



The microscopic Twisted Mass Dirac spectrum and the spectral determination of the LECs of Wilson $\chi\text{-}\mathsf{PT}$

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Work in progress with Krzysztof Cichy (Universität Frankfurt) and Kim Splittorff (NBI) .

Incorporating cutoff effects (UV) in the low-energy EFT describing the IR behavior?

Cutoff effects (UV) break chiral symmetry which determines the $\ensuremath{\mathsf{IR}}$ behavior

• combined extrapolation in $a \to 0$ and $m_{\pi} \to m_{\pi}^{phys}$

- accounts for non-analyticities in a absent in a polynomial continuum extrapolation e.g. a² in chiral logs observed by MILC
- relations between cutoff effects in different quantities
- non-perturbartive info regarding the phase structure of the theory

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Wilson Chiral Perturbation Theory for Twisted Mass fermions

- Wilson term breaks χ symmetry explicitly
- Lattice spacing effects lead to new terms in χPT

Sharpe and Singleton (1998), Rupak and Shoresh (2002), Bär, Rupak and Shoresh (2004)

- ϵ regime where in the thermodynamic, chiral and continuum limit $mV\Sigma$, $z_tV\Sigma$ and a^2VW_i kept fixed.
- At order a^2 it involves three Low Energy Constants (LECs)

$$Z_{N_f}(m,z;a) = \int_{\mathcal{M}} dU \ e^{-S[U]}$$

where the action is

$$S = -\frac{m}{2} \Sigma V \operatorname{tr} (U + U^{\dagger}) - \frac{z_t}{2} \Sigma V \operatorname{tr} (\tau_3 U - \tau_3 U^{\dagger}) + a^2 V W_6 [\operatorname{tr} (U + U^{\dagger})]^2 + a^2 V W_7 [\operatorname{tr} (U - U^{\dagger})]^2 + a^2 V W_8 \operatorname{tr} (U^2 + U^{\dagger 2})$$

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Quoting Heiri Leutwyler

"Please do not be content with reaching physical quark masses. Extract the dependence on them, determine the LECs!" Heiri Leutwyler at Chiral Dynamics 2015, Pisa

pion mass splittings

Herdoiza et al (2013)

lattice determination of the pion scattering lengths

Aoki et al (2008) and Bernardoni et al (2011)

mixed action setup

Cichy et al (2012)

 \blacksquare matching analytical predictions for the spectrum of D to lattice data for fixed ν in a finite volume

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They have square watermelons in Japan - they stack better.



Lattice setup

• $N_{\rm f} = 2 + 1 + 1$ ETMC configurations

- Iwasaki gauge action
- 1 ensemble with $\beta = 1.95$, a = 0.078 fm $a\mu = 0.0055$ with $m_{\pi} \approx 390$ MeV with $32^3 \times 64$ volume with a physical extent of 2.5 fm $m_{\pi}L \approx 5$
- very large statistics \sim 5000 confs out of them \sim 200 have $\nu=$ 0, \sim 400 have $|\nu|=1,2$ and \sim 300 have $|\nu|=3$
- field theoretical computation of the top. charge using the Gradient Flow Lüscher (2008)

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Analytical derivation of $ho_5^ u(\lambda^5, \overline{z_t}; a)$

$$Z_{3|1}^{\nu}(\mathcal{Z};a) = \int dU_{Gl(3|1)} \operatorname{Sdet}(iU)^{\nu} e^{\frac{i}{2}\operatorname{Str}(\mathcal{Z}[U+U^{-1}]) + a^{2}\operatorname{Str}(U^{2}+U^{-2})}$$

with
$$\mathcal{Z} \equiv \operatorname{diag}(iz_t, -iz_t, z, \tilde{z})$$

Splittorff and Verbaarschot (2012)

$$G_{3|1}^{\nu}(z, z_t; a) = \lim_{\tilde{z} \to z} \frac{d}{dz} \mathcal{Z}_{3|1}^{\nu}(iz_t, -iz_t, z, \tilde{z}; a)$$
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Comparison RMT vs lattice for $\nu = 0$



dynamical lattice simulation with $\hat{z}_t = 38.5 \ \hat{a} = 0.7$ vs analytical result by Splittorff and Verbaarschot (2012) Savas Zafeiropoulos TM Dirac Spectrum and RMT

Comparison RMT vs lattice for $\nu = 2$



dynamical lattice simulation with $\hat{z}_t = 29.82$ $\hat{a} = 1.12$ vs analytical result by Splittorff and Verbaarschot (2012)

u	0	1	2	combined
$\Sigma^{1/3}$ [MeV]	289.0(2.7)	272.3(4.1)	270.8(6.8)	271.1(7.3)
$W_8 \ [r_0^6 W_0^2]$	0.0021(12)	0.0055(19)	0.0064(12)	0.0064(12)

Comparison with other (ETMC) results continuum value $\Sigma^{1/3} = 290 \pm 11$ MeV

computed by the method of spectral projectors Cichy et al (2012) our results are very close to this continuum value (hint for small cutoff effects)

 $W_8[r_0^6W_0^2] = 0.0064(2)(24)$ from the mixed action setup Cichy et al (2012) $W_8[r_0^6W_0^2] = 0.0138(22)$ from the pion mass splittings Herdoiza et al (2012)

- diagonalization of 1300 confs IDRIS-CNRS BGQ in Paris, cost of 5Mh
- larger volumes? smaller pion mass? smaller lattice spacing?
- \blacksquare there is another long ETMC ensemble with $m_\pi \sim 200 {\rm MeV}$ and $V = 48^3 \times 96$
- \blacksquare compared to the used one $m_\pi \sim 390 {
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Stay Tuned!



for upcoming results ...

Thank you for your attention!

Many thanks to Andreas Athenodorou and Jac Verbaarschot

Extraction through the pion mass splittings

Münster (2004), Scorzato (2004), Sharpe et al (2004), Herdoiza et al (2013)

$$M_{\pi^{\pm}}^{2} = 2B_{0}\mu_{\ell}$$

$$M_{\pi^{0}}^{2} = 2B_{0}\mu_{\ell} - 8a^{2} (2w_{6}' + w_{8}')$$

$$M_{\pi^{(0,c)}}^{2} = 2B_{0}\mu_{\ell} - 8a^{2} w_{8}'$$

 w_k^\prime defined through the Wilson LEC W_k^\prime by

$$w'_k = \frac{16W_0^2 W'_k}{f^2} \qquad (k = 6, 8)$$

Let's consider the following mass-splittings

$$M_{\pi^{\pm}}^2 - M_{\pi^{(0,c)}}^2 = 8a^2 w_8'$$
$$\frac{1}{2} \left(M_{\pi^{(0,c)}}^2 - M_{\pi^0}^2 \right) = 8a^2 w_6'$$

$$c_2 = -\frac{32W_0^2}{f^2}(2W_6' + W_8')$$

Pion scattering lengths in WchPT

Aoki et al (2008) and Bernardoni et al (2011)

two-pion scattering process

$$\pi^{\alpha}(p) + \pi^{\beta}(k) \longrightarrow \pi^{\gamma}(p') + \pi^{\delta}(k') .$$
$$A(s, t, u) = \frac{1}{f^2} (s - M_0^2 - 2c_2 a^2) .$$
$$a_0^0 = \frac{7}{32\pi f^2} \left(M_0^2 - \frac{5}{7} 2c_2 a^2 \right)$$
$$a_0^2 = \frac{1}{16\pi f^2} \left(M_0^2 + 2c_2 a^2 \right)$$

Furchner (2010), Cichy et al (2012)

$$M_{SS,\pm}^2 = 2B_0\mu$$

$$M_{SS,0,conn}^2 = 2B_0\mu - \hat{a}^2 \frac{32}{f^2} W_8'$$

$$M_{VV}^2 = 2B_0m_{ov},$$

$$M_{VS}^2 = B_0(m_{ov} + \mu) + \hat{a}^2 \frac{4}{f^2} W_M - \hat{a}^2 \frac{8}{f^2} W_8'$$