Radial distributions of the axial density and the $B^{*\prime}B\pi$ coupling

Antoine Gérardin

In collaboration with Benoit Blossier

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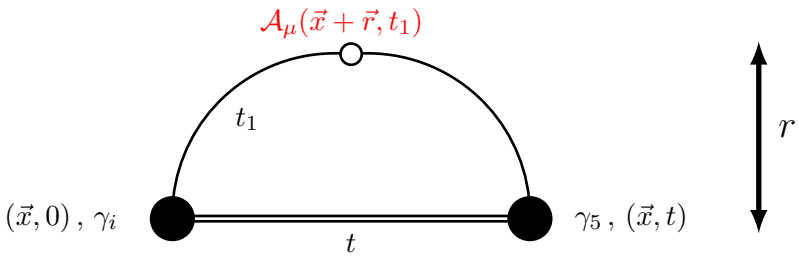
Introduction

- The radial distributions of the axial current are defined by $(n, m = 1, 2, ...)$:

$$f^{(mn)}_{\gamma_\mu\gamma_5}(\vec{r}) = \langle\, B_m | A_\mu(\vec{r}) | B^*_n(\lambda)\,\rangle \quad , \quad B_n = n^{\text{th}} \text{ radial excitation}$$



The axial current $A_\mu = \overline{q}\gamma_\mu\gamma_5 q$ (probe) is inserted at a distance r from the heavy quark Q

- We work in the static limit of HQET: the heavy quark (Q) is static
- Ground states (B, B^*) and excited states (B', $B^{*\prime}$) analysis : $n, m = 1, 2$
- Motivations
  - → The heavy-light meson is a good starting point, we can also compare our results with quark models
  - → Extract some excited state properties on the lattice (not only the spectrum but also the couplings)
  - → In particular, the radial distributions are related to form factors via a Fourier transform [Becirevic et al. (2012)] (application to the $g_{B^*B\pi}$ and $g_{B^{*\prime}B\pi}$ couplings)
  - → From a lattice point of view : insights on volume effects

Computation

- Lattice setup:

  - Two degenerated dynamical quarks ($N_f = 2$, $m_u = m_d$)
  - $O(a)$ improved Wilson-Clover Fermions (to reduce lattice artefacts)
  - 3 lattice spacings a :

    (0.048, 0.065, 0.075) $<$ 0.1 fm
  - pion masses in the range [280 MeV, 440 MeV]

CLS

based

- We need to compute the following two and three-point correlation functions on the lattice

$$C^{(3)}_{\gamma_\mu\gamma_5}(t,t_1;\vec{r}) \;=\; \langle\, \mathcal{P}(t;\vec{x})\, \mathcal{A}_\mu(t_1;\vec{x}+\vec{r})\, \mathcal{V}^\dagger_k(0;\vec{x})\,\rangle\,,$$

$$C^{(2)}_{\mathcal{P}}(t) = \sum_{\vec{x}} \langle\, \mathcal{P}(t,\vec{x})\, \mathcal{P}^\dagger(0,\vec{x})\,\rangle \quad , \quad C^{(2)}_{\mathcal{V}}(t) = \frac{1}{3}\sum_{\vec{x},k} \langle\, \mathcal{V}_k(t,\vec{x})\, \mathcal{V}^\dagger_k(0,\vec{x})\,\rangle$$

- $\mathcal{P}(t,\vec{x})$ and $\mathcal{V}_k(t,\vec{x})$ are interpolating operators with the quantum numbers of a pseudoscalar and vector mesons respectively

$$\mathcal{R}_{\gamma_\mu\gamma_5}(t,t_1,\vec{r}) = \frac{C^{(3)}_{\gamma_\mu\gamma_5}(t,t_1;\vec{r})}{\left(C^{(2)}_{\mathcal{P}}(t)\; C^{(2)}_{\mathcal{V}}(t)\right)^{1/2}} \xrightarrow[t\gg t_1\gg 1]{} f^{(11)}_{\gamma_\mu\gamma_5}(\vec{r}) = \sum_\lambda \langle B(\vec{p})|A_\mu(\vec{r})|B^*(\vec{p}\,',\lambda)\rangle\, \epsilon^*_i(\vec{p}\,',\lambda)$$

$\longrightarrow$ One obtains the ground state distribution (B and B^*) at large (euclidean) time

Excited states contribution

How to isolate the contribution from excited states ? (In particular $\langle B|A_\mu(\vec{r})|B^{*\prime}(\lambda)\rangle$)

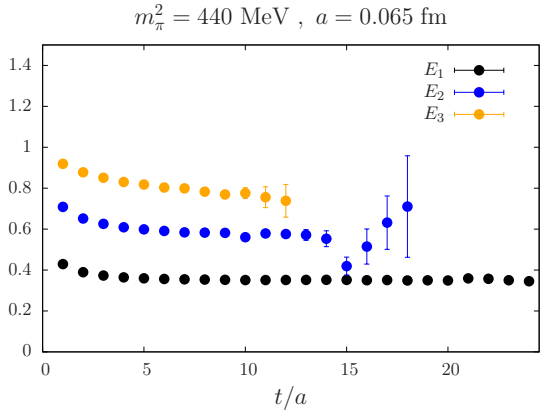
- Excited state contributions are exponentially suppressed compared to the ground state contribution
- Solution: use different interpolating operators $\mathcal{P}^{(i)}$ and $\mathcal{V}_k^{(j)}$ with different overlaps with the excited states
  $\longrightarrow$ matrix of correlators : $C_{ij}^{(2)}(t)$
  $\longrightarrow$ generalized eigenvalues and eigenvectors $\quad C^{(2)}(t)\, v_n(t,t_0) = \lambda_n(t,t_0)\, C^{(2)}(t_0)\, v_n(t,t_0)$
- This allows to disentangle the contribution from the ground state and from the different exited states
  $\longrightarrow$ Generalized eigenvalues and eigenvectors of the two-point correlation function
- Exemple : spectrum of the heavy-light mesons in the static limit of HQET

$$E_n^{\text{eff}}(t) = \log\left(\frac{\lambda_n(t)}{\lambda_n(t+1)}\right) \xrightarrow[t\gg 1]{} E_n$$

$\longrightarrow$ The mass are extracted from the plateaus

$\longrightarrow$ B and B^* are degenerate (E_1)

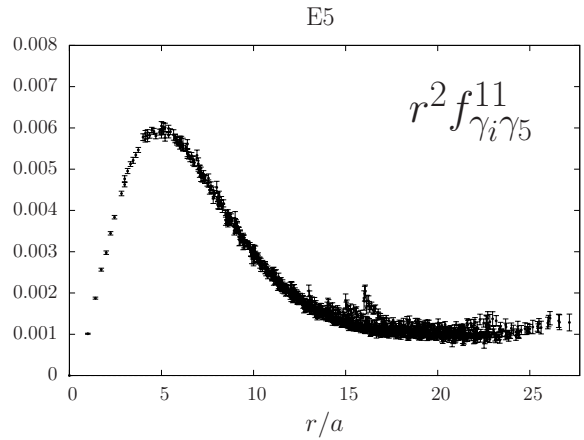
$\longrightarrow$ B' and $B^{*\prime}$ are the first radial excitations (E_2)

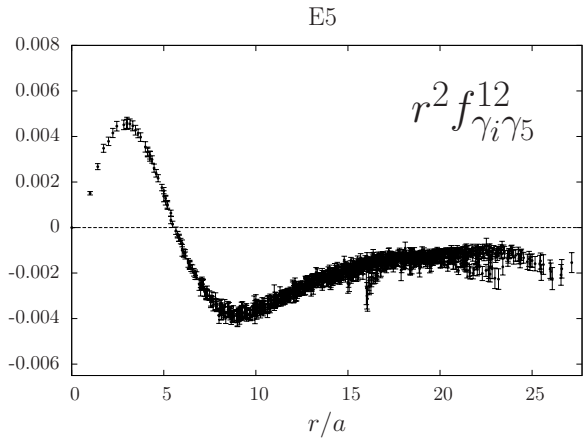


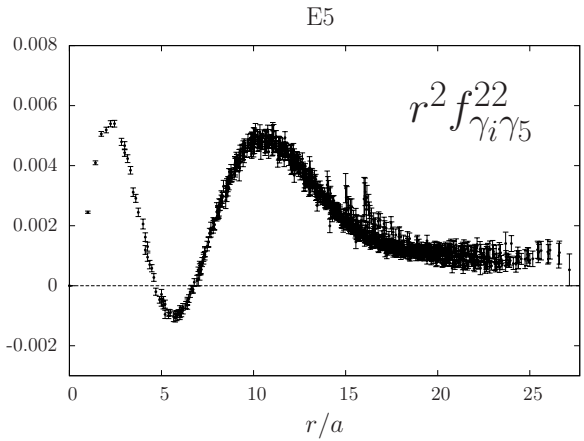
- Matrix elements : (Generalized eigenvalue problem) GEVP [Bulava et. al, '11] [Blossier et. al, '13]

$$\mathcal{R}_{mn}^{\text{GEVP}}(t,t_1;\vec{r}) \;=\; f_{\gamma_\mu\gamma_5}^{(mn)}(\vec{r}) \;+\; \mathcal{O}\left(e^{-\Delta_{N+1,m}t_2}, e^{-\Delta_{N+1,n}t_1}\right) \quad , \quad \Delta_{N+1,n} = E_{N+1} - E_n$$

Spatial component of the radial distributions: raw data







$$f^{(mn)}_{\gamma_i\gamma_5}(\vec{r}) = \langle\, B_m | A_i(\vec{r}) | B^*_n(\lambda)\,\rangle$$

- E5 : $a = 0.065$ fm and $m_\pi = 440$ MeV
- $\#r = 969 - 2925$ for $L/a = 32 - 48$ respectively
- exponential fall-off
- node for excited states
- " fishbone" structure at large radii
- preliminary results

Volume effects

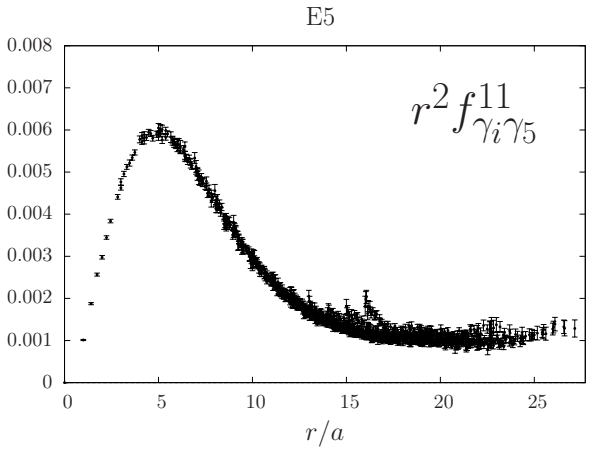
Lattice with periodic boundary conditions in space directions [Negele, '94]

$$a^3 f^{\text{lat}}_{\gamma_i\gamma_5}(\vec{r}) = \sum_{\vec{n}} a^3 \widetilde{f}_{\gamma_i\gamma_5}(\vec{r}+\vec{n}L) \quad , \quad n_i \in \mathbb{Z}\,,$$

- L is the size of the lattice
- a is the lattice spacing

Two kinds of volume effects are expected:

- $f^{\text{lat}}_{\gamma_i\gamma_5}(\vec{r})$ is the sum of all periodic images contributions : $\vec{n} = (0,0,0)\,,\ (0,0,1)\,,\ \ldots$
- $\widetilde{f}_{\gamma_i\gamma_5}(\vec{r})$ can still differs from the infinite volume distribution $f_{\gamma_i\gamma_5}(\vec{r})$ due to interactions with periodic images



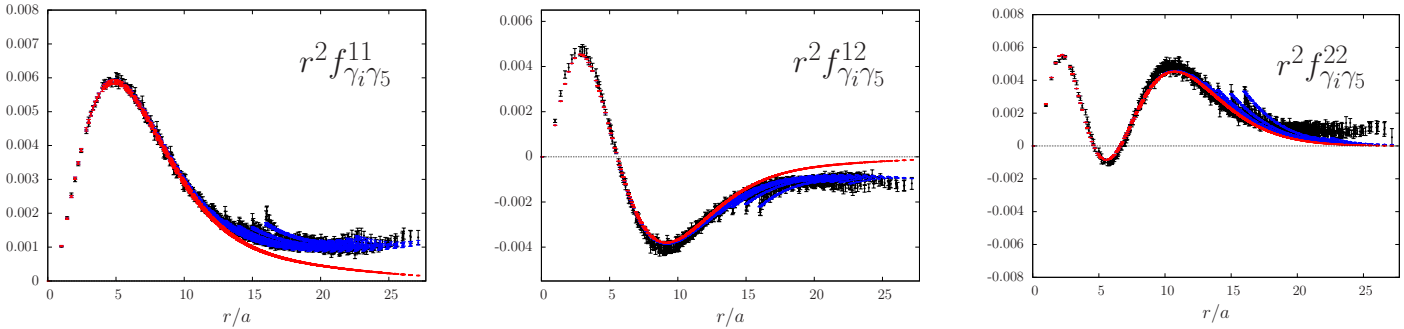
- $L/a = 32$
- $r^2 f^{11}_{\gamma_i\gamma_5}(\vec{r}) \neq 0$ for $r = L/2$ $\Rightarrow$ overlap of the tails
  $\Rightarrow$ "fishbone" structure
- We neglect interactions with periodic images
  $\Rightarrow \widetilde{f}_{\gamma_i\gamma_5}(\vec{r}) \approx f_{\gamma_i\gamma_5}(\vec{r})$ (even in the overlap region)
- Typically volume effect decrease as $\mathcal{O}(e^{-m_\pi L})$ and we always have $Lm_\pi > 4$

To remove these volume effects, we assume a functional form and fit the data with

$$f^{(mn)}_{\gamma_i\gamma_5}(\vec{r}) = P_{mn}(r)\, r^\beta \, \exp\left(-r/r_0\right)\,,$$

where $P_{mn}(r)$ is a polynomial function

Volume effects: results for E5 ($a = 0.065$ fm and $m_\pi = 440$ MeV)



- Does not affect the computation of the couplings g_{mn} or form factors at discrete lattice momenta (as long as the distribution vanishes for $r > L$)

$$g = \sum_{\vec{r}} f^{\text{lat}}_{\gamma_i\gamma_5}(\vec{r}) = \int \mathrm{d}^3 r\, f_{\gamma_i\gamma_5}(\vec{r})$$

$\rightarrow$ the contribution coming from periodic images compensates exactly the missing part of the tail for $r > L/2$.

- However, it affects quantities like $\langle r^2 \rangle_A$ or form factors at non-lattice momenta

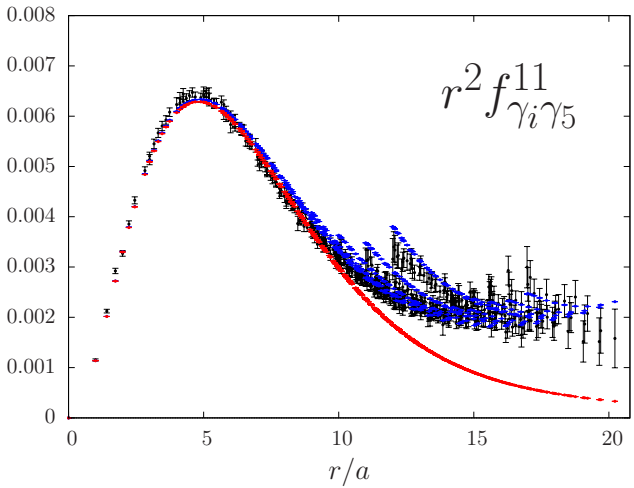
$$\langle r^2 \rangle_\Gamma = \frac{\displaystyle\int_0^\infty \mathrm{d}r\, r^4\, f^{(11)}_\Gamma(r)}{\displaystyle\int_0^\infty \mathrm{d}r\, r^2\, f^{(11)}_\Gamma(r)}$$

- We have made the assumption that the tail of the distribution is not distorted by interactions

Volume effects: results for E5 ($a = 0.065$ fm and $m_\pi = 440$ MeV)

Test volume effects on a new lattice ensemble :

- Same lattice spacing ($a = 0.065$ fm)
- Same pion mass ($m_\pi = 440$ MeV)
- But smaller volume : $L/a = 24$ instead of 32



- fit the raw data of the smaller volume ensemble ($L/a = 24$) using the same fit parameters ($L/a = 32$)
- the fit (blue points) still matches the data of the smaller ensemble (in black)

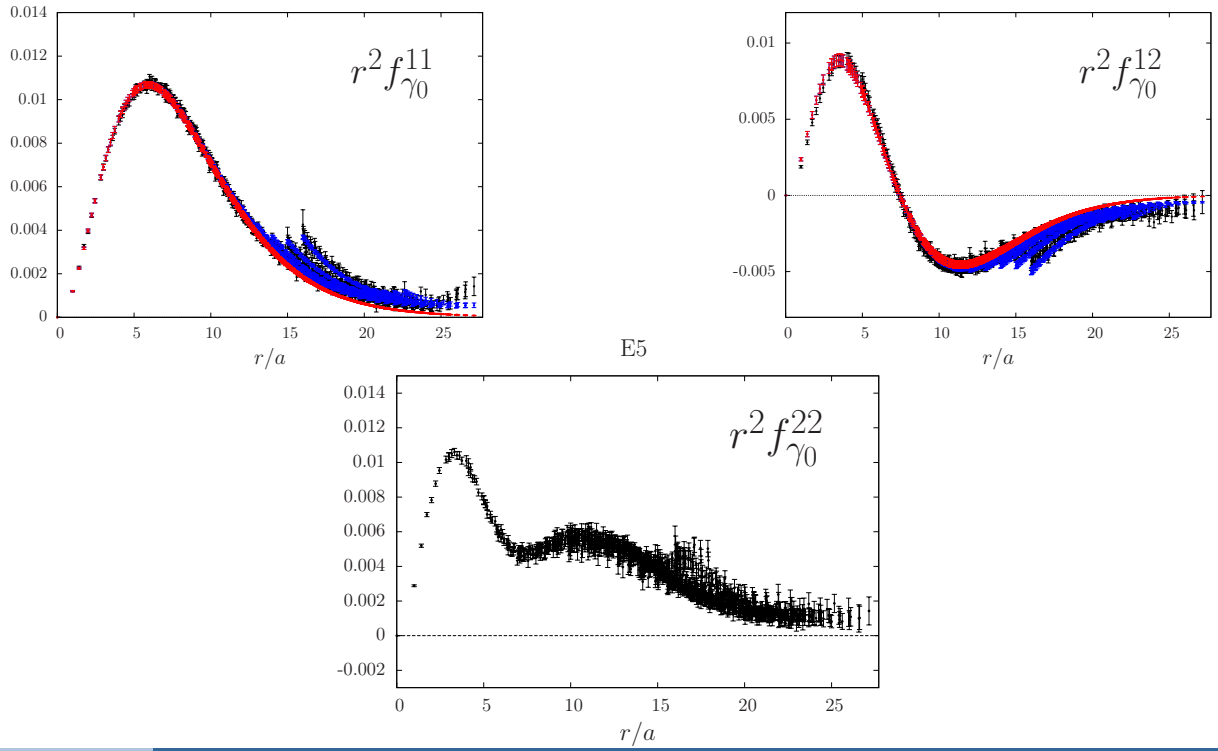
$\longrightarrow$ the deformation of the tail is negligible at our level of precision

Vector distributions and the renormalization constant Z_V

- We would like to check whether or not we are able to safely isolate the first excited state
- Therefore, we have computed the vector (or charge) radial distributions. Similarly, they are defined by

$$f_{\gamma_0}^{(mn)}(\vec{r}) = \langle B_m(\vec{p})| \left(\overline{\psi}_l \gamma_0 \psi_l\right)(\vec{r}) |B_n(\vec{p}\,')\rangle \quad , \quad (\gamma_\mu\gamma_5 \leftrightarrow \gamma_0)$$

- Results for E5 ($m_\pi = 440$ MeV, $a = 0.065$ fm)



Vector distributions and the renormalization constant Z_V

$$f_{\gamma_0}^{(mn)}(\vec{r}) = \langle B_m(\vec{p})| \left(\overline{\psi}_l \gamma_0 \psi_l\right)(\vec{r}) |B_n(\vec{p}\,')\rangle$$

- Sum the vector radial distributions over all values of $\vec{r}$:

$$c_{mn} = \sum_{\vec{r}} f_{\gamma_0}^{(mn)}(\vec{r})$$

mn	11	22	12	23
c_{mn}	1.311(17)	1.212(52)	0.015(32)	−0.010(35)

- At finite lattice spacing ($a \neq 0$) the vector current conservation is broken
- One defines the renormalization constant Z_V to restore the symmetry : $Z_V\, c_{11} = 1$
- With our data :

$$Z_V = 0.763(12) \quad \text{at} \quad \beta = 5.3 \quad (a = 0.065 \text{ fm})$$

→ Compatible with the non-perturbative estimate $Z_V = 0.750(5)$ from the ALPHA Collaboration

[DellaMorte, '05] [Fritzsch, '12]

→ $Z_V\, c_{22}$ is also very close to one

→ c_{12} and c_{23} are compatible with zero: we safely isolate the ground state and first excited state

- The results are compatible with the interpretation of an excited state

Sum rules : g_{11}, g_{12} and g_{22}

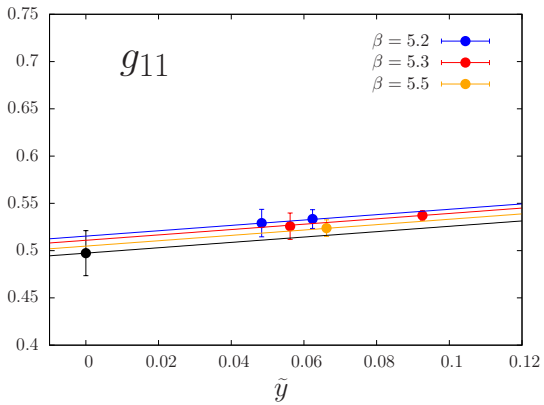
The sum over r of the spatial component of the radial distributions gives the form factor at $q^2 = q^2_{\max} = m_{B^*_m} - m_{B_n}$

$$g_{mn} = \sum_{\vec{r}} f^{(mn)}_{\gamma_i\gamma_5}(\vec{r}) = \langle B^0_m(\vec{0})|A_k(0)|B^{*+}_n(\vec{0},\lambda)\rangle$$

The renormalized $\mathcal{O}(a)$-improved couplings are then given by

$$\overline{g}_{mn} = Z_A(1 + b_A a m_q)\, g_{mn}$$

→ Z_A is the light axial vector current renormalisation constant [DellaMorte et. al, '08] [Fritzsch et. al, '12]
→ b_A is an improvement coefficient



$\tilde{y} = m_\pi^2/(8\pi^2 f_\pi^2)$

Extrapolations to the physical point :

→ $\overline{g}_{11} = 0.499(24)(?)$

→ $\overline{g}_{12} = -0.161(45)(?)$

→ $\overline{g}_{22} = 0.363(38)(?)$

(preliminary, only naive extrapolations)

- The results are perfectly compatible with previous lattice calculations [Bernardoni et. al, '14] [Blossier et. al, '13]
- $g_{11} = \hat{g}$ is related to the $g_{B^*B\pi}$ coupling in the static limit ($q^2_{\max} \approx 0$)
- However, for g_{12} we obtain the result at $q^2_{\max} = m_{B^{*\prime}} - m_B \neq 0$

Properties of the radial distributions

- First moment of the ground state radial distributions (square radius)

$$\langle r^2 \rangle_\Gamma = \frac{\int_0^\infty \mathrm{d}r\, r^4\, f_\Gamma^{(11)}(r)}{\int_0^\infty \mathrm{d}r\, r^2\, f_\Gamma^{(11)}(r)}$$

$$\begin{aligned} \Gamma = 1 \quad &: \quad \langle r^2 \rangle_M = 0.215(9)\ \mathrm{fm}^2 \\ \Gamma = \gamma_0 \quad &: \quad \langle r^2 \rangle_C = 0.345(6)\ \mathrm{fm}^2 \\ \Gamma = \gamma_i\gamma_5 \quad &: \quad \langle r^2 \rangle_A = 0.254(6)\ \mathrm{fm}^2 \end{aligned}$$

(preliminary)

$$\langle r^2 \rangle_M < \langle r^2 \rangle_A < \langle r^2 \rangle_C$$

- $g_{12} \ll g_{11} = \hat{g}$: it can be understood by the presence of a node for the excited state

- Position of the node for $f_{\gamma_i\gamma_5}^{(12)}(\vec{r}) = \langle\, B|A_i(\vec{r})|B^{*\prime}(\lambda)\,\rangle$

	$a = 0.075$ fm		$a = 0.065$ fm		$a = 0.048$ fm
m_π	330 MeV	280 MeV	440 MeV	310 MeV	340 MeV
r_n [fm]	0.371(6)	0.369(6)	0.369(4)	0.371(3)	0.358(4)

$\rightarrow$ no dependance on the lattice spacing / pion mass at our level of precision

Multihadron thresholds

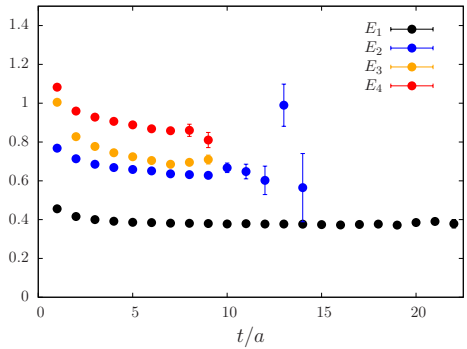
- Within our lattice setup, the radial excitation of the vector meson ($B^{*\prime}$) lies near the multiparticles threshold ($B_1^*\pi$) where B_1^* represents the axial B meson.

a (fm)	m_π (MeV)	$a(m_{B^{*\prime}} - m_B)$	$a(m_{B_1^*} - m_B + m_\pi)$
0.075	330	0.253(7)	0.281(4)
	280	0.235(8)	0.248(4)
0.065	440	0.225(10)	0.278(6)
	310	0.213(11)	0.233(3)
0.048	340	0.166(9)	0.176(3)

The mass of the axial B meson in the static limit is extracted from our previous study [Blossier et al. (2014)]

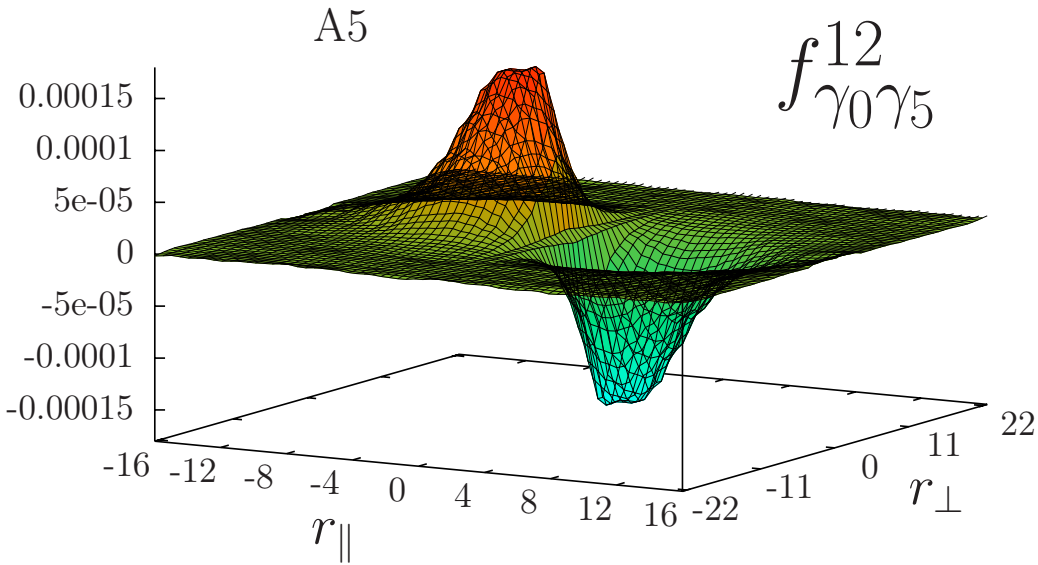
We are always bellow threshold but the decay is potentially dangerous

- Our interpolating operators ($\mathcal{V}_k^{(j)}$) couple (in principle) to all states with the same quantum numbers $\longrightarrow$ so we could couple to this state.
- However, the position of the node is remarkably stable and does not depend on the pion mass contrary to what would be expected in the case of a mixing with the multiparticles states.
- Using more interpolating operators (with better overlap to the multi-particle state), it seems that we are able to disentangle the two states (spectrum)
- We are currently analyzing the corresponding radial distributions ...



Time component of the axial density

- The time component is given by $f^{(12)}_{\gamma_0\gamma_5}(\vec{r}) = \sum_\lambda \langle B(\vec{p})|A_0(\vec{r})|B^{*\prime}(\vec{p}\,',\lambda)\rangle\, \epsilon^*_i(\vec{p}\,',\lambda)$
- The distribution is odd with respect to r_i (i is the direction of the polarisation vector)
- The raw data are averaged over the plane orthogonal to the direction i



- Odd with respect to r_i $\Rightarrow$ $\langle B(p)|A^0|B^{*\prime}(p',\lambda)\rangle = \sum_{\vec{r}} f^{(12)}_{\gamma_0\gamma_5}(\vec{r}) = 0$ as expected from the form factor decomposition in the zero recoil confiration (static heavy quarks)

Application to the $g_{B^{*\prime}B\pi}$ coupling

- The $g_{B^{*\prime}B\pi}$ coupling is a coupling involving an excited state
- It is defined by the following on-shell matrix element

$$\left\langle B(p')\,\pi^+(q)\,|\,B^{*\prime}(p',\epsilon^{(\lambda)})\right\rangle \;=\; -g_{B^{*\prime}B\pi}\times q_\mu\,\epsilon^\mu(p')$$

Pseudoscalar B meson

Radially excited vector $B^{*\prime}$ meson

- Using the LSZ reduction and the PCAC relation, we are left with the following matrix element which can be computed on the lattice:

$$q^\mu\,\langle B^0(p)|A_\mu(0)|B^{*\prime +}(p+q)\rangle \;=\; g_{B^{*\prime}B\pi}\,(\epsilon\cdot q)\times\frac{f_\pi m_\pi^2}{m_\pi^2-q^2}\;+\;\ldots$$

- We would like to compute $q^\mu\,\langle B^0(p)|A_\mu(0)|B^{*\prime +}(p+q)\rangle$ as $q^2\to 0$ (chiral limit)
- On the lattice with static heavy quarks: zero recoil kinematic configuration ($\vec{p}=\vec{p}\,'=\vec{0}$)

  $\rightarrow$ Simulations correspond to $q^2=q^2_{\max}=(m_{B^{*\prime}}-m_B)^2\neq 0$ (far from the chiral limit)

  $\rightarrow$ One should extrapolate the form factor to $q^2=0$
- This can be done by taking the Fourier transform of the radial distribution [Becirevic et al. (2012)]
- Requires the knowledge of the spatial component $f^{(12)}_{\gamma_i\gamma_5}(\vec{r})$ but also of the time component $f^{(12)}_{\gamma_0\gamma_5}(\vec{r})$

Conclusion

- We have computed the radial distributions of the axial vector density for the ground state and the first excited state

  $\longrightarrow$ Heavy-light system in the static limit

  $\longrightarrow$ We have both the spatial and the time component

  $\longrightarrow$ We have five lattice ensembles to study discretization and quark mass effects

  $\longrightarrow$ Volume effects seems negligible at our level of precision

- We are potentially near the two-body decay threshold

  $\longrightarrow$ the position of the node is stable

  $\longrightarrow$ we are using a larger basis to isolate the two contributions

- Results are still preliminary

- The next step is to compute the $g_{B^{*\prime}B\pi}$ coupling at $q^2 = 0$ (Fourier transform of the distribution)

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Thank you