Nuclear Reaction from a Structure Point of View

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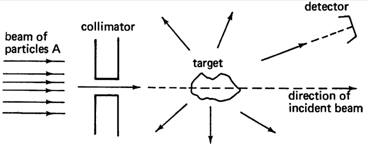
NUCLEON SCATTERING

Reminder on cross section

- Consider a beam of particles hitting a thin sheet of material
- $N_i = J_i S$ incident particles hit the surface per second
- N_c outgoing particles counted per second (only count particles belonging to an outgoing channel c. For instance elastic channel: detection of a particle with the same energy than the incident particle)
- Probability P_c of reaction: $P_c = \frac{N_c}{N_i} = \frac{N_c}{J_i S}$



- The cross section σ_c is an effective area associated to one target nucleus, that provides a measure of the probability of reaction in the channel c.
- $\Sigma_c = \sigma_c N_t$ ($N_t = nSdx$ number of target nuclei) is the portion of the surface S which, when hit by the incident particle, will lead to the reaction channel c.

$$P_c = \frac{\Sigma_c}{S} = \frac{N_c}{J_i S}, \quad \sigma_c = \frac{N_c}{N_t}\frac{1}{J_i} = \frac{\text{reaction rate}}{\text{incident flux}}$$

Optical potential

- Problem
  - Nucleon scattering from a target nucleus is a many-body problem with A bound nucleons and a scattered one: very difficult...
  - Many body problem approximated by a two-body problem

$$\left(\frac{\hbar^2}{2\mu}\nabla^2 + V(r)\right)\phi(\mathbf{r}) = E\phi(\mathbf{r})$$

    with $V(\mathbf{r})$ a one-body effective potential
- Requirements
  - V should describe the direct reaction in a nuclear collision and should give the energy averaged scattering amplitude
  - V should take into account in an effective way all the inelastic channels
- Solution
  - Complex one-body potential: $V(\mathbf{r}) = U(\mathbf{r}) + iW(\mathbf{r})$
  - Real part: simple refraction of the incident wave
  - Imaginary part models flux loss during the elastic scattering process

Absorption by a complex potential

Probability current:

$$\mathbf{j(r)} = -i\frac{\hbar}{2\mu}\left(\phi^*(\mathbf{r})\nabla\phi(\mathbf{r}) - \phi(\mathbf{r})\nabla\phi^*(\mathbf{r})\right)$$

Schrödinger Equation:

$$\left(\frac{\hbar^2}{2\mu}\nabla^2 + (U(r) + iW(r))\right)\phi(\mathbf{r}) = E\phi(\mathbf{r})$$

$\phi^*(\mathbf{r}) \times \{S.E.\} - \phi(\mathbf{r})\,\{S.E.\}^*$:

Flux variation: $\nabla.\mathbf{j} = \frac{i}{\hbar}(V^* - V(r))|\phi(r)|^2 = \frac{2}{\hbar}W(r)|\phi(r)|^2$

Negative imaginary potential: flux absorption

Schrödinger equation with a spherical potential

$$H|\psi\rangle = (T+V)|\psi\rangle = E|\psi\rangle$$

$$\int \langle \mathbf{r}|(T+V)|\mathbf{r}'\rangle\langle \mathbf{r}'|\psi\rangle d\mathbf{r}' = E\langle \mathbf{r}|\psi\rangle$$

Kinetic part

$$\begin{aligned}
T &= \frac{\mathbf{P}^2}{2m} \\
\langle \mathbf{r}|\mathbf{P}^2|\mathbf{r}'\rangle &= \int \langle \mathbf{r}|\mathbf{P}^2|\mathbf{p}\rangle\langle \mathbf{p}|\mathbf{r}'\rangle d\mathbf{p} \\
&= \int \langle \mathbf{r}|\mathbf{p}\rangle \mathbf{p}^2 \langle \mathbf{p}|\mathbf{r}'\rangle d\mathbf{p} \\
&= \frac{1}{(2\pi\hbar)^3}\int \mathbf{p}^2 e^{\frac{i}{\hbar}\mathbf{p}.(\mathbf{r}\text{-}\mathbf{r}')} d\mathbf{p} \\
&= -\hbar^2\delta''(\mathbf{r}\text{-}\mathbf{r}') \\
\langle \mathbf{r}|T|\psi\rangle &= -\frac{\hbar^2}{2m}\Delta\psi(\mathbf{r})
\end{aligned}$$

Potential part

$$\begin{aligned}
\langle \mathbf{r}|V|\psi\rangle &= \int d\mathbf{r}'\langle \mathbf{r}|V|\mathbf{r}'\rangle\langle \mathbf{r}'|\psi\rangle \\
\langle \mathbf{r}|V|\mathbf{r}'\rangle &\equiv V(\mathbf{r},\mathbf{r}')
\end{aligned}$$

Local potential

$$V(\mathbf{r},\mathbf{r}') = V(r)\delta(\mathbf{r},\mathbf{r}')$$

Schrödinger equation with a spherical potential

$$-\frac{\hbar^2}{2m}\Delta\psi(\mathbf{r}) + \int d\mathbf{r}'\, V(\mathbf{r},\mathbf{r}')\psi(\mathbf{r}') = E\psi(\mathbf{r})$$

Spherical coordinates,

$$\left.\begin{aligned}\Delta \equiv &\quad p_r^2 + \frac{\mathbf{l}^2}{r^2} \\ p_r^2 = &\quad -\hbar^2 \frac{1}{r}\frac{d^2}{dr^2} r\end{aligned}\right\} \langle \mathbf{r}|T|\psi\rangle = \left[-\frac{\hbar^2}{2m}\frac{1}{r}\frac{d^2}{dr^2}r + \frac{\mathbf{l}^2}{2mr^2}\right]\psi(\mathbf{r})$$

Using the following multipole expansions and projecting on $|ljm\rangle$

$$\psi(\mathbf{r}) = \sum_{ljm}\frac{u_{jlm}(r)}{r}\mathcal{Y}_{jl}^{m}(\hat{\mathbf{r}}) \quad \text{and} \quad \nu_{ljm}(r,r') = \iint d\hat{\mathbf{r}}\,d\hat{\mathbf{r}}'\,\mathcal{Y}_{jl}^{m}(\hat{\mathbf{r}})V(\mathbf{r},\mathbf{r}')\mathcal{Y}_{jl}^{m\dagger}(\hat{\mathbf{r}}')$$

Integro-differential Schrödinger equation

$$-\frac{\hbar^2}{2m}\left[\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2}\right]u_{ljm}(r) + \int dr'\, r\nu_{ljm}(r,r')r'\, u_{jlm}(r') = E\; u_{ljm}(r)$$

Phase shift determination

Integro-differential Schrödinger equation

$$-\frac{\hbar^2}{2m}\left[\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\right]u_{ljm}(r)+\int dr' r\nu_{ljm}(r,r')r' u_{jlm}(r') = E\ u_{ljm}(r)$$

Equations can be expressed on a radial mesh with h the step. The potential is negligible at $R_{max} = h \times N$.

$$\begin{aligned} u(r) &\longrightarrow u_i \\ \frac{d^2}{dr^2}u(r) &\longrightarrow \frac{u_{i+1}-2u_i+u_{i-1}}{h^2} \\ \nu(r,r') &\longrightarrow \nu_{ij} \end{aligned}$$

Schrödinger equation reads

$$\left[\begin{pmatrix} -2 & 1 & & & \\ 1 & -2 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & -2 & 1 \\ & & & 1 & -2 \end{pmatrix} + \begin{pmatrix} M_{1,1} & \cdots & & \\ \vdots & \ddots & & \\ & & & \\ & & \ddots & \vdots \\ & & \cdots & M_{N,N} \end{pmatrix}\right]\begin{pmatrix} u_1 \\ \vdots \\ \\ u_N \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ -1 \end{pmatrix}$$

Conditions at the limits: $u_0 = 0,\ u_{N+1} = 1,\ M_{i,N+1} = 0$

Phase shift determination

$$\sum_k \mathcal{M}_{i,k} u_k = \begin{pmatrix} 0 \\ \vdots \\ \\ 0 \\ -1 \end{pmatrix}$$

Solution merges from matrix inversion

$$u_i = -\left(\mathcal{M}^{-1}\right)_{i,N}$$

Solution can further be re-injected into Schrödinger equation with better precision and iterated until the needed precision is obtained.

Phase shift determination

Connection to asymptotic solutions

$$u_{lj}(r) \underset{r\to+\infty}{=} C[\cos(\delta_{lj})j_l(kr) - \sin(\delta_{lj})n_l(kr)]$$

$$\text{avec} \qquad k^2 = -(2m/\hbar^2)\times E$$

with j_l, n_l Bessel and Neumann spherical functions.

Normalisation by a Dirac in energy

$$C = \sqrt{\frac{1}{\pi}\frac{2m}{\hbar^2 k}}$$

Phase shift is obtained from

$$\frac{u'_N}{u_N} = \frac{\cos(\delta_{lj})j'_l(kR_{max}) - \sin(\delta_{lj})n'_l(kR_{max})}{\cos(\delta_{lj})j_l(kR_{max}) - \sin(\delta_{lj})n_l(kR_{max})}$$

Phase shift determination

Connection to asymptotic solutions

$$u_{lj}(r) \underset{r\to+\infty}{=} C[\cos(\delta_{lj})j_l(kr) - \sin(\delta_{lj})n_l(kr)]$$

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Normalisation by a Dirac in energy

$$C = \sqrt{\frac{1}{\pi}\frac{2m}{\hbar^2 k}}$$

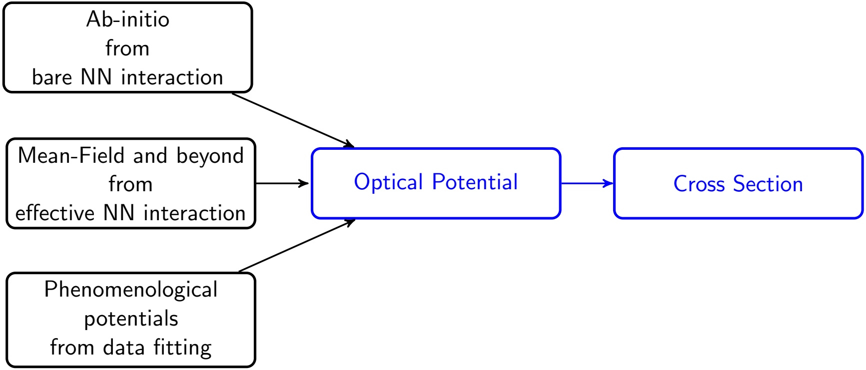
Phase shift

$$\tan(\delta_{lj}) = \frac{u_N j_l'(kR_{max}) - u_N' j_l(kR_{max})}{u_N n_l'(kR_{max}) - u_N' n_l(kR_{max})}$$

From optical potential to reaction observables

Cross Section (without spin)

$$\sigma_{el} = \frac{\pi}{k^2} \sum_{\ell} |1 - S_\ell|^2 \quad \text{with} \quad S_\ell = e^{i2\delta_\ell}$$



Mean-Field and beyond from effective NN interaction → Optical Potential

Goals

- Build an optical potential from an effective NN interaction
- Consistent use of the effective NN interaction
- Self-consistency

Tools

- Green's functions formalism
- Gogny D1S phenomenological effective interaction

EFFECTIVE NN INTERACTION

Effective NN interaction: Pros and cons

Pros

- Phenomenological account of short range correlations
- Simple shape
- Energy independent
- Extended reach of EDF approaches

Cons

- Simple shape
- Validity out of the parametrization range
- Loss of the contact with more fundamental theories

Skyrme and Gogny interactions

Skyrme interaction

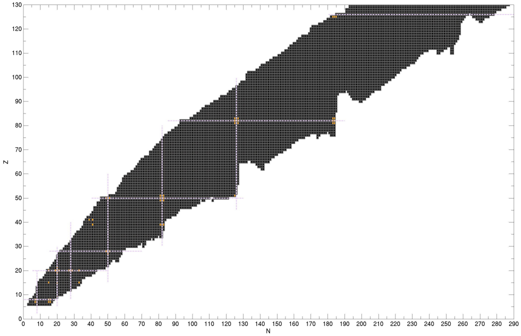
Zero-range interaction

Gogny interaction

Finite-range interaction
(Brink and Boeker)

Extended reach of EDF approaches

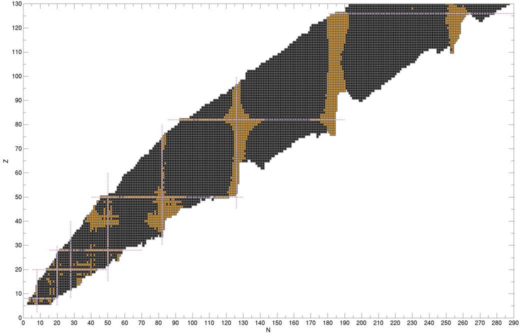
Spherical Hartree-Fock (∼30 nuclei)



Calculations with Gogny D1S interaction (S. Hilaire and J.P. Ebran)

Extended reach of EDF approaches

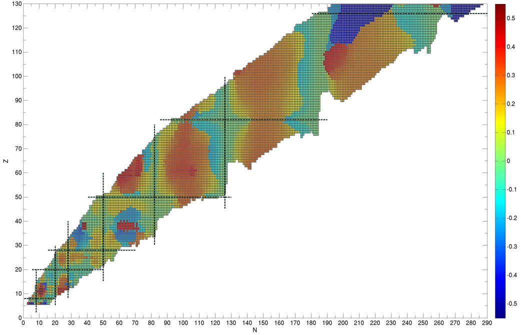
Spherical Hartree-Fock-Bogoliubov (∼300 nuclei)



Calculations with Gogny D1S interaction (S. Hilaire and J.P. Ebran)

Extended reach of EDF approaches

Axially-deformed Hartree-Fock-Bogoliubov ($\sim$6000 nuclei)



Calculations with Gogny D1S interaction (S. Hilaire and J.P. Ebran)

GREEN'S FUNCTIONS

Definitions

The state $|\alpha, t_0\rangle$ of a particle with quantum numbers α at time t_0 evolves in

$$|\alpha, t_0; t\rangle = e^{-\frac{i}{\hbar}H(t-t_0)}|\alpha, t_0\rangle$$

at a time t ($t > t_0$) and for a time-independent Hamiltonian.

$$\begin{aligned}
\psi(\mathbf{r}, t) &= \langle \mathbf{r}|\alpha, t_0; t\rangle = \langle \mathbf{r}|e^{-\frac{i}{\hbar}H(t-t_0)}|\alpha, t_0\rangle \\
&= \int d\mathbf{r'}\langle \mathbf{r}|e^{-\frac{i}{\hbar}H(t-t_0)}|\mathbf{r'}\rangle\langle \mathbf{r'}|\alpha, t_0\rangle \\
&= i\hbar \int d\mathbf{r'}\, G(\mathbf{r}, \mathbf{r'}; t-t_0)\psi(\mathbf{r'}, t_0)
\end{aligned}$$

where G is referred to as

Propagator or Green's Function

$$G(\mathbf{r}, \mathbf{r'}; t-t_0) = -\frac{i}{\hbar}\langle \mathbf{r}|e^{-\frac{i}{\hbar}H(t-t_0)}|\mathbf{r'}\rangle$$

Propagator or Green's Function

$$G(\mathbf{r},\mathbf{r'};t-t_0) = -\frac{i}{\hbar}\langle \mathbf{r}|e^{-\frac{i}{\hbar}H(t-t_0)}|\mathbf{r'}\rangle$$

$$\psi(\mathbf{r},t) = i\hbar\int d\mathbf{r'}\,G(\mathbf{r},\mathbf{r'};t-t_0)\psi(\mathbf{r'},t_0)$$

The wave function at **r** and t is determined by the wave function at the original time t_0, receiving contributions from all **r'** weighted by the amplitude G.

Operators and Statistics

Second quantization

$\psi^\dagger(\mathbf{r}, t)$ creates a particle at $(\mathbf{r}, t)$

$\psi(\mathbf{r}, t)$ annihilates a particle at $(\mathbf{r}, t)$

Bose-Einstein statistics (-)/Fermi-Dirac statistics (+)

$$\left[\psi^\dagger(\mathbf{r}, t), \psi^\dagger(\mathbf{r}', t)\right]_\pm = 0$$

$$\left[\psi^\dagger(\mathbf{r}, t), \psi(\mathbf{r}', t)\right]_\pm = 0$$

$$\left[\psi(\mathbf{r}, t), \psi^\dagger(\mathbf{r}', t)\right]_\pm = \delta(\mathbf{r} - \mathbf{r}')$$

$$G(\mathbf{r},\mathbf{r'};t-t_0) = -\frac{i}{\hbar}\langle\mathbf{r}|e^{-\frac{i}{\hbar}H(t-t_0)}|\mathbf{r'}\rangle = -\frac{i}{\hbar}\langle 0|a_{\mathbf{r}}e^{-\frac{i}{\hbar}H(t-t_0)}a^\dagger_{\mathbf{r'}}|0\rangle$$
$$= -\frac{i}{\hbar}\sum_{nn'}\langle 0|a_{\mathbf{r}}|n\rangle\langle n|e^{-\frac{i}{\hbar}H(t-t_0)}|n'\rangle\langle n'|a^\dagger_{\mathbf{r'}}|0\rangle$$

One-body propagator in second quantization

$$G(1,1') = -i\langle 0|\mathcal{T}(\psi(1)\psi^\dagger(1'))|0\rangle$$

$\mathcal{T}$ is the time ordering operator and $1 \equiv \mathbf{r}_1, t_1$

Ex: $$\mathcal{T}(\psi(1)\psi^\dagger(1')) = \psi(1)\psi^\dagger(1') \quad \text{if } t_1 > t_{1'}$$
$$= -\psi^\dagger(1')\psi(1) \quad \text{if } t_1 < t_{1'}$$

$$G(\mathbf{r},\mathbf{r'};t-t_0) = -\frac{i}{\hbar}\langle\mathbf{r}|e^{-\frac{i}{\hbar}H(t-t_0)}|\mathbf{r'}\rangle = -\frac{i}{\hbar}\langle 0|a_{\mathbf{r}}e^{-\frac{i}{\hbar}H(t-t_0)}a^{\dagger}_{\mathbf{r'}}|0\rangle$$

$$= -\frac{i}{\hbar}\sum_{nn'}\langle 0|a_{\mathbf{r}}|n\rangle\langle n|e^{-\frac{i}{\hbar}H(t-t_0)}|n'\rangle\langle n'|a^{\dagger}_{\mathbf{r'}}|0\rangle$$

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Particle propagator $t_1 > t_{1'}$

$$G_1(1,1') = i\langle 0|\psi(1)\psi^{\dagger}(1')|0\rangle$$

Hole propagator $t_1 < t_{1'}$

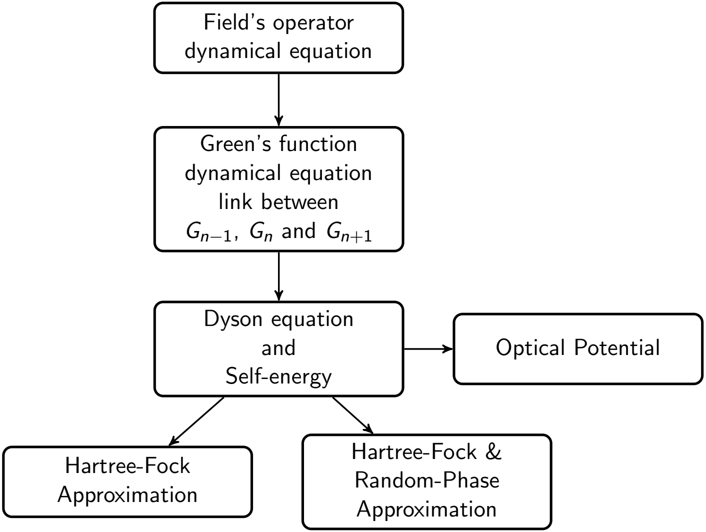
$$G_1(1,1') = -i\langle 0|\psi^{\dagger}(1')\psi(1)|0\rangle$$

n-body Green's function

$$G_n = (-i)^n \langle 0|\mathcal{T}\{\psi(1)...\psi(n)\psi^\dagger(n')...\psi^\dagger(1')\}|0\rangle$$

Green's functions are average value
of
creation and annihilation operators

Road map


Field's operator
dynamical equation
Green's function
dynamical equation
link between
G_{n-1}, G_n and G_{n+1}
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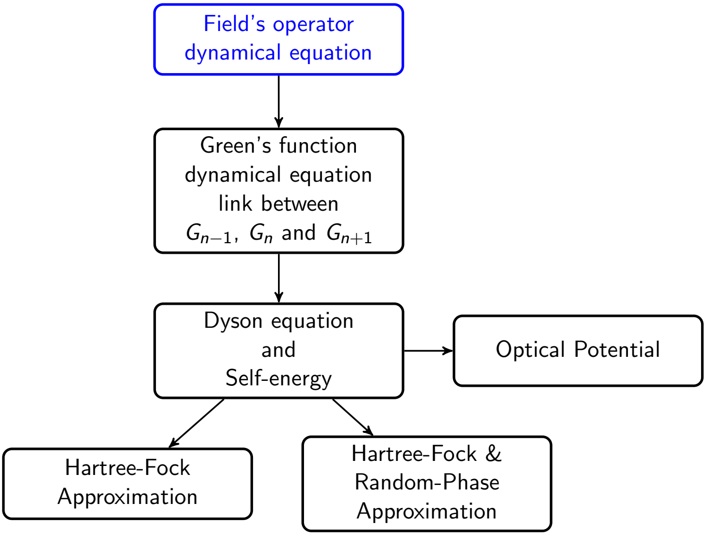
Field's operator dynamical equation
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Field's operator dynamical equation

Heisenberg picture

$$\begin{cases} \text{Time-dependent operators} \\ \text{Time-independent state vectors} \end{cases}$$

Schrödinger picture

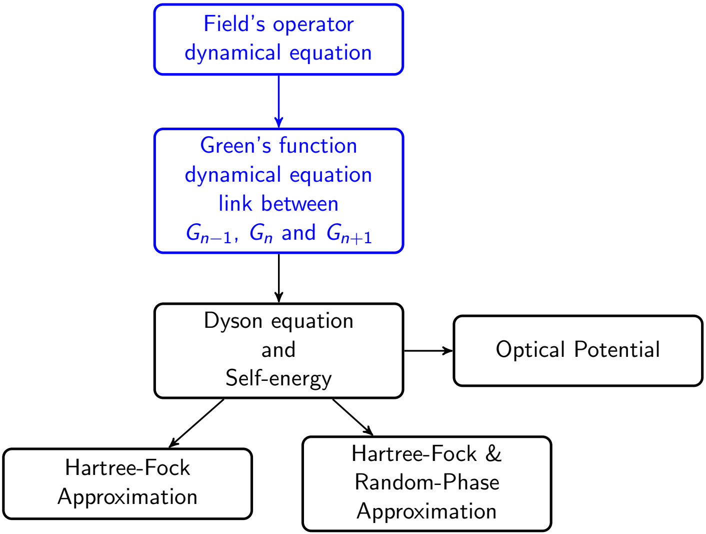
$$\begin{cases} \text{Time-independent operators} \\ \text{Time-dependent state vectors} \end{cases}$$

$$\langle \psi'_S(t) | \hat{O}_S | \psi_S(t) \rangle = \langle \psi'_H | \underbrace{e^{i\hat{H}t/\hbar} \hat{O}_S e^{-i\hat{H}t/\hbar}}_{\hat{O}_H(t)} | \psi_H \rangle$$

$$i\hbar \frac{\partial \hat{O}_H(t)}{\partial t} = e^{i\hat{H}t/\hbar} [\hat{O}_S, \hat{H}] e^{-i\hat{H}t/\hbar} = [\hat{O}_H(t), \hat{H}]$$

Equation of motion for an operator $\hat{O}_H(t)$ in Heisenberg picture

$$i\hbar \frac{\partial \hat{O}_H(t)}{\partial t} = \left[\hat{O}_H(t), H \right]$$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Green's function dynamical equation: one-body case

Equation of motion
for an operator $\hat{O}_H(t)$
in Heisenberg picture

$$i\hbar\frac{\partial \hat{O}_H(t)}{\partial t} = \left[\hat{O}_H(t), H\right]$$

Hamiltonian in second quantization

$$\begin{aligned} H &= T + V \\ T &= \frac{\hbar^2}{2m}\int \psi^\dagger(x)\Delta\psi(x)dx \\ V &= \frac{1}{2}\int \psi^\dagger(x)\psi^\dagger(x')v(x,x')\psi(x')\psi(x)dxdx' \end{aligned}$$

$$i\frac{\partial\psi(x)}{\partial t} = -\frac{1}{2m}\Delta\psi(x) + \int dx'' v(x,x'')\psi^\dagger(x'')\psi(x'')\psi(x)$$

$$i\frac{\partial\psi(x)}{\partial t} = -\frac{1}{2m}\Delta\psi(x) + \int dx'' v(x,x'')\psi^\dagger(x'')\psi(x'')\psi(x)$$

$\times\psi^\dagger(x')$ from the right and applying T

Keeping in mind the definition...

One-body Green's function

$$G(1,1') = -i\langle 0|\mathcal{T}(\psi(1)\psi^\dagger(1'))|0\rangle$$

$$i\langle 0|\mathcal{T}\left(\frac{\partial}{\partial t}\psi(x)\psi^\dagger(x')\right)|0\rangle = -\frac{1}{2m}\langle 0|\mathcal{T}\left(\Delta\psi(x)\psi^\dagger(x')\right)|0\rangle + \int dx'' v(x,x'')\langle 0|\mathcal{T}\left(\psi^\dagger(x'')\psi(x'')\psi(x)\psi^\dagger(x')\right)|0\rangle$$

$\mathcal{T}$ commutes with Δ

$\Delta G_1(x,x')$

$$i\langle 0|\mathcal{T}\left(\frac{\partial}{\partial t}\psi(x)\psi^\dagger(x')\right)|0\rangle = -\frac{1}{2m}\langle 0|\mathcal{T}\left(\Delta\psi(x)\psi^\dagger(x')\right)|0\rangle + \int dx'' v(x,x'')\langle 0|\mathcal{T}\left(\psi^\dagger(x'')\psi(x'')\psi(x)\psi^\dagger(x')\right)|0\rangle$$

$\mathcal{T}$ doesn't commute with $\frac{\partial}{\partial t}$

Two-body Green's function

$G_2(x''x;x''_+x')$

$$\mathcal{T}(\psi(x)\psi^\dagger(x')) = \theta(t-t')\psi(x)\psi^\dagger(x') - \theta(t'-t)\psi^\dagger(x')\psi(x)$$

$$\frac{\partial}{\partial t}\left\{\mathcal{T}(\psi(x)\psi^\dagger(x'))\right\} = \delta(x-x') + \mathcal{T}(\frac{\partial}{\partial t}\psi(x)\psi^\dagger(x'))$$

$$\frac{\partial}{\partial t}G_1(x,x') = \delta(x-x') + \langle 0|\mathcal{T}(\frac{\partial}{\partial t}\psi(x)\psi^\dagger(x'))|0\rangle$$

$$\left(i\frac{\partial}{\partial t}+\frac{1}{2m}\Delta\right)G_1(x,x')=\delta(x-x')-i\int dx''v(x,x'')G_2(x''x;x''_+x')$$

Definition of the free propagator

$$\left(i\frac{\partial}{\partial t}+\frac{1}{2m}\Delta\right)G_0(x,x')=\delta(x-x')$$

$$\left(i\frac{\partial}{\partial t}+\frac{1}{2m}\Delta\right)G_1(x,x')=\left(i\frac{\partial}{\partial t}+\frac{1}{2m}\Delta\right)G_0(x,x')$$
$$-i\int dx''dx'''\left(i\frac{\partial}{\partial t}+\frac{1}{2m}\Delta\right)G_0(x,x''')v(x''',x'')G_2(x''x;x''_+x')$$

$$\left(i\frac{\partial}{\partial t}+\frac{1}{2m}\Delta\right)G_1(x,x')=\delta(x-x')$$
$$-i\int dx''dx'''\delta(x-x''')v(x''',x'')G_2(x''x;x''_+x')$$

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$$-i\int dx''dx'''\left(i\frac{\partial}{\partial t}+\frac{1}{2m}\Delta\right)G_0(x,x''')v(x''',x'')G_2(x''x;x''_+x')$$

Dynamical equation for G_1

$$G_1(1,1') = G_0(1,1') - i \int d2d3 G_0(1,2) v(2,3) G_2(23;1'3^+)$$

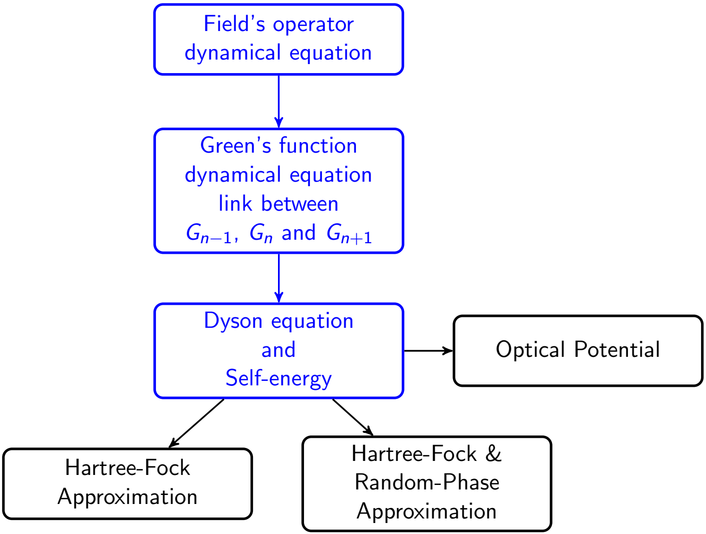
The dynamical equation for the one-body Green's function connects G_0, G_1 and G_2.

Dynamical equation for G_n

$$\left(i\frac{\partial}{\partial_1}+\frac{1}{2m}\Delta_1\right)G_n(1...n;1'...n') = \left\{G_{n-1}(2...n;2'...n')\delta(1-1')\right\}_{sym} - i\int dm\ v(1,m)G_{n+1}(1...n,m;1'...n'm^+)$$

where $\{\ \ \}_{sym}$ stands for the summation of the terms where $1'$ is replaced by $2', ..., n'$ with a $\pm$ signe corresponding to the parity of the permutation

(For the complete demo, see Fetter & Walecka...).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Dyson equation

$$G_1(1,1') = G_0(1,1') + \int d2d3\, G_0(1,2)\underbrace{\Sigma(2,3)}_{\text{Self-energy}} G_1(3,1')$$

Self-energy

$$\int d3\Sigma(2,3)G_1(3,1') = -i\int d3v(2,3)G_2(23,1'3^+)$$

Dynamical equation for G_1

$$G_1(1,1') = G_0(1,1') - i\int d2d3 G_0(1,2)v(2,3)G_2(23;1'3^+)$$

Dyson equation

$$G_1(1,1') = G_0(1,1') + \int d2d3 G_0(1,2) \underbrace{\Sigma(2,3)}_{\text{Self-energy}} G_1(3,1')$$

Self-energy

$$\int d1'd3\Sigma(2,3)G_1(3,1')G_1^{-1}(1',4) = -i\int d1'd3 v(2,3)G_2(23,1'3^+)G_1^{-1}(1',4)$$

Dynamical equation for G_1

$$G_1(1,1') = G_0(1,1') - i\int d2d3\, G_0(1,2)v(2,3)G_2(23;1'3^+)$$

Dyson equation

$$G_1(1,1') = G_0(1,1') + \int d2d3\, G_0(1,2)\underbrace{\Sigma(2,3)}_{\text{Self-energy}} G_1(3,1')$$

Self-energy

$$\int d1'd3\Sigma(2,3)\overbrace{G_1(3,1')G_1^{-1}(1',4)}^{\delta(3,4)} = -i\int d1'd3v(2,3)G_2(23,1'3^+)G_1^{-1}(1',4)$$

Dynamical equation for G_1

$$G_1(1,1') = G_0(1,1') - i \int d2d3 G_0(1,2) v(2,3) G_2(23;1'3^+)$$

Dyson equation

$$G_1(1,1') = G_0(1,1') + \int d2d3 G_0(1,2) \underbrace{\Sigma(2,3)}_{\text{Self-energy}} G_1(3,1')$$

Self-energy

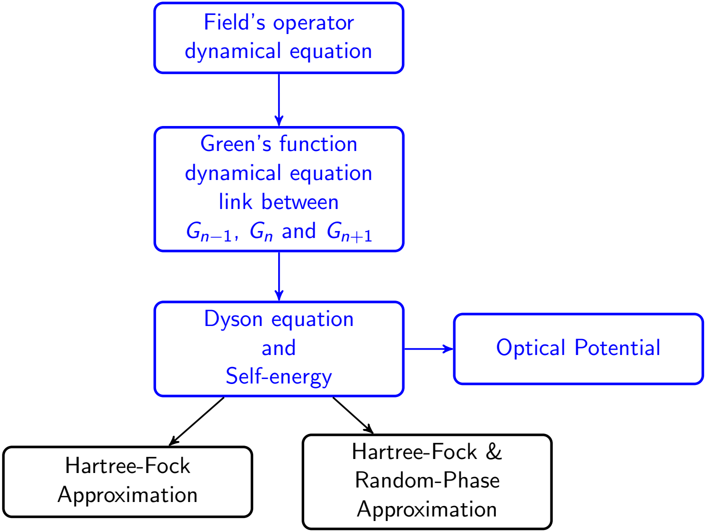
$$\Sigma(2,3) = -i \int d4d5 v(2,4) G_2(24,54^+) G_1^{-1}(5,3)$$

Self-energy

$$\Sigma(2,3) = -i \int d4d5 v(2,4) G_2(24, 54^+) G_1^{-1}(5,3)$$

- Self-energy is exactly determined starting from a two-body interaction.
- G_2 is connected to G_1 and G_3 and so on...

Need for approximations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Dyson equation

$$G_1(1,1') = G_0(1,1') - \int d2d3G_0(1,2)\Sigma(2,3)G_1(3,1')$$

Dyson equation

$$G_1(1,1') = G_0(1,1') - \int d2d3 G_0(1,2)\Sigma(2,3)G_1(3,1')$$

$$\left(\frac{\partial}{\partial t} + \frac{1}{2m}\Delta\right) \mapsto \text{Dyson equation}$$

$$\left(\frac{\partial}{\partial t} + \frac{1}{2m}\Delta\right) G_1(x,x') = \delta(x,x') - \int dx''\Sigma(x,x'')G_1(x'',x')$$

Dyson equation

$$G_1(1,1') = G_0(1,1') - \int d2d3G_0(1,2)\Sigma(2,3)G_1(3,1')$$

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$$\text{FT}\left[\left(\frac{\partial}{\partial t} + \frac{1}{2m}\Delta\right) \mapsto \text{Dyson equation}\right]$$

$$\left(\varepsilon - \frac{p^2}{2m}\right) G_1(\mathbf{r},\mathbf{r'};\varepsilon) = \delta(\mathbf{r},\mathbf{r'}) - \int d\mathbf{r''}\Sigma(\mathbf{r},\mathbf{r''};\varepsilon)G_1(\mathbf{r''},\mathbf{r'};\varepsilon)$$

$$\text{FT}\left[\left(\frac{\partial}{\partial t} + \frac{1}{2m}\Delta\right) \mapsto \text{Dyson equation}\right]$$

$$\left(\varepsilon - \frac{p^2}{2m}\right) G_1(\mathbf{r}, \mathbf{r'}; \varepsilon) = \delta(\mathbf{r}, \mathbf{r'}) - \int d\mathbf{r''}\, \Sigma(\mathbf{r}, \mathbf{r''}; \varepsilon) G_1(\mathbf{r''}, \mathbf{r'}; \varepsilon)$$

One-body Green's function

$$G_1(x, x') = -i\langle 0|\mathcal{T}(\psi(x)\psi^\dagger(x'))|0\rangle$$

Field's operators

$$\begin{aligned} \psi^\dagger(x) &= \sum_\lambda \phi^*_\lambda(\mathbf{r}) a^\dagger_\lambda(t) \\ \psi(x) &= \sum_\lambda \phi_\lambda(\mathbf{r}) a_\lambda(t) \end{aligned}$$

$$\text{FT}\left[\left(\frac{\partial}{\partial t}+\frac{1}{2m}\Delta\right)\mapsto \text{Dyson equation}\right]$$

$$\left(\varepsilon-\frac{p^2}{2m}\right)G_1(\mathbf{r},\mathbf{r'};\varepsilon)=\delta(\mathbf{r},\mathbf{r'})-\int d\mathbf{r''}\,\Sigma(\mathbf{r},\mathbf{r''};\varepsilon)G_1(\mathbf{r''},\mathbf{r'};\varepsilon)$$

One-body Green's function

$$G_1(x,x')=\sum_{\lambda\lambda'}\phi_\lambda(\mathbf{r})\phi^*_{\lambda'}(\mathbf{r'})G_{\lambda\lambda'}(t-t')$$

Field's operators

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$\mathrm{FT}(G_1)$

$$G_1(\mathbf{r},\mathbf{r'};\varepsilon)=\sum_{\lambda\lambda'}\phi_\lambda(\mathbf{r})\phi^*_{\lambda'}(\mathbf{r'})G_{\lambda\lambda'}(\varepsilon)$$

Field's operators

$$\psi^\dagger(x)=\sum_\lambda \phi^*_\lambda(\mathbf{r})a^\dagger_\lambda(t)$$
$$\psi(x)=\sum_\lambda \phi_\lambda(\mathbf{r})a_\lambda(t)$$

$$\mathrm{FT}\left[\left(\frac{\partial}{\partial t}+\frac{1}{2m}\Delta\right)\mapsto \text{Dyson equation}\right]$$

$$\left(\varepsilon-\frac{p^2}{2m}\right)\sum_{\lambda\lambda'}\phi_\lambda(\mathbf{r})\phi^*_{\lambda'}(\mathbf{r'})G_{\lambda\lambda'}(\varepsilon)=\delta(\mathbf{r},\mathbf{r'})$$
$$-\int d\mathbf{r''}\,\Sigma(\mathbf{r},\mathbf{r''};\varepsilon)\sum_{\lambda\lambda'}\phi_\lambda(\mathbf{r})\phi^*_{\lambda'}(\mathbf{r'})G_{\lambda\lambda'}(\varepsilon)$$

$\mathrm{FT}(G_1)$

$$G_1(\mathbf{r},\mathbf{r'};\varepsilon)=\sum_{\lambda\lambda'}\phi_\lambda(\mathbf{r})\phi^*_{\lambda'}(\mathbf{r'})G_{\lambda\lambda'}(\varepsilon)$$

Field's operators

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$$-\int d\mathbf{r''}\Sigma(\mathbf{r},\mathbf{r''};\varepsilon)\sum_{\lambda\lambda'}\phi_\lambda(\mathbf{r})\phi^*_{\lambda'}(\mathbf{r'})G_{\lambda\lambda'}(\varepsilon)$$

$$\int d\mathbf{r}d\mathbf{r'}\phi^*_{\lambda_3}(\mathbf{r})\phi_{\lambda_4}(\mathbf{r'})\text{FT}\left[\left(\frac{\partial}{\partial t}+\frac{1}{2m}\Delta\right)\mapsto \text{Dyson equation}\right]$$

$$\sum_{\lambda_1}\left\{\varepsilon\delta_{\lambda_1\lambda_3}-\int d\mathbf{r}\phi^*_{\lambda_3}(\mathbf{r})\frac{p^2}{2m}\phi_{\lambda_1}(\mathbf{r})\right.$$

$$\left.+\int d\mathbf{r}\phi^*_{\lambda_3}(\mathbf{r})\int d\mathbf{r''}\Sigma(\mathbf{r},\mathbf{r''};\varepsilon)\phi_{\lambda_1}(\mathbf{r''})\right\}G_{\lambda_1\lambda_4}(\varepsilon)=\delta_{\lambda_3\lambda_4}$$

$$\int d\mathbf{r}d\mathbf{r'}\phi^*_{\lambda_3}(\mathbf{r})\phi_{\lambda_4}(\mathbf{r'})\mathrm{FT}\left[\left(\frac{\partial}{\partial t}+\frac{1}{2m}\Delta\right)\mapsto \text{Dyson equation}\right]$$

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Let's consider a set of wave functions ϕ_λ that diagonalizes it

$$[\varepsilon - E_\lambda(\varepsilon)]\,G_{\lambda\lambda'}(\varepsilon)=\delta_{\lambda\lambda'}$$

hence

$$\langle\lambda_3|\frac{p^2}{2m}+\int d\mathbf{r''}\Sigma(\mathbf{r},\mathbf{r''};\varepsilon)|\lambda_1\rangle = E_{\lambda_1}(\varepsilon)\delta_{\lambda_3\lambda_1}$$

The set of wave functions ϕ_λ obeys

$$\frac{p^2}{2m}\phi_\lambda(\mathbf{r})+\int d\mathbf{r''}\Sigma(\mathbf{r},\mathbf{r''};\varepsilon)\phi_\lambda(\mathbf{r''})=E_\lambda(\varepsilon)\phi_\lambda(\mathbf{r})$$

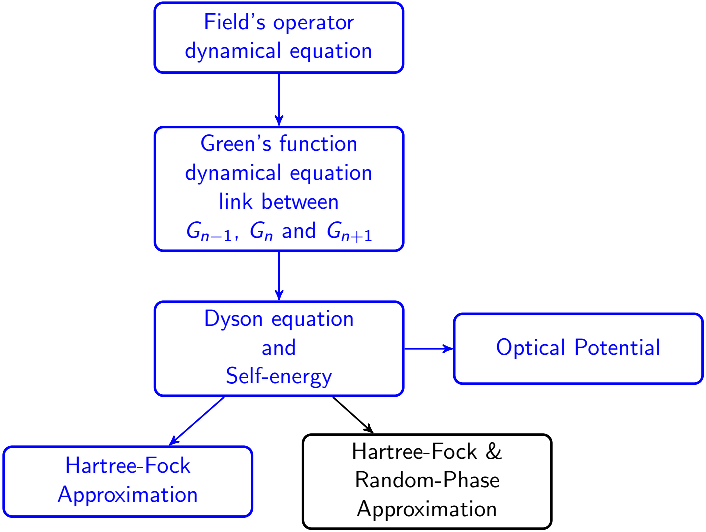
Schrödinger equation

$$\frac{p^2}{2m}\phi_\lambda(\mathbf{r},\varepsilon) + \int d\mathbf{r'}\Sigma(\mathbf{r},\mathbf{r'};\varepsilon)\phi_\lambda(\mathbf{r'},\varepsilon) = E(\varepsilon)\phi_\lambda(\mathbf{r},\varepsilon)$$

ϕ's are the wave functions of a particle experiencing
a potential Σ which is non-local and energy dependent

Optical potential is connected to the Fourier transform of Self-energy

itself connected to the two-body interaction.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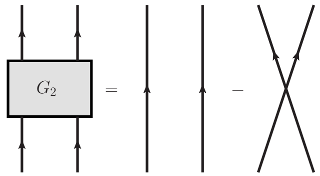
Dynamical equation for G_1

$$G_1(1,1') = G_0(1,1') - i\int d2d3G_0(1,2)v(2,3)G_2(23,1'3^+)$$

Hartree-Fock approximation



1. Two-body correlations are neglected
2. G_2 becomes an antisymmetrized product of G_1's

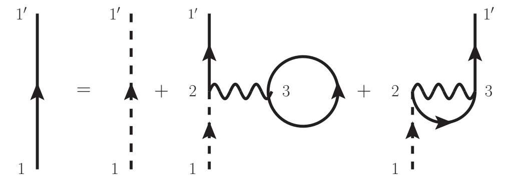
Dynamical equation for G_1 within HF approximation

$$G_1^{HF}(1,1') = G_0(1,1') - i\int d2d3G_0(1,2)v(2,3)\Big(G_1^{HF}(2,1')\\ G_1^{HF}(3,3^+) - G_1^{HF}(2,3^+)G_1^{HF}(3,1')\Big)$$

Dynamical equation for G_1 within HF approximation

$$G_1^{HF}(1,1') = G_0(1,1') - i\int d2d3 G_0(1,2)v(2,3)\Big(G_1^{HF}(2,1')\, G_1^{HF}(3,3^+) - G_1^{HF}(2,3^+)G_1^{HF}(3,1')\Big)$$

Hartree-Fock Diagrammatic



Infinite sum of *'bubbles'* and *'oysters'*

Exact Self-energy

$$\Sigma(2,3) = -i \int d4d5v(2,4)G_2(24,54^+)G_1^{-1}(5,3)$$

Self-energy at the HF approximation

$$\Sigma^{HF}(2,3) = -i \int d4d5v(2,4)\left(G_1(2,5)G_1(4,4^+) - G_1(2,4^+)G_1(4,5)\right)G_1^{-1}(5,3)$$

Exact Self-energy

$$\Sigma(2,3) = -i\int d4d5v(2,4)G_2(24,54^+)G_1^{-1}(5,3)$$

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Exact Self-energy

$$\Sigma(2,3) = -i \int d4d5v(2,4)G_2(24,54^+)G_1^{-1}(5,3)$$

Self-energy at the HF approximation

$$\Sigma^{HF}(2,3) = -i \int d4v(2,4)\left(\delta(2,3)G_1(4,4^+) - G_1(2,4^+)\delta(4,3)\right)$$

Exact Self-energy

$$\Sigma(2,3) = -i \int d4d5v(2,4)G_2(24,54^+)G_1^{-1}(5,3)$$

Self-energy at the HF approximation

$$\Sigma^{HF}(2,3) = -i \int d4v(2,4)\delta(2,3)G_1(4,4^+) + i\ v(2,3)G_1(2,3)$$

Exact Self-energy

$$\Sigma(2,3) = -i \int d4d5v(2,4)G_2(24,54^+)G_1^{-1}(5,3)$$

Self-energy at the HF approximation

$$\Sigma^{HF}(2,3) = -i \int d4v(2,4)\delta(2,3)G_1(4,4^+) + i\ v(2,3)G_1(2,3)$$

Schrödinger equation

$$\frac{p^2}{2m}\phi_\lambda(\mathbf{r},\varepsilon) + \int d\mathbf{r'}\ \underbrace{\Sigma^{HF}(\mathbf{r},\mathbf{r'};\varepsilon)}_{\textbf{FT of Self-energy}}\ \phi_\lambda(\mathbf{r'},\varepsilon) = E(\varepsilon)\phi_\lambda(\mathbf{r},\varepsilon)$$

Self-energy at the HF approximation

$$\Sigma^{HF}(2,3) = -i \int d4 v(2,4)\delta(2,3)G_1(4,4^+) + i\ v(2,3)G_1(2,3)$$

One-body Green's function

$$G_1(x,x') = \sum_{\lambda\lambda'} \phi_\lambda(\mathbf{r})\phi^*_{\lambda'}(\mathbf{r'})G_{\lambda\lambda'}(t-t')$$

Occupation numbers

$$G_{\lambda\lambda}(t-t'=+0) = -i(1-m_\lambda)$$
$$G_{\lambda\lambda}(t-t'=-0) = i\ m_\lambda$$
$$m_\lambda = \langle\psi_0|a^\dagger_\lambda a_\lambda|\psi_0\rangle$$

Fourier transform of Σ^{HF} with $v(x,x') = v(\mathbf{r}-\mathbf{r'})\delta(t-t')$

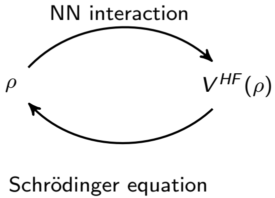
$$\begin{aligned}\Sigma^{HF}(\mathbf{r},\mathbf{r''};\varepsilon) &= \delta(\mathbf{r},\mathbf{r''})\int d\mathbf{r'} v(\mathbf{r},\mathbf{r'})\sum_\lambda m_\lambda \phi^*_\lambda(\mathbf{r'})\phi_\lambda(\mathbf{r'}) \\ &- v(\mathbf{r},\mathbf{r''})\sum_\lambda m_\lambda\phi^*_\lambda(\mathbf{r})\phi_\lambda(\mathbf{r''}) \\ &= \delta(\mathbf{r},\mathbf{r''})\int d\mathbf{r'} v(\mathbf{r},\mathbf{r'})\rho(\mathbf{r'}) - v(\mathbf{r},\mathbf{r''})\rho(\mathbf{r},\mathbf{r''})\end{aligned}$$

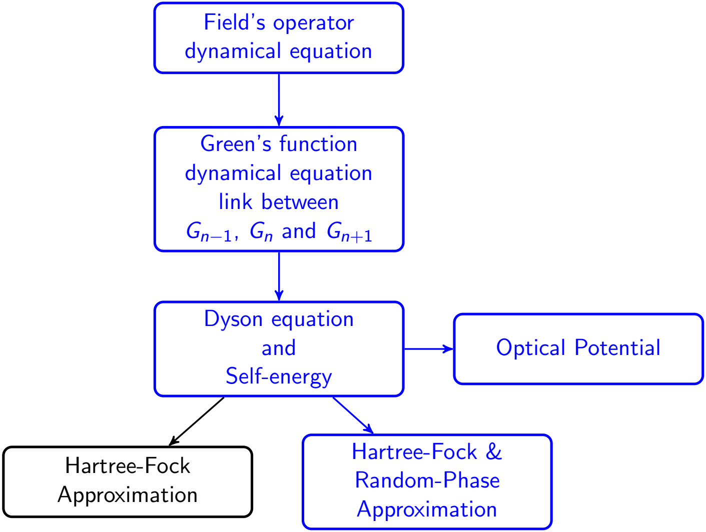
Schrödinger equation

$$\frac{p^2}{2m}\phi_\lambda(\mathbf{r},\varepsilon) + \int d\mathbf{r'}\,V^{HF}(\mathbf{r},\mathbf{r'};\varepsilon)\phi_\lambda(\mathbf{r'},\varepsilon) = E(\varepsilon)\phi_\lambda(\mathbf{r},\varepsilon)$$

HF potential

$$V^{HF}(\mathbf{r},\mathbf{r''};\varepsilon) = \delta(\mathbf{r},\mathbf{r''})\int d\mathbf{r'}\,v(\mathbf{r},\mathbf{r'})\rho(\mathbf{r'}) - v(\mathbf{r},\mathbf{r''})\rho(\mathbf{r},\mathbf{r''})$$

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SOME CONSIDERATIONS ABOUT G_2

n-body Green's function

$$G_n = (-i)^n \langle 0 | \mathcal{T}\{\psi(1)...\psi(n)\psi^\dagger(n')...\psi^\dagger(1')\} | 0 \rangle$$

Two-body Green's function

$$G_2(12;1'2') = -\langle 0|\mathcal{T}\{\psi(1)\psi(2)\psi^\dagger(2')\psi^\dagger(1')\}|0\rangle$$

Different meanings according to times relative order

2p-propagator

$$t_1, t_2 > t_{1'}, t_{2'}$$

$(A+2)$-particle system

2h-propagator

$$t_1, t_2 < t_{1'}, t_{2'}$$

$(A-2)$-particle system

ph-propagator

$$t_1, t_{2'} < t_2, t_{1'}$$
$$t_2, t_{1'} < t_1, t_{2'}$$

Excited states of the A-particle system

Two-body Green's function

$$G_2(12;1'2') = -\langle 0|\psi(1)\psi(2)\psi^\dagger(2')\psi^\dagger(1')|0\rangle$$

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Excited states of the A-particle system

Two-body Green's function

$$G_2(12;1'2') = -\langle 0|\psi^\dagger(2')\psi^\dagger(1')\psi(1)\psi(2)|0\rangle$$

2p-propagator

$t_1, t_2 > t_{1'}, t_{2'}$

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Excited states of the A-particle system

Two-body Green's function

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Excited states of the A-particle system

Two-body Green's function

$$G_2(12;1'2') = -\langle 0|\mathcal{T}\{\psi(1)\psi(2)\psi^\dagger(2')\psi^\dagger(1')\}|0\rangle$$

Field's operators

$$\psi^\dagger(x) = \sum_\lambda \phi^*_\lambda(\mathbf{r})a^\dagger_\lambda(t)$$

$$\psi(x) = \sum_\lambda \phi_\lambda(\mathbf{r})a_\lambda(t)$$

Two-body Green's function in 'λ-representation'

$$G_2(x_1x_2;x_{1'}x_{2'}) = \sum_{\substack{\lambda_1\lambda_2\\ \lambda_{1'}\lambda_{2'}}} \phi_{\lambda_1}(\mathbf{r}_1)\phi_{\lambda_2}(\mathbf{r}_2)G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(t_1t_2;t_{1'}t_{2'})\phi^*_{\lambda_{1'}}(\mathbf{r'}_1)\phi^*_{\lambda_{2'}}(\mathbf{r'}_2)$$

with

$$G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(t_1t_2;t_{1'}t_{2'}) = -\langle 0|\mathcal{T}(a_{\lambda_1}(t_1)a_{\lambda_2}(t_2)a^\dagger_{\lambda'_2}(t'_2)a^\dagger_{\lambda'_1}(t'_1))|0\rangle$$

Two-body Green's function in 'λ-representation'

$$G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(t_1 t_2; t_{1'} t_{2'}) = -\langle 0|\mathcal{T}(a_{\lambda_1}(t_1)a_{\lambda_2}(t_2)a^\dagger_{\lambda'_2}(t'_2)a^\dagger_{\lambda'_1}(t'_1))|0\rangle$$

pp/hh-propagator

$$t_1 = t_2 = t$$
$$t'_1 = t'_2 = t'$$

Heisenberg operator

$$a_\lambda(t) = e^{i\hat{H}t/\hbar} a_\lambda e^{-i\hat{H}t/\hbar}$$

Two-body Green's function pp/hh

$$G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(t, t') = -\langle 0|\mathcal{T}(a_{\lambda_1}(t)a_{\lambda_2}(t)a^\dagger_{\lambda'_2}(t')a^\dagger_{\lambda'_1}(t'))|0\rangle$$

Two-body Green's function in 'λ-representation'

$$G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(t_1 t_2; t_{1'} t_{2'}) = -\langle 0|\mathcal{T}(a_{\lambda_1}(t_1)a_{\lambda_2}(t_2)a^\dagger_{\lambda'_2}(t'_2)a^\dagger_{\lambda'_1}(t'_1))|0\rangle$$

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$$a_\lambda(t) = e^{i\hat{H}t/\hbar}a_\lambda e^{-i\hat{H}t/\hbar}$$

Two-body Green's function pp/hh

$$\begin{aligned} G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(t,t') &= -\langle 0|a_{\lambda_1}(t)a_{\lambda_2}(t)a^\dagger_{\lambda'_2}(t')a^\dagger_{\lambda'_1}(t')|0\rangle \qquad t' < t \\ &= -\langle 0|a^\dagger_{\lambda'_2}(t')a^\dagger_{\lambda'_1}(t')a_{\lambda_1}(t)a_{\lambda_2}(t)|0\rangle \qquad t' > t \end{aligned}$$

Two-body Green's function in 'λ-representation'

$$G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(t_1 t_2; t_{1'} t_{2'}) = -\langle 0|\mathcal{T}(a_{\lambda_1}(t_1)a_{\lambda_2}(t_2)a^\dagger_{\lambda'_2}(t'_2)a^\dagger_{\lambda'_1}(t'_1))|0\rangle$$

pp/hh-propagator

$$t_1 = t_2 = t$$
$$t'_1 = t'_2 = t'$$

Heisenberg operator

$$a_\lambda(t) = e^{i\hat{H}t/\hbar} a_\lambda e^{-i\hat{H}t/\hbar}$$

Two-body Green's function pp/hh ($\hbar = 1$)

$$\begin{aligned} G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(t,t') &= -\langle 0|e^{i\hat{H}t} a_{\lambda_1} a_{\lambda_2} e^{-i\hat{H}t} e^{i\hat{H}t'} a^\dagger_{\lambda'_2} a^\dagger_{\lambda'_1} e^{-i\hat{H}t'}|0\rangle \qquad t' < t \\ &= -\langle 0|e^{i\hat{H}t'} a^\dagger_{\lambda'_2} a^\dagger_{\lambda'_1} e^{-i\hat{H}t'} e^{i\hat{H}t} a_{\lambda_1} a_{\lambda_2} e^{-i\hat{H}t}|0\rangle \qquad t' > t \end{aligned}$$

Two-body Green's function in 'λ-representation'

$$G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(t_1 t_2; t_{1'} t_{2'}) = -\langle 0|\mathcal{T}(a_{\lambda_1}(t_1) a_{\lambda_2}(t_2) a^\dagger_{\lambda'_2}(t'_2) a^\dagger_{\lambda'_1}(t'_1))|0\rangle$$

pp/hh-propagator

$$t_1 = t_2 = t$$
$$t'_1 = t'_2 = t'$$

Heisenberg operator

$$a_\lambda(t) = e^{i\hat{H}t/\hbar} a_\lambda e^{-i\hat{H}t/\hbar}$$

Two-body Green's function pp/hh ($\hbar = 1$)

$$G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(t,t') = -e^{iE_0(t-t')}\langle 0|a_{\lambda_1} a_{\lambda_2} e^{-i\hat{H}(t-t')} a^\dagger_{\lambda'_2} a^\dagger_{\lambda'_1}|0\rangle \qquad t' < t$$

$$= -e^{-iE_0(t-t')}\langle 0|a^\dagger_{\lambda'_2} a^\dagger_{\lambda'_1} e^{-i\hat{H}(t'-t)} a_{\lambda_1} a_{\lambda_2}|0\rangle \qquad t' > t$$

Two-body Green's function pp/hh ($\hbar = 1$)

$$G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(t,t') = -e^{iE_0(t-t')}\langle 0|a_{\lambda_1}a_{\lambda_2}e^{-i\hat{H}(t-t')}a^\dagger_{\lambda'_2}a^\dagger_{\lambda'_1}|0\rangle \quad t' < t$$

$$= -e^{-iE_0(t-t')}\langle 0|a^\dagger_{\lambda'_2}a^\dagger_{\lambda'_1}e^{-i\hat{H}(t'-t)}a_{\lambda_1}a_{\lambda_2}|0\rangle \quad t' > t$$

$$G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(t,t') \underset{t'<t}{=} -\sum_n e^{iE_0(t-t')}\langle 0|a_{\lambda_1}a_{\lambda_2}|\psi_n\rangle\langle\psi_n|e^{-i\hat{H}(t-t')}a^\dagger_{\lambda'_2}a^\dagger_{\lambda'_1}|0\rangle$$

$$\underset{t'>t}{=} -\sum_m e^{-iE_0(t-t')}\langle 0|a^\dagger_{\lambda'_2}a^\dagger_{\lambda'_1}|\psi_m\rangle\langle\psi_m|e^{-i\hat{H}(t'-t)}a_{\lambda_1}a_{\lambda_2}|0\rangle$$

Assuming,

ψ_n state of (N+2)-system

$$E_n = E_n(N+2) - E_0(N)$$

ψ_m state of (N-2)-system

$$E_m = E_m(N-2) - E_0(N)$$

Two-body Green's function pp/hh ($\hbar = 1$)

$$
\begin{aligned}
G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(t,t') &= -e^{iE_0(t-t')}\langle 0|a_{\lambda_1}a_{\lambda_2}e^{-i\hat{H}(t-t')}a^\dagger_{\lambda'_2}a^\dagger_{\lambda'_1}|0\rangle \qquad t' < t \\
&= -e^{-iE_0(t-t')}\langle 0|a^\dagger_{\lambda'_2}a^\dagger_{\lambda'_1}e^{-i\hat{H}(t'-t)}a_{\lambda_1}a_{\lambda_2}|0\rangle \qquad t' > t
\end{aligned}
$$

$$
\begin{aligned}
G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(t,t') &\underset{t'<t}{=} -\sum_n e^{-iE_n(t-t')}\langle 0|a_{\lambda_1}a_{\lambda_2}|\psi_n\rangle\langle\psi_n|a^\dagger_{\lambda'_2}a^\dagger_{\lambda'_1}|0\rangle \\
&\underset{t'>t}{=} -\sum_m e^{iE_m(t-t')}\langle 0|a^\dagger_{\lambda'_2}a^\dagger_{\lambda'_1}|\psi_m\rangle\langle\psi_m|a_{\lambda_1}a_{\lambda_2}|0\rangle
\end{aligned}
$$

Assuming,

ψ_n state of (N+2)-system

$$E_n = E_n(N+2) - E_0(N)$$

ψ_m state of (N-2)-system

$$E_m = E_m(N-2) - E_0(N)$$

Two-body Green's function pp/hh ($\hbar = 1$)

$$G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(t,t') = -e^{iE_0(t-t')}\langle 0|a_{\lambda_1}a_{\lambda_2}e^{-i\hat{H}(t-t')}a^\dagger_{\lambda_2'}a^\dagger_{\lambda_1'}|0\rangle \qquad t' < t$$

$$= -e^{-iE_0(t-t')}\langle 0|a^\dagger_{\lambda_2'}a^\dagger_{\lambda_1'}e^{-i\hat{H}(t'-t)}a_{\lambda_1}a_{\lambda_2}|0\rangle \qquad t' > t$$

$$G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(t,t') \underset{t'<t}{=} -\sum_n e^{-iE_n(t-t')}X^{(n)*}_{\lambda_1\lambda_2}X^{(n)}_{\lambda_1'\lambda_2'}$$

$$\underset{t'>t}{=} -\sum_m e^{iE_m(t-t')}Y^{(m)*}_{\lambda_{1'}\lambda_{2'}}Y^{(m)}_{\lambda_1\lambda_2}$$

pp/hh-Amplitudes

$$X^{(n)}_{ab} = \langle\psi_n|a^\dagger_a a^\dagger_b|0\rangle$$
$$Y^{(m)}_{ab} = \langle\psi_m|a_a a_b|0\rangle$$

Assuming,

ψ_n state of (N+2)-system

$$E_n = E_n(N+2) - E_0(N)$$

ψ_m state of (N-2)-system

$$E_m = E_m(N-2) - E_0(N)$$

Two-body Green's function pp/hh

$$
\begin{aligned}
G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(t,t') \quad &\underset{t'<t}{=} \quad -\sum_n e^{-iE_n(t-t')} X^{(n)*}_{\lambda_1\lambda_2} X^{(n)}_{\lambda_1'\lambda_2'} \\
&\underset{t'>t}{=} \quad -\sum_m e^{iE_m(t-t')} Y^{(m)*}_{\lambda_{1'},\lambda_{2'}} Y^{(m)}_{\lambda_1\lambda_2}
\end{aligned}
$$

FT(Two-body Green's function) pp/hh

$$
G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(\omega) \quad = \quad -i \sum_{n(N+2)} \frac{X^{(n)*}_{\lambda_1\lambda_2} X^{(n)}_{\lambda_1'\lambda_2'}}{\omega - E_n + i\eta} + i \sum_{m(N-2)} \frac{Y^{(m)*}_{\lambda_{1'},\lambda_{2'}} Y^{(m)}_{\lambda_1\lambda_2}}{\omega + E_m - i\eta}
$$

Useful results for the following...

FT(Two-body Green's function) pp/hh

$$G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(\omega) = -i\sum_{n(N+2)}\frac{X^{(n)*}_{\lambda_1\lambda_2}X^{(n)}_{\lambda'_1\lambda'_2}}{\omega - E_n + i\eta} + i\sum_{m(N-2)}\frac{Y^{(m)*}_{\lambda_{1'}\lambda_{2'}}Y^{(m)}_{\lambda_1\lambda_2}}{\omega + E_m - i\eta}$$

FT(Two-body Green's function) ph/hp

$$G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(\omega) = -i\sum_{n}\frac{\chi^{(n)*}_{\lambda_{1'}\lambda_1}\chi^{(n)}_{\lambda_2\lambda_{2'}}}{\omega - E_n + i\eta} + i\sum_{n}\frac{\chi^{(n)*}_{\lambda_{2'}\lambda_2}\chi^{(n)}_{\lambda_1\lambda_{1'}}}{\omega + E_n - i\eta}$$

ph/hp-Amplitudes & Energy

$$\chi^{(n)}_{ab} = \langle\psi_n|a_a a^\dagger_b|0\rangle$$

$$E_{n=E_n(N)-E_0(N)}$$

ph/hp-propagator

$$t_1 = t'_1 + 0$$

$$t_2 = t'_2 + 0$$

SLOW CONVERGENCE OF PERTURBATION SERIE
BUILT ON TOP OF G_0

LET'S BUILT ONE ON TOP OF G_1

Let's get rid of G_0 from...

Dynamical equations for G_2

$$G_2(12;1'2') = G_0(1,1')G_1(2,2') - G_0(1,2')G_1(2,1') - i\int d3d4\, G_0(1,3)v(3,4)G_3(324;1'2'4^+)$$

Let's get rid of G_0 from...

Dynamical equations for G_2

$$G_2(12;1'2') = G_0(1,1')G_1(2,2') - G_0(1,2')G_1(2,1') - i\int d3d4\, G_0(1,3)v(3,4)G_3(324;1'2'4^+)$$

$\int d1\, G_0^{-1}(5,1)G_2(12,1'2')$

$$\int d1\, G_0^{-1}(5,1)G_2(12;1'2') = \int d1\, G_0^{-1}(5,1)G_0(1,1')G_1(2,2') - \int d1\, G_0^{-1}(5,1)G_0(1,2')G_1(2,1') - i\int d3d4d1\, G_0^{-1}(5,1)G_0(1,3)v(3,4)G_3(324;1'2'4^+)$$

Let's get rid of G_0 from...

Dynamical equations for G_2

$$G_2(12;1'2') = G_0(1,1')G_1(2,2') - G_0(1,2')G_1(2,1') - i\int d3d4\,G_0(1,3)v(3,4)G_3(324;1'2'4^+)$$

$\int d1\,G_0^{-1}(5,1)G_2(12,1'2')$

$$\int d1\,G_0^{-1}(5,1)G_2(12;1'2') = \delta(5,1')G_1(2,2') - \delta(5,2')G_1(2,1') - i\int d3d4\,\delta(5,3)v(3,4)G_3(324;1'2'4^+)$$

And determine G_0^{-1}

Dynamical equations for G_1

$$G_1(1,1') = G_0(1,1') - i\int d3d4\, G_0(1,3)v(3,4)G_2(34,1'4^+)$$

$$\int d1d1' G_0^{-1}(5,1)G_1(1,1')G_1^{-1}(1',6)$$

$$\int d1d1' G_0^{-1}(5,1)G_1(1,1')G_1^{-1}(1',6) = \int d1d1' G_0^{-1}(5,1)G_0(1,1')G_1^{-1}(1',6)$$
$$- i\int d3d4d1d1' G_0^{-1}(5,1)G_0(1,3)v(3,4)G_2(34,1'4^+)G_1^{-1}(1',6)$$

And determine G_0^{-1}

Dynamical equations for G_1

$$G_1(1,1') = G_0(1,1') - i\int d3d4\, G_0(1,3)v(3,4)G_2(34,1'4^+)$$

$\int d1d1'\, G_0^{-1}(5,1)G_1(1,1')G_1^{-1}(1',6)$

$$\int d1\, G_0^{-1}(5,1)\delta(1,6) = \int d1'\delta(5,1')G_1^{-1}(1',6)$$
$$- i\int d3d4d1'\delta(5,3)v(3,4)G_2(34,1'4^+)G_1^{-1}(1',6)$$

And determine G_0^{-1}

Dynamical equations for G_1

$$G_1(1,1') = G_0(1,1') - i \int d3d4\, G_0(1,3) v(3,4) G_2(34,1'4^+)$$

$\int d1d1' G_0^{-1}(5,1) G_1(1,1') G_1^{-1}(1',6)$

$$G_0^{-1}(5,6) \quad = \quad G_1^{-1}(5,6) \;-\; i \int d4d1' v(5,4) G_2(54,1'4^+) G_1^{-1}(1',6)$$

$\int d1 G_0^{-1}(5,1) G_2(12,1'2')$

$$\int d1 G_0^{-1}(5,1) G_2(12;1'2') = \delta(5,1') G_1(2,2') - \delta(5,2') G_1(2,1') - i \int d3d4 \delta(5,3) v(3,4) G_3(324;1'2'4^+)$$

$\int d1 d1' G_0^{-1}(5,1) G_1(1,1') G_1^{-1}(1',6)$

$$G_0^{-1}(5,6) = G_1^{-1}(5,6) - i \int d4 d1' v(5,4) G_2(54,1'4^+) G_1^{-1}(1',6)$$

$\int d1 G_0^{-1}(5,1) G_2(12,1'2') \longleftarrow G_0^{-1}(5,1)$

$$\int d1 G_1^{-1}(5,1) G_2(12;1'2') = \delta(5,1') G_1(2,2') - \delta(5,2') G_1(2,1') - i \int d4 v(5,4) G_3(524;1'2'4^+) + i \int d1d3d4 v(5,4) G_2(54,34^+) G_1^{-1}(3,1) G_2(12;1'2')$$

$$\int d1 G_0^{-1}(5,1) G_2(12,1'2') \longleftarrow G_0^{-1}(5,1)$$

$$\int d1 G_1^{-1}(5,1) G_2(12;1'2') = \delta(5,1') G_1(2,2') - \delta(5,2') G_1(2,1')$$
$$- i \int d4 v(5,4) G_3(524;1'2'4^+)$$
$$+ i \int d1d3d4 v(5,4) G_2(54,34^+) G_1^{-1}(3,1) G_2(12;1'2')$$

$$\int d1 d5 G_1(7,5) G_0^{-1}(5,1) G_2(12,1'2')$$

$$\int d1 d5 G_1(7,5) G_1^{-1}(5,1) G_2(12;1'2') = \int d5 G_1(7,5) \delta(5,1') G_1(2,2')$$
$$- \int d5 G_1(7,5) \delta(5,2') G_1(2,1')$$
$$- i \int d4 d5 G_1(7,5) v(5,4) G_3(524;1'2'4^+)$$
$$+ i \int d1d3d4 d5 G_1(7,5) v(5,4) G_2(54,34^+) G_1^{-1}(3,1) G_2(12;1'2')$$

$$\int d1 G_0^{-1}(5,1) G_2(12,1'2') \longleftarrow G_0^{-1}(5,1)$$

$$\int d1 G_1^{-1}(5,1) G_2(12;1'2') = \delta(5,1') G_1(2,2') - \delta(5,2') G_1(2,1')$$
$$- i \int d4 v(5,4) G_3(524;1'2'4^+)$$
$$+ i \int d1d3d4 v(5,4) G_2(54,34^+) G_1^{-1}(3,1) G_2(12;1'2')$$

$$\int d1 d5 G_1(7,5) G_0^{-1}(5,1) G_2(12,1'2')$$

$$\int d1 d5 G_1(7,5) G_1^{-1}(5,1) G_2(12;1'2') = \int d5 G_1(7,5) \delta(5,1') G_1(2,2')$$
$$- \int d5 G_1(7,5) \delta(5,2') G_1(2,1')$$
$$- i \int d4 d5 G_1(7,5) v(5,4) G_3(524;1'2'4^+)$$
$$+ i \int d1d3d4d5 G_1(7,5) v(5,4) G_2(54,34^+) G_1^{-1}(3,1) G_2(12;1'2')$$

$$\int d1 G_0^{-1}(5,1) G_2(12,1'2') \longleftarrow G_0^{-1}(5,1)$$

$$\int d1 G_1^{-1}(5,1) G_2(12;1'2') = \delta(5,1') G_1(2,2') - \delta(5,2') G_1(2,1')$$
$$- i \int d4 v(5,4) G_3(524;1'2'4^+)$$
$$+ i \int d1d3d4 v(5,4) G_2(54,34^+) G_1^{-1}(3,1) G_2(12;1'2')$$

$$\int d1d5 G_1(7,5) G_0^{-1}(5,1) G_2(12,1'2')$$

$$\int d1 \delta(7,1) G_2(12;1'2') = G_1(7,1') G_1(2,2')$$
$$- G_1(7,2') G_1(2,1')$$
$$- i \int d4d5 G_1(7,5) v(5,4) G_3(524;1'2'4^+)$$
$$+ i \int d1d3d4d5 G_1(7,5) v(5,4) G_2(54,34^+) G_1^{-1}(3,1) G_2(12;1'2')$$

$$\int d1 G_0^{-1}(5,1) G_2(12,1'2') \longleftarrow G_0^{-1}(5,1)$$

$$\int d1 G_1^{-1}(5,1) G_2(12;1'2') = \delta(5,1') G_1(2,2') - \delta(5,2') G_1(2,1')$$
$$- i \int d4 v(5,4) G_3(524;1'2'4^+)$$
$$+ i \int d1 d3 d4 v(5,4) G_2(54,34^+) G_1^{-1}(3,1) G_2(12;1'2')$$

$$\int d1 d5 G_1(7,5) G_0^{-1}(5,1) G_2(12,1'2')$$

$$G_2(72;1'2') = G_1(7,1') G_1(2,2')$$
$$- G_1(7,2') G_1(2,1')$$
$$- i \int d4 d5 G_1(7,5) v(5,4) G_3(524;1'2'4^+)$$
$$+ i \int d1 d3 d4 d5 G_1(7,5) v(5,4) G_2(54,34^+) G_1^{-1}(3,1) G_2(12;1'2')$$

Finally, we get

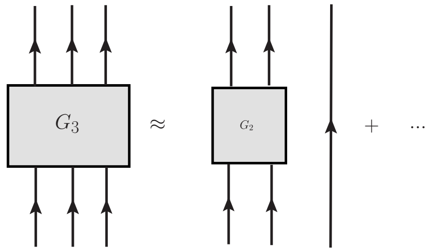
G_2 as a function G_1, G_2 and G_3

$$
\begin{aligned}
G_2(72;1'2') = G_1(7,1')G_1(2,2') - G_1(7,2')G_1(2,1') \\
- i \int d4d5\, G_1(7,5)v(5,4) \left\{ G_3(524;1'2'4^+) \right. \\
\left. - \int d1d3\, G_2(54,34^+)G_1^{-1}(3,1)G_2(12;1'2') \right\}
\end{aligned}
$$

We find the Hartree-Fock terms corrected by higher order terms

Need for an approximation to deal with G_3

G_3 approximated as the antisymmetrized sum of $G_1 G_2$ products



We neglect interaction between the correlated pair of particles and the third particle

G_3 approximated as the antisymmetrized sum of $G_1 G_2$ products

$$
\begin{aligned}
G_3(123,1'2'3') \quad \approx \quad & G_1(1,1')G_2(23;2'3') + G_1(2,2')G_2(13;1'3') \\
+ \quad & G_1(3,3')G_2(12;1'2') - G_1(1,2')G_2(23;1'3') \\
- \quad & G_1(1,3')G_2(23;2'1') - G_1(2,1')G_2(13;2'3') \\
- \quad & G_1(2,3')G_2(13;1'2') - G_1(3,1')G_2(12;2'3') \\
- \quad & G_1(3,2')G_2(12;1'3') - 2G_3^{(0)}(123;1'2'3')
\end{aligned}
$$

$G_3^{(0)}$ is the free contribution to G_3

$$
\begin{aligned}
G_3^{(0)}(123,1'2'3') \quad = \quad & G_1(1,1')G_1(2,2')G_1(3,3') - G_1(1,1')G_1(2,3')G_1(3,2') \\
- \quad & G_1(1,2')G_1(2,1')G_1(3,3') + G_1(1,2')G_1(2,3')G_1(3,1') \\
- \quad & G_1(1,3')G_1(2,2')G_1(3,1') + G_1(1,3')G_1(2,1')G_1(3,2')
\end{aligned}
$$

Now we have an approximation for G_3, we deal with...

G_2 as a function G_1, G_2 and G_3

$$G_2(72;1'2') = G_1(7,1')G_1(2,2') - G_1(7,2')G_1(2,1') - i\int d4d5\, G_1(7,5)v(5,4)\left\{G_3(524;1'2'4^+) - \int d1d3\, G_2(54,34^+)G_1^{-1}(3,1)G_2(12;1'2')\right\}$$

$$-\int d1d3\, G_2(54,34^+)G_1^{-1}(3,1)G_2(12;1'2') \text{ ?}$$

$$-\int d1d3G_2(54,34^+)G_1^{-1}(3,1)G_2(12;1'2')$$

$$\int d1G_1^{-1}(5,1)G_2(12;1'2') = \delta(5,1')G_1(2,2') - \delta(5,2')G_1(2,1')$$
$$- i\int d4v(5,4)G_3(524;1'2'4^+)$$
$$+ i\int d1d3d4v(5,4)G_2(54,34^+)G_1^{-1}(3,1)G_2(12;1'2')$$

$$-\int d1d3G_2(54,34^+)G_1^{-1}(3,1)G_2(12;1'2')$$

$$\int d1d5G_2(64,54^+)G_1^{-1}(5,1)G_2(12;1'2') = \int d5G_2(64,54^+)\delta(5,1')G_1(2,2')$$

$$-\int d5G_2(64,54^+)\delta(5,2')G_1(2,1')$$

$$-i\int d4d5G_2(64,54^+)v(5,4)G_3(524;1'2'4^+)$$

$$+i\int d1d3d4d5G_2(64,54^+)v(5,4)G_2(54,34^+)G_1^{-1}(3,1)G_2(12;1'2')$$

$$-\int d1d3\,G_2(54,34^+)G_1^{-1}(3,1)G_2(12;1'2')$$

$$\begin{aligned}\int d1d5\,G_2(64,54^+)G_1^{-1}(5,1)G_2(12;1'2') &= G_2(64,1'4^+)G_1(2,2') \\ &- G_2(64,2'4^+)G_1(2,1') \\ &- i\int d4d5\,G_2(64,54^+)v(5,4)G_3(524;1'2'4^+) \\ &+ i\int d1d3d4d5\,G_2(64,54^+)v(5,4)G_2(54,34^+)G_1^{-1}(3,1)G_2(12;1'2')\end{aligned}$$

$$-\int d1d3G_2(54,34^+)G_1^{-1}(3,1)G_2(12;1'2')$$

$$\int d1d5G_2(64,54^+)G_1^{-1}(5,1)G_2(12;1'2') = G_2(64,1'4^+)G_1(2,2') - G_2(64,2'4^+)G_1(2,1') - i\int d4d5G_2(64,54^+)v(5,4)G_3(524;1'2'4^+) + i\int d1d3d4d5G_2(64,54^+)v(5,4)G_2(54,34^+)G_1^{-1}(3,1)G_2(12;1'2')$$

Once again we neglect correlations between G_1 and G_2 and between two G_2's.

$$-\int d1d3G_2(54,34^+)G_1^{-1}(3,1)G_2(12;1'2')$$

$$\int d1d5G_2(64,54^+)G_1^{-1}(5,1)G_2(12;1'2') \approx G_2(64,1'4^+)G_1(2,2') - G_2(64,2'4^+)G_1(2,1')$$

G_2 as a function G_1, G_2 and G_3

$$G_2(72;1'2') = G_1(7,1')G_1(2,2') - G_1(7,2')G_1(2,1') - i\int d4d5\,G_1(7,5)v(5,4)\left\{G_3(524;1'2'4^+) - \int d1d3\,G_2(54,34^+)G_1^{-1}(3,1)G_2(12;1'2')\right\}$$

Approximation #1

$$\begin{aligned} G_3(123,1'2'3') \approx\; & G_1(1,1')G_2(23;2'3') + G_1(2,2')G_2(13;1'3') \\ +\; & G_1(3,3')G_2(12;1'2') - G_1(1,2')G_2(23;1'3') \\ -\; & G_1(1,3')G_2(23;2'1') - G_1(2,1')G_2(13;2'3') \\ -\; & G_1(2,3')G_2(13;1'2') - G_1(3,1')G_2(12;2'3') \\ -\; & G_1(3,2')G_2(12;1'3') - 2G_3^{(0)}(123;1'2'3') \end{aligned}$$

Approximation #2

$$\begin{aligned} \int d1d5\,G_2(64,54^+)G_1^{-1}(5,1)G_2(12;1'2') \approx\; & G_2(64,1'4^+)G_1(2,2') \\ -\; & G_2(64,2'4^+)G_1(2,1') \end{aligned}$$

Approximated version of G_2

$$
\begin{aligned}
G_2(12;1'2') &= G_1(1,1')G_1(2,2') - G_1(1,2')G_1(2,1') \\
&- i\int d3d4\, G_1(1,3)v(3,4)\Big[G_1(3,1')G_2(24;2'4^+) \\
&+ G_1(2,2')G_2(34;1'4^+) - G_1(3,2')G_2(24;1'4^+) \\
&- G_1(3,4)G_2(24;2'1') - G_1(2,1')G_2(34;2'4^+) \\
&- G_1(2,4)G_2(34,1'2') - G_1(4,1')G_2(23;2'4) \\
&- G_1(4,2')G_2(32;1'4) - 2G_3^{(0)}(324;1'2'4^+)\Big]
\end{aligned}
$$

Neglecting correlations between G_1 and G_2 and between two G_2's.

ph-propagator and RPA equations

ph-propagator (G_2 with $t_1 = t_{1'} + 0$ and $t_2 = t_{2'} + 0$)

Two-body Green's function

$$G_2(12;1'2') = -\langle 0|\mathcal{T}\{\psi(1)\psi(2)\psi^\dagger(2')\psi^\dagger(1')\}|0\rangle$$

Approximated version of G_2

$$\begin{aligned}
G_2(12;1'2') &= G_1(1,1')G_1(2,2') - G_1(1,2')G_1(2,1') \\
&- i\int d3d4\, G_1(1,3)v(3,4)\left[G_1(3,1')G_2(24;2'4^+)\right. \\
&+ G_1(2,2')G_2(34;1'4^+) - G_1(3,2')G_2(24;1'4^+) \\
&- G_1(3,4)G_2(24;2'1') - G_1(2,1')G_2(34;2'4^+) \\
&- G_1(2,4)G_2(34,1'2') - G_1(4,1')G_2(23;2'4) \\
&- \left. G_1(4,2')G_2(32;1'4) - 2G_3^{(0)}(324;1'2'4^+)\right]
\end{aligned}$$

ph-propagator and RPA equations

ph-propagator (G_2 with $t_1 = t_{1'} + 0$ and $t_2 = t_{2'} + 0$)

Two-body Green's function

$$G_2(12;1'2') = -\langle 0|\mathcal{T}\{\psi(1)\psi(2)\psi^\dagger(2')\psi^\dagger(1')\}|0\rangle$$

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&+ G_1(2,2')G_2(34;1'4^+) - G_1(3,2')G_2(24;1'4^+) \\
&- G_1(3,4)G_2(24;2'1') - G_1(2,1')G_2(34;2'4^+) \\
&- G_1(2,4)G_2(34,1'2') - G_1(4,1')G_2(23;2'4) \\
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\end{aligned}$$

with $v(3,4) = v(r_3, r_4)\delta(t_3, t_4)$.

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$$G_2(12;1'2') = -\langle 0|\mathcal{T}\{\psi(1)\psi(2)\psi^\dagger(2')\psi^\dagger(1')\}|0\rangle$$

ph-G_2

$$\begin{aligned} G_2(1^+2^+;1'2') &= G_1(1^+,1')G_1(2^+,2') - G_1(1,2')G_1(2,1') \\ &- i\int d3d4\, G_1(1,3)v(3,4)\left[G_1(3,1')G_2(24;2'4^+)\right. \\ &- \left. G_1(4,1')G_2(23;2'4)\right] \end{aligned}$$

with $v(3,4) = v(r_3,r_4)\delta(t_3,t_4)$.

Now we have isolated ph-contributions

FT(Two-body Green's function) ph/hp in 'λ-representation'

$$G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(\omega) = -i\sum_n \frac{\chi^{(n)*}_{\lambda_{1'},\lambda_1}\chi^{(n)}_{\lambda_2\lambda_{2'}}}{\omega - E_n + i\eta} + i\sum_n \frac{\chi^{(n)*}_{\lambda_{2'},\lambda_2}\chi^{(n)}_{\lambda_1\lambda_{1'}}}{\omega + E_n - i\eta}$$

ph-G_2

$$\begin{aligned} G_2(1^+2^+;1'2') &= G_1(1^+,1')G_1(2^+,2') - G_1(1,2')G_1(2,1') \\ &- i\int d3d4\, G_1(1,3)v(3,4)\left[G_1(3,1')G_2(24;2'4^+)\right. \\ &- \left. G_1(4,1')G_2(23;2'4)\right] \end{aligned}$$

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ph-G_2 in 'λ-representation'

$$\begin{aligned} G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(t_1-t_2) &= G_{\lambda_1}(+0)G_{\lambda_2}(+0)\delta_{\lambda_1\lambda_{1'}}\delta_{\lambda_2\lambda_{2'}} \\ &- G_{\lambda_1}(t_1-t_2)G_{\lambda_2}(t_2-t_1)\delta_{\lambda_1\lambda_{2'}}\delta_{\lambda_2\lambda_{1'}} \\ &+ i\sum_{\lambda_3\lambda_4}\int_{-\infty}^{+\infty} dt'\, G_{\lambda_1}(t_1,t')G_{\lambda_{1'}}(t',t_1) \\ &\times \langle\lambda_1\lambda_3|v(3,4)|\lambda_4\lambda_{1'}\rangle_A G_{\lambda_3\lambda_2;\lambda_4\lambda_{2'}(t'-t_2)} \end{aligned}$$

with $v(3,4) = v(r_3,r_4)\delta(t_3,t_4)$.

Let's redefine ...

$$G^{II}_{\lambda_1\lambda_{1'},\lambda_2\lambda_{2'}}(t_1 - t_2) = G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(t_1 - t_2)$$

Thus

$G^{II} = ph - G_2$ in 'λ-representation'

$$\begin{aligned} G^{II}_{\lambda_1\lambda_{1'},\lambda_2\lambda_{2'}}(t_1 - t_2) &= -G_{\lambda_1}(t_1 - t_2)G_{\lambda_2}(t_2 - t_1)\delta_{\lambda_1\lambda_{2'}}\delta_{\lambda_2\lambda_{1'}} \\ &+ i\sum_{\lambda_3\lambda_4}\int_{-\infty}^{+\infty} dt'\, G_{\lambda_1}(t_1, t')G_{\lambda_{1'}}(t', t_1) \\ &\times \langle\lambda_1\lambda_3|v(3,4)|\lambda_4\lambda_{1'}\rangle_A G^{II}_{\lambda_3\lambda_2;\lambda_4\lambda_{2'}(t'-t_2)} \end{aligned}$$

FT(Two-body Green's function) ph/hp in 'λ-representation'

$$G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(\omega) = -i\sum_n \frac{\chi^{(n)*}_{\lambda_{1'}\lambda_1}\chi^{(n)}_{\lambda_2\lambda_{2'}}}{\omega - E_n + i\eta} + i\sum_n \frac{\chi^{(n)*}_{\lambda_{2'}\lambda_2}\chi^{(n)}_{\lambda_1\lambda_{1'}}}{\omega + E_n - i\eta}$$

$G^{II} = ph - G_2$ in 'λ-representation'

$$\begin{aligned} G^{II}_{\lambda_1\lambda_{1'},\lambda_2\lambda_{2'}}(t_1 - t_2) &= -G_{\lambda_1}(t_1 - t_2)G_{\lambda_2}(t_2 - t_1)\delta_{\lambda_1\lambda_{2'}}\delta_{\lambda_2\lambda_{1'}} \\ &+ i\sum_{\lambda_3\lambda_4}\int_{-\infty}^{+\infty} dt'\, G_{\lambda_1}(t_1, t')G_{\lambda_{1'}}(t', t_1) \\ &\times \langle\lambda_1\lambda_3|v(3,4)|\lambda_4\lambda_{1'}\rangle_A G^{II}_{\lambda_3\lambda_2;\lambda_4\lambda_{2'}(t'-t_2)} \end{aligned}$$

FT(Two-body Green's function) ph/hp in 'λ-representation'

$$G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(\omega) = -i\sum_n \frac{\chi^{(n)*}_{\lambda_{1'}\lambda_1}\chi^{(n)}_{\lambda_2\lambda_{2'}}}{\omega - E_n + i\eta} + i\sum_n \frac{\chi^{(n)*}_{\lambda_{2'}\lambda_2}\chi^{(n)}_{\lambda_1\lambda_{1'}}}{\omega + E_n - i\eta}$$

$G^{II} = ph - G_2$ in 'λ-representation'

$$\sum_n \left(\frac{\chi^{(n)*}_{i'i}\chi^{(n)}_{j'j}}{\omega - E_n + i\eta} - \frac{\chi^{(n)}_{ii'}\chi^{(n)*}_{ii'}}{\omega + E_n - i\eta} \right) = \frac{(1-m_i)m_{i'} - m_i(1-m_{i'})}{\omega - \varepsilon_i + \varepsilon_{i'} + i\eta}$$
$$\left(\delta_{ij}\delta_{i'j'} + \sum_{nkl} \langle il|v|i'k\rangle \left[\frac{\chi^{(n)*}_{lk}\chi^{(n)}_{j'j}}{\omega - E_n + i\eta} - \frac{\chi^{(n)}_{kl}\chi^{(n)*}_{jj'}}{\omega + E_n - i\eta} \right] \right)$$

FT(Two-body Green's function) ph/hp in 'λ-representation'

$$G_{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}(\omega) = -i\sum_n \frac{\chi^{(n)*}_{\lambda_{1'}\lambda_1}\chi^{(n)}_{\lambda_2\lambda_{2'}}}{\omega - E_n + i\eta} + i\sum_n \frac{\chi^{(n)*}_{\lambda_{2'}\lambda_2}\chi^{(n)}_{\lambda_1\lambda_{1'}}}{\omega + E_n - i\eta}$$

ph RPA equations

$$(E_n - \varepsilon_i + \varepsilon_{i'})\chi^{(n)}_{i'i} - \sum_{kk'}\langle ik'|v|i'k\rangle_A \chi^{(n)}_{kk'} - \sum_{kk'}\langle ik|v|i'k'\rangle_A \chi^{(n)}_{kk'} = 0$$

$$(E_n - \varepsilon_i + \varepsilon_{i'})\chi^{(n)}_{ii'} + \sum_{kk'}\langle ik|v|i'k'\rangle_A \chi^{(n)}_{k'k} + \sum_{kk'}\langle ik'|v|i'k\rangle_A \chi^{(n)}_{kk'} = 0$$

ph RPA equations

$$(E_n - \varepsilon_i + \varepsilon_{i'})\chi^{(n)}_{i'i} - \sum_{kk'} \langle ik'|v|i'k\rangle_A \chi^{(n)}_{kk'} - \sum_{kk'} \langle ik|v|i'k'\rangle_A \chi^{(n)}_{kk'} = 0$$

$$(E_n - \varepsilon_i + \varepsilon_{i'})\chi^{(n)}_{ii'} + \sum_{kk'} \langle ik|v|i'k'\rangle_A \chi^{(n)}_{k'k} + \sum_{kk'} \langle ik'|v|i'k\rangle_A \chi^{(n)}_{kk'} = 0$$

E_n excited energy of the target nucleus

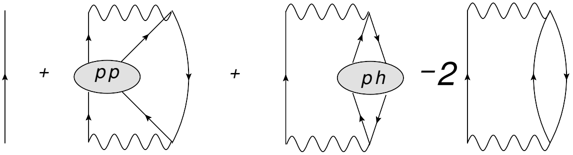
χ's 'occupation' of each ph pair

Exact Self-energy

$$\Sigma(2,3) = -i \int d4d5v(2,4)G_2(24,54^+)G_1^{-1}(5,3)$$

Self-energy at the HF+RPA approximation

$$
\begin{aligned}
\Sigma_1(1,1') &= \Sigma_{HF}(1,1') + \Sigma_{pp}(1,1') + \Sigma_{ph}(1,1') - 2\Sigma^{(2)}(1,1') \\
\Sigma_{HF}(1,1') &= iv(1,1')G_1^{HF}(1,1') - i\delta(1,1')\int d2 v(1,2) G_1^{HF}(2;2^+) \\
\Sigma_{pp}(1,1') &= \int d3d4 v(1,3) G_1^{HF}(4,3) G_2(13;1'4) v(4,1') \\
\Sigma_{ph}(1,1') &= -\int d3d4 v(1,3)\Big[G_1^{HF}(1,1')G_2(34;3^+4^+) \\
&\quad - G_1^{HF}(1,4)G_2(43;1'3^+) - G_1^{HF}(3,1')G_2(41;4^+3) \\
&\quad - G_1^{HF}(3,4)G_2(14;1'3)\Big] v(4,1')
\end{aligned}
$$



Schrödinger equation

$$\frac{p^2}{2m}\phi_\lambda(\mathbf{r},\varepsilon) + \int d\mathbf{r}' V^{HF}(\mathbf{r},\mathbf{r}';\varepsilon)\phi_\lambda(\mathbf{r}',\varepsilon) = E(\varepsilon)\phi_\lambda(\mathbf{r},\varepsilon)$$

HF potential

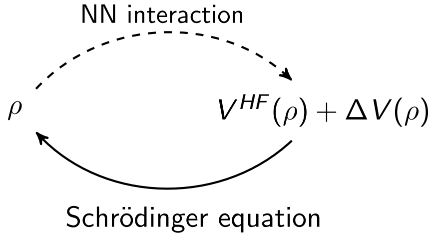
$$V^{HF}(\mathbf{r},\mathbf{r}'';\varepsilon) = \delta(\mathbf{r},\mathbf{r}'')\int d\mathbf{r}' v(\mathbf{r},\mathbf{r}')\rho(\mathbf{r}') - v(\mathbf{r},\mathbf{r}'')\rho(\mathbf{r},\mathbf{r}'')$$

RPA potential

$$\begin{aligned} V^{RPA}(\mathbf{r},\mathbf{r}',E) &= \lim_{\eta\to 0^+} \sum_{N\neq 0, ijkl} \sum\!\!\!\!\!\!\int_{\lambda} \chi_{ij}^{(N)} \chi_{kl}^{(N)} \\ &\times \left(\frac{n_\lambda}{E-\epsilon_\lambda+E_N-i\eta} + \frac{1-n_\lambda}{E-\epsilon_\lambda-E_N+i\eta} \right) \\ &\times F_{ij\lambda}(\mathbf{r})F^*_{kl\lambda}(\mathbf{r}') \end{aligned}$$

with

$$F_{ij\lambda}(\mathbf{r}) = \int d^3\mathbf{r}_1 \phi_i^*(\mathbf{r}_1) v(\mathbf{r},\mathbf{r}_1)[1-P]\phi_\lambda(\mathbf{r})\phi_j(\mathbf{r}_1)$$


NN interaction
ρ
$V^{HF}(\rho) + \Delta V(\rho)$
Schrödinger equation

SCRPA

Optical potential

The optical potential as a possible connection between different levels of phenomenology

- Phenomenological optical potential
- Potentials based on phenomenological effective NN interaction (Gogny, Skyrme...)
- Ab-initio potentials based on phenomenological bare NN interaction

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- Ab-initio potentials based on phenomenological bare NN interaction

Possibility of fruitful exchanges between those communities

Ab initio potential

- Nuclear matter method (50 MeV - 1 GeV)
- Resonating Group Method / No Core Shell Model (light nuclei and weak energy)
- Green's function Monte Carlo (light nuclei and weak energy)
- Self-consistent Green's function (doubly magic nuclei)
- Gorkov-SCGF (around doubly magic nuclei)
- Coupled cluster (doubly magic nuclei)

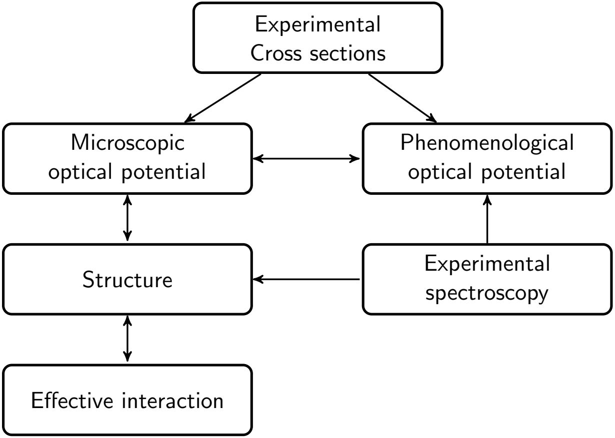
Potential based on effective interaction

- Nuclear Structure Method developed by N. Vinh Mau
- Recent interest (Orsay, Hanoï, Japan, Milano, China, Bruyères, Russia)

Phenomenological optical potential

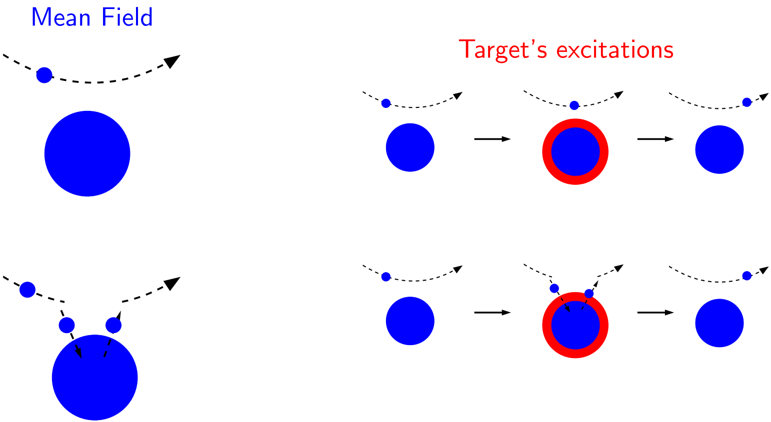
- Precision required for the evaluations
- Contrained by numerous calculations using reaction codes: TALYS, EMPIRE
- Predictivity outside the range parametrization
- Parametrization of non local dispersive potentials
- Issues induced by localisation procedures : effet Perey, dépendance spurieuse en énergie

NUCLEAR STRUCTURE METHOD FOR SCATTERING


Experimental
Cross sections
Microscopic
optical potential
Phenomenological
optical potential
Structure
Experimental
spectroscopy
Effective interaction

Nuclear Structure Method (NSM)

$$V = V^{HF} + \Delta V^{RPA}$$



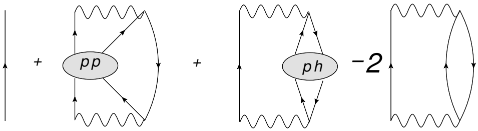
(N. Vinh Mau, Theory of nuclear structure (IAEA, Vienna) p. 931 (1970),
G. Blanchon, M. Dupuis, H.F. Arellano et N. Vinh Mau, PRC 91, 014612 (2015))

Nuclear Structure Method

Optical potential

$$V = V^{HF} + V^{PP} + V^{RPA} - 2V^{(2)}$$

Bare
Interaction

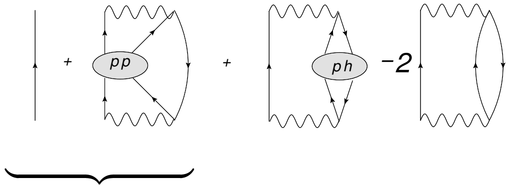


Nuclear Structure Method

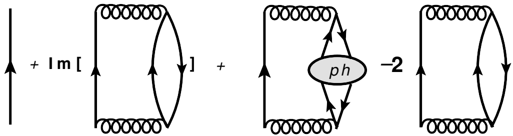
Optical potential

$$V = V^{HF} + V^{PP} + V^{RPA} - 2V^{(2)}$$

Bare Interaction



Effective Interaction



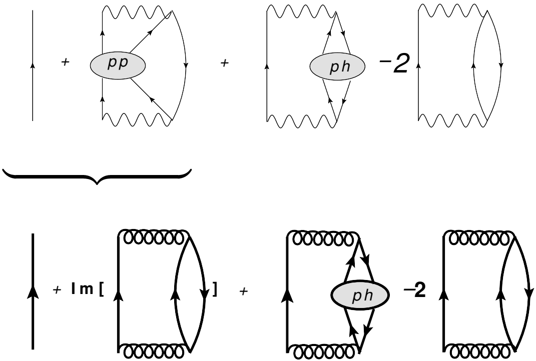
Nuclear Structure Method

Optical potential

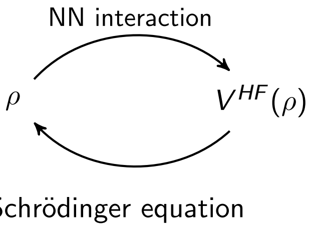
$$V = V^{HF} + V^{PP} + V^{RPA} - 2V^{(2)}$$

Bare Interaction

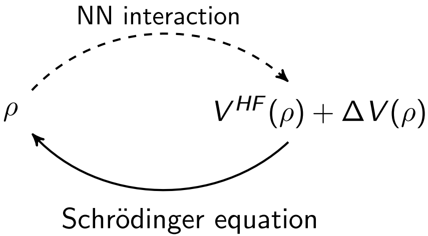
Gogny Interaction



Self-consistency

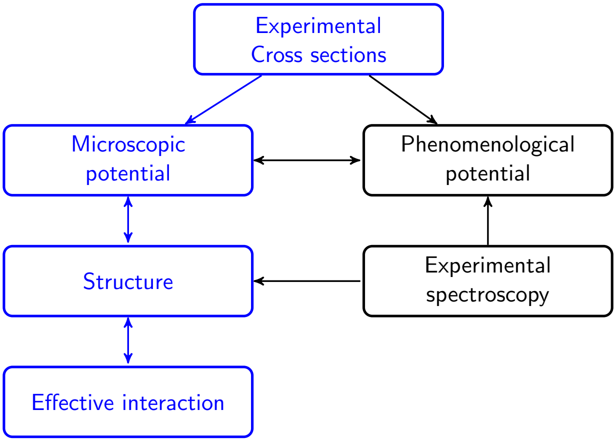


SCHF



SCRPA

Elastic scattering n/p + ^{40}Ca

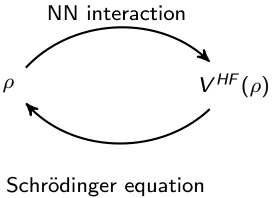


Schrödinger equation

$$\frac{p^2}{2m}\phi_\lambda(\mathbf{r},\varepsilon) + \int d\mathbf{r'}\,V^{HF}(\mathbf{r},\mathbf{r'};\varepsilon)\phi_\lambda(\mathbf{r'},\varepsilon) = E(\varepsilon)\phi_\lambda(\mathbf{r},\varepsilon)$$

HF potential

$$V^{HF}(\mathbf{r},\mathbf{r''};\varepsilon) = \delta(\mathbf{r},\mathbf{r''})\int d\mathbf{r'}\,v(\mathbf{r},\mathbf{r'})\rho(\mathbf{r'}) - v(\mathbf{r},\mathbf{r''})\rho(\mathbf{r},\mathbf{r''})$$



HF potential shape

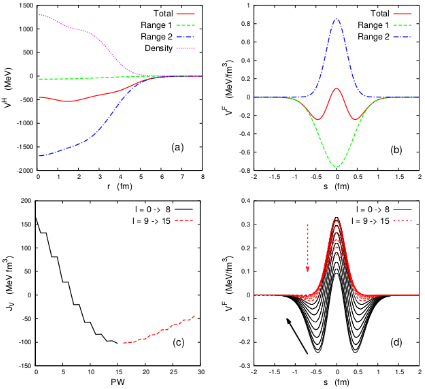
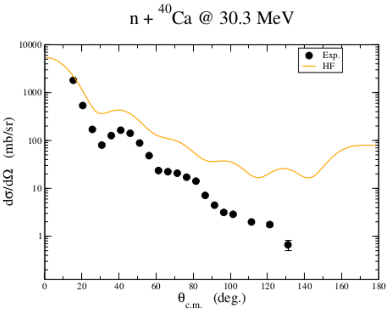
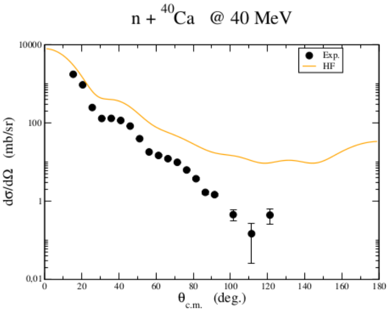


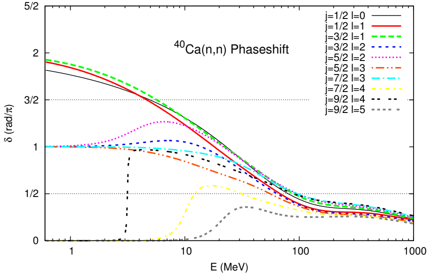
Fig. 15. Contributions for n + ^{40}Ca to: (a) to the Hartree local potential (V^H): Total (solid line), first range of D1S (dashed line), second range of D1S (dash-dotted line) and density term (dotted line). (b) First partial wave of the nonlocal Fock term at $r = r' = 4.3$ fm: Total (solid line), first range of D1S (dashed line) and second range of D1S (dash-dotted line). (c) Volume integral of the Fock potential as a function of partial wave: Negative slope (solid line), positive slope (dashed line). (d) Same as (c) for the Fock components nonlocality at $r = r' = 4.3$ fm.

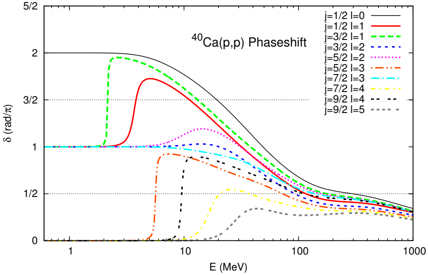
HF cross section





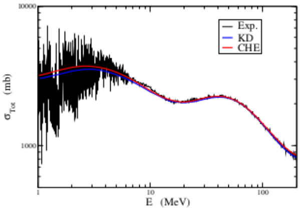
HF phaseshift n/p+^{40}Ca

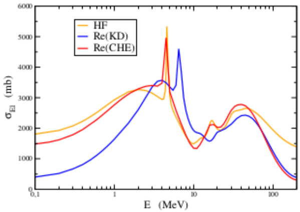




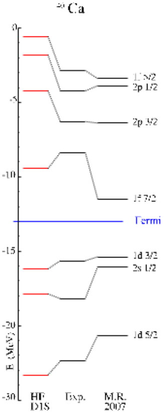
- Single particle resonances when $\delta = n\pi/2$ (n impair).
- Exact treatment of the intermediate wave ϕ_λ.
- Strong impact on ΔV_{RPA}
- Levinson theorem and total cross section

Total cross section n+^{40}Ca





Bound states HF/D1S Exp. CHE



- V^{HF} gives the main contribution to the real part of the potential

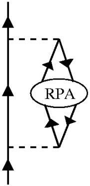
(B. Morillon and P. Romain, Phys. Rev. C 70, 014601 (2004).) $\rightarrow$ dispersive potential

(A. J. Koning and J. P. Delaroche, Nuclear Physics A 713, 231 (2003).)

ph-RPA potential

$$\Delta V_{RPA} = \mathrm{Im}\left[V^{(2)}\right] + V^{RPA} - 2V^{(2)}$$

$$V^{RPA}(\mathbf{r},\mathbf{r}',E) = \lim_{\eta\to 0^+} \sum_{N\neq 0, ijkl} \sum\!\!\!\!\!\!\int_{\lambda} \chi_{ij}^{(N)} \chi_{kl}^{(N)} \times \left(\frac{n_\lambda}{E-\epsilon_\lambda + E_N - i\Gamma(E_N)} + \frac{1-n_\lambda}{E-\epsilon_\lambda - E_N + i\Gamma(E_N)} \right) F_{ij\lambda}(\mathbf{r}) F^*_{kl\lambda}(\mathbf{r}')$$



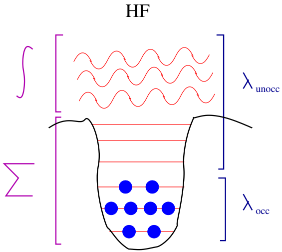
with

$$F_{ij\lambda}(\mathbf{r}) = \int d^3\mathbf{r}_1 \phi_i^*(\mathbf{r}_1) v(\mathbf{r},\mathbf{r}_1)[1-P]\phi_\lambda(\mathbf{r})\phi_j(\mathbf{r}_1)$$

- ϕ are HF wave functions.
- Bound as well as continuum states are taken into account for the intermediate state ϕ_λ.
- Target excitations are obtained from the spherical RPA/D1S code.

  Blaizot, et al., NPA 265, 315 (1976).

  Berger, et al., Comp. Phys. Com. 63, 365 (1991).



$V^{(2)}$ potential

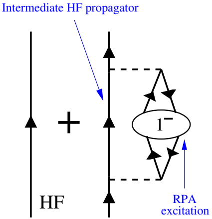
Uncorrelated particle-hole potential

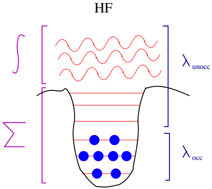
$$\begin{aligned} V^{(2)}(\mathbf{r},\mathbf{r}',E) &= \frac{1}{2}\sum_{ij}\sum\!\!\!\!\!\!\int_{\lambda}\left(\frac{n_i(1-n_j)n_\lambda}{E-\epsilon_\lambda+E_{ij}-i\Gamma(E_{ij})}\right. \\ &+ \left.\frac{n_j(1-n_i)(1-n_\lambda)}{E-\epsilon_\lambda-E_{ij}+i\Gamma(E_{ij})}\right)F_{ij\lambda}(\mathbf{r})F^*_{kl\lambda}(\mathbf{r}') \end{aligned}$$

with $E_{ij}=\varepsilon_i-\varepsilon_j$.

Coupling to a single excited state

- p+^{40}Ca scattering
- Potential: $V^{HF} + \mathrm{Im}(V^{RPA})$
- Coupling to the first 1^- state of ^{40}Ca with $E_{1^-} = 9.7$ MeV





Coupling to a single excited state

- p+^{40}Ca scattering
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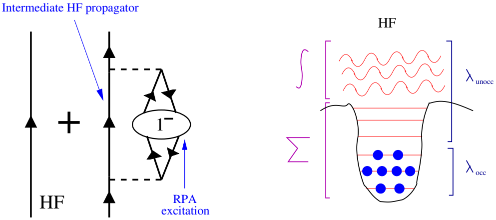
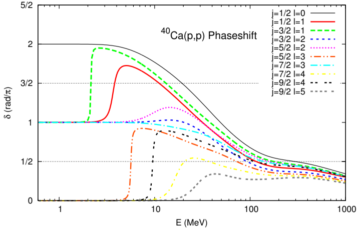
- Coupling to the first 1^- state of ^{40}Ca with $E_{1^-} = 9.7$ MeV



- HF phaseshift



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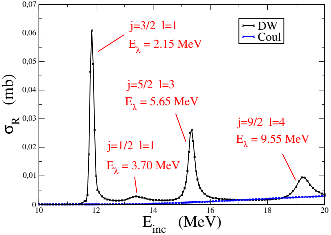
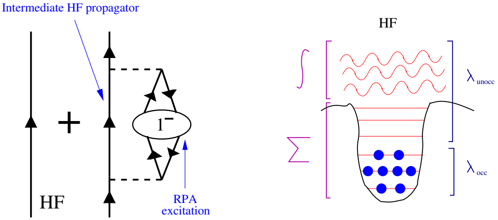
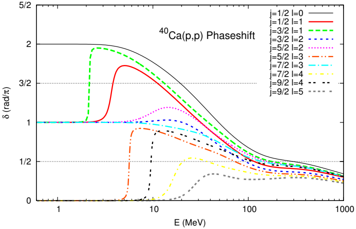




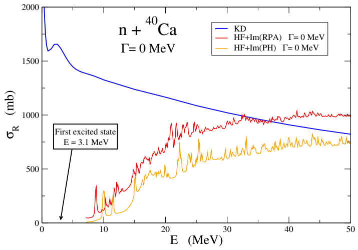
- Importance of the intermediate single particle resonances
- Strong impact on reaction cross section

- HF phaseshift



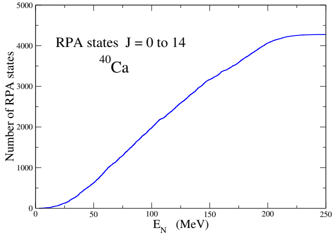
Effect of HF intermediate propagator

- σ_R from $V_{HF} + \mathrm{Im}(V_{RPA})$
- σ_R from $V_{HF} + \mathrm{Im}(V_{PH})$



$\rightarrow$ Effect of the HF resonances on $\mathrm{Im}(V_{RPA})$

- Zero width calculation:
  - $\sigma_R = 0$ for incident energies below the energy of the first excited state of the target nucleus
- ^{40}Ca RPA states $J = 0 \rightarrow 8$



$$S = \langle S \rangle + \widehat{S}$$

Averaged cross section

$$\langle \sigma_E \rangle = \frac{\pi}{k^2} \langle |1 - S|^2 \rangle$$
$$\langle \sigma_R \rangle = \frac{\pi}{k^2} \langle 1 - |S|^2 \rangle$$
$$\langle \sigma_T \rangle = \frac{\pi}{k^2} \langle 1 - Re[S] \rangle$$

Averaged potential

$$\bar{\sigma_E} = \frac{\pi}{k^2} |1 - \langle S \rangle|^2$$
$$\bar{\sigma_R} = \frac{\pi}{k^2} (1 - |\langle S \rangle|^2)$$
$$\bar{\sigma_T} = \frac{\pi}{k^2} (1 - Re[\langle S \rangle])$$

$$\langle \sigma_E \rangle = \bar{\sigma_E} + \sigma_{CE}$$
$$\langle \sigma_R \rangle = \bar{\sigma_R} - \sigma_{CE}$$
$$\langle \sigma_T \rangle = \bar{\sigma_T}$$

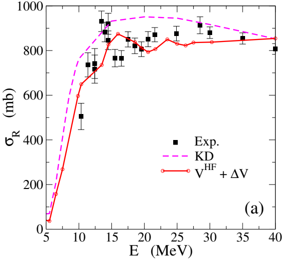
Compound elastic

$$\sigma_{CE} = \frac{\pi}{k^2} \langle |\widehat{S}|^2 \rangle$$

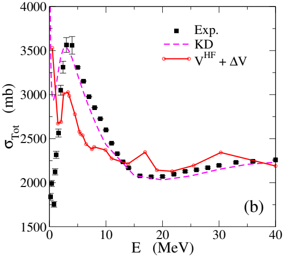
- TALYS: Hauser-Feshbach/ Koning-Delaroche
- particularly relevant for neutron scattering below 10 MeV

Integral cross sections n/p + ^{40}Ca

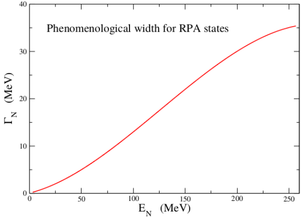
- p + ^{40}Ca



- n + ^{40}Ca



- Coupling to 4500 excited states of the target (J = 0 à 14) given by a RPA code projected on oscillator basis.
- Use of phenomenological width for the excited states of the target.



Integral cross sections n/p + ^{40}Ca

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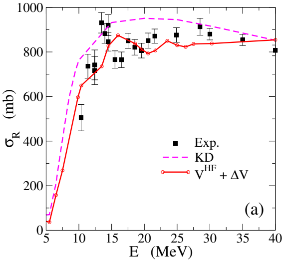
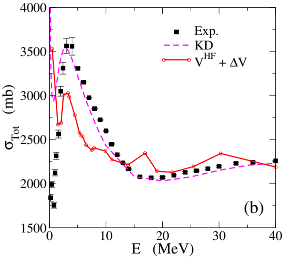


- n + ^{40}Ca



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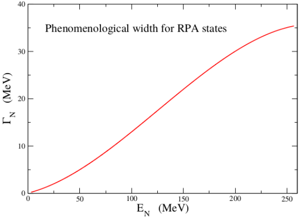
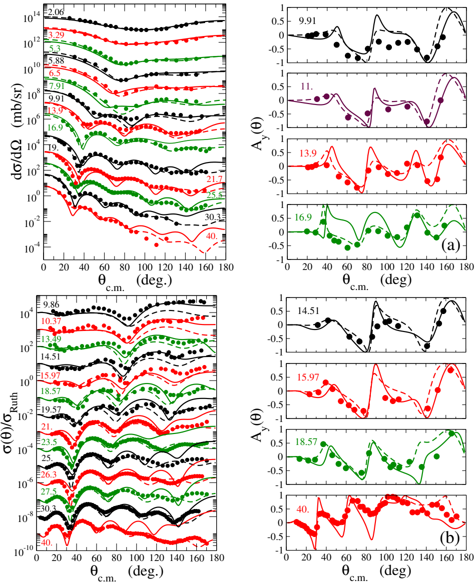


- In the future we would like a microscopic determination of energy widths and shifts: 2p-2h coupling

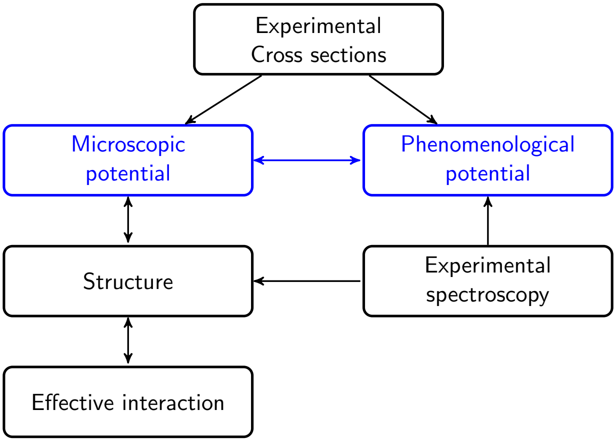
Cross section and Analysing powers n/p+^{40}Ca



NSM (full line)
Koning-Delaroche (dashed line)

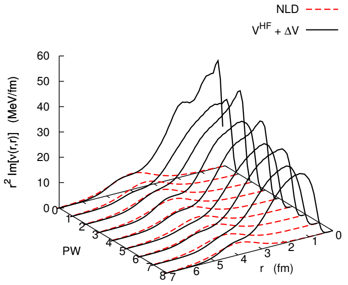
- Good agreement with cross section data below 30 MeV.
- In terms of energy regime, NSM is complementary to g-matrix approaches.
- Good agreement with analysing powers data: correct behaviour of the "spin-orbit" term of the potential.
- Effective interaction fitted with structure data + fission barriers

Microscopic and phenomenological potentials



Potential for n + ^{40}Ca @ 10 MeV

- NSM potential
- Non local dispersive potential fitted on all the available data for ^{40}Ca
  $\nu_{lj}(r,r') = \iint d\hat{\mathbf{r}}d\hat{\mathbf{r}}'\mathcal{Y}_{jl}^{m}(\hat{\mathbf{r}})V(\mathbf{r},\mathbf{r'})\mathcal{Y}_{jl}^{m\dagger}(\hat{\mathbf{r}}')$



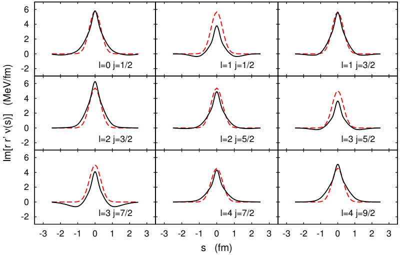
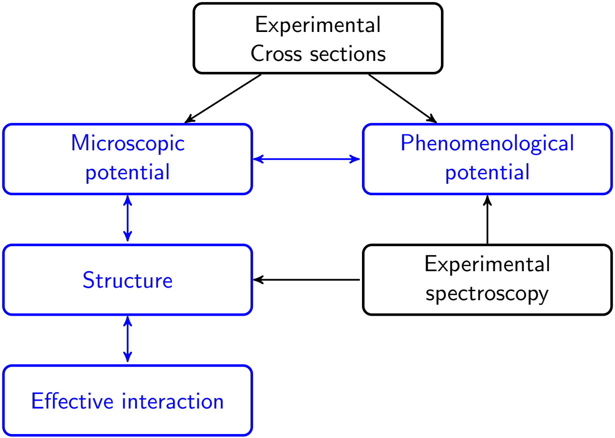


Figure: $s = |\mathbf{r} - \mathbf{r'}|$

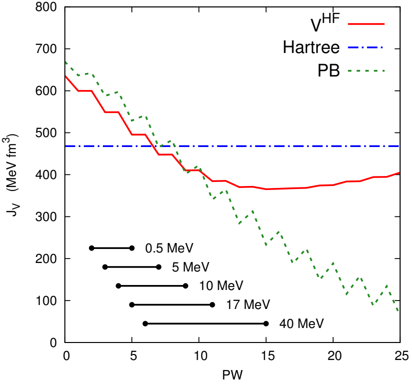
M.H. Mahzoon, R.J. Charity, W.H. Dickhoff, H. Dussan, S.J. Waldecker, Phys. Rev. Lett. 112, 162503 (2014)

Phenomenological potential and effective interaction



Phenomenological potential and D1S effective interaction

Volume integral: $J_V^{lj} = \frac{-4\pi}{A} \int dr\ r^2 \int dr' r'^2 \nu_{lj}(r,r')$



- Perey Buck optical potential with gaussian non locality and energy independent.

Phenomenological potential and D1S effective interaction

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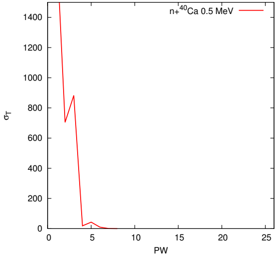

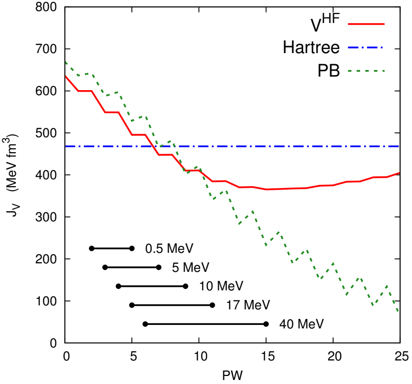

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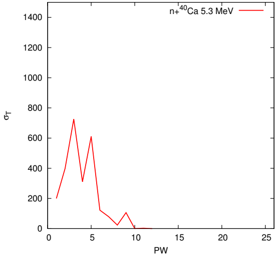

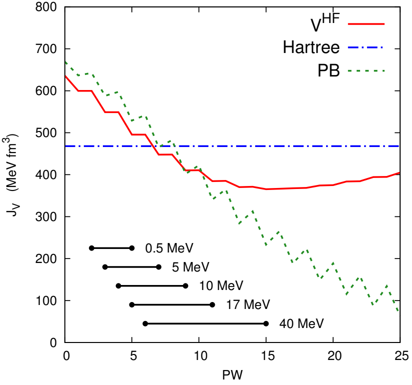

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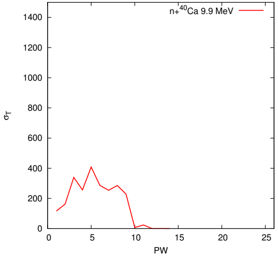

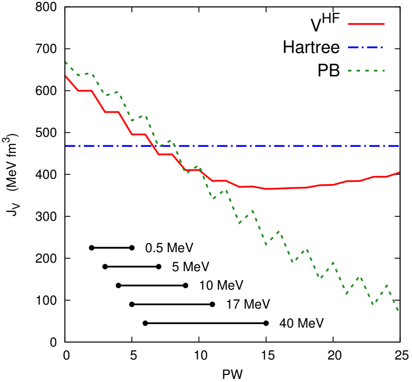

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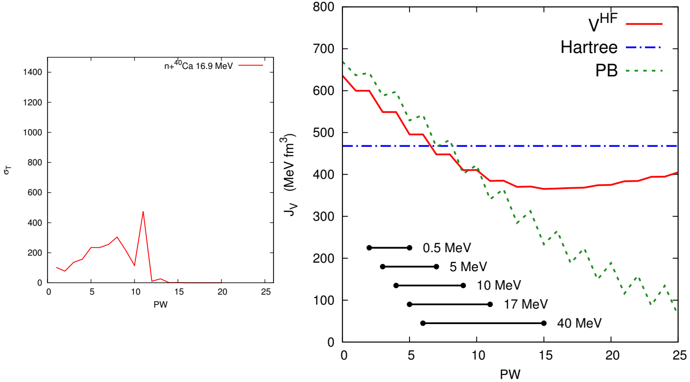

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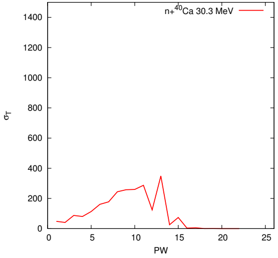

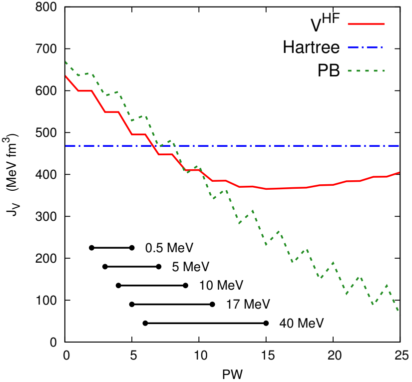

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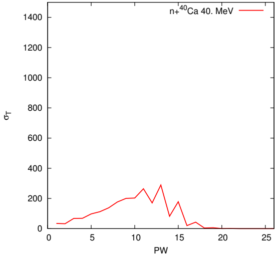

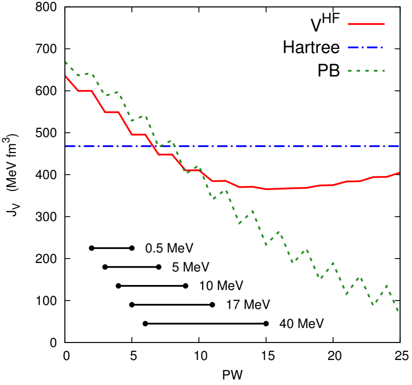

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HF potential shape

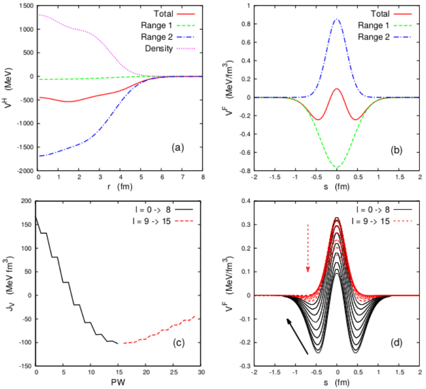

Fig. 15. Contributions for n + ^{40}Ca to: (a) to the Hartree local potential (V^H): Total (solid line), first range of D1S (dashed line), second range of D1S (dash-dotted line) and density term (dotted line). (b) First partial wave of the nonlocal Fock term at $r = r' = 4.3$ fm: Total (solid line), first range of D1S (dashed line) and second range of D1S (dash-dotted line). (c) Volume integral of the Fock potential as a function of partial wave: Negative slope (solid line), positive slope (dashed line). (d) Same as (c) for the Fock components nonlocality at $r = r' = 4.3$ fm.

FURTHER READINGS

- *Quelques applications du formalisme des fonctions de Green à l'étude des noyaux*,
  N. Vinh Mau
- *Quantum Theory of Many-Particle Systems*,
  Fetter and Walecka.
- *A Guide to Feynman Diagrams in the Many-Body Problem*,
  Mattuck.
- *Quantum Statistical Mechanics: Green's Function Methods in Equilibrium and Non-Equilibrium Problems*,
  Kadanoff.
- *The nuclear many-body problem*,
  Ring and Schuck.