) have shown that gluons are massive.
- We consider an effective massive theory to reproduce lattice results: Curci-Ferrari (CF) model. [G. Curci, R. Ferrari, Nuovo Cim. A 32, (1976)] [M. Tissier, N. Wschebor, Phys. Rev. D84 (2011)]

CF MODEL

- Infrared safe RG trajectories down to zero momentum.
- One-loop calculations are in very good agreement with lattice results.



- The effective mass naturally arises in a new quantization scheme which consistently deals with Gribov ambiguities in Landau gauge. [J. Serreau, M. Tissier, Phys. Lett. B712 (2012)]
- Most of the gluonic non-perturbative dynamics is accurately captured by the effective mass. [M. Peláez, M. Tissier, N. Wschebor, Phys. Rev. D 88 (2013)]
- Does a similar massive approach allow for a perturbative description of YM correlators at finite temperature around the phase transition?

Preliminary: massive thermal YM

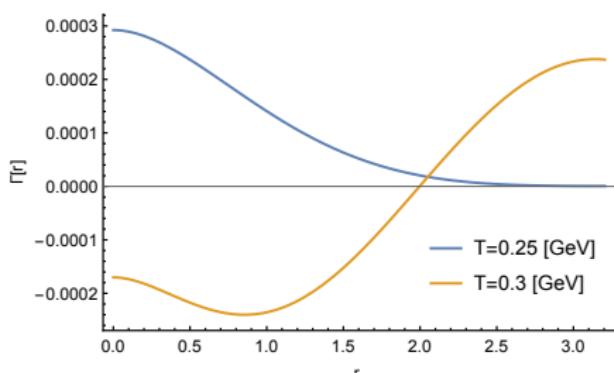
MASSIVE LANDAU GAUGE AT ONE-LOOP ORDER: $SU(2)$

[U. REINOSA, J. SERREAU, M. TISSIER, N. WSCHEBOR, PHYS. REV. D89 (2014)]

- Ghost and magnetic gluon in agreement with lattice simulations.
- Electric gluon close to T_c does not describe well the lattice results.

ORDER PARAMETER FOR THE PHASE TRANSITION: \bar{A}_μ , BACKGROUND FIELD

- Analogy with mean field theory of the Ising model.
- Perturbation theory around the background: $a_\mu = A_\mu - \bar{A}_\mu$.



Two-loop potential $\Gamma[\bar{A}]$ in $SU(2)$.

[U. Reinosa, J. Serreau , M. Tissier , N. Wschebor,
Phys. Rev. D91 (2015)]

- $\Gamma[\bar{A}]$ is minimum for $\bar{A} = \frac{rT}{g} \neq 0$.
- Landau ($\bar{A} = 0$) is a maximum!
- Phase transition is observed perturbatively.

The Landau-DeWitt gauge

BACKGROUND FIELD METHODS TO KEEP TRACK OF THE RELEVANT (CENTER) SYMMETRY

[J. BRAUN, H. GIES, J. M. PAWLICKI, PHYS. LETT. B 684 (2010)]

- Gauge condition $\underbrace{\partial_\mu a_\mu - ig[\bar{A}_\mu, a_\mu]}_{\bar{D}_\mu a_\mu} = 0.$
- $SU(2)$ gauge group with $\bar{A}_\mu = \bar{A}\delta_{\mu 0}\delta^{3a}t^a \rightarrow$ one color direction is singled out.
- \Rightarrow No more diagonal matrix form (the usual δ^{ab} structure) in the basis $\{t^a\}$.

COLOR CHARGE κ AND CANONICAL (WEYL) BASIS $\{t^\kappa\}$

- $\{t^a\} \rightarrow \{t^\kappa\} = \{t^0, t^+, t^-\}$, (like for spin).
- \bar{D}_μ diagonal in this basis: $\bar{D}_\mu \rightarrow \partial_\mu^\kappa = \partial_\mu + g\kappa\delta_{\mu 0}\bar{A}.$

In Fourier space: $\partial_\mu^\kappa \rightarrow i(K_\mu + g\kappa\bar{A}\delta_{\mu 0}) \Rightarrow$ shifts the Matsubara frequencies by $g\kappa\bar{A}$. Define generalized momentum:

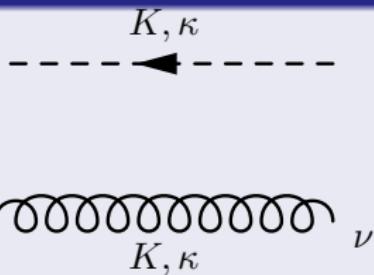
$$K_\mu^\kappa = K_\mu + g\kappa\bar{A}\delta_{\mu 0}$$

Massive Landau-DeWitt action

$$S_{\bar{A}} = \int_x \text{tr} \left\{ \frac{1}{2} F_{\mu\nu} F_{\mu\nu} + \mathbf{m^2} a_\mu a_\mu + 2\bar{D}_\mu \bar{c} D_\mu c + 2ih\bar{D}_\mu a_\mu \right\}.$$

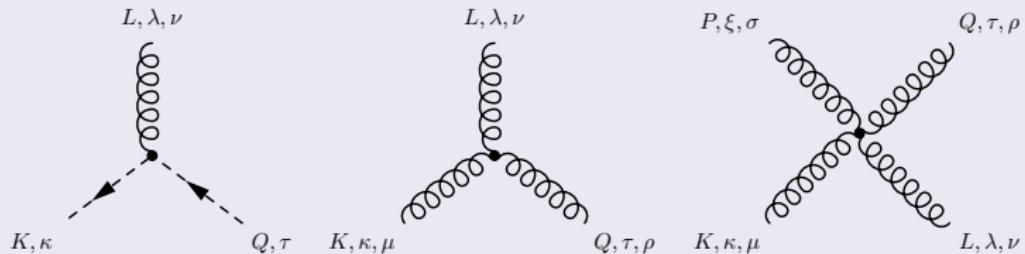
- Mass term introduced such as it does not break the background field gauge symmetry:
 $S_{\bar{A}} = S_{\bar{A}^U}$, \bar{A}^U gauge transformed of \bar{A} .
This insures that the center symmetry is not explicitly broken.

PROPAGATORS


$$G_{gh}^\kappa(K) = \frac{1}{(K^\kappa)^2}$$
$$G_{\mu\nu}^\kappa(K) = \frac{P_{\mu\nu}^\perp(K^\kappa)}{\mathbf{m^2} + (K^\kappa)^2}$$

Feynman rules

INTERACTION VERTICES



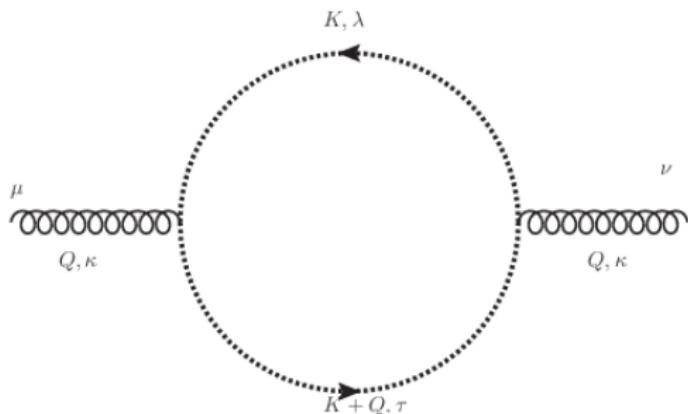
- E.g. ghost-antighost-gluon vertex: $gf^{\kappa\lambda\tau}K_\mu^\kappa$.

COLOR CHARGE CONSERVATION

- ⇒ Shift conservation at vertices: $\kappa + \tau + \lambda = 0$.
- ⇒ Conservation of generalized momenta $K^\kappa + Q^\tau + L^\lambda = 0$.

In the $SU(2)$ case, the structure constant is $\epsilon^{0+-} = 1$.

One-loop computations



$$\begin{aligned}\Pi_{\mu\nu}^{gh}(Q^\kappa) &= \sum_K \sum_{\lambda, \tau=\{0, +, -\}} f^{\tau(-\kappa)(-\lambda)} f^{\lambda\kappa(-\tau)} (K+Q)_\mu^\tau K_\nu^\lambda \frac{1}{((K+Q)^\tau)^2 (K^\lambda)^2} \\ &= \sum_K \sum_{\lambda=\{0, +, -\}} \frac{|f^{\lambda\kappa(-\kappa-\lambda)}|^2 (K^\lambda + Q^\kappa)_\mu K_\nu^\lambda}{(K^\lambda + Q^\kappa)^2 (K^\lambda)^2}.\end{aligned}$$

Background field dependence in loop contributions

One-loop expressions are expressed as simple sum-integrals.

- All Matsubara sums (and angular integrals) can be performed analytically.
- Tadpole-like:

$$\begin{aligned} J^{(m,\kappa)} &= \sum_K \frac{1}{\mathbf{m}^2 + (K^\kappa)^2} \\ &= \int_k \frac{n((\epsilon_{\mathbf{m},k} - i\kappa g \bar{\mathbf{A}})/T) + n((\epsilon_{\mathbf{m},k} + i\kappa g \bar{\mathbf{A}})/T) + 1}{2\epsilon_{\mathbf{m},k}}. \\ \epsilon_{\mathbf{m},k} &= \sqrt{\mathbf{m}^2 + k^2} & n(x) &= \frac{1}{e^x - 1} \end{aligned}$$

- Low temperature phase $g\bar{A}/T = \pi \Rightarrow n(x - i\pi) = -\frac{1}{e^x + 1}$ behaves like fermions!
- Many other integrals are involved, e.g. Sunset-like:

$$I_{\mu\nu}^{(\alpha,\kappa),(\beta,\lambda)}(Q) = \sum_K K_\mu^\kappa K_\nu^\kappa \frac{1}{\alpha^2 + (K^\kappa)^2} \frac{1}{\beta^2 + (K^\kappa + Q^\lambda)^2}.$$

- The background field acts as an imaginary chemical potential.

One-loop propagators

$$\Pi_{\mu\nu}^\kappa(K) = \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3}$$

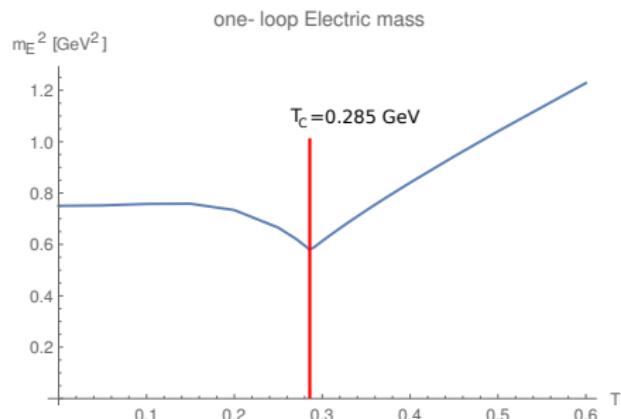
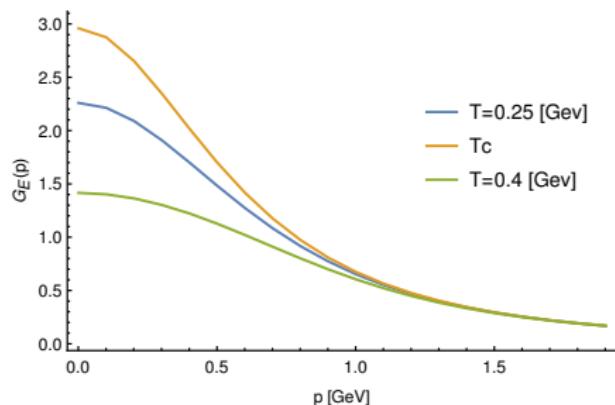
$$\Sigma^\kappa(Q) = \text{Diagram 4}$$

$$\begin{aligned} \Sigma^\kappa(Q) &= \sum_{\tau, \lambda} C_{\kappa\tau\lambda} \frac{(m^2 - (Q^\kappa)^2)}{4m^2} \left(J^{(m, \lambda)} - J^{(0, \lambda)} \right) \\ &\quad + \frac{((Q^\kappa)^2 + m^2)^2}{4m^2} I^{(m, \lambda)(0, \tau)} - \frac{(Q^\kappa)^4}{4m^2} I^{(0, \lambda)(0, \tau)} \\ &\quad + \frac{\omega_Q^\kappa}{2m^2} \left(\hat{J}_0^{(m, \lambda)} - \hat{J}_0^{(0, \lambda)} \right). \end{aligned}$$

- Smooth limit $\bar{A} \rightarrow 0, T \rightarrow 0$.
- Only vacuum divergences.

Evaluate these contributions at \bar{A} that minimizes $\Gamma[\bar{A}]$ computed at two-loop order.

Preliminary results



- Non-monotonic behavior around T_c (similar to lattice results in the Landau gauge).

- Cusp in the Electric mass at T_c due to the phase transition.

Conclusion and outlook

MASSIVE LANDAU-DEWITT GAUGE

- ① \bar{A} order parameter for the phase transition.
- ② Massive extension of the LDW gauge (mass related to issues of Gribov copies). [J. Serreau, M. Tissier, Phys. Lett. B712 (2012)], [J. Serreau, M. Tissier, A. Tresmontant Phys. Rev. D89 (2014)], [J. Serreau M. Tissier, A. Tresmontant, appears in Phys. Rev. D, arXiv:1505.07270]
- ③ Vacuum study showed renormalizability, asymptotic freedom,...

UP TO NOW

- ① Access perturbatively the phase transition (potential Γ up to two loops).
- ② Propagators at one loop. Many modes ($\kappa = 0, +, -$) \rightarrow rich phenomenology.

OUTLOOK

- ① Lattice simulations in this gauge. Might help for stability issues.
- ② $SU(3)$, spectral functions, renormalization group trajectories, Polyakov loop.

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