Energy in massive gravity

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Phys.Rev. D90 (2014) 12, 124090

II. Stability of cosmological solutions

C.Mazuet, M.S.V., to appear, arXiv:1503.03042

I. Energy in the dRGT theory

dRGT theory

Physical metric $g_{\mu\nu}$ and reference metric $f_{\mu\nu}$

$$S = M_{\rm Pl}^2 \int \left(\frac{1}{2} R - m^2 \mathcal{U} \right) \sqrt{-g}\, d^4x$$

with

$$\mathcal{U} = b_0 + b_1 \sum_a \lambda_a + b_2 \sum_{a<b} \lambda_a \lambda_b + b_3 \sum_{a<b<c} \lambda_a \lambda_b \lambda_c + b_4 \lambda_0 \lambda_1 \lambda_2 \lambda_3$$

where b_k are parameters and λ_a are eigenvalues of the matrix

$$\boxed{\gamma^\mu_{\ \nu} = \sqrt{g^{\mu\alpha} f_{\alpha\nu}}}$$

/de Rham, Gabadadze, Tolley 2010/

How to compute the energy ?

Hamiltonian formulation

After the ADM decomposition

$$\begin{aligned} ds_g^2 &= -N^2 dt^2 + \gamma_{ik}(dx^i + N^i dt)(dx^k + N^k dt) \\ ds_f^2 &= -dt^2 + \delta_{ik} dx^i dx^k \end{aligned}$$

the Lagrangian

$$\mathcal{L} = \left(\frac{1}{2} R - m^2 \mathcal{U}\right) \sqrt{-g}$$

becomes

$$\mathcal{L} = \frac{1}{2}\sqrt{\gamma}\, N \left(K_{ik}K^{ik} - K^2 + R^{(3)}\right) - m^2 \mathcal{V}(N^\mu, \gamma_{ik}) + \text{total derivative}$$

where $\mathcal{V} = \sqrt{\gamma}\, N\, \mathcal{U}$ and the second fundamental form

$$K_{ik} = \frac{1}{2N} \left(\dot{\gamma}_{ik} - \nabla_i^{(3)} N_k - \nabla_k^{(3)} N_i\right)$$

Variables are γ_{ik} and $N^\mu = (N, N^k)$.

Hamiltonian

Conjugate momenta

$$\pi^{ik} = \frac{\partial \mathcal{L}}{\partial \dot{\gamma}_{ik}} = \frac{1}{2}\sqrt{\gamma}(K^{ik} - K\gamma^{ik}), \qquad \boxed{p_{N_\mu} = \frac{\partial \mathcal{L}}{\partial \dot{N}^\mu} = 0} \quad \text{constraints}$$

$\Rightarrow$ N^ν are non-dynamical $\Rightarrow$ phase space is spanned by 12 variables (π^{ik}, γ_{ik}) = 6 DoF. Hamiltonian

$$\boxed{H = \pi^{ik}\dot{\gamma}_{ik} - \mathcal{L} = N^\mu \mathcal{H}_\mu(\pi^{ik}, \gamma_{ik}) + m^2 \mathcal{V}(N^\mu, \gamma_{ik})}$$

with

$$\mathcal{H}_0 = \frac{1}{\sqrt{\gamma}}(2\pi_{ik}\pi^{ik} - (\pi^k_k)^2) - \frac{1}{2}\sqrt{\gamma}R^{(3)}, \quad \mathcal{H}_k = -2\nabla^{(3)}_i \pi^i_k$$

Secondary constraints

$$-\dot{p}_{N_\mu} = \frac{\partial \mathcal{H}}{\partial N^\mu} = \mathcal{H}_\mu(\pi^{ik}, \gamma_{ik}) + m^2 \frac{\partial \mathcal{V}(N^\mu, \gamma_{ik})}{\partial N^\mu} = 0$$

Degrees of freedom, $m = 0$

$$\frac{\partial \mathcal{H}}{\partial N^\mu} = \mathcal{H}_\mu(\pi^{ik}, \gamma_{ik}) + m^2 \frac{\partial \mathcal{V}(N^\mu, \gamma_{ik})}{\partial N^\mu} = 0$$

If $m = 0$ this gives 4 constraints

$$\mathcal{H}_\mu(\pi^{ik}, \gamma_{ik}) = 0$$

They are first class

$$\{\mathcal{H}_\mu, \mathcal{H}_\nu\} \sim \mathcal{H}_\alpha$$

and generate gauge symmetries, one can impose 4 gauge conditions, there remain 4 independent phase space variables

$$12 - 4 - 4 = 4 = 2 \times (2 \text{ DoF}) \quad \Rightarrow \quad 2 \text{ graviton polarizations}$$

Energy vanishes on the constraint surface (up to a surface term)

$$H = N^\mu \mathcal{H}_\mu = 0$$

Degrees of freedom, $m \neq 0$

$$\frac{\partial \mathcal{H}}{\partial N^\mu} = \mathcal{H}_\mu(\pi^{ik}, \gamma_{ik}) + m^2 \frac{\partial \mathcal{V}(N^\mu, \gamma_{ik})}{\partial N^\mu} = 0$$

For $m \neq 0$ these 4 equations which determine 3 shifts $N^k(\pi^{ik}, \gamma_{ik})$ but the lapse N remains undetermined since

$$\mathrm{rank}\left(\frac{\partial \mathcal{V}(N^\mu, \gamma_{ik})}{\partial N^\mu \partial N^\nu}\right) = 3.$$

Inserting $N^k(\pi^{ik}, \gamma_{ik})$ to the Hamiltonian gives

$$\boxed{\mathcal{H} = \mathcal{E}(\pi^{ik}, \gamma_{ik}) + N\mathcal{C}(\pi^{ik}, \gamma_{ik})}$$

$$\Rightarrow \quad \text{two constraints} \quad \mathcal{C} = 0, \quad \mathcal{S} = \{H, \mathcal{C}\} = 0$$

$\Rightarrow$ there are $12 - 2 = 10 = 2 \times (5$ DoF$)$ The energy density is $\mathcal{E}(\pi^{ik}, \gamma_{ik})$ computed on the constraint surface. No explicit expressions for $\mathcal{E}, \mathcal{C}, \mathcal{S}$.

Restricting to the s-sector

Spherical symmetry

$$
\begin{aligned}
ds_g^2 &= -N^2 dt^2 + \frac{1}{\Delta^2}(dr + \beta dt)^2 + R^2 d\Omega^2 \\
ds_f^2 &= -dt^2 + dr^2 + r^2 d\Omega^2
\end{aligned}
$$

N, β, R, Δ depend on t, r. Lapse N and shift β are non-dynamical. Dynamical variables are Δ, R and their momenta

$$
p_\Delta = \frac{\partial \mathcal{L}}{\partial \dot{\Delta}}, \qquad p_R = \frac{\partial \mathcal{L}}{\partial \dot{R}},
$$

Phase space is 4-dimensional, spanned by $(R, \Delta, p_R, p_\Delta)$.

Hamiltonian

$$H = N\mathcal{H}_0 + \beta\mathcal{H}_r + m^2\mathcal{V}$$

where

$$\begin{aligned} \mathcal{H}_0 &= \frac{\Delta^3}{4R^2}p_\Delta^2 + \frac{\Delta^2}{2R}p_\Delta p_R + \Delta R R'^2 + 2R(\Delta R')' - \frac{1}{\Delta} \\ \mathcal{H}_r &= \Delta'_\Delta + 2\Delta' p_\Delta + R' p_R \end{aligned}$$

and the potential

$$\mathcal{V} = \frac{NR^2P_0}{\Delta} + \frac{R^2P_1}{\Delta}\sqrt{(\Delta N + 1)^2 - \beta^2} + R^2P_2$$

with

$$P_n = b_n + 2b_{n+1}\frac{r}{R} + b_{n+2}\frac{r^2}{R^2}$$

Number of DoF

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial N} &= \mathcal{H}_0 + m^2 \frac{\partial \mathcal{V}}{\partial N} = 0, \\ \frac{\partial \mathcal{H}}{\partial \beta} &= \mathcal{H}_r + m^2 \frac{\partial \mathcal{V}}{\partial \beta} = 0. \end{aligned}$$

- If $m = 0 \Rightarrow$ two first class constraints, $\mathcal{H}_0 = 0$ and $\mathcal{H}_r = 0 \Rightarrow$ $4 - 2 - 2 = 0$ DoF $\Rightarrow$ no dynamics = Birkhoff theorem
- If $m \neq 0 \Rightarrow$ the second equations determines β,

$$\beta = (\Delta N + 1) \frac{\Delta \mathcal{H}_r}{Y},$$

while the fist one gives the constraint

$$\mathcal{C}(\Delta, R, p_\Delta, p_R) = 0$$

Hamiltonian and constraints

$$H = \mathcal{E} + N\mathcal{C}, \qquad \mathcal{E} = \frac{Y}{\Delta} + m^2 R^2 P_2$$

where the primary constraint

$$\mathcal{C} = \mathcal{H}_0 + Y + m^2 \frac{R^2 P_0}{\Delta} \quad \text{with} \quad Y \equiv \sqrt{(\Delta \mathcal{H}_r)^2 + (m^2 R^2 P_1)^2}$$

while the secondary constraint

$$\begin{aligned} \mathcal{S} &= \{\mathcal{C}, H\} = \frac{m^4 R^2 P_1^2}{2Y} (\Delta p_\Delta + R\, p_R) - Y \left(\frac{\Delta \mathcal{H}_r}{Y}\right)' \\ &- \frac{\Delta^2 p_\Delta}{2R} \left\{ \frac{m^4}{2\Delta Y} \partial_R (R^4 P_1^2) + m^2 \partial_R (R^2 P_2) \right\} \\ &- \frac{m^2 \mathcal{H}_r}{Y} \left\{ \Delta (R^2 P_2)' + R^2 \partial_r (P_0 - \Delta P_2) \right\} = 0 \end{aligned}$$

$\Rightarrow 4 - 2 = 2 \times 1$ DoF. Energy $E = \int_0^\infty \mathcal{E} dr$ *assuming* $\mathcal{C} = \mathcal{S} = 0$.
PF limit is OK.

Kinetic energy sector

Three-metric is flat but the momenta do not vanish (with $x = mr$)

$$\Delta = 1, \quad R = r, \quad p_\Delta = \frac{\sqrt{xz}}{m}, \qquad p_R = -\frac{(xz + 4x^4 f)}{2x\sqrt{xz}},$$

two constraints reduce to

$$\begin{aligned} \frac{dz}{dx} &= 4\,x^2 f + 2x\sqrt{xz}\,F\,, \\ \frac{df}{dx} &= \frac{4\,(1-c_3)\,zf - 4x^3 f - 3z}{4x\sqrt{xz}}\,F - \frac{2}{x}\,F^2\,, \end{aligned}$$

with $F = \pm\sqrt{f(f+2)}$. Since $F^2 = f(f+2) \geq 0 \Rightarrow$ two branches: either $f > 0$ or $f < -2$, the energy

$$\boxed{\mathcal{E}_0 = x^2 f}$$

being either non-negative or strictly negative.
Solutions with $\mathcal{E} < 0$ are not globally defined and singular.

Potential energy sector

3-metric is not flat but the momenta vanish

$$p_\Delta = p_R = 0, \quad \Delta = g/h, \quad R = hr \qquad \Rightarrow \quad \mathcal{S} = 0,$$

the first constraint becomes

$$\begin{aligned} \mathcal{C}(h,g) &= h'' + \frac{2}{x}\,h' - \frac{h'^2}{2h} + \frac{(xh)'g'}{xg} - \frac{h(1-g^2)}{2x^2g^2} \\ &- \frac{h(2-3h)}{2g} - \frac{h(1-6h+6h^2)}{2g^2} = 0, \end{aligned}$$

while the energy density

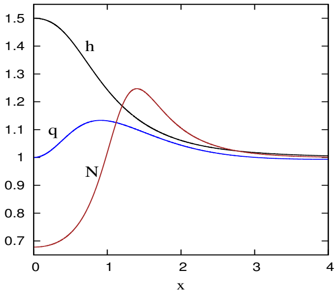
$$\mathcal{E}(h,g) = \frac{x^2h^2(3h-g-2)}{g}\,.$$

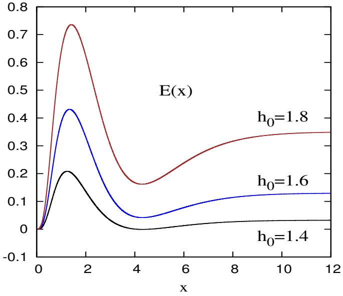
Again there are two solution branches.

Positive energy branch

flat space: $h = g = 1, \quad g_{\mu\nu} = f_{\mu\nu} = \eta_{\mu\nu} \quad \Rightarrow \quad \mathcal{E} = 0$

and its globally regular deformations:





Energy is positive for globally regular, asymptotically flat fields.

Negative energy branch

tachyon space: $h = \frac{1}{2}, \quad g = 1, \quad g_{\mu\nu} = \frac{1}{4} f_{\mu\nu} = \frac{1}{4} \eta_{\mu\nu}$

$$\boxed{\mathcal{E} = -\frac{3}{8} x^2} \qquad m_{\rm FP}^2 = -\frac{1}{2} m^2$$

Its deformations all have negative and infinite energy; they are not asymptotically flat. There are also asymptotically flat negative energy solutions, but they are singular.

One can continuously deform positive energy fields to negative energy fields, but the energy then shows a pole ⇒ the two branches are completely disjoint.

The negative energies cannot affect the physical sector.

Summary of part I

- The energy is positive in the physical sector of the theory.
- Other sectors are unphysical as they show ghost-like features – negative energies and tachyons.
- The physical sector is protected from the unphysical ones by a potential barrier and cannot affected by negative energies $\Rightarrow$ Minkowski space is stable.

Remarks

- (A) The energy is claimed to be always positive if the parameters are chosen as $b_k \sim \delta_k^1$ /Comelli and Pilo 2012/
- (B) The energy is claimed to be positive for a special family of massive gravity theories with $\underline{5+1 \text{ DoF}}$ whose potential is chosen such that $g_{\mu\nu}$ fulfills the Lorentz condition $\partial^\mu g_{\mu\nu} = 0$ /Ogievetsky, Polubarinov 1965/. This family contains (A) as a special case.

II. Stability of cosmological solutions

arXiv.1503.03042

Problems of dRGT cosmologies – commonly accepted facts

- The theory does not admit spatially flat FLRW.
- There is a spatially open FLRW but it is unstable.
- It follows that the theory should probably be abandoned in favour of its extensions (bigravity, quasidilaton, ...)

Problems of dRGT cosmologies – commonly accepted facts

- The theory does not admit spatially flat FLRW. (?)

- There is a spatially open FLRW but it is unstable. (?)

- It follows that the theory should probably be abandoned in favour of its extensions (bigravity, quasidilaton, ...) (?)

de Sitter in dRGT

dRGT admits infinitely many solution for which $g_{\mu\nu}$ is the standard de Sitter while $f_{\mu\nu} = \eta_{ab}\partial_\mu\Phi^a\partial_\nu\Phi^b$ where Φ^a fulfill a non-linear PDE. This fact is not widely reckognized.

Koyama, Niz, Tasinato, 2011
Chamseddine and M.S.V., 2011
d'Amico, de Rham, Dubovsky, Gabadadze, Pirskhalava, 2011
Gumrukcuoglu, Lin, Mukohyama, 2011
Gratia, Hu, Wyman, 2011
M.S.V., 2012
Kobayashi, Siino, Yamaguchi, Yoshida, 2012
Khosravi, Niz, Koyama, Tasinato, 2013

de Sitter space

Hyperbolid

$$-X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 = \alpha^2$$

in 5D Minkowski space with the metric

$$ds^2 = -dX_0^2 + dX_1^2 + dX_2^2 + dX_3^2 + dX_4^2$$

The geometry induced on the hyperboloid fulfills the 4D equations

$$G_{\mu\nu} + \Lambda\, g_{\mu\nu} = 0$$

with

$$\Lambda = \frac{3}{\alpha^2}$$

Changing coordinates gives FLRW cosmologies.

Embedding de Sitter to dRGT within the Gordon ansatz

$$\boxed{f_{\mu\nu} = u^2(g_{\mu\nu} + (1-\zeta^2)V_\mu V_\nu)}$$

with

$$g^{\mu\nu}V_\mu V_\nu \equiv V^\mu V_\nu = -1$$

and

$$P_1(u) = 0$$

where

$$P_m(u) = b_m + 2b_{m+1}\,u + b_{m+2}\,u^2$$

implies that

$$T^\mu_\nu = -\Lambda\,\delta^\mu_\nu$$

with

$$\Lambda = m^2 P_0(u) \qquad \Rightarrow \qquad G^\mu_\nu + \Lambda\,\delta^\mu_\nu = 0.$$

/Baccetti, Martin-Moruno, Visser 2012/

Massive gravity cosmologies

- Physical metric, $\Lambda = 3/\alpha^2 = m^2 P_0(u)$,

$$\begin{aligned} ds_g^2 &= \alpha^2 \left\{-dt^2 + dr^2 + dx^2 + dy^2 + dz^2\right\} \qquad (g) \\ 1 &= -t^2 + r^2 + x^2 + y^2 + z^2 \equiv -t^2 + r^2 + R^2 \end{aligned}$$

- reference metric

$$ds_f^2 = u^2\alpha^2 \left\{-dT^2(t,r) + dx^2 + dy^2 + dz^2\right\} \qquad (f)$$

with $P_1(u) = 0$. One has

$$ds_f^2 = u^2(ds_g^2 + dt^2 - dr^2 - dT^2)$$

which is compatible with the Gordon ansatz if

$$\partial_\mu t \partial_\nu t - \partial_\mu r \partial_\nu r - \partial_\mu T \partial_\nu T = (1-\zeta^2) V_\mu V_\nu$$

and this determines ζ, V_μ if only $T(t,r)$ fulfills

$$\boxed{(\partial_t T)^2 - (\partial_r T)^2 = 1}$$

Massive gravity cosmologies

- Physical metric, $\Lambda = 3/\alpha^2 = m^2 P_0(u)$,

$$\begin{aligned} ds_g^2 &= \alpha^2 \left\{-dt^2 + dr^2 + dx^2 + dy^2 + dz^2\right\} \qquad (g) \\ 1 &= -t^2 + r^2 + x^2 + y^2 + z^2 \equiv -t^2 + r^2 + R^2 \end{aligned}$$

- reference metric, $P_1(u) = 0$,

$$ds_f^2 = u^2\alpha^2 \left\{-dT^2(t,r) + dx^2 + dy^2 + dz^2\right\} \qquad (f)$$

with

$$\boxed{(\partial_t T)^2 - (\partial_r T)^2 = 1}$$

$\Rightarrow$ there are infinitely many solutions with the same (g) but different (f). Only $\boxed{T = t}$ has been studied.

$T = t$ in flat slicing

Setting

$$t = \sinh\tau + \frac{\rho^2}{2}\, e^\tau, \quad r = \cosh\tau - \frac{\rho^2}{2}\, e^\tau, \quad R = \sqrt{x^2 + y^2 + z^2} = \rho e^\tau$$

gives the spatially-flat FLRW g-metric with $a(\tau) = e^\tau$,

$$ds_g^2 = \alpha^2 \left(-d\tau^2 + a(\tau)(d\rho^2 + \rho^2 d\Omega^2)\right),$$

while

$$ds_f^2 \;\; = \;\; u^2\alpha^2 \left\{-dT^2(\tau,\rho) + dR^2 + R^2 d\Omega^2\right\}$$

with

$$T(\tau,\rho) = \frac{1}{2}\int \frac{d\tau}{\dot{a}(\tau)} + \frac{1}{2}(1+\rho^2)\, a(\tau)$$

$\Rightarrow$ f-metric is inhomogeneous $\Rightarrow$ no spatially flat FLRW

/d'Amico, de Rham, Dubovsky, Gabadadze, Pirtskhalava, 2011/

$T = t$ in the open slicing

Setting

$$t = \sinh\tau\cosh\rho, \quad r = \cosh\tau, \quad R = \sinh\tau\sinh\rho$$

gives, with $a(\tau) = \sinh(\tau)$,

$$\begin{aligned} ds_g^2 &= \alpha^2\left(-d\tau^2 + a^2(\tau)(d\rho^2 + \sinh^2\rho d\Omega^2)\right), \\ ds_f^2 &= u^2\alpha^2\left(-\dot{a}^2(\tau) + a^2(\tau)(d\rho^2 + \sinh^2\rho d\Omega^2)\right). \end{aligned}$$

The two metrics share the same symmetries $\Rightarrow$ manifest FLRW.

/Gumrukcuoglu, Lin, Mukohyama, 2011/

However, this solution is completely equivalent to its flat version.

Lesson: there can be non-manifest common isometries

$T = t$ in the static slicing

Setting

$$t = \sqrt{1-\rho^2}\,\sinh(\tau), \quad r = \sqrt{1-\rho^2}\,\cosh(\tau), \quad R = \rho$$

gives, with $a(\tau) = \sinh(\tau)$,

$$\begin{aligned} ds_g^2 &= \alpha^2 \left(-(1-\rho^2)d\tau^2 + \frac{d\rho^2}{1-\rho^2} + \rho^2 \rho d\Omega^2) \right) \\ ds_f^2 &= u^2\alpha^2 \left(-dT^2(\tau,\rho) + d\rho^2 + \rho^2 d\Omega^2 \right) \end{aligned}$$

with

$$T = \sqrt{1-\rho^2}\,\sinh(\tau)$$

f-metric is not invariant under the action of the timelike de Sitter isometry $\partial/\partial\tau$. This is probably the reason why the solution is unstable (if one replaces flat f by dS).

/de Felice, Gumrukcuoglu, Mukohyama, 2012/

What about other solutions of $(\partial_t T)^2 - (\partial_r T)^2 = 1$?

$(\partial_t T)^2 - (\partial_r T)^2 = 1$

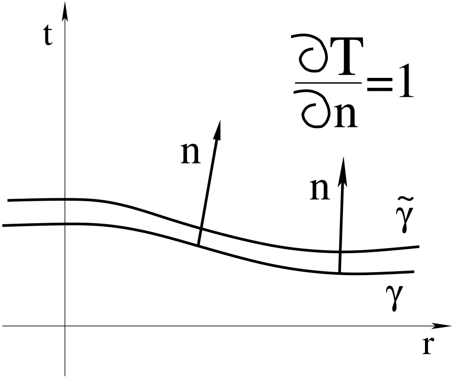
- fairly general solution

$$
\begin{aligned}
T &= \cosh(\xi)\, t + \sinh(\xi)\, r + W(\xi)\,, \\
0 &= \sinh(\xi)\, t + \cosh(\xi)\, r + \frac{dW(\xi)}{d\xi}\,,
\end{aligned}
$$

Difficult to explicitly resolve with respect to $T(t, r)$.

$(\partial_t T)^2 - (\partial_r T)^2 = 1$

- method of characteristics



$(\partial_t T)^2 - (\partial_r T)^2 = 1$

- separation of variables. E.g. in static coordinates

$$\frac{1}{\Sigma}\left(\frac{\partial T}{\partial \tau}\right)^2 - \frac{\Sigma}{1-\Sigma}\left(\frac{\partial T}{\partial \rho}\right)^2 = 1$$

gives static solutions

$$T = \sqrt{1+q^2}\,\tau + \int \frac{\rho\, d\rho}{\Sigma}\sqrt{q^2+\rho^2}\,,$$

$\Rightarrow$ f-metric is static.

Static solutions

A one-parameter family labeled by $q \geq 0$. For $q = 0$ one has

$$\begin{aligned} ds_g^2 &= \alpha^2 \{-\Sigma\, dV^2 + 2dVd\rho + \rho^2 d\Omega^2\}, \\ ds_f^2 &= u^2\alpha^2 \{-dV^2 + 2dVd\rho + \rho^2 d\Omega^2\}. \end{aligned}$$

with

$$V = t + \int \frac{d\rho}{1-\rho^2}$$

Only for these solutions the canonical Killing energy is time-independent.

Energy for non-trivial Stuckelbergs

Arbitrary slicing

$$\begin{aligned} ds_g^2 &= \alpha^2 \left\{ -N^2 d\eta^2 + \frac{1}{\Delta^2} \left(d\chi + \beta \, d\eta \right)^2 + R^2 \, d\Omega^2 \right\}, \\ ds_f^2 &= \alpha^2 u^2 \left\{ -dT^2 + dR^2 + R^2 \, d\Omega^2 \right\} \end{aligned}$$

The energy is

$$E[\eta, T] = u^2 \alpha^4 P_2(u) \int R^2 \left(\dot{T} R' - \dot{R} T' \right) d\chi$$

integrating over the hypersurface Σ_η of constant η. The energy depends on choice of the solution $T(\eta, \chi)$ and also on Σ_η. In the unitary gauge, $\eta = T$ and $\chi = R$, one obtains unitary energy

$$E[T, T] = u^2 \alpha^4 P_2(u) \int \chi^2 d\chi$$

This is conserved – with respect to the unitary time that is individual for each chosen solution $T(\eta, \chi)$.

Killing energy

A geometrically distinguished choice is the Killing time τ

$$E[\tau, T] = u^2 \alpha^4 P_2(u) \int \partial_\tau T \, \rho^2 d\rho$$

however this is time-independent only for the static solutions with $\partial_\tau T = const.$

Conjecture: any deformation around a given static solution

$$g^{(0)}_{\mu\nu} \to g^{(0)}_{\mu\nu} + \delta g_{\mu\nu}$$

increases the Killing energy as compared to the background value $E[\tau, T] \Rightarrow$ static solutions are stable.

Summary of part II

- In dRGT theory there are infinitely many de Sitter solutions labeled by $T(t,r)$ subject to $(\partial_t T)^2 - (\partial_r T)^2 = 1$.

- Solutions can be FLRW ($\Rightarrow$ g and f have common rotational and translational symmetries) in a non-manifest way.

- Stability of these solutions remains an open issue. One can compare their energy with the energy of their deformations.

- All solutions have time-independent canonical unitary energy defined with respect to the unitary time.

- There is a distinguished set of solutions which are invariant under the action of the timelike de Sitter isometry. Only for them the canonical Killing energy is time-idependent. The energy of this solutions is conjectured to be minimal as compared to the energy of their deformations, hence the solutions are conjectured to be stable.

Problems of dRGT cosmologies ?

- The theory does not admit spatially flat FLRW – but there is a spatially flat solution with 3 translational and 3 rotational Killings, hence it is homogeneous and isotropic.

- There is a spatially open FLRW but it is unstable – however, nobody has demonstrated the instability, even perturbatively.

- It follows that the theory should probably be abandoned in favour of its extensions (bigravity, quasidilaton, ...) but the theory contains infinitely many de Sitter solutions which should be studied.