# Triple Gauge Couplings from Higgs data

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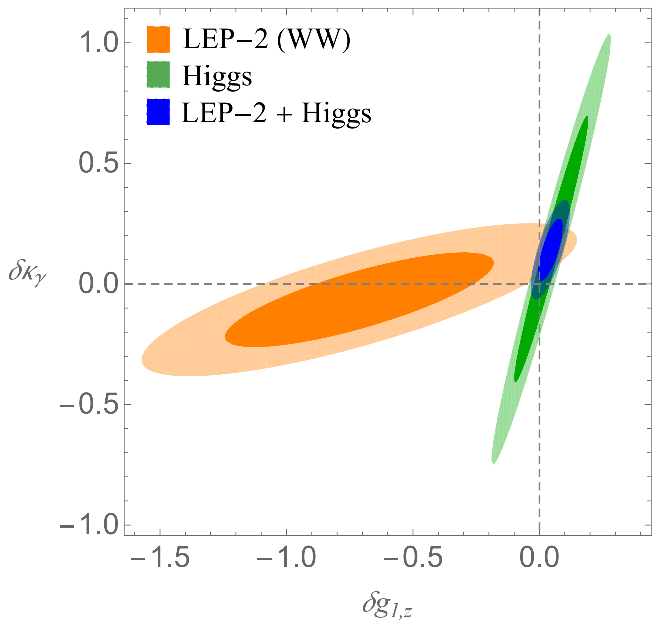
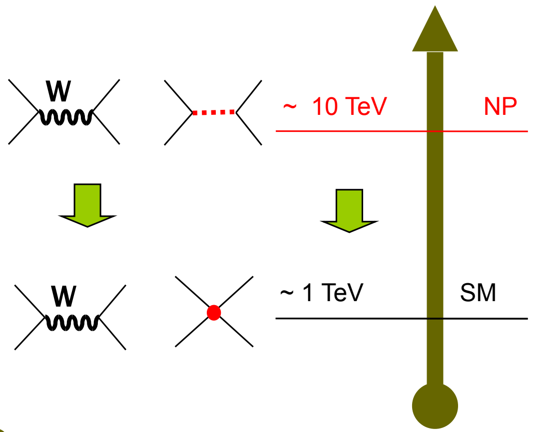



*[Falkowski, MGA, Greljo & Marzocca, Phys. Rev. Lett. 116 (2016), 011801]*

# EFT at the EW scale




$$\mathcal{L} = \mathcal{L}(\phi, \phi_\Lambda)$$

$$\mathcal{L}_{eff.}(x) = \mathcal{L}_{SM}(x) + \frac{1}{\Lambda}\cancel{\mathcal{L}_5}(x) + \frac{1}{\Lambda^2}\mathcal{L}_6(x) + \dots$$

$$\boxed{\sum \alpha_i \mathcal{O}_i}$$

**EFT = Symmetries + Fields**

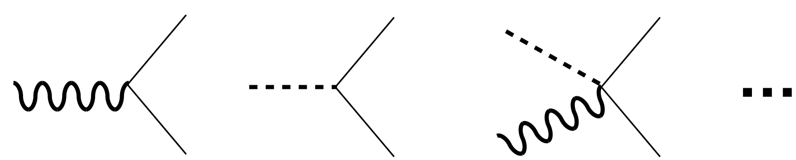
- Lorentz;
- SU(2) x U(1);
- Flavour sym?
- B, L;

- SM fields
- h SU(2) doublet
- No light NP

**α: Wilson coefficients (UV physics)**
**59 dim-6 operators**
*[Buchmuller & Wyler'1986, Leung et al.'1986, Grzadkowksi et al., 2010]*

**Example:**

$$(\varphi^\dagger i D_\mu \varphi)(l_p \gamma^\mu l_r)$$



# EFT at the EW scale

- The analysis (bkg, PDFs, FF, simulations, …) is done once and for all!

- The output of an EFT analysis is the input of model-dependent studies.

$$\frac{\alpha_6^{(i)}}{\Lambda^2} = f_i\left(g_{NP}, M_{NP}\right)$$

Useful only if…

- Global analysis (all operators present *simultaneously*);
- Consistent bounds: small quadratic effects (bases are equivalent only at linear order!)

$$\begin{aligned} \mathcal{A} &\sim \mathcal{A}_{SM}\left(1 + \alpha_6 \frac{x}{\Lambda^2} + \alpha_8 \frac{x^2}{\Lambda^4} + \dots\right) \\ \mathcal{O} &\sim \mathcal{O}_{SM}\left(1 + \alpha_6 \frac{x}{\Lambda^2} + \left(\alpha_6^2 + \alpha_8\right) \frac{x^2}{\Lambda^4} + \dots\right) \end{aligned}$$

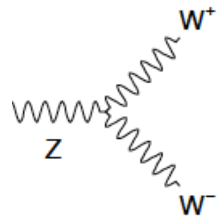
***x = (v, E)***

**Validity of the EFT:**
**E << Λ**

# Triple Gauge Couplings

- In the SM, they are fixed by gauge couplings:

$$\begin{aligned}\mathcal{L}_{\rm TGC}^{\rm SM} =& ie\left[A_{\mu\nu}\, W^+_\mu W^-_\nu + \left(W^+_{\mu\nu} W^-_\mu - W^-_{\mu\nu} W^+_\mu\right) A_\nu\right] \\ &+ i g_L c_\theta \left[\left(W^+_{\mu\nu} W^-_\mu - W^-_{\mu\nu} W^+_\mu\right) Z_\nu + Z_{\mu\nu}\, W^+_\mu W^-_\nu\right]\end{aligned}$$




- D=6 operators modify them:

$$\begin{aligned}\mathcal{L}_{\rm tgc}^{D=6} =& ie\left[\delta\kappa_\gamma A_{\mu\nu}\, W^+_\mu W^-_\nu + \tilde{\kappa}_\gamma \tilde{A}_{\mu\nu}\, W^+_\mu W^-_\nu\right] \\ &+ i g_L c_\theta \left[\delta g_{1,z} \left(W^+_{\mu\nu} W^-_\mu - W^-_{\mu\nu} W^+_\mu\right) Z_\nu + \delta\kappa_z Z_{\mu\nu}\, W^+_\mu W^-_\nu + \tilde{\kappa}_z \tilde{Z}_{\mu\nu}\, W^+_\mu W^-_\nu\right] \\ &+ i\frac{e}{m_W^2}\left[\lambda_\gamma W^+_{\mu\nu} W^-_{\nu\rho} A_{\rho\mu} + \tilde{\lambda}_\gamma W^+_{\mu\nu} W^-_{\nu\rho} \tilde{A}_{\rho\mu}\right] + i\frac{g_L c_\theta}{m_W^2}\left[\lambda_z W^+_{\mu\nu} W^-_{\nu\rho} Z_{\rho\mu} + \tilde{\lambda}_z W^+_{\mu\nu} W^-_{\nu\rho} \tilde{Z}_{\rho\mu}\right]\end{aligned}$$

*[de Rujula et al. NPB384 (1992), Hagiwara et al. PRD48 (1993)]*

- TGC = f ( WC )
  Example: Higgs basis…

$$\delta g_{1,z} = \frac{1}{2(g_L^2 - g_Y^2)}\left[c_{\gamma\gamma} e^2 g_Y^2 + c_{z\gamma}(g_L^2 - g_Y^2) g'^2 - c_{zz}(g_L^2 + g_Y^2) g_Y^2 - c_{z\Box}(g_L^2 + g_Y^2) g_L^2\right]$$

$$\delta\kappa_\gamma = -\frac{g_L^2}{2}\left(c_{\gamma\gamma}\frac{e^2}{g_L^2 + g_Y^2} + c_{z\gamma}\frac{g_L^2 - g_Y^2}{g_L^2 + g_Y^2} - c_{zz}\right),$$

$$\tilde{\kappa}_\gamma = -\frac{g_L^2}{2}\left(\tilde{c}_{\gamma\gamma}\frac{e^2}{g_L^2 + g_Y^2} + \tilde{c}_{z\gamma}\frac{g_L^2 - g_Y^2}{g_L^2 + g_Y^2} - \tilde{c}_{zz}\right),$$

$$\delta\kappa_z = \delta g_{1,z} - t_\theta^2 \delta\kappa_\gamma$$

$$\tilde{\kappa}_z = - t_\theta^2 \tilde{\kappa}_\gamma$$

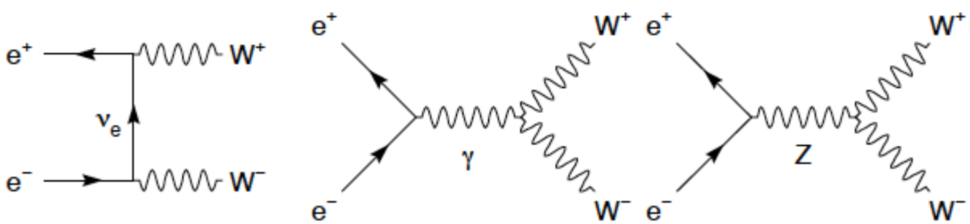
$$\lambda_\gamma = \lambda_z$$

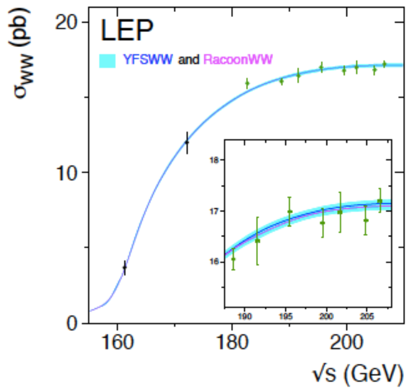
$$\tilde{\lambda}_\gamma = \tilde{\lambda}_z$$

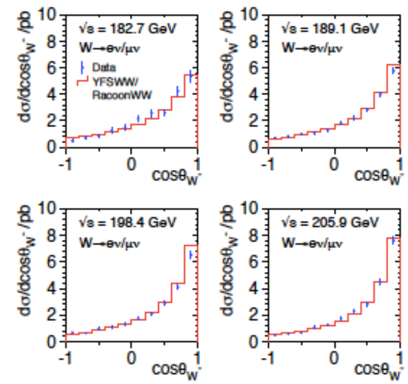
# TGC bounds from LEP2: $e^+e^- \to W^+W^-$

- Total and differential cross-sections measured in LEP2 (at different c.o.m. energies);

- At tree-level we have (SM & EFT):




- $\sigma_{WW}$, $d\sigma_{WW}$ = SM + f ( WC ) ≈ SM + f ( $\delta g_{1z}$ , $\delta\kappa_\gamma$ , $\lambda_z$ )

**Other NP contributions (e.g. in Vff ) are strongly bounded using LEP1 data**

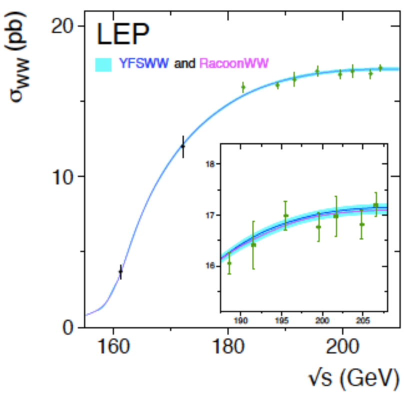
*[Falkowski, Riva, 2015, Efrati, Falkowski, Soreq, 2015]*

# TGC bounds from LEP2

$e^+e^- \rightarrow W^+W^-$ data
($\sigma_{WW}$, $d\sigma_{WW}$)

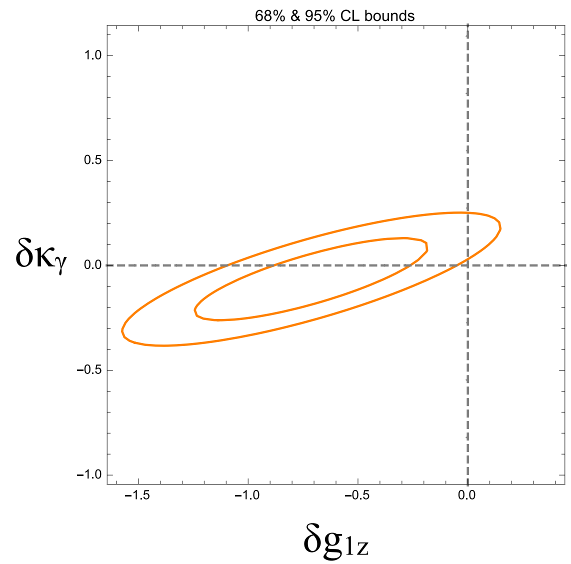
$\sigma_{WW}$, $d\sigma_{WW} \approx$ SM + f ( $\delta g1z$ , $\delta\kappa\gamma$ , $\lambda z$ )

Bounds on
$\delta g_{1z}$ , $\delta\kappa_\gamma$ , $\lambda_z$




$$\begin{pmatrix} \delta g_{1,z} \\ \delta\kappa_\gamma \\ \lambda_z \end{pmatrix} = \begin{pmatrix} -0.71 \pm 0.35 \\ -0.07 \pm 0.13 \\ 0.69 \pm 0.40 \end{pmatrix}, \qquad \rho = \begin{bmatrix} 1 & 0.79 & -0.993 \\ & 1 & -0.84 \\ & & 1 \end{bmatrix}$$

Accidental flat direction
along $\lambda z \approx \delta g1z$




[Brooijmans et al, arXiv:1405.1617]

[Falkowski & Riva, JHEP 1502 (2015)]

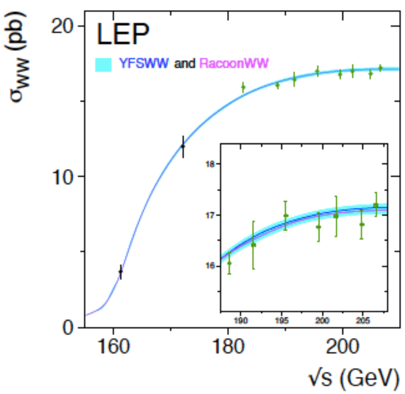
[Falkowski, MGA, Greljo & Marzocca, PRL 116 (2016)]

# TGC bounds from LEP2

$e^+e^- \to W^+W^-$ data ($\sigma_{WW}$, $d\sigma_{WW}$)

$\sigma_{WW}$, $d\sigma_{WW} \approx$ SM + f ( $\delta g_{1z}$ , $\delta\kappa_\gamma$ , $\lambda_z$ )

Bounds on $\delta g_{1z}$ , $\delta\kappa_\gamma$ , $\lambda_z$

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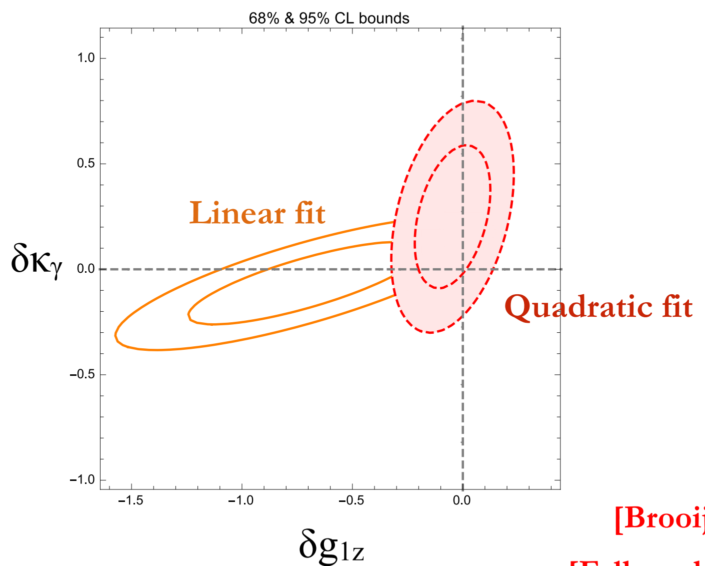
Accidental flat direction along $\lambda_z \approx \delta g_{1z}$




[Brooijmans et al, arXiv:1405.1617]

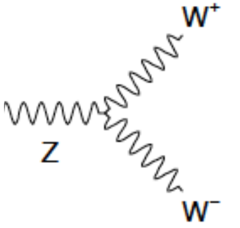
[Falkowski & Riva, JHEP 1502 (2015)]

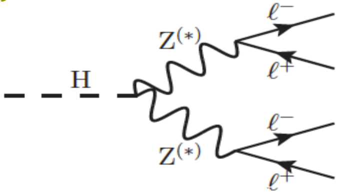
[Falkowski, MGA, Greljo & Marzocca, PRL 116 (2016)]

# How can Higgs data help?

- Symmetries relate these 2 sectors:
  the eff. operators affecting them are not orthogonal (nonzero overlap *in any basis*)
  One *always* has operators contributing to both, as e.g.

$$gg' H^\dagger \sigma^i H \, W^i_{\mu\nu} B_{\mu\nu}$$




**Exploited before, but with not all operators &/or not consistently at O($\Lambda^{-2}$)**
[Pomarol-Riva'2013, Corbett et al'2013, Dumont et al'2013, …]

# How can Higgs data help?

- LEP2 WW production (CP-cons)
  $\sigma_{WW}, d\sigma_{WW} = SM + f(WC) \approx SM + f(\delta g_{1z}, \delta\kappa_\gamma, \lambda_z)$

- Higgs signal strengths (CP-cons)
  $\mu_i = SM + f(WC) \approx SM + f(c_1, c_2, \ldots, c_9)$

$$\mu_{X;Y} = \frac{\sigma(pp \to X)}{\sigma(pp \to X)_{\rm SM}} \frac{\Gamma(h \to Y)}{\Gamma(h \to Y)_{\rm SM}} \frac{\Gamma(h \to {\rm all})_{\rm SM}}{\Gamma(h \to {\rm all})}$$

**Other NP contributions (e.g. in Vff ) are strongly bounded using LEP1 data**
[Falkowski, Riva, 2015, Efrati, Falkowski, Soreq, 2015]

- In principle we have enough constraints to extract all:
  diff. Higgs production mechanism (ggF, VBF, Vh, …) & decay modes ($\gamma\gamma$, $\gamma Z$, 4l, …).

- Relation between TGC & anomalous Higgs couplings:
  $\delta g_{1z} = f(c_1, c_2, \ldots, c_9)$
  $\delta\kappa_\gamma = g(c_1, c_2, \ldots, c_9)$
  $\lambda_z$ = unconstrained

Higgs measurements imply bounds on TGC, & viceversa!
(again: global analysis is needed)

# Higgs data analysis

[See A. Falkowski, arXiv:1505.00046]

- Observables: signal-strengths;
- Higgs basis used (irrelevant);
- Only linear effects included
  (quadratic effects will be included as a check)
- All D=6 operators included simultaneously;
- Flavor universality assumed [MFV ≈ $U(3)^5$ ];
- LO calculation of the ratios $\sigma/\sigma_{SM} = 1 + \ldots$
  $\Gamma/\Gamma_{SM} = 1 + \ldots$
- Example:

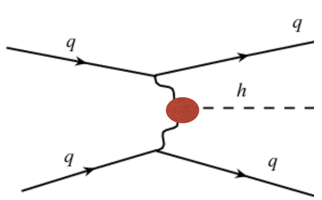
| Channel | $\mu_{\text{ATLAS}}$ | $\mu_{\text{CMS}}$ | Production |
|---|---|---|---|
| $\gamma\gamma$ | $1.17^{+0.28}_{-0.26}$ | $1.12^{+0.25}_{-0.22}$ | cats. |
| $Z\gamma$ | $2.7^{+4.6}_{-4.5}$ | $-0.2^{+4.9}_{-4.9}$ | total |
| $ZZ^*$ | $1.46^{+0.40}_{-0.34}$ | $1.00^{+0.29}_{-0.29}$ | 2D |
| $WW^*$ | $1.18^{+0.24}_{-0.21}$ | $0.83^{+0.21}_{-0.21}$ | 2D |
| | $2.1^{+1.9}_{-1.6}$ | - | Wh |
| | $5.1^{+4.3}_{-3.1}$ | - | Zh |
| | - | $0.80^{+1.09}_{-0.93}$ | Vh |
| $\tau\tau$ | $1.44^{+0.42}_{-0.37}$ | $0.91^{+0.28}_{-0.28}$ | 2D |
| | - | $0.87^{+1.00}_{-0.88}$ | Vh |
| $bb$ | $1.11^{+0.65}_{-0.61}$ | - | Wh |
| | $0.05^{+0.52}_{-0.49}$ | - | Zh |
| | - | $0.89^{+0.47}_{-0.44}$ | Vh |
| | - | $2.8^{+1.6}_{-1.4}$ | VBF |
| | $1.5^{+1.1}_{-1.1}$ | $1.2^{+1.6}_{-1.5}$ | tth |
| $\mu\mu$ | $-0.7^{+3.7}_{-3.7}$ | $0.8^{+3.5}_{-3.4}$ | total |
| multi-$\ell$ | $2.1^{+1.4}_{-1.2}$ | $3.8^{+1.4}_{-1.4}$ | tth |

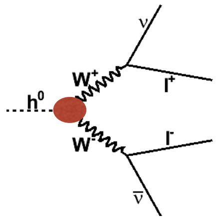
ggH
ttH
VBF
Wh
Zh

ggH+ttH
VBF+Vh

$$\frac{\sigma_{VBF}}{\sigma^{\text{SM}}_{VBF}} \simeq 1 + 2\delta c_z - 2.25 c_{z\Box} - 0.83 c_{zz} + 0.30 c_{z\gamma} + 0.12 c_{\gamma\gamma}.$$

$$\frac{\Gamma_{2\ell 2\nu}}{\Gamma^{\text{SM}}_{2\ell 2\nu}} \simeq 1 + 2\delta c_z + 0.67 c_{z\Box} + 0.05 c_{zz} - 0.17 c_{z\gamma} - 0.05 c_{\gamma\gamma}$$



# Higgs data analysis

Higgs data (signal strengths) → Bounds on 9 (linear combinations of) WC → Bounds on $\delta g_{1z}$ , $\delta \kappa_\gamma$

| Channel | $\mu_{\rm ATLAS}$ | $\mu_{\rm CMS}$ | Production |
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| $\gamma\gamma$ | $1.17^{+0.28}_{-0.26}$ | $1.12^{+0.25}_{-0.22}$ | cats. |
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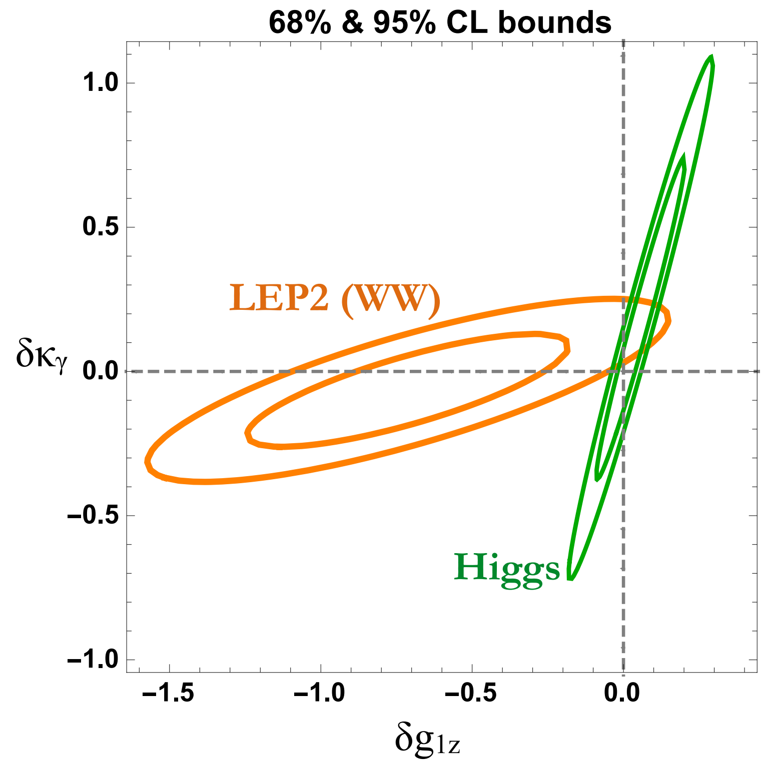
| | $2.1^{+1.9}_{-1.6}$ | - | Wh |
| | $5.1^{+4.3}_{-3.1}$ | - | Zh |
| | - | $0.80^{+1.09}_{-0.93}$ | Vh |
| $\tau\tau$ | $1.44^{+0.42}_{-0.37}$ | $0.91^{+0.28}_{-0.28}$ | 2D |
| | - | $0.87^{+1.00}_{-0.88}$ | Vh |
| $bb$ | $1.11^{+0.65}_{-0.61}$ | - | Wh |
| | $0.05^{+0.52}_{-0.49}$ | - | Zh |
| | - | $0.89^{+0.47}_{-0.44}$ | Vh |
| | - | $2.8^{+1.6}_{-1.4}$ | VBF |
| | $1.5^{+1.1}_{-1.1}$ | $1.2^{+1.6}_{-1.5}$ | tth |
| $\mu\mu$ | $-0.7^{+3.7}_{-3.7}$ | $0.8^{+3.5}_{-3.4}$ | total |
| multi-$\ell$ | $2.1^{+1.4}_{-1.2}$ | $3.8^{+1.4}_{-1.4}$ | tth |

→ WC bounds available in different basis in the paper →



# Higgs data analysis

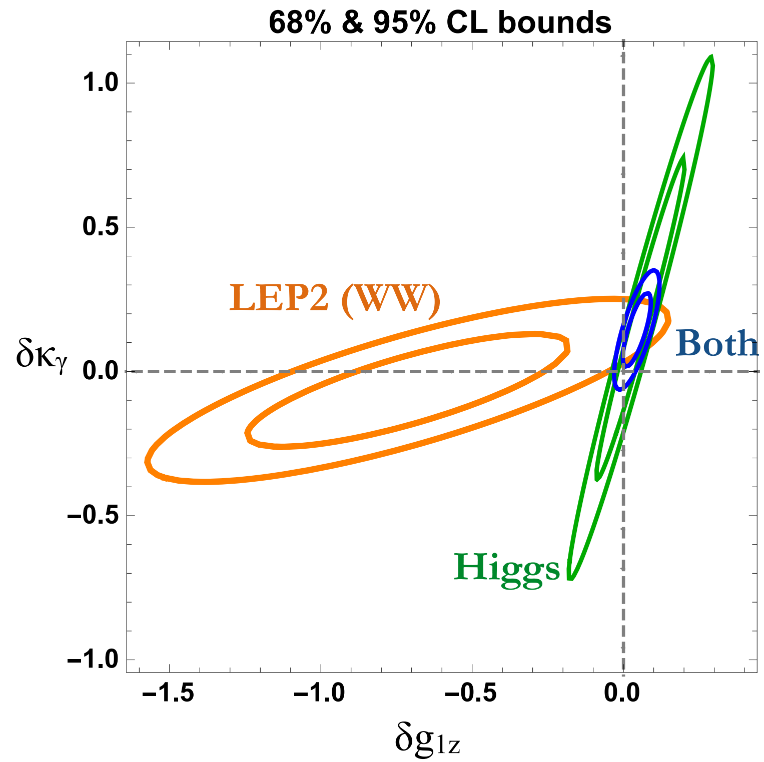
**Higgs data (signal strengths)** → **Bounds on 9 (linear combinations of) WC** → **Bounds on $\delta g_{1z}$ , $\delta\kappa_\gamma$**

+ LEP2 data ($e^+e^- \to W^+W^-$)

[+ $\lambda_z$ ]

[+ $\lambda_z$ ]

- Clear complementarity;
- %-level bounds on TGC!
- The impact of the inclusion of quadratic effects on these bounds is small; (not true for the separated datasets!)

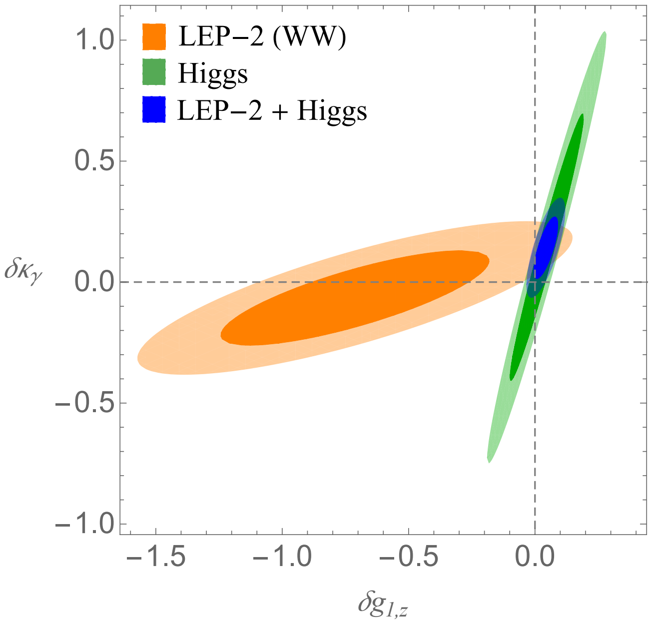



$$\begin{pmatrix} \delta g_{1,z} \\ \delta\kappa_\gamma \\ \lambda_z \end{pmatrix} = \begin{pmatrix} 0.043 \pm 0.031 \\ 0.142 \pm 0.085 \\ -0.162 \pm 0.073 \end{pmatrix}, \qquad \rho = \begin{pmatrix} 1 & 0.74 & -0.85 \\ 0.74 & 1 & -0.88 \\ -0.85 & -0.88 & 1 \end{pmatrix}.$$

# Summary

- Combining Higgs data and WW production at LEP we obtained, for the first time, strong and consistent bounds on anomalous Triple Gauge Couplings.
- Our results can be used to set bounds in specific models in a simple way!
- It shows clearly how the Higgs boson has become a precision tool to search for New Physics.
- The addition of direct TGC measurements at the LHC should improve these bounds, but some work is required [*backup slide*];

*[Falkowski, MGA, Greljo & Marzocca, Phys. Rev. Lett. 116 (2016), 011801]*




**Merci beaucoup!**

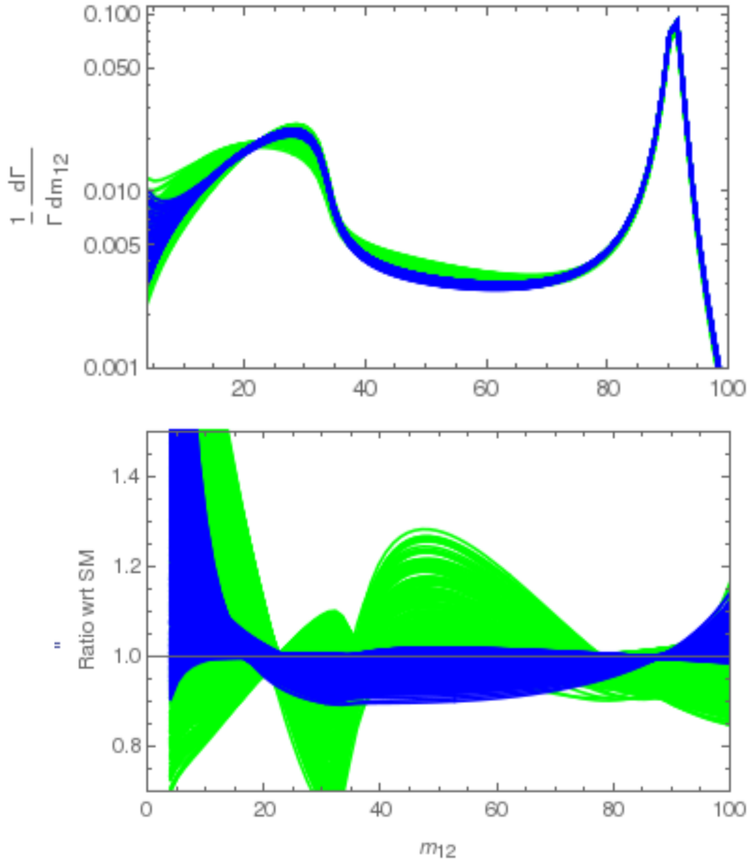
$$\begin{pmatrix} \delta g_{1,z} \\ \delta\kappa_\gamma \\ \lambda_z \end{pmatrix} = \begin{pmatrix} 0.043 \pm 0.031 \\ 0.142 \pm 0.085 \\ -0.162 \pm 0.073 \end{pmatrix}, \qquad \rho = \begin{pmatrix} 1 & 0.74 & -0.85 \\ 0.74 & 1 & -0.88 \\ -0.85 & -0.88 & 1 \end{pmatrix}.$$

# Backup slides

# Consequences for $h \to e^+ e^- \mu^+ \mu^-$

[MGA, Greljo, Isidori & Marzocca, EPJC75 (2015), 128]

[MGA, Greljo, Isidori & Marzocca, EPJC75 (2015), 341]




Higgs (μ)

Higgs (μ) + LEP2

LEP I

$$\epsilon_{Zf} = \sqrt{g^2+g'^2}\,\left(\cancel{\delta g^{Zf}} - (c_\theta^2 T_f^3 + s_\theta^2 Y_f)\,\delta g_{1,z} + t_\theta^2 Y_f\,\delta\kappa_\gamma\right),$$

$$\delta\epsilon_{ZZ} = \cancel{\delta\epsilon_{\gamma\gamma}} + \frac{c_{2\theta}}{s_\theta c_\theta}\cancel{\delta\epsilon_{Z\gamma}} - \frac{1}{c_\theta^2}\delta\kappa_\gamma$$

h→γγ h→Zγ

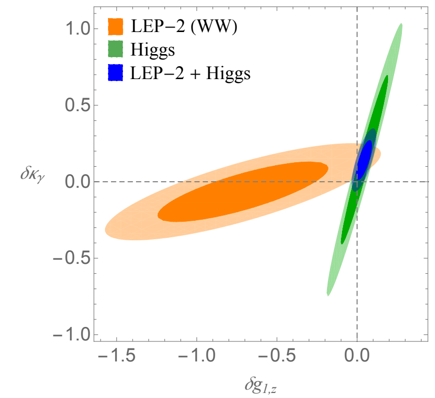
**Higgs decay pseudo-observables better constrained**

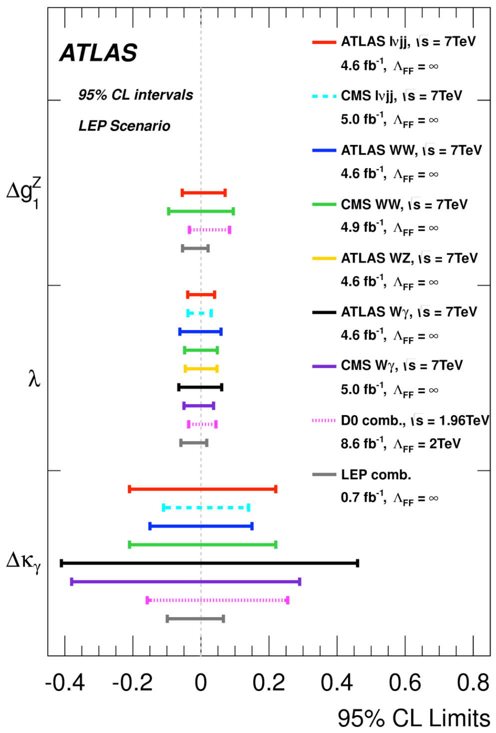
$$\begin{pmatrix} \kappa_{ZZ} = 0.85 \pm 0.17 \\ \epsilon_{Z\ell_L} = -0.0001 \pm 0.0078 \\ \epsilon_{Z\ell_R} = -0.025 \pm 0.015 \\ \kappa_{Z\gamma} = 0.96 \pm 1.6 \\ \kappa_{\gamma\gamma} = 0.88 \pm 0.19 \end{pmatrix}, \quad \rho = \begin{pmatrix} 1 & .72 & .60 & .19 & .83 \\ \cdot & 1 & .35 & -.16 & .62 \\ \cdot & \cdot & 1 & .02 & .47 \\ \cdot & \cdot & \cdot & 1 & .20 \\ \cdot & \cdot & \cdot & \cdot & 1 \end{pmatrix}$$

# TGC measurements @ LHC

Why LHC WW/WZ searches are not in the plot?

They TGC sensitivity seems significant…




We need…

- All 3 TGC present, correlations provided;
- Analysis performed consistently at $O(1/\Lambda^2)$;
  - Trivial implementation, but…
  - Introduction of kinematical cuts likely needed
    (new wrt LEP, where energy of partonic collisions was fixed and not very high)

NOTES:
If an observable is very sensitive to the quadratic term…
- it does not mean that it is useless (it is quite likely a very sensitive NP probe), but we cannot do the analysis “once and for all” à la EFT…
- Analyses including quadratic terms are OK for certain class of models (strong coupling);

# More details…

## Vh & quadratic terms

- For Vh production, quadratic terms are comparable to linear terms; (small impact on TGC bounds, though)
- Sensitivity to $O(\Lambda^{-4})$ reduced using $\mu\ (m_{Vh}) < 400$ GeV.

## Results available in different bases

Higgs basis

$$\begin{pmatrix} \delta c_z \\ c_{zz} \\ c_{z\Box} \\ c_{\gamma\gamma} \\ c_{z\gamma} \\ c_{gg} \\ \delta y_u \\ \delta y_d \\ \delta y_e \\ \lambda_z \end{pmatrix} = \begin{pmatrix} -0.02 \pm 0.17 \\ 0.69 \pm 0.42 \\ -0.32 \pm 0.19 \\ 0.009 \pm 0.015 \\ 0.002 \pm 0.098 \\ -0.0052 \pm 0.0027 \\ 0.57 \pm 0.30 \\ -0.24 \pm 0.35 \\ -0.12 \pm 0.20 \\ -0.162 \pm 0.073 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -.04 & -.21 & -.76 & -.15 & .15 & .12 & .88 & .71 & -.22 \\ \cdot & 1 & -.96 & .37 & .19 & .03 & .04 & -.12 & -.31 & -.88 \\ \cdot & \cdot & 1 & -.17 & -.10 & -.07 & -.06 & -.10 & .12 & .93 \\ \cdot & \cdot & \cdot & 1 & .20 & -.12 & -.07 & -.79 & -.74 & -.13 \\ \cdot & \cdot & \cdot & \cdot & 1 & -.01 & -.01 & -.15 & -.18 & -.10 \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 & -.87 & .26 & .17 & -.07 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & .13 & .11 & -.06 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & .81 & -.11 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & .09 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 \end{pmatrix}$$

Warsaw basis

$$\begin{pmatrix} c_H & = 0.11 \pm 0.15 \\ c_T & = 0.034 \pm 0.021 \\ c_{WB} & = 0.34 \pm 0.20 \\ c_{WW} & = 0.69 \pm 0.43 \\ c_{BB} & = 0.69 \pm 0.42 \\ c_{GG} & = -0.0052 \pm 0.0027 \\ \hat{c}_u & = 0.65 \pm 0.32 \\ \hat{c}_d & = -0.16 \pm 0.23 \\ \hat{c}_e & = -0.03 \pm 0.13 \\ c_{3W} & = 0.63 \pm 0.29 \end{pmatrix}$$

$$\begin{pmatrix} 1 & .51 & .38 & 0.43 & .34 & -.11 & .37 & -.62 & -.01 & .16 \\ \cdot & 1 & .97 & .94 & .96 & .00 & .22 & -.13 & -.17 & .79 \\ \cdot & \cdot & 1 & .97 & .97 & .03 & .16 & .01 & -.16 & .88 \\ \cdot & \cdot & \cdot & 1 & .89 & .03 & .18 & -.01 & -.16 & .87 \\ \cdot & \cdot & \cdot & \cdot & 1 & .03 & .14 & .01 & -.15 & .84 \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 & -.87 & .31 & .11 & .07 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & -.19 & .03 & .07 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & .37 & .18 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & -.11 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 \end{pmatrix}$$