Fluctuations in high-energy scattering

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Outline

- ***Picture of a hadron at high energy***

  Heuristic discussion of quantum fluctuations

  Evolution of hadronic states towards higher energies: the color dipole model

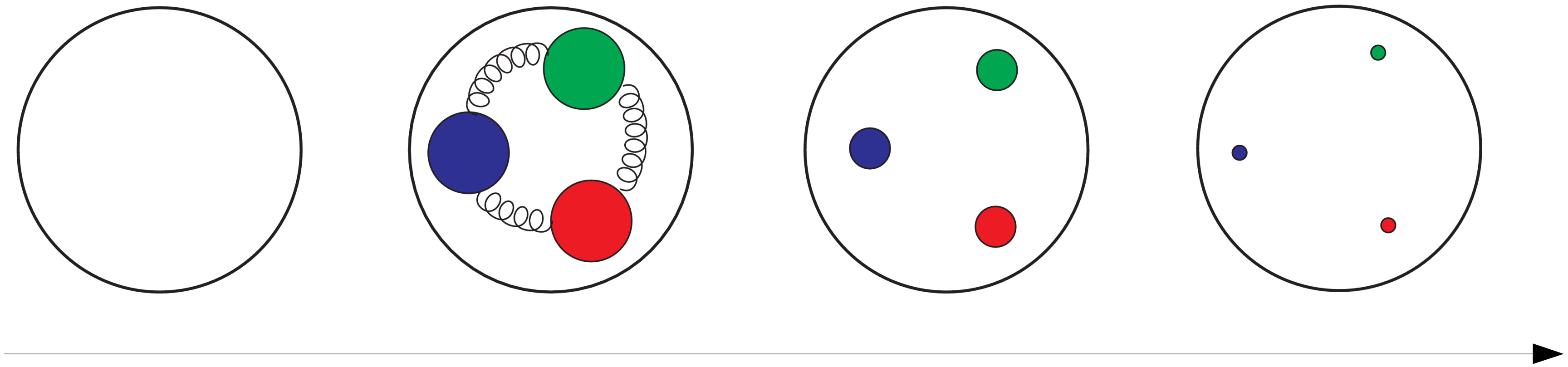
- ***Dipole-nucleus scattering***

  Total cross section and the statistics of extremes

  Multiplicity fluctuations and probability distribution of the (integrated) gluon density

How a (slow) hadron looks at different resolutions

(Over)simplified picture



[Larger momentum
= shorter distance
= higher “resolution”]

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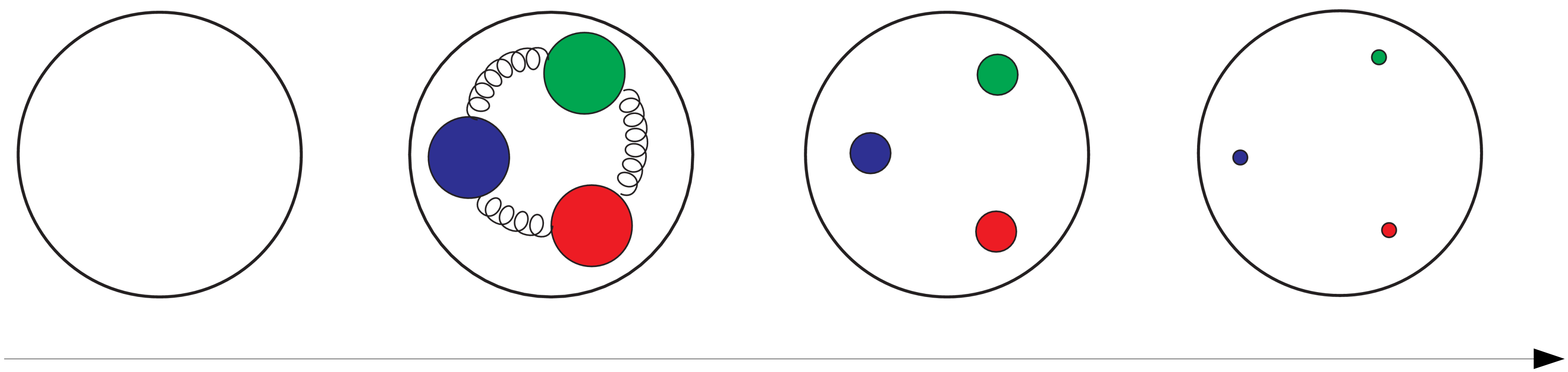

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Large distances (several fm)

Colorless extended object
(size ~ 1 fm)

A fraction of a fm

"Constituent"
quarks

$$\alpha_s = O(1)$$

Strongly coupled quantum field theory
No analytical methods...

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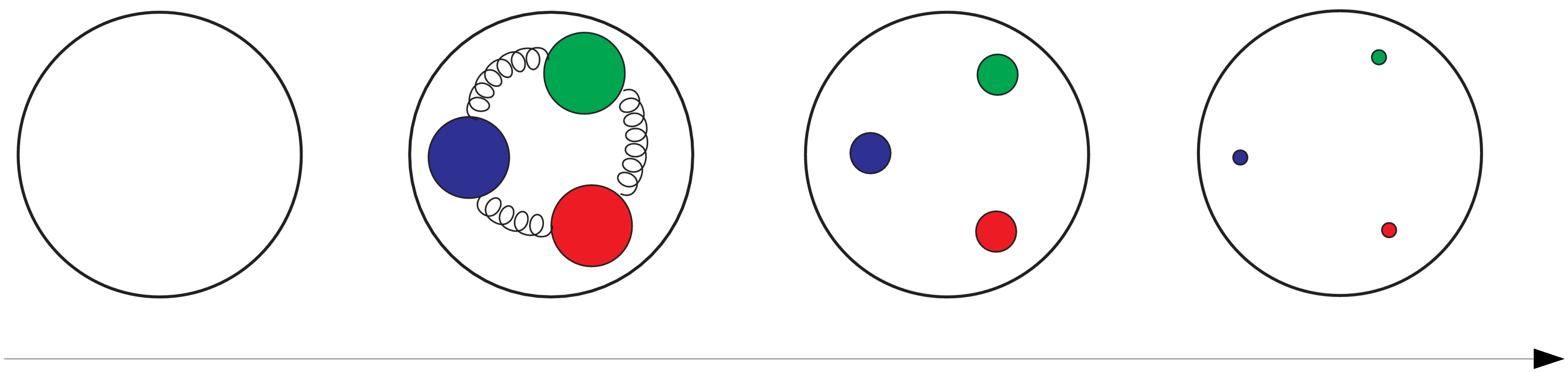

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$$\alpha_s \ll 1$$

The interactions are small corrections
Perturbation theory applies!

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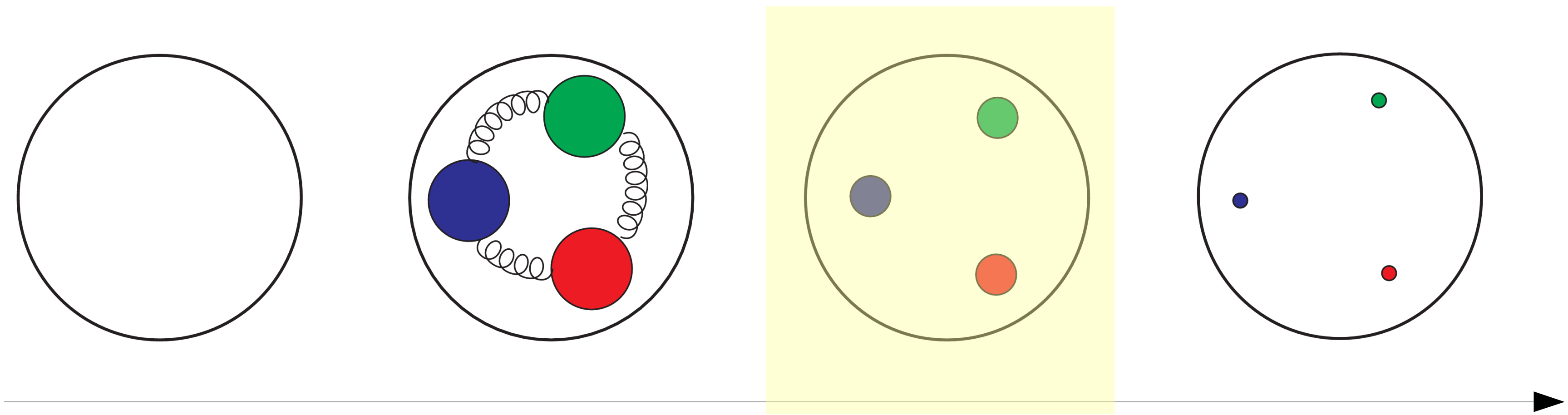

<u>This talk</u>: momenta of a few GeV

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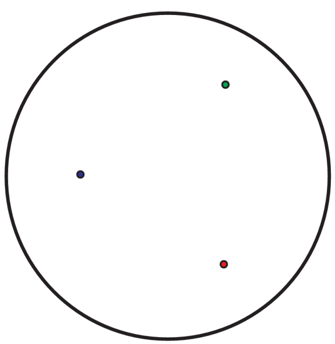
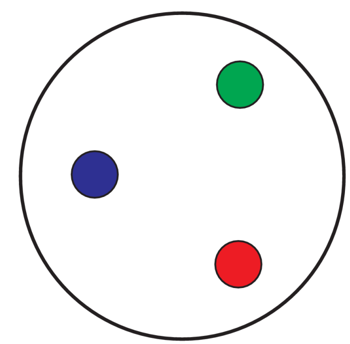
Quantum dynamics

Proton in its ground state
= 3 pointlike quarks

Quantum dynamics



$$U(-\infty, t) = T e^{i\int_{-\infty}^{t} d\tau L_I(\tau)}$$



Perturbative expansion: $= 1 + i\int_{-\infty}^{t} d\tau L_I(\tau) + i^2 \int_{-\infty}^{t} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 L_I(\tau_1) L_I(\tau_2) + \cdots$

t

t=$-\infty$

Time at which the state is observed

Proton in its ground state
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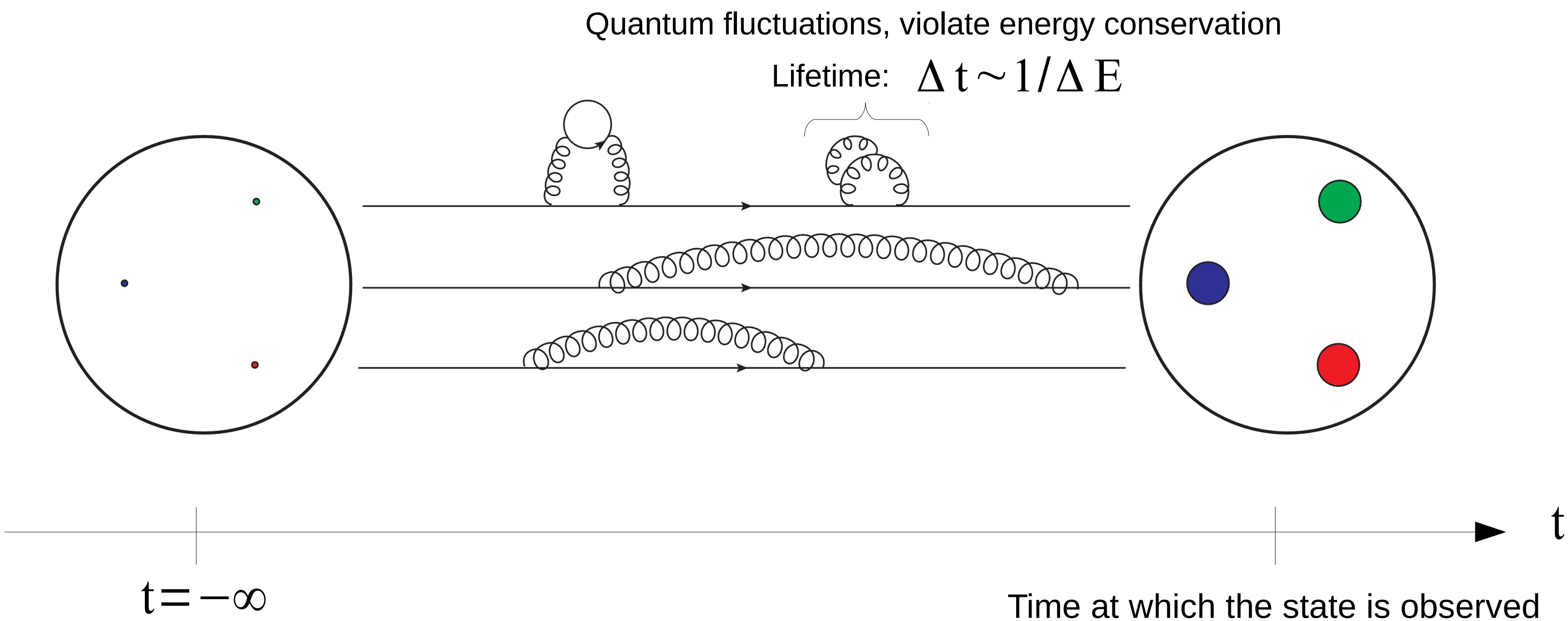
Quantum dynamics



Proton in its ground state
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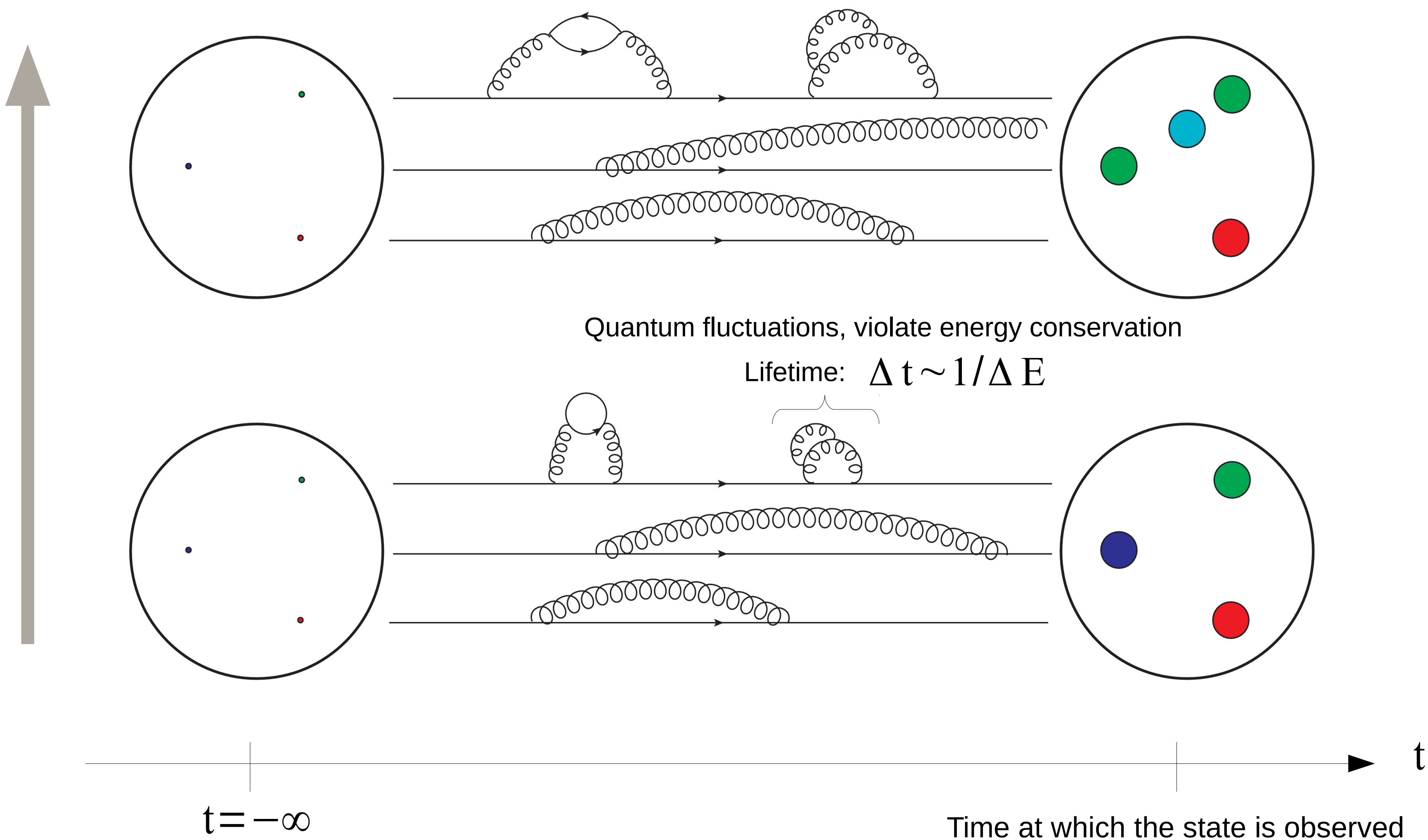
Quantum dynamics

Faster hadron in the frame
of the observer:

Fluctuations are longer-lived due to Lorentz time dilation!



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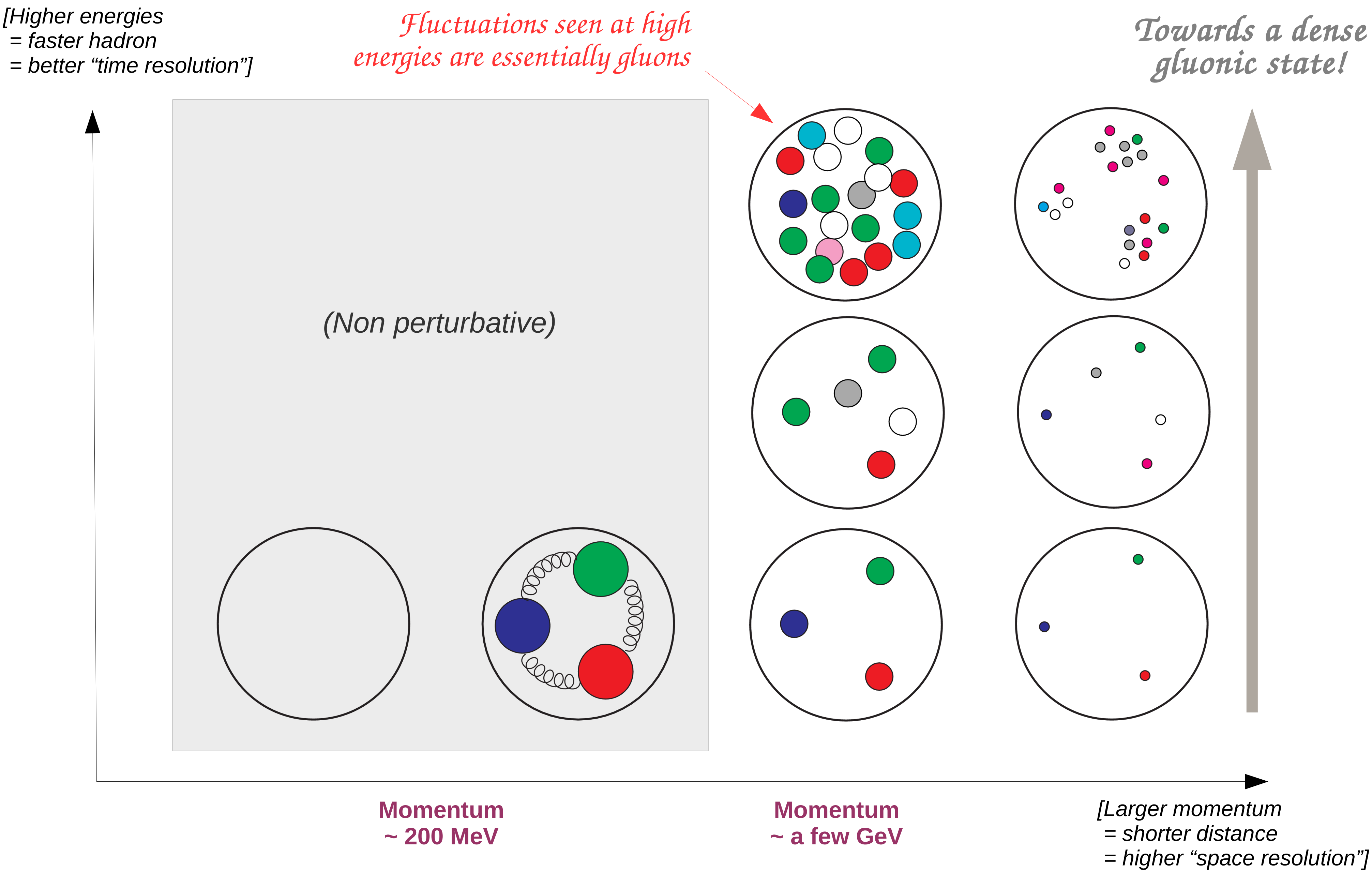
Picture of a hadron



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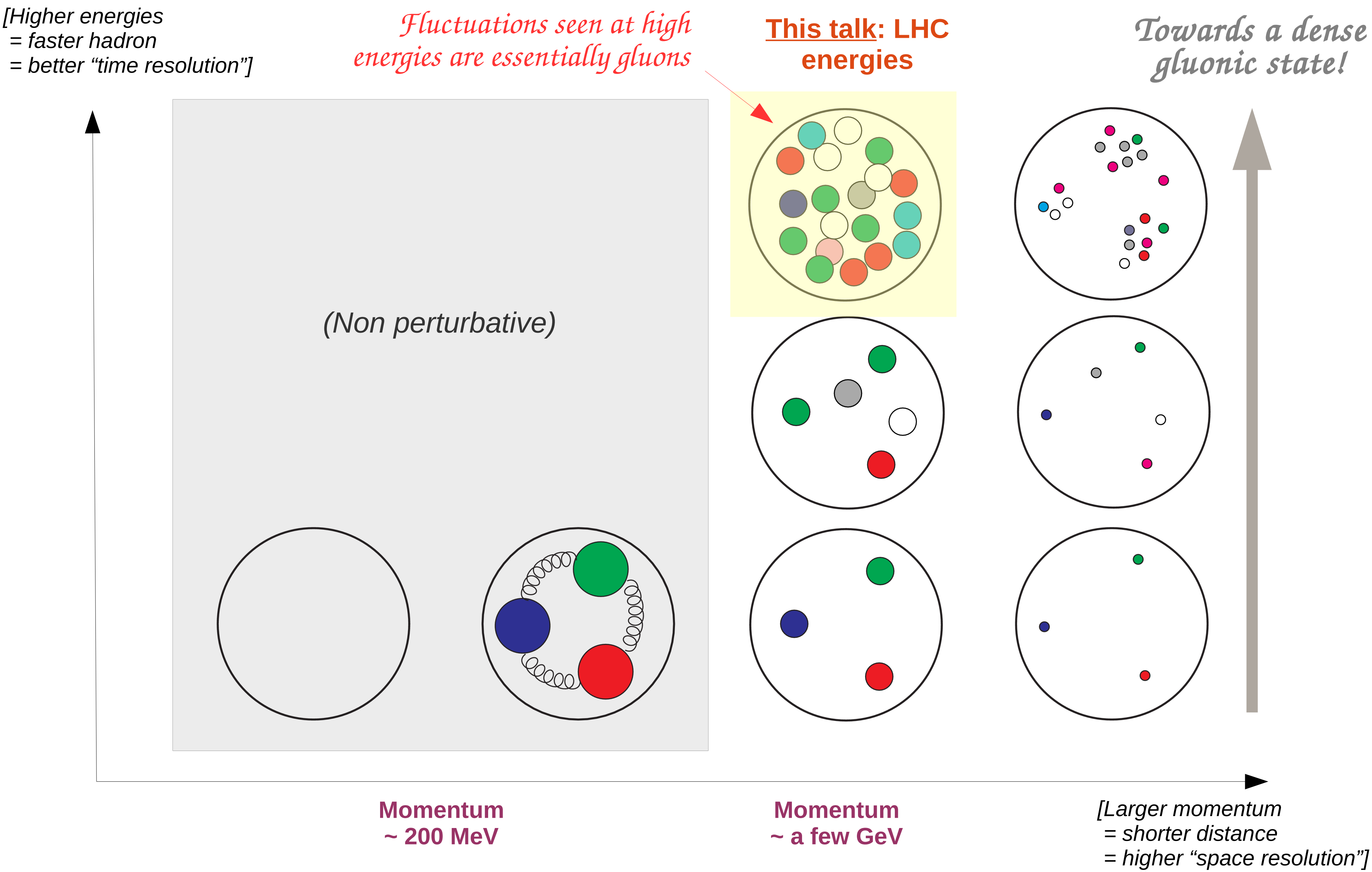
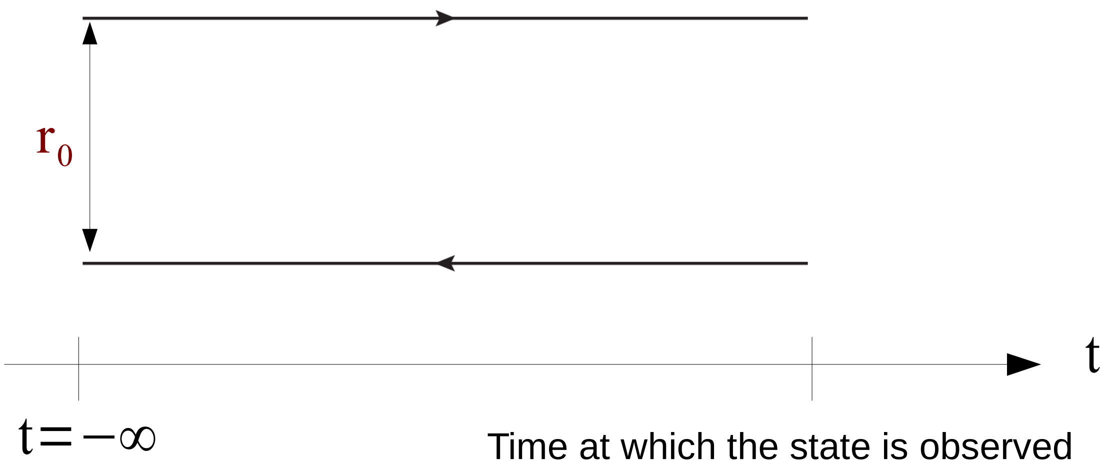


QCD calculation: the color dipole model

*To simplify, we start with a **color neutral quark-antiquark pair** (=meson) of given transverse size.*



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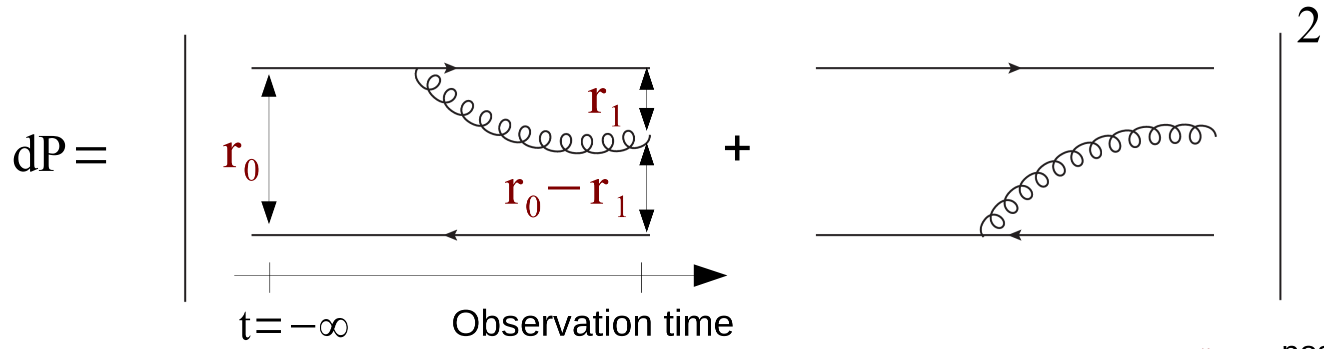
Probability of observing 1 gluon fluctuation when one increases the rapidity ***y*** *by* ***dy****:*



$r_1 =$ position of the gluon with respect to the quark

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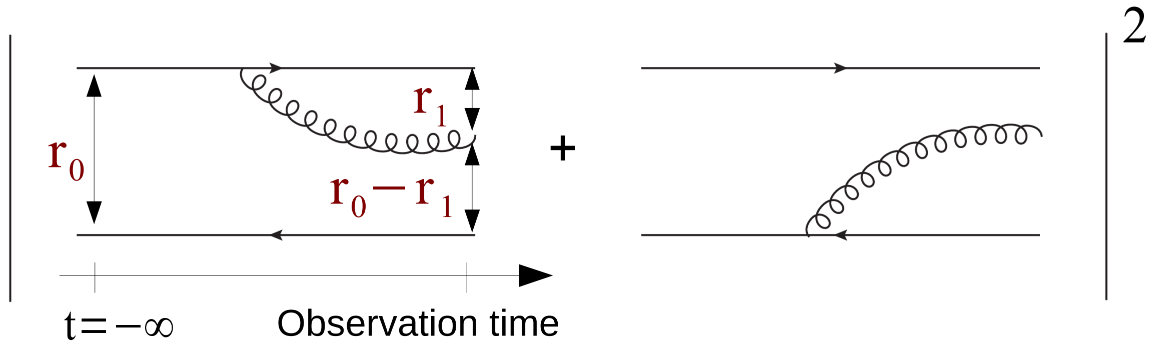

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$$= \quad dy \, \frac{\alpha_s (N_c^2 - 1)}{\pi N_c} \frac{r_0^2}{r_1^2 (r_0 - r_1)^2} \frac{d^2 r_1}{2\pi}$$

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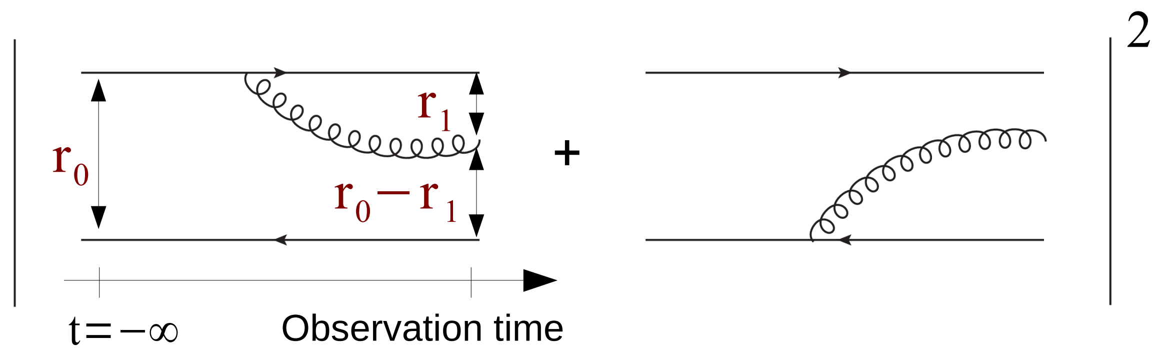

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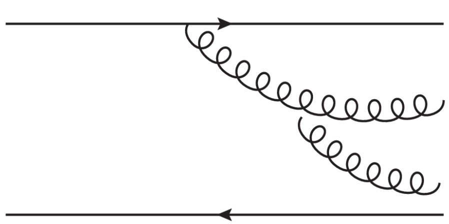
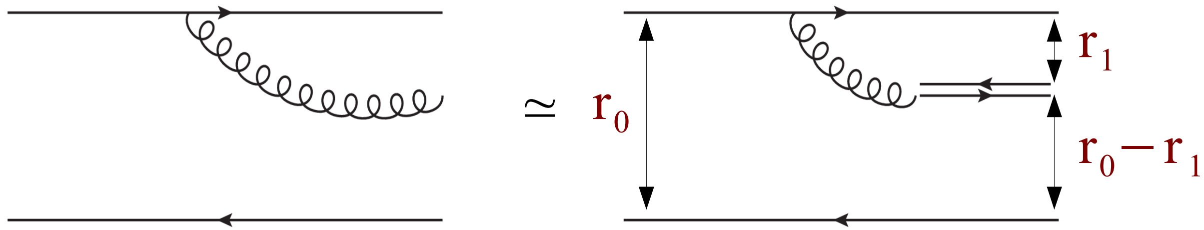
When $y \sim \dfrac{1}{\alpha_s (N_c^2 - 1)/(\pi N_c)}$ *, the probability to observe a fluctuation with* $|r_1| \sim |r_0|$ *is O(1)*

One needs to consider higher-order fluctuations:



QCD calculation: the color dipole model

Trick: ***large number-of-color limit!***



Gluon emission is interpreted as a color-dipole splitting, with proba.

$$dP = dy\,\frac{\alpha_s N_c}{\pi}\,\frac{r_0^2}{r_1^2 (r_0 - r_1)^2}\,\frac{d^2 r_1}{2\pi}$$

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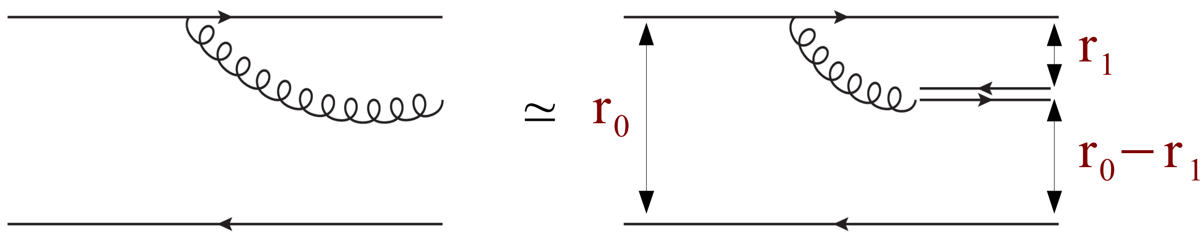

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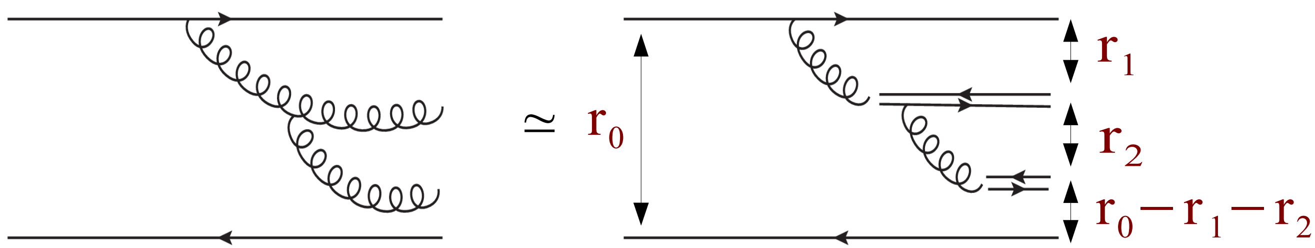
Higher-order fluctuations are generated by a branching process:

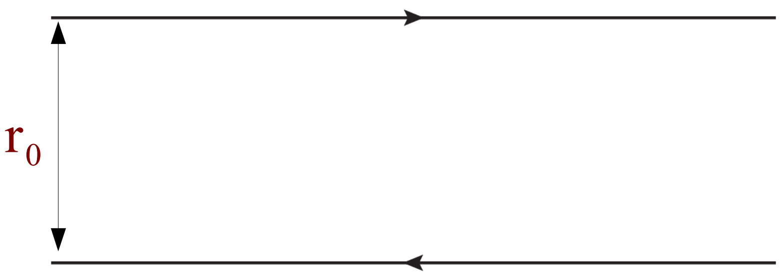


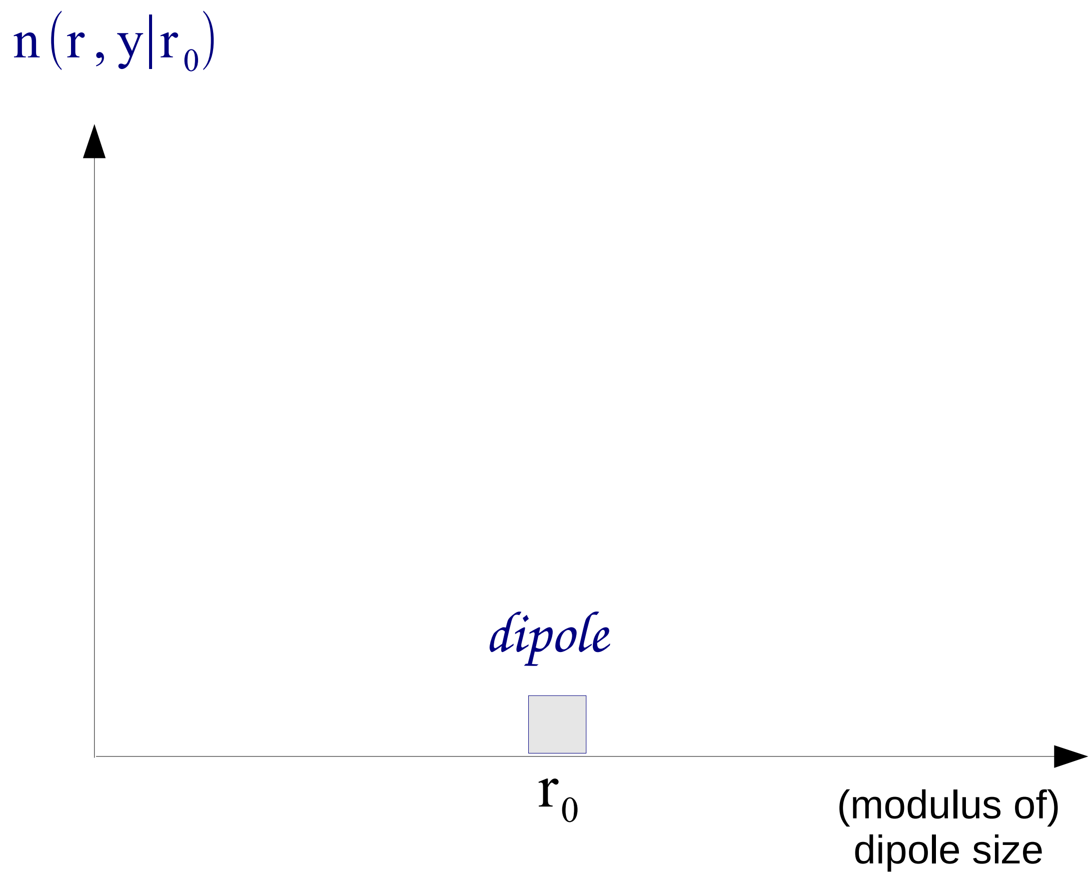
Two successive dipole branchings

Gluon density at momentum scale k ~ density of dipoles of size 1/k

How the dipole model works

$$dP = dy\,\frac{\alpha_s N_c}{\pi}\,\frac{r_0^2}{r_1^2 (r_0 - r_1)^2}\,\frac{d^2 r_1}{2\pi}$$



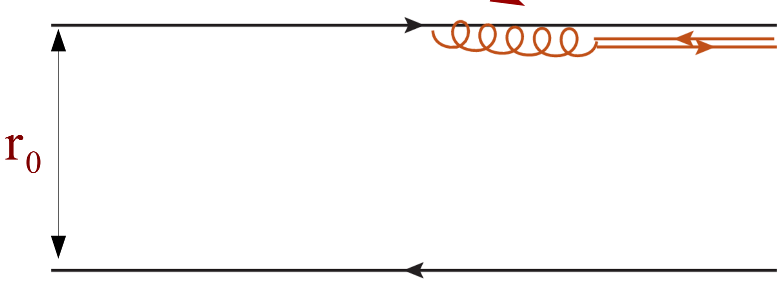


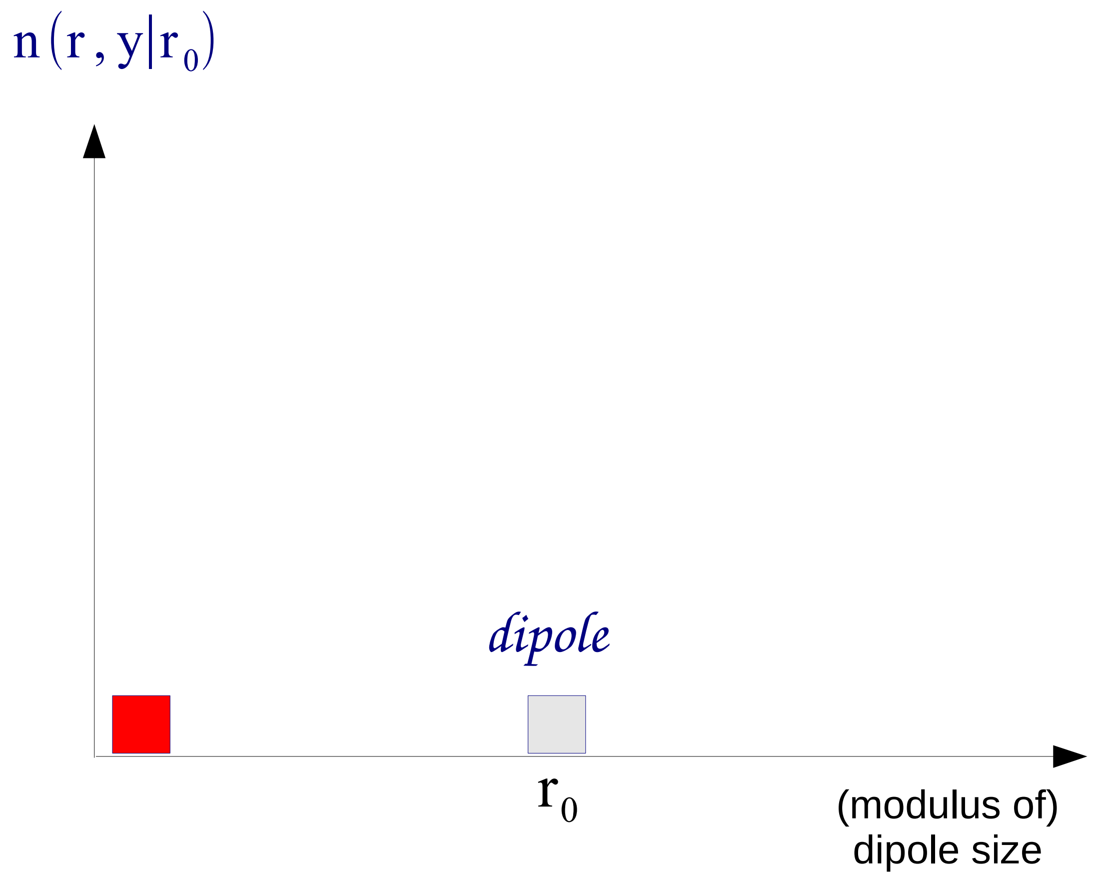
$$y = 0$$

How the dipole model works

$$dP = dy\,\frac{\alpha_s N_c}{\pi}\,\frac{r_0^2}{r_1^2 (r_0 - r_1)^2}\,\frac{d^2 r_1}{2\pi}$$

Singular when the gluon is close
to the quark or to the antiquark
Collinear singularity





$y \geq 0$

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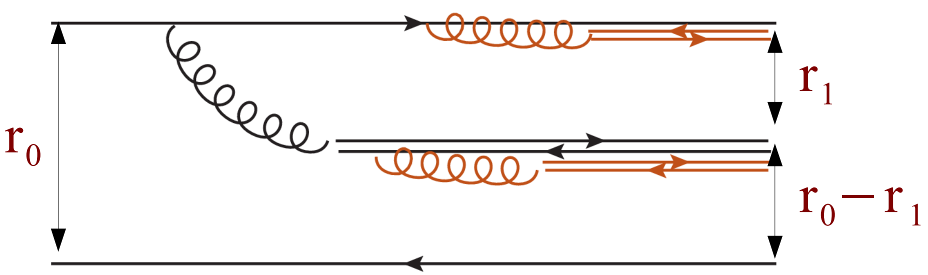

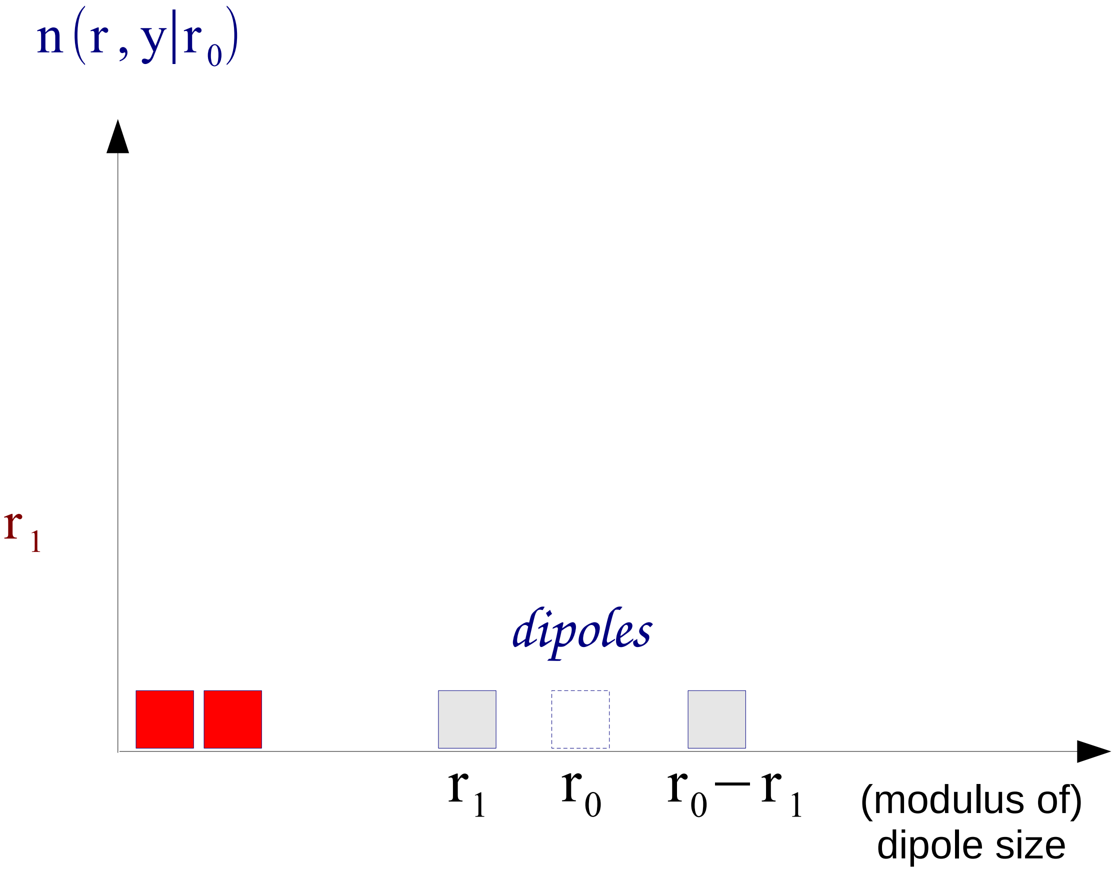

$$y \sim \frac{1}{\alpha_s N_c / \pi}$$

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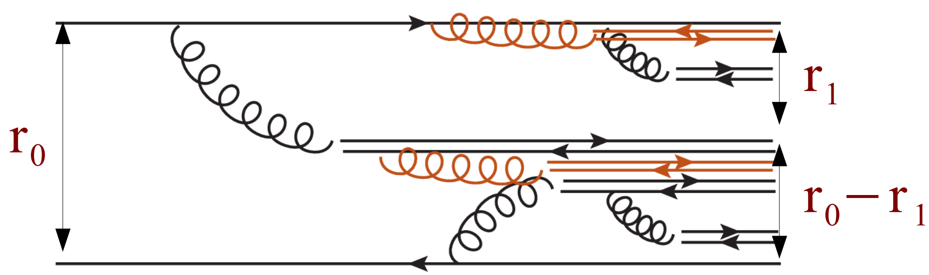

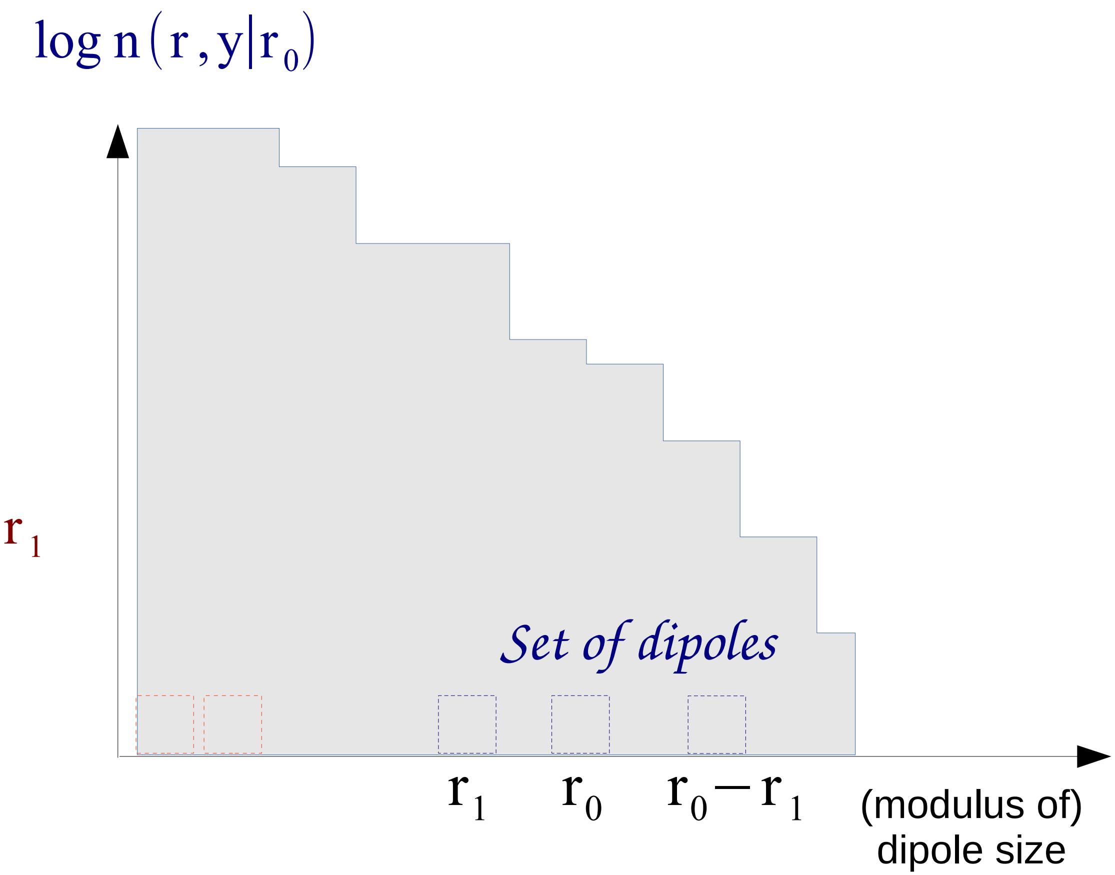

The number of dipoles grows exponentially with rapidity through a (nonlocal) branching process

$$y \gg \frac{1}{\alpha_s N_c / \pi}$$

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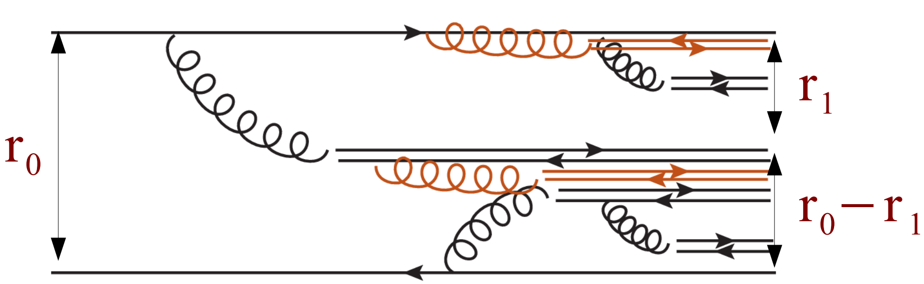
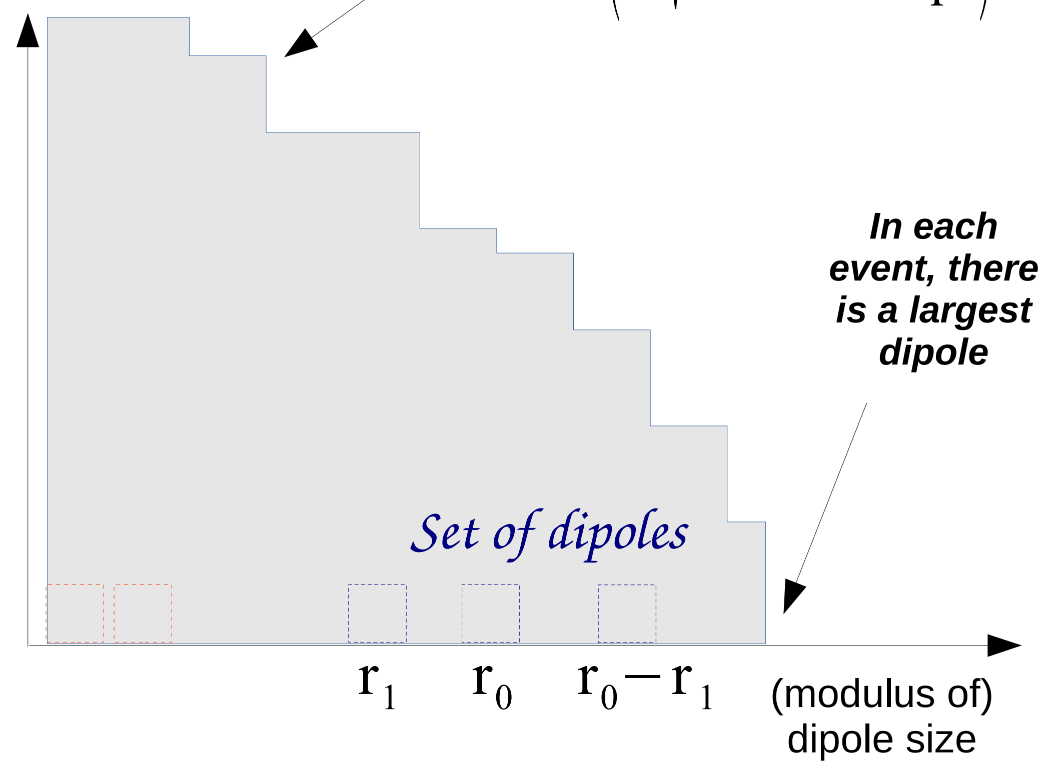


$\log n(r, y | r_0)$

$$n \sim \exp\left(2\sqrt{\frac{\alpha N_c}{\pi}\, y \log \frac{1}{r^2}} \right)$$



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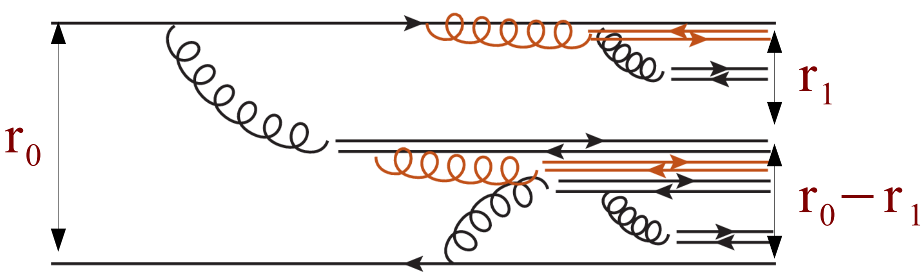

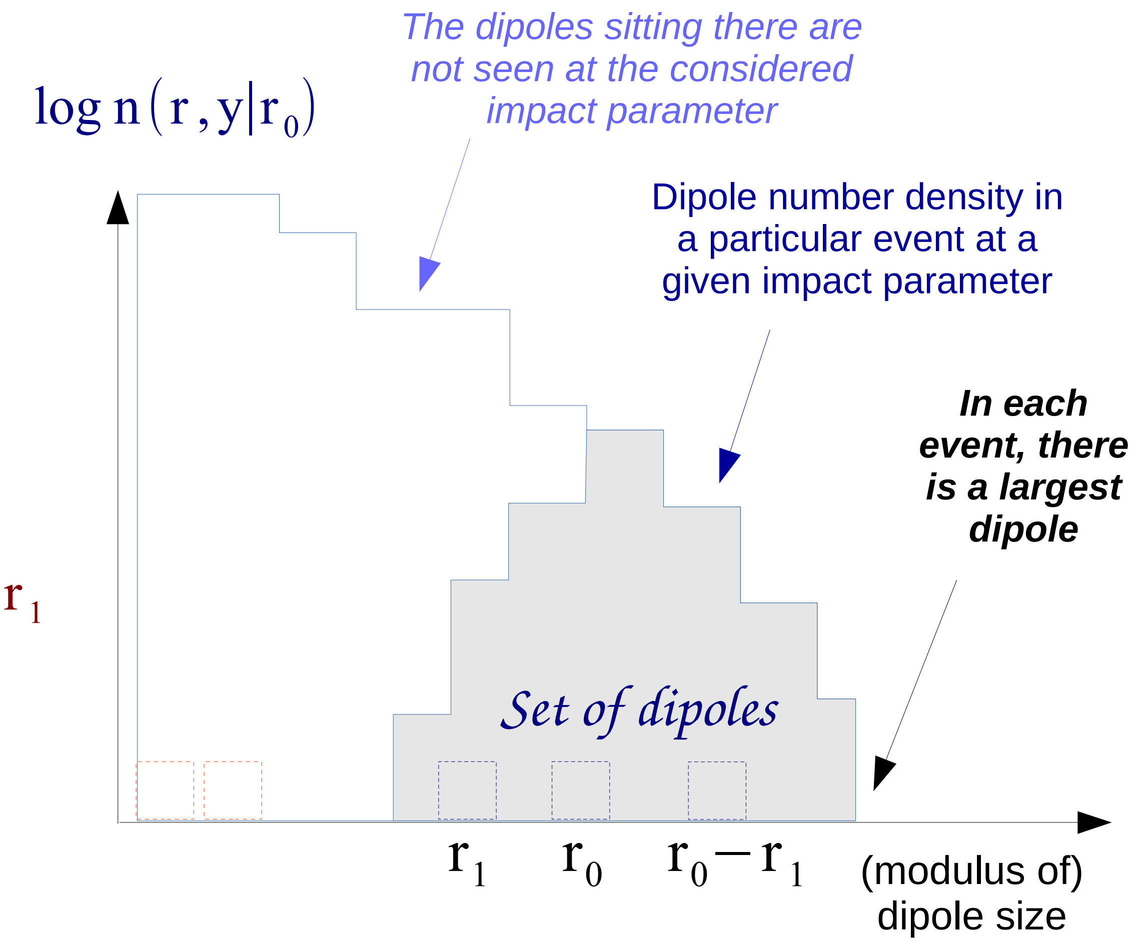

The number of dipoles grows exponentially with rapidity through a (nonlocal) branching process,

$$y \gg \frac{1}{\alpha_s N_c / \pi}$$

which at each fixed impact parameter, is a (local) branching random walk

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  Evolution of hadronic states towards higher energies: the color dipole model

- **Dipole-nucleus scattering**

  Total cross section and the statistics of extremes

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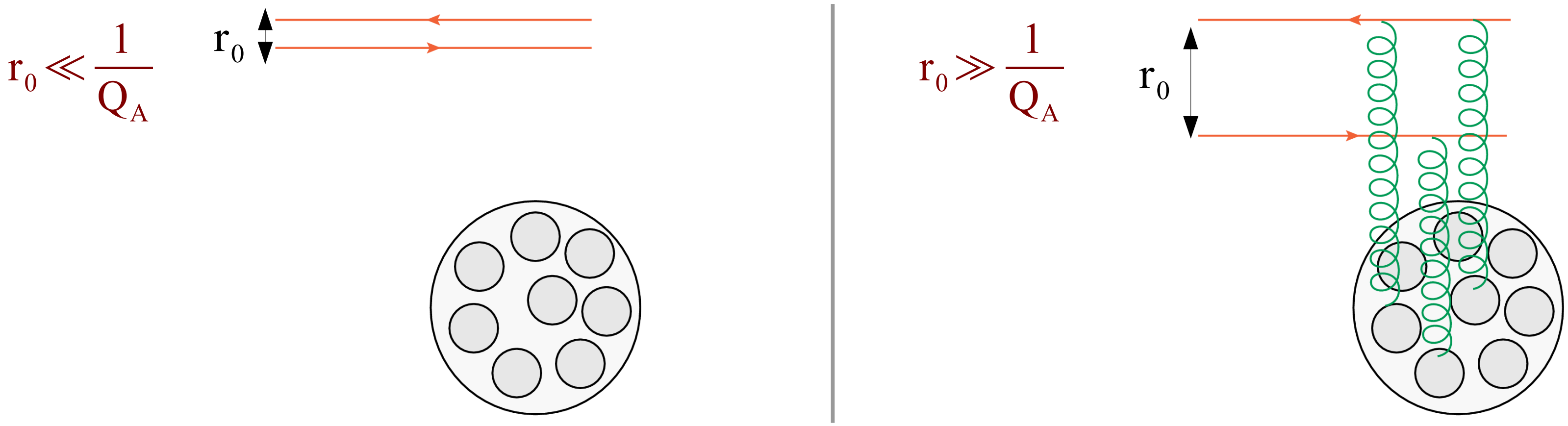
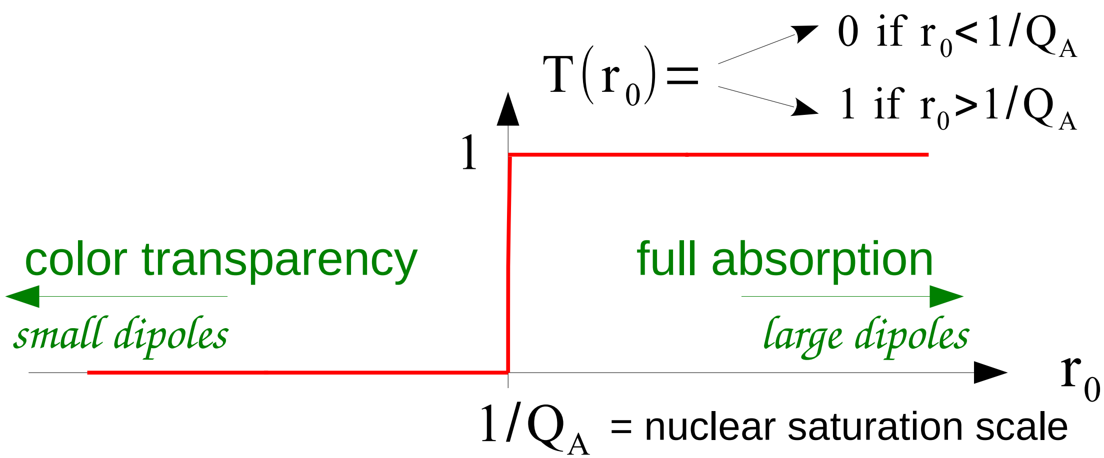
Dipole-nucleus scattering: total cross section

Scattering of dipoles of different sizes at low rapidity

A nucleus is a dense object, characterized by a scale Q_A *, which is essentially* ***transparent*** *to dipoles of size smaller than* $1/Q_A$ *and fully* ***absorptive*** *to larger dipoles:*

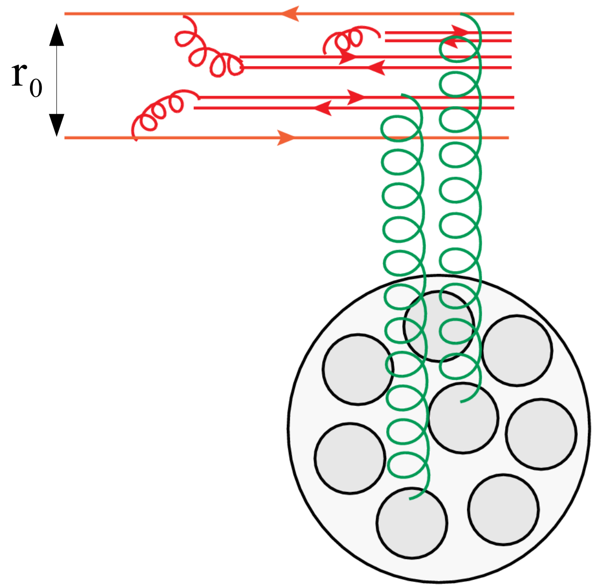


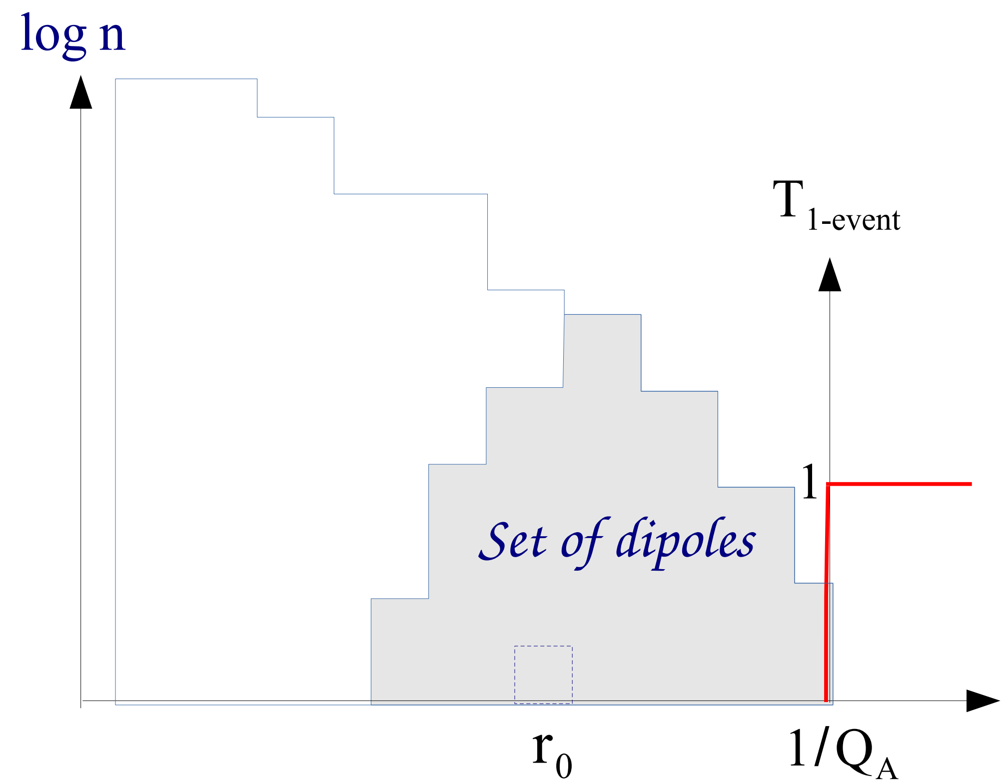
Dipole-nucleus cross section (fixed impact parameter) = scattering probability:



Dipole-nucleus scattering: total cross section

Scattering of a small dipole at high rapidity





This particular evolved quantum state scatters if and only if at least one dipole at the time of the interaction is ***larger*** *than the inverse nuclear saturation momentum.*

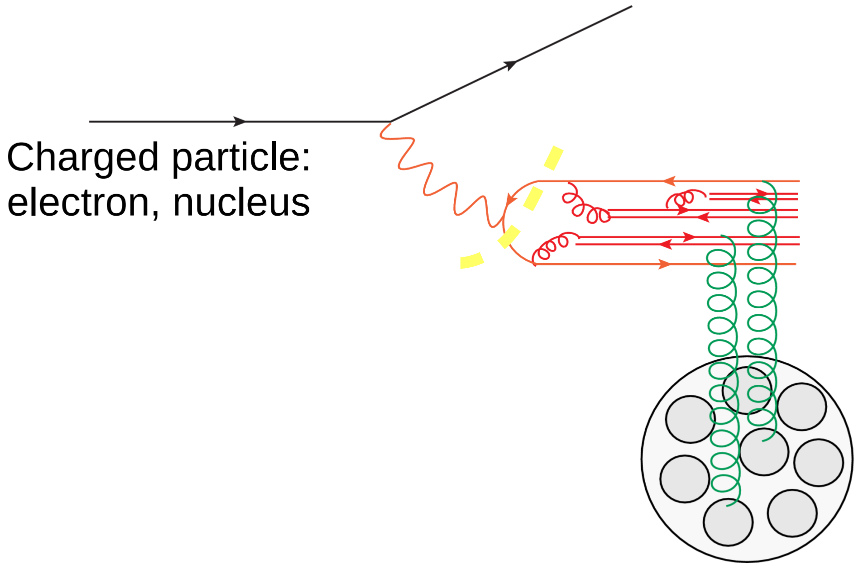
The measured amplitude is the average over events: $T = \langle T_{1\text{-event}} \rangle_{\text{events}}$

The scattering amplitude is the probability that the largest dipole is larger than $1/Q_A$

Solves the Balitsky-Kovchegov equation...

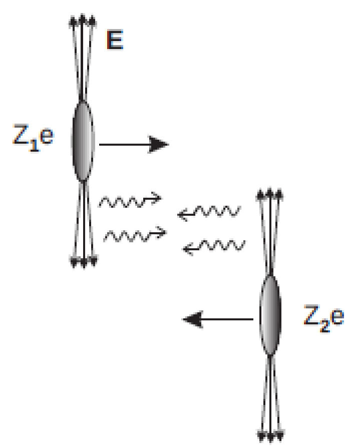
...but also connects with more general branching random walks

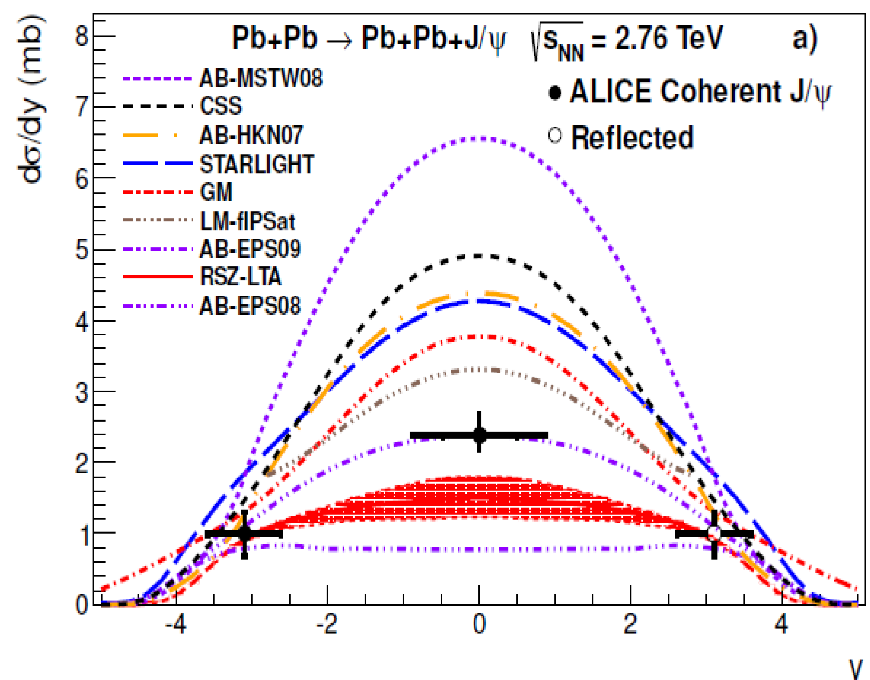
Experimentally: "deep-inelastic scattering"



- *At the LHC, "kind of" DIS off nuclei: ultraperipheral AA collisions!*

$$\gamma A \rightarrow J/\psi A'$$





Dipole-nucleus scattering: total multiplicity

Scattering of a large dipole at high rapidity

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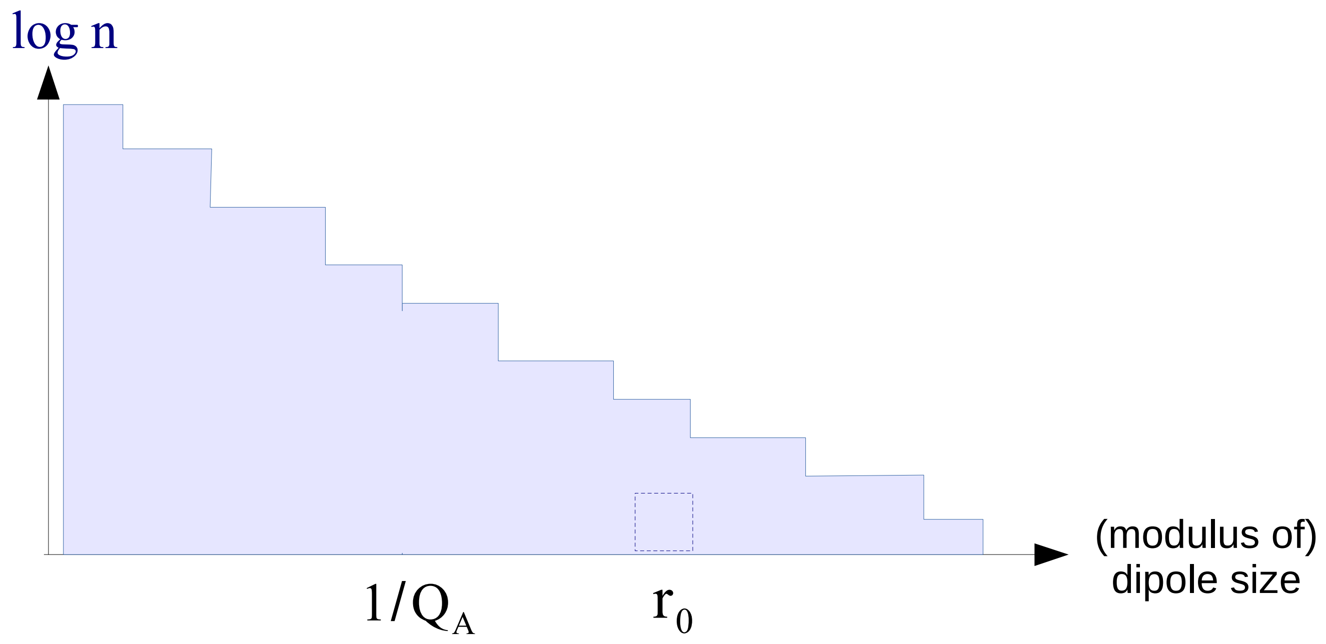

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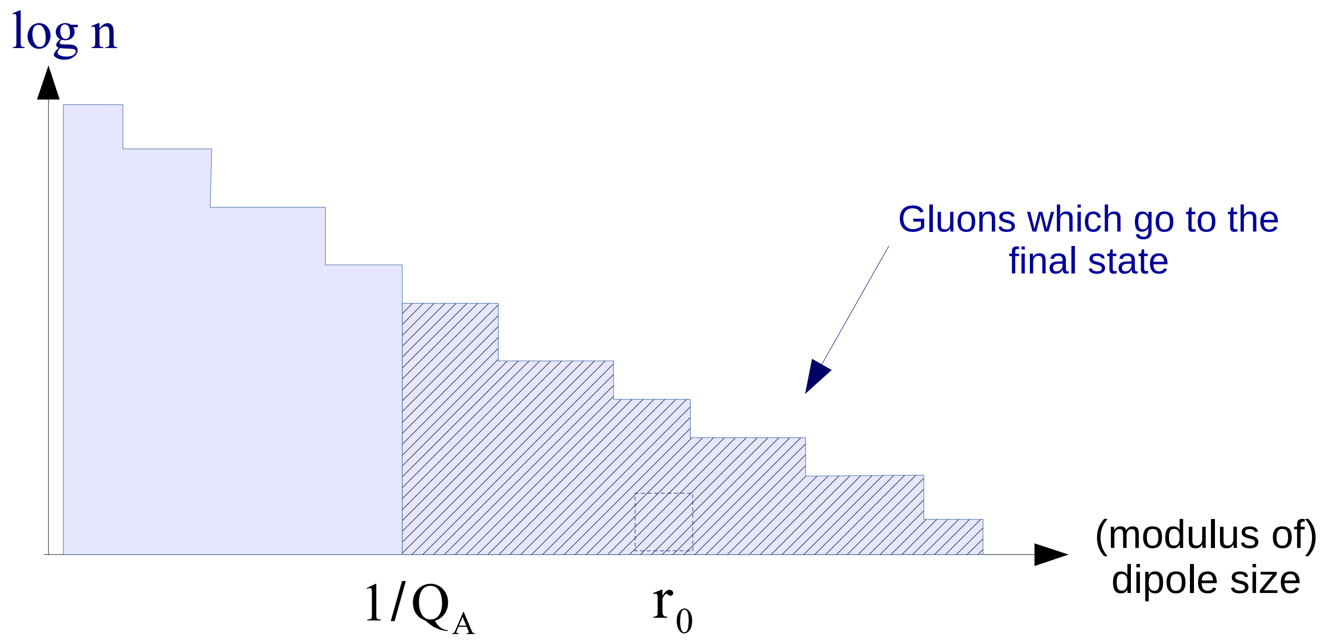

The ***gluons which go to the final state****, i.e. which are freed in the scattering, correspond to* ***dipoles which have a size larger than the inverse saturation scale of the nucleus.***

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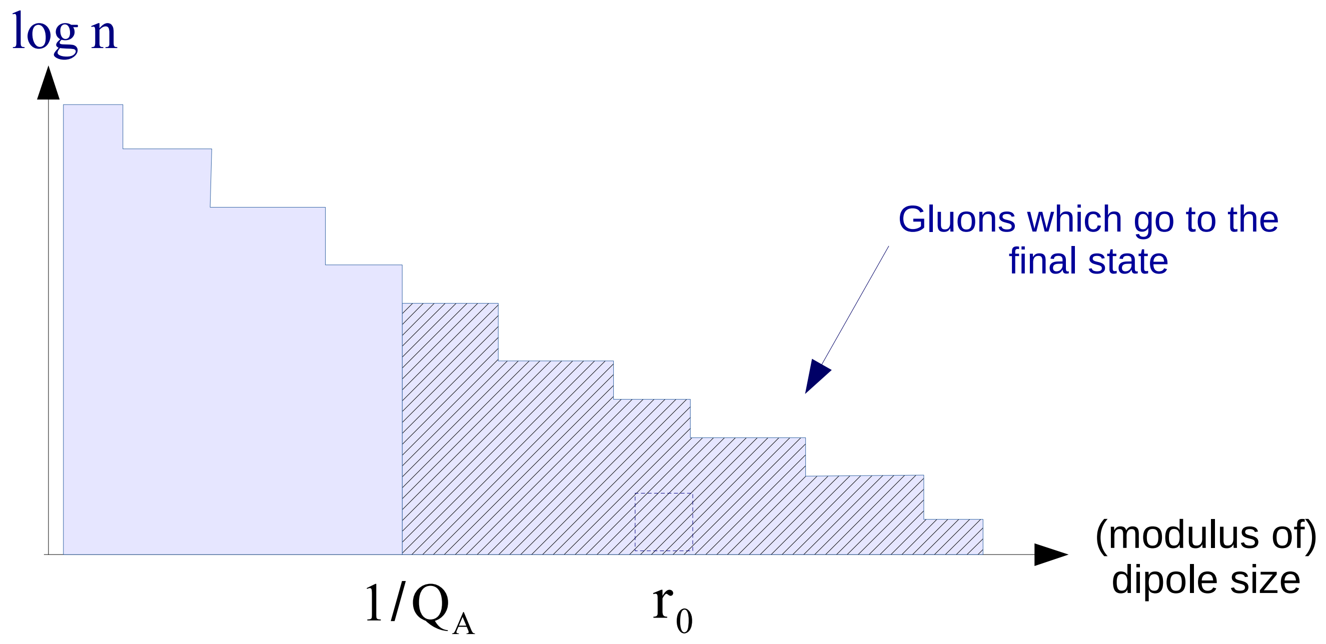

The ***gluons which go to the final state****, i.e. which are freed in the scattering, correspond to* ***dipoles which have a size larger than the inverse saturation scale of the nucleus.***

The multiplicity measured in the proton fragmentation region in an event is the gluon number density at the scale Q_A in the corresponding realization of the QCD evolution.

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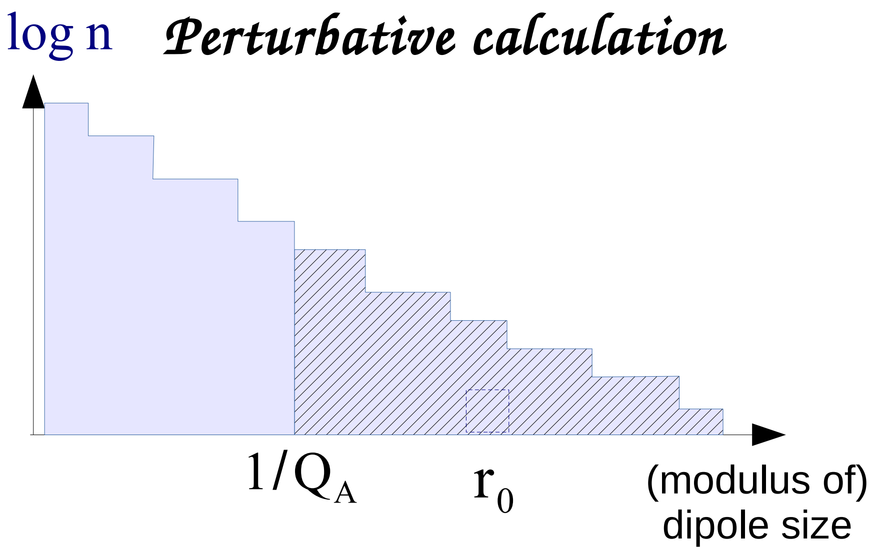
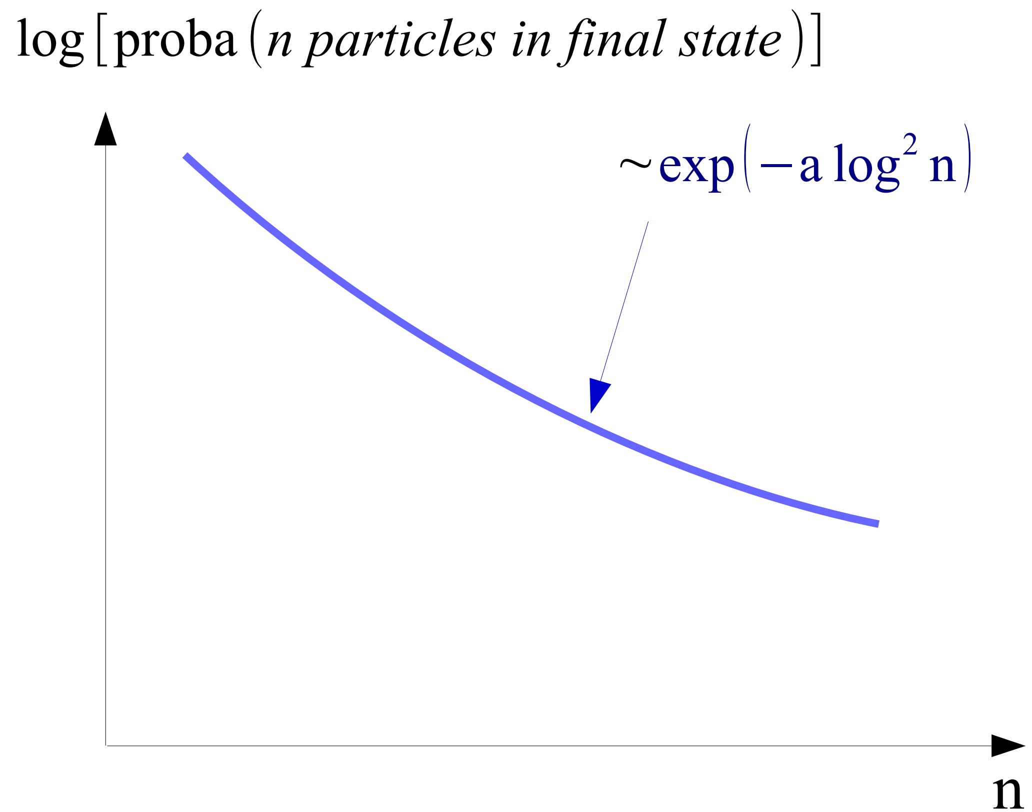
Perturbative calculation





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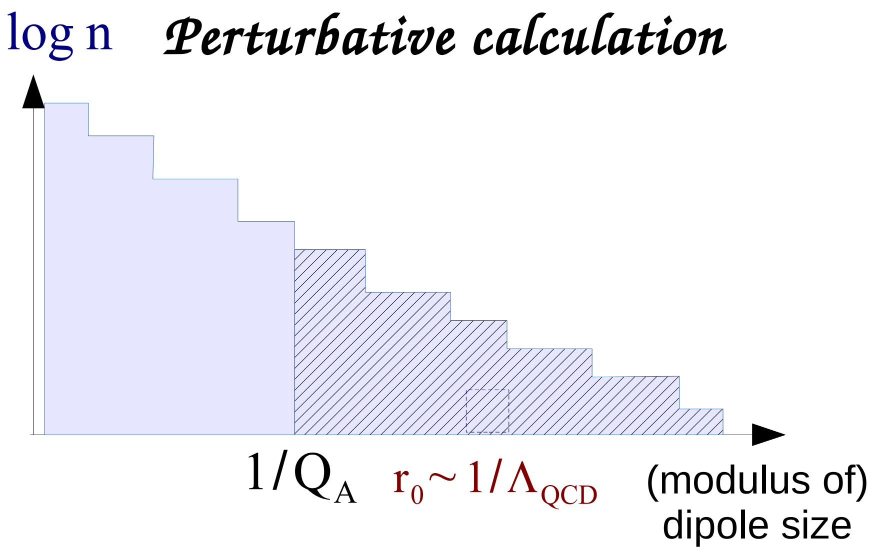
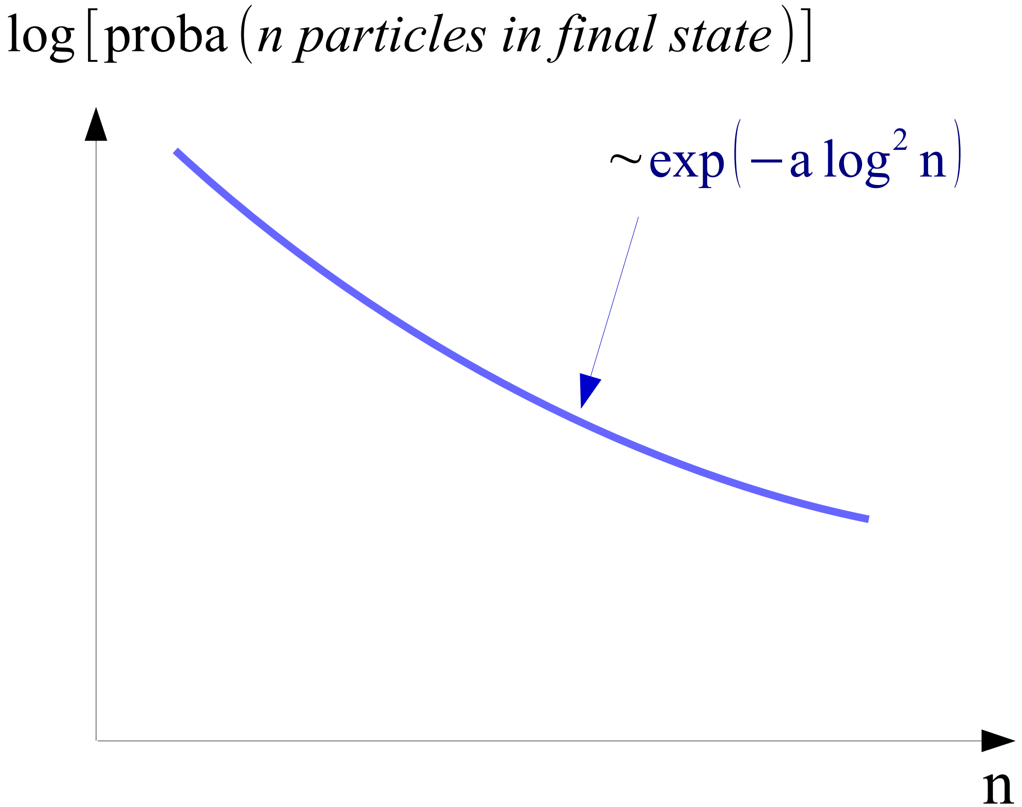


But in pA, the dipole is a proton!



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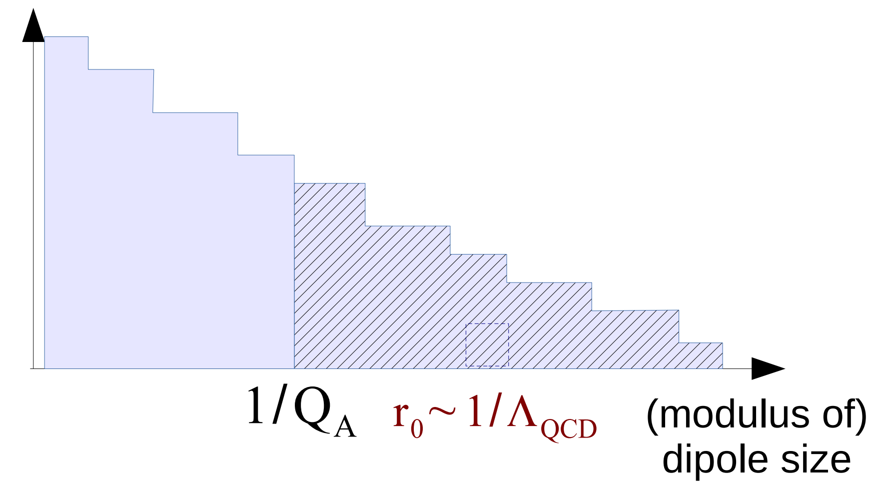

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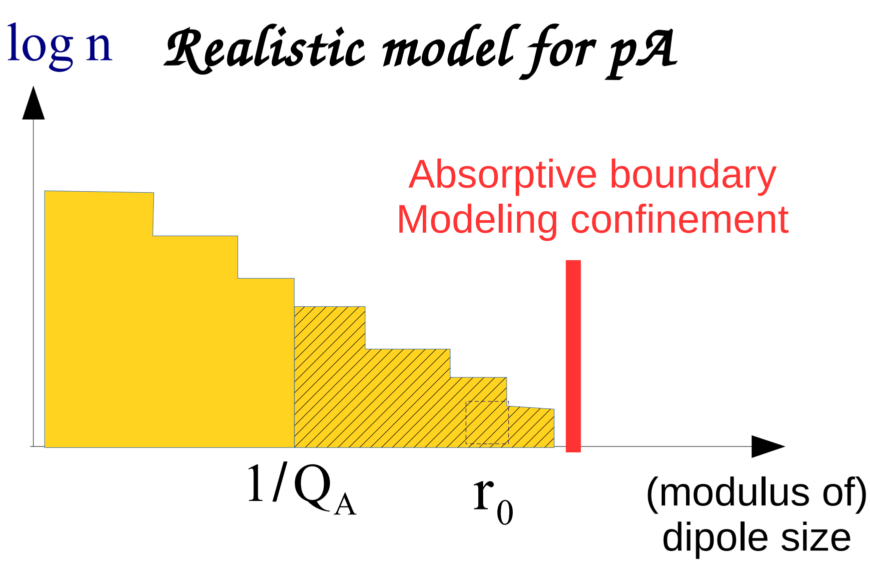
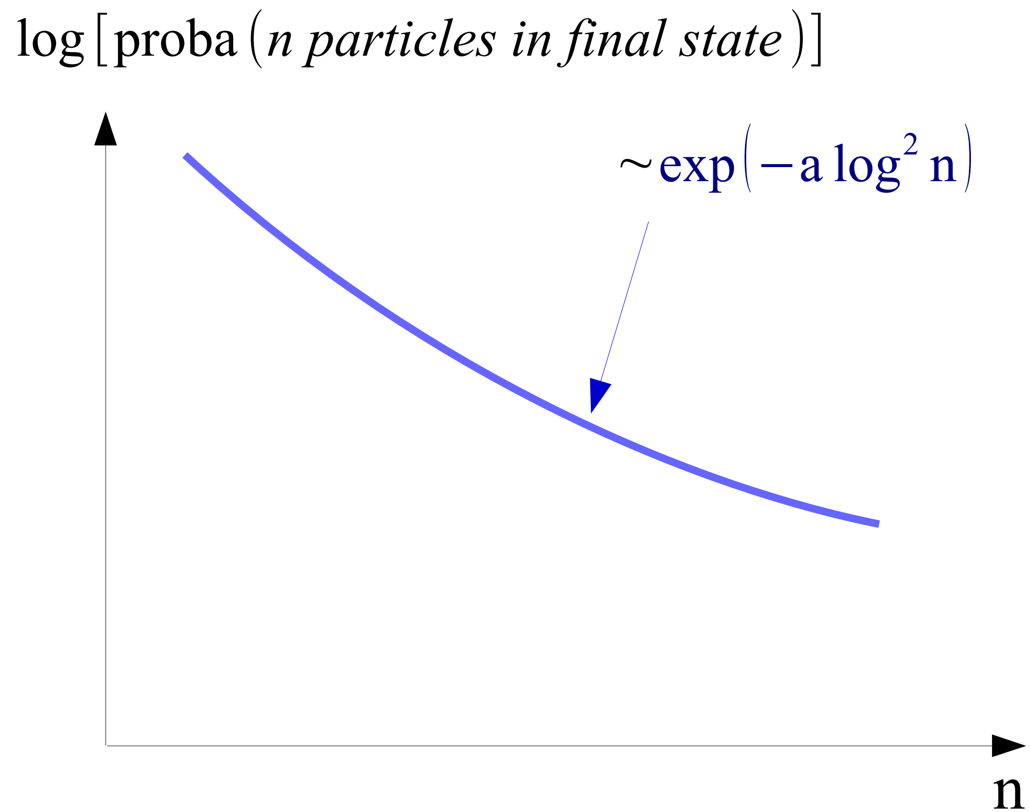
Realistic model for pA





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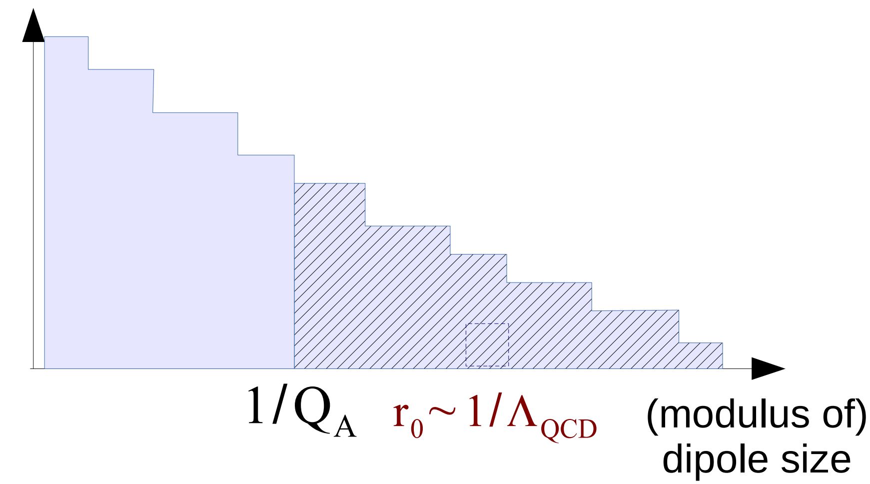

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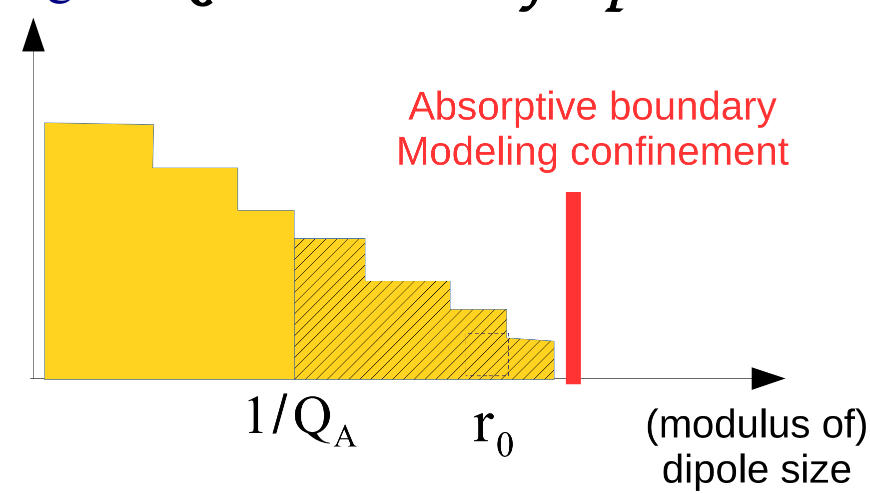
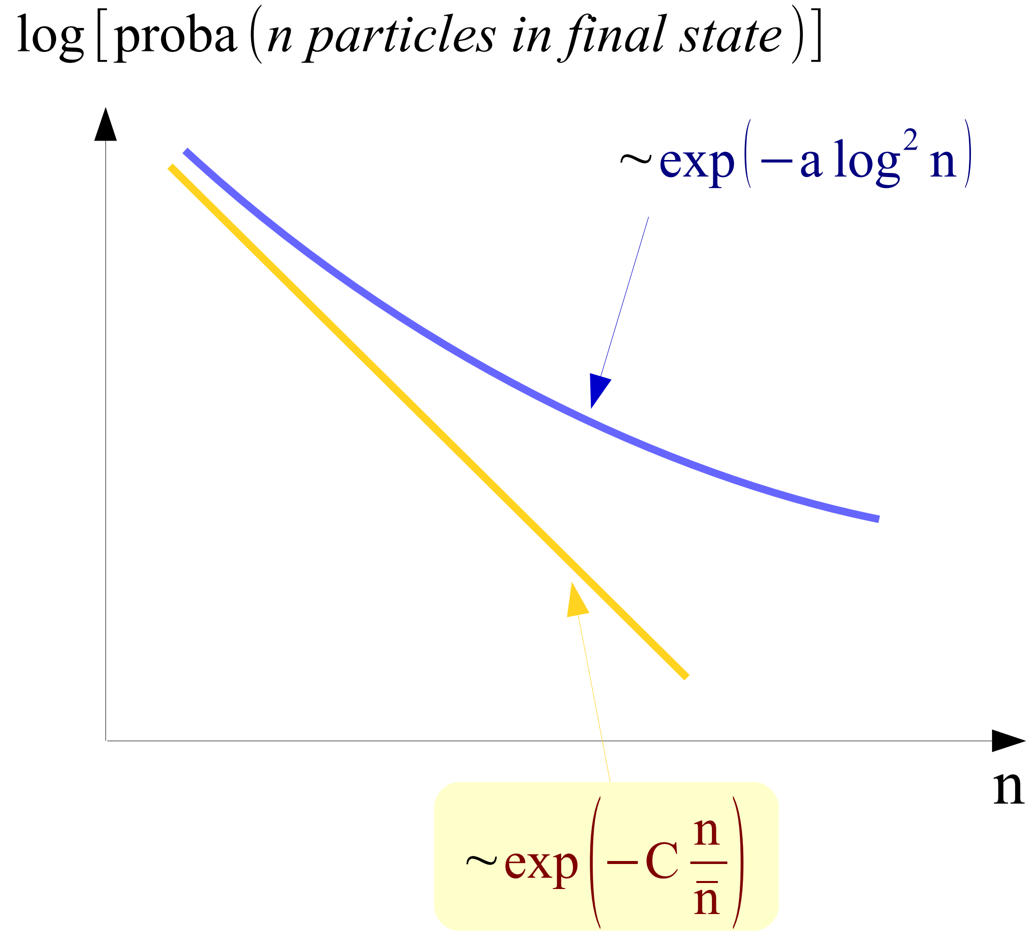
Realistic model for pA



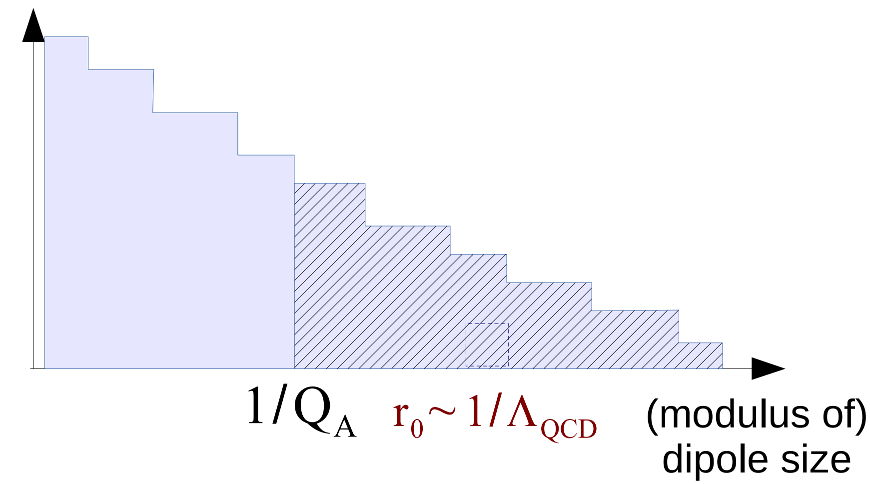


Consistent with the data!

Dipole-nucleus scattering: total multiplicity

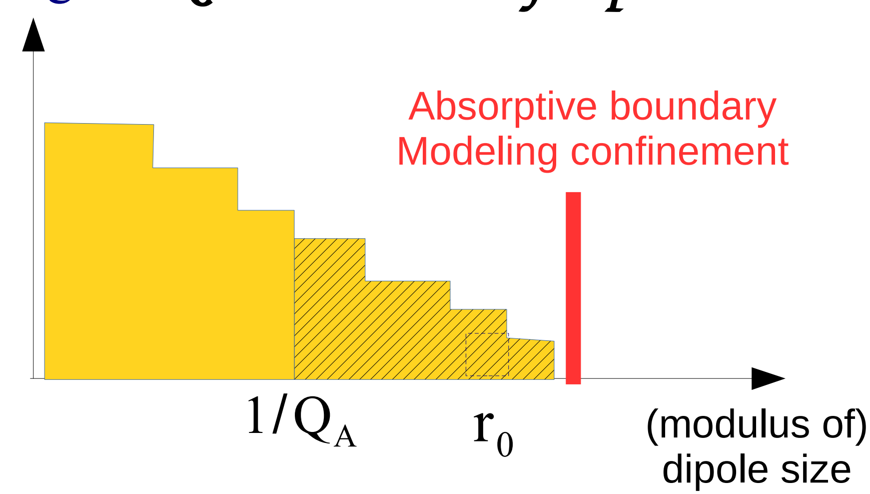
Scattering of a large dipole at high rapidity

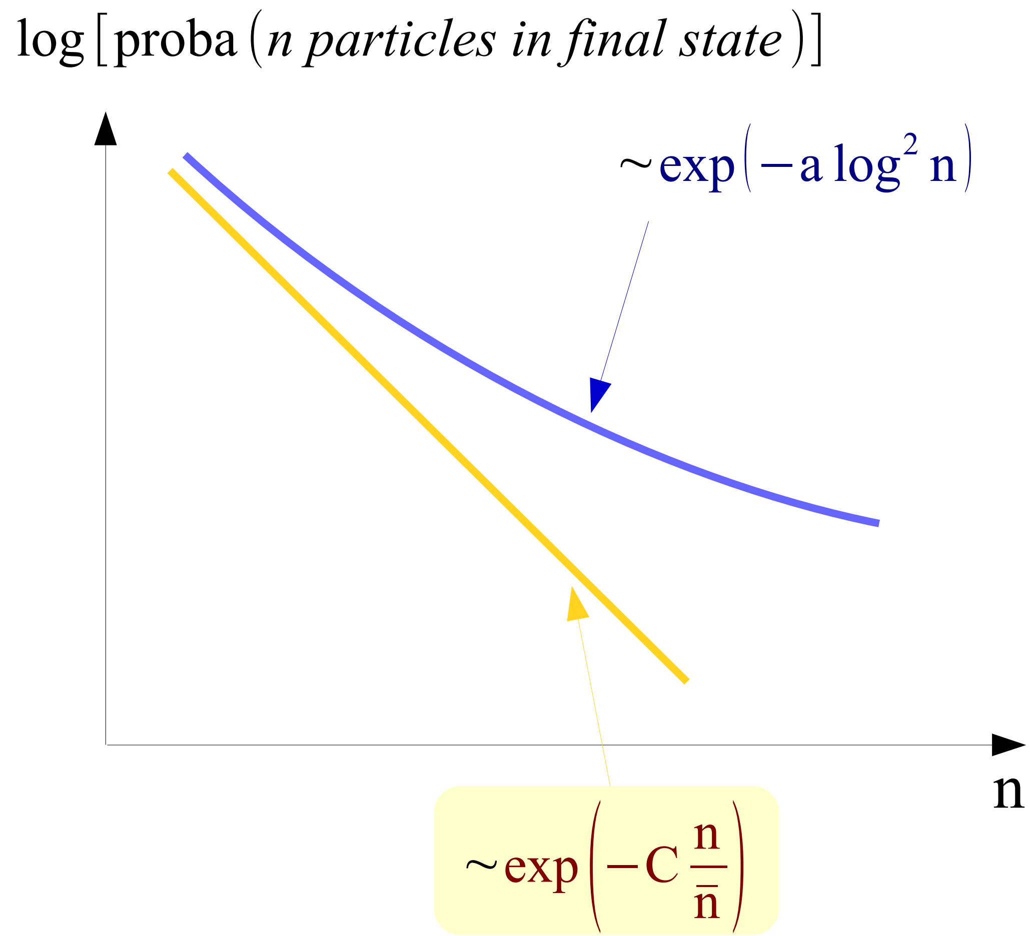
Perturbative calculation



But in pA, the dipole is a proton!

Realistic model for pA





Consistent with the data!

How much does C depend on the details of confinement?

Summary

- At high energies, **hadrons look like dense states of gluons** (sometimes called "color glass condensates"), very far from the valence picture. This is a property of QCD.
- The evolution of hadronic wave functions towards high energy can be computed in QCD. The **color dipole model** is a convenient implementation of this evolution.

- The scattering cross section of a small dipole off a nucleus at high energy can be seen as a measurement of the probability distribution of the size of the largest dipole generated by the QCD evolution of the dipole → **boundary of a branching random walk** .

  This is an interesting and active field in mathematics (→ theorems!)

  Mueller, SM (2004-...)

  To appear (2016): A. Kohara, SM

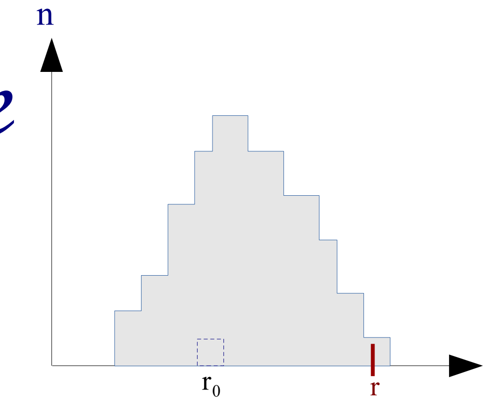
- **Fluctuations of the multiplicity in pA scattering in the proton fragmentation region can be related to the event-by-event fluctuations of the total integrated gluon density!** pA data at the LHC is a great opportunity to study these fluctuations!

  To appear (2016): Tseh Liou, A.H. Mueller, S. Munier

Backup

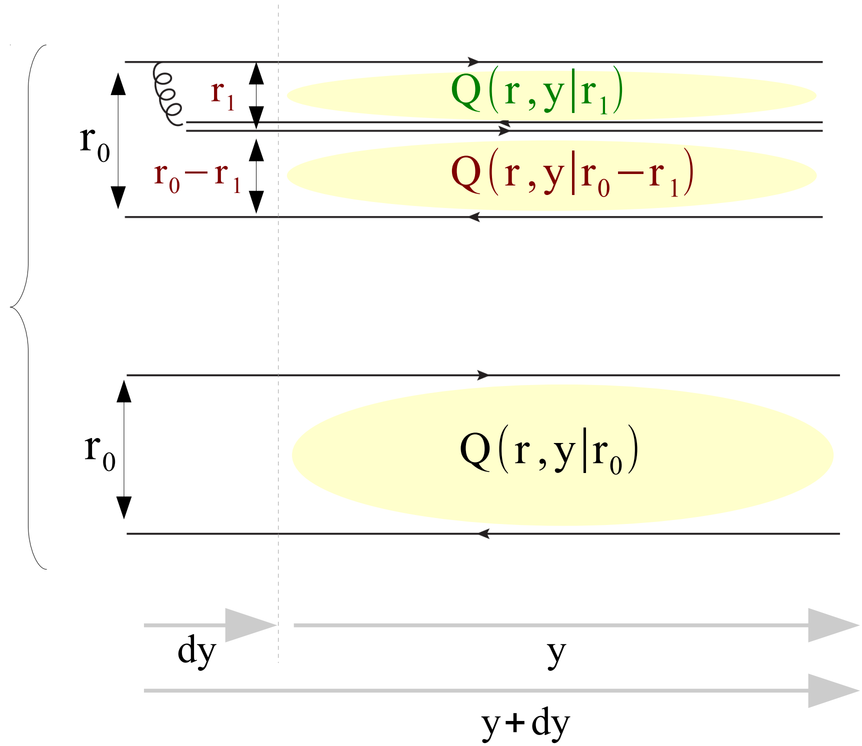
- The Balitsky-Kovchegov equation and its solution
- Formulation of DIS in the dipole model and in the target restframe

Probability distribution of the largest size



$Q(r,y|r_0)$ **=probability that all dipoles have a size smaller than r**

$Q(r,y+dy|r_0) \;=$



proba

$$dP = dy\,\frac{\alpha_s N_c}{\pi}\,\frac{r_0^2}{r_1^2(r_0-r_1)^2}\,\frac{d^2 r_1}{2\pi}$$

proba. $1-\int dP$

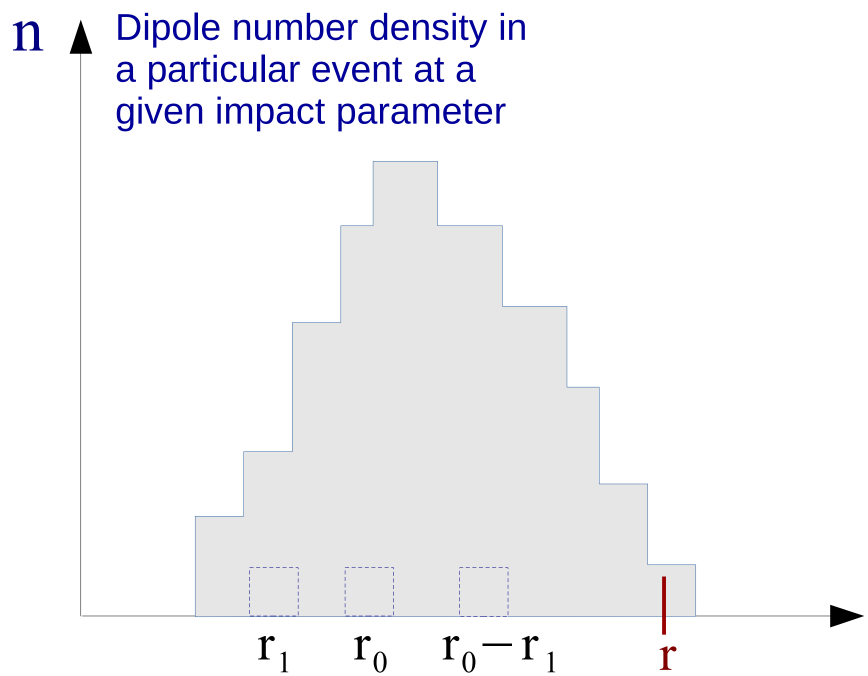
$$Q(r,y+dy|r_0) = \int dP\,[Q(r,y|r_1)\times Q(r,y|r_0-r_1)] + \left(1-\int dP\right) Q(r,y|r_0)$$

$$\frac{\partial}{\partial y} Q(r,y|r_0) = \frac{\alpha_s N_c}{\pi}\int \frac{d^2 r_1}{2\pi}\,\frac{r_0^2}{r_1^2(r_0-r_1)^2}\,[Q(r,y|r_1)\times Q(r,y|r_0-r_1) - Q(r,y|r_0)]$$

Balitsky-Kovchegov (nonlinear) equation!

The BK equation and its solution

$T = 1 - Q =$ **probability that at least one dipole has a size larger than r**

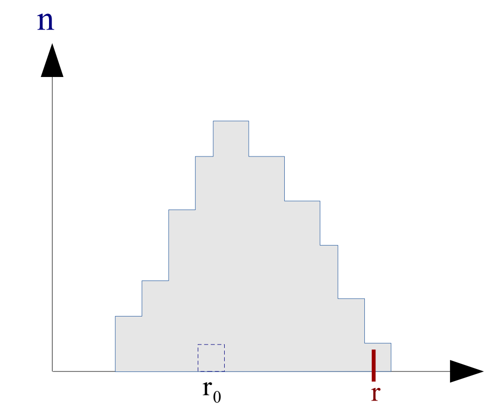


$$\partial_y T(r, y | r_0) = \frac{\alpha_s N_c}{\pi} \int \frac{d^2 r_1}{2\pi} \frac{r_0^2}{r_1^2 (r_0 - r_1)^2} \Big[\underbrace{T(r, y | r_1) + T(r, y | r_0 - r_1) - T(r, y | r_0)}_{\text{Linear part: BFKL equation}} - T(r, y | r_1) T(r, y | r_0 - r_1) \Big]$$

Nonlinear integro-differential equation

The BK equation and its solution

$T=1-Q=$ **probability that at least one dipole has a size larger than r**

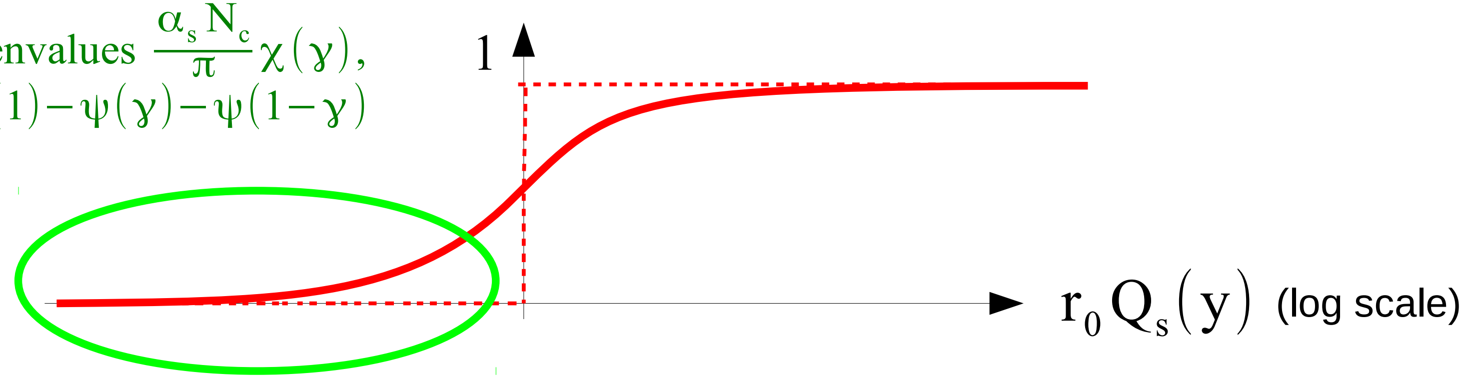


$$\partial_y T(r,y|r_0)=\frac{\alpha_s N_c}{\pi}\int\frac{d^2 r_1}{2\pi}\frac{r_0^2}{r_1^2(r_0-r_1)^2}\Big[\underbrace{T(r,y|r_1)+T(r,y|r_0-r_1)-T(r,y|r_0)}_{\text{Linear part: BFKL equation}}-T(r,y|r_1)\,T(r,y|r_0-r_1)\Big]$$

eigenfunctions $T(r|r_0)\sim(r_0^2/r^2)^{\gamma}$,

eigenvalues $\frac{\alpha_s N_c}{\pi}\chi(\gamma)$,

$\chi(\gamma)\equiv 2\psi(1)-\psi(\gamma)-\psi(1-\gamma)$



$$T(r,y|r_0)\underset{y\gg\frac{1}{\alpha_s N_c}}{\sim}\text{function of }(r_0 Q_s(y))$$

Traveling wave property

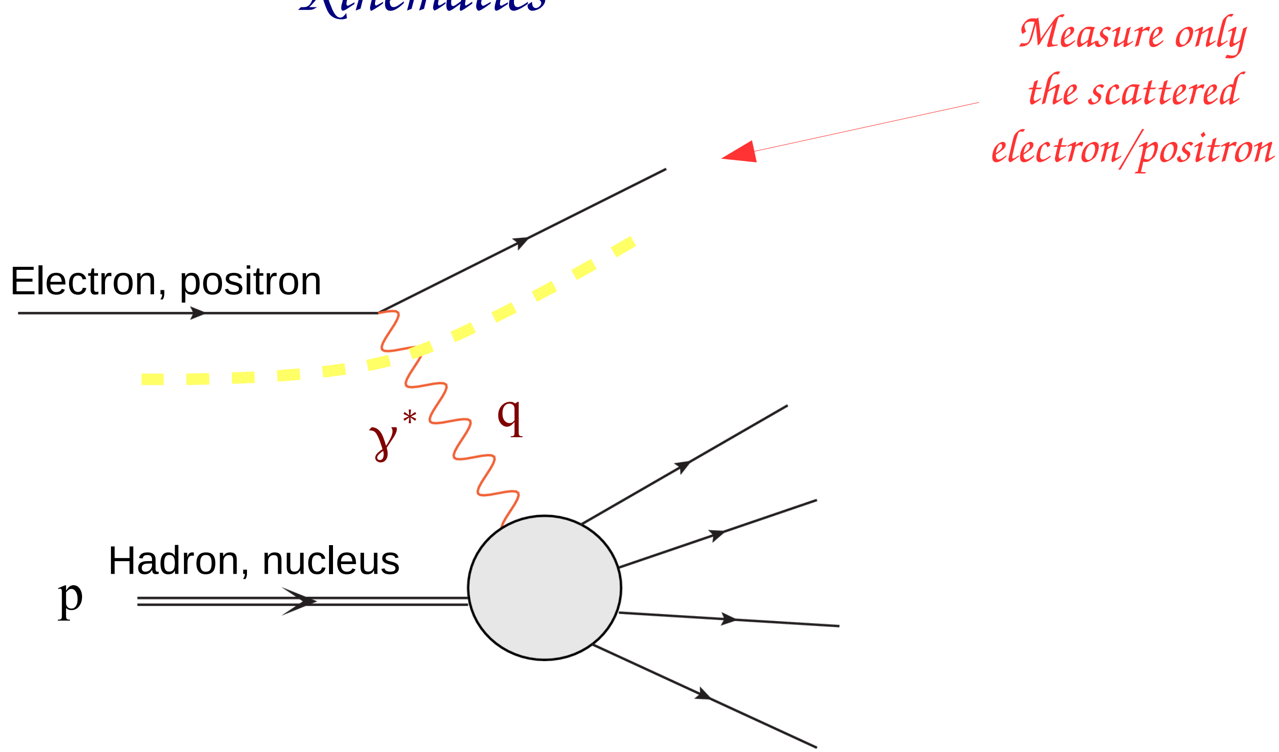
$$T\underset{r_0 Q_s(y)\ll 1}{\sim}\ln\frac{1}{r_0^2 Q_s^2(y)}\left(r_0^2 Q_s^2(y)\right)^{\gamma_0}$$

$$Q_s^2(y)\simeq Q_A^2\, e^{\frac{\alpha_s N_c}{\pi}\chi'(\gamma_0)\,y}$$

γ_0 solves $\gamma_0\chi'(\gamma_0)=\chi(\gamma_0)$

Deep-inelastic scattering

Kinematics



Variables: $p,q \;\Rightarrow\; Q^2 \equiv -q^2, x_{Bj} \equiv \dfrac{Q^2}{2\,p\cdot q}$

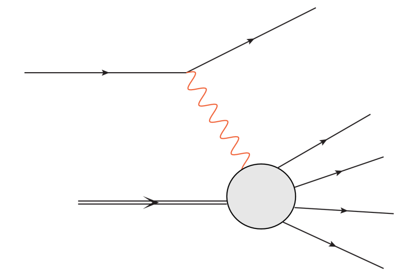
Rapidity: $y = \ln \dfrac{1}{x_{Bj}} \simeq \ln \dfrac{(p+q)^2}{Q^2}$

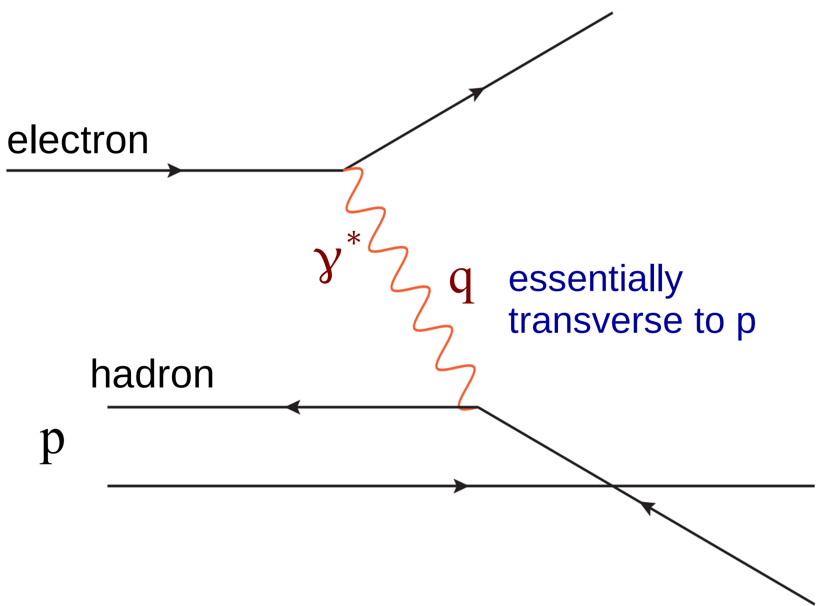
Large y = small x = high energy

Observable: $\sigma^{\gamma^* h}\left(Q^2, x_{Bj}\right)$

Deep-inelastic scattering

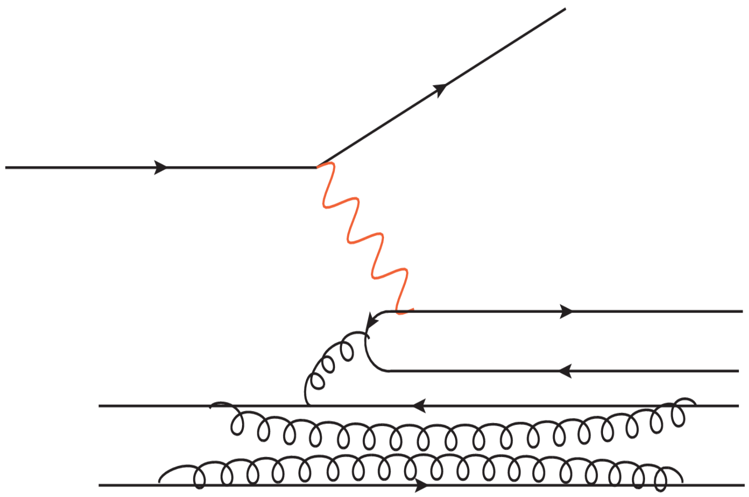
Picture in the Bjorken frame





Low rapidity y

$$y = \ln \frac{1}{x_{Bj}}$$



Higher rapidity y

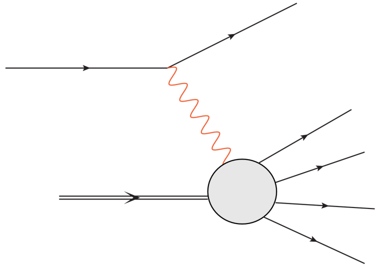
Parton model formula "improved":

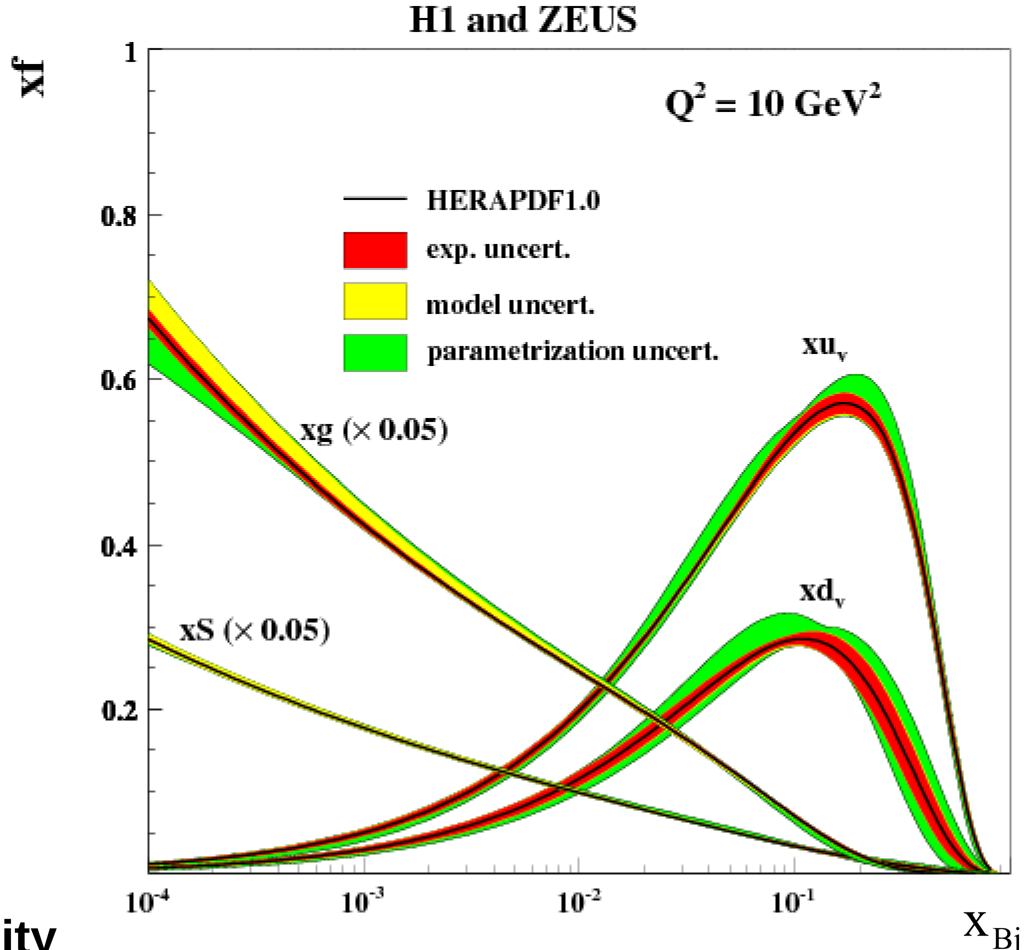
$$\sigma^{\gamma^* h}\left(Q^2, x_{Bj}\right) = \frac{4\pi^2 \alpha_{em}}{Q^2} \sum_q e_q^2 \left[\underbrace{x_{Bj}\, q\left(x_{Bj}, Q^2\right)}_{\text{(Mean) integrated } \cancel{\text{(valence)}} \text{ quark density}} + x_{Bj}\, \bar{q}\left(x_{Bj}, Q^2\right) \right]$$

~~~ Bjorken scaling (pointlike quarks)~~
~~~

Deep-inelastic scattering

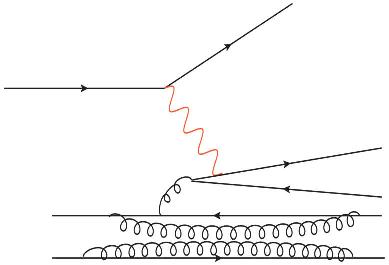
Picture in the Bjorken frame

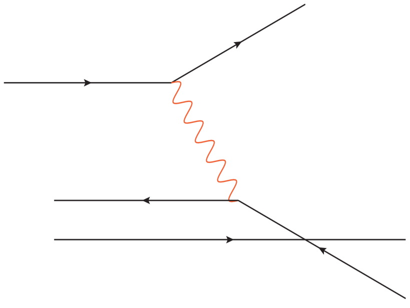




High rapidity

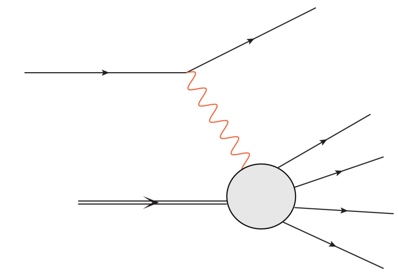
Low rapidity

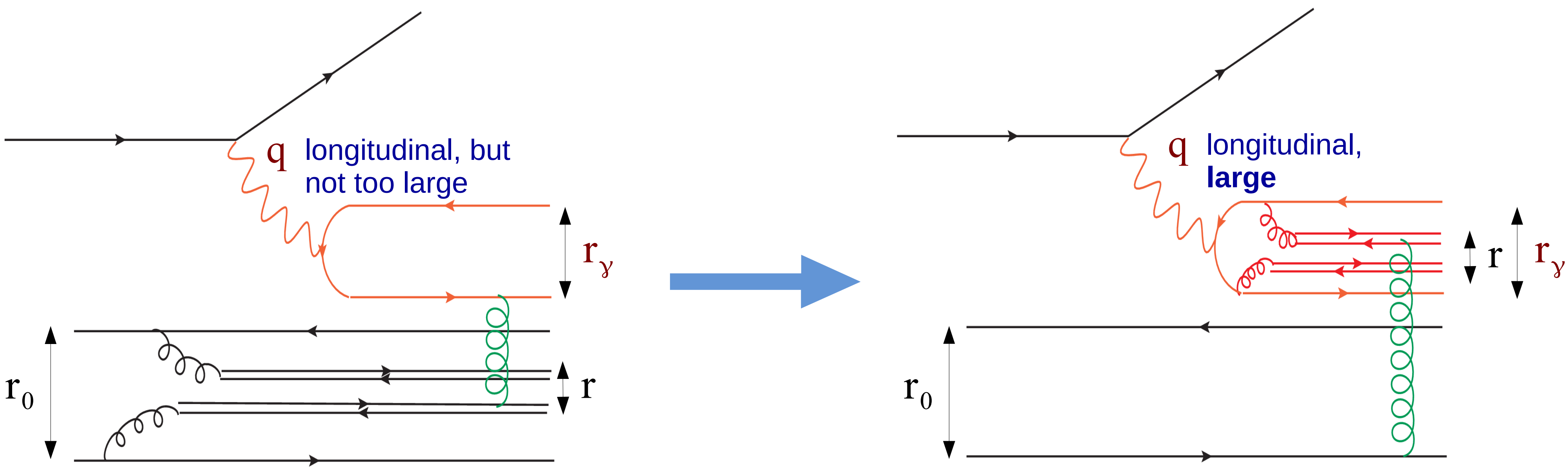




Deep-inelastic scattering at high energy

Picture in the target restframe





Dipole frame **Target restframe**

$$\sigma^{\gamma^* h}\left(Q^2, x_{Bj}\right) = \int d^2 r_\gamma \, \mathrm{Proba}\left(\gamma^* \to q\,\bar{q}\,(r_\gamma)\right) \times \begin{cases} \int d^2 r \, \sigma^{dd}(r_\gamma, r) \times \langle n(r, y | r_0)\rangle & \text{(dipole frame)} \\ \quad \| & \\ \int d^2 r \, \langle n(r, y | r_\gamma)\rangle \times \sigma^{dd}(r, r_0) & \text{(target restframe)} \end{cases}$$

In the target frame, DIS "measures" the mean dipole density in the photon