

Bound states of annihilating dark matter

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RPP, Annecy
January 2016

Motivation

- **Bound states arise in theories with long-range interactions**, interactions mediated by massless or light particles.
- **Long-range interactions appear in a variety of DM theories**
 - Self-interacting DM
 - Asymmetric DM
 - DM explanations of galactic positrons
 - DM explanations of IceCube PeV neutrinos
 - Little hierarchy problem, e.g. twin Higgs models
 - Sectors with stable particles in String Theory

Hidden sector DM

- **WIMP DM with $m_{\text{DM}} > \text{few TeV} !$** [Hisano et al. 2002]
 - Minimal DM [Cirelli et al.]
 - LHC implications for SUSY
 - Direct/Indirect detection bounds

- **Large logarithmic corrections:**

$$\delta\sigma/\sigma \sim \alpha \ln (m_{\text{DM}} / m_{\text{mediator}})$$

→ resummation techniques etc.

- **Non-perturbative effects:**

Sommerfeld enhancement in the non-relativistic regime.

Usually invoked for DM annihilation into radiation, but in fact affects *all* processes with same initial state.

- **More processes:**

Radiative formation of bound states [Sommerfeld enhanced]

- **Asymmetric DM → stable bound states**
 - Kinetic decoupling of DM from radiation, in the early universe
 - DM self-scattering in haloes (screening)
 - Indirect detection signals (radiative level transitions)
 - Direct detection signals (screening, inelastic scattering)
- **Symmetric DM → unstable bound states**
formation + decay = extra annihilation channel
 - Relic abundance [von Harling, KP (2014); Ellis et al. (2015)]
 - Indirect detection

Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

Processes

Toy model – Dark QED: **Dirac fermions** (X, \bar{X}) of mass m ,
coupled to a massless dark photon γ , with dark fine-structure constant α .

Very important parameter: $\zeta = \alpha / v_{\text{rel}}$

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Annihilation
 $X + \bar{X} \rightarrow \gamma + \gamma$

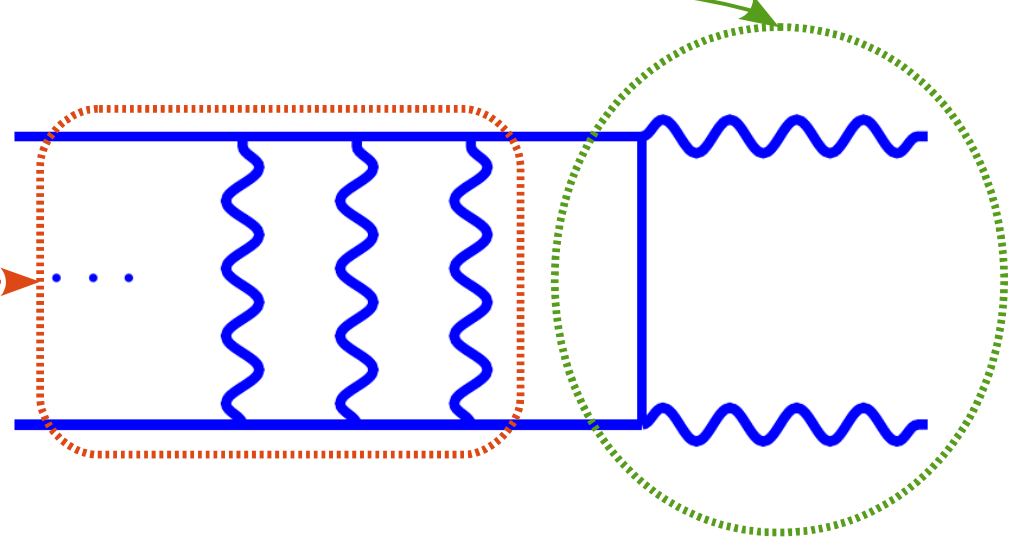
$$\sigma_{\text{ann}} v_{\text{rel}} = \sigma_0 S_{\text{ann}}(\zeta)$$

$$\sigma_0 = \pi \alpha^2 / m^2$$

$$S_{\text{ann}}(\zeta) = \frac{2\pi\zeta}{1 - e^{-2\pi\zeta}}$$

$$S_{\text{ann}}(\zeta \ll 1) \simeq 1$$

$$S_{\text{ann}}(\zeta \gtrsim 1) \simeq 2\pi\zeta$$



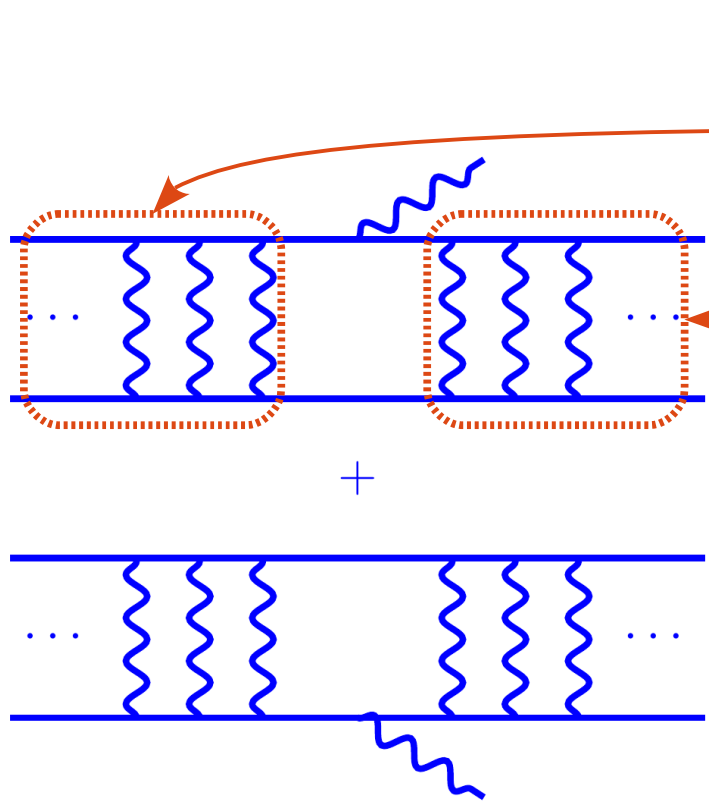
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[von Harling, KP (2014)]

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Bound state formation and decay

$X + \bar{X} \rightarrow (X \bar{X})_{\text{bound}} + \gamma$

$(X \bar{X})_{\text{bound}} \rightarrow 2\gamma \text{ or } 3\gamma$

$$\sigma_{BSF} v_{\text{rel}} = \sigma_0 S_{BSF}(\zeta)$$

$$\sigma_0 = \pi \alpha^2 / m^2$$

$$S_{BSF}(\zeta) = \left[\frac{2^9}{3 e^{4\zeta \operatorname{arccot}(\zeta)}} \frac{\zeta^4}{(1+\zeta^2)^2} \right] \frac{2\pi\zeta}{1-e^{-2\pi\zeta}}$$

$$S_{BSF}(\zeta \ll 1) \simeq \frac{2^9 \zeta^4}{3} \ll 1$$

$$S_{BSF}(\zeta \gtrsim 1) \simeq \frac{2^9}{3 e^4} \times 2\pi\zeta \simeq \mathbf{3.13} \times S_{\text{ann}}$$

Relic density of symmetric DM with long-range interactions

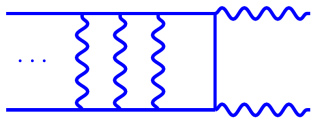
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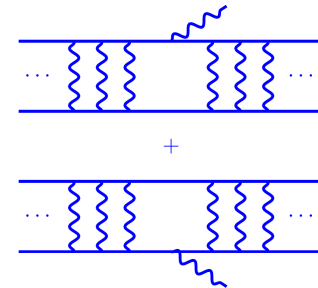
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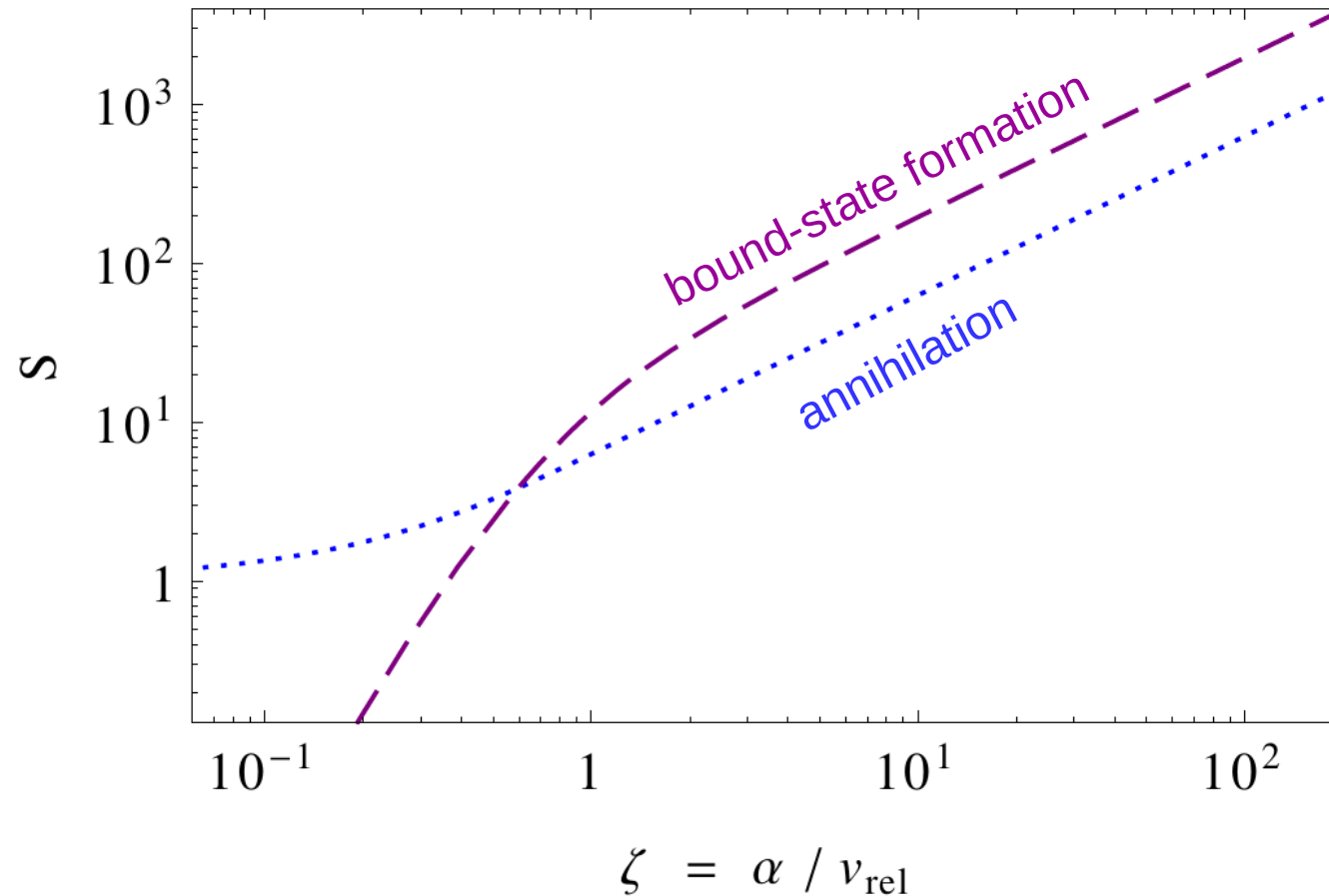
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Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

Rates



BSF dominates over annihilation everywhere the Sommerfeld effect is important ($\zeta > 1$) !

Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

Boltzmann equations

$$\frac{dn_X}{dt} + 3H n_X = -\left(n_X^2 - n_X^{eq\ 2}\right) \langle \sigma_{ann} \mathbf{v}_{rel} \rangle - n_X^2 \langle \sigma_{BSF} \mathbf{v}_{rel} \rangle + (n_{\uparrow\downarrow} + n_{\uparrow\uparrow}) \Gamma_{ion}$$

$$\frac{dn_{\uparrow\downarrow}}{dt} + 3H n_{\uparrow\downarrow} = + \frac{1}{4} n_X^2 \langle \sigma_{BSF} \mathbf{v}_{rel} \rangle - n_{\uparrow\downarrow} (\Gamma_{ion} + \Gamma_{decay, \uparrow\downarrow})$$

$$\frac{dn_{\uparrow\uparrow}}{dt} + 3H n_{\uparrow\uparrow} = + \frac{3}{4} n_X^2 \langle \sigma_{BSF} \mathbf{v}_{rel} \rangle - n_{\uparrow\uparrow} (\Gamma_{ion} + \Gamma_{decay, \uparrow\uparrow})$$

$$(X \bar{X})_{\uparrow\downarrow} \rightarrow 2\gamma:$$

$$\Gamma_{decay, \uparrow\downarrow} = \alpha^5 (m/2)$$

$$(X \bar{X})_{\uparrow\uparrow} \rightarrow 3\gamma:$$

$$\Gamma_{decay, \uparrow\uparrow} = \frac{4(\pi^2 - 9)}{9\pi} \alpha^6 (m/2)$$

$$(X \bar{X})_{\uparrow\downarrow \text{ or } \uparrow\uparrow} + \gamma \rightarrow X + \bar{X}:$$

$$\Gamma_{ion}(T) = \frac{2}{(2\pi)^3} 4\pi \int_0^\infty d\omega \frac{\omega^2}{e^{\omega/T} - 1} \sigma_{ion}(\omega)$$

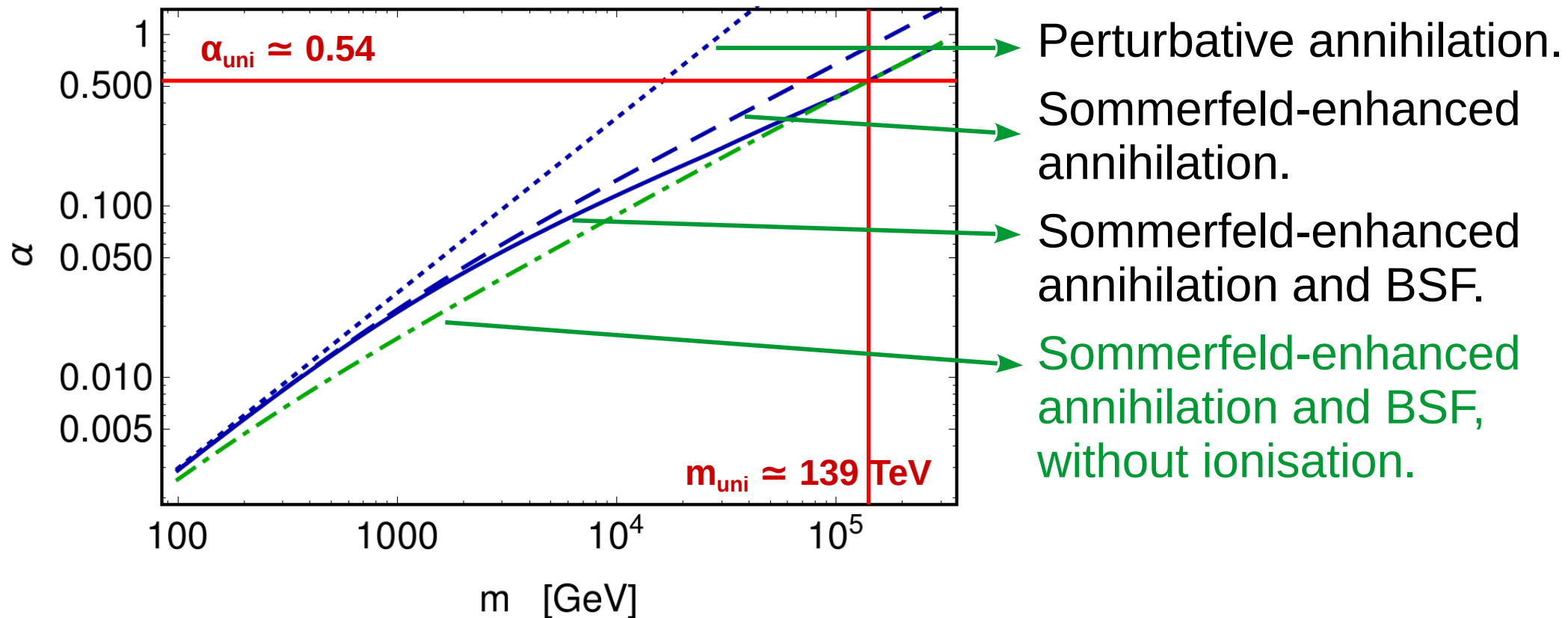
related to σ_{BSF}

BSF important when
 $\Gamma_{decay} > \Gamma_{ion}(T)$

Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

Determination of $\alpha(m)$ or $m(\alpha)$



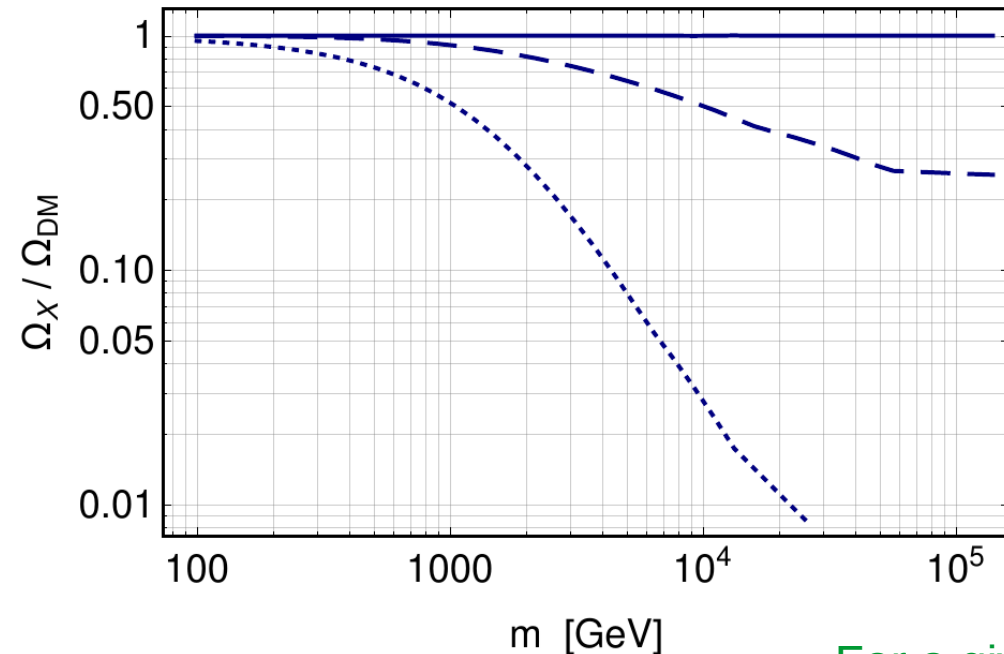
Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

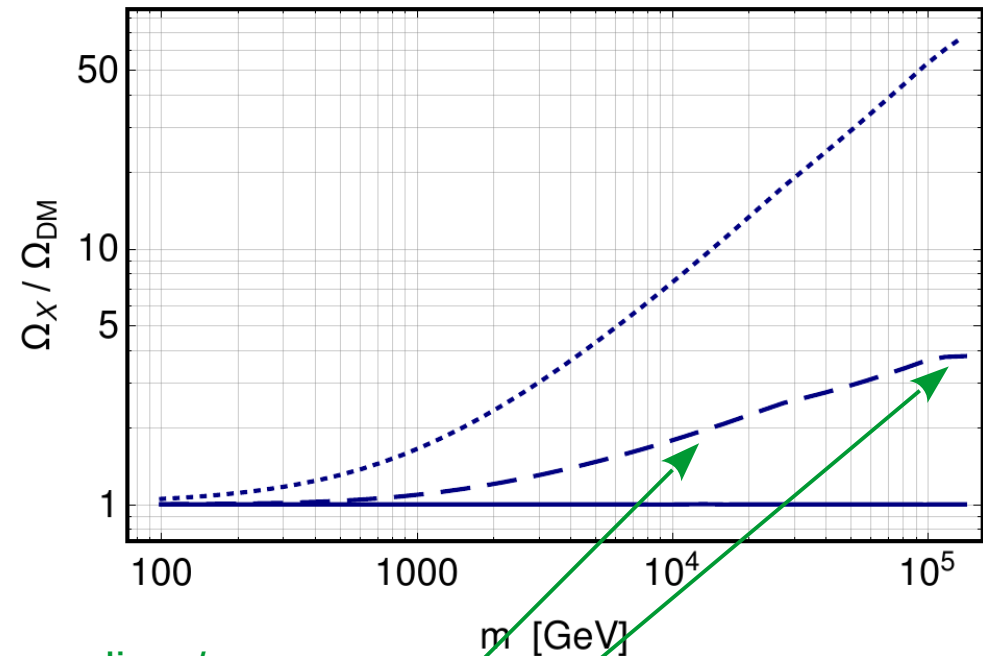
Effect on DM relic density

Much larger than experimental uncertainty of 1% !

Various determinations of α , plugged into full Boltzmann Eqs.



α determined from full Boltzmann Eqs, plugged into “partial” Boltzmann Eqs.



For a given coupling / mass, SE annihilation alone results in

$$\Omega_X / \Omega_{DM} \approx 2 \text{ @ } 15 \text{ TeV}$$

$$\Omega_X / \Omega_{DM} \approx 4 \text{ @ } 139 \text{ TeV}$$

Indirect detection of symmetric DM

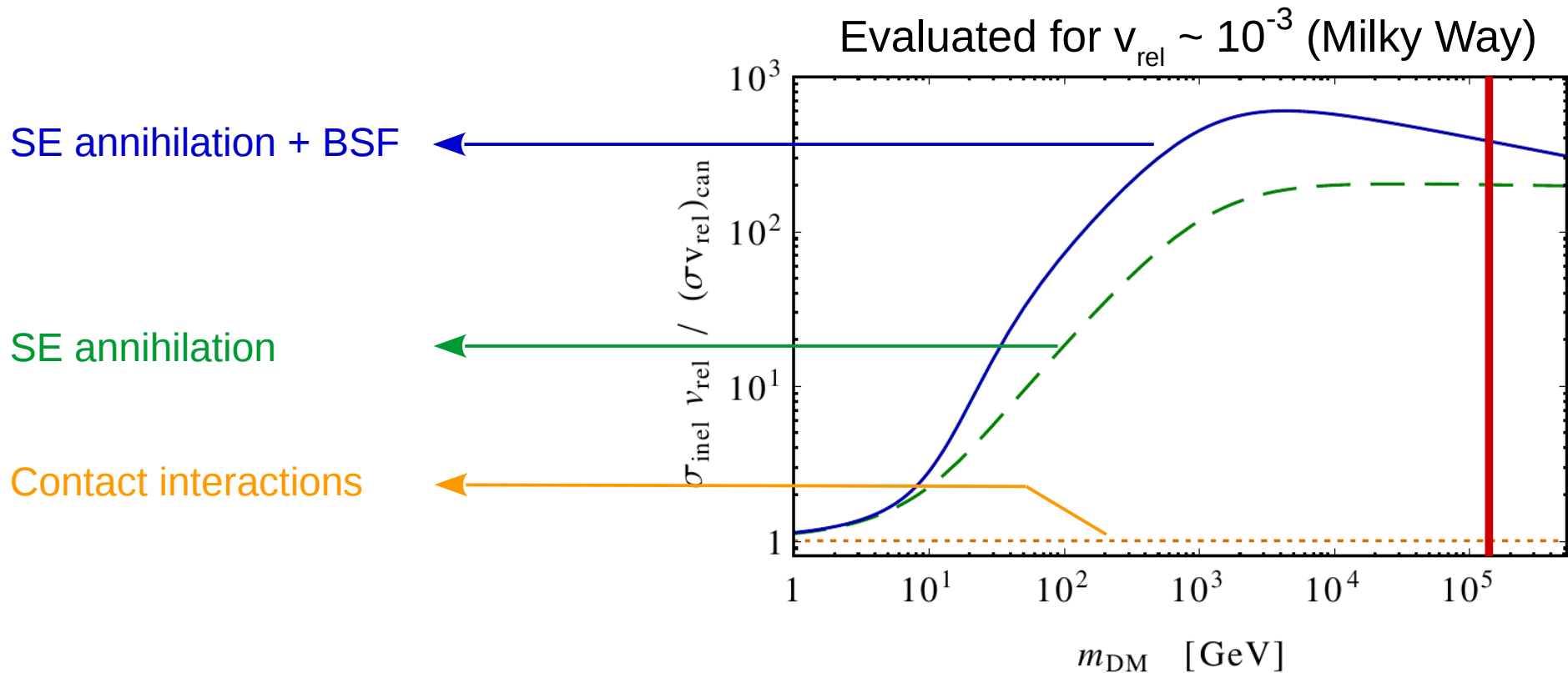
BSF implies:

- Enhanced signal rate, $\sigma_{\text{BSF}} > \sigma_{\text{ann}}$.
- Features in the spectrum:

$$X + \bar{X} \rightarrow (X\bar{X})_{\text{bound}} + \gamma \quad [E = \alpha^2 m_X / 4]$$

$$(X\bar{X})_{\text{bound}, \uparrow\downarrow} \rightarrow 2\gamma \quad [E = 2 \times m_X]$$

$$(X\bar{X})_{\text{bound}, \uparrow\uparrow} \rightarrow 3\gamma \quad [E : \text{extended spectrum}]$$



Why?

- Massive mediator – Yukawa potential.
- Different interactions, e.g. scalar mediator.
- Non-Abelian non-confining theories, e.g. electroweak interactions.

How?

QFT formalism (instead of QM), for bound-state-related processes, in weakly-coupled theories with long-range interactions.

Non-relativistic limit relevant for cosmo/astro DM applications.

- Can accommodate non-Abelian interactions, e.g. EW interactions.
- Allows systematic inclusion of higher-order corrections in the coupling strength and in the momentum transfer.

[KP, Postma, Wiechers (2015)]

Are bound-state effects relevant to WIMP dark matter?

TeV-scale WIMPs

- The Sommerfeld effect is important
⇒ BSF is likely too.
- In a Yukawa potential, bound states exist if $m_{\text{mediator}} \lesssim (m_X/2) \alpha$. Take:

$$m_X \rightarrow m_{\text{WIMP}}, m_{\text{mediator}} \rightarrow m_W, \alpha \rightarrow \alpha_2$$

WIMPonium exists if
 $m_{\text{WIMP}} \gtrsim 5 \text{ TeV}$

Sub-TeV WIMPs

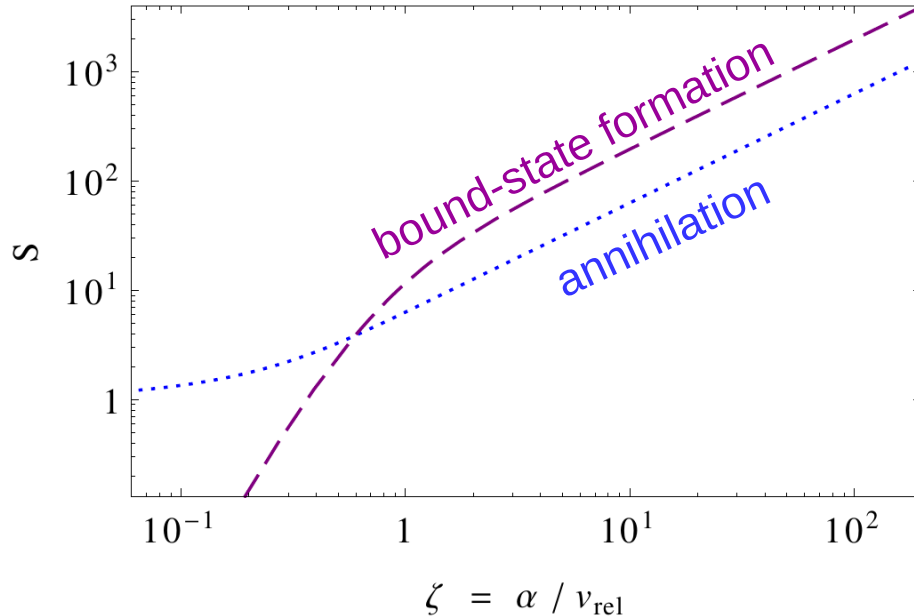
- Weak interactions manifest as contact type.
- In some MSSM scenarios, the NLSP decays after the LSP freeze-out
→ NLSP density itself important.
- Coloured NLSPs
→ strong coupling, massless meds
→ Sommerfeld effect important
→ BSF potentially important

Extra slides

Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

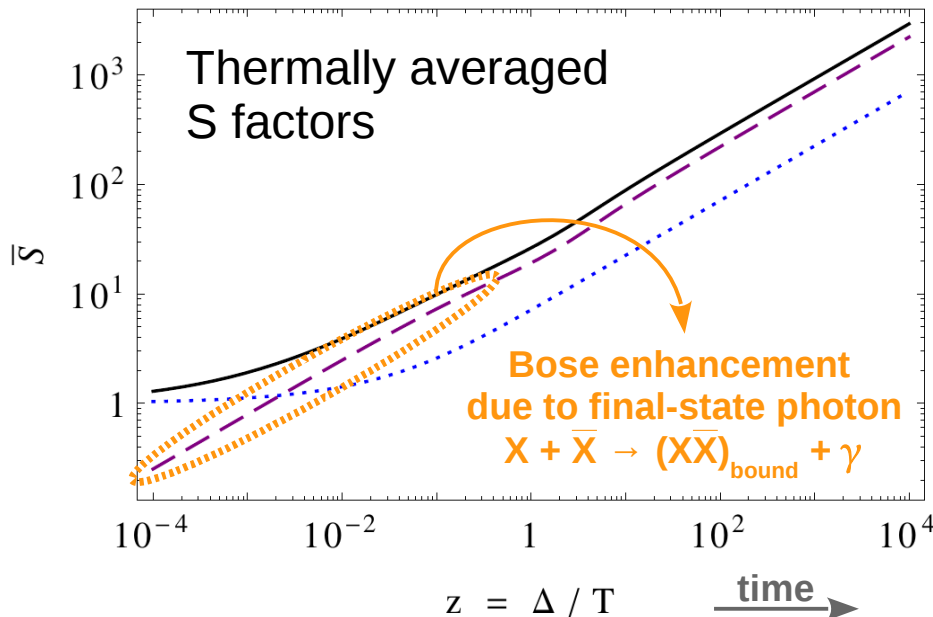
Rates



$$\zeta \equiv \frac{\text{Bohr momentum}}{\text{relative momentum}} = \frac{\mu \alpha}{\mu v_{\text{rel}}}$$

(reduced mass $\mu = m/2$)

$\sigma_{\text{BSF}} v_{\text{rel}} > \sigma_{\text{ann}} v_{\text{rel}}$
 everywhere the Sommerfeld effect
 is important ($\zeta > 1$).



Time parameter :

$$z \equiv \frac{\text{binding energy } [\Delta]}{T} \sim \frac{(1/2) \mu \alpha^2}{(1/6) \mu \langle v_{\text{rel}}^2 \rangle} \sim \langle \zeta^2 \rangle$$

$\langle \sigma_{\text{BSF}} v_{\text{rel}} \rangle > \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle$ even at $z \ll 1$,
 but

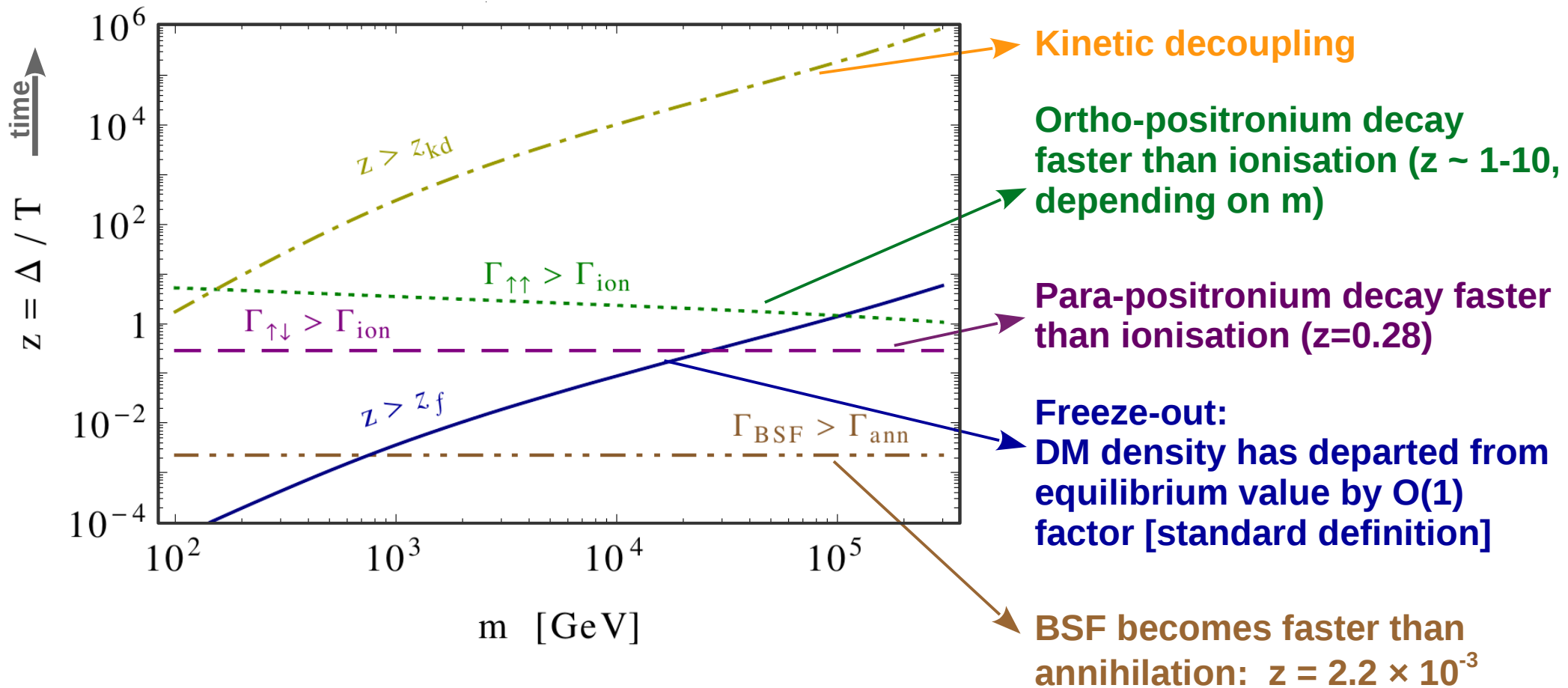
BSF can deplete DM only at $z \gtrsim 1$,
 when disassociation of bound states
 becomes unimportant.

Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

Timeline

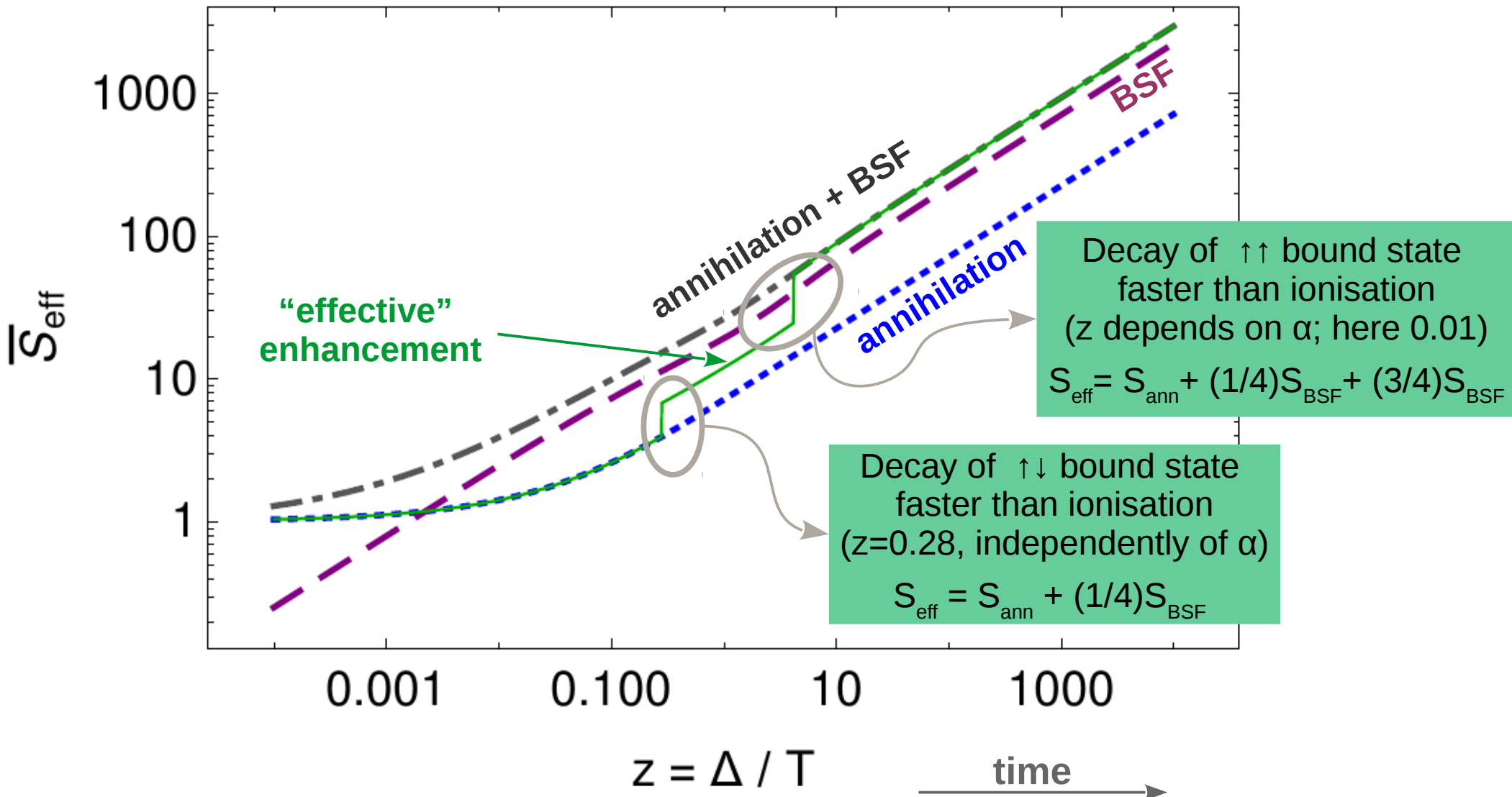
$\alpha = \alpha(m)$ fixed from relic abundance [see results]



Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

“Effective”
enhancement



Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

Partial-wave unitarity

Saturation of interaction probability at large couplings.

$$\sigma_{inel, J} v_{rel} \leq \frac{(2J+1) 4\pi}{m^2} v_{rel}$$

feature of long-range inelastic processes

- **Implies upper limit on mass of thermal relic DM.** [Griest, Kamionkowski (1990)]
- **Can be realised only if DM possesses long-range interactions.**
S-wave processes: $m < m_{\text{UNI}} = 83 \text{ TeV} \rightarrow 139 \text{ TeV}$ (non-self-conjugate DM)
[von Harling, KP (2014)]
- **All partial waves must have the same velocity dependence close to the unitarity limit.** Confirmed by explicit calculations for long-range interactions.
 - × For annihilation, higher $J \Rightarrow$ higher powers of α . [Cassel (2009)]
 - × For BSF, higher partial waves give significant contribution, e.g. BSF with vector emission: $\mathcal{M} \propto \sin \theta \Rightarrow J=0: 62\%, J=2: 24\% \dots$
 \Rightarrow Unitarity limit on m_{DM} even higher? [KP, Postma, Wiechers (2015)]

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feature of
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- **Unitarity realised perturbatively for $\alpha \sim 0.5$** , i.e. well below the perturbativity limit ($\alpha \sim \pi$ or 4π).

At large α ($\alpha \gg v_{rel}$):

$$number_J \times \frac{\pi \alpha^2}{m^2} \frac{\alpha}{v_{rel}} \leq \frac{(2J+1) 4\pi}{m^2 v_{rel}} \Rightarrow \alpha \lesssim 0.5$$