

# Exclusive Photoproduction of Quarkonia in QCD at NLO

Lech Szymanowski

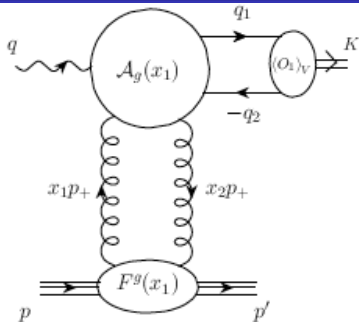
National Centre for Nuclear Research (NCBJ)  
Warsaw, Poland

IHP, Paris, 24-25 September 2015

*based on Eur. Phys. J. C 34 (2004) 297, Err.: C75 (2015) 2, 75*

in collaboration with **D. Yu. Ivanov** (Institut of Mathematics, Novosibirsk)  
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- Method of calculation
- Comparison with data and phenomenology



$$\Delta = p' - p, \quad P = \frac{p + p'}{2}, \quad t = \Delta^2,$$

$$(q - \Delta)^2 = K^2 = M^2, \quad \zeta = \frac{M^2}{W^2}.$$

$$q = \frac{(W^2 - m_N^2)}{2(1 + \xi)W} n_-,$$

$$p = (1 + \xi)W n_+ + \frac{m_N^2}{2(1 + \xi)W} n_-,$$

$$p' = (1 - \xi)W n_+ + \frac{(m_N^2 + \Delta^2)}{2(1 - \xi)W} n_- + \Delta_\perp,$$

$$\Delta = -2\xi W n_+ + \left( \frac{\xi m_N^2}{(1 - \xi^2)W} + \frac{\Delta^2}{2(1 - \xi)W} \right) n_- + \Delta_\perp.$$

(:

- quarkonium vertex:

$$v_i(q_2) \bar{u}_j(q_1) \rightarrow \frac{\delta_{ij}}{4N_c} \left( \frac{\langle O_1 \rangle_V}{m} \right)^{1/2} \not{\epsilon}_V^* (K + M).$$

- Factorised amplitude

$$\mathcal{M} = \left( \frac{\langle O_1 \rangle_V}{m} \right)^{1/2} \sum_{p=g,q,\bar{q}} \int_0^1 dx_1 A_H^p(x_1, \mu_F^2) \mathcal{F}_\zeta^p(x_1, t, \mu_F^2).$$

$$x_1 = \frac{x + \xi}{1 + \xi}, \quad x_2 = \frac{x - \xi}{1 + \xi}.$$

- gluonic GPDs in the leading twist 2

$$\begin{aligned}
 & F^g(x, \xi, t) \\
 &= \frac{1}{(Pn_-)} \int \frac{d\lambda}{2\pi} e^{ix(Pz)} n_{-\alpha} n_{-\beta} \\
 &\quad \times \langle p' | G^{\alpha\mu} \left(-\frac{z}{2}\right) G_{\mu}^{\beta} \left(\frac{z}{2}\right) | p \rangle \Big|_{z=\lambda n_-} \\
 &= \frac{1}{2(Pn_-)} \left[ \mathcal{H}^g(x, \xi, t) \bar{u}(p') \not{n}_- u(p) \right. \\
 &\quad \left. + \mathcal{E}^g(x, \xi, t) \bar{u}(p') \frac{i\sigma^{\alpha\beta} n_{-\alpha} \Delta_{\beta}}{2m_N} u(p) \right]
 \end{aligned}$$

- in the axial gauge

$$\begin{aligned}
 & \int \frac{d\lambda(Pn_-)}{2\pi} e^{ix(Pz)} \langle p' | A_{\mu}^a \left(-\frac{z}{2}\right) A_{\nu}^b \left(\frac{z}{2}\right) | p \rangle \Big|_z \\
 &= \frac{\delta^{ab}}{N_c^2 - 1} \left( \frac{-g_{\mu\nu}^{\perp}}{2} \right) \frac{F^g(x, \xi, t)}{(x - \xi + i\varepsilon)(x + \xi - i\varepsilon)}.
 \end{aligned}$$

$$z = \lambda n_-$$



- Results in LO

$$\mathcal{M} = \frac{4\pi\sqrt{4\pi\alpha} e_q (e_V^* e_\gamma)}{N_c \xi} \left( \frac{\langle O_1 \rangle_V}{m^3} \right)^{1/2} \int_{-1}^1 dx [T_g(x, \xi) F^g(x, \xi, t) + T_q(x, \xi) F^{q,S}(x, \xi, t)]$$

$$F^{q,S}(x, \xi, t) = \sum_{q=u,d,s} F^q(x, \xi, t)$$

- Results in LO

$$\mathcal{M} = \frac{4\pi\sqrt{4\pi\alpha} e_q (e_V^* e_\gamma)}{N_c \xi} \left( \frac{\langle O_1 \rangle_V}{m^3} \right)^{1/2} \int_{-1}^1 dx [T_g(x, \xi) F^g(x, \xi, t) + T_q(x, \xi) F^{q,S}(x, \xi, t)]$$

$$F^{q,S}(x, \xi, t) = \sum_{q=u,d,s} F^q(x, \xi, t)$$

$$T_g(x, \xi) = \frac{\xi}{(x - \xi + i\epsilon)(x + \xi - i\epsilon)} \mathcal{A}_g \left( \frac{x - \xi + i\epsilon}{2\xi} \right)$$

$$T_q(x, \xi) = \mathcal{A}_q \left( \frac{x - \xi + i\epsilon}{2\xi} \right). \quad (2)$$

$$\mathcal{A}_g^{(0)}(y) = \alpha_S(1 + \epsilon),$$

$$\mathcal{A}_q^{(0)}(y) = 0.$$

# Hard-scattering amplitudes at NLO

- Method of calculation: unitarity and dispersion relations

$$\tilde{s} = x_1 s \quad \zeta = \frac{M^2}{W^2}$$
$$y = \frac{\tilde{s} - M^2}{M^2} = \frac{x_2}{\zeta} = \frac{x - \xi}{2\xi}$$

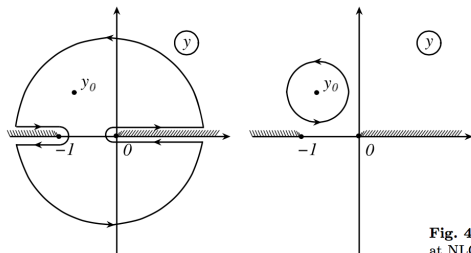
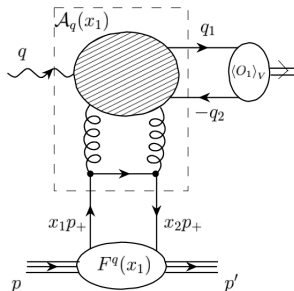


Fig. 4. The analytical properties of the partonic amplitudes at NLO in the complex plane of  $y = x_2/\zeta$

$$\tilde{s} \leftrightarrow \tilde{u} \quad \Rightarrow \quad y \leftrightarrow -(1 + y)$$

- unsubtracted dispersion relation

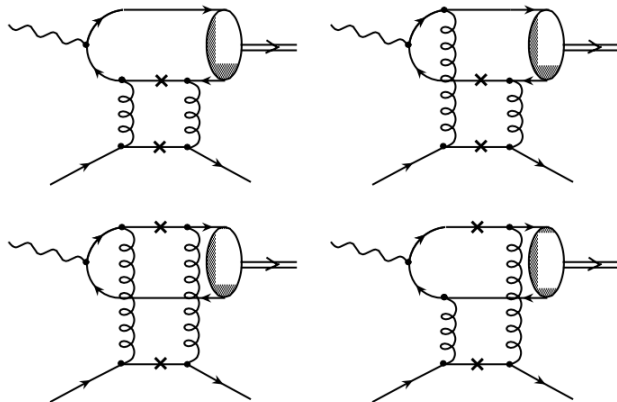
$$\mathcal{A}_q^{(1)}(y) = \frac{1}{\pi} \int_0^\infty dz \operatorname{Im} \mathcal{A}_q^{(1)}(z) \left( \frac{1}{z-y} - \frac{1}{z+y+1} \right)$$



**Fig. 3.** The light quark contribution to heavy meson photo-production

- non-polarised quark GPDs

$$\begin{aligned}
 F^q(x, \xi, t) &= \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{i\lambda(Pz)} \langle p' | \bar{q} \left( -\frac{z}{2} \right) \not{n}_- q \left( \frac{z}{2} \right) | p \rangle \Big|_z \\
 &= \frac{1}{2(Pn_-)} \left[ \mathcal{H}^q(x, \xi, t) \bar{u}(p') \not{n}_- u(p) \right. \\
 &\quad \left. + \mathcal{E}^q(x, \xi, t) \bar{u}(p') \frac{i\sigma^{\alpha\beta} n_{-\alpha} \Delta_\beta}{2m_N} u(p) \right]
 \end{aligned}$$



**Fig. 5.** The  $\tilde{s}$ -channel cut diagrams for the quark amplitude

$$\mathcal{A}_q^{(1)}(y) = \frac{\alpha_S^2 C_F}{(4\pi)^{1+\epsilon} \Gamma(1+\epsilon)} \left( \frac{4m^2}{\mu^2} \right)^\epsilon \mathcal{I}_q(y).$$

$$\begin{aligned} \mathcal{I}_q(y) = & \frac{2}{\epsilon} (1+2y) \left( \frac{\ln(-y)}{1+y} - \frac{\ln(1+y)}{y} \right) \\ & - \pi^2 \frac{13(1+2y)}{24y(1+y)} + \frac{4 \ln 2}{1+2y} + 2 \frac{\ln(-y) + \ln(1+y)}{1+2y} \\ & + 2(1+2y) \left( \frac{\ln^2(-y)}{1+y} - \frac{\ln^2(1+y)}{y} \right) \\ & + \frac{3-4y+16y(1+y)}{2y(1+y)} \text{Li}_2(1+2y) \\ & - \frac{7+4y+16y(1+y)}{2y(1+y)} \text{Li}_2(-1-2y), \end{aligned} \quad ($$

where

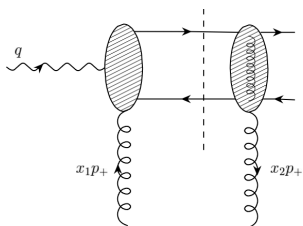
$$\text{Li}_2(z) = - \int_0^z \frac{dt}{t} \ln(1-t). \quad ($$

- subtracted dispersion relation

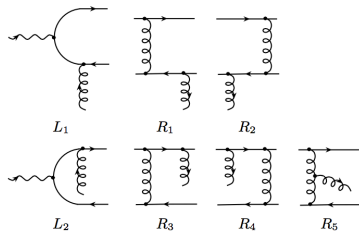
$$\mathcal{A}_g^{(1)}(y) - \mathcal{A}_g^{(1)}(0) = \frac{1}{\pi} \int_0^{\infty} dz \operatorname{Im} \mathcal{A}_g^{(1)}(z) \times \left( \frac{y}{z(z-y)} - \frac{y}{(z+y)(z+y+1)} \right).$$

- three s-channel cuts:  $\bar{Q} - Q$ ,  $\bar{Q} - g$ ,  $Q - g$

- $\bar{Q}Q$  cut

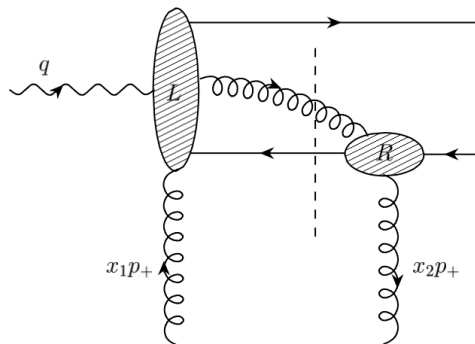


**Fig. 6.** The contribution of the  $\bar{Q}Q$  intermediate state to the gluonic amplitude



**Fig. 7.** The left and the right effective vertices for the  $\bar{Q}Q$  cut

- $\bar{Q}g$  cut



**Fig. 8.** The contribution of the  $\bar{Q}g$  intermediate state to the gluonic amplitude

# Hard-scattering amplitudes at NLO: Gluon CF

- $\bar{Q}g$  cut cntd

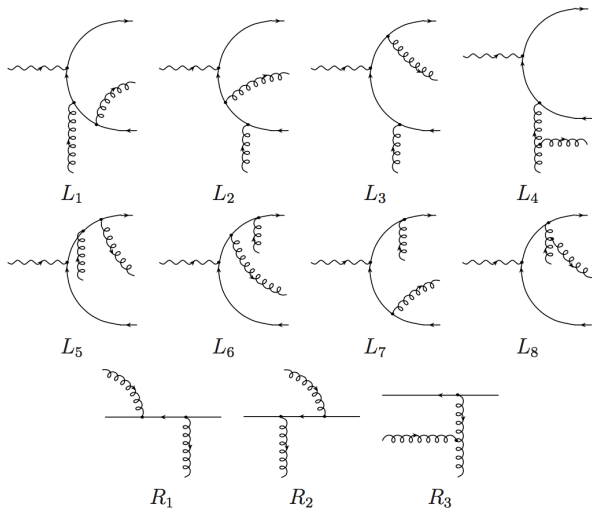


Figure 9: The left and the right effective vertices for the  $\bar{Q}g$ -cut.

- subtracted dispersion relation

$$\mathcal{A}_g^{(1)}(y) - \mathcal{A}_g^{(1)}(0) = \frac{1}{\pi} \int_0^{\infty} dz \operatorname{Im} \mathcal{A}_g^{(1)}(z) \times \left( \frac{y}{z(z-y)} - \frac{y}{(z+y)(z+y+1)} \right).$$

- three s-channel cuts:  $\bar{Q} - Q$ ,  $\bar{Q} - g$ ,  $Q - g$
- determination of the subtraction constant  $\mathcal{A}_g^{(1)}(0)$ :

Low theorem from QED applied to QCD for soft gluon with  $x_2 \rightarrow 0$

- renormalization

# Hard-scattering amplitudes at NLO: Gluon CF

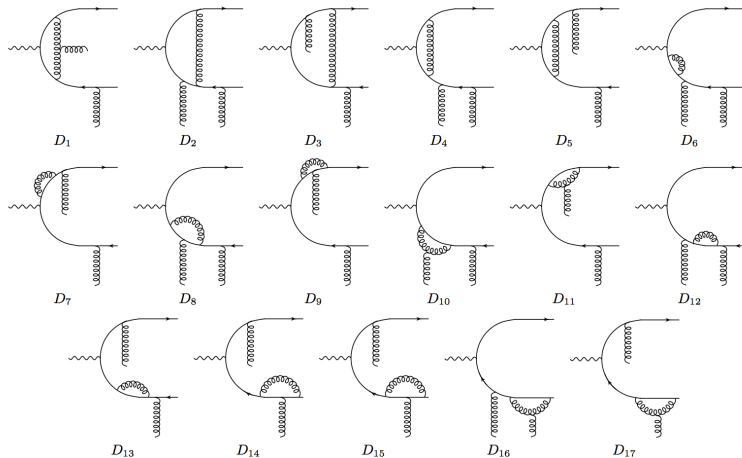
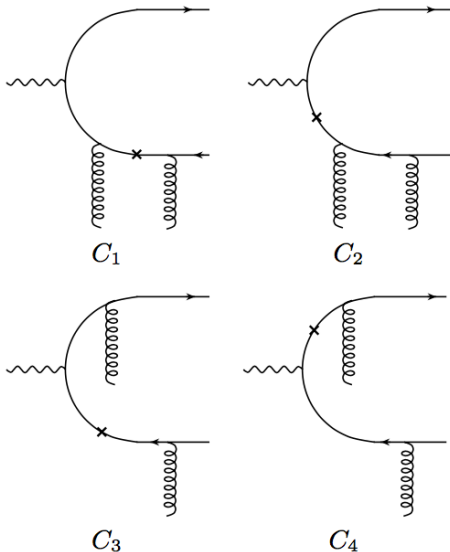
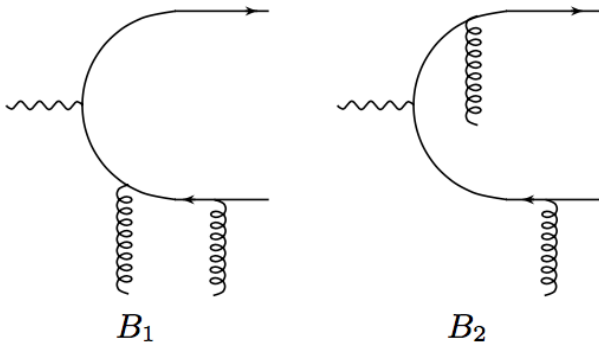


Fig. 10. Diagrams  $D_i$ , describing the radiation of a soft gluon from the on-shell antiquark line



**Fig. 11.** Mass counterterm diagrams which have an antiquark pole in the soft gluon limit



**Fig. 12.** LO antiquark pole diagrams

**Table 1.** Contributions to  $\mathcal{I}_g(0)$  of diagrams  $D_1, \dots, D_{11}$

Diagram	$\epsilon^{-2}$	$\epsilon^{-1}$	$\epsilon^0$
$D_1^z$	$-\frac{3}{8}(c_1 - c_2)$	$-\frac{3}{8}(c_1 - c_2)$	$(c_1 - c_2) \left( \frac{\pi^2}{16} + \frac{5 \ln(2)}{4} \right)$
$D_1^{\text{add}}$	$-\frac{3}{8}(c_1 - c_2)$	$-\frac{3}{8}(c_1 - c_2)$	$(c_1 - c_2) \left( -\frac{1}{2} + \frac{\pi^2}{4} + \frac{7 \ln(2)}{2} \right)$
$D_2^z$	0	$-\frac{1}{4}c_2$	$c_2 \left( -\frac{3}{4} + \frac{\pi^2}{32} + \frac{\ln(2)}{2} \right)$
$D_2^{\text{add}}$	0	$-\frac{3}{4}c_2$	$c_2 \left( -\frac{9}{4} - \frac{3\pi^2}{32} + \frac{13 \ln(2)}{4} \right)$
$D_3^z$	0	0	$c_2 \left( -\frac{1}{4} - \frac{\pi^2}{32} + \ln(2) \right)$
$D_3^{\text{add}}$	0	0	$c_2 \left( \frac{1}{4} + \frac{3\pi^2}{32} - \frac{\ln(2)}{4} \right)$
$D_4^z$	0	$-\frac{1}{4}c_1$	$c_1 \left( -\frac{1}{4} + \frac{\pi^2}{16} \right)$
$D_4^{\text{add}}$	0	$-\frac{1}{4}c_1$	$c_1 \left( -\frac{1}{8} + \frac{\pi^2}{32} - \frac{\ln(2)}{2} \right)$
$D_5^z$	0	0	0
$D_5^{\text{add}}$	0	0	$c_1 \left( \frac{1}{8} - \frac{\pi^2}{32} - \frac{\ln(2)}{2} \right)$
$D_6^z$	0	$\frac{5}{8}c_1$	$c_1 \left( -\frac{1}{8} - \frac{\ln(2)}{4} \right)$
$D_6^{\text{add}}$	0	$\frac{13}{16}c_1$	$c_1 \left( -\frac{1}{8} - \frac{\ln(2)}{8} \right)$
$D_7^z$	0	0	$c_1 \frac{3}{8}$
$D_7^{\text{add}}$	0	$\frac{3}{16}c_1$	$c_1 \left( -\frac{3}{8} + \frac{\ln(2)}{8} \right)$
$D_8^z$	0	$-\frac{1}{4}c_2$	$c_2 \left( -\frac{1}{4} + \frac{\pi^2}{16} - \frac{\ln(2)}{2} \right)$
$D_8^{\text{add}}$	0	$-\frac{1}{4}c_2$	$c_2 \left( \frac{1}{4} - \frac{\pi^2}{16} - \frac{3 \ln(2)}{4} \right)$
$D_9^z$	0	0	0
$D_9^{\text{add}}$	0	0	$-c_2 \frac{\ln(2)}{4}$
$D_{10}^z$	$\frac{1}{8}(c_1 - c_2)$	$-\frac{3}{8}(c_1 - c_2)$	$(c_1 - c_2) \left( \frac{1}{2} - \frac{\ln(2)}{2} \right)$
$D_{10}^{\text{add}}$	$-\frac{3}{8}(c_1 - c_2)$	$-\frac{3}{8}(c_1 - c_2)$	$-(c_1 - c_2) \left( \frac{1}{2} + \frac{3 \ln(2)}{8} \right)$
$D_{11}^z$	0	0	0
$D_{11}^{\text{add}}$	0	0	$-(c_1 - c_2) \frac{3 \ln(2)}{8}$

**Table 2.** Contributions to  $\mathcal{I}_g(0)$  of  $D_{12}, \dots, D_{17}$  and the mass counterterm diagrams

Diagram	$\epsilon^{-1}$	$\epsilon^0$
$(D_{12} + C_1 \frac{\delta m}{m} + D_{14} + D_{16})$	$-\frac{1}{4}(c_1 - c_2)$	$c_1 \left( \frac{1}{2} + \frac{\ln(2)}{2} \right) - c_2 \left( \frac{1}{4} + \frac{\ln(2)}{2} \right)$
$(D_{13} + C_3 \frac{\delta m}{m} + D_{15} + D_{17})$	0	$-(c_1 - c_2) \frac{1}{4}$
$C_2^z \frac{\delta m}{m}$	$-\frac{3}{8}c_1$	$c_1 \left( \frac{1}{8} + \frac{3\ln(2)}{4} \right)$
$C_2^{\text{add}} \frac{\delta m}{m}$	$-\frac{9}{16}c_1$	$c_1 \frac{9\ln(2)}{8}$
$C_4^z \frac{\delta m}{m}$	0	$-c_1 \frac{3}{8}$
$C_4^{\text{add}} \frac{\delta m}{m}$	$-\frac{3}{16}c_1$	$c_1 \left( \frac{1}{4} + \frac{3\ln(2)}{8} \right)$

$$T_q(x, \xi) = \frac{\alpha_S^2(\mu_R) C_F}{2\pi} f_q \left( \frac{x - \xi + i\epsilon}{2\xi} \right),$$

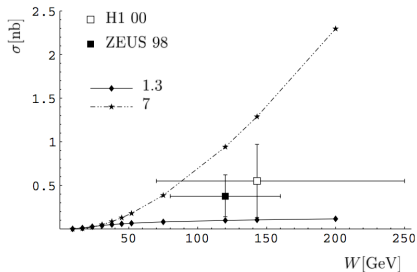
$$\begin{aligned} f_q(y) = & \ln \left( \frac{4m^2}{\mu_F^2} \right) (1 + 2y) \left( \frac{\ln(-y)}{1 + y} - \frac{\ln(1 + y)}{y} \right) - \pi^2 \frac{13(1 + 2y)}{48y(1 + y)} + \frac{2 \ln 2}{1 + 2y} \\ & + \frac{\ln(-y) + \ln(1 + y)}{1 + 2y} + (1 + 2y) \left( \frac{\ln^2(-y)}{1 + y} - \frac{\ln^2(1 + y)}{y} \right) \\ & + \frac{3 - 4y + 16y(1 + y)}{4y(1 + y)} Li_2(1 + 2y) - \frac{7 + 4y + 16y(1 + y)}{4y(1 + y)} Li_2(-1 - 2y) \end{aligned}$$

$$y = \frac{x_1 s - M^2}{M^2} = \frac{x_2}{\zeta} = \frac{x - \xi}{2\xi}$$

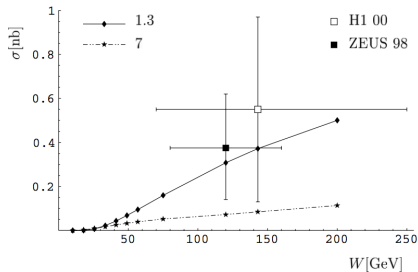
$$\zeta = \frac{M^2}{W^2}$$

$$\begin{aligned}
 T_g(x, \xi) &= \frac{\xi}{(x - \xi + i\varepsilon)(x + \xi - i\varepsilon)} \left[ \alpha_S(\mu_R) + \frac{\alpha_S^2(\mu_R)}{4\pi} f_g \left( \frac{x - \xi + i\varepsilon}{2\xi} \right) \right] \\
 f_g(y) &= 4(c_1 - c_2)(1 + 2y(1 + y)) \left( \frac{\ln(-y)}{1 + y} - \frac{\ln(1 + y)}{y} \right) \left( \ln \frac{4m^2}{\mu_F^2} - 1 \right) + \beta_0 \ln \frac{\mu_R^2}{\mu_F^2} \\
 &\quad + 4(c_1 - c_2)(1 + 2y(1 + y)) \left( \frac{\ln^2(-y)}{1 + y} - \frac{\ln^2(1 + y)}{y} \right) - 8c_1 \\
 &\quad - \pi^2 \left( \frac{2 + y(1 + y)(25 + 88y(1 + y))}{48y^2(1 + y)^2} c_1 + \frac{10 + y(1 + y)(7 - 52y(1 + y))}{24y^2(1 + y)^2} c_2 \right) \\
 &\quad - \left[ c_1 \frac{1 + 6y(1 + y)(1 + 2y(1 + y))}{y(1 + y)(1 + 2y)^2} + c_2 \frac{(1 + 2y)^2}{y(1 + y)} \right] \ln(2) \\
 &\quad + \pi \frac{\sqrt{-y(1 + y)}}{y(1 + y)} \left( \frac{7}{2} c_1 - 3c_2 \right) \\
 &\quad + 2c_2 \frac{\sqrt{-y(1 + y)}}{y(1 + y)} \left( \frac{1 + 4y}{1 + y} \arctan \sqrt{\frac{-y}{1 + y}} + \frac{3 + 4y}{y} \arctan \sqrt{\frac{1 + y}{-y}} \right) \\
 &\quad - \frac{\arctan^2 \sqrt{\frac{-y}{1 + y}}}{2y(1 + y)} \left( (7 + 4y)c_1 - 2 \frac{1 + 2y - 2y^2}{1 + y} c_2 \right) \\
 &\quad - \frac{\arctan^2 \sqrt{\frac{1 + y}{-y}}}{2y(1 + y)} \left( (3 - 4y)c_1 - 2 \frac{3 + 6y + 2y^2}{y} c_2 \right) \\
 &\quad + 2a_1(y) \ln(-y) + 2a_1(-1 - y) \ln(1 + y) \\
 &\quad + 2a_2(y) Li_2(1 + 2y) + 2a_2(-1 - y) Li_2(-1 - 2y),
 \end{aligned}$$

$$c_1 = C_F \quad c_2 = C_F - \frac{C_A}{2} = -\frac{1}{2N_c}$$



**Fig. 13.** The cross section for  $\Upsilon$  photoproduction; the theoretical predictions at LO for the scales  $\mu_F = \mu_R = [1.3, 7]$  GeV (ranging from bottom to top), and the data are from ZEUS [5] and H1 [6]



**Fig. 14.** The cross section of the  $\Upsilon$  photoproduction; theoretical predictions at NLO for the scales  $\mu_F = \mu_R = [1.3, 7]$  GeV (ranging from top to bottom), and the data from ZEUS [5] and H1 [6]

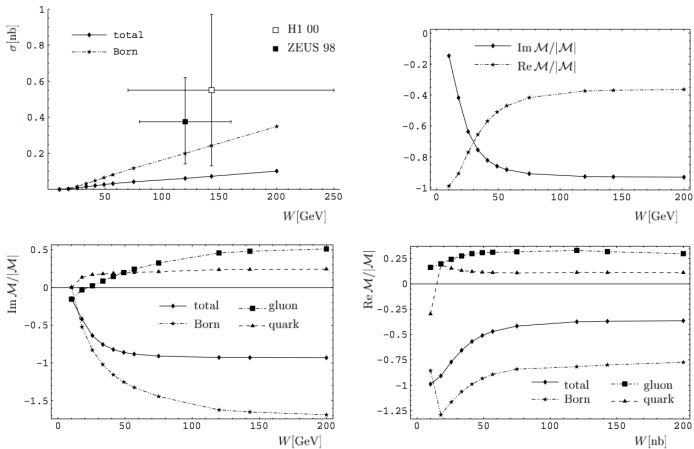
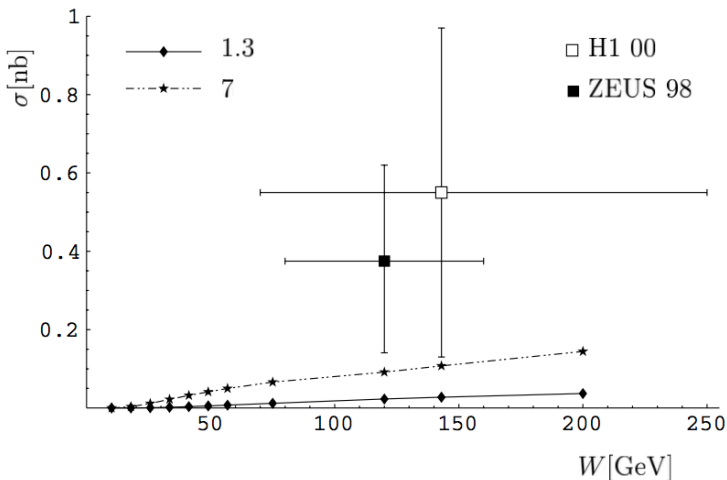
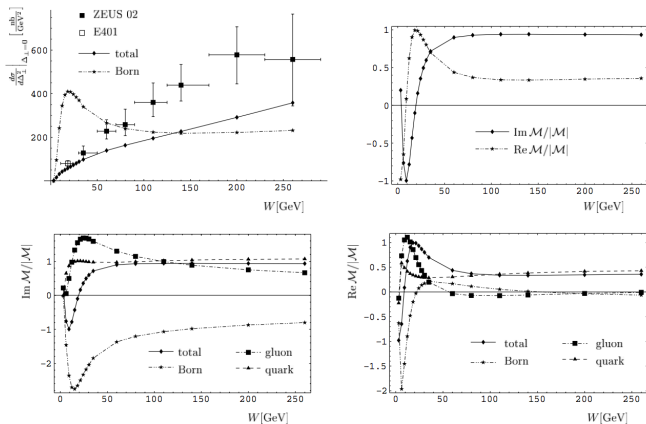


Fig. 15.  $\mathcal{T}$  photoproduction, NLO prediction for  $\mu_F = \mu_R = 4.9$  GeV and its decomposition into different contributions; s

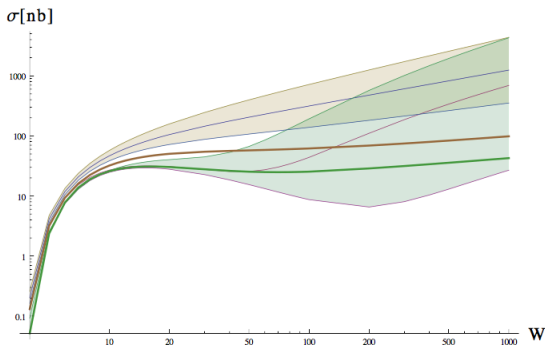


**Fig. 16.** The cross section for  $\gamma$  photoproduction, and NLO predictions for the scales  $\mu_F = [1.3, 7]$  GeV,  $\mu_R = 5.9$  GeV



**Fig. 17.** The differential cross section for  $J/\psi$  photoproduction, NLO predictions for  $\mu_F = \mu_R = 1.52 \text{ GeV}$ , and the data from E401 [1] and ZEUS [7]. The labeling of the curves is the same as in Fig. 15

# Photoproduction cross section - LO and NLO

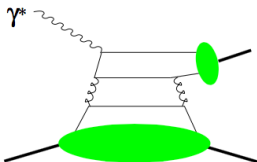
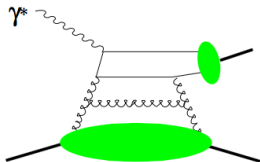


**Figure :** Photoproduction cross section as a function of  $W = \sqrt{s_{\gamma p}}$  for  $\mu_F^2 = M_{J/\psi}^2 \times \{0.5, 1, 2\}$ - LO and NLO. Thick lines for LO and NLO for  $\mu_F^2 = 1/4 M_{J/\psi}^2$ .

- ▶ Jones & Martin & Ryskin & Teubner, arXiv:1507.06942. Choice of the factorization scale.
- ▶ Why NLO corrections are large at small  $x_B$ ?  
large contribution comes from

$$Im A^g \sim H^g(\xi, \xi) + \frac{3\alpha_s}{\pi} \left[ \log \frac{M_V^2}{\mu_F^2} - \log 4 \right] \int_{\xi}^1 \frac{dx}{x} H^g(x, \xi)$$

$H^g(x, \xi) \sim xg(x) \sim const$ , therefore  $\int dx/x H^g(x, \xi) \sim \log(1/\xi) H^g(\xi, \xi)$



At higher orders powers of energy log are generated

$$\mathcal{I}m A^g \sim H^g(\xi, \xi) + \int_{\xi}^1 \frac{dx}{x} H^g(x, \xi) \sum_{n=1} C_n(L) \frac{\bar{\alpha}_s^n}{(n-1)!} \log^{n-1} \frac{x}{\xi}$$

$C_n(L)$  - polynomials of  $L = \log \frac{Q^2}{\mu_F^2}$ , maximum power is  $L^n$

- ▶ for DIS a technique suggested by Catani, Ciafaloni and Hautmann; [Catani, Hautmann '94]
- ▶ One can calculate  $C_n(L)$  in  $D = 4 + 2\epsilon$  dimensions.
- ▶ Consistently with collinear factorization, in terms of corrections to coeff. functions and anomalous dimensions, in  $\overline{MS}$  scheme
- ▶ The method used in DIS can be generalized to exclusive, nonforward processes.

Coeff. functions at small  $x$  / their Mellin moments at  $N \rightarrow 0$

$$L = \log \left( \frac{Q^2}{\mu_F^2} \right), \quad x = \frac{\bar{\alpha}_s}{N}$$

Result for  $J/\Psi, \Upsilon$

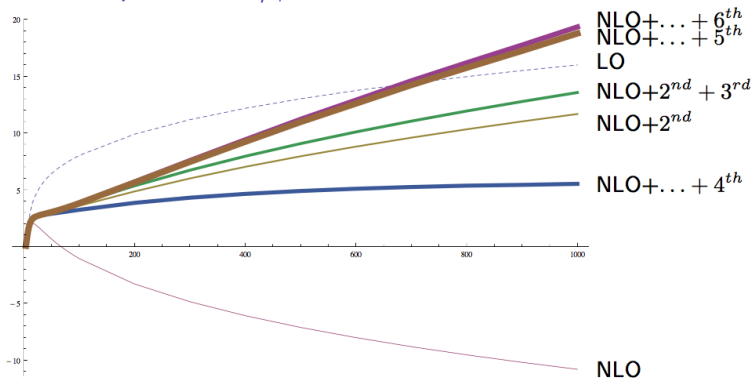
$$1 + x(L - \log 4) + \frac{x^2}{6} (\pi^2 + 3 \log^2 4 + 3L(L - \log 16)) + \dots + \mathcal{O}(x^{10})$$

$F_L$  - [Catani, Hautmann '94]

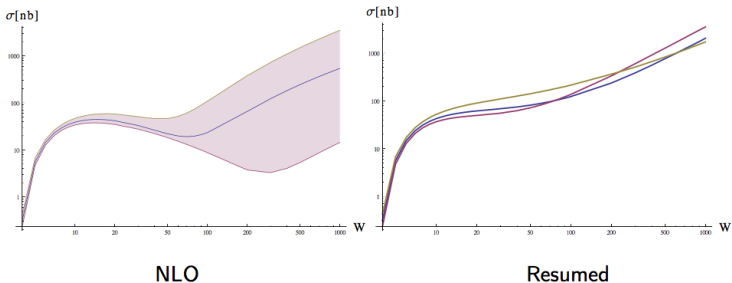
$$\mu_F^2 = Q^2$$

$$F_L: \quad 1 - \frac{1}{3}x + 2.13x^2 + 2.27x^3 + 0.434x^4 + \dots$$

$$J/\Psi, \Upsilon: \quad 1 - 1.39x + 2.61x^2 + 0.481x^3 - 4.96x^4 + \dots$$

Resummed amplitude for  $J/\psi$ 

Imaginary part of the amplitude for photoproduction of heavy mesons as a function of  $W = \sqrt{s_{\gamma p}}$  for  $\mu_F^2 = M_{J/\psi}^2$

Resummed cross section for  $J/\psi$ 

Photoproduction cross section as a function of  $W = \sqrt{s_{\gamma p}}$  for  
 $\mu_F^2 = M_{J/\psi}^2 \times \{0.5, 1, 2\}$

- NLO calculations permit to check correctness of the all-order-proof of QCD factorisation
- NLO corrections are very important for quarkonia photoproduction
- High energy resummation needed for quarkonia photoproduction