Westfälische Wilhelms-Universität Münster
institut für theoretische physik
GRK 2149
ALPHA Collaboration

Heavy Quark Masses from Lattice QCD

New Possibilities in Physics of Quarkonia

Workshop @ Institut Henri Poincaré, Paris, France, September 24 – 25, 2015

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September 25, 2015

Motivation

- Quark masses belong to the fundamental parameters of the Standard Model (SM)
- They enter in many applications to phenomenology and Beyond-SM physics $\rightarrow$ prominently: Higgs partial widths
  - Couplings are proportional to quark masses
  - Main source of uncertainty in partial widths comes from m_c, m_b and α_s, as argued in [Lepage, Mackenzie & Peskin, arXiv:1404.0319]
  - significant impact of m_c, m_b on precision Higgs physics at future experiments (LHC and particularly ILC)
  - e.g., reducing a to $0.023\,\mathrm{fm}$ brings parametric errors for the Higgs couplings below those expected from the full ILC

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  - e.g., reducing a to 0.023 fm brings parametric errors for the Higgs couplings below those expected from the full ILC
- Here: summarize current status of lattice results for charm and bottom masses, emphasizing the specific challenges and focusing on a *subjective* selection of approaches to them

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How to define quark masses?

- Quarks are not asymptotic (physical) states
- As the (running) strong coupling constant, quark masses are renormalization scheme and scale dependent:

  $\overline{m}_f^{\mathrm{X}}(\mu) \qquad f = \mathrm{u}, \mathrm{d}, \mathrm{s}, \ldots \qquad \mathrm{X} = \text{scheme} = \overline{\mathrm{MS}}, \text{RI-MOM}, \text{SF}, \ldots$
- It's customary practice to quote renormalized quark masses as

  $\overline{m}_f^{\overline{\mathrm{MS}}}(\mu_{\mathrm{ref}}) \qquad$ e.g. $\mu_{\mathrm{ref}} = 2\,\mathrm{GeV}$ or $\mu_{\mathrm{ref}} = \overline{m}_{\mathrm{c}}, \overline{m}_{\mathrm{b}}$ for $f = \mathrm{c}, \mathrm{b}$

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  in order to enable comparisons / averages among different (continuum & lattice) approaches of their determination
- Lattice QCD calculations of light ($f = \mathrm{u/d,s,c}$) quark masses typically employ the *partially conserved axial current (PCAC)* definition:

$$m_{\mathrm{PCAC}} = \frac{\langle \alpha \,|\partial_\mu A_\mu^a(x)|\, \beta\rangle}{2\langle \alpha \,|P^a(x)|\, \beta\rangle}$$

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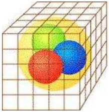
Lattice QCD — ‘Ab initio’ tool based on FIs & MC

$$\mathcal{L}_{\mathrm{QCD}}\left[g_0, m_f\right] = -\frac{1}{2g_0^2}\,\mathrm{Tr}\left\{F_{\mu\nu}F_{\mu\nu}\right\} + \sum_{f=\mathrm{u,d,s,\ldots}} \overline{\psi}_f\left\{\gamma_\mu\left(\partial_\mu + g_0 A_\mu\right) + m_f\right\}\psi_f$$

$$\underbrace{\begin{bmatrix} f_{\mathrm{K}} \\ m_\pi \\ m_{\mathrm{K}} \\ m_{\mathrm{D}} \\ m_{\mathrm{B}} \end{bmatrix}}_{\text{Experiment}} \quad \xrightarrow{\mathcal{L}_{\mathrm{QCD}}\left[g_0, m_f\right]} \quad \underbrace{\begin{bmatrix} \Lambda_{\mathrm{QCD}} \\ M_{\mathrm{u}}, M_{\mathrm{d}} \\ M_{\mathrm{s}} \\ M_{\mathrm{c}} \\ M_{\mathrm{b}} \end{bmatrix}}_{\text{QCD parameters (RGIs)}} \quad + \quad \underbrace{\begin{bmatrix} f_{\mathrm{D}} \\ f_{\mathrm{B}} \\ B_{\mathrm{K}}, B_{\mathrm{B}} \\ \xi \\ \ldots \end{bmatrix}}_{\text{Predictions}}$$



Main sources of systematic uncertainties in LQCD computations:

- Part of the vacuum polarization effects is missed, as long as u, d, s (and ideally also c) sea quarks are not incorporated
  $\rightarrow$ today’s LQCD computations use $N_{\mathrm{f}} = 2,\, 2+1$ and even $2+1+1$

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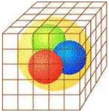

Main sources of systematic uncertainties in LQCD computations:

- Extrapolations to $m_{\rm u,d}$ guided by χPT to connect to physical world
- Discretization errors, notably from heavy quarks: $\mathrm{O}\left[(am_{\rm Q})^n\right]$ effects
  $\rightarrow$ # $\gtrsim$ 3 lattice spacings needed to take continuum limit, $a \rightarrow 0$
- Perturbative vs. non-perturbative renormalization

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Determination of quark masses on the lattice

Usually, the *bare quark masses* are *input parameters* to lattice simulations in the (practical) sense that are tuned to reproduce physical quantities, for instance:

$$
\begin{aligned}
m_{\mathrm{u,d}}^{(0)} &\longrightarrow m_\pi^2 \\
m_{\mathrm{s}}^{(0)} &\longrightarrow m_{\mathrm{K}}^2 \\
m_{\mathrm{c}}^{(0)} &\longrightarrow m_{\eta_{\mathrm{c}}},\, m_{\mathrm{D}_{(\mathrm{s})}}
\end{aligned}
$$

Conversion from lattice input parameters to *renormalized quark masses* amounts to renormalization: $\overline{m}_f^{\overline{\mathrm{MS}}}(\mu) = Z_m^{\overline{\mathrm{MS}}}(\mu, a^{-1}) \times m_f^{(0)}$

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Tuning to be performed at several a, defining a trajectory (*line of constant physics*), along which the continuum limit $a \to 0$ is taken

- Remaining physics is then a prediction of QCD
- Parameters can be varied away from the physical values
  $\to$ understand quark mass effects, quantify systematics, etc.

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Challenges of treating heavy quarks on the lattice

Why are there, all in all, less lattice results available in the heavy sector (m_c, m_b) than in the light sector ($m_{u,d,s}$) ?

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- m_c
  - requires $am_c^{(0)} < 1$ to keep discretization effects under control
  - sufficiently large # of sites to minimize finite-volume effects

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- m_c
  - requires $am_c^{(0)} < 1$ to keep discretization effects under control
  - sufficiently large # of sites to minimize finite-volume effects
- m_b
  - even more difficult to include b-quarks in LQCD computations, since $am_b^{(0)} \ll 1$ required
  - a fully relativistic b not yet feasible with today's CPU resources (attempts by HPQCD via highly-improved actions exist, though) $\rightarrow$ effective field theories: NRQCD, HQET

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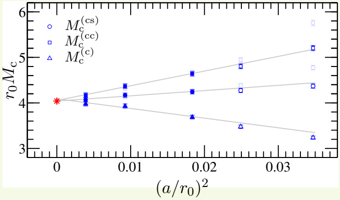
Illustration: Cutoff effects in the charm sector

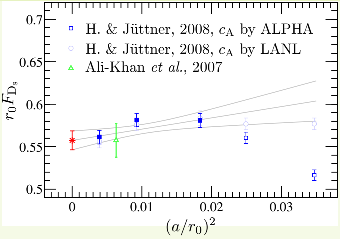
High-precision computation of m_c & $F_{\mathrm{D_s}}$ in quenched QCD ($N_\mathrm{f} = 0$)

- Large volume and small lattice spacings: $a \approx (0.09 - 0.03)$ fm
- $\mathrm{O}(a, am_{\mathrm{q,c}})$ cutoff effects relevant & removed NP'ly [ALPHA Collaboration]
- Control of the CL via scaling study down to very fine lattices

Lattice artefacts may be large

[H. & Jüttner, JHEP0905(2009)101]





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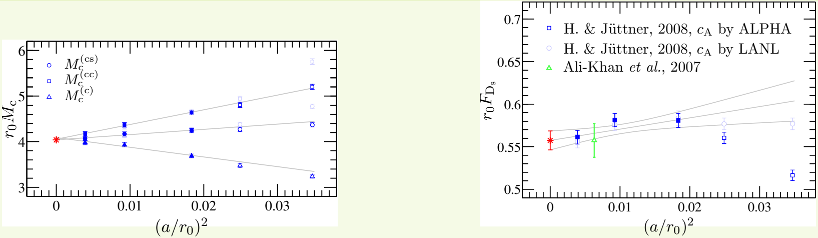
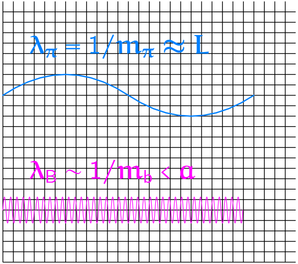


$\Rightarrow$ *Warning* from F_{D_s}:
Symanzik programme works for charm, but $a < 0.08$ fm seems mandatory (note: small lattice spacings are challenging for $N_f > 0$)

Why effective theories in the bottom sector?



- Light quarks: too light
  - Widely spread objects
  - Finite-volume errors via light π's
- b-quark: too heavy
  - Extremely localized object
  - B-mesons with a propagating b need fine resolutions ($am_b \ll 1$):
    - large discretization errors
    - "they fall through the lattice"

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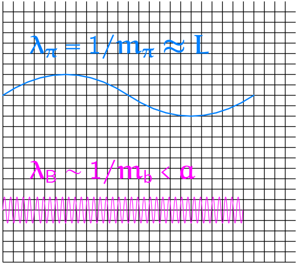

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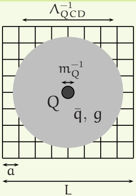
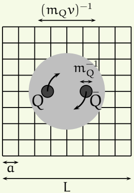
Heavy Quark Effective Theory — Non-Relativistic QCD



Resort to discretized low-energy *effective* theory for the b: HQET (for heavy-light systems) or NRQCD (for heavy quarkonia, too), differing in how they classify interactions as dictated by underlying dynamics of hl- and hh-hadrons



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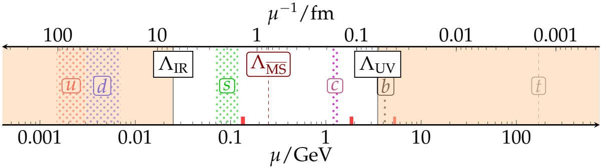
Issue with lattice B-physics: A multi-scale problem

Predictivity in a quantum field theory relies upon a large scale ratio

interaction range $\ll$ physical length scales

momentum cutoff $\gg$ physical mass scales : $\Lambda_{\mathrm{cut}} \sim a^{-1} \gg E_i, m_j$



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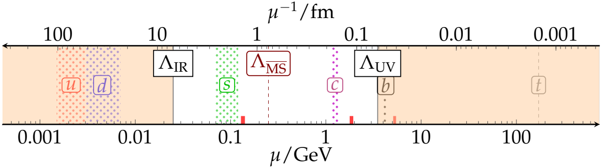

Hierarchy of disparate physical scales difficult to cover simultaneously:

$$\Lambda_{\mathrm{IR}} = L^{-1} \ll m_\pi, \ldots, m_{\mathrm{D}}, m_{\mathrm{B}} \ll a^{-1} = \Lambda_{\mathrm{UV}}$$

$$\downarrow \qquad\qquad \downarrow$$

$$\left\{ \mathrm{O}(e^{-Lm_\pi}) \Rightarrow L \gtrsim \frac{4}{m_\pi} \sim 6\,\mathrm{fm} \right\} \curvearrowright \; L/a \gtrsim 120 \; \curvearrowleft \left\{ am_{\mathrm{D}} \lesssim \frac{1}{2} \Rightarrow a \approx 0.05\,\mathrm{fm} \right\}$$

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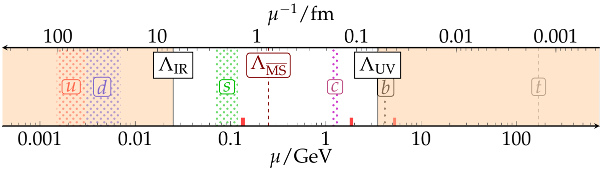

Propagation of the charm quark still resolvable, but the dominant b-quark mass scale ($m_{\mathrm{b}}/m_{\mathrm{c}} \sim 4$) has to be separated / removed from the others in a sound ways before handling the theory in numerical simulations

$\Rightarrow$ demands effective theories & clever techniques such as the Heavy Quark Effective Theory formulation for the b-quark in heavy-light systems (a few more details later ...)

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Outline

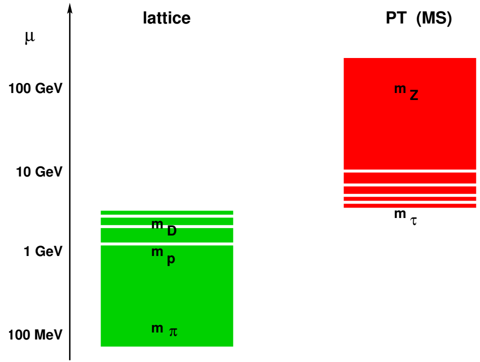
1. Motivation & Challenges
2. Non-perturbative quark mass renormalization and m_c
3. m_b from non-perturbative HQET at $O(1/m_h)$
4. Current status of lattice results on m_c and m_b
5. Conclusions & Prospects

Non-perturbative quark mass renormalization and m_c

NP quark mass renormalization: Strategy

To translate a lattice input parameter/bare quark mass to a renormalized one amounts to determine a renormalization factor, but we already know:

Multiple scale problems difficult for a numerical / lattice treatment



$1/L \ll \mu, E, |\mathbf{p}| \ll 1/a$ $\qquad$ $\mu_{\mathrm{pert}} \gg \Lambda_{\mathrm{QCD}}$

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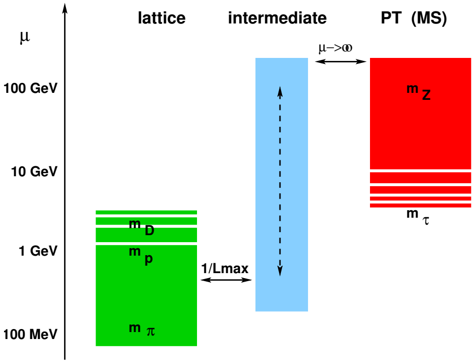


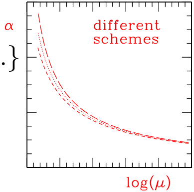
intermediate = Schrödinger functional, finite-volume scheme

Scale dependence of QCD parameters

- Renormalization group (RG) equations, $\bar{g} \equiv \bar{g}(\mu)$

$$\mu \frac{\partial \bar{g}}{\partial \mu} = \beta(\bar{g}) \quad \overset{\bar{g}\to 0}{\sim} \quad -\bar{g}^3 \{b_0 + b_1\bar{g}^2 + b_2\bar{g}^4 + \ldots\}$$

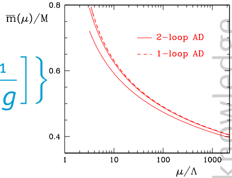
$$\mu \frac{\partial \overline{m}}{\partial \mu} = \tau(\bar{g})\, \overline{m} \qquad \tau(\bar{g}) \quad \overset{\bar{g}\to 0}{\sim} \quad -\bar{g}^2 \{d_0 + d_1\bar{g}^2 + \ldots\}$$



- Solution leads to *exact* equations in a mass-indep. scheme

$$\Lambda \equiv \mu\,(b_0\bar{g}^2)^{-b_1/(2b_0^2)}\, e^{-1/(2b_0\bar{g}^2)} \times \exp\left\{-\int_0^{\bar{g}} dg \left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g}\right]\right\}$$

$$M \equiv \overline{m}(\mu)\,(2b_0\bar{g}^2)^{-d_0/(2b_0)} \exp\left\{-\int_0^{\bar{g}} dg \left[\frac{\tau(g)}{\beta(g)} - \frac{d_0}{b_0 g}\right]\right\}$$



RG invariant quark mass

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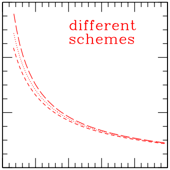

- Solution leads to *exact* equations in a mass-indep. scheme
- *Simple* relations between different renormalization schemes:

$$\mathrm{S} \;\to\; \mathrm{S}' \qquad\qquad \alpha \;\to\; \alpha' = \alpha + c\,\alpha^2 + \mathrm{O}(\alpha^3)$$

implies that M is scale & scheme independent

$$\frac{\Lambda_{\mathrm{S}'}}{\Lambda_{\mathrm{S}}} = \mathrm{e}^{\frac{c}{4\pi b_0}}\,, \quad \frac{\overline{m}_{\mathrm{S}'}(\mu)}{\overline{m}_{\mathrm{S}}(\mu)} = 1 + \mathrm{O}\left(\alpha(\mu)\right) \stackrel{\mu\to\infty}{\longrightarrow} 1 \;\Rightarrow\; M_{\mathrm{S}'} = M_{\mathrm{S}} \equiv M$$

$\Rightarrow$ choose a *convenient* scheme (i.e. via some physical coupling) to compute M

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The basic equation for the RGI quark mass

For a lattice QCD computation of light quark masses as well as the charm mass, define the running mass through the PCAC relation:

$$\partial_\mu A_\mu^{\mathrm{cs}} = (m_\mathrm{s} + m_\mathrm{c})\, P^{\mathrm{cs}} \qquad \begin{cases} A_\mu^{\mathrm{cs}} = \bar{s}\,\gamma_\mu\gamma_5\, c : & \text{axial vector current} \\ P^{\mathrm{cs}} = \bar{s}\,\gamma_5\, c : & \text{pseudoscalar density} \end{cases}$$

Upon renormalization in the lattice regularized theory $(g_0 \leftrightarrow a)$:

$$\underbrace{\overline{m}_\mathrm{s}(\mu) + \overline{m}_\mathrm{c}(\mu)}_{\text{renormalized \& running}} = \frac{Z_\mathrm{A}(g_0)}{Z_\mathrm{P}(g_0,\mu)} \times \underbrace{\frac{\langle 0 \,|\, \partial_\mu A_\mu^{\mathrm{cs}} \,|\, \mathrm{D}_\mathrm{s}^+(\mathbf{p}=0)\rangle}{\langle 0 \,|\, P^{\mathrm{cs}} \,|\, \mathrm{D}_\mathrm{s}^+(\mathbf{p}=0)\rangle}}_{m_\mathrm{s} + m_\mathrm{c} = \frac{F_{\mathrm{D_s}} m_{\mathrm{D_s}}^2}{G_{\mathrm{D_s}}} \,:\ \text{bare PCAC}}$$

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- Scale & scheme dependence via renormalization factor Z_P, which is poorly convergent in PT
  $\rightarrow$ NP estimate needed [ALPHA Collaboration 2005 $(N_\mathrm{f}=2)$ & under way $(N_\mathrm{f}=3)$]
  Z_A is fixed by imposing a chiral Ward identity from Euclidean current algebra [ALPHA Collaboration 2005 $(N_\mathrm{f}=2)$ & under way $(N_\mathrm{f}=3)$]

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Now split up the problem according to this generic strategy:

$$M_{\mathrm{c}} = \frac{M}{\overline{m}(\mu)}\,\overline{m}_{\mathrm{c}}(\mu) = \frac{M}{\overline{m}(\mu)}\,\frac{Z_{\mathrm{A}}(g_0)}{Z_{\mathrm{P}}(g_0,\mu)}\,\underbrace{m_{\mathrm{c}}(g_0)}_{\text{bare PCAC}}$$

$$\equiv Z_{\mathrm{M}}(g_0)\times m_{\mathrm{c}}(g_0) \qquad (\text{recall: } g_0 \leftrightarrow a)$$

$$Z_{\mathrm{M}}(g_0) = \frac{M}{\overline{m}(\mu)}\,\frac{Z_{\mathrm{A}}(g_0)}{Z_{\mathrm{P}}(g_0,\mu)} = \underbrace{\frac{M}{\overline{m}(\mu_{\mathrm{pert}})}}_{\text{PT}}\,\underbrace{\frac{\overline{m}(\mu_{\mathrm{pert}})}{\overline{m}(\mu_{\mathrm{had}})}}_{\text{NP}}\,\underbrace{\frac{Z_{\mathrm{A}}(g_0)}{Z_{\mathrm{P}}(g_0,\mu_{\mathrm{had}})}}_{\text{"easy"}}$$

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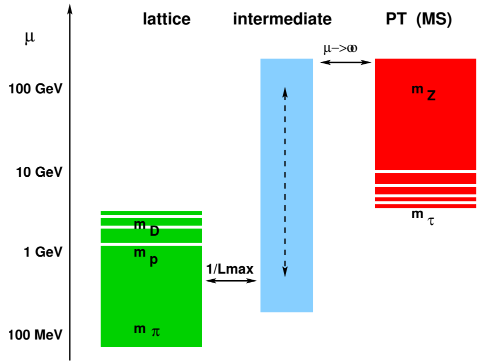

Basic equation for the RGI quark mass [ALPHA Collaboration 1999,2005,... $(N_f = 0, 2, 3)$]

$$Z_M(g_0) = \underbrace{\frac{Z_A(g_0)}{Z_P(g_0,\mu_{had})}}_{\text{lattice}} \underbrace{\frac{\overline{m}(\mu_{pert})}{\overline{m}(\mu_{had})}}_{\textbf{NP}\text{: interm. \& CL}} \underbrace{\frac{M}{\overline{m}(\mu_{pert})}}_{\text{PT}}$$

$$M_f = Z_M(g_0)\, m_f(g_0) \qquad (\text{here: } f = \mathrm{u/d, s, c})$$

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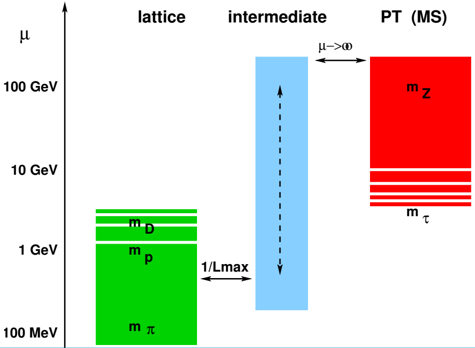
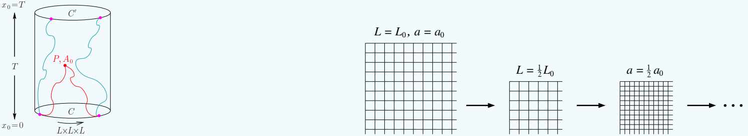


NP calculation of $\overline{m}(\mu_{\rm pert})/\overline{m}(\mu_{\rm had})$ in the *intermediate SF scheme*

Schrödinger Functional = QCD in a cylindric box (Dirichlet BCs in x_0)



⇒ NP running from (finite-vol.) correlators & recursive technique, $\mu = L^{-1}$

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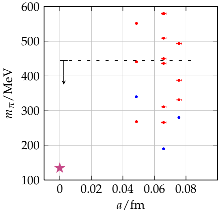
Application: The charm quark's mass in $N_\mathrm{f} = 2$ QCD

[ALPHA Collaboration, H., von Hippel, Schaefer & Virotta, PoS LATTICE2013 (2013) 475, arXiv:1312.7693]

- Coordinated Lattice Simulations ensembles

CLS based

Computational setup:
Wilson plaquette gauge action,
$N_\mathrm{f} = 2$ mass degenerate NP'ly $\mathrm{O}(a)$ improved Wilson sea quarks with
$Lm_\pi \gtrsim 4$, $(190 \lesssim m_\pi \lesssim 440)$ MeV,
$a \in \{0.065\,\mathrm{fm}\,,\ 0.048\,\mathrm{fm}\}$



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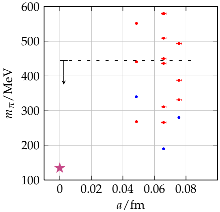

- Scale setting through f_K, the "physical point" being defined by:
  $m_{\pi,\text{phys}} = 134.8$ MeV, $m_{K,\text{phys}} = 494.2$ MeV, $f_{K,\text{phys}} = 155$ MeV
  (in the isospin-symmetric limit with QED effects removed)

- Partially quenched setup: strange quark's mass κ_s fixed via

  $am_{\text{PCAC}}(\kappa_l, \kappa_s) = \mu_s$ where μ_s is defined from m_K^2/f_K^2

- Fixing of the hopping parameter of the (valence) charm quark, κ_c, through the physical D_s-meson mass:

$$m_{D_s} = m_{D_s,\text{phys}} = 1968\,\text{MeV} \qquad [\,(am_{D_s})^2 \text{ linearly interpolated in } 1/\kappa_c\,]$$

- Correlation functions of PS density and axial vector current:

$$f_{PP}^{ij}(x_0) = -a^3 \sum_{\mathbf{x}} \langle P^{ij}(x) P^{ji}(0) \rangle \quad , \quad f_{AP}^{ij}(x_0) = -a^3 \sum_{\mathbf{x}} \langle A_0^{ij}(x) P^{ji}(0) \rangle$$

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- Fixing of the hopping parameter of the (valence) charm quark, $\kappa_{\rm c}$, through the physical $\rm D_s$-meson mass:

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- *Renormalized PCAC mass* $m_{\rm R}^{cf}$ in terms of the *bare* one, m_{cf} :

$$m_{\rm R}^{cf} = \frac{Z_{\rm A}(1 + \bar{b}_{\rm A} a m_{\rm sea} + \tilde{b}_{\rm A} a m_{cf})}{Z_{\rm P}(1 + \bar{b}_{\rm P} a m_{\rm sea} + \tilde{b}_{\rm P} a m_{cf})} \times m_{cf} \qquad\qquad f = \mathrm{s}, \mathrm{u}/\mathrm{d}$$

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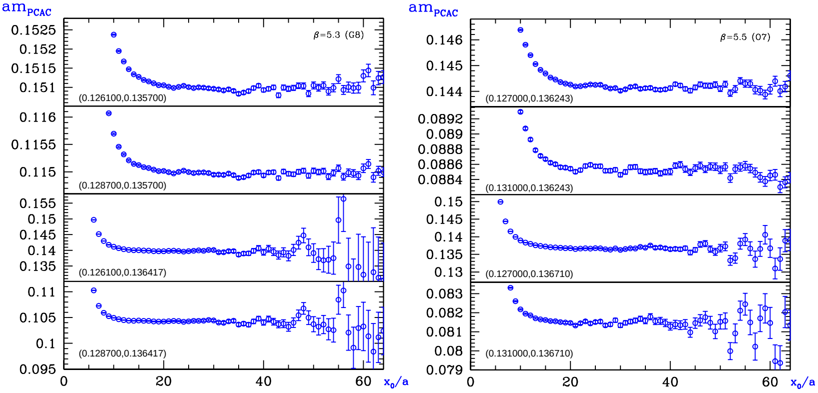
- Consider 3 ways to obtain the *RGI charm quark's mass*:

$$\tfrac{1}{2}\,(M_c + M_f) = \frac{M}{\overline{m}}\, m_R^{cf} \qquad [\,M/\overline{m}\,,\, M_f \text{ known (ALPHA Collaboration)}\,]$$

$$M_c = \frac{M}{\overline{m}}\,\frac{Z_A}{Z_P}\, Z\,(1 + b_m a m_{q,c})\, m_{q,c} \qquad m_{q,c} = \frac{1}{2}\left(\frac{1}{\kappa_c} - \frac{1}{\kappa_{\mathrm{crit}}}\right)$$

Local bare O(a) improved PCAC quark mass

$$m_{cf}(x_0) = \frac{\frac{1}{2}\left(\partial_0 + \partial_0^*\right) f_{AP}(x_0) + c_A\, a\, \partial_0^* \partial_0\, f_{PP}(x_0)}{2\, f_{PP}(x_0)} \qquad f = s, u/d$$



$\rightarrow$ take timeslice averages over plateau region to extract bare m_{cf}

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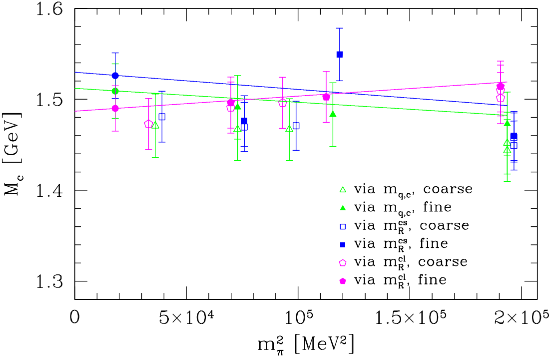
Joint chiral and continuum limit extrapolations of $m_{\rm R}$ resp. M:

$$m_{\rm R}^{\rm cf}(m_\pi, a) = B + C\,m_\pi^2 + D\,a^2 \qquad f = {\rm s, u/d}$$



Preliminary result (final analysis in progress):

$$M_{\rm c} = 1.51(4)\ {\rm GeV} \quad \overset{\text{4-loop conversion}}{\Longrightarrow} \quad \overline{m}_{\rm c}^{\overline{\rm MS}}(\overline{m}_{\rm c}^{\overline{\rm MS}}) = 1.274(36)\ {\rm GeV}$$

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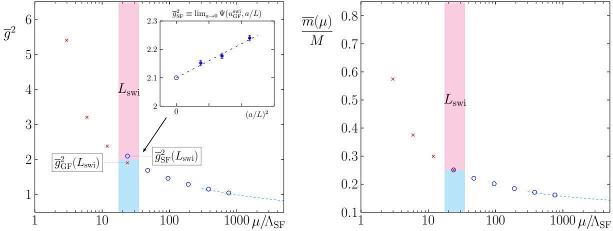
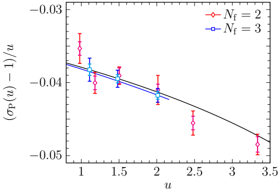
In Progress: $M/\overline{m}$ in three-flavour QCD

[ALPHA Collaboration, Campos, Fritzsch, Pena, Preti, Ramos & Vladikas, PoS LATTICE2015 (2015), arXiv:1508.06939]

Aim:
systematic reduction of
the error $\Delta[M/\overline{m}] = 1.1\%$ ($N_f = 2$) as much as possible



- Revised strategy for $N_f = 3$:
  standard (recursive) finite-size scaling method in the SF accomodating for NP scheme-switch $\bar{g}^2_{\rm SF} \longleftrightarrow \bar{g}^2_{\rm GF}$
- $\Delta[M/\overline{m}] \lesssim 0.5\%$ appears within reach



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Status of CLS $N_\mathrm{f} = 2+1$ large volume simulations

CLS effort [CLS based, Bruno et al., JHEP 02 (2015) 043, arXiv:1411.3982]

- $N_\mathrm{f} = 2+1$ flavours of NP'ly $\mathrm{O}(a)$ improved Wilson fermions, Lüscher-Weisz (aka tree-level Symanzik-impr.) gauge action
- NP c_sw & c_A determined, Z_A in progress [ALPHA Collaboration 2013 – 2015]
- Open boundary conditions $\Rightarrow$ no topology freezing as $a \to 0$ (gauge field generation with "openQCD" code [Lüscher & Schaefer, 2012])
- 4 lattice spacings $a \approx 0.05\,\mathrm{fm}, \ldots, 0.085\,\mathrm{fm}$; $m_\pi \gtrsim$ "physical"

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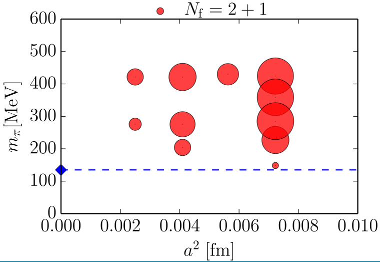

- ▶ Ensembles as of $\sim$ summer 2015
- ▶ Next step: scale setting via PS decay constants, the goal is 1% accuracy [S. Schaefer @ Lattice 2015]
- ▶ Various physics applications to come, e.g. for charm: M_c, $f_{D_{(s)}}$, ...

m_{b} from non-perturbative HQET at $\mathrm{O}(1/m_{\mathrm{h}})$

Non-perturbative (NP) HQET at $\mathrm{O}(1/m_\mathrm{h})$ — Why NP?

- Effective field theories such as HQET, formulated on the lattice, exhibit power divergences in a (induced by operator mixing)

$$(am_\mathrm{h})^{-n} \quad : \qquad \frac{g_0^{2l}}{a^n} \sim \frac{1}{\ln^l(a\Lambda_\mathrm{QCD})\, a^n} \quad , \quad n = 1, 2$$

  that must be subtracted *NP'ly* to have continuum limit
  (note: for NRQCD, the continuum limit does *not* exist ...)
- Power $(1/m_\mathrm{h})$ corrections are only defined, when the leading term is computed non-perturbatively

$$\left(\alpha(m_\mathrm{h})\right)^l \sim \left\{\frac{1}{2b_0 \ln(m_\mathrm{h}/\Lambda_\mathrm{QCD})}\right\}^l \overset{m_\mathrm{h}\to\infty}{\gg} \frac{\Lambda_\mathrm{QCD}}{m_\mathrm{h}}$$

- "Late" asymptotics of QCD pert. theory for heavy-light physics

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- "Late" asymptotics of QCD pert. theory for heavy-light physics

All this is taken care of by *non-perturbative HQET*

✓ NP matching of HQET and QCD ✓ No predictions are lost

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Non-perturbative (NP) HQET at $\mathrm{O}(1/m_\mathrm{h})$

HQET = (continuum) asymptotic expansion of QCD in $\frac{\Lambda_\mathrm{QCD}}{m_\mathrm{b}} \ll 1$:

[Eichten, 1988; Eichten & Hill, 1990]

$$\overline{\psi}_\mathrm{b} \left\{\gamma_\mu D_\mu + m_\mathrm{b}\right\} \psi_\mathrm{b}$$

lowest (static) order | **1st order correction in $1/m_\mathrm{b}$**

$$\rightarrow \quad \mathcal{L}_\mathrm{HQET}(x) = \boxed{\overline{\psi}_\mathrm{h}(x)\, D_0\, \psi_\mathrm{h}(x)} \; \boxed{-\,\omega_\mathrm{kin}\mathcal{O}_\mathrm{kin}(x) - \omega_\mathrm{spin}\mathcal{O}_\mathrm{spin}(x)}$$

$$\mathcal{O}_\mathrm{kin}(x) = \overline{\psi}_\mathrm{h}(x)\,\mathbf{D}^2\,\psi_\mathrm{h}(x)$$

$$\mathcal{O}_\mathrm{spin}(x) = \overline{\psi}_\mathrm{h}(x)\,\boldsymbol{\sigma}\cdot\mathbf{B}\,\psi_\mathrm{h}(x)$$

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Non-perturbative (NP) HQET at $O(1/m_h)$

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[Eichten, 1988; Eichten & Hill, 1990]

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$$\rightarrow \quad \mathcal{L}_{HQET}(x) = \boxed{\overline{\psi}_h(x)\, D_0\, \psi_h(x)} \; \boxed{-\,\omega_{kin}\mathcal{O}_{kin}(x) - \omega_{spin}\mathcal{O}_{spin}(x)}$$

$$\mathcal{O}_{kin}(x) = \overline{\psi}_h(x)\, \mathbf{D}^2\, \psi_h(x)$$

$$\mathcal{O}_{spin}(x) = \overline{\psi}_h(x)\, \boldsymbol{\sigma}\cdot\mathbf{B}\, \psi_h(x)$$

$1/m$–terms:
appear as local operator insertions in correlation functions
(expand the functional integral weight in $1/m$) $\Rightarrow$ renormalizability

$$\langle O \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[\varphi]\, e^{-S_{rel} - a^4 \sum_x \mathcal{L}_{stat}(x)}\, O \left\{ 1 - a^4 \sum_x \mathcal{L}^{(1)}(x) + \ldots \right\}$$

- $\mathcal{O}_{kin}$: kin. energy from heavy quark's residual motion $(\omega_{kin} \sim \frac{1}{2m_b})$
- $\mathcal{O}_{spin}$: chromomagnetic interaction with gluon field $(\omega_{spin} \sim \frac{1}{2m_b})$

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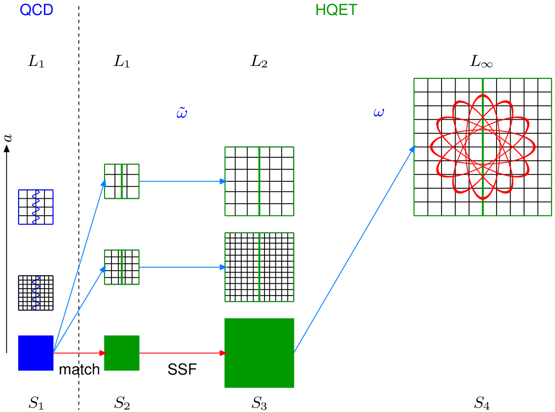
Non-perturbative matching between HQET and QCD

[ALPHA Collaboration, H. & Sommer, 2004, ..., Blossier et al., JHEP 09 (2012) 132, arXiv:1203.6516]

A finite-volume, recursive strategy:



Matching volume: $L_1 \approx 0.5\,\mathrm{fm} \;\to\; am_{\mathrm{h}} \ll 1$, relativistic b-quark feasible

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Non-perturbative matching between HQET and QCD

[ALPHA Collaboration, notation from Della Morte, Dooling, H., Hesse & Simma, JHEP 05 (2014) 060, arXiv:1312.1566]

Idea of NP matching: QCD "=" HQET in the sense

$$\Phi_i^{\mathrm{QCD}}(L, m_\mathrm{h}, 0) \overset{!}{=} \Phi_i^{\mathrm{HQET}}(L, m_\mathrm{h}, a) \equiv \eta_i(L, a) + \varphi_i^j(L, a)\,\omega_j(M, a) + \mathrm{O}\left(\tfrac{1}{m_\mathrm{h}^2}\right)$$

structure : $\varphi = \begin{pmatrix} \varphi_1^1 & * & * & 0 & 0 \\ 0 & * & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 \\ 0 & * & 0 & * & 0 \\ 0 & * & 0 & 0 & * \end{pmatrix}$ $m_\mathrm{h} = M$: RGI mass

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Matching conditions:

- renormalized QCD quantities $\Phi_i^{\rm QCD}$, bare HQET correlators φ_i^j, static-order HQET terms η_i — and the *HQET parameters* ω_i

$\omega_i \in \left\{ m_{\rm bare}\,,\, \omega_{\rm kin}\,,\, \omega_{\rm spin}\,,\, c_{\rm A}^{(1)}\,,\, Z_{\rm A}^{\rm HQET}\,,\, \ldots \right\}$ incl. all heavy-light currents

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Matching conditions:

- renormalized QCD quantities Φ_i^{QCD}, bare HQET correlators φ_i^j, static-order HQET terms η_i — and the *HQET parameters* ω_i
- CL $a \to 0$ can be taken in QCD (l.h.s.) due to small volume

$\Rightarrow$ calculate HQET Parameters $\omega_i(M, a)$ (absorbing log. & power divergences), which inherit NP QCD quark mass dependence

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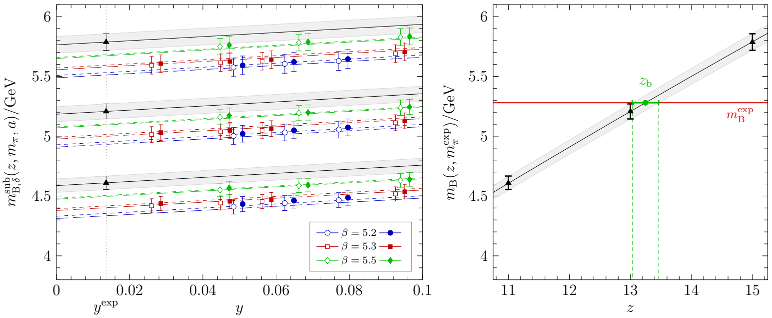
Application: The bottom quark's mass in $N_\mathrm{f} = 2$ QCD

[ALPHA Collaboration, Bernardoni, Blossier, Bulava, Della Morte, Fritzsch, Garron, Gérardin, H., von Hippel, Simma & Sommer, PLB 730 (2014) 171, arXiv:1311.5498]

Combined chiral and continuum extrapolations according to

$$m^{\mathrm{sub}}_{\mathrm{B},\delta}(z,y,a) \;=\; B(z) + C\,(y - y^{\mathrm{exp}}) + D_\delta a^2 \;\;,\;\; y \equiv m_\pi^2/\,(8\pi^2 f_\pi^2)$$

$$m^{\mathrm{sub}}_{\mathrm{B},\delta}(z,y,a) \;\equiv\; m_{\mathrm{B},\delta}(z,m_\pi,a) + \frac{3\widehat{g}^2}{16\pi}\left(\frac{m_\pi^3}{f_\pi^2} - \frac{(m_\pi^{\mathrm{exp}})^3}{(f_\pi^{\mathrm{exp}})^2}\right) \;\;,\;\; z = L_1 M$$



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- Employ previously computed HQET parameters ω_i and HQET energies extracted on large-volume simulation ensembles with 3 lattice spacings $a \in \{0.075\,\mathrm{fm}\,,\ 0.065\,\mathrm{fm}\,,\ 0.048\,\mathrm{fm}\}$ and impose NLO HQET expansion of m_B :

CLS based

$$m_\mathrm{bare} + E^\mathrm{stat} + \omega_\mathrm{kin} E^\mathrm{kin} + \omega_\mathrm{spin} E^\mathrm{spin} \quad , \quad m_\mathrm{B}(z, m_\pi^\mathrm{exp}, 0)\big|_{z=z_\mathrm{b}} \overset{!}{=} m_\mathrm{B}^\mathrm{exp}$$

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- This fixes $z_{\mathrm{b}} = L_1 M_{\mathrm{b}}$ and yields the result:

$$M_{\mathrm{b}} = 6.58(17)\ \mathrm{GeV} \quad \xRightarrow{\text{4-loop conversion}} \quad \overline{m}_{\mathrm{b}}^{\overline{\mathrm{MS}}}(\overline{m}_{\mathrm{b}}^{\overline{\mathrm{MS}}}) = 4.21(11)\ \mathrm{GeV}$$

N_{f}	Ref.	M	$\overline{m}_{\overline{\mathrm{MS}}}(\overline{m}_{\overline{\mathrm{MS}}})$	$\overline{m}_{\overline{\mathrm{MS}}}(4\,\mathrm{GeV})$	$\overline{m}_{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})$	$\Lambda_{\overline{\mathrm{MS}}}$[MeV]
0	[36]	6.76(9)	4.35(5)	4.39(6)	4.87(8)	238(19) [69]
2	this work	6.58(17)	4.21(11)	4.25(12)	4.88(15)	310(20) [55]
5	PDG13 [1]	7.50(8)	4.18(3)	4.22(4)	4.91(5)	212(8) [1]

(error dominated by ω_i, $M/\overline{m}$; goal to improve this for $N_{\mathrm{f}} = 2+1$ ...)

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N_f	Ref.	M	$\overline{m}_{\overline{\mathrm{MS}}}(\overline{m}_{\overline{\mathrm{MS}}})$	$\overline{m}_{\overline{\mathrm{MS}}}(4\,\mathrm{GeV})$	$\overline{m}_{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})$	$\Lambda_{\overline{\mathrm{MS}}}$[MeV]
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(error dominated by ω_i, $M/\overline{m}$; goal to improve this for $N_\mathrm{f} = 2+1$ ...)

- NP HQET works in practice; it has tiny NLO $(1/m_\mathrm{h})$ corrections (e.g., static limit computation: $[\overline{m}_\mathrm{b}^{\overline{\mathrm{MS}}}(\overline{m}_\mathrm{b}^{\overline{\mathrm{MS}}})]^\mathrm{stat} = 4.21(11)\ \mathrm{GeV}$)
- More results on: (semi-)leptonic B decay constants & spectroscopy

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Current status of lattice results on m_{c} and m_{b}

Overview of computational approaches

One would like to reduce uncertainties through ...

- ... determinations from different & independent formulations (continuum & lattice)
- ... multiple determinations within a given formulation

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Overview of computational approaches

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Continuum QCD approaches to determine heavy quark masses

- Variants of QCD sum rules:
  relativistic, non-relativistic, Borel, momentum, ...
  (charm & bottom)
- Fitting DIS scattering data, decay spectra, ... to predictions of continuum perturbation theory
  (charm & bottom)
- They all involve perturbation theory (PT) at some point
  $\rightarrow$ is it still reliable in the energy domain where it is applied?

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Overview of computational approaches

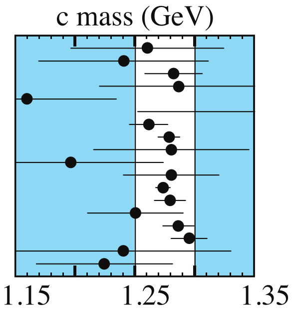
Lattice QCD approaches to determine heavy quark masses

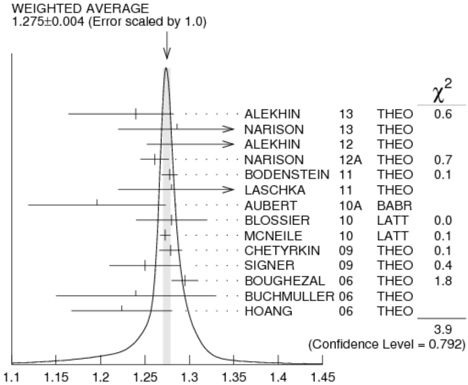
- Via input from hadron spectroscopy with relativistic quarks (charm; e.g. ETMC, ALPHA)
- Current-current correlator method with HISQ discretization: comparing the (continuum limit of) time-moments of a LQCD heavyonium correlator to continuum QCD PT for the vacuum polarization function (charm & bottom; HPQCD)
- Interpolation resp. ratio method: interpolation between data on the (relativistic) heavy-light meson-to-quark mass ratio in the charm mass region and its exactly known static limit (bottom; ETMC)
- Non-perturbative HQET including $1/m_{\mathrm{h}}$–terms (bottom; ALPHA)
- Via binding energies of b-hadrons in NRQCD (bottom; HPQCD)

m_c

Particle Data Group 2014 (from continuum determinations):

$$\overline{m}_c^{\overline{\mathrm{MS}}}(\overline{m}_c^{\overline{\mathrm{MS}}}) = 1.275(25)\ \mathrm{GeV}$$

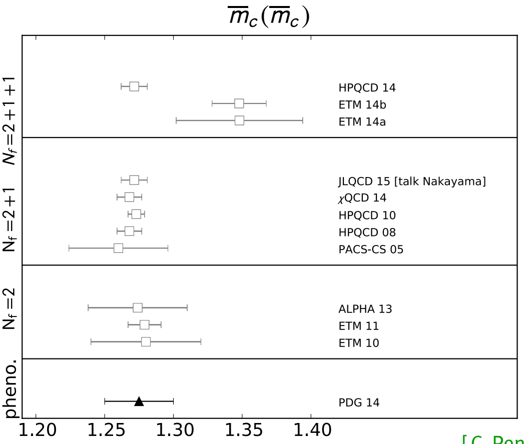




[2014 Review of Particle Physics: K.A. Olive et al. (PDG), Chin. Phys. C, 38, 090001 (2014)]

m_c: Status @ Lattice 2015

Almost no new results after F. Sanfilippo's review @ Lattice 2014



[C. Pena @ Lattice 2015]

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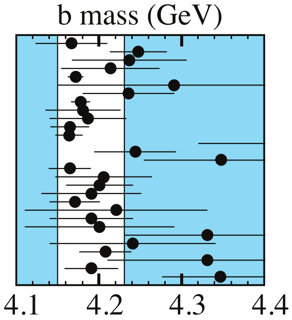
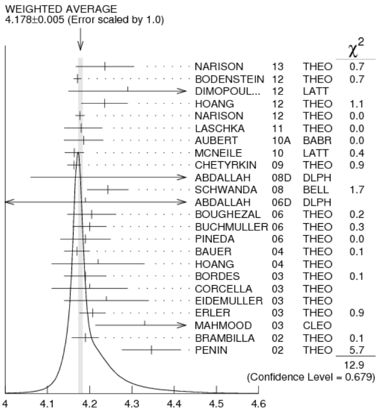
m_b

Particle Data Group 2014 (from continuum determinations):

$$\overline{m}_b^{\overline{\rm MS}}(\overline{m}_b^{\overline{\rm MS}}) \;=\; 4.18(3)\ \text{GeV}$$





[2014 Review of Particle Physics: K.A. Olive et al. (PDG), Chin. Phys. C, 38, 090001 (2014)]

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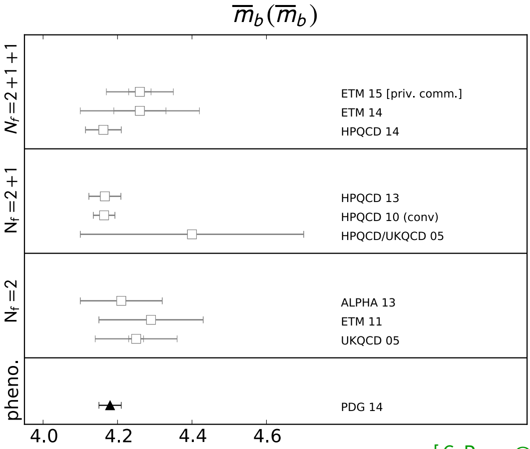
m_b: Status @ Lattice 2015

Almost no new results after F. Sanfilippo's review @ Lattice 2014



[C. Pena @ Lattice 2015]

Conclusions & Prospects

- Accurate determinations of quark masses are of crucial importance for (B)SM physics
- LQCD formulation provides a genuinely non-perturbative and thus very controlled framework to compute quark masses
  - systematically improveable, uncertainties can be quantified
  - non-perturbative improvement & renormalization, as well as simulations at fine enough lattice spacings to take the CL, are decisive elements for solid results with credible error budgets
  - mass *ratios*: precise result on $\overline{m}_i^{\overline{\mathrm{MS}}}(\mu)$ translatable to $\overline{m}_j^{\overline{\mathrm{MS}}}(\mu)$
  - multiple complementary approaches allow assessing the different systematics involved and checking their consistency
- In the future one may expect ...
  - ... more independent calculations by different groups
  - ... better systematics, precision, $N_{\mathrm{f}} = 2 \to 2+1 \to 2+1+1, \ldots$

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